Measurement of Centrality Dependence of Elliptic Flow for Identified Hadrons in Au + Au Collisions at $\sqrt{s_{NN}} = 200$ GeV

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Abstract

Quark-gluon plasma (QGP) is a new state of matter comprised of the deconfined quarks and gluons. It is expected to exist in the early Universe, a few \(\mu\)sec after the Big Bang or in a core of neutron star. Ultra-relativistic heavy ion collision is a unique tool to reach such a very hot and/or dense state on the earth.

Elliptic flow is one of the most promising probe to explore the early stage of heavy ion collisions. It is defined by the second harmonics of the azimuthal anisotropy \(v_2\) with respect to the reaction plane. Once the local thermal equilibrium is attained, elliptic flow is determined through (1) the initial geometry overlap, and (2) the initial density profile. Therefore, elliptic flow could be sensitive to the possible local thermal equilibrium in the produced matter.

The elliptic flow of identified hadrons has been measured for a broad range of centrality selection and up to transverse momentum \(p_T = 4\) GeV/c in the PHENIX experiment at the Relativistic Heavy Ion Collider (RHIC) in Au + Au collisions. The statistics is increased by a factor 20 compared to previous measurements, and it enable us to study detailed centrality dependence of \(v_2\) for identified hadrons. Particle identification has been performed with Time-Of-Flight Counter and Electro-Magnetic Calorimeter by measuring mass square calculated from flight time, flight path length, and momentum \((\pi^\pm, K^\pm, p/\bar{p}, d/d\bar{d})\) and invariant mass reconstruction from the decay product \((\phi \rightarrow K^+K^-)\). The magnitude of \(v_2\) has been measured with the event plane method. Event plane is determined by the Beam-Beam Counter (BBC) located at forward and backward rapidities, and the elliptic flow of produced hadrons are obtained at the Central arm spectrometer at mid rapidity with respect to the event plane. The large rapidity interval, \(|\Delta \eta| \sim 3\), between the Central arm and the BBC has an advantage to reduce the possible non-flow effects, which are correlations among particles and are not oriented from the reaction plane and becomes dilute the true signal of elliptic flow.

Distinct features of the results are follows;

- The transverse momentum dependence of \(v_2\) shows the mass ordering at low \(p_T\), i.e. smaller \(v_2\) for heavier hadrons at a given \(p_T\) \((v_2(\pi) > v_2(K) > v_2(p) \geq v_2(\phi) > v_2(d))\).

- At higher \(p_T\), however, \(v_2\) for mesons saturate earlier than that for baryons while the \(v_2\) for baryons are still increasing with \(p_T\) \((v_2(\pi) \approx v_2(K) \approx v_2(\phi) < v_2(p) \approx v_2(d))\).

- \(v_2\) increase with centrality. The centrality dependence of \(v_2\) is qualitatively consistent with that of initial geometry overlap (eccentricity) estimated by Glauber Monte Carlo simulation.
In order to study the relation between the initial spatial anisotropy (eccentricity, $\langle \varepsilon \rangle$) and final elliptic flow, the eccentricity scaling of $v_2$ has been studied for a wide range of centrality. We have observed that the participant eccentricity, which is defined by the principal axes of participating nucleons, is the relevant geometric quantity to explain the scaling of $v_2/\langle \varepsilon \rangle$ across Au + Au and Cu + Cu collisions. This result suggests that $v_2/\langle \varepsilon \rangle$ is determined by the number density of nucleons even if the system size is different.

We have developed an extended Blast-wave model, in which collective flow is defined by the gradient of density profile, in order to study the sensitivity of the initial density profile to $v_2$. Freeze-out temperature ($T$) and the radial flow velocity ($\beta_T$) have been extracted for both transverse momentum spectra and $v_2$, independently. We have observed that extracted $T$ from $v_2$ are about 100 – 200 MeV larger than that from spectra and the results obtained from $N_{\text{coll}}$ density profile have always smaller $\chi^2$/NDF for both spectra and $v_2$. This result may suggest that the $v_2$ is developed by the number of collisions among the constituents since the number of collisions are closely related to the degree of thermalization. The average radial flow velocity is almost same for spectra and $v_2$. In order to see the sensitivity of the eccentricity in the measured $v_2$, we have performed the Blast-wave fitting with the simple 1D in-plane expansion of the system. By expanding the initial density profile, we have observed that $T$ from $v_2$ fit decreases with the eccentricity, while those from spectra fit and the radial flow velocity are almost unchanged. $T$ from $v_2$ fit is as large as the chemical freeze-out temperature if we assume that the kinetic freeze-out takes place at the $\langle \varepsilon \rangle$ obtained by the azimuthal HBT analysis. Larger $T$ from $v_2$ fit than that from spectra fit at the kinetic freeze-out may suggest that the freeze-out of $v_2$ could be earlier than that of spectra. These results are consistent with the picture of the collective in-plane expansion, where the initial eccentricity is quenched and the magnitude of $v_2$ is developed through the time evolution.

The quark number scaling of $v_2$ has been further examined. We have observed that quark number scaling of $v_2$ works for $p_T/n_q > 1$ GeV/$c$, where $n_q$ denote the number of constituent quarks in each hadron ($n_q = 2$ for mesons, 3 for baryons and 6 for deuterons). However, clear mass dependence has been observed for $p_T/n_q < 1$ GeV/$c$. By assuming that $v_2$ is driven by the transverse kinetic energy ($KE_T = m_T - m_0$) instead of $p_T$, we have observed that the quark number scaling with $KE_T$ holds for $\pi$, $K$ and $p$ in all centrality classes and for almost all $KE_T$ range, except for low $KE_T$, $KE_T/n_q < 0.3$ GeV. Since the pressure gradient is directly linked to the transverse kinetic energy, this results could suggest that the collective pressure gradient is the driving force of elliptic flow. We have also observed that the quark number scaling with $KE_T$ works for $\phi$ and $d$ in minimum bias and in other centrality bins. Since $\phi$ mesons do not suffer from the hadronic interactions, the observation of the quark number scaling of $v_2$ for $\phi$ mesons could indicate the partonic collectivity in the pre-hadronic phase of heavy ion collisions.
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Chapter 1

Introduction

The ultimate goal of the experiments with relativistic heavy ion collisions is to study the properties of the quark-gluon plasma (QGP), which is a new state of matter at very high density and/or very high temperatures. It is expected that a new state of matter may be created under such extreme conditions, where quarks and gluons are no longer confined inside hadrons and can move freely. The relativistic heavy ion collision offer a unique opportunity to achieve those conditions on the earth. In this chapter, we introduce the quantum chromodynamics (QCD) which is the essential theory to describe the relativistic heavy ion collisions, and introduce the major feature of the the experimental observables at the Relativistic Heavy Ion Collider (RHIC).

1.1 Quantum Chromodynamics (QCD)

Quantum Chromodynamics (QCD) is the theory to describe the strong forces between quarks, where they are the fundamental building blocks of the matter and carry the color charge analogous to the electric charge in quantum electrodynamics (QED). In QCD, gluons are the force medians and they carry the strong force, as photons carry the electromagnetic force in QED. While photons carry no electric charge, gluons also carry color charge and they can interact among themselves.

The classical Lagrangian density for QCD is

$$L_{cl} = \sum_{f}^{N_f} \bar{q}_f (i\gamma^\mu D_\mu - m_f) q_f - \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu}$$

(1.1)

where $q_f$ is the quark field of flavor $f$ and mass $m_f$ ($f$ runs from 1 to 3). The covariant derivative, $D_\mu$ is

$$D_\mu = \partial_\mu + ig\frac{\lambda_a}{2} A_\mu^a$$

(1.2)

where $\lambda_a$ is the eight Gell-Mann matrices. $F^a_{\mu\nu}$ is the gluon field strength tensor defined as

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$$

(1.3)
where $A_\mu^a$ is the gluon field ($a$ runs from 1 to 8), and $f_{abc}$ is the structure constants of the SU(3) group. $g$ is defined as $g \equiv \sqrt{4\pi\alpha_s}$, and $\alpha_s$ is the coupling constant of the strong force, which represent the strength of the interactions between quark-gluon, and gluon-gluon interactions.

At short distances, the running coupling constant for the strong force becomes small by the anti-screening feature of the color charge because the gluons are not neutral in color. This property is known as asymptotic freedom. Asymptotic freedom means that the typical length scale decreases (or the momentum scale increases) the coupling constants decreases. Because of the smaller coupling constants at smaller distance, pertubative QCD (pQCD) calculation can only be performed for interactions with large momentum transfers ($Q$). As one can see in Fig. 1.1, the measurements of running coupling constants are in very good agreement with the calculations by pQCD. Although pQCD works very well involving larger momentum transfers, it cannot be used to calculate for the processes with larger distances (or smaller momentum transfers).

A typical form of phenomenological QCD $q\bar{q}$ pair potential is

$$V_{q\bar{q}} = -\frac{a(r)}{r} + Kr$$

(1.4)
where \( r \) is the distance between \( q \) and \( \bar{q} \), and \( K \) is the string tension between quark pair. The first term is the color Coulomb potential, and the second term is the linear confining potential which is the unique properties of QCD: the potential increase linearly with the increase of distance.

In the strong coupling regime, the energy to separate two quarks increases linearly with increasing the distance between them. A new \( q\bar{q} \) pair is produced when the energy contained between the two quarks becomes more than the twice of the rest momentum of a quark. Therefore, no deconfined quarks have ever been observed. It is known as confinement of quarks.

In order to study the strong coupling regime where the perturbative QCD calculations is not valid, Lattice QCD calculations is considered as a strong tool to perform with numerical path integrals of the QCD Lagrangian on a four-dimensional Euclidean lattice with box size \( L \) and lattice spacing \( a \). A modern thermodynamical lattice QCD at finite temperature and density suggests that quarks and gluons are deconfined if sufficiently high temperature and/or density are reached.

Fig. 1.2 shows the energy density divided by \( T^4 \) as a function of temperature scaled by the critical temperature \( T_c \) calculated by Lattice QCD [2]. Current calculations shows that \( \epsilon/T^4 \) increases rapidly around a critical temperature \( T_c \simeq 155 - 175 \) MeV. And its values of critical temperatures corresponds to an critical energy density \( \epsilon_c \simeq 0.5 - 1 \) GeV/fm\(^3\) [2]. Because \( \epsilon/T^4 \) corresponds to the number of degrees of freedom, this rapid increase of \( \epsilon/T^4 \) indicate a transition to a new state of matter, namely quark-gluon plasma (QGP).

![Figure 1.2](image_url)

Figure 1.2: The energy density divided by \( T^4 \) (\( \epsilon/T^4 \)) as a function of temperature scaled by the critical temperature \( T_c \) calculated in Lattice QCD simulation [2]. The arrows on the right side indicate the values of \( \epsilon/T^4 \) for the Stefan-Boltzmann limit.
1.2 Relativistic Heavy Ion Collisions

Table 1.1: Summary of current and proposed heavy ion programs with facilities, the typical ion beams, and the center of mass energy per nucleon pair

<table>
<thead>
<tr>
<th>Machine</th>
<th>Location</th>
<th>Ion beam</th>
<th>Start of experimental program</th>
</tr>
</thead>
<tbody>
<tr>
<td>AGS</td>
<td>BNL</td>
<td>$^{16}$O, $^{28}$Si</td>
<td>Oct, 1986</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$^{197}$Au</td>
<td>Apr, 1992</td>
</tr>
<tr>
<td>SPS</td>
<td>CERN</td>
<td>$^{16}$O, $^{32}$S</td>
<td>Sep, 1986</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$^{208}$Pb</td>
<td>Nov, 1994</td>
</tr>
<tr>
<td>RHIC</td>
<td>BNL</td>
<td>$^{197}$Au</td>
<td>2000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$^{197}$Au</td>
<td>2001</td>
</tr>
<tr>
<td></td>
<td></td>
<td>d + $^{197}$Au</td>
<td>2003</td>
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<tr>
<td></td>
<td></td>
<td>$^{197}$Au</td>
<td>2004</td>
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<td></td>
<td>$^{197}$Cu</td>
<td>2005</td>
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<td></td>
<td></td>
<td>$^{197}$Au</td>
<td>2007</td>
</tr>
<tr>
<td>LHC</td>
<td>CERN</td>
<td>$^{208}$Pb</td>
<td>2007 (projected)</td>
</tr>
</tbody>
</table>

As seen in Chapter 1.1, QCD predicts a very hot and dense matter consists of deconfined quarks and gluons, quark-gluon plasma (QGP). Relativistic heavy ion collisions is considered to be a unique tool to create such a state of matter under extremely high temperature and/or density on the earth. QGP phase is also expected to exist in the early Universe (after a few micro sec after the Big-Bang), and in the interior of neutron stars. Therefore the observation and study the matter under such extreme conditions has an impact not only on the nuclear physics, but also on the astrophysics and the high-energy physics.

Since 1980’s various experiments have taken place both at the Brookhaven National Laboratory (BNL) and European Organization for Nuclear Research (CERN). Table 1.1 summarize the current and proposed heavy ion programs at BNL and CERN. While both AGS and SPS were provided beams with fixed target experiments, RHIC and LHC are the colliders, where two heavy ions accelerate up to nearly the speed of light and collide each other.

The relativistic heavy ion collisions are dynamic processes with typical time scales of an order 10 fm/c. Even if the QGP is created in collisions, the system expands and cools rapidly back to a hadron gas through a QCD phase transition. In order to probe the formation of the QGP, the signals which is sensitive to the QCD phase transitions should be observed as many as possible.

1.2.1 Collision Geometry

In relativistic heavy ion collisions, the geometry of the collisions can be defined by the participant spectator model. Fig. 1.3 shows a schematic view of heavy ion collision between symmetric Lorentz contracted projectile and target nuclei in the center of mass frame. The impact parameter $b$ is the distance between the center of nuclei and characterize the centrality of collision. The nucleons taking part in the primary collisions
are called as \textbf{participants} and the rest that are not participated in the collisions are called as \textbf{spectators}.

In most heavy ion experiments, the impact parameter is estimated by measuring the size of the participants and/or the spectators. The participants and the spectators are well separated experimentally because the spectator keeps its longitudinal velocity and mostly emitted in the forward (backward) rapidity, while the secondary particles from participants are peaked around mid-rapidity.

Once the impact parameter of the collision is determined, the Glauber Model [3] provides the number of participant nucleons ($N_{\text{part}}$), number of nucleon-nucleon collisions ($N_{\text{coll}}$), and the spatial eccentricity ($\varepsilon$) for a given impact parameter. These quantities can be calculated analytically or numerically under the following assumptions;

- Collisions of two nuclei are expressed in terms of the individual interactions of the constituent nucleons.
- At high energies, nucleons travel on straight line trajectories and are essentially undeflected.
- Inelastic nucleon-nucleon cross-section is independent of the number of collisions for a nucleon underwent before.

Analytical expressions of these quantities can be found in Appendix A.

What is the relation between these quantities and the experimental observables? In proton-nucleus collisions, the total multiplicity scales with the number of participants
\( N_{\text{part}} \) (in other words, "wounded nucleons") \([4]\). In nucleus-nucleus collisions, it is also found that the total multiplicity is proportional to be \( N_{\text{part}} \) \([5]\). \( N_{\text{part}} \) is scaled with the volume of the interaction region, therefore the total multiplicity is given by

\[
\frac{dN}{dy} \propto N_{\text{part}} \propto A. \tag{1.5}
\]

For processes involving large momentum transfer (hard scattering processes), all nucleon-nucleon collisions are assumed to be independent because of their small cross-sections. Therefore, the cross-sections for hard-scattering processes should scale with the number of binary nucleon-nucleon collisions.

Perfect fluid hydrodynamics suggest that initial anisotropy in the coordinate space are directly converted into the momentum anisotropy in the final momentum space. Since hydrodynamic model always assumes the local thermal equilibrium, the relation between initial spatial eccentricity and the final momentum anisotropy could provide the signal of possible thermalization in the early stage of heavy ion collisions.

### 1.2.2 Time Evolution

![Figure 1.4: A sketch of the space-time picture of a relativistic heavy-ion collision.](image)

Fig. 1.4 shows a simplified space-time evolution of a heavy ion collision which consists of 4 stages; (i) a parton cascade stage, (ii) a QGP phase, (iii) an interacting hadron gas phase and (iv) a free hadron stage.
**Parton cascade stage: \(0 < \tau < \tau_0\)**

Several models are proposed to describe the dynamics of initial parton-parton scattering in heavy ion collisions: the color-string models [6], color glass condensate [7], and perturbative QCD models [8]. The parton production mechanism in parton cascade stage, however, is not well understood, and it is being actively studied both from theoretical and experimental point of view.

**QGP phase and QCD phase transition: \(\tau_0 < \tau < \tau_f\)**

The frequent scatterings of the partons leads to the local thermal equilibrium at \(\tau_0\). Once the local thermal equilibrium is attained, the relativistic hydrodynamics can be used to describe the evolution of the system. The basic equations of relativistic hydrodynamics are the conservation of the energy-momentum tensor and the baryon number

\[
\partial_\mu T^{\mu
u} = 0 \tag{1.6}
\]

\[
\partial_\mu j_B^\mu = 0 \tag{1.7}
\]

where \(T^{\mu\nu}\) is the energy momentum tensor, and \(j_B^\mu\) is the baryon number current. In the perfect fluid approximation, they are given by

\[
T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu - g^{\mu\nu}P \tag{1.8}
\]

\[
j_B^\mu = n_B u^\mu \tag{1.9}
\]

\[
u^\mu = \gamma(1, v_x, v_y, v_z) \tag{1.10}
\]

where \(\epsilon\) is the local energy density, \(P\) is the local pressure, \(n_B\) is the baryon number density, and \(u^\mu\) is the fluid four-velocity. There are 6 unknown variables: \(\epsilon\), \(P\), \(n_B\), and \(v_x, v_y, v_z\), and the conservation laws in Eq. (1.6) and (1.7) contains 5 independent equations. The equation of state (EOS) relating \(\epsilon\) and \(P\) provides an additional equation to solve the space time evolution of six thermodynamical variables.

Once an equation of state is choosen, one can solve the set of equations until the system undergoes a freeze-out at \(\tau = \tau_f\) with the full numerical integration using the full 3D hydrodynamics, or with assuming some symmetry and simplify the equations in just one or two dimensions.

**Freeze-out and free hadrons stage: \(\tau_f < \tau\)**

The plasma expansion lead the drop of temperature, eventually hadronization takes place and relative number of species of the emitted particles is fixed at chemical freeze-out temperature. The particles are rescattering each other until the hadronic interactions no longer occured. Kinetic freeze-out happens if the kinetic equilibrium is no longer maintained, and no further hadronic interarctions occur until the free streaming particles are detected.

Only the hadrons from the free hadrons stage can be detected in the heavy ion experiments. It is very challenging to probe the early stage of the heavy ion collisions with hadrons measured in the final stage.
1.3 Major Features of Experimental Observables at RHIC

In this section, we present the selected experimental observables obtained at RHIC and discuss the relation to the collision dynamics and the bulk properties of QGP.

1.3.1 Achieved Energy Density

![Graph showing Bjorken energy density as a function of number of participants (Np) for different center-of-mass energies](image)

Figure 1.5: $\epsilon_{Bj} \cdot \tau$ as a function of number of participants ($N_p$) in different $\sqrt{s_{NN}}$ [9].

The average transverse energy of particles ($dE_T/dy$) can be used to estimate the energy density achieved in the heavy ion collisions with the Bjorken formula [9]

$$
\epsilon_{Bj} = \frac{1}{\pi R^2 \tau} \frac{dE_T}{dy}
$$

(1.11)

where $\tau$ is the formation time and $R$ is the radius of nucleus. Eq. (1.11) is derived from perfect fluid hydrodynamics with free streaming particles at $\tau$, which is defined as the proper time when the system reaches local thermal equilibrium.

Fig. 1.5 shows the Bjorken energy density for three different center-of-mass energies calculated from the measured $dE_T/dy$. For the 5% most central collisions, $\epsilon_{Bj} \cdot \tau$ was $2.2 \pm 0.2$, $4.7 \pm 0.5$, and $5.4 \pm 0.6$ GeV fm$^{-2}$c$^{-1}$ for $\sqrt{s_{NN}} = 19.6$, 130 and 200 GeV, respectively. An estimate with $\tau = 0.6$ (1.0) fm gives $\epsilon_{Bj} = 9$ (5.4) GeV/fm$^3$ for $\sqrt{s_{NN}} = 200$ GeV, which is larger than the critical energy density $\epsilon_c \sim 1$ GeV/fm$^3$ predicted...
from Lattice QCD calculations. Thus, the energy density of the matter created at top RHIC energy is well above the threshold for QGP formation.

### 1.3.2 Radial Flow

![Figure 1.6: Centrality dependence of transverse mass $m_T$ distributions for $\pi^\pm$, $K^\pm$ and $p(\bar{p})$ in Au + Au collisions at $\sqrt{s_{NN}} = 200$ GeV. The lines on each spectra represent the fitting results with $m_T$ exponential function [10].](image)

The produced hadrons carry informations about the collision dynamics and the history for entire space-time evolution of the system, so that the measurements of the transverse momentum distributions of identified hadrons could be an essential tool to study the collision dynamics.

Fig. 1.6 shows the transverse mass spectra for identified particles in different centrality selections. The transverse mass spectra can be described by an exponential shape

\[
E \frac{d^3 \sigma}{dp^3} \propto \exp \left( -\frac{m_T}{T} \right)
\]

where $m_T = \sqrt{m_0^2 + p_T^2}$ denotes the transverse mass of hadrons, $m_0$ is the hadron mass, $p_T$ is the transverse momentum, and $T$ is the inverse slope parameter. The inverse slope parameter is often interpreted as the temperature of the system. In the high energy
\( p + p/p + A \) collisions, the inverse slope parameters are common \((T \approx 150 \text{ MeV})\) for various particle species \([11]\). This phenomenon is known as ”\( m_T \) scaling” and suggest that the spectra of hadrons with different masses would have similar slopes.

![Figure 1.7: Centrality and mass dependence of inverse slope parameter \( T \) in Au + Au collisions at \( \sqrt{s_{\text{NN}}} = 200 \text{ GeV} \). The dotted lines represent a linear fit of the data for each centrality bin with Eq. (1.14) \([10]\).](image)

Fig. 1.7 summarize the centrality and particle type dependence of the inverse slope parameters in Au + Au collisions. The inverse slope parameters increase with increasing particles mass in all centrality bins. For central collisions, the slope is more rapidly increasing for heavier particles. These mass dependence of slope parameters are not observed in \( p + p/p + A \) collisions. Such mass dependences are considered as the evidence of common outward radial flow created by the strong interaction among the produced particles, and well describe the phenomenological hydrodynamical model (blast-wave model) \([12]\). The collective radial flow are incorporated into the trasverse mass spectra as

\[
\frac{dN}{m_T dm_T} = Am_T \int_0^{\infty} r dr I_0 \left( \frac{p_T \sinh \rho}{T_f} \right) K_1 \left( \frac{m_T \cosh \rho}{T_f} \right)
\]

(1.13)

where \( I_0 \) and \( K_1 \) represent the modified Bessel functions of first and second kind respectively, \( \rho = \tanh^{-1} \beta(r) \) denotes the transverse rapidity, \( \beta(r) \) is the radial flow velocity, and \( T_f \) is the kinetic freeze-out temperature. In the limit of \( m \gg T_f, m \gg p_T \), and \( T_f \gg m/\beta^2 \) the inverse slope parameter becomes

\[
T_{\text{eff}} \simeq T_f + \frac{1}{2} m \langle \beta \rangle^2.
\]

(1.14)

Eq. (1.14) shows the heavier the particles, the more they gain momentum or energy from the radial flow velocity, and thus the effective temperature becomes larger.
CHAPTER 1. INTRODUCTION

1.3.3 Azimuthal Anisotropy

Azimuthal anisotropic emission of particles in momentum-space is expected to be sensitive to the early stage of collisions. In non-central collisions, the initial overlap of two nuclei in the transverse plane becomes almond shape as depicted in Fig. 1.8. The reaction plane is defined as the plane where the directions of beam and the vector connecting the center of both nuclei (impact parameter). The azimuthal anisotropy of emitted particles is quantitatively evaluated by using Fourier expansion series as

\[ E \frac{d^3 N}{dp^3} = \frac{1}{2\pi} \frac{dN}{p_T dp_T dy} (1 + 2v_1 \cos(\phi_{lab} - \Psi) + 2v_2 \cos(2[\phi_{lab} - \Psi]) + \ldots) \quad (1.15) \]

where \( \phi_{lab} \) is the azimuthal angle of emitted particles in the fixed laboratory frame, \( \Psi \) denotes the azimuthal angle of the reaction plane, and \( v_n \) is the magnitude of each harmonics [13]. In this thesis, we only focus on elliptic flow \( (v_2) \), which is the 2nd harmonics of Fourier expansion in Eq. (1.15).

There are several reasons why elliptic flow is thought to be sensitive to the early stage of heavy ion collisions.

1. Thermalization

The magnitude of elliptic flow is strongly influenced by the relation between the mean free path \( \lambda \) and the typical length scale of the system \( R \). If thermalization is achieved, that is \( \lambda \ll R \), the magnitude of elliptic flow is proportional to the initial eccentricity \( (v_2 \propto \varepsilon) \). Since the ratio of \( \lambda/R \) is characterized as the degrees of thermalization (Knudsen number), the ratio \( v_2/\varepsilon \) could be an indicator of possible thermalization in the early stage of heavy ion collisions.
2. Sensitive to the equation of state

In the hydrodynamical picture, the pressure gradient is the driving force of the elliptic flow. In non-central collisions, the pressure gradients between the produced matter and the external vacuum is steeper in the direction of reaction plane (in-plane) than in the direction of perpendicular to the reaction plane (out-of-plane). The larger in-plane pressure gradient convert the initial spatial eccentricity into the in-plane elliptic flow in the final momentum space. The pressure gradient is closely related to the EOS so that the emission pattern of elliptic flow could be sensitive to the existence of the QGP phase in the early stage.

The magnitude of elliptic flow could also be sensitive to the phase transition. If the phase transition is of the first order, the pressure stays constants during the phase transition. This results in vanishing the speed of sound \( c_s = \sqrt{\partial P/\partial \epsilon} \) (softening the EOS). Hence, the magnitude of elliptic flow significantly reduces if the softening of EOS happens.

3. Self-quenching signal with time

The observed elliptic flow is sensitive to the time scale of equilibrirum. The system expands radially before the hydrodynamical evolution, thus the initial spatial anisotropy is reduced by the time when equilibrium is achieved. The observed \( v_2 \) could be diluted if equilibration does not occur early times of collisions. In the late evolution stage, the stronger in-plane pressure gradient could also lead the system expanding more rapidly in that direction, therefore reducing the initial spatial anisotropy.

Fig. 1.9 shows the PHENIX results of \( v_2 \) for identified hadrons as a function of \( p_T \) in minimum bias events. The data shows that for \( p_T < 2 \text{ GeV/c} \), the particles with lighter mass have a larger \( v_2 \) for a given \( p_T \), which is in good agreement with the hydrodynamical model calculation. Since the hydrodynamical model assumes the very rapid thermal equilibrium (\( \tau \simeq 0.6 \text{ fm/c} \)), the results of \( v_2 \) suggest that the local thermal equilibrium could be attained at very early time in the heavy ion collisions. A striking feature is that the observed \( v_2 \) for \( p_T > 2 \text{ GeV/c} \) of \( p \) and \( \bar{p} \) are larger than that of \( \pi \) and \( K \). This trend is in sharp contrast to the its \( p_T \) dependence of hydrodynamical model calculation, which would predict to keep the same mass ordering for entire \( p_T \) range. Such a behavior of \( v_2 \) is predicted by the quark coalescence/recombination mechanism [15].

Assuming the quarks and antiquarks distributions are the same, the invariant spectrum of mesons and baryons are proportional to the product of the invariant spectra of constituents in the coalescence model [16]. The hadron spectra at mid-rapidity are

\[
\frac{dN_M}{d^2p_T} (p_T) = C_M(p_T) \left( \frac{dN_q}{d^2p_T} (p_T/2) \right)^2 \tag{1.16}
\]

\[
\frac{dN_B}{d^2p_T} (p_T) = C_B(p_T) \left( \frac{dN_q}{d^2p_T} (p_T/3) \right)^3 \tag{1.17}
\]

where \( N_M, N_B, N_q \) denote the yield of mesons, baryons and quarks, \( C_M \) and \( C_B \) are the probabilities for \( q\bar{q} \rightarrow \text{meson} \) and \( qqq \rightarrow \text{baryon coalescence} \) [15]. If partons have only
elliptic flow, i.e.,

$$\frac{dN_q}{p_T dp_T d\phi_q} = \frac{1}{2\pi} \frac{dN_q}{p_T dp_T} (1 + 2v_{2,q} \cos(2\phi_q))$$

(1.18)

where $\phi_q$ is the azimuthal angle of partons relative to the direction of reaction plane. $v_2$ is defined as

$$v_2 = \frac{\int_0^{2\pi} d\phi \cos(2\phi) dN/p_T dp_T d\phi}{\int_0^{2\pi} d\phi dN/p_T dp_T d\phi}$$

(1.19)

then from Eq. (1.16), one immediately obtains the elliptic flow of meson and baryons with the assumption of $v_2^q \ll 1$

$$v_2^M(p_T) \approx 2v_{2,q} \left(\frac{p_T}{2}\right)$$

(1.20)

$$v_2^B(p_T) \approx 3v_{2,q} \left(\frac{p_T}{3}\right)$$

(1.21)

where $p_T$ is the transverse momentum of hadrons.

Fig. 1.10 demonstrates the quark number scaling of $v_2$ as a function of $p_T/n_q$ in minimum bias events from PHENIX and STAR experiments, where $n_q$ denotes the number of constituent quarks in each hadron [17]. The scaled $v_2$ values above 0.6 GeV/c lie on a universal curve for all particle species, except for $\pi$. The deviation for $\pi$ may be caused by the resonance decay contributions [18, 19], or it may be difficult to describe the pion production by a constituent quark model because the assumed constituent quark masses are significantly larger than current quark masses [15, 20].
This results support the picture of hadron production via the coalescence of constituent quarks with collective anisotropic flow and suggest that the elliptic flow has been established in the partonic phase in the heavy ion collisions. One could see that the quark number scaling of $v_2$ breaks for $p_T < 0.6\text{ GeV/c}$. There is another scaling variable, $KE_T = m_T - m_0$ (GeV), which can describe the scaling of $v_2$ in entire $p_T$ range up to $p_T = 4 - 5\text{ GeV/c}$. The validity of quark number scaling of $v_2$ with $KE_T$ will be discussed in the thesis.

### 1.3.4 Blast-wave model

As we already discussed in Section 1.2.2 and 1.3.3, the space-time evolution of the QGP phase can be described by relativistic hydrodynamics. And the calculation of hydrodynamical model is consistent with the results of $v_2$ for $p_T < 2\text{ GeV/c}$. Since the hydrodynamical model assumes local thermal equilibrium, the agreement of $v_2$ with hydrodynamical model prediction indicates that the local thermal equilibrium is attained very rapidly ($\tau \sim 1\text{ fm/c}$).

In order to describe the evolution of heavy ion collisions, full 3-D hydrodynamical simulation should be performed with Eq. (1.6) - (1.10). Instead of employing full 3-D hydrodynamical simulation, one could solve the set of equations analytically with some assumptions. Blast-wave model is one of the hydro-inspired model which help us to verify the configuration just after the thermal kinetic freeze-out.
Figure 1.11: Transverse momentum spectra for positive (left) and negative (right) $\pi$, $K$, $p$ in 0 – 5 % centrality (top) and 60 – 92 % centrality (bottom). Solid lines represent the fitting results by Blast-wave model.

Fig. 1.11 shows transverse momentum spectra for $\pi$, $K$ and $p$ in most central (0 – 5 %) and peripheral (60 – 92 %) events [21]. The calculation by Blast-wave model is in good agreement with the data as shown by the solid lines in the figure. The parameters are extracted by fitting single transverse momentum spectra for $\pi$, $K$ and $p$ simultaneously with the Blast-wave model. It is found that the freeze-out temperature (average radial flow velocity) increase (decrease) from central to peripheral collisions. This anti-correlation of $T$ and $\langle \beta_T \rangle$ is consistent with the collective expansion picture.

Several blast-wave framework also tries to fit the $v_2$ with additional parameters: $\varepsilon$, $\beta_2$, where $\varepsilon$ is the spatial eccentricity at the freeze-out, and $\beta_2$ is the 2nd harmonic coefficient of radial flow velocity or radial transverse rapidity [22]. Since these models consider the bulk properties where the kinetic freeze-out takes place, the parameters are extracted by fitting single particle spectra and $v_2$ simultaneously.

However, $v_2$ is thought to be more sensitive to the early stage of heavy ion collisions compared to the single particle spectra, one can expect that $v_2$ has different sensitivities on the parameters from transverse momentum spectra. Thus, one could extract bulk properties, such as temperature and radial flow velocity, in the early stage of collisions by comparing with measured $v_2$ and blast-wave model.
1.4 Thesis Motivation

In this chapter, we reviewed several observables which is thought to be the signatures of the QGP. The estimated Bjorken energy density is larger than 5 GeV/fm$^3$, which is well above the critical energy density predicted by the Lattice QCD calculations. The results of single particle transverse momentum spectra are consistent with the description by the model with collective transverse expansion.

The results of $v_2$ for identified hadrons at $p_T < 2$ GeV/c are in good agreement with the calculation of ideal hydrodynamical model with very rapid thermalization, $\tau_0 \sim 1$ fm/c. It indicate that the thermal equilibrium have been attained for early stage of heavy ion collisions at RHIC. For $p_T > 2$ GeV/c, however, the deviation from hydrodynamic model is observed and that trend of $v_2$ can be well described with the quark coalescence mechanism. This results indicate that the elliptic flow was developed in the partonic phase of heavy ion collisions.

In this thesis, we present the measurements of centrality dependence of identified hadron elliptic flow in $\sqrt{s_{NN}} = 200$ GeV Au + Au collisions at RHIC-PHENIX experiment. Our main goals are as follows;

1. Study the relation between initial geometry overlap (eccentricity) and final momentum elliptic flow.

2. Study the sensitivity of initial density profile to the elliptic flow by Blast-wave model, and extract freeze-out temperature ($T$) and radial flow velocity ($\beta_T$).

3. Test the validity of the quark number scaling of $v_2$ for several different centrality with $\pi$, $K$, $p$, $d$ and $\phi$.

As we already shown in 1.3.3, elliptic flow is expected to be proportional to the eccentricity if the local thermal equilibrium is achieved. Therefore, it is important to measure the centrality dependence of elliptic flow in order to understand how the initial eccentricity is converted into the final elliptic flow. We examine the eccentricity scaling of $v_2$ in Au + Au and Cu + Cu collisions with several different definitions of eccentricity, and also study the sensitivity of scaled $v_2$ with different density profile, namely number of participant and number of collision density.

The blast-wave model successfully describes the transverse momentum spectra in heavy ion collisions. Several blast-wave models also describe the elliptic flow by simultaneous fitting with both spectra and $v_2$. However, magnitude of $v_2$ saturate earlier than $p_T$ spectra so that $v_2$ may have different sensitivity to the early stage of heavy ion collisions. The sensitivity of $v_2$ to the density profile are studied and extract bulk thermodynamic properties, such as freeze-out temperature and radial flow velocity, by the extended blast-wave model. It takes into account the density and velocity profiles estimated by Glauber model, while the usual blast-wave model assume that density is constant inside the overlap zone and velocity profile is proportional to the transverse radius $r = \sqrt{x^2 + y^2}$.

Quark number scaling of $v_2$ suggest that the existence of universal curve for hadrons with light quarks for $p_T/n_q > 0.6$ GeV/c, and that the coalescence of constituent quarks
could be dominant particle production at that $p_T$ range. Quark number scaling of $v_2$ has been studied in minimum bias events. However, the magnitude of $v_2$ strongly depends on the collision centrality. Therefore, it is crucial to test the validity of quark number scaling for each centrality, from central to peripheral collisions. We study the validity of the quark number scaling of $v_2$ for identified hadrons in the measured centrality range.

More concrete evidence of quark coalescence would be provided by the measurement of $v_2$ for $\phi$ meson. Because the lifetime of $\phi$ meson in vacuum is larger ($\approx 45$ fm/c) compared to the typical length scales of the medium ($\sim 10$ fm), and the cross-section for scattering of strange hadrons by non-strange hadrons are small ($\sim 9$ mb) [23]. Thus, if elliptic flow was developed in a phase involving hadrons interacting with their hadronic cross sections, one would expect that $v_2$ of the $\phi$ could be significantly smaller than that of other hadrons. On the other hand, if the elliptic flow was established in the pre-hadronic phase, the $\phi$ meson provide an important test for the quark number scaling as we already discussed in the last section. Since its mass is similar to that of the proton, its $v_2$ should be additive with the $v_2$ of the two constituent quarks.
Chapter 2

Experimental Apparatus

2.1 Relativistic Heavy Ion Collider (RHIC)

Figure 2.1: (a) A schematic of the RHIC complex. (b) The layout of the detectors around the RHIC tunnel.

The Relativistic Heavy Ion Collider (RHIC) [24, 25] is located at Brookhaven National Laboratory (BNL) and is the highest energy collider in the world. RHIC is capable of colliding a wide variety of particle species from $A = 1$ (protons) to $A \sim 200$ (gold), at present. One obtains energies up to 100 GeV per nucleon for Au + Au collisions and up to 250 GeV for protons. The designed luminosity $^1$ is $2 \times 10^{26} \text{ cm}^{-2} \text{ s}^{-1}$ for Au ions and $1.4 \times 10^{31} \text{ cm}^{-2} \text{ s}^{-1}$ for protons. A schematic of the RHIC complex is shown in Fig. 2.1.

The RHIC consists of two quasi-circular concentric accelerator/storage rings on a common horizontal plane, one ("Blue Ring") for clockwise and the other ("Yellow Ring") for counter-clockwise beams. The circumference of RHIC ring is 3.8-km. An existing

$^1$the number of interactions per unit time per unit cross-section
chain of hadron accelerators, i.e., the Tandem Van de Graff, the Booster, and the Alternating Gradient Synchrotron (AGS) are used as the heavy ion injector to the collider rings.

### 2.2 PHENIX Detector Overview

The Pioneering High Energy Nuclear Interaction eXperiment (PHENIX) is one of the large experiments at RHIC. The PHENIX detector comprises four instrumented spectrometers or arms and three global detectors [26]. The detector consists of a number of subsystems. The rapidity and \( \phi \) coverages and other features of these subsystems used in this thesis is given in Table 2.1.

Table 2.1: Summary of PHENIX detector subsystems used in this thesis

<table>
<thead>
<tr>
<th>Element</th>
<th>( \Delta \eta )</th>
<th>( \Delta \phi )</th>
<th>Purpose and special features</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Magnet</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Central (CM)</td>
<td>( \pm 0.35 )</td>
<td>360°</td>
<td>Up to 1.15 T m</td>
</tr>
<tr>
<td><strong>Global detectors</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Beam-beam (BBC)</td>
<td>( \pm (3.0 \text{ to } 3.9) )</td>
<td>360°</td>
<td>Start timing, fast vertex</td>
</tr>
<tr>
<td>ZDC</td>
<td>( \pm 2 \text{ mrad} )</td>
<td>360°</td>
<td>Minimum bias trigger</td>
</tr>
<tr>
<td><strong>Tracking</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Drift Chamber (DC)</td>
<td>( \pm 0.35 )</td>
<td>90°( \times 2 )</td>
<td>Good momentum and mass resolution ( \Delta m/m = 1.0 % \text{ at } m = 1 \text{ GeV} )</td>
</tr>
<tr>
<td>Pad Chambers (PC)</td>
<td>( \pm 0.35 )</td>
<td>90°( \times 2 )</td>
<td>Pattern recognition, tracking</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>for nonbend direction</td>
</tr>
<tr>
<td><strong>Particle identification</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Time-of-Flight (TOF)</td>
<td>( \pm 0.35 )</td>
<td>45°</td>
<td>Good hadron identification, ( \sigma &lt; 100 \text{ ps} )</td>
</tr>
<tr>
<td>PbSc EMCal</td>
<td>( \pm 0.35 )</td>
<td>90°( +45° )</td>
<td>For both calorimeters, photon and electron detection and energy measurement</td>
</tr>
</tbody>
</table>

Table 2.2 provide the recorded integrated luminosity at PHENIX and the statistics on the total number of events achieved by PHENIX in Runs 1 through 5.
Table 2.2: Summary of PHENIX data sets acquired in RHIC Runs 1 through 5.

<table>
<thead>
<tr>
<th>Run</th>
<th>Year</th>
<th>Species</th>
<th>$\sqrt{s_{NN}}$ (GeV)</th>
<th>$\int Ldt$</th>
<th>$N_{tot}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>01</td>
<td>2000</td>
<td>Au + Au</td>
<td>130</td>
<td>1 $\mu$b$^{-1}$</td>
<td>10 M</td>
</tr>
<tr>
<td>02</td>
<td>2001/2002</td>
<td>Au + Au</td>
<td>200</td>
<td>24 $\mu$b$^{-1}$</td>
<td>170 M</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$p + p$</td>
<td>200</td>
<td>6.35 pb$^{-1}$</td>
<td>3.7 G</td>
</tr>
<tr>
<td>03</td>
<td>2002/2003</td>
<td>$d + Au$</td>
<td>200</td>
<td>2.74 nb$^{-1}$</td>
<td>5.5 G</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$p + p$</td>
<td>200</td>
<td>0.15 pb$^{-1}$</td>
<td>3.7 G</td>
</tr>
<tr>
<td>04</td>
<td>2003/2004</td>
<td>Au + Au</td>
<td>200</td>
<td>241 $\mu$b$^{-1}$</td>
<td>1.5 G</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Au + Au</td>
<td>62.4</td>
<td>9 $\mu$b$^{-1}$</td>
<td>58 M</td>
</tr>
<tr>
<td>05</td>
<td>2004/2005</td>
<td>Cu + Cu</td>
<td>200</td>
<td>3 nb$^{-1}$</td>
<td>8.6 G</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Cu + Cu</td>
<td>62.4</td>
<td>0.19 nb$^{-1}$</td>
<td>0.4 G</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Cu + Cu</td>
<td>22.5</td>
<td>2.7 $\mu$b$^{-1}$</td>
<td>9 M</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$p + p$</td>
<td>200</td>
<td>3.8 pb$^{-1}$</td>
<td>85 B</td>
</tr>
</tbody>
</table>

2.3 PHENIX Central Magnet (CM)

The PHENIX magnet system [27] is composed of three spectrometer magnets with warm iron yokes and water-cooled copper coils. The Central Magnet (CM) is energized by two pairs of concentric coils, which are "Outer" and "Inner" coils, and provides a field around the interaction vertex that is parallel to the beam. A schematic drawings of magnet system is shown in Fig. 2.2.

The data in this thesis only consisted of the "Inner + Outer" configuration. Since the field is a very good approximation phi-symmetric and axial so that most of the bending of charged particle's trajectory occurs in phi, and not in theta. The parameters for the central magnet are shown in Table 2.3.

2.4 The Global Detectors

The global properties of the heavy ion collisions, including the collision vertex along the beam direction, the trigger and timing information, the collision centrality, the multiplicity, and the event plane are characterized by a set of global detectors around the beam line. In this thesis, the Zero Degree Calorimeters (ZDC), and the Beam Beam Counters (BBC) are used.

2.4.1 Zero Degree Calorimeter (ZDC)

Fig. 2.3 A) shows an overhead drawing of the PHENIX interaction region. A pair of ZDC (Zero Degree Calorimeter) [28] are located on either side of the interaction region, 18 m away, and behind the DX magnet in order to provide universal characterization of
Figure 2.2: Line drawings of the PHENIX magnets, shown in perspective and cut away to show the interior structures. Arrows indicate the beam line of the colliding beams in RHIC.

Table 2.3: Parameters for the PHENIX Central Magnet

<table>
<thead>
<tr>
<th>Parameter</th>
<th>CM Coils</th>
<th>CM Coils</th>
</tr>
</thead>
<tbody>
<tr>
<td>Field configuration</td>
<td>Axial</td>
<td>Axial</td>
</tr>
<tr>
<td>Field integral (T-m)</td>
<td>0.43 to 1.15 (Θ=90°)</td>
<td>0.78 (Θ=90°)</td>
</tr>
<tr>
<td>Wt. (metric tons)</td>
<td>421</td>
<td>421</td>
</tr>
<tr>
<td>Pseudorapidity coverage</td>
<td>-0.35 &lt; η &lt; 0.35</td>
<td>-0.35 &lt; η &lt; 0.35</td>
</tr>
<tr>
<td>Polar angle coverage</td>
<td>70° &lt; Θ &lt; 110°</td>
<td>70° &lt; Θ &lt; 110°</td>
</tr>
<tr>
<td>Amp-turns</td>
<td>541,000</td>
<td>248,000</td>
</tr>
<tr>
<td>Power (kW)</td>
<td>928</td>
<td>600</td>
</tr>
<tr>
<td>Average coil temp. (°C)</td>
<td>23.8(I)/32.1(O)</td>
<td>32.1</td>
</tr>
</tbody>
</table>
heavy ion collisions. Because of the DX magnet, any charged particles are swept away before hitting the ZDC. Fig. 2.3 B) shows the locations of neutrons, gold, and protons after going through the DX magnet.

The ZDC is a small hadron calorimeter, consisting of layers of tungsten plates and scintillator slabs as shown in Fig. 2.4 and detects neutron multiplicities from the heavy ion collisions, giving one of the collision centrality measures. The ZDC pair at each crossing point is also used as a luminosity monitor in steering the beams to collide.

Figure 2.3: A) An overhead view of the interaction region showing the location of the ZDCs as either end, just after the DX magnets and in the crotch where the two rings merge. B) A frontal view of the ZDC and beam-pipe with the locations of neutrons, gold and proton after they are swept by the DX magnet.

2.4.2 Beam-Beam Counter (BBC)

The main role of the BBC (Beam-Beam Counter) [29] is

- to provide the start time for the TOF measurements
- to produce the signal for the PHENIX LVL1 trigger
- to measure the collision vertex point along the beam axis

The BBC consists of two identical sets of counters installed on both sides of the collision point along the beam axis. The BBC is placed 144 cm from the collision point and surround the beam pipe, which covered a pseudorapidity range from 3.0 to 3.9 over the full azimuth. Fig. 2.5 shows the photographs of BBC. Each element of BBC is one-inch diameter mesh-dynode photomultiplier tubes (Hamamatsu R6178) equipped
Figure 2.4: A photo of 1 module of the ZDC. The top shows the PMMA fibers which are sandwiched between tungsten plates. These fibers generate and guide cerenkov light to the Hamamatsu R329-2 PMT. The red arrow on the left shows the impact position of the beam.
with 3 cm quartz on the head of the PMT as Cherenkov radiator as shown in Fig. 2.5 (a). Each BBC is composed of 64 PMT elements (Fig. 2.5 (b)) and is installed on the mounting structure surrounding the beam pipe as shown in Fig. 2.5 (c).

The BBC was made to satisfy the following requirements;

1. The BBC must have a capability to function over a large dynamic range from 1 to 30 MIP’s in order to cover from p+p to central Au + Au collisions.

2. The BBC is required to be radiation hard because the location of BBC, which is around the beam pipe near the collision point, is very high-level radiation area.

3. The BBC needs to work in a high magnetic field environment ($\sim 3$ kG) since the BBC is installed just behind the central magnet.

For both ZDC and BBC, the start time and z-vertex position are determined by using the measured time difference between the south and north detectors and known distance between the two detectors. The start time ($\mathit{T}_0$) and the z-vertex position ($\mathit{z}_{\text{vtx}}$) are calculated as

$$\mathit{T}_0 = (\mathit{T}_1 + \mathit{T}_2)/2$$  \hspace{0.5cm} (2.1)
$$\mathit{z}_{\text{vtx}} = c \cdot (\mathit{T}_1 - \mathit{T}_2)/2$$  \hspace{0.5cm} (2.2)

where $\mathit{T}_1$ and $\mathit{T}_2$ are the average timing of particles in each counter and $c$ is the speed of light. Typically, the ZDC z-vertex is measured with a resolution of 2.5 cm with an intrinsic timing resolution of 150 ps, and the resolution of BBC z-vertex is 0.6 cm with timing resolution of 40 ps [30].
2.5 The Central Arm Spectrometers

The PHENIX central arm spectrometers consists of two arms, which is east arm (right) and west arm (left). Each arm covers $90^\circ$ in azimuth and $|\eta| < 0.35^2$. As shown in the Fig. 2.6, each spectrometers consists of layers of tracking and particle identification subsystems.

![PHENIX Detector](image)

Figure 2.6: The layout of PHENIX central arm spectrometers viewed along the beam axis in Run4 configuration.

The PHENIX central arm spectrometers have three main tracking device, which is Drift Chamber (DC), Pad Chambers (PC), and Time Expansion Chamber (TEC), to measure the momentum of charged particles, reconstruct invariant masses of particle pairs and contributes to particle identification. The three tracking subdetectors are optimized for different purposes.

1. The Drift Chamber (DC) provide high resolution $p_T$ measurements and position information used to match tracks for outer subsystems.

2. Three Pad Chambers (PC) designated PC1, PC2, and PC3 provide three dimensional position measurements along the straight line trajectories for charged hadrons outside the magnetic field. PC1 also determine the three-dimensional momentum vector by providing polar angle $\theta$ for charged tracks at the exit of the DC.

3. The Time Expansion Chamber (TEC) provides additional tracking and particle identification.

$^{2}$Pseudorapidity, $\eta = -\ln (\tan (\theta/2))$. $\theta$ is polar angle.
Particle identification is provided by Ring Imaging CHerenkov counter (RICH), Time Expansion Chamber (TEC), Time-Of-Flight (TOF), Lead Scintillator Electromagnetic Calorimeter (PbSc), and Lead Glass Electromagnetic Calorimeter (PbGl). RICH provide excellent separation of electrons from hadrons over a wide range of momenta in $0.2 < p < 5.0 \text{ GeV/c}$ or greater. TEC is an additional tracking device which helps with momentum resolution at high $p_T$, and provides electron identification below 2.5 GeV/c through a measurement of $dE/dx$. The main role of TOF is to provide good hadron separation out to 2.4 (4.0) GeV/c for $\pi/K$ (K/p) with about 100 ps timing resolution. The EMCal system (PbSc and PbGl) provide a measurement of energy and spatial position of photons and electrons.

In the following sections, we introduce the PHENIX central arm spectrometers focusing on the subsystems which are used in this analysis: DC, PC, PbSc, and TOF. More details about the PHENIX detector can be found in [31, 32, 33].

### 2.5.1 The Drift Chamber (DC)

![Figure 2.7: Drawing of the Drift Chamber Frame.](image)

The Drift Chamber (DC) [34] is located in the east and west arm at a radial distance of $2.02 < R < 2.46 \text{ m}$, respectively. One of them is the mirror copy of each other, and each DC covers 90° in azimuth and 1.8 m along the z direction which corresponds to $|\eta| < 0.35$. The schematic drawing of DC frame is shown in Fig. 2.7.
DC is intended to measure momentum of charged particles with high resolution, \( \Delta p_T/p_T \approx 0.5\% \). In order to achieve high momentum resolution, the DC has to satisfy the following requirements:

- Single wire resolution better than 150 \( \mu \text{m} \) in the \( r-\phi \).
- Single wire efficiency better than 99 \%.
- Single wire two track separation better than 1.5 mm.
- Spatial resolution in the \( z \) direction better than 2 mm.

![Diagram of wire orientations](image)

**Figure 2.8:** The layout of wire position within one sector and inside the anode plane (left). A schematic diagram, top view, of the stereo wire orientation (right).

DC frame can be divided in 20 identical sectors each covering 4.5\(^\circ\) in \( \phi \). Every sector is filled with six modules of different types: X1, U1, V1, X2, U2, and V2 as shown in Fig. 2.8.

Each module contains 4 sense (anode) planes and 4 cathode planes forming cells with a 2 to 2.5 cm drift space in the \( \phi \) direction. The X1 and X2 wire cells are running in parallel to the beam to perform precise track measurements in \( r-\phi \). The U and V layers begin in one sector on one side of the frame and end in neighboring sectors on the opposite side. Angle between U, V stereo wires and X wires is about \( \pm 6^\circ \) and measure \( z \) coordinate of the track. Each of the X and U, V stereo cells contain 12 and 4 anode
(sense) wires, respectively so that there are 40 drift cells in the DC located at different radii. The sense wires (anode) are also electrically isolated in the middle by a low mass kapton strip. The number of readout channels is, therefore, doubled, about $3200 \times 2 = 6400$ channels, for each arm. It is necessary to reduce the track density on a single wire and perform reliable pattern recognition for the largest track multiplicities at RHIC.

2.5.2 The Pad Chamber (PC)

The Pad Chambers (PC) [35] are multi-wire proportional chambers. The three layers of Pad Chambers are located at the radial distance of 2.5 m (PC1), 4.2 m (PC2), and 4.9 m (PC3) from the interaction region as shown in Fig. 2.6. The PC system determines space points along the straight line particle trajectories since the PCs are located well outside the magnetic field ($R > 2.4$ m). The PCs are the only non-projective detectors in the central arm tracking system, and thus are critical elements of the pattern recognition. PC1 is also essential for determining the 3-dimensional momentum vector by providing the z coordinate at the exit of the DC.

Each detector contains a single plane of wires inside a gas volume bounded by two cathode planes as shown in Fig. 2.9. One cathode is finely segmented into an array of pixels. The charge induced on a number of pixels when a charged particle starts an avalanche on an anode wire, is read out through specially designed readout electronics. The design of pixels is driven by the need for good position resolution in the z-coordinate and a low occupancy even in the high track multiplicities. The design goal for the position resolution was 4 mm so that an anode wire spacing of about 8 mm was motivated. Finally, for a geometrical reasons, a spacing of 8.4 mm was chosen. A cell area of $8.4 \times 8.4$ mm$^2$ was adopted since a square cell geometry was desired.
A special pad design was invented, where each cell contains three pixels and an avalanche must be sensed by all three pixels to form a valid hit in the cell in order to reduce the amount of electric and other noise. This arrangement is, however, costly in terms of electronic channels. Thus, the interleaved pixels were ganged together as shown in the Fig. 2.10. Nine pixels are connected to a group and to a common readout channel, such that the three pixels in a cell are always connected to different but neighbouring channels and each cell is defined by its unique channel triplet. This solution saves a factor of nine in readout channels compared to readout of every pixel and a factor of three compared to a readout pad geometry where a cell is the actual electrode connected to an electronics channel.

The performance of PCs are summarized in the Table 2.4 [36].

<table>
<thead>
<tr>
<th>Chamber</th>
<th>Wire distance (mm)</th>
<th>$z$ resolution (mm)</th>
<th>perp. resolution (mm)</th>
<th>radiation thickness</th>
</tr>
</thead>
<tbody>
<tr>
<td>PC1</td>
<td>8.4</td>
<td>1.7</td>
<td>2.5</td>
<td>1.2 %</td>
</tr>
<tr>
<td>PC2</td>
<td>13.6</td>
<td>3.1</td>
<td>3.9</td>
<td>2.4 %</td>
</tr>
<tr>
<td>PC3</td>
<td>16.0</td>
<td>3.6</td>
<td>4.6</td>
<td>2.4 %</td>
</tr>
</tbody>
</table>

### 2.5.3 Lead Scintillator Electromagnetic Calorimeter (PbSc)

The primary role of the Electromagnetic Calorimeter (EMCal) [37] in PHENIX is to provide a measurement of the energies and spatial positions of photons and electrons. The EMCal system consists of 2 differently designed subsystems. The first is a shishlik type sampling calorimeter consisting of 15552 lead and scintillator (PbSc) towers. The other is a homogeneous detector consists of 9216 elements of lead-glass (PbGl). They both have good timing and energy resolutions, although the PbSc has better timing and the PbGl has better spatial and energy resolution. In addition, the PbSc has 0.85 nuclear
interaction length in depth so that it has some sensitivity for hadron measurements even though the PbSc was designed as an electromagnetic calorimeter. Having two detectors with different systematics increases the confidence level of the physics results. We will discuss design and operational parameters of the PbSc calorimeter which used in this analysis. More details of PbGl can be found in [37].

Figure 2.11: Interior view of PbSc calorimeter module showing a stack of scintillator and lead plates, wavelength fiber readout and leaky fiber inserted in the central hole.

The PbSc towers contains 66 sampling cells consisting of alternating tiles of Pb and scintillator. These cells are optically connected by 36 longitudinally penetrating wavelength shifting fibers for light collection. Fig. 2.11 shows a "module" which is mechanically grouped 4 towers together into a single structural entity. 36 of these modules are held together to form a "supermodule". 18 supermodule make a "sector". All major PbSc design parameters are listed in Table 2.5.

The nominal energy resolution of PbSc is

$$\frac{\sigma(E)}{E} = 8.1\%/\sqrt{E} \mp 2.1\%$$  

The resolution was determined by electron test beams at BNL and CERN under ideal conditions as shown in Fig. 2.12.

Fig. 2.13 shows the timing resolution of PbSc. the PbSc timing resolution is nearly constant at $\sigma \sim 120$ ps for electrons and protons, and $\sigma \sim 270$ ps for pions for energy deposits in the PbSc larger than 0.5 GeV. For the real data in Run-4 period, the timing resolution is achieved about 400 ps for pions, which provides the particle separation up to $p_T \sim 1$ GeV$/c$ for $\pi/K$, and up to $p_T \sim 2$ GeV$/c$ for $K/p$. 
Table 2.5: Individual Pb-Scintillator Calorimeter Tower Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lateral Segmentation</td>
<td>$5.535 \times 5.535 \text{ cm}^2$</td>
</tr>
<tr>
<td>Active Sampling Cells</td>
<td>66</td>
</tr>
<tr>
<td>Scintillator</td>
<td>Polystyrene (1.5% PT / 0.01% POPOP), 0.4 cm</td>
</tr>
<tr>
<td>Absorber</td>
<td>Pb, 0.15 cm</td>
</tr>
<tr>
<td>Cell Thickness</td>
<td>0.56 cm $(0.277 \times \text{X}_0)$</td>
</tr>
<tr>
<td>Active Depth</td>
<td>37.5 cm</td>
</tr>
<tr>
<td>Radiation Length</td>
<td>18</td>
</tr>
<tr>
<td>Nuclear Interaction Length</td>
<td>0.85</td>
</tr>
<tr>
<td>WLS Fiber</td>
<td>BCF-99-29a, 0.1 cm</td>
</tr>
<tr>
<td>WLS Fibers per Tower</td>
<td>36</td>
</tr>
<tr>
<td>PMT Type</td>
<td>FEU115M, MELS, Russia, 3.0 cm</td>
</tr>
<tr>
<td>Photocathode</td>
<td>Sb-K-Na-Cs</td>
</tr>
<tr>
<td>Luminous Sensitivity</td>
<td>$\geq 80 \mu\text{a}/\text{lm}$</td>
</tr>
<tr>
<td>Rise Time (20%-80%)</td>
<td>$\leq 5 \text{ ns}$</td>
</tr>
</tbody>
</table>

Figure 2.12: PbSc energy resolution obtained by beam tests at CERN and BNL. The dashed line shows a fit to the linear formula $\sigma(E)/E = 1.2\% + 6.2\%/\sqrt{E}$. The dashed-dotted line shows the fit to the quadratic formula $\sigma(E)/E = 2.1\% \oplus 8.1\%/\sqrt{E}$. 
Figure 2.13: PbSc timing resolution for different particles. Top figure shows lineshape for 1 GeV/c electrons, pions, and protons. Bottom shows timing resolution in the momentum range $0.3 < p < 1.0$ GeV/c.
2.5.4 The Time-of-Flight Counter (TOF)

The Time-Of-Flight (TOF) system [33] provides a primary device for charged hadron identification in PHENIX and is designed to achieve a very clear particle separation out to high momentum, i.e. $\pi$/K separation up to 2.4 GeV/$c$ and K/p separation up to 4.0 GeV/$c$ with 100 ps timing resolution.

The TOF is located at a radial distance of 5.1 m from the collision vertex in the east arm. It covers $70^\circ \leq \theta \leq 110^\circ$, which corresponds to $|\eta| < 0.35$, and 30° in azimuth. The TOF slat is a plastic scintillator (Bicron BC404) with PMT (Hamamatsu R3478S) at both ends. Two different lengths of scintillator (637.7 and 439.9 mm) are used in order to avoid geometrical conflicts between the PMT’s of neighboring slats. 96 of these slats are grouped to a TOF wall called as ”panel” as shown in Fig. 2.14. 10 panels make a TOF system, thus total 960 slats and 1920 channels of PMT’s are installed. The segmentation of TOF system is determined to minimize the probability of double hit. Assuming the rapidity density of the charged particle to be $dN_{ch}/dy = 1500$, the charged particle multiplicity on the TOF is expected to be 9. In order to keep the occupancy below 10 %, the segmentation should be about 1000 and the required area of each segment at a radial distance of 5.1 m from the interaction points is 100 cm$^2$.

Particle identification for charged particles is performed by combining the information of the BBC, DC, PC, and TOF. Fig. 2.15 shows scatter plot of inverse momentum
times charge with respect to the flight time of charged particles in minimum bias Au + Au collisions. This figure clearly demonstrates the particle identification capability of the TOF detector. During Run-4 period, the actual timing resolution of TOF system is about 125 ps, which is enable us to separate $\pi/K$ up to 2 GeV/$c$, and $K/p$ up to 3.2 GeV/$c$.

![PHENIX High Resolution TOF](image)

Figure 2.15: Flight time of charged hadrons as a function of inverse momentum in minimum bias Au + Au collisions at $\sqrt{s_{NN}} = 200$ GeV.
Chapter 3

Data Reduction

In this chapter, we present the minimum bias trigger and centrality determination (Section 3.1), event plane determination with BBC (Section 3.2), track reconstruction and momentum determination by the DC (Section 3.3), particle identification by the TOF (Section 3.4), how to extract the $v_2$ for identified hadrons (Section 3.6), and systematic uncertainties on $v_2$ (Section 3.7).

3.1 Event selection

3.1.1 Minimum Bias Trigger

Minimum bias trigger is defined by selecting following events on the BBC and the ZDC:

1. BBC
   - At least two PMTs are fired in each BBC and the collision vertex $z_{vertex}$ satisfy $|z_{vertex}| < 38 \text{ cm}$ by the online BBC Level-1 trigger.
   - $|z_{vertex}| < 30 \text{ cm}$ is required by offline analysis.

2. ZDC
   - At least one forward neutron is detected in each of the two ZDC.

The trigger efficiency for minimum bias Au + Au collisions is studied for both of the BBC and the ZDC [38, 39]. The extracted trigger efficiencies are given by

$$\epsilon_{BBC} = 92.3\% \pm 0.4\%(\text{stat.}) \pm 1.6\%(\text{sys.})$$

$$\epsilon_{ZDC|BBC} = 99^{+1.0\%}_{-1.5\%}$$

$$\epsilon_{\text{minbias}} = \epsilon_{BBC} \times \epsilon_{ZDC|BBC} = 91.4^{+2.5\%}_{-3.0\%}$$

where $\epsilon_{BBC}$ is the BBC trigger efficiency estimated by HIJING event generator [40]. $\epsilon_{ZDC|BBC}$ is the ZDC trigger efficiency for BBC Level-1 triggers, and $\epsilon_{\text{minbias}}$ is the minimum bias trigger efficiency with the coincidence of BBC and ZDC. Therefore, PHENIX minimum bias trigger selection can cover 91.4\% of the Au + Au inelastic cross sections.
### 3.1.2 Centrality Determination

![Graph showing centrality determination using the ZDC total energy and the BBC charge sum.](image)

Figure 3.1: Centrality determination using the ZDC total energy and the BBC charge sum. See text for details how to determine the centrality in ZDC-BBC space.

In order to study the collision geometry dependence of the measured anisotropic flow we should know the relationship between the impact parameter and centrality since we cannot measure impact parameter directly. In PHENIX experiments, the centrality is determined by the measured correlation between the charge deposited in the BBC and the energy deposited in the ZDC.

In Fig. 3.1 the centrality angle $\phi_{\text{cent}}$ is determined in the ZDC energy - BBC charge space, where $\phi_{\text{cent}}$ is the angular position of the event defined as

$$\phi_{\text{cent}} = \tan^{-1}\left(\frac{(Q^{BBC} - Q_0^{BBC})/Q_{max}^{BBC}}{E_{\text{ZDC}}/E_{\text{ZDC max}}}ight)$$

(3.4)

where $Q^{BBC}$ and $E^{ZDC}$ are measured charge sum in the BBC and measured total energy in the ZDC, respectively, $Q_{max}^{BBC}$ is the maximum charge sum measured in the BBC and is equal to 1700, $Q_0^{BBC} = 0.15 \cdot Q_{max}^{BBC}$ is the position along the BBC axis from which the angle $\phi_{\text{cent}}$ is determined, and $E_{\text{ZDC max}}$ is the maximum energy deposited in the ZDC and is 8500 GeV. This event with $\phi_{\text{cent}}$ is grouped into the centrality class defined by lower and upper bounds, $\phi_{\text{min}}$ and $\phi_{\text{max}}$ if $\phi_{\text{min}} < \phi_{\text{cent}} < \phi_{\text{max}}$. Since the performance of the BBC and ZDC is changed during the period of Run4, we apply the time dependent correction for both BBC charge and ZDC energy [41].

After centrality is determined we estimate number of participating nucleons ($N_{\text{part}}$) and number of binary collisions ($N_{\text{coll}}$) by using Glauber model based on Monte Carlo simulations. Further details of Glauber model can be found in Appendix A.
3.2 Event Plane

In this section, we introduce the Fourier expansion of azimuthal particle distribution and its properties with respect to the reaction plane. And we also introduce event plane which is the estimate of the true reaction plane determined by using the signal of flow itself.

3.2.1 Fourier Expansion of Azimuthal Distribution

Since the azimuthal distribution of emitted particles \( dN/d\phi \) is the periodic function with \( 2\pi \) fundamental period, it is natural to expand azimuthal distribution into Fourier series with \( 2\pi \) period.

\[
\frac{dN}{d\phi} = \frac{x_0}{2\pi} + \frac{1}{\pi} \sum_{n=1}^{\infty} \left( x_n \cos (n\phi) + y_n \sin (n\phi) \right)
\]

\[
= \frac{x_0}{2\pi} \left( 1 + 2 \sum_{n=1}^{\infty} \left( \frac{x_n}{x_0} \cos (n\phi) + \frac{y_n}{x_0} \sin (n\phi) \right) \right) \tag{3.5}
\]

The Fourier coefficients \( x_n \) and \( y_n \) can be obtained by integrating \( dN/d\phi \) with weights proportional to \( \cos (n\phi) \) and \( \sin (n\phi) \). Here, we introduce the following notation,

\[
\langle O \rangle = \frac{\int d\phi \ O \times dN/d\phi}{\int d\phi \ dN/d\phi} \tag{3.6}
\]

where \( O \) denotes some observables. Since there is only a finite number of particles in each event, the integral become simply sums over particles found in the event

\[
x_n = \int_0^{2\pi} d\phi \ \frac{dN}{d\phi} \cos (n\phi) = \sum_{i=0}^{M} w_i \cos (n\phi_i) \equiv Q_x \tag{3.7}
\]

\[
y_n = \int_0^{2\pi} d\phi \ \frac{dN}{d\phi} \sin (n\phi) = \sum_{i=0}^{M} w_i \sin (n\phi_i) \equiv Q_y \tag{3.8}
\]

where \( i \) runs over all particles \( (M) \) used to determine the event plane, \( \phi_i \) is the azimuthal angle of the emitted \( i \)-th particle and \( w_i \) is the weight \( (p_T, \text{ multiplicity etc}) \) to minimize the dispersion of event plane (i.e. maximize event plane resolution). We define the following two-dimentional vector \( \mathbf{Q} = (Q_x, Q_y) \) called as a flow vector.

If we assume \( \phi \) in Eq. (3.5) is defined relative to the reaction plane, then \( dN/d\phi \) becomes an even function and we can omit \( y_n \) terms since the integration would be zero in Eq. (3.8),

\[
\frac{dN}{d\phi} = \frac{x_0}{2\pi} \left( 1 + 2 \sum_{n=1}^{\infty} \frac{x_n}{x_0} \cos (n\phi) \right) = \frac{x_0}{2\pi} \left( 1 + 2 \sum_{n=1}^{\infty} v_n \cos (n[\phi_{lab} - \Psi]) \right) \tag{3.9}
\]
where $\phi_{lab}$ is the azimuthal angle of fixed orientation in the experiment, $\Psi$ is the azimuthal angle of true reaction plane and $v_n = x_n/x_0$ is the magnitude of anisotropy. We introduce the following two variables,

$$v_n^{obs} = \sqrt{x_n^2 + y_n^2}, \quad x_0 = M \langle w \rangle$$

(3.10)

$$\Psi_n = \frac{1}{n} \tan^{-1} \left( \frac{y_n}{x_n} \right), \quad 0 \leq \Psi_n \leq \frac{2\pi}{n}$$

(3.11)

From Eq. (3.10) and (3.11), measured azimuthal distribution $r^m(\phi)$ can be given by

$$r^m(\phi_{lab}) = \frac{x_0}{2\pi} \left( 1 + 2 \sum_{n=1}^{\infty} \left( \frac{x_n}{x_0} \cos (n\phi_{lab}) + \frac{y_n}{x_0} \sin (n\phi_{lab}) \right) \right)$$

$$= \frac{x_0}{2\pi} \left( 1 + 2 \sum_{n=1}^{\infty} \left( v_n^{obs} \cos (n\Psi_n) \cos (n\phi_{lab}) + v_n^{obs} \sin (n\Psi_n) \sin (n\phi_{lab}) \right) \right)$$

$$= \frac{x_0}{2\pi} \left( 1 + 2 \sum_{n=1}^{\infty} v_n^{obs} \cos (n[\phi_{lab} - \Psi_n]) \right)$$

(3.12)

Comparing to Eq. (3.9), and (3.12) one can see that $\Psi_n$ gives event plane, which is the estimate of an azimuthal angle of true reaction plane. It is reconstructed from the reaction products event-by-event basis. The reconstructed plane (event plane) differs in general from the true reaction plane by an error $\Delta \Psi$. Thus, the measured azimuthal angle of event plane $\Psi_n$ is related to the true azimuthal angle of reaction plane $\Psi$ by $\Psi_n = \Psi + \Delta \Psi$. Averaging over many events, one obtains the following relation between the measured and true Fourier coefficients [42]:

$$v_n^{obs} = \langle \cos (n[\phi_{lab} - \Psi_n]) \rangle$$

$$= \langle \cos (n[\phi_{lab} - \Psi] - n[\Psi_n - \Psi]) \rangle$$

$$= \langle \cos (n[\phi_{lab} - \Psi]) \cdot \cos (n\Delta \Psi) \rangle + \langle \sin (n[\phi_{lab} - \Psi]) \cdot \sin (n\Delta \Psi) \rangle$$

$$= \langle \cos (n[\phi_{lab} - \Psi]) \rangle \langle \cos (n\Delta \Psi) \rangle$$

$$= v_n \langle \cos (n\Delta \Psi) \rangle$$

(3.13)

from line 3 to 4 we assume that $\phi_{lab} - \Psi$ and $\Delta \Psi$ are statistically independent. And we use the reflection symmetry of $\phi_{lab} - \Psi$ and $\Delta \Psi$, i.e. average sine term vanish under that condition. This assumption is valid for the system with large multiplicity. In section 3.2.4, we derive the analytical formula of event plane resolution, $\langle \cos (n\Delta \Psi) \rangle$, when the multiplicity is large ($M \gg 1$) in a selected window.

### 3.2.2 Event Plane Determination

Since an azimuthal angle of true reaction plane is unknown, we have to determine estimated reaction plane (event plane) experimentally. In this analysis, the BBC is
used to determine an event plane for each event. The BBC have several advantages to determine the event plane:

- The BBC has full azimuthal coverage while central arm has only half of full azimuth.

- The BBC is located from 144 cm from the collision points and this corresponds to a pseudo-rapidity $|\eta| = 3.0$ – 3.9. This rapidity gap helps to reduce non-flow contributions, which is the correlations not originated from the reaction plane, such as di-jet correlations, resonance decays, and Bose-Einstein correlations.

- The BBC has a very good stability during the entire RUN period.

Event plane is calculated at both south BBC (BBCS) and north BBC (BBCN) by the Eq. (3.14) - (3.16)

$$2\Psi_{\text{obs}} = \tan^{-1}\left(\frac{Q_y}{Q_x}\right)$$  \hspace{1cm} (3.14)

$$Q_x = \sum_{i=0}^{64} w_i \cos{(2\phi_i)}$$  \hspace{1cm} (3.15)

$$Q_y = \sum_{i=0}^{64} w_i \sin{(2\phi_i)}$$  \hspace{1cm} (3.16)

where $\Psi_{\text{obs}}$ is measured azimuthal angle of event plane, $Q_x$ and $Q_y$ is the projection of event plane to $x$ and $y$ axes respectively, $\phi_i$ is the azimuthal angle of each PMT, and $w_i$ is the weight. We choose charge value on each PMT as a weight. Fig. 3.2 shows the typical response in one event for both south and north BBC. The solid red lines represent the azimuthal angle of measured (uncorrected) event plane. Normally, measured event plane is not always flat because of imperfect detector acceptance, dead channels in BBC’s, beam condition and so on. In order to correct those effects, we have 2 calibration steps for event plane. Details of these calibration will be discussed in the next section.

### 3.2.3 Event Plane Calibration

As we already mentioned in previous section, measured event plane distribution is not always flat because of several effects. In this section, we introduce the detail of calibration for event plane.
CHAPTER 3. DATA REDUCTION

Figure 3.2: Event display how to determine the azimuthal angle of event plane in south BBC (left) and north BBC (right). Solid red lines show the azimuthal angle of measured event plane in this particular event.

Re-centering Calibration

“Re-centering calibration” is defined as

\[ 2\Psi_{\text{cor}} = \tan^{-1} \left( \frac{Q_{y \text{cor}}}{Q_{x \text{cor}}} \right) \]  \hspace{1cm} (3.17)
\[ Q_{x \text{cor}} = \frac{Q_x - \langle Q_x \rangle}{\sigma_x} \]  \hspace{1cm} (3.18)
\[ Q_{y \text{cor}} = \frac{Q_y - \langle Q_y \rangle}{\sigma_y} \]  \hspace{1cm} (3.19)

where \( \langle Q_{x,y} \rangle \) are the mean of \( Q_x \) and \( Q_y \) distributions, and \( \sigma_{x,y} \) are the width of \( Q_x \) and \( Q_y \). As we can see in the Fig. 3.3 the width of flow vector become wider in more peripheral bins, while mean is unchanged for all centrality bins. The mean and width of flow vectors are extracted by fitting \( Q_x \) and \( Q_y \) distributions with gaussian, and parameterized as a function of centrality.

Fig. 3.4 shows the mean and width of flow vectors as a function of centrality. Since detector response of BBC’s have very good stability, mean and width does not change during the entire period of run.

Flattening Calibration

Fig. 3.5 show the event plane distribution determined in BBC SOUTH + NORTH. One can see that the event plane with re-centering calibration (blue histogram) is almost
Figure 3.3: $Q_x$ (left) and $Q_y$ (right) distributions for several centrality bins in BBC SOUTH + NORTH. Centrality bins are 0 – 10% (solid black), 10 – 30% (dashed red), 30 – 60% (dotted blue), and 60 – 92% (dashed-dotted magenta).

Figure 3.4: Mean (left) and width (right) of flow vectors as a function of centrality. Open black (solid red) symbols show the mean and width of $Q_x$ ($Q_y$).
CHAPTER 3. DATA REDUCTION

Figure 3.5: Event plane distribution for BBC SOUTH + NORTH, without correction (black), with re-centering calibration (blue), and with re-centering + flattening calibration (red). Top right figure is expanded the same event plane distribution to see the flatness of event plane.

flat but there is still small non-flat components as shown in top right part in the figure. In order to remove the remaining non-flatness of event plane, we perform ”Flattening calibration” which is defined by

\[
\begin{align*}
    n \Delta \Psi & \equiv \sum_{k=1}^{k_{\text{max}}} [A_k \cos (kn \Psi^{\text{cor}}) + B_k \sin (kn \Psi^{\text{cor}})] \\
n \Psi & \equiv n \Psi^{\text{cor}} + n \Delta \Psi
\end{align*}
\] (3.20) (3.21)

The coefficients \(A_k\) and \(B_k\) can be obtained by requiring that \(k\)-th Fourier moment of the \(\Psi\) distribution is vanished, i.e., requiring isotropic distribution for \(\Psi\). Assuming that the correction \(\Delta \Psi\) is small,

\[
\begin{align*}
    \langle \cos (kn \Psi) \rangle & = \langle \cos (kn \Psi^{\text{cor}} + kn \Delta \Psi) \rangle \\
& = \langle \cos (kn \Psi^{\text{cor}}) \cdot \cos (kn \Delta \Psi) \rangle - \langle \sin (kn \Psi^{\text{cor}}) \cdot \sin (kn \Delta \Psi) \rangle \\
& \approx \langle \cos (kn \Psi^{\text{cor}}) \rangle - \langle \sin (kn \Psi^{\text{cor}}) \cdot \langle kn \Delta \Psi \rangle \rangle \\
& = \langle \cos (kn \Psi^{\text{cor}}) \rangle - k B_k \langle \sin^2 (kn \Psi^{\text{cor}}) \rangle \\
& = \langle \cos (kn \Psi^{\text{cor}}) \rangle - \frac{k B_k}{2} \\
& = 0 \\
\therefore B_k & = \frac{2}{k} \langle \cos (kn \Psi^{\text{cor}}) \rangle
\end{align*}
\] (3.22)
Similarly,
\[
\langle \sin (k\Psi) \rangle = \langle \sin (k\Psi^{cor} + k\Delta \Psi) \rangle \\
= \langle \sin (k\Psi^{cor}) \cdot \cos (k\Delta \Psi) \rangle + \langle \cos (k\Psi^{cor}) \cdot \sin (k\Delta \Psi) \rangle \\
\approx \langle \sin (k\Psi^{cor}) \rangle + \langle \cos (k\Psi^{cor}) \cdot (k\Delta \Psi) \rangle \\
= \langle \sin (k\Psi^{cor}) \rangle + kA_k \langle \cos^2 (k\Psi^{cor}) \rangle \\
= \langle \sin (k\Psi^{cor}) \rangle + \frac{kA_k}{2} \left( \cdots \langle \cos^2 (k\Psi^{cor}) \rangle = \frac{1}{2} \right) \\
= 0 \\
\therefore A_k = -\frac{2}{k} \langle \sin (k\Psi^{cor}) \rangle
\]
(3.23)

\[
\Psi = \Psi^{cor} + \Delta \Psi
\]
(3.24)

\[
\Delta \Psi = \sum_k (A_k \cos (2k\Psi^{cor}) + B_k \sin (2k\Psi^{cor}))
\]
(3.25)

\[
A_k = -\frac{2}{k} \langle \sin (2k\Psi^{cor}) \rangle
\]
(3.26)

\[
B_k = \frac{2}{k} \langle \cos (2k\Psi^{cor}) \rangle
\]
(3.27)

where \(\Psi^{cor}\) is corrected event plane in Eq. (3.17), and \(\Delta \Psi\) is the correction factor from flattening calibration, \(k\) is degree of Fourier expansion, and brackets denote the average over all particles in all events. One can see that the event plane distribution with the flattening calibration (red histogram in Fig. 3.5) becomes flat. We perform flattening calibration run by run basis since the condition of BBC and beam could be changed for different runs.

### 3.2.4 Event Plane Resolution

In the real experiment, only finite number of particles are emitted and detected in each collision. This introduces some fluctuations in observing the anisotropy on an event-by-event basis. Even if the distribution is azimuthally isotropic, statistical fluctuations can lead to non-zero coefficients \(v_n\).

We first show the following three assumptions in order to simplify the problem.

1. All particles in the same event and in the same rapidity window can be treated as being independent.
2. The total number of particles in the selected window is relatively large \((M \gg 1)\)
3. The magnitude of flow is not fluctuate in the same impact parameter or centrality

Under these assumptions, the event plane resolution can be expressed as [13]

\[
\langle \cos (k\theta_n) \rangle = \sqrt{\frac{\pi}{2}} \bar{\chi}_n e^{-\bar{\chi}_n^2/2} \left[ I_{(k-1)/2} \left( \frac{\bar{\chi}_n^2}{2} \right) + I_{(k+1)/2} \left( \frac{\bar{\chi}_n^2}{2} \right) \right]
\]
(3.28)
Fig. 3.6 shows the event plane resolution as a function of $\chi$ for different $k$, $k = 1, 2, 3, 4$. Asymptotic results are also plotted by blue lines ($\chi \ll 1$) from Eq. (B.10), and red lines ($\chi \gg 1$) from Eq. (B.11).

Since we have two independent event plane from SOUTH and NORTH BBC, we can estimate the event plane resolution by measuring the relative azimuthal angle $\Delta \Psi_n^{BBC} \equiv n(\Psi_n^{BBCS} - \Psi_n^{BBCN})$. Because acceptance of each BBC is same, the multiplicity for each BBC is equal to be half of total multiplicity in both BBC’s, $N^{BBCS} = N^{BBCN} = N/2$, the corresponding statistical fluctuation $\sigma_n$ and $\bar{\chi}_n$ should be:

$$\sigma_n^{BBCS} = \sigma_n^{BBCN} = \frac{1}{\sqrt{N^{BBCS,N}} \langle w^2 \rangle} = \frac{\sqrt{2} \langle w^2 \rangle}{\sqrt{N} \langle w \rangle^2} = \sqrt{2} \sigma_n^{BBC}$$ \hspace{1cm} (3.29)

$$\chi_n^{BBCS} = \chi_n^{BBCN} = \frac{\bar{\chi}_n}{\sigma_n^{BBCS,N}} = \frac{\bar{\chi}_n}{\sqrt{2} \sigma_n^{BBC}} = \frac{\chi_n^{BBC}}{\sqrt{2}}$$ \hspace{1cm} (3.30)

One can obtain $\chi_n$ by measuring $\langle \cos (k\Delta \Psi_n^{BBC}) \rangle = \langle \cos (kn[\Psi_n^{BBCS} - \Psi_n^{BBCN}]) \rangle$:

$$\langle \cos (k\Delta \Psi_n^{BBC}) \rangle = \langle \cos (kn[\Psi_n^{BBCS} - \Psi_n^{BBCN}]) \rangle = \langle \cos (kn[\Psi_n^{BBCS} - \Psi]) \rangle \langle \cos (kn[\Psi_n^{BBCN} - \Psi]) \rangle$$

$$= \frac{\pi}{8} \chi_n^2 e^{-\chi_n^2/2} \left[ I_{(k-1)/2} \left( \frac{\chi_n^2}{4} \right) + I_{(k+1)/2} \left( \frac{\chi_n^2}{4} \right) \right]^2$$ \hspace{1cm} (3.31)

Fig. 3.7 shows the event plane resolution as a function of centrality. As one can see
Figure 3.7: Event plane resolution as a function of centrality measured by the correlation between SOUTH and NORTH BBC event plane.

\( \langle \sin (2\Delta \Psi^{BC}) \rangle = 0 \) for entire centrality bins as we expected from the reflection symmetry of \( \Delta \Psi \). Since the maximum value of \( \langle \cos (2\Delta \Psi^{BC}) \rangle \) (open black) is less than 0.1, the approximation of small anisotropy limit \( (\chi \ll 1) \) works very well (see the lines for \( k = 1 \) in Fig. 3.6). Thus, if \( \chi \ll 1 \) the event plane resolution is reduced to

\[
\langle \cos (k\Delta \Psi^{BC}) \rangle \approx \frac{\pi}{8} \chi_n^2 e^{-\chi_n^2/2} \frac{1}{\Gamma \left( \frac{k+1}{2} \right)^2} \left( \frac{\chi_n^2}{8} \right)^{k-1} \left[ 1 + \frac{2}{k+3} \left( \frac{\chi_n^2}{8} \right) \right]^2
\]

\[
\approx \frac{\pi}{8^k \Gamma \left( \frac{k+1}{2} \right)^2 \chi_n^{2k}}
\]

\[
= \frac{1}{2^k} \langle \cos (k\theta_n) \rangle^2
\]

\[
\therefore \langle \cos (\theta_n) \rangle \approx \sqrt{2} \langle \cos (\Delta \Psi^{BC}) \rangle \quad (k = 1)
\]  

3.2.5 Event Plane QA

After we perform full calibrations, the stability of event plane is checked for each run. Since the direction of event plane should be random, average cosine and sine of event plane azimuthal angle (\( \langle \cos (n\Psi) \rangle \), \( \langle \sin (n\Psi) \rangle \)) should be zero. Examples of event plane QA are shown in Fig. 3.8. As one can see(2,2\Psi)\rangle \) and \( \langle \sin (2\Psi) \rangle \) are zero for entire event for a given run if the event plane calibration is perfect (right figure). However, if
you look at the left figure there is still time dependent non-flatness for some runs after the full event plane calibration. These bad runs are less than 1% fraction of total data so it seems to not affect our final results, but we exclude these runs from analysis.

3.3 Global Track Selection

3.3.1 Track Reconstruction

A typical track in the DC main bend plane \((r - \phi)\) is illustrated in Fig. 3.9. The coordinates in the DC are \(\phi\) and \(\alpha\), where \(\phi\) is the azimuthal angle at the intersection of the track with a ”reference radius” at the mid-radius of the DC, and \(\alpha\) is the inclination of the track at that point. In principle, \(\phi\) and \(\alpha\) are equivalent to a slope and intercept; the main difference is that \(\phi\) and \(\alpha\) are limited to a given range of possible value while slope and intercept are not. Right hand side in Fig. 3.9 shows the track in the \(r - z\) plane, perpendicular to the bend plane. Because of the magnetic field is along the beam (z-) direction, tracks usually have a very small bend in this plane. The coordinates used in this projection are \(z_{ed}\), the \(z\) coordinate of the intersection point, and \(\beta\), the inclination of the track at the reference radius.

In order to find a track, hits produced in the detector by the same charged particle have to be found and combined. The tracking is done separately in the \(r - \phi\) and the \(r - z\) plane. The track reconstruction in \(r - \phi\) is realized using a combinatorial hough transform technique [32], where any pair of hits can be mapped to a point in the space defined by azimuthal angle \(\phi\) and track bending angle \(\alpha\).

In this analysis, we require the following conditions for the tracks: (1) Found the hits for X1 and X2 wire in DC, (2) Found the unique hit in UV wire in PC1, (3) Found the (unique) hit in PC1. These requirements corresponds that the DC track quality bit is equal to be 31 or 63.
3.3.2 Momentum Determination

The $\alpha$ measured in the DC is closely related to the field integral along the track trajectory. For tracks emitted perpendicular to the beam axis, this relation can be approximated as

$$\alpha \simeq \frac{K}{p_T}$$  \hspace{1cm} (3.33)

where $K = 101$ mrad GeV/$c$ is the effective field integral [43]. From Eq. (3.33), one can derive the following form between the momentum resolution and the angular resolution

$$\frac{\delta p}{p} = \frac{\delta \alpha}{\alpha} = \frac{1}{K} \sqrt{\left(\frac{\sigma_	ext{ms}}{\beta}\right)^2 + (\sigma_\alpha p)^2}$$  \hspace{1cm} (3.34)

where $\delta \alpha$ is the measured angular spread, $\sigma_	ext{ms}$ and $\sigma_\alpha$ is the contribution from multiple scattering and from angular resolution of the DC, respectively.

3.3.3 Matching Tracks to Outer Detectors

Tracks are reconstructed by DC-PC1 and projected to the outer tracking detectors by the track model. These detectors are two dimensional walls extending in $r - \phi$ and $z$ directions. Each of them provides a 3-dimensional hit at the detector wall. A wide window around the track intersection point with the detector plane is searched for a list
of candidate hits. The one with closest distance to the intersection point is identified as the hit associated with the track.

For primary tracks, the distance in both the $r - \phi$ and the $z$ direction between track projection point and the associated hit position is approximately Gaussian with a width as

$$\sigma_{\text{match}} = \sqrt{\sigma_{\text{detector}}^2 + \left(\frac{\sigma_{\text{ms}}}{p\beta}\right)^2}$$

(3.35)

where $\sigma_{\text{detector}}$ is the finite detector resolution, which include the DC pointing resolution and the detector spatial resolution, and $\sigma_{\text{ms}}$ is the multiple scattering contribution. The mean of the residual distribution ($\text{mean}_{\text{match}}$) is typically small compared to $\sigma_{\text{match}}$ after detector alignment. A non-zero value of $\text{mean}_{\text{match}}$ usually results from imperfect detector alignment or the magnetic field map used by the track model. These imperfections can lead to a momentum and charge sing dependence of $\text{mean}_{\text{match}}$.

The matching distribution needs to be parameterized separately in all tracking detectors for both positive and negative charged particles in $r - \phi$ and $z$ directions, and for collision centrality. The mean and width of matching distribution are extracted by a Gaussian fit, and parameterized as a function of $p_T$. Fig. 3.10 shows $\text{mean}_{r-\phi}$, $\text{mean}_z$, $\sigma_{r-\phi}$, $\sigma_z$ as a function of $p_T$ and their parameterization for TOF.

![Figure 3.10: TOF matching variables for $r - \phi$ and $z$ as a function of $p_T$ and charge. Top figure show $\text{mean}_{r-\phi}$ and $\text{mean}_z$, and bottom show $\sigma_{r-\phi}$ and $\sigma_z$.](image)
3.4 Particle Identification

In the section 3.4.1 we describe particle identification for $\pi$, $K$, $p$ and $d$ with Time-Of-Flight detector by using mass square distributions, and describe $\phi$ meson identification with invariant mass technique in the section 3.4.3.

3.4.1 Mass square distribution

Mass square is given by

$$m^2 = p^2 \left( \frac{1}{\beta^2} - 1 \right) = p^2 \left( \left( \frac{c \cdot t}{L} \right)^2 - 1 \right) \quad (3.36)$$

where $p$ is the particle momentum (GeV/c), and velocity $\beta$ is expressed with the measured time of flight ($t$), the measured flight path length ($L$), and the speed of light ($c = 29.98 \text{ cm/ns}$).

The width of the mass squared peak depends on both the momentum and time-of-flight resolutions. An analytic form for the width of mass square as a function of momentum resolution $\delta_p$ and timing resolution $\sigma_T$ is determined as follows;

$$\sigma_{m^2}^2 = \left( \frac{\partial m^2}{\partial p} \right)^2 \delta_p^2 + \left( \frac{\partial m^2}{\partial t} \right)^2 \sigma_T^2$$

$$= \left( 2p \left( \frac{1}{\beta^2} - 1 \right) \right)^2 \delta_p^2 + \left( 2t \left( \frac{pc}{L} \right)^2 \right)^2 \sigma_T^2$$

$$= \left( \frac{2m^2}{p} \right)^2 \delta_p^2 + \left( \frac{2cp}{L} \sqrt{m^2 + p^2} \right)^2 \sigma_T^2$$

$$= \left( \frac{\delta_p}{p} \right)^2 (4m^4) + \left( \frac{\sigma_T c}{L} \right)^2 \left[ 4p^2(m^2 + p^2) \right]. \quad (3.37)$$

Using Eq. (3.34) the mass square width can be written as

$$\sigma_{m^2}^2 = \frac{1}{K^2} \left[ \left( \frac{\sigma_{ms}}{\beta} \right)^2 + (\sigma_{\alpha} p)^2 \right] (4m^4) + \left( \frac{\sigma_{c} c}{L} \right)^2 \left[ 4p^2(m^2 + p^2) \right]$$

$$= \frac{\sigma_{\phi}^2}{K^2} (4m^4 p^2) + \frac{\sigma_{ms}^2}{K^2} \left[ 4m^4 \left( 1 + \frac{m^2}{p^2} \right) \right] + \left( \frac{\sigma_{c} c}{L} \right)^2 \left[ 4p^2(m^2 + p^2) \right]. \quad (3.38)$$

A typical mass square distribution is shown in Fig. 3.11. The centroid and width of the mass square for each particle species are extracted by a Gaussian fit. The width is parameterized as a function of $p_T$ by Eq. (3.38) with $\sigma_{\alpha}$, $\sigma_{ms}$ and $\sigma_{c}$ as free parameters. The fitting are done for 3 separated Run categories, where

1. $++$ field configuration : RUN 105218 - 111593
2. $++$ field configuration : RUN 111603 - 115780
Figure 3.11: Mass square distribution for momentum of 1.5 - 2 GeV/c in TOF.

Figure 3.12: Momentum dependence of the centroid (left) and the width (right) of mass square distributions for $\pi$, $K$ and $p$. 
The momentum dependence of the centroid and width are shown in Fig. 3.12. Dashed blue lines correspond expected values of the centroid from Particle Data Book value and the width from Eq. (3.38) with one of the parameterization as shown below. The resulting parameters are extracted for each RUN category [44].

Table 3.1: Fitting results of mass square width for 3 different RUN categories

<table>
<thead>
<tr>
<th>RUN categories</th>
<th>$\sigma_\alpha$ (mrad)</th>
<th>$\sigma_{ms}$ (mrad)</th>
<th>$\sigma_t$ (ps)</th>
</tr>
</thead>
<tbody>
<tr>
<td>++ field (1)</td>
<td>1.213</td>
<td>1.276</td>
<td>120</td>
</tr>
<tr>
<td>++ field (2)</td>
<td>1.192</td>
<td>1.269</td>
<td>120</td>
</tr>
<tr>
<td>-- field (3)</td>
<td>1.313</td>
<td>1.242</td>
<td>120</td>
</tr>
</tbody>
</table>

Figure 3.13: The contribution of the 1 $\sigma$ mass square width for $\pi$ (left) and $p$ (right). Each contribution is shown by different colors around the expected mass square value. The parameterization of each width is taken from the category (1).

Fig. 3.13 shows the contributions to the mass square width for $\pi$ and $p$. Since the $\pi$ mass is small, the width of mass square is dominated by $\sigma_t$ in the measured momentum range. For protons, the three terms have different contributions at different momenta. The multiple scattering term dominates at low momentum, i.e. $p < 1$ GeV/c. The momentum resolution term $\sigma_\alpha$ and $\sigma_t$ are dominate for intermediate momentum region ($p = 1 - 1.5$ GeV/c), and $\sigma_t$ starts to become dominant contribution at the high momentum ($p > 1.5$ GeV/c).

### 3.4.2 Momentum Calibration

Usually, the magnetic field map used by the track model does not reflect the real magnetic field properly. It leads to a systematic momentum shift which is reflected by a
shift of the centroid of mass square distribution. This shift $\Delta m^2$ can be calculated from the momentum shift $\Delta p$ as

$$m^2_{\text{exp}} = p^2_{\text{exp}} \left( \frac{1}{\beta^2} - 1 \right)$$

$$= (p + \Delta p)^2 \left( \frac{1}{\beta^2} - 1 \right)$$

$$\approx p \left( \frac{1}{\beta^2} - 1 \right) + 2p\Delta p \left( \frac{1}{\beta^2} - 1 \right)$$

$$= m^2 + 2\frac{\Delta p}{p} m^2$$

$$\therefore \Delta m^2 = m^2_{\text{exp}} - m^2 = 2m^2 \cdot \left( \frac{\Delta p}{p} \right)$$

(3.39)

where $p_{\text{exp}}$ and $m^2_{\text{exp}}$ is the expected momentum and mass square value, and these are connected to measured momentum and mass square value with $\Delta p$ and $\Delta m^2$ as $p_{\text{exp}} = p + \Delta p$ and $\Delta m^2 = m^2 + \Delta m^2$. Since momentum shift $\Delta p$ is typically small, a few %, we omit $\Delta^2 p$ term from 2nd to 3rd line in Eq. (3.39). Because $\Delta m^2$ is proportional to the particle $m^2$, a heavy particles such as the protons are more sensitive to the momentum scale $\Delta p$.

A shift of the beam position from the nominal origin of PHENIX detector will produce a systematic offset in mean $\alpha$ value, which can be parameterized as a function of DC azimuthal angle $\phi$ as:

$$\Delta \alpha = \frac{\Delta x \cdot \sin \phi}{R_{\text{DC}}} + \frac{\Delta y \cdot \cos \phi}{R_{\text{DC}}}$$

(3.40)

where $R_{\text{DC}} = 220$ cm is the reference radius of the DC, $\Delta x$ and $\Delta y$ are the beam offset. This small change in the beam position can introduce a systematic shift in track $\alpha$ angle, thus affects the measured momentum as follows;

$$\frac{\Delta p}{p} = \frac{\Delta \alpha}{\alpha} = -p \frac{\Delta \alpha}{K}$$

(3.41)

Since this change affects the momentum of positive and negative particles in opposite direction, we can determine $\Delta x$ and $\Delta y$ from the systematic shift of the measured mass square.

In order to perform a quantitative analysis for momentum scale and beam position offset, we define the modified mass square mean variables of protons and anti-protons as

$$M^+ = \frac{m^2_p + m^2_{\bar{p}}}{2} - m^2_{p(PDG)}$$

(3.42)

$$M^- = \frac{m^2_p - m^2_{\bar{p}}}{2}$$

(3.43)
where \( m_p^2 \) and \( m_{\bar{p}}^2 \) is the measured mean value of mass square for proton and anti-proton, and \( m_{p(PDG)}^2 = (0.9383)^2 \) is the mass square of the proton PDG value. From Eq. (3.39) and (3.43) momentum scale can be rewritten by using \( M^+ \) as

\[
\frac{\Delta p}{p} = \frac{\Delta m^2}{2m^2} = \frac{M^+}{2m^2}
\]

Beam position offset can be directly estimated from the intersection point where \( M^- \) is equal to be 0. Since \( \Delta x \) is not sensitive to the shift of mass square, we always set to \( \Delta x = 0 \) in this analysis. Fig. 3.14 show the extracted \( M^+ \) and \( M^- \) for \( 1 < p < 1.5 \text{ GeV/c} \). Momentum range of \( 1 - 1.5 \text{ GeV/c} \) is choosen to make \( M^+ \) and \( M^- \) stable where \( \alpha \) resolution is dominant. One can see that \( M^+ \) is constant over all \( \Delta y \) and \( M^- \) depends on \( \Delta y \) linearly. From these results, we conclude that

- Momentum scale factor of 2.2% is obtained, thus we have to scale down the momentum by 1.022
- \( \Delta y \) is about -0.01 and -0.07 for ++ and -- field configuration, respectively.

![Figure 3.14](image)

Figure 3.14: Modified mass square, \( M^+ \) and \( M^- \), as a function of \( \Delta y \) in \( p = 1 - 1.5 \text{ GeV/c} \) for (a) ++ field configuration and (b) -- field configuration. Momentum scale and \( \Delta y \) is determined for each magnetic configuration.

### 3.4.3 Invariant mass distribution of \( \phi \)

Since a \( \phi \) meson decays with \( \tau \sim 1/\Gamma = 1/4.26 \text{ (MeV}^{-1}\text{)} \sim 46 \text{ (fm/c)} \), we cannot directly measure \( \phi \) mesons. Thus, we identify \( \phi \) mesons via \( K^+ + K^- \) decay channel by using invariant mass technique. Charged kaons are identified with Time-Of-Flight detector and EM Calorimeter. We use relatively looser track and PID cuts (3 \( \sigma \)) for \( \phi \) analysis compared to \( \pi, K \) and \( p \) due to the low statistics of \( \phi \) mesons. Details of track and PID cuts by TOF and EMC for \( \phi \) mesons can be found in Section 3.5.
Invariant mass $m_{\text{inv}}$ is obtained by

$$m_{\text{inv}} = p_1 \cdot p_2 = E_1 E_2 - \mathbf{p}_1 \cdot \mathbf{p}_2$$

(3.45)

$$E = \sqrt{m_K^2 + p^2}, \quad \mathbf{p} = \sqrt{p_x^2 + p_y^2 + p_z^2}$$

(3.46)

$$\phi_{\text{pair}} = \tan^{-1}\left(\frac{p_{y,1} + p_{y,2}}{p_{x,1} + p_{x,2}}\right)$$

(3.47)

where $E$ is the total energy, $p$ is the momentum, $m_K = 0.4937$ GeV/$c^2$ is the mass of kaons, and $\phi_{\text{pair}}$ is the azimuthal angle of $p_T$ for $\phi$ meson.

Fig. 3.15 show the invariant mass distributions in minimum bias event. Since we calculate all possible combinations of $K^+$ and $K^-$ for a given event, there are always combinatorial background which do not originate from $\phi$ mesons. The combinatorial background contributions are subtracted with event mixing technique, where $K^+$ is taken from event $i$, and $K^-$ from event $j$ ($i \neq j$). Combinatorial background distributions are normalized for $m_{\text{inv}} = 1.2 - 1.3$ GeV/$c^2$ well separated from the $\phi$ meson peak.

Fig. 3.16 show the invariant mass distributions after subtracted combinatorial background from foreground. Signal distributions are fitted by the Breit-Wigner + constant functions;

$$\frac{dN}{dm_{\text{inv}}} = \frac{1}{2\pi} \frac{p_0 \cdot \Gamma^2}{(m_{\text{inv}} - m_0)^2 + \Gamma^2/4} + p_1$$

(3.48)

where $p_0$, $m_0$, $\Gamma$ and $p_1$ are the free parameters. Extracted $m_0$ is consistent with the expected value, $m_\phi = 1.019$ within the error bars. $\Gamma$ values are about 6 MeV, which is larger than expected width $\Gamma_\phi \sim 4$ MeV, due to the effect of mass resolution. We extract the $\phi$ meson yield by integrating within $\pm 12$ MeV around $\phi$ meson peak as indicated dashed red lines in the figure.
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Figure 3.15: Invariant mass distributions via $\phi \rightarrow K^+ + K^-$ for $p_T = 2 - 3$ GeV/c in minimum bias events. Event plane bins are divided into $-\pi < 2(\phi - \Psi) < -2\pi/3$, $-2\pi/3 < 2(\phi - \Psi) < -\pi/3$, $-\pi/3 < 2(\phi - \Psi) < 0$, $0 < 2(\phi - \Psi) < \pi/3$, $\pi/3 < 2(\phi - \Psi) < 2\pi/3$, and $-2\pi/3 < 2(\phi - \Psi) < \pi$ from top to bottom figures. Solid black and dashed red lines show the foreground and combinatorial background invariant mass distributions, respectively.
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Figure 3.16: Signal invariant mass distributions via $\phi \rightarrow K^+ + K^-$ after subtracted combinatorial background for $p_T = 2 - 3$ GeV/$c$ in minimum bias events. Solid blue lines show the fitting results by Breit-Wigner + constant background functions.
### 3.5 Cut Conditions

Table 3.2 summarizes the cut criteria used in the analysis.

<table>
<thead>
<tr>
<th>Cut Value</th>
<th>Global Event Selection</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trigger</td>
<td>Minimum bias</td>
</tr>
<tr>
<td>Z-vertex</td>
<td>BBC $</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Track Selection (Common)</th>
</tr>
</thead>
<tbody>
<tr>
<td>DC Track quality</td>
</tr>
<tr>
<td>DC $z_{ed}$</td>
</tr>
<tr>
<td>$p_T$ cut off</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Track Selection (TOF: $\pi$, $K$, $p$ and $d$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TOF energy loss</td>
</tr>
<tr>
<td>TOF matching cut</td>
</tr>
<tr>
<td>PID cut ($\pi$, $K$ and $p$)</td>
</tr>
<tr>
<td>(see Fig. 3.19)</td>
</tr>
<tr>
<td>PID cut ($d$)</td>
</tr>
<tr>
<td>$p_T$ cut off</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Track Selection ($\phi$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TOF</td>
</tr>
<tr>
<td>matching cut</td>
</tr>
<tr>
<td>PID cut</td>
</tr>
<tr>
<td>$p_T$ cut off</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>PbSc</th>
</tr>
</thead>
<tbody>
<tr>
<td>energy cut</td>
</tr>
<tr>
<td>PC3 + PbSc matching cut</td>
</tr>
<tr>
<td>PID cut</td>
</tr>
<tr>
<td>$p_T$ cut off</td>
</tr>
</tbody>
</table>
Figure 3.17: \( zed \) distribution, the \( z \) coordinate of the intersection point at DC. Dashed red lines show the \( z \) position where \( zed = \pm 75 \) cm.

Figure 3.18: Energy loss in TOF scintillators as a function of \( \beta \). Solid red line shows the energy loss cut, \( 0.0002 + 0.0014 \times \beta^{-2} \), to remove the background contributions with low energy loss.
Figure 3.19: Mass square vs charge \cdot momentum. Solid black lines represent the PID cuts for $\pi$, $K$ and $p$ ($2 \sigma$ PID + $2 \sigma$ veto cuts).

Figure 3.20: Mass square distribution for $K^+$ (red) and $K^-$ (blue) measured in EMC for $p = 0.6 - 0.9$ GeV/c.
3.6 Extraction of Elliptic flow, $v_2$

The magnitude of $v_2$'s are obtained with event plane method as introduced in Section 3.2. The azimuthal distributions are measured in central arm ($|\eta| < 0.35$) relative to the event plane measured at BBC ($|\eta| = 3.0 - 3.9$). The finite multiplicity fluctuation lead the dispersion of the event plane, so that the observed $v_2$'s are always smaller than the true $v_2$ values. We, therefore, need to correct the measured $v_2$ values with event plane resolution;

$$v_2 = \frac{v_2^{\text{obs}}}{\sigma_{EP}} \approx \frac{\langle \cos (2[\phi - \Psi_{2}^{\text{BBC}}]) \rangle}{\sqrt{2} \langle \cos (2[\Psi_{2}^{\text{BBCS}} - \Psi_{2}^{\text{BBCN}}]) \rangle}$$

(3.49)

where $v_2$ denote the true $v_2$ value, $v_2^{\text{obs}}$ is the observed $v_2$, and $\sigma_{EP}$ is the event plane resolution measured by the correlation between SOUTH and NORTH BBC. As we already discussed in Section 3.2.4, the event plane resolution of BBC can be well approximated by $\sqrt{2} \langle \cos (2\Delta\Psi) \rangle$.

In this section, we present the method to extract $v_2$ for $\pi$, $K$ and $p$ (Section 3.6.1), deuterons (Section 3.6.2) and $\phi$ meson (Section 3.6.3).

3.6.1 Extraction for $v_2(\pi)$, $v_2(K)$ and $v_2(p)$

![Figure 3.21: Centrality dependence of $dN/d(\phi - \Psi)$ distributions for $\pi^+ + \pi^-$ in $p_T = 0.8 - 1.2$ GeV/c. Vertical axis is scaled to see the difference of each centrality bin.](image)

Fig. 3.21 shows the centrality dependence of $dN/d(\phi - \Psi)$ distributions for $\pi^+ + \pi^-$ at $p_T = 0.8 - 1.2$ GeV/c. The measured $v_2$'s are extracted by fitting $dN/d(\phi - \Psi)$
distributions with Fourier expansion of azimuthal distributions;

\[
\frac{dN}{d(\phi - \Psi)} = N(1 + 2v_2^{obs} \cos (2(\phi - \Psi_{BBC})))
\]  

(3.50)

where \(N\) and \(v_2^{obs}\) are free parameters. For \(p_T > 2\ \text{GeV/c}\), \(\pi\) and \(K\) start to overlap each other we should consider the contaminations for each particle species. And we also need to take into account the background contribution for \(p_T > 3\ \text{GeV/c}\), arising from the DC tracking. These results and the estimated systematic error will be discussed in the Section 3.7.

### 3.6.2 Extraction for \(v_2(d)\)

The following two methods are used to extract deuteron \(v_2\):

- **Subtraction method**
- **Mass square fit method**

The signal extraction in mass square distribution, centrality and \(p_T\) dependence of \(S/B\) ratio for deuteron are shown below, then we discuss how to extract \(v_2\) from these methods.

![Deuteron mass square distribution in minimum bias events. From top to bottom, \(p_T\) range is 1.0 – 1.5 GeV/c, 1.5 – 2.0 GeV/c, 2.0 – 2.5 GeV/c, 2.5 – 3.0 GeV/c, and 3.0 – 4.0 GeV/c. Solid blue line represent the Gaussian + Exponential function for fitting deuteron peak and background. Dashed red line is the contribution from background, and dashed black line is signal distribution after subtract background contribution.](image-url)
Fig. 3.22 show mass square distribution of deuteron in minimum bias events with 2 \( \sigma \) TOF + PC3 matching cuts. Signal is extracted by fitting the distribution with gaussian + exponential, then we integrate the yield in \( 3 < M^2 < 4 \) (GeV/c\(^2\))^2.

![Figure 3.23: Signal to background ratio of deuterons as a function of \( p_T \) in each centrality.](image)

Signal to background ratio is obtained in each centrality and \( p_T \) bin, and is shown in Fig. 3.23. The yield of deuterons and \( S/B \) ratio are summarized in table 3.3 and used later for calculating signal \( v_2 \).
Table 3.3: Summary of centrality dependence of $S/B$ for $d + \bar{d}$

<table>
<thead>
<tr>
<th>$p_T$</th>
<th>Signal</th>
<th>Background</th>
<th>$S/B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>centrality 0 - 20 %</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$1 &lt; p_T &lt; 2$ GeV/c</td>
<td>$27804 \pm 166$</td>
<td>$8916 \pm 94$</td>
<td>$3.12 \pm 0.04$</td>
</tr>
<tr>
<td>$2 &lt; p_T &lt; 3$ GeV/c</td>
<td>$22206 \pm 149$</td>
<td>$3706 \pm 60$</td>
<td>$5.99 \pm 0.11$</td>
</tr>
<tr>
<td>$3 &lt; p_T &lt; 4$ GeV/c</td>
<td>$4869 \pm 69$</td>
<td>$1883 \pm 43$</td>
<td>$2.59 \pm 0.07$</td>
</tr>
<tr>
<td>centrality 20 - 40 %</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$1 &lt; p_T &lt; 2$ GeV/c</td>
<td>$16131 \pm 127$</td>
<td>$2257 \pm 47$</td>
<td>$7.14 \pm 0.16$</td>
</tr>
<tr>
<td>$2 &lt; p_T &lt; 3$ GeV/c</td>
<td>$10723 \pm 103$</td>
<td>$1292 \pm 35$</td>
<td>$8.30 \pm 0.24$</td>
</tr>
<tr>
<td>$3 &lt; p_T &lt; 4$ GeV/c</td>
<td>$1957 \pm 44$</td>
<td>$840 \pm 28$</td>
<td>$2.33 \pm 0.10$</td>
</tr>
<tr>
<td>centrality 40 - 60 %</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$1 &lt; p_T &lt; 2$ GeV/c</td>
<td>$6203 \pm 78$</td>
<td>$506 \pm 22$</td>
<td>$12.24 \pm 0.57$</td>
</tr>
<tr>
<td>$2 &lt; p_T &lt; 3$ GeV/c</td>
<td>$2800 \pm 52$</td>
<td>$322 \pm 17$</td>
<td>$8.69 \pm 0.51$</td>
</tr>
<tr>
<td>$3 &lt; p_T &lt; 4$ GeV/c</td>
<td>$432 \pm 20$</td>
<td>$203 \pm 14$</td>
<td>$2.12 \pm 0.18$</td>
</tr>
<tr>
<td>centrality 60 - 92 %</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$1 &lt; p_T &lt; 2$ GeV/c</td>
<td>$1237 \pm 35$</td>
<td>$76 \pm 8$</td>
<td>$16.25 \pm 1.92$</td>
</tr>
<tr>
<td>$2 &lt; p_T &lt; 3$ GeV/c</td>
<td>$398 \pm 19$</td>
<td>$51 \pm 7$</td>
<td>$7.73 \pm 1.15$</td>
</tr>
<tr>
<td>$3 &lt; p_T &lt; 4$ GeV/c</td>
<td></td>
<td>$-$</td>
<td></td>
</tr>
<tr>
<td>centrality 20 - 60 %</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$1 &lt; p_T &lt; 2$ GeV/c</td>
<td>$22324 \pm 149$</td>
<td>$2787 \pm 52$</td>
<td>$8.01 \pm 0.16$</td>
</tr>
<tr>
<td>$2 &lt; p_T &lt; 3$ GeV/c</td>
<td>$13522 \pm 116$</td>
<td>$1628 \pm 40$</td>
<td>$8.31 \pm 0.22$</td>
</tr>
<tr>
<td>$3 &lt; p_T &lt; 4$ GeV/c</td>
<td>$2389 \pm 48$</td>
<td>$1062 \pm 32$</td>
<td>$2.25 \pm 0.08$</td>
</tr>
<tr>
<td>Minimum bias</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$1 &lt; p_T &lt; 2$ GeV/c</td>
<td>$51462 \pm 226$</td>
<td>$11739 \pm 108$</td>
<td>$4.38 \pm 0.04$</td>
</tr>
<tr>
<td>$2 &lt; p_T &lt; 3$ GeV/c</td>
<td>$36149 \pm 190$</td>
<td>$5389 \pm 73$</td>
<td>$6.71 \pm 0.10$</td>
</tr>
<tr>
<td>$3 &lt; p_T &lt; 4$ GeV/c</td>
<td>$7282 \pm 85$</td>
<td>$3033 \pm 55$</td>
<td>$2.40 \pm 0.05$</td>
</tr>
</tbody>
</table>
Subtraction Method

In subtraction method, deuteron $v_2$ is extracted by the following equations:

$$v_2^{measured} = \frac{Sv_2^d + Bv_2^B}{S+B} = Rv_2^d + (1-R)v_2^B, \quad R = \frac{S}{S+B} \tag{3.51}$$

$$v_2^d = \left(1 + \frac{B}{S}\right)v_2^{measured} - \frac{B}{S}v_2^B \tag{3.52}$$

where $S(B)$ is the number of deuterons (background) in $M^2 = 3-4$ (GeV/c$^2$)$^2$, $v_2^{measured}$ is measured $v_2$, $v_2^B$ is background $v_2$ estimated in $1.5-3$ (GeV/c$^2$)$^2$ and $4-6$ (GeV/c$^2$)$^2$ mass square windows as shown by the shaded yellow area in Fig. 3.24, and $v_2^d$ is deuteron $v_2$. Measured $v_2$ is calculated by $\langle \cos 2(\phi_{lab} - \Psi) \rangle$, where $\phi_{lab}$ is the azimuthal angle of deuterons, $\Psi$ is combined BBC event plane, and bracket denote average over all events and tracks.

![Figure 3.24](image_url)

Figure 3.24: Measured deuteron $v_2$ is calculated in $M^2 = 3-4$ (GeV/c$^2$)$^2$ (solid blue line), and background $v_2$ are estimated in $M^2 = 1.5-3$ (GeV/c$^2$)$^2$, and $4-6$ (GeV/c$^2$)$^2$ (yellow histograms).

Fig. 3.25 show the centrality dependence of $v_2(d)$. For comparison, $v_2(\pi)$, $v_2(K)$, and $v_2(p)$ are plotted together with $v_2(d)$. In 40 – 60 % centrality bin, we cannot estimate the background $v_2$ values for $M^2 = 4-6$ (GeV/c$^2$)$^2$ due to the limited statistics.

Fig. 3.26 shows the deuteron $v_2$ after subtracting background contributions. Solid black circles and red triangles are extracted by using background $v_2$ estimated in $M^2 = 1.5-3$ (GeV/c$^2$)$^2$ and $4-6$ (GeV/c$^2$)$^2$, respectively.
Figure 3.25: Comparison of $v_2(p_T)$ for deuterons by subtraction method in different centrality classes. Solid blue circles represent the measured $v_2$, open black squares and red triangles are background $v_2$ estimated in $M^2 = 1.5 - 3$ (GeV/$c^2$)$^2$ and $4 - 6$ (GeV/$c^2$)$^2$, respectively.

Figure 3.26: Comparison of $v_2^d(p_T)$ by subtraction method in different centrality classes. Solid black circles and red triangles represent the $v_2(d)$ after subtracting the background $v_2$ which is estimated in $M^2 = 1.5 - 3$ and $4 - 6$ (GeV/$c^2$)$^2$, respectively.
Mass Square Fit Method

![Figure 3.27: (Left) $R$ (black) and $1 - R$ (red) as a function of mass square in $2 < p_T < 3$ GeV/c in minimum bias event. (Right) $v_2^{obs} = \langle \cos (2[\phi_{lab} - \Psi_{2}^{BBC}] \rangle$ as a function of mass square. Blue line shows the fitting result, and black and red dashed line show signal $(S/(S + B) \times v_2^d)$ and background $(B/(S + B) \times v_2^B)$ contribution.](image)

$v_2(d)$ are also measured by simultaneous fitting with both deuteron yield and $v_2$ as a function of mass square (Mass square fit method). In Fig. 3.27, signal $(R)$ and background ratio $(1 - R)$ are plotted as a function of mass square (left). Right figure shows measured uncorrected $v_2$ as a function of mass square. We perform fitting by using Eq. (3.51) with fixed deuteron and background yield, which are determined in Fig. 3.27. In this plot, background $v_2$ $(v_2^B)$ is assumed to be constant over all mass square. We have performed fitting by assuming linear function for background $v_2$ but the results are unchanged.

Fig. 3.28 shows comparison of deuteron $v_2$ between subtraction method and mass square fit method in different centrality selection. One can see the results of $v_2$ from two different method is consistent within statistical error bars.

Fig. 3.29 shows comparison of deuteron $v_2$ from mass square fit method between ”TOF + PC3 matching cuts” and ”TOF matching cuts only” in different centrality selection. The results are consistent with each other, except for lowest $p_T$ bin.

We choose the results from mass square fit method as final deuteron $v_2$ since this method takes into account mass square dependent $S/B$. The systematic error on $v_2(d)$ are estimated from the comparison between different cuts and methods. The evaluated errors are summarized in Section 3.7.3.
Figure 3.28: Comparison of $v_2^d(p_T)$ between subtraction (black circles) and mass square fit method (red triangles).

Figure 3.29: Comparison of $v_2^d(p_T)$ between mass square fit method with TOF + PC3 matching cuts (red triangles) and with TOF matching only (blue crosses).
3.6.3 Extraction for $v_2(\phi)$

Two different techniques are used to extract $\phi$ meson $v_2$, which is **Subtraction method**, and **Invariant mass fit method**. We introduce details of these method in following sections, and present the results from both methods.

**Event mixing and pair cuts**

Event mixing technique is used in order to evaluate the yield of $\phi$ meson. The following list is the standard event categories to make mixed events with similar global characteristics as real events:

- 20 bins for centrality (5 % step)
- 12 bins for $z$-vertex (5 cm step)
- 20 bins for event plane ($\pi/20$ step)

Event mixing are not performed across the different runs. Pair cuts are applied to remove ghost tracks in Drift Chamber. Intruder cuts are also used for TOF and PbSc to remove track merging effects. Here is the list of pair cuts.

- $|\Delta z_{DCH}| > 5.0$ cm, $|\Delta \phi_{DCH}| > 0.03$ rad
- $|\Delta R_{PC1}| > 7$ cm

**Pair flow coefficients, $v^\text{pair}_n$**

Recently, N. Borghini et al introduce model independent observables that describe the dependence in azimuth of two particle correlations [45]. According to [45] the probability distribution of a sample of pairs of particles in some range of $p_{T1}$, $p_{T2}$, $y_1$, $y_2$, $\Delta \phi^\text{pair} \equiv \phi_2 - \phi_1$ can be written

$$p(\phi^\text{pair} - \Psi) = \frac{1}{2\pi} \sum_{n=-\infty}^{+\infty} v^\text{pair}_n e^{in(\phi^\text{pair} - \Psi)}$$  \hspace{1cm} (3.53)$$

where $\Psi$ is the true (unknown) azimuth of reaction plane in the lab frame, $\phi^\text{pair}$ is the azimuthal angle of the total transverse momentum $p_{T1} + p_{T2}$, and $v^\text{pair}_n$ is ”pair-flow” coefficients defined as $v^\text{pair}_n = \langle e^{-in(\phi^\text{pair} - \Psi)} \rangle$, with the normalization $v^\text{pair}_0 = 1$. Eq. (3.53) can be replaced as

$$p(\phi^\text{pair} - \Psi) = \frac{1}{2\pi} \left( \sum_{n=-\infty}^{+\infty} [v^\text{pair}_{c,n} \cos (n[\phi^\text{pair} - \Psi]) + v^\text{pair}_{s,n} \sin (n[\phi^\text{pair} - \Psi])] \right)$$ \hspace{1cm} (3.54)$$

where the real coefficients $v^\text{pair}_{c,n} = \langle \cos (n[\phi^\text{pair} - \Psi]) \rangle$ and $v^\text{pair}_{s,n} = \langle \sin (n[\phi^\text{pair} - \Psi]) \rangle$ are related to the complex $v^\text{pair}_n$ by the relation $v^\text{pair}_n = v^\text{pair}_{c,n} - i v^\text{pair}_{s,n}$. In the particular case of $\phi$ meson, symmetry with respect to reaction plane for $\phi^\text{pair}$ implies $v^\text{pair}_{s,n} = 0$, except for experimental biases and fluctuations.
Subtraction Method

In $\phi \rightarrow K^+ + K^-$ analysis, $(K^+, K^-)$ pairs are sorted into bins of invariant mass $m_{inv}$, and pair azimuth $\phi_{pair}$ with respect to the event plane $\Psi$. Then, $v_2(\phi)$ can be extracted from the following steps;

1. **Count total number of $\phi$ yield**

   Counts number of $\phi$ yield in each invariant mass bin ($N_{\phi}(m_{inv})$) after separates it into an uncorrelated part ($N_b(m_{inv})$) centered around the expected mass;

   \[
   N_{pair}(m_{inv}) = N_{\phi}(m_{inv}) + N_b(m_{inv}) \quad (3.55)
   \]

   where $N_{pair}(m_{inv})$ is the number of pairs in each invariant mass bin. Background part is estimated by the event mixing technique. Yield is given by the integral of the correlated part $N_{\phi}(m_{inv})$ over $m_{inv}$.

2. **Repeat step 1 for each $\phi_{pair} - \Psi$ bin**

3. **Analyze pair flow coefficients**

   Pair flow coefficients $v_{c,n}^\phi$ can be extracted by fitting the azimuthal distribution of $\phi$ by the Eq. (3.50)

   Invariant mass distributions are shown in Fig. 3.15 and 3.16 (see Section 3.4.3). Extracted yield of $\phi$ meson are plotted for each $\phi_{pair} - \Psi$ bins, then we fit the distribution by the Fourier expansion of azimuthal distributions in Eq. (3.50). The extracted $\Delta \phi_{pair}$ distribution is plotted in Fig. 3.30 Dashed black lines are fitting results by Eq. (3.50).
Figure 3.30: Extracted $\Delta \phi_{\text{pair}}$ distribution of $\phi$ mesons for different $p_T$ selection minimum bias event. These are fitted by the Eq. (3.50), which is shown by the dashed black lines in the figure.
CHAPTER 3. DATA REDUCTION

Invariant Mass Fit Method

In subtraction method, the decomposition between the background and the peak is performed independently for several bins (typically, 10–20) in $\phi_{pair} - \Psi_{BBC}$. But in invariant mass fit method, the decomposition is only performed twice. First step is exactly same as step 1 in subtraction method except for not need to separate $\phi_{pair} - \Psi_{BBC}$ bins. Next step is analyzing pair flow coefficients from the Eq. (3.56);

$$N_{pair}(m_{inv})v_{c,n}(m_{inv}) = N_{b}(m_{inv})v_{c,n}^{(b)}(m_{inv}) + N_{\phi}(m_{inv})v_{c,n}^{\phi}$$

$$N_{pair}(m_{inv})v_{s,n}(m_{inv}) = N_{b}(m_{inv})v_{s,n}^{(b)}(m_{inv}) + N_{\phi}(m_{inv})v_{s,n}^{\phi} \quad (3.56)$$

As we mentioned in 3.6.3, $v_{s,n}^{pair} = 0$ if there are no experimental biases and fluctuations. If the background consists of uncorrelated particles, one also has $v_{s,n}^{(b)} = 0$. These can be used for checking the validity of the procedure.

Fig. 3.32 show invariant mass distribution, ratio of signal ($S$) and background yield ($B$) to total yield ($S + B$), and measured $v_2$. $v_{s,2} = \langle \sin(2[\phi_{pair} - \Psi_{B}^{BBC}]) \rangle$ (pink triangles) is also plotted in bottom figure. One can see that $v_{s,2}$ is zero for the mass window we are interested in. We perform the fitting by using Eq. (3.56), which is shown by black line in the figure. Background $v_2$ is assumed to be second polynominal function, i.e. $v_2^{b} = p_0 + p_1 m_{inv} + p_2 m_{inv}^2$. The shape of $S/(S + B)$ and $B/(S + B)$ are fixed, thus, number of free parameters for the fitting is 4, one is for measured $v_2$, and others ($p_0, p_1, p_2$) are for background $v_2$.

The results are shown in Fig. 3.33 after repeated the fitting for each ($p_T$, centrality) bins. Results from subtraction method are also plotted for comparison, and both results
CHAPTER 3. DATA REDUCTION

Figure 3.32: Top: Invariant mass distribution for $p_T = 2 - 3$ GeV/c in minimum bias event. Middle: The ratio $S/(S + B)$ (black) and $B/(S + B)$ (red) vs invariant mass. Bottom: measured $v_2 (= \langle \cos (2[\phi_{\text{pair}} - \Psi_{2}^{BBC}]) \rangle$) vs invariant mass. Black line shows the fitting result by Eq. (3.56). Red and blue line show the signal and background contributions from the fitting result.
are in good agreement within the statistical error bars.

For the stability check of invariant mass fit method, we perform fitting the following conditions:

- Fit background $v_2$ by linear function (default is quadratic function)
- Fit $v_2$ in $m_{inv} = 0.99 - 1.2$ GeV/$c^2$ (default is $m_{inv} = 0.99 - 1.1$ GeV/$c^2$)
- Fit $v_2$ by using histogram of $S/(S + B)$ and $B/(S + B)$ (default is Breit-Wigner + constant function)

The results are shown in Fig. 3.34. Largest difference of default invariant mass fit method is coming from subtraction method. The deviation from default fitting method is included in the final systematic error (see Section 3.7).

![Figure 3.33: Centrality dependence of $\phi$ meson $v_2(p_T)$ by invariant mass fit method.](image-url)
Figure 3.34: Comparison of $v_2(p_T)$ extracted by invariant mass fit method and subtraction method. Solid black circles are default invariant mass fit method, open red triangles are fit method with linear background $v_2$, open pink rhombus are fit method with $m_{inv} = 0.99 - 1.2$ GeV/c, open green crosses are fit method with histograms of S/B ratio, and solid blue triangles are subtraction method.
### 3.7 Systematic Uncertainties

#### 3.7.1 Systematic error of BBC event plane

Table 3.4: Summary of systematic error of BBC event plane

<table>
<thead>
<tr>
<th>Centrality</th>
<th>$\sigma_{EP}$</th>
<th>$\sigma_{Flat\text{-}procedure}$</th>
<th>$\sigma_{Flat\text{-}coefficient}$</th>
<th>$\sigma_{Run}$</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 - 5 %</td>
<td>10 %</td>
<td>17 %</td>
<td>1.9 %</td>
<td>20.4 %</td>
<td></td>
</tr>
<tr>
<td>5 - 10 %</td>
<td>2.1 %</td>
<td>11 %</td>
<td>1.9 %</td>
<td>12.4 %</td>
<td></td>
</tr>
<tr>
<td>10 - 15 %</td>
<td>3.3 %</td>
<td>4 %</td>
<td>0.8 %</td>
<td>7.2 %</td>
<td></td>
</tr>
<tr>
<td>15 - 20 %</td>
<td>3.5 %</td>
<td>2 %</td>
<td>0.8 %</td>
<td>6.5 %</td>
<td></td>
</tr>
<tr>
<td>20 - 30 %</td>
<td>3.2 %</td>
<td>1 %</td>
<td>5 %</td>
<td>6.0 %</td>
<td></td>
</tr>
<tr>
<td>30 - 40 %</td>
<td>2.5 %</td>
<td>1 %</td>
<td>0.6 %</td>
<td>5.7 %</td>
<td></td>
</tr>
<tr>
<td>40 - 50 %</td>
<td>1.2 %</td>
<td>1 %</td>
<td>1.3 %</td>
<td>5.4 %</td>
<td></td>
</tr>
<tr>
<td>50 - 60 %</td>
<td>1 %</td>
<td>2 %</td>
<td>2.1 %</td>
<td>5.9 %</td>
<td></td>
</tr>
<tr>
<td>60 - 90 %</td>
<td>2 %</td>
<td>13 %</td>
<td>7.6 %</td>
<td>16.0 %</td>
<td></td>
</tr>
<tr>
<td>0 - 10 %</td>
<td>3.9 %</td>
<td>17 %</td>
<td>1.9 %</td>
<td>18.2 %</td>
<td></td>
</tr>
<tr>
<td>10 - 20 %</td>
<td>3.2 %</td>
<td>4 %</td>
<td>1.9 %</td>
<td>7.4 %</td>
<td></td>
</tr>
<tr>
<td>0 - 20 %</td>
<td>2.1 %</td>
<td>4 %</td>
<td>1.0 %</td>
<td>6.8 %</td>
<td></td>
</tr>
<tr>
<td>20 - 40 %</td>
<td>3.0 %</td>
<td>1 %</td>
<td>5 %</td>
<td>5.9 %</td>
<td></td>
</tr>
<tr>
<td>40 - 60 %</td>
<td>1 %</td>
<td>2 %</td>
<td>1.3 %</td>
<td>5.6 %</td>
<td></td>
</tr>
<tr>
<td>20 - 60 %</td>
<td>2.9 %</td>
<td>2 %</td>
<td>0.5 %</td>
<td>6.1 %</td>
<td></td>
</tr>
<tr>
<td>0 - 50 %</td>
<td>2.2 %</td>
<td>2 %</td>
<td>0.5 %</td>
<td>5.8 %</td>
<td></td>
</tr>
<tr>
<td>Minimum bias</td>
<td>2.3 %</td>
<td>3 %</td>
<td>0.8 %</td>
<td>6.3 %</td>
<td></td>
</tr>
</tbody>
</table>

Systematic error of BBC event plane are summarized in Table 3.4. We evaluate the systematic error from (1) comparison of $v_2$ with respect to the different BBC event plane, such as BBC SOUTH, NORTH, and combined SOUTH + NORTH plane ($\sigma_{EP}$), (2) comparison of $v_2$ for different flattening procedure ($\sigma_{Flat\text{-}procedure}$), (3) comparison of $v_2$ for different number of flattening coefficients ($\sigma_{Flat\text{-}coefficient}$), and (4) comparison of $v_2$ for several Run group ($\sigma_{Run}$).

Fig. 3.35 show the $\langle v_2 \rangle$ as a function of centrality for different choice of BBC event plane. We estimate the systematic error from the difference between SOUTH, NORTH and combined plane for each centrality bin.

Fig. 3.36 shows the $\langle v_2 \rangle$ as a function of centrality for different flattening procedure in Run2 [46]. In Run2, we have performed the BBC event plane calibration by (1) Ring by ring gain correction, (2) Re-centering of flow vector by using measured average cosine and sine, and (3) Higher order fourier flattening. This procedure is denoted as standard in Fig. 3.36. In modified flattening procedure, instead of performing step 2 we remove the special 4 PMT’s from each BBC in order to make event plane resolution uniform in azimuthal direction. More details can be found in [46], section 1.

The error $\sigma_{Flat\text{-}coefficient}$ comes from the fact that current calibration constants for event plane are not good enough to make event plane flat. This is because the calibration
Figure 3.35: Average $v_2$ as a function of centrality for $\pi^+ + \pi^-$. Integration is performed in $0.2 < p_T < 4$ GeV/c. Open circles show $v_2$ with BBC SOUTH event plane, solid triangles are NORTH plane, and open crosses are combined SOUTH and NORTH event plane.

Figure 3.36: Average $v_2$ as a function of centrality for non-identified hadrons. Integration is performed in $0.2 < p_T < 5$ GeV/c. See text and [46] for the details of flattening procedure.
was done for relatively smaller data sets (∼300 k events for each run). We compare the result between current calibration constants in data base and improved flattening calibration by David Winter, which is done by using entire data sets. We find that 5% difference of \( v_2 \) for non-identified charged hadrons almost independent on centrality.

Fig. 3.37 shows the comparison of \( v_2 \) for different run group with improved flattening corrections as we mention in previous paragraph. We divide Run-4 data sets into 8 group to see any difference between them. Each group has approximately 100 M event except for group 7 (∼50 M event). Run range is listed below for each group;

- Group 0 : 109189 < RUN < 112124
- Group 1 : 112128 < RUN < 114927
- Group 2 : 114929 < RUN < 116425
- Group 3 : 116427 < RUN < 117297
- Group 4 : 117303 < RUN < 118024
- Group 5 : 118028 < RUN < 119440
- Group 6 : 119448 < RUN < 120416
- Group 7 : 120419 < RUN < 122223

The systematic error is about 1 – 2% for 0 – 60%, and 8% for 60 – 92% centrality bin.

Figure 3.37: (Left) Average \( v_2 \) for charged hadrons as a function of centrality. Integrated \( p_T \) range is \( 0.5 - 5 \) GeV/c. We divide entire data sets to 7 run groups, G0–G7 in order to see the stability of \( v_2 \). (Right) The ratio of \( \langle v_2 \rangle \) as a function of centrality for several run groups.
3.7.2 Systematic error on $v_2(\pi)$, $v_2(K)$, and $v_2(p)$

The source of systematic error on $v_2$ for $\pi$, $K$, and $p$ are evaluated from: (1) Cut criteria, (2) Feed down effect on proton $v_2$ (upper systematic error, see the section below) (3) Background (upper systematic error), and (4) Mis-identification of particles.

Systematic error of cut criteria

We have studied $v_2$ by changing several cuts, such as matching cut, PID cut, and energy loss cut, in reasonable range to see the effect of each cut. We finally found that the difference of $v_2$ from each cut is about 1\%, thus total systematic errors from the cut criteria is $\sqrt{0.01^2 \times 3} \sim 1.7\%$.

Systematic error of feed down effect on proton $v_2$

We have done very detailed study of feed down effect on proton $v_2$. The effect of feed down from $\Lambda$ decay has been studied by assuming that the observed proton $v_2$ is composed of $v_2$ for direct protons with that for protons from $\Lambda$ feed down. We further assume that the $v_2$ for direct proton is equal to that for protons. The $v_2$ for protons from $\Lambda$ feed down has been estimated by decay kinematics with measured $\Lambda$ transverse momentum spectra and $v_2$. Since the decay protons has smaller $p_T$ compared to the original $\Lambda$ and the magnitude of $v_2$ is approximately proportioanl to $p_T^2$ for baryons, $v_2$ for decay protons is always larger than that of measured protons. Thus, we only add the upper systematic error from $\Lambda$ feed down. We found that the effect of feed down on proton $v_2$ is about 11\% independent on $p_T$, up to 1.5 GeV/c. For $p_T > 1.5$ GeV/c, the contribution of $v_2$ from $\Lambda$ feed down is negligible.

Systematic error of background contribution

From the analysis for charged hadron transverse momentum spectra [48], background contribution was found to be dominant in high $p_T$ ($p_T \geq 4 - 5$ GeV/c). Below $p_T = 3$ GeV/c, the background contribution is very small ($\sim 1\%$) in charged hadron $v_2$ analysis. However, for $4 < p_T < 5$ GeV/c ($p_T > 5$ GeV/c), we found that 8\% (17\%) correction is needed from background contributions [47]. We study background contribution for $\pi, K, p$ separately since the background contribution and those $v_2$ might be different for different particle species. The procedure how to extract the signal $v_2$ is similar to that in [47], i.e.

1. Fitting $\Delta R$ distribution by Gaussian + $2^{nd}$ polynomial, and then fix the shape of signal and background ($\Delta R \equiv \sqrt{(\Delta \phi)^2 + (\Delta z)^2}$).

2. Fit $v_2$ vs $\Delta R$ by Eq. (3.57), and extract $v_2^S$

$$v_2^{\text{measured}} = \frac{N_S v_2^S + N_B v_2^B}{N_S + N_B} \quad (3.57)$$

$$v_2^S = \frac{N_S + N_B v_2^{\text{measured}} - N_B v_2^B}{N_S} \quad (3.58)$$
Figure 3.38: (Left) TOF $\Delta R$ distribution for $\pi$ (top), $K$ (middle), and $p$ (bottom) at $p_T = 2.5 - 3$ GeV/c in minimum bias event. Solid blue lines represent the fitting result, and dashed black (red) lines are signal (background) contribution. (Right) Measured $v_2$ vs $\Delta R$ for $\pi$, $K$, and $p$. Solid blue lines are the result of fitting by Eq. (3.57). Dashed black (red) lines are $v_2^S \times N^S/(N^S + N^B)$ ($v_2^B \times N^B/(N^S + N^B)$).
Fig. 3.38 demonstrates $v_2$ extraction from $\Delta R$ distribution. We confirm that the result are unchanged by using the different background shape, such as $3^{rd}$ or $4^{th}$ polynomial, for the fitting of $\Delta R$ and $v_2$.

![Graph showing $v_2$ as a function of $p_T$ in minimum bias event for $\pi$, $K$, and $p$. Open triangle is background $v_2$ ($v_2^B$), open cross is measured $v_2$ ($v_2^{\text{measured}}$), and solid circle is corrected $v_2$ ($v_2^S$).]

Figure 3.39: $v_2$ as a function of $p_T$ in minimum bias event for $\pi$, $K$, and $p$. Open triangle is background $v_2$ ($v_2^B$), open cross is measured $v_2$ ($v_2^{\text{measured}}$), and solid circle is corrected $v_2$ ($v_2^S$).

Fig. 3.39 show the result of $v_2$ with background correction for $\pi$, $K$, and $p$. Since the background $v_2$ is usually smaller than observed $v_2$ due to the smearing of track or random association, corrected $v_2$ become larger than measured $v_2$. One can see such a trend for all particles species in the measured $p_T$ range. Thus, we take upper systematic error from background contributions.

Fig. 3.40 shows the systematic error of background contribution as a function of $p_T$ in different centrality bins. Systematic error is larger in higher $p_T$ bin as we expected, but the magnitude of error is relatively smaller than that of non-identified hadrons. This is mainly because $S/B$ ratio is enhanced by requiring the particle identification in TOF.

Table 3.5 shows the summary of systeamic error of background contribution for $\pi$, $K$, and $p$.

**Systematic error of mis-identification**

It is important to understand the contribution of mis-identified particles, especially for higher $p_T$. Because of the finite timing resolution particles become overlapping each other for high $p_T$ in mass square. For example, $\pi$ and $K$ cannot be well separated for $p_T > 2 \text{ GeV/c}$.

Fig. 3.41 demonstrates how to evaluate the yield of contamination for each particle species in minimum bias events. We perform fitting the mass square distribution by (3+1) gaussian, where the extra gaussian is used for background. Then we compare
Figure 3.40: Systematic error from background contribution for $\pi^+ + \pi^-$ (left), $K^+ + K^-$, $p + \bar{p}$ in $0 - 60\%$ centrality bin, and minimum bias.

Table 3.5: Summary of background systematic error for $\pi$, $K$ and $p$. In $p_T = 2 - 2.5$ GeV/c, 1\% systematic error is added for $\pi$, $K$ and $p$.

<table>
<thead>
<tr>
<th>Centrality (%)</th>
<th>$\pi$ (%)</th>
<th>$K$ (%)</th>
<th>$p$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_T$ (GeV/c)</td>
<td>2.5 - 3</td>
<td>3 - 4</td>
<td>2.5 - 3</td>
</tr>
<tr>
<td>0 - 5%</td>
<td>3%</td>
<td>7%</td>
<td>3%</td>
</tr>
<tr>
<td>5 - 10%</td>
<td>1%</td>
<td>6%</td>
<td>2%</td>
</tr>
<tr>
<td>10 - 15%</td>
<td>1%</td>
<td>4%</td>
<td>1%</td>
</tr>
<tr>
<td>15 - 20%</td>
<td>1%</td>
<td>3%</td>
<td>1%</td>
</tr>
<tr>
<td>20 - 30%</td>
<td>1%</td>
<td>2%</td>
<td>1%</td>
</tr>
<tr>
<td>30 - 40%</td>
<td>1%</td>
<td>3%</td>
<td>1%</td>
</tr>
<tr>
<td>40 - 50%</td>
<td>1%</td>
<td>2%</td>
<td>1%</td>
</tr>
<tr>
<td>50 - 60%</td>
<td>1%</td>
<td>1%</td>
<td>1%</td>
</tr>
<tr>
<td>60 - 92%</td>
<td>1%</td>
<td>21%</td>
<td>1%</td>
</tr>
<tr>
<td>0 - 10%</td>
<td>2%</td>
<td>7%</td>
<td>3%</td>
</tr>
<tr>
<td>10 - 20%</td>
<td>1%</td>
<td>4%</td>
<td>1%</td>
</tr>
<tr>
<td>0 - 20%</td>
<td>2%</td>
<td>5%</td>
<td>2%</td>
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<tr>
<td>20 - 40%</td>
<td>1%</td>
<td>2%</td>
<td>1%</td>
</tr>
<tr>
<td>40 - 60%</td>
<td>1%</td>
<td>2%</td>
<td>1%</td>
</tr>
<tr>
<td>20 - 60%</td>
<td>1%</td>
<td>2%</td>
<td>1%</td>
</tr>
<tr>
<td>0 - 50%</td>
<td>1%</td>
<td>4%</td>
<td>2%</td>
</tr>
<tr>
<td>Minimum bias</td>
<td>1%</td>
<td>4%</td>
<td>1%</td>
</tr>
</tbody>
</table>
Figure 3.41: Mass square distribution for $p_T = 2.5 - 3$ GeV/$c$ (left), and $p_T = 3 - 4$ GeV/$c$ (right) in minimum bias event. Solid black line shows fitting result by 4 gaussian (3 for $\pi$, $K$ and $p$, 1 for background). We perform fitting by fixing mean value of $\pi$, $K$, $p$ mass square, and constrain $\sigma$ to the expected width in the measured $p_T$ range. Dashed red, green, blue lines are the contribution of each $\pi$, $K$, and $p$. Shaded grey histograms show the yield of each particle with 2 $\sigma$ PID cuts. The yellow histograms are the contamination for each particle.

Figure 3.42: Systematic error from mis-identification for $\pi^\pm$ (left), $K^\pm$ (middle) and $p(\bar{p})$ (right) in each centrality bin.
extracted gaussian of each particle species (dashed line in the figure) with measured particles with 2 $\sigma$ PID cuts (grey histograms), and evaluate the contamination (yellow histograms). The contribution from contamination are negligible below 2.5 GeV/$c$, so we do not take into account the systematic error below that $p_T$ range. The contamination for each particle species are evaluated for each centrality bin.

The systematic error on $v_2$ from the contribution of contaminations are mostly centrality independent and the magnitude of errors are 1 % for 0 – 60 % centrality bins, and 2 % for 60 – 92 % (see Fig. 3.42).

### 3.7.3 Systematic error on $v_2$ for $d + \bar{d}$, and $\phi$

**Systematic error on $v_2$ for $d + \bar{d}$**

Fig. 3.43 shows the ratio of $v_2$ for deuterons as a function of $p_T$ from different method and cuts. The systematic error values are calculated for each ($p_T$, centrality) bin by taking the quadratic sum of deviation of $v_2$ value.

![Graphs showing the ratio of $v_2$ for deuterons as a function of $p_T$ for different centrality bins.](image)

Figure 3.43: The ratio of $v_2$ for deuterons as a function of $p_T$ for different centrality bins. The denominator is $v_2$ values from mass square fit method with TOF + PC3 matching cuts.

**Systematic error on $v_2$ for $\phi$**

The systematic error on $\phi$ meson $v_2$ are estimated from Fig. 3.34 and summarized in Table 3.7.
### Table 3.6: Summary of the relative systematic error on $d + \bar{d} \nu_2$ for each ($p_T$, centrality)

<table>
<thead>
<tr>
<th>Centrality (%)</th>
<th>$p_T$ (GeV/c)</th>
<th>1 - 1.5</th>
<th>1.5 - 2</th>
<th>2 - 2.5</th>
<th>2.5 - 3</th>
<th>3 - 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 - 20 %</td>
<td>37.7 (%)</td>
<td>3.0 (%)</td>
<td>13.7 (%)</td>
<td></td>
<td></td>
<td>13.7 (%)</td>
</tr>
<tr>
<td>20 - 40 %</td>
<td>82.9 (%)</td>
<td>41.4 (%)</td>
<td>20.0 (%)</td>
<td>18.4 (%)</td>
<td>12.9 (%)</td>
<td></td>
</tr>
<tr>
<td>40 - 60 %</td>
<td>7.4 (%)</td>
<td>2.4 (%)</td>
<td>26.8 (%)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20 - 60 %</td>
<td>72.4 (%)</td>
<td>20.3 (%)</td>
<td>11.9 (%)</td>
<td>19.1 (%)</td>
<td>8.3 (%)</td>
<td></td>
</tr>
<tr>
<td>Minimum bias</td>
<td>53.4 (%)</td>
<td>24.4 (%)</td>
<td>13.7 (%)</td>
<td>8.8 (%)</td>
<td>14.5 (%)</td>
<td></td>
</tr>
</tbody>
</table>

### Table 3.7: Summary of the relative systematic error on $\phi \nu_2$ for each ($p_T$, centrality)

<table>
<thead>
<tr>
<th>Centrality (%)</th>
<th>$p_T$ (GeV/c)</th>
<th>1 - 2</th>
<th>2 - 3</th>
</tr>
</thead>
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<tr>
<td>0 - 20 %</td>
<td>34.9 (%)</td>
<td>44.7 (%)</td>
<td></td>
</tr>
<tr>
<td>20 - 40 %</td>
<td>12.6 (%)</td>
<td>6.8 (%)</td>
<td></td>
</tr>
<tr>
<td>40 - 60 %</td>
<td>10.6 (%)</td>
<td>12.3 (%)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Centrality (%)</th>
<th>1 - 1.5</th>
<th>1.5 - 1.9</th>
<th>1.9 - 2.4</th>
<th>2.4 - 3.5</th>
</tr>
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<tbody>
<tr>
<td>20 - 60 %</td>
<td>74.5 (%)</td>
<td>6.9 (%)</td>
<td>3.2 (%)</td>
<td>23.4 (%)</td>
</tr>
<tr>
<td>Minimum bias</td>
<td>137.5 (%)</td>
<td>2.3 (%)</td>
<td>8.2 (%)</td>
<td>21.1 (%)</td>
</tr>
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</table>
Chapter 4

Experimental Results

In this chapter, we present the experimental results of $v_2$ for identified hadrons in Au + Au collisions at $\sqrt{s_{NN}} = 200$ GeV. In Section 4.1, transverse momentum dependence of $v_2$ for $\pi$, $K$ and $p$ in minimum bias trigger are shown and the validity of our results are examined. Centrality dependence of $v_2(p_T)$ for $\pi$, $K$ and $p$ are shown in Section 4.2. The results of average $v_2$ and their evaluation are presented in Section 4.3. And in Section 4.4, the results of $v_2$ for deuterons and $\phi$ mesons are shown and compared to that for other hadrons.

4.1 Transverse momentum dependence of $v_2$ for $\pi$, $K$ and $p$ in Minimum bias events

In Run4, the total number of events is about 600 M events, by a factor 20 larger compared to Run2 (30 M events). This significant increase of statistics enable us to study the detail centrality dependence of $v_2$ for identified hadrons as we will show in the next section.

Fig. 4.1 shows $v_2$ for $\pi$, $K$ and $p$ as a function of transverse momentum $p_T$ in minimum bias events. Comparing the Run2 results as already shown in Fig. 1.9, we can extend the $p_T$ reach for $\pi$ and $p$ up to 4 GeV/c by estimating the contamination for each particle carefully. And we can also significantly reduce the statistical error with the high statistics Run-4 data set. For $p_T < 2$ GeV/c, measured $v_2$ values are smaller for heavier particle species, i.e., $v_2(\pi) > v_2(K) > v_2(p)$. For $p_T > 2$ GeV/c, however, $v_2(\pi)$ and $v_2(K)$ are saturated while $v_2(p)$ is still increasing (and saturating) with $p_T$, i.e., $v_2(p) > v_2(\pi) \sim v_2(K)$.

As we discussed in 3.2.1, azimuthal distributions related to the event plane are symmetric with $2(\phi_{lab} - \Psi_2^{BBC}) \rightarrow -2(\phi_{lab} - \Psi_2^{BBC})$, so that the $\langle \sin (2[\phi_{lab} - \Psi_2^{BBC}]) \rangle$ values are expected to be zero. If there are experimental bias and fluctuations, it could be deviated from zero. Thus, one could use $\langle \sin (2[\phi_{lab} - \Psi_2^{BBC}]) \rangle$ values for checking the validity of the analysis. As one can see that the $\langle \sin (2[\phi - \Psi_2^{BBC}]) \rangle$ for all particle species are zero in Fig. 4.1, we confirm that our procedure to extract $v_2$ with respect to the BBC event plane are correct.
Figure 4.1: $v_2$ as a function of $p_T$ for $\pi$, $K$ and $p$ in minimum bias events. Shaded bands indicates systematic errors for each particle. Open symbols around $v_2 = 0$ represent $\langle \sin (2[\phi_{lab} - \Psi_2^{BBC}]) \rangle$.

Figure 4.2: Comparison between Run4 and previous Run2 results of $v_2(p_T)$ in minimum bias events. Open symbols show the results obtained in Run2 [14]. Only statistical errors are shown.
The consistency are also checked by comparing the previous Run2 results. Fig. 4.2 shows comparison of the $v_2(p_T)$ between Run4 and Run2 [14] (same as Fig. 1.9) in minimum bias events. One can see that the both results agrees well within the error bars.

### 4.2 Centrality dependence of $v_2(\pi, K, p)$

Centrality dependence of $v_2(p_T)$ and $\langle \sin (2[\phi_{lab} - \Psi_{BBC}^B] \rangle$ values for $\pi$, $K$ and $p$ are shown in Fig. 4.3 - 4.5. Average sine values are consistent with zero for all particle species and all centrality bins as we expected. Clear increase of $v_2(p_T)$ can be observed for all particle species, that is, the magnitude of $v_2$ increase with increasing centrality.

Fig. 4.6 - 4.8 show the ratio of $v_2$ for negative to positive particles as a function of $p_T$. One can see that the ratio of $v_2$ for $\pi$ and $K$ is consistent with unity for all centrality bins. On the other hand, $v_2(\bar{p})/v_2(p)$ is smaller than unity for mid-central collisions.

In order to see the difference of $v_2$ between positive and negative particles, the average values of the ratio’s are 0.992 ± 0.002, 1.001 ± 0.006, and 0.966 ± 0.005 for $\pi$, $K$ and $p$ respectively. The average value for $p$ is $3.4 \pm 0.5$ % smaller than 1 in the measured $p_T$ range. There are several sources which induce the difference of $v_2$ between anti-proton and proton;

- Feed-down decays from resonances ($\Lambda, \Xi$ etc)
- Baryon transport or baryon stopping in the very beginning of collisions.

The effect of $\Lambda$ feed down decays on proton $v_2$ has been studied by a fast MC simulation. In this simulation, we have assumed that total yield of protons is the sum of direct protons and protons from feed down decays of $\Lambda$, and that $v_2$ for direct protons are equal to that of anti-protons. We have also assumed the $v_2$ for $\Lambda$ and $\bar{\Lambda}$ is equal. Resulting $v_2$ for $p$ and $\bar{p}$ can be expressed as

$$v_2(p) = (1 - R_f)v_2(p)^{dir} + R_f v_2(p)^{feed} \quad (4.1)$$

where $R_f$ is the fraction of protons from $\Lambda$ feed down in all measured protons, $v_2(p)^{dir}$ is the $v_2$ for direct protons, and $v_2(p)^{feed}$ is the $v_2$ for protons from $\Lambda$ feed down decays. The value of $R_f$ is taken from [10]. We have added 10 % uncertainties in the $v_2$ for direct protons, and also taken into account the systematic error from $R_f$ ($\sim 12$ %).

Top panel in Fig. 4.10 shows the $v_2$ for $p$ and $\bar{p}$ from the fast MC simulation. We found that the difference of $v_2$ between $p$ and $\bar{p}$ can be explained by the feed down decays from $\Lambda$ due to the large systematic errors from direct protons and $R_f$. In our current assumptions, $R_f$ is the only source to cause the difference between $p$ and $\bar{p}$. However, $v_2$ for $\Lambda$ and $\bar{\Lambda}$ could be different from each other. It is difficult to study more quantitatively how the $v_2$ for $p$ and $\bar{p}$ are different due to the large systematic uncertainties for $v_2$ of direct proton and for $R_f$. 
Figure 4.3: $v_2$ vs $p_T$ for different centrality bins for $\pi^+$ (solid circles) and $\pi^-$ (open circles). Yellow band around $v_2 = 0$ shows the absolute systematic errors. $\langle \sin (2[\phi_{lab} - \Psi_2^{BBC}]) \rangle$ are also plotted by grey symbols.
Figure 4.4: $v_2$ vs $p_T$ for different centrality bins for $K^+$ (solid squares) and $K^-$ (open squares). Yellow band around $v_2 = 0$ shows the absolute systematic errors. $\langle \sin (2[\phi_{lab} - \Psi^{BBC}_2]) \rangle$ are also plotted by grey symbols.
Figure 4.5: $v_2$ vs $p_T$ for different centrality bins for $p$ (solid triangles) and $\bar{p}$ (open triangles). Yellow band around $v_2 = 0$ shows the absolute systematic errors. $\langle \sin (2[\phi_{lab} - \Psi_2^{BBC}]) \rangle$ are also plotted by grey symbols.
Figure 4.6: Ratio of $v_2 (v_2^-/v_2^+)$ vs $p_T$ for different centrality bins. Solid lines show the average values of the ratios in $p_T = 0.5 - 3$ GeV/$c$, and yellow bands show the statistical error from the fitting. Only statistical errors are shown.
Figure 4.7: Ratio of $v_2 (v_2(K^-)/v_2(K^+))$ vs $p_T$ for different centrality bins. Solid lines show the average values of the ratios in $p_T = 0.5 - 3$ GeV/$c$, and yellow bands show the statistical error from the fitting. Only statistical errors are shown.
Figure 4.8: Ratio of $v_2$ ($v_2(\bar{p})/v_2(p)$) vs $p_T$ for different centrality bins. Solid lines show the average values of the ratios in $p_T = 0.5 - 3$ GeV/c, and yellow bands show the statistical error from the fitting. Only statistical errors are shown.
Figure 4.9: Average values of the ratio of $v_2$ in $p_T = 0.5 - 3$ GeV/c as a function of $N_{\text{part}}$. The average ratio value on $v_2(p)$ is shown by light blue bands. $R$ values in the figure denote the fitting results of the ratio over entire $N_{\text{part}}$ for each particle.

One can estimate the $v_2$ for net proton by assuming that the $v_2$ is same for the pair producing proton and anti-proton. The $v_2$ for the net protons is given by

$$v_2^{\text{net}}(p) = \frac{1}{N_p - N_{\bar{p}}}(N_p v_2(p) - N_{\bar{p}} v_2(\bar{p})).$$

Fig. 4.11 shows the extracted $v_2$ for the net proton. We found that $v_2$ for the net proton is $9 \pm 5\%$ larger than that of $\bar{p}$ in $5 - 40\%$ centrality. Baryon transport is one of the scenario which lead to the difference of $v_2$ between $p$ and $\bar{p}$. Larger $v_2$ for baryons may evolve with the multiple scattering through they are transported to mid-rapidity. While $v_2$ is typically smaller in forward rapidity, it is unknown what the mechanism produce large $v_2$ in that very short time scale.
Figure 4.10: (Top) $v_2$ as a function of $p_T$ for $p$ (red circles) and $\bar{p}$ (blue triangles) calculated from $v_2$ from direct proton and protons from $\Lambda$ feed down decays. Solid black lines (and open black circles) is the input $v_2$ for direct protons. Yellow bands include the systematic error which is estimated by allowing for 10% variation for $v_2$ of direct protons. (Bottom) Ratio of $v_2(\bar{p})/v_2(p)$ as a function of $p_T$. Dashed lines represent 10% systematic errors from direct protons, and systematic errors are assumed to be uncorrelated. We do not plot the additional 12% systematic error from $R_f$ (see text).
Figure 4.11: (Left) Comparison of $v_2$ for net proton, $p$ and $\bar{p}$ as a function of $N_{\text{part}}$. Average $v_2$ is extracted in $p_T = 0.5 - 4$ GeV/c. (Right) The ratio of $v_2$ ($v_2(\text{net } p)/v_2(\bar{p})$) as a function of $N_{\text{part}}$. Dashed lines represent the fitting results of the ratio by a straight line. Only statistical errors are shown.

### 4.3 Average $v_2$, $\langle v_2 \rangle$, for $\pi$, $K$ and $p$

Average $v_2$ over measured $p_T$ range, which we denote $\langle v_2 \rangle$, of $\pi$, $K$ and $p$ are calculated as

$$
\langle v_2 \rangle = \frac{\int_0^{p_T} dN/dp_T \times v_2(p_T)}{\int_0^{p_T} dN/dp_T} = \frac{\sum_i dN_i/dp_T \times v_2^i(p_T)}{\sum_i dN_i/dp_T}
$$

(4.3)

where $dN/dp_T$ is the transverse momentum distributions, and $v_2(p_T)$ is the differential $v_2$ as a function of $p_T$. Since we measure both $p_T$ spectra and $v_2$ in the limited $p_T$ range ($0.2 < p_T < 4$ GeV/c for $\pi$, $0.3 < p_T < 3$ GeV/c for $K$, $0.5 < p_T < 4$ GeV/c for $p$), the integral in Eq. (4.3) are replaced to the sum of data points as one see in the $3^{rd}$ term. We estimate $v_2$ and $dN/dp_T$ for lower $p_T$ region by extrapolating the fitting results (see below) to $p_T \rightarrow 0$. Higher $p_T$ range are also extrapolated for both $v_2$ and $dN/dp_T$ but they do not contribute the $\langle v_2 \rangle$ for all particle species, thus we just integrate the results up to the maximum of measured $p_T$.

Fig. 4.12 – 4.14 show transverse momentum spectra for $\pi$, $K$ and $p$ as a function of centrality [10]. We parameterize the $p_T$ spectra by the following functions:

$$
f_\pi(p_T) = A \cdot \left( \frac{p_0}{p_T + p_0} \right)^n \quad \text{(for } \pi)$$

(4.4)

$$
f_K(p_T) = A \cdot e^{-m_T/p_T} \quad \text{(for } K)$$

(4.5)

where $A$, $p_0$, $n$, and $T$ are the free parameters. For protons, blast-wave model (Eq. (1.13)) is used to parameterize their $p_T$ spectra. Different parameterizations of $p_T$ spectra are also used to evaluate the systematic error on $\langle v_2 \rangle$, i.e. the $p_T$ spectra for $\pi$
Figure 4.12: (Left) $p_T$ spectra for $\pi^+ + \pi^-$ from central (top) to peripheral (bottom) collision. Solid black lines represent power-law fitting results by Eq. (4.4). (Right) The ratio of data to fitting results as a function of $p_T$.

Figure 4.13: (Left) $p_T$ spectra for $K^+ + K^-$ from central (top) to peripheral (bottom) collision. Solid black lines represent $m_T$ exponential fitting results by Eq. (4.5). (Right) The ratio of data to fitting results as a function of $p_T$. 
and $p$ are fitted by $m_T$ exponential function, and that for $K$ is fitted by $p_T$ exponential function.

Fig. 4.15 shows the fitting results of $v_2(\pi)$, $v_2(K)$ and $v_2(p)$ as a function of $p_T$ for difference centrality classes. To extrapolate the data to low and high $p_T$, we use the following functions

$$f(p_T) = a \cdot n_q \left( \frac{1}{1 + e^{-(p_T/n_q-b)/c}} - \frac{1}{1 + e^{b/c}} \right)$$  \hfill (4.6)

$$f(p_T) = a \cdot p_T^n$$  \hfill (4.7)

where $a$, $b$, $c$ and $n$ are the free parameters, and $n_q$ is the number of quarks for each hadrons (2 for mesons, 3 for baryons). The empirical formula from Eq. (4.6) well describe the $p_T$ dependence of $v_2$ from low to intermediate $p_T$ [18]. As we already mentioned, the contributions from high $p_T$ does not change the final results of $\langle v_2 \rangle$, Eq. (4.7) gives reasonable estimate of $v_2$ at low $p_T$.

Fig. 4.16 shows the results of $\langle v_2 \rangle$ as a function of $N_{part}$. In the measured $p_T$ range, measured $v_2$ values are used to calculate $\langle v_2 \rangle$, and in the lower $p_T$ range where we don’t have the data points of $v_2$ extrapolated values from Eq. (4.6) is used. One can see $\langle v_2 \rangle$ values are increasing in smaller $N_{part}$ region (peripheral events) and are saturating in $N_{part} < 100$.

Fig. 4.17 shows the ratio of $\langle v_2 \rangle$ as a function of $N_{part}$ for $\pi$, $K$ and $p$. Systematic errors are evaluated from the maximum difference of $\langle v_2 \rangle$ for each centrality. Systematic
Figure 4.15: $v_2(p_T)$ for $\pi$, $K$ and $p$ in different centrality bins. Fitting results by Eq. (4.6) and (4.7) are plotted in left and right panels, respectively.
Figure 4.16: \( N_{\text{part}} \) dependence of \( \langle v_2 \rangle \) for \( \pi \), \( K \) and \( p \).

Figure 4.17: Ratio of \( \langle v_2 \rangle \) as a function of \( N_{\text{part}} \) for different parameterizations of \( v_2(p_T) \) and \( p_T \) spectra.
errors are summarized in Table 4.1. We assume the flat distribution of $\langle v_2 \rangle$ for each centrality bin and then evaluate the systematic error as

$$\sigma^{sys}(\langle v_2 \rangle) = \frac{2 \times |D_{max}|}{\sqrt{12}} \quad (4.8)$$

where $|D_{max}|$ is the maximum difference of between the measured $\langle v_2 \rangle$ and the reference $\langle v_2 \rangle$.

![Graph](image)

Figure 4.18: $\langle v_2 \rangle$ as a function of $N_{part}$ for $\pi$ (left), $K$ (middle) and $p$ (right) with absolute systematic error (yellow bands around $v_2 = 0$). Systematic error is calculated by quadratic sum of systematic error from the different procedures to extract $\langle v_2 \rangle$ and errors from Section 3.7.
Table 4.1: Summary of systematic error on $\langle v_2 \rangle$. $|D_{max}|$ is given by the maximum difference of $\langle v_2 \rangle$.

<table>
<thead>
<tr>
<th>Centrality</th>
<th>$\pi^+ + \pi^-$</th>
<th>$K^+ + K^-$</th>
<th>$p + \bar{p}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$</td>
<td>D_{max}</td>
<td>$</td>
</tr>
<tr>
<td>0 - 5 %</td>
<td>0.0016</td>
<td>0.1 %</td>
<td>0.0791</td>
</tr>
<tr>
<td>5 - 10 %</td>
<td>0.0048</td>
<td>0.3 %</td>
<td>0.1052</td>
</tr>
<tr>
<td>10 - 15 %</td>
<td>0.0106</td>
<td>0.6 %</td>
<td>0.0953</td>
</tr>
<tr>
<td>15 - 20 %</td>
<td>0.0137</td>
<td>0.8 %</td>
<td>0.1053</td>
</tr>
<tr>
<td>20 - 30 %</td>
<td>0.0202</td>
<td>1.2 %</td>
<td>0.1033</td>
</tr>
<tr>
<td>30 - 40 %</td>
<td>0.0319</td>
<td>1.8 %</td>
<td>0.1010</td>
</tr>
<tr>
<td>40 - 50 %</td>
<td>0.0370</td>
<td>2.1 %</td>
<td>0.1000</td>
</tr>
<tr>
<td>50 - 60 %</td>
<td>0.0397</td>
<td>2.3 %</td>
<td>0.1008</td>
</tr>
<tr>
<td>60 - 92 %</td>
<td>0.0477</td>
<td>2.8 %</td>
<td>0.1011</td>
</tr>
</tbody>
</table>

4.4 Centrality and $p_T$ dependence of $v_2$ for $d + \bar{d}$ and $\phi$

Fig. 4.19 shows $v_2$ as a function of $p_T$ for $d + \bar{d}$ in minimum bias events. For $p_T < 2$ GeV/c, the magnitude of $v_2$ for deuterons is smaller than that of other hadrons. This trend of $v_2$ is consistent with the picture of hydrodynamic calculation with common velocity field, that is $v_2$ of heavier particles become smaller for a given $p_T$. For $p_T > 2$ GeV/c, on the other hand, one can see that $v_2(d)$ is consistent with $v_2$ for other hadrons.

Fig. 4.20 shows $v_2$ as a function of $p_T$ for $\phi$ mesons in minimum bias events. The magnitude of $v_2$ for $\phi$ mesons is smaller than that of other hadrons in $p_T < 1.5$ GeV/c, while $v_2$ is consistent with that for other hadrons in $p_T > 1.5$ GeV/c.

Fig. 4.21 shows centrality dependence of $v_2(d)$ and $v_2(\phi)$ as a function of $p_T$. Similar trend of $v_2$ are observed for the data in 20 % centrality steps as in minimum bias events, while the statistical errors are large especially for higher $p_T$ bins.
Figure 4.19: $v_2$ as a function of $p_T$ for $d + \bar{d}$ in minimum bias events. $v_2$ of $\pi$, $K$ and $p$ are also plotted for comparison. Yellow bands show the systematic error on $v_2(d)$.

Figure 4.20: $v_2$ as a function of $p_T$ for $\phi$ mesons in minimum bias events. $v_2$ of $\pi$, $K$ and $p$ are also plotted for comparison. Yellow bands show the systematic error on $v_2(\phi)$. 
Figure 4.21: Comparison of $v_2(p_T)$ between $d + \bar{d}$ and $\phi$ in different centrality bins. Yellow bands show the systematic error on $v_2(d)$, and light blue bands around $v_2 = 0$ show the systematic error on $v_2(\phi)$. 
Chapter 5

Discussions

In the previous chapter, we have found that:

- Mass ordering of $v_2$, i.e. $v_2$ for heavier particles is larger for a given $p_T$ at $p_T < 2$ GeV/$c$, which is consistent with the hydrodynamical model calculations.

- For $p_T > 2$ GeV/$c$, $v_2$ for heavier particle species, such as protons, deuterons are larger compared to those for mesons.

- $v_2$ increases from central to peripheral collisions. Centrality dependence of $v_2$ is qualitatively consistent with that of the eccentricity.

In Section 5.1, we discuss degree of thermalization and its relation between spatial eccentricity and elliptic flow more quantitatively with the eccentricity estimated by the Monte Carlo simulation with Glauber model. In Section 5.2, the sensitivity of initial density profile to measured $v_2$ is discussed by using extended Blast-wave parameterization. Finally, in Section 5.3, we discuss the partonic collectivity with the quark number scaling of $v_2$.

5.1 Eccentricity Scaling of $v_2$

As we discussed in Section 1.3.3, final momentum anisotropy is sensitive to the degree of thermalization. In peripheral collisions, where the emitted particles can escape from the overlap zone without interacting with other particles, the system is close to free streaming. In the case of free streaming (collisionless limit: $\lambda \to \infty$), the magnitude of elliptic flow is \cite{49}

$$\langle v_2 \rangle \propto \frac{\langle \varepsilon \rangle dN}{\langle S \rangle dy}$$

(5.1)

where $\langle \varepsilon \rangle$ is the spatial eccentricity (several definitions of eccentricity can be used, see Appendix A.2), $\langle S \rangle$ is the transverse area of collision zone, and $dN/dy$ is the rapidity density of total (charged + neutral) produced particles. Bracket $\langle \rangle$ for $v_2$, $\varepsilon$ and $S$ denote the average over all events for all particles. In this free streaming limit, the ratio of $\langle v_2 \rangle / \langle \varepsilon \rangle$ linearly increase with the transverse number density $(1/ \langle S \rangle dN/dy)$. Since
$dN/dy$ increases in more central collisions, the ratio of $\langle v_2 \rangle / \langle \varepsilon \rangle$ increases with more central events for a fixed collision system.

The transverse number density can be related to the degree of thermalization, i.e. the Knudsen number $K$ [50]

$$
\frac{1}{K} = \frac{R}{\lambda} = R\sigma n(\tau) \sim \frac{c_\sigma}{c} \frac{dN}{dS dy}
$$

(5.2)

where $\lambda$ is the mean free path of produced particles, $R$ is the transverse size of the system, $\sigma$ is the cross section among the produced particles, $n(\tau)$ is the number density of produced particles at "time" $\tau$, and $c$, $c_s$ are the speed of light and speed of sound, respectively. The above equation is valid for $\tau \leq R/c_s$, where the transverse size of the system does not change significantly [50]. In the hydrodynamical limit: $\lambda \to 0$, that is $1/K \gg 1$, the ratio $\langle v_2 \rangle / \langle \varepsilon \rangle$ becomes constant [51]. The relation between the measured $v_2$ and Knudsen number can be given by [50]

$$
\frac{\langle v_2 \rangle}{\langle \varepsilon \rangle} = \frac{\langle v_2^h \rangle}{\langle \varepsilon \rangle} \frac{1}{1 + K/K_0}
$$

(5.3)

where $\langle v_2^h \rangle$ is the limiting values of $v_2$ when $K \to 0$, which corresponds to the $v_2$ from hydrodynamics, and $K_0$ is the parameter of order unity, whose precise value can only be determined through a transport calculation.

Therefore, the possible thermalization achieved in the early stage of heavy ion collisions could be investigated with detailed comparison between spatial eccentricity in initial coordinate space and elliptic flow in final momentum space.

First, we discuss the eccentricity scaling of $\langle v_2 \rangle$ (average $v_2$) by using standard eccentricity among the Au + Au and Cu + Cu collisions. Second, we introduce the more accurate definitions of eccentricity (participant eccentricity), which is taking into account the fluctuation of positions for participating nucleons, and discuss the scaling relation between $\langle v_2 \rangle$ and participant eccentricity in Au + Au and Cu + Cu collisions. Last, we discuss the difference of eccentricity between different averaging and compare the results of scaled $\langle v_2 \rangle$ for different definitions of eccentricity.

### 5.1.1 $\langle v_2 \rangle$ in Au + Au and Cu + Cu Collisions and Standard Eccentricity

Fig. 5.1 shows the $\langle v_2 \rangle$ for charged hadrons as a function of $N_{part}$ in Au + Au and Cu + Cu collisions at $\sqrt{s_{NN}} = 200$ GeV. The behavior of $\langle v_2 \rangle$ is qualitatively similar to that of eccentricity, that is the magnitude of $\langle v_2 \rangle$ is smaller for larger $N_{part}$ and $\langle v_2 \rangle$ in Au + Au is larger than that of Cu + Cu for a given $N_{part}$ (see Fig. A.3).

In order to study the relation between the $\langle v_2 \rangle$ and the eccentricity more quantitatively, the eccentricity scaling of $v_2$ is examined for both Au + Au and Cu + Cu collisions. We first use standard eccentricity $\langle \varepsilon_{\text{std}} \rangle$, which is defined by the fixed axes in the coordinate space, as defined in Eq. (A.7) and (A.8). Fig. 5.2 show the $N_{part}$ dependence of the ratio $\langle v_2 \rangle / \langle \varepsilon_{\text{std}} \rangle$. One can clearly see that the scaled $\langle v_2 \rangle$ rapidly
Figure 5.1: $\langle v_2 \rangle$ for charged hadrons as a function of $N_{\text{part}}$ in Au + Au and Cu + Cu collisions (corresponding centrality range is 0 - 60%) at $\sqrt{s_{\text{NN}}} = 200$ GeV. Average $v_2$ values are obtained by integrating $v_2(p_T)$ over $p_T = 0.2 - 5.0$ GeV/$c$. Yellow and light blue bands represent the systematic errors on $v_2$ in Au + Au and Cu + Cu, respectively.

Figure 5.2: $\langle v_2 \rangle / \langle \varepsilon_{\text{std}} \rangle$ as a function of $N_{\text{part}}$ in Au + Au and Cu + Cu collisions. Solid (open) data points show the results of $\langle v_2 \rangle$ divided by $\langle \varepsilon_{\text{std}} \rangle$ weighting with $N_{\text{part}}$ ($N_{\text{coll}}$) density profile.
increase around $N_{\text{part}} \sim 100$ especially for Cu + Cu and $\langle v_2 \rangle / \langle \varepsilon_{\text{std}} \rangle$ is not scaled among the different colliding systems. Since $\langle \varepsilon_{\text{std}} \rangle$ is defined with the fixed axes in coordinate space, it does not take into account the event-by-event position fluctuations of participating nucleons in principle. Such fluctuations lead to the difference between fixed axes in the reference frame and the principal axes determined by the participant nucleons. Therefore, $\langle \varepsilon_{\text{std}} \rangle$ could underestimate the magnitude of eccentricity. The scaling breaks between Au + Au and Cu + Cu with $\langle \varepsilon_{\text{std}} \rangle$ suggest that such fluctuations are dominant especially for Cu + Cu collisions.

Such a behavior is also found for $\langle \varepsilon_{\text{coll}} \rangle$ as shown by the open data points in Fig. 5.2. One also see relatively smaller $\langle v_2 \rangle / \langle \varepsilon_{\text{coll}} \rangle$ compared to $\langle v_2 \rangle / \langle \varepsilon_{\text{part}} \rangle$. This is simply because the steeper $N_{\text{coll}}$ density distribution gives larger eccentricity (about $1.5 - 2$ times larger in most central, close to unity in most peripheral, see Fig. A.7 and A.8).

### 5.1.2 Eccentricity Scaling of $\langle v_2 \rangle$ with Participant Eccentricity

![Figure 5.3: Comparison of $\langle \varepsilon \rangle$ as a function of impact parameter in Au + Au. Red circles, black triangles and blue crosses are $\langle \varepsilon_{\text{std}} \rangle$, $\langle \varepsilon_{\text{var}} \rangle$, and $\langle \varepsilon_2 \rangle$, respectively. Eccentricity is calculated with $N_{\text{part}}$ weight.](image)

To take into account the position fluctuations of participant nucleons, we calculate $\langle \varepsilon_{\text{var}} \rangle$ (**participant eccentricity**) which is defined by the axes determined with the distributions of participant nucleons. We also consider $\langle \varepsilon_2 \rangle$ (**event plane eccentricity**) which is basically same as $\langle \varepsilon_{\text{var}} \rangle$ but is defined by subtracting auto-correlation between participating nucleons and the event plane. Details definitions of these $\langle \varepsilon \rangle$ can be found in Appendix A.2.
Fig. 5.3 compare the eccentricity as a function of impact parameter in Au + Au collisions. $\langle \varepsilon_{\text{std}} \rangle$ becomes zero at most central since the radius of x and y is equal. $\langle \varepsilon_{\text{std}} \rangle$ increase with increasing impact parameter, and become decreasing from $b \geq 2R_{\text{Au}}$ due to the effect of surface diffuseness. On the other hand, $\langle \varepsilon_{\text{var}} \rangle$ and $\langle \varepsilon_2 \rangle$ have finite value even at most central since they include the event-by-event position fluctuations. Because $\langle \varepsilon_{\text{var}} \rangle$ is defined by the event plane determined with the participating nucleons, it becomes unity in the limit of $N_{\text{part}} \rightarrow 1$ due to the auto-correlation. For $\langle \varepsilon_2 \rangle$, the effect of auto-correlation is strongly suppressed at most peripheral, and the magnitude is approaching to the $\langle \varepsilon_{\text{std}} \rangle$.

![Graph](image)

Figure 5.4: Eccentricity scaling of $\langle v_2 \rangle$ as a function of $N_{\text{part}}$. (a) with $\langle \varepsilon_{\text{var}} \rangle$, (b) with $\langle \varepsilon_2 \rangle$. Note that the scale of vertical axis is different from Fig. 5.2.

Fig. 5.4 show the $\langle v_2 \rangle/\langle \varepsilon_{\text{var}} \rangle$ (left) and $\langle v_2 \rangle/\langle \varepsilon_2 \rangle$ (right) as a function of $N_{\text{part}}$. As one can see that eccentricity scaling works for Au + Au and Cu + Cu with $\langle \varepsilon_{\text{var}} \rangle$ as shown in left figure. However, if one takes $\langle \varepsilon_2 \rangle$, eccentricity scaling seems to break at most central Cu + Cu bin. There might be several reasons why the eccentricity scaling of $\langle v_2 \rangle$ with $\langle \varepsilon_2 \rangle$ breaks; first there may be remaining auto-correlations in the measured $\langle v_2 \rangle$, and second we may oversubtract auto-correlations from $\langle \varepsilon_{\text{var}} \rangle$.

5.1.3 $\langle \varepsilon \rangle$ vs $\varepsilon\{2\}$

We measured the magnitude of $v_2$ by the event plane method, $v_2\{EP_2\} = \langle \cos 2[\phi - \Psi] \rangle$, which is equivalent to that by two-particle azimuthal correlations between emitted particles $v_2\{2\} = \sqrt{\cos 2[\phi_1 - \phi_2]}$ [52]. Both methods yield $v_2\{EP_2\} \approx v_2\{2\} = \sqrt{\langle v_2^2 \rangle}$ when $v_2$ fluctuate event-by-event. If we assume $v_2$ in each event is proportional to $\varepsilon$, one expects $v_2\{EP_2\} \approx v_2\{2\} = \sqrt{\langle \varepsilon^2 \rangle}$ [53]. Therefore, $\varepsilon\{2\} \equiv \sqrt{\langle \varepsilon^2 \rangle}$ is thought to be more natural choice of eccentricity compared to the $\langle \varepsilon \rangle$, as long as $v_2$ is determined by the event plane determined with the emitted particles from the participant nucleons.

Fig. 5.5 show eccentricity scaling of $\langle v_2 \rangle$ with $\varepsilon\{2\}$ as a function of $N_{\text{part}}$. Since the magnitude of $\varepsilon\{2\}$ is almost independent of the definitions of eccentricity (see Fig. A.5 and A.6), scaled $\langle v_2 \rangle$ is almost same with different definitions of eccentricity for a given
Figure 5.5: Comparison of $\langle v_2 \rangle / \varepsilon \{2\}$ as a function of $N_{part}$. From left to right figures, $\langle \varepsilon_{std} \rangle$, $\langle \varepsilon_{var} \rangle$, and $\langle \varepsilon_2 \rangle$ are used for scaling, respectively. Grey data points show the $\langle v_2 \rangle / \langle \varepsilon \rangle$ (standard averaging).

centrality. One can also see that the $\langle v_2 \rangle / \varepsilon \{2\} < \langle v_2 \rangle / \langle \varepsilon \rangle$ (grey data points), which is due to $\varepsilon \{2\} > \langle \varepsilon \rangle$ (see Fig. A.5 and A.6). In our current systematic error on $\langle v_2 \rangle$ and $\varepsilon$, it is difficult to conclude which is the best quantity to explain the scaling of $\langle v_2 \rangle$.

For all definitions of eccentricity, scaled $v_2$ is relatively smaller and flatter by $\langle \varepsilon \rangle$ weighting with $N_{coll}$ density profile. Since the hydrodynamics is scale invariant, $\langle v_2 \rangle / \langle \varepsilon \rangle$ becomes independent of the system size if the complete local thermal equilibrium is established. Therefore, flatter $\langle v_2 \rangle / \langle \varepsilon \rangle$ with $N_{coll}$ density might suggest that the local thermal equilibrium is attained, although there is a possibility that this flatter behavior of $\langle v_2 \rangle / \langle \varepsilon \rangle$ is just a coincidence.
5.2 Interpretation with Blast-wave Picture

The elliptic flow could be one of the most sensitive signatures in the early stage of heavy ion collisions. Therefore, detailed comparison between measured $v_2$ and hydrodynamical model can shed light on the thermodynamic bulk properties of the system and the collision dynamics.

Blast-wave model parameterization is similar to the freeze-out configuration obtained from hydrodynamical model but the physical parameters of the configuration are treated as free parameters. It is widely used to describe the temperature and common velocity field at the kinetic freeze-out by comparing with the measured single particle spectra and $v_2$. However, one could expect $v_2$ is more sensitive to the early stage before the kinetic freeze-out takes place so that the transverse momentum spectra and $v_2$ may have different sensitivities to the dynamics in the heavy ion collisions.

In this section, we show the independent fitting results of single particle spectra and $v_2$ by extended Blast-wave model, and discuss the extracted parameters and its relation of collision dynamics.

5.2.1 Extended Blast-wave Model

In order to study the sensitivity of initial density profile to measured $v_2$, we perform the fitting with Blast-wave parameterization. In the standard Blast-wave framework, density distribution is assumed to be constant and velocity profile is linear as a function of radius in the transverse directions. In our extended Blast-wave model, we use more realistic density and velocity distributions inspired from the Hydrodynamical model. That is, the density distribution is determined by the initial overlap density ($N_{part}$ or $N_{coll}$ density profile), and the velocity distribution is calculated by its gradient distributions. As we already discussed in Section 5.1, $N_{coll}$ density profile is more closely related to the degrees of thermalization, one could expect that $v_2$ may be described in $N_{coll}$ density distributions than in $N_{part}$ density profile.

The basic assumptions of our Blast-wave model are listed below;

- Longitudinal boost invariance
- An instant freeze-out takes place just after the collision on a hyper-surface $\Sigma$ at a proper time $\tau$
- An instant freeze-out is independent of $r$ at $\tau = \text{const.}$
  - Use $N_{part}$ or $N_{coll}$ overlap density distributions from Glauber model
  - Initial spatial eccentricity $\varepsilon$ is fixed by initial overlap geometry, and $\varepsilon$ is calculated by

$$
\varepsilon = \frac{\sum_i N_i (y_i^2 - x_i^2)}{\sum_i N_i (x_i^2 + y_i^2)}
$$

(5.4)

where $N_i$ is the number density ($N_{part}$ or $N_{coll}$) at (x, y), and $x_i, y_i$ are the x and y positions for $i^{th}$ participant.
• Radial boost velocity is assumed to be proportional to the density gradient distributions
  
  - The magnitude of radial boost velocity is fixed by the magnitude of density gradient
  
  - The boost direction (azimuthal angle of boost) is fixed by the direction of density gradient, thus the azimuthal dependence of boost $\beta_n$ ($n^{th}$ fourier harmonics, $n = 2, 4, ...$) is automatically determined by the gradient of density distributions. $\beta_n$ is obtained by

$$\beta_n = \frac{\sum_i N_i G_i \cos (n \phi_i)}{\sum_i N_i G_i}$$

where $G_i$ is the gradient of the density profile at $(x, y)$ and $\phi_i$ is the azimuthal angle with respect to the reaction plane for $i^{th}$ participant. We only consider $\beta_2$ since the higher harmonics ($n \geq 4$) is negligible.

• No chemical freeze-out

• Freeze-out temperature is independent of the position for particles $(x, y)$, $T(x, y) = T$, that is kinetic freeze-out takes place for all hadrons at the same time

There are 2 free parameters in our model: 1) freeze-out temperature ($T$), and 2) surface radial boost velocity ($\beta_T$).

![N_{part} density, 20 - 30 % centrality](image1)

![N_{part} gradient, 20 - 30 % centrality](image2)

Figure 5.6: (Left) $N_{part}$ density distribution in 20 - 30 % centrality. (Right) $N_{part}$ gradient distribution in 20 - 30 % centrality.

Fig. 5.6 and 5.7 show $N_{part}$ and $N_{coll}$ density distributions (left), gradient of density (right) in 20 - 30 % centrality, respectively used in our model. One can see that the
width of $x$ direction in $N_{coll}$ distribution is smaller than that of $N_{part}$ distribution. Since the density is larger in the smaller $r$, number of collisions for each participant nucleon become larger in small $r$. That is why $N_{coll}$ distribution become steeper compared to the $N_{part}$ distribution.

Fig. 5.8 show the $N_{part}$ and $N_{coll}$ density distributions and gradient arrows in 20 - 30 % centrality bin. As one already sees in Fig. 5.6 and 5.7, the steeper $N_{coll}$ distribution lead to larger gradient, i.e. larger velocity, for a given ($x$, $y$) position compared to $N_{part}$ distributions. The direction of arrows are used as the azimuthal angles of boost direction.

Fig. 5.9 show the projections of $N_{part}$ density and its gradient as a function of $r$ for several centrality classes. One can see that the density distributions in in-plane are steeper than out-of-plane and that trend is more visible for peripheral collisions.

The more details of our Blast-wave parameterization are described in Appendix C.
Figure 5.8: (a) $N_{\text{part}}$ density distributions, and (b) $N_{\text{coll}}$ density distributions in 20 - 30% centrality. The direction of arrows indicates the direction of gradient, and the length of arrows is the magnitude of gradient. The maximum value of gradient is normalized to 1 in these figures.

Figure 5.9: (Left) Centrality dependence of projections of $N_{\text{part}}$ density distributions into x-axis (solid red) and y-axis (dashed black). (Right) Centrality dependence of projections of $N_{\text{part}}$ gradient distributions, i.e. velocity distributions, into x-axis (solid red) and y-axis (dashed black).
5.2.2 Fitting Results

Figure 5.10: Fitting results for \( \pi, K \) and \( p \) by Blast-wave model in 20 – 30 % centrality. Single particle spectra and \( v_2 \) are fitted independently. Fitting is performed by minimizing \( \chi^2 \) for \( \pi, K \) and \( p \) simultaneously. Left and right panel show the results with \( N_{\text{part}} \) and \( N_{\text{coll}} \) density, respectively. Yellow bands represents the systematic error on \( v_2(p) \), and green dashed lines are the systematic error on \( v_2(K) \). Solid lines on the data points represent the fitting results in the \( p_T \) range denoted in Table 5.1, and doted lines extrapolate fitting results in lower and higher \( p_T \) range.

Fig. 5.10 show the fitting results for both spectra and \( v_2 \) in 20 – 30 % centrality bin by Blast-wave model. The data points of \( p_T \) spectra is taken from [10], and results of \( v_2 \) are obtained in this thesis. Fitting is performed for \( \pi, K \) and \( p \) simultaneously. Fitting range for each particle species are summarized in Table 5.1. Higher \( p_T \) values are determined by requiring \( m_T - m_0 < 1 \text{ GeV/c} \). We exclude the low \( p_T \) pions from spectra because there are significant contributions from resonance decays. Minimization of \( \chi^2 \) is performed including the systematic error. The systematic error on \( v_2(\pi) \) are not plotted in the figure but the magnitude of the error is comparable to that of \( K \). One can see that the fitting for \( \pi \) is not so good especially by using \( N_{\text{part}} \) density distribution, so we also fit the spectra and \( v_2 \) for \( K \) and \( p \) excluding \( \pi \) as shown in Fig. 5.11.

Table 5.2 summarize the extracted parameters. Average radial flow velocity, \( \langle \beta_T \rangle \), is
Figure 5.11: Fitting results for $K$ and $p$ by Blast-wave model in 20 – 30 % centrality.

Table 5.1: Fitting range used in the Blast-wave fit.

<table>
<thead>
<tr>
<th>Particle species</th>
<th>spectra</th>
<th>$v_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi$</td>
<td>0.5 - 1.13 GeV/c</td>
<td>0.2 - 1.13 GeV/c</td>
</tr>
<tr>
<td>$K$</td>
<td>0.4 - 1.40 GeV/c</td>
<td>0.3 - 1.40 GeV/c</td>
</tr>
<tr>
<td>$p$</td>
<td>0.6 - 1.70 GeV/c</td>
<td>0.5 - 1.70 GeV/c</td>
</tr>
</tbody>
</table>
Table 5.2: Summary of extracted $T$, $\langle \beta_T \rangle$ and $\chi^2$/NDF in 20 – 30 % centrality.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$\pi + K + p$</th>
<th>$K + p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density profile</td>
<td>$N_{part}$</td>
<td>$N_{coll}$</td>
</tr>
<tr>
<td>Statistical error only : spectra fit</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T$ (MeV)</td>
<td>114.5 ± 1.9</td>
<td>103.2 ± 1.5</td>
</tr>
<tr>
<td>$\langle \beta_T \rangle$</td>
<td>0.432 ± 0.002</td>
<td>0.506 ± 0.003</td>
</tr>
<tr>
<td>$\chi^2$/NDF</td>
<td>491.8/52 = 9.5</td>
<td>83.2/52 = 1.6</td>
</tr>
<tr>
<td>Statistical error only : $v_2$ fit</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T$ (MeV)</td>
<td>213.2 ± 2.4</td>
<td>311.8 ± 0.7</td>
</tr>
<tr>
<td>$\langle \beta_T \rangle$</td>
<td>0.450 ± 0.005</td>
<td>0.582 ± 0.001</td>
</tr>
<tr>
<td>$\chi^2$/NDF</td>
<td>4761.1/25 = 1836.2/25 = 70.5/16 = 82.7/16 = 5.2</td>
<td>190.4</td>
</tr>
<tr>
<td>Include systematic error on $v_2$ for $\chi^2$ minimization</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T$ (MeV)</td>
<td>209.4 ± 2.4</td>
<td>305.5 ± 4.4</td>
</tr>
<tr>
<td>$\langle \beta_T \rangle$</td>
<td>0.442 ± 0.005</td>
<td>0.578 ± 0.004</td>
</tr>
<tr>
<td>$\chi^2$/NDF</td>
<td>179.5/25 = 7.2</td>
<td>64.5/25 = 2.6</td>
</tr>
</tbody>
</table>

calculated by integrating the gradient distribution weighted with density profile. Several features are listed below.

- **Temperature**
  - For spectra, consistent values are obtained from both $N_{part}$ and $N_{coll}$ density.
  - For $v_2$, temperature in $N_{coll}$ density is about 100 MeV larger than $N_{part}$ density.
  - Temperatures obtained from $v_2$ fitting are about 100 - 200 MeV higher than that from spectra.

- **$\langle \beta_T \rangle$**
  - Larger $\langle \beta_T \rangle$ in $N_{coll}$ density for both spectra and $v_2$.

- **$\chi^2$/NDF**
  - Better $\chi^2$/NDF in $N_{coll}$ density profile for both spectra and $v_2$.
  - Better $\chi^2$/NDF if one exclude $\pi$ from fitting.

As one can see that the results obtained from $K$ and $p$ fit give always better $\chi^2$/NDF compared to that from $\pi$, $K$ and $p$ fit. Larger temperature from the fitting of $v_2$ could suggest that the magnitude of $v_2$ saturate in early time compared to the time scale of kinetic freeze-out which can be obtained by the spectra fit only. In order to see
Figure 5.12: 3 \( \sigma \) \( \chi^2 \) contour for temperature vs average radial flow velocity with \( N_{part} \) density profile. Dashed and solid black line show the 1 and 2 \( \sigma \) contour, and solid red (blue) line represent 3 \( \sigma \) contour line for \( v_2 \) (spectra).

Figure 5.13: 3 \( \sigma \) \( \chi^2 \) contour for temperature vs average radial flow velocity with \( N_{coll} \) density profile.
the difference of temperature and radial flow velocity between spectra and $v_2$ visually, contour lines of $\chi^2$ are plotted in temperature with respect to the radial flow velocity.

Fig. 5.12 and 5.13 show the $3 \sigma$ contour lines for temperature with respect to the average radial flow velocity for both spectra (blue) and $v_2$ (red) with $N_{\text{part}}$ and $N_{\text{coll}}$ density profile. The contours of $\chi^2$ are obtained from $\pi$, $K$ and $p$ fitting. Slightly different minimum $\chi^2$ positions are due to the finite bin size of temperature and $\langle \beta_T \rangle$. One clearly sees that the minimum position of $\chi^2$ is quite different between spectra and $v_2$ for both results.

From the next section, we only consider the results from $K$ and $p$ fitting since the shape of $v_2(p_T)$ for $\pi$ from Blast-wave model does not well reproduce the data, and it gives better $\chi^2$/NDF for both spectra and $v_2$ compared to that from $\pi$, $K$ and $p$ fitting.

### 5.2.3 Sensitivity to the Eccentricity

In order to study the effective dynamical evolution of the system, the eccentricity value is varied by expanding the initial density distribution given from Glauber model. For the sake of simplicity, we only consider the expansion of x-direction, i.e. the direction of reaction plane. Since the pressure gradient is largest in that direction, this simple expansion gives naively correct picture of the system evolution.

![Figure 5.14](image)

**Figure 5.14**: Transverse length dependence of eccentricity and $\beta_2$ in 20−30 % centrality (left). Right figure shows the ratio of $\varepsilon$ to $\beta_2$ as a function of $((R) - \langle R_0 \rangle)/\langle R_0 \rangle$.

Fig. 5.14 shows the eccentricity weighted by $N_{\text{part}}$ (solid red circles) and $N_{\text{coll}}$ (solid red triangles) as a function of normalized transverse length. $\langle R \rangle$ variable is defined as

$$\frac{1}{\langle R \rangle^2} = \frac{1}{\langle x^2 \rangle} + \frac{1}{\langle y^2 \rangle}$$

(5.6)
\( \langle R \rangle \) is more natural choice than the rms radius of the system since \( v_2 \) is driven by the pressure gradient [50]. \( \langle R_0 \rangle \) value is the average transverse length in the initial density profile given by the Glauber model.

![Figure 5.15: Temperature \( T \) as a function of \( \langle R \rangle - \langle R_0 \rangle \)/\( \langle R_0 \rangle \) obtained from \( N_{\text{coll}} \) density profile. Solid red and open black symbols show the results from spectra and \( v_2 \), respectively. Open triangles show the result from \( v_2 \) fitting with statistical error only. Yellow band represent the lower and upper values of chemical freeze-out temperature \( T_{\text{ch}} = 143 - 184 \) (MeV) [56, 57]. Black dashed line show the \( \langle \varepsilon \rangle \) value taken from azimuthal HBT analysis at STAR experiment [58]. Top x-axis denote the corresponding values of \( \langle \varepsilon \rangle \).

Fig. 5.15 shows the temperature \( T \) as a function of \( \langle R \rangle - \langle R_0 \rangle \)/\( \langle R_0 \rangle \) in 20 - 30% centrality with \( N_{\text{coll}} \) density profile. The temperature from spectra fit are unchanged with \( \langle R \rangle \) as it should since transverse momentum spectra does not depend on the eccentricity. And the temperature values are smaller than that at chemical freeze-out temperature which is given in [56, 57]. This result support that the temperature obtained from spectra fit could reflect the conditions where the kinetic freeze-out takes place. If one look at the Fig. 5.16 and 5.17, average radial flow velocity \( \langle \beta_T \rangle \) and \( \chi^2/\text{NDF} \) are also unchanged so much for spectra fit.

However, the result of temperature from \( v_2 \) fit significantly decrease with \( \langle R \rangle - \langle R_0 \rangle \)/\( \langle R_0 \rangle \) and become same temperature value from spectra fit around \( \langle \varepsilon \rangle \sim 0.06 \). For \( \langle \beta_T \rangle \) values, it initially 0.1 larger than that from spectra fit, and coincide in large \( \langle R \rangle - \langle R_0 \rangle \)/\( \langle R_0 \rangle \). The \( \chi^2/\text{NDF} \) values are slowly decreasing with \( \langle R \rangle - \langle R_0 \rangle \)/\( \langle R_0 \rangle \) and take the minimum value around \( \langle R \rangle - \langle R_0 \rangle \)/\( \langle R_0 \rangle \sim 0.22 \) \( \langle \varepsilon \rangle \sim 0.06 \). One also see that the results about temperature and average radial flow velocity from \( \chi^2 \) minimization with statistical error unchanged compared to that obtained with statistical and systematic
CHAPTER 5. DISCUSSIONS

Dashed lines in Fig. 5.15 – 5.17 represent the $\langle \varepsilon \rangle$ value obtained from azimuthal HBT analysis at STAR [58]. We define its value as the time when the kinetic freeze-out takes place. Corresponding $\langle (R - R_0) \rangle / \langle R_0 \rangle$ value at kinetic freeze-out is about 0.13. If one compare the temperature of $v_2$ and spectra at the kinetic freeze-out, the temperature from $v_2$ is as large as $T_{ch}$, and larger than that of spectra. This result suggest that the ”freeze-out” of $v_2$ could occur at the chemical freeze-out and its freeze-out is earlier than that of spectra. The result obtained by changing the eccentricity is consistent with the picuture of the collective in-plane expansion, where the initial eccentricity is quenched and the $v_2$ is developed through the expansion of the system with time.

Figure 5.16: Average radial flow velocity $\langle \beta_T \rangle$ as a function $\langle (R - R_0) \rangle / \langle R_0 \rangle$ obtained from $N_{coll}$ density profile.
Figure 5.17: $\chi^2$/NDF as a function ($\langle R \rangle - \langle R_0 \rangle$)/$\langle R_0 \rangle$ obtained from $N_{col}$ density profile.

5.2.4 Robustness of Fitting Results

In order to check the robustness of the fitting, we have checked several systematics of the fitting results as follows;

- Vary the size of system
- Vary velocity profile for a given density distribution.
- Use different parameterization for Woods-saxon density profile
  
  - $\sigma_{NN} = 42$ mb, $a = 0.53$ fm (default)
  - $\sigma_{NN} = 37$ mb, $a = 0.53$ fm
  - $\sigma_{NN} = 47$ mb, $a = 0.53$ fm
  - $\sigma_{NN} = 42$ mb, $a = 0.43$ fm
  - $\sigma_{NN} = 42$ mb, $a = 0.63$ fm

- Different centrality window, $10 - 20$ % and $30 - 40$ %.

System Size

Since $v_2$ does not depend on the absolute scale of $R_x$ and $R_y$, where $R_x(y)$ denote the radius of x (y) direction, only depends on the ratio of size for x and y ($R_y/R_x$), fitting results should not be changed with the system size. We confirmed that the results are unchanged for both spectra and $v_2$ with reasonable range of the system size.
Parameterization of Woods-saxon density profile

Table 5.3: Summary of temperatures and average velocity for different parameterizations from \( v_2 \) fit

<table>
<thead>
<tr>
<th>Type</th>
<th>( \langle \varepsilon \rangle )</th>
<th>( \beta_2 )</th>
<th>( T ) (MeV)</th>
<th>( \langle \beta_T \rangle )</th>
<th>( \chi^2/NDF )</th>
</tr>
</thead>
<tbody>
<tr>
<td>default</td>
<td>0.262</td>
<td>0.212</td>
<td>222.2 ( \pm ) 3.8</td>
<td>0.436 ( \pm ) 0.012</td>
<td>47.8/16 = 3.0</td>
</tr>
<tr>
<td>(1)</td>
<td>0.270</td>
<td>0.220</td>
<td>232.5 ( \pm ) 3.9</td>
<td>0.444 ( \pm ) 0.011</td>
<td>42.4/16 = 2.7</td>
</tr>
<tr>
<td>(2)</td>
<td>0.254</td>
<td>0.204</td>
<td>213.2 ( \pm ) 3.7</td>
<td>0.427 ( \pm ) 0.012</td>
<td>53.3/16 = 3.3</td>
</tr>
<tr>
<td>(3)</td>
<td>0.311</td>
<td>0.245</td>
<td>234.8 ( \pm ) 3.9</td>
<td>0.441 ( \pm ) 0.012</td>
<td>63.0/16 = 4.0</td>
</tr>
<tr>
<td>(4)</td>
<td>0.216</td>
<td>0.180</td>
<td>202.9 ( \pm ) 3.6</td>
<td>0.423 ( \pm ) 0.011</td>
<td>38.5/16 = 2.4</td>
</tr>
</tbody>
</table>


<table>
<thead>
<tr>
<th>( N_{\text{coll}} ) weight</th>
<th>( \langle \varepsilon \rangle )</th>
<th>( \beta_2 )</th>
<th>( T ) (MeV)</th>
<th>( \langle \beta_T \rangle )</th>
<th>( \chi^2/NDF )</th>
</tr>
</thead>
<tbody>
<tr>
<td>default</td>
<td>0.311</td>
<td>0.272</td>
<td>329.9 ( \pm ) 8.3</td>
<td>0.582 ( \pm ) 0.008</td>
<td>15.3/16 = 1.0</td>
</tr>
<tr>
<td>(1)</td>
<td>0.311</td>
<td>0.272</td>
<td>329.9 ( \pm ) 8.3</td>
<td>0.582 ( \pm ) 0.008</td>
<td>15.3/16 = 1.0</td>
</tr>
<tr>
<td>(2)</td>
<td>0.311</td>
<td>0.272</td>
<td>329.9 ( \pm ) 8.3</td>
<td>0.582 ( \pm ) 0.008</td>
<td>15.3/16 = 1.0</td>
</tr>
<tr>
<td>(3)</td>
<td>0.354</td>
<td>0.300</td>
<td>330.6 ( \pm ) 8.0</td>
<td>0.587 ( \pm ) 0.009</td>
<td>19.8/16 = 1.2</td>
</tr>
<tr>
<td>(4)</td>
<td>0.269</td>
<td>0.248</td>
<td>320.2 ( \pm ) 8.2</td>
<td>0.575 ( \pm ) 0.008</td>
<td>12.9/16 = 0.8</td>
</tr>
</tbody>
</table>

Different parameterizations of Woods-saxon density distribution are used to check the stability of the extracted parameters. Table 5.3 summarize the results of extracted parameters from \( v_2 \) fit from different parameterizations for the Glauber model, where (1) \( \sigma_{NN} = 37 \text{ mb} \), (2) \( \sigma_{NN} = 47 \text{ mb} \), (3) \( a = 0.43 \text{ fm} \), and (4) \( a = 0.63 \text{ fm} \). \( \sigma_{NN} \) and \( a \) denote the inelastic \( p+p \) cross sections and surface diffuseness parameters, respectively. Comparing the results from default parameterization given in Table 5.2, the temperature (average velocity) is about 10 (6) % changed for \( N_{\text{part}} \) weight, while for \( N_{\text{coll}} \) weight the difference is relatively smaller, about 4 % for both \( T \) and \( \langle \beta_T \rangle \). The difference obtained from this study are included in the systematic errors on extracted parameters.

**Velocity Profile**

The sensitivity of the extracted parameters are studied by varying the velocity profile distribution for the density distribution where the \( \chi^2/NDF \) is minimum, \((\langle R \rangle - \langle R_0 \rangle)/\langle R_0 \rangle \sim 0.22\). The velocity distribution is modified by adding \( 1 \pm 2\beta_2 \cos (2\phi) \) to the original velocity profile in Glauber model, where \( \beta_2 \) is \( 2^{nd} \) harmonics of radial flow velocity introduced in Eq. (5.5) for \( n = 2 \). We found that \( \chi^2/NDF \) become larger if the velocity profile varied by the \( \beta_2 \) parameter for both \( N_{\text{part}} \) and \( N_{\text{coll}} \) density, i.e. the minimum \( \chi^2/NDF \) is obtained at the initial velocity profile given by Glauber model. This result could suggest that the \( v_2 \) is sensitive to the velocity profile which is determined by the initial density overlap.
Different centrality

Other centrality bins, namely 10 – 20 % and 30 – 40 % centrality bins, are also fitted to see the centrality dependence of extracted parameters, and the results are shown in Table 5.4. $\langle \varepsilon \rangle$ and $\beta_2$ is calculated by Eq. (5.4) and Eq. (5.5), respectively.

Table 5.4: Summary of temperatures and average velocity for different centrality classes. Results of 20 – 30 % centrality are taken from Table 5.2.

<table>
<thead>
<tr>
<th>Centrality (%)</th>
<th>$N_{\text{part}}$ weight</th>
<th>$N_{\text{coll}}$ weight</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\langle \varepsilon \rangle$</td>
<td>$\beta_2$</td>
</tr>
<tr>
<td>0 - 10 %</td>
<td>0.061</td>
<td>0.049</td>
</tr>
<tr>
<td>10 - 20 %</td>
<td>0.178</td>
<td>0.141</td>
</tr>
<tr>
<td>20 - 30 %</td>
<td>0.262</td>
<td>0.212</td>
</tr>
<tr>
<td>30 - 40 %</td>
<td>0.331</td>
<td>0.276</td>
</tr>
</tbody>
</table>
5.3 Partonic Collectivity

Hadron production via quark coalescence mechanism predicts the universal scaling of $v_2$ from thermalized partons

$$v_2^h(p_T) \simeq n_q \times v_q^2(p_T/n_q)$$

(5.7)

where $n_q$ is the number of constituent quarks in hadrons, $v_2^h$ and $v_q^2$ are hadron $v_2$ at $p_T$ and quark $v_2$ at $p_T/n_q$, respectively. This quark number scaling indicates that the elliptic flow is developed in the early partonic stage of heavy ion collisions.

In Section 5.3.1, we first discuss the quark number scaling of $v_2$ for several different particle species in minimum bias events. Second, the validity of quark number scaling of $v_2$ is also discussed for different centrality classes in Section 5.3.2.

5.3.1 Quark Number Scaling in Minimum Bias Events

![Graph showing $v_2/n_q$ as a function of $p_T/n_q$ (left) and $m_T - m_0$ (right) for identified hadrons in minimum bias events. Yellow bands show the absolute systematic error on $\pi^+ + \pi^-$. Dashed lines represent the simultaneous fitting results for $\pi$, $K$, and $p$ by 4th polynomial functions. Bottom figures show the ratios of data to fitting results of $v_2$.](image)

Fig. 5.18: $v_2$ as a function of $p_T$ (left) and $m_T - m_0$ (right) divided by number of quarks for each hadrons ($n_q = 2$ for mesons and $n_q = 3$ for protons) in minimum bias events. Yellow bands show the absolute systematic error on $\pi^+ + \pi^-$. Dashed lines represent the simultaneous fitting results for $\pi$, $K$, and $p$ by 4th polynomial functions. Bottom figures show the ratios of data to fitting results of $v_2$.

Fig. 5.18 shows the $v_2/n_q$ as a function of $p_T/n_q$ (left) and $KE_T/n_q$ (right) for identified hadrons in minimum bias events. The scaled $v_2$ values are fitted by polynomial function in order to test the validity of the quark number scaling. The ratio of data to fitting result is plotted in the bottom panels. One immediately find that the scaling
breaks at $p_T/n_q < 1$ GeV/c, and the ratio is larger for lighter hadrons (i.e., $\pi > K > p$). As we already discussed in Section 1.3.3, the resonance decay contributions could be one of the possible explanations about the deviation of $\pi$ from the quark number scaling [18].

Recently, the PHENIX collaboration found that the quark number scaling of $v_2$ with transverse kinetic energy $K E_T = m_T - m_0$ (GeV) holds full measured $K E_T$ range [54]. If the elliptic flow is driven by the hydrodynamical pressure gradient, the magnitude of $v_2$ is expected to scale with $K E_T$ since the pressure gradient is directly linked to the collective kinetic energy of the emitted particles.

The scaling of $v_2$ with $K E_T$ is plotted in the right panel. The fitting is also performed by polynomial function and take the ratio as shown in right bottom panel. The ratio shows excellent scaling in the measured $K E_T$ range compared to the $p_T$ scaling in left panel, except for very low $K E_T$. This result support the picture that the elliptic flow is driven by the hydrodynamical collective pressure gradient.

Figure 5.19: Quark number scaling of $v_2$ for $d$ ($n_q = 6$) and $\phi$ ($n_q = 2$) as a function of $p_T$ (left) and $m_T - m_0$ (right) in minimum bias events. Yellow and blue bands around $v_2 = 0$ represent the absolute systematic error on $v_2$ for $d$ and $\phi$, respectively. Dashed lines are the fitting results by $4^{th}$ polynomial functions. Bottom figures show the ratio of data to fitting of $v_2$.

Elliptic flow for $\phi$ mesons could provide the evidence of partonic collectivity in the early pre-hadronic stage of heavy ion collisions. Because of their small hadronic cross sections with non-strange hadrons and their relatively longer lived life time, $\phi$ meson is expected to reach the detector with almost no interactions with the other hadrons in the later hadronic stage. Thus, if quark number scaling holds to $v_2$ for $\phi$ meson, it should
reflect the elliptic flow of thermal s-quark which is developed in the partonic stage of the system.

One could test the validity of this hypothesis that $v_2$ of each constituent quark is additive in the coalescence mechanism by measuring deuteron $v_2$. At RHIC energies, the dominant production mechanism of deuterons is considered as hadron coalescence with $p$ and $n$, and in fact the measurement of the yield for $d$ and $\bar{d}$ support the hadron coalescence of $p$ and $n$ [55]. Thus, the measurement of $v_2$ for deuterons could be a good probe to test the additive scaling relations among the constituents, i.e. first with respect to the $v_2$ of its constituent of hadrons, and second with respect to the $v_2$ of the constituent quarks of those hadrons ($n_q = 2 \times 3$).

Fig. 5.19 show the comparison between quark number scaling of $v_2$ for $\phi$ mesons and deuterons and those for $\pi, K$ and $p$ as a function of $p_T$ and $m_T - m_0$ in minimum bias events. Scaled $v_2$ of deuterons are in good agreement with the protons as shown in left panel. This result support that $v_2(d)$ is additive with respect to their constituent hadrons. And scaled $v_2$ for $\phi$ mesons are also consistent with the other hadrons within the errors. Right panel shows universal scaling of $v_2$ with $K E_T$ and $d$ and $\phi$ lie on the same line together with the other hadrons, although the statistical and systematic errors on $\phi$ mesons are large. This results are the strong indication that the elliptic flow is established under the conditions where the partonic degrees of freedom is relevant.

### 5.3.2 Centrality Dependence of Quark Number Scaling of $v_2$

The validity of quark number scaling has also studied in measured centrality classes. Fig. 5.20 show the quark number scaling of $v_2(K E_T)$ from central to peripheral events. And Fig. 5.21 show the ratio of $v_2$(Data) to $v_2$(Fit) as a function of $K E_T/n_q$. One can see that the quark number scaling of $v_2$ holds for all centrality bin within the systematic errors, except for below $K E_T/n_q \simeq 0.3$ GeV, which is corresponds to $K E_T \simeq 0.6$ (0.9) GeV for mesons (protons). Large deviations at low $K E_T/n_q$ from central to mid-central show the similar mass dependence as we see in $v_2(p_T)$ at low $p_T$, i.e. $v_2(\pi) \geq v_2(K) > v_2(p)$. This deviations could be due to the radial flow in the hadronic stage after the chemical freeze-out.

Fig. 5.22 show the quark number scaling of $v_2$ for $d$ and $\phi$ compared to $\pi, K$ and $p$ in 3 different centrality bins. Although the statistical error bars on $d$ and $\phi$ mesons are large, the quark number scaling of $v_2$ with $K E_T$ also holds for $d$ and $\phi$ in $0 - 60$ % centrality.
Figure 5.20: $v_2/n_q$ vs $(KE_T)/n_q$ for $\pi$, $K$ and $p$ in different centrality classes. Yellow bands around $v_2/n_q = 0$ denote the systematic error on $v_2(\pi)$. Dashed lines represent the simultaneous fitting results for $\pi$, $K$ and $p$ by 4th polynominal functions. Open data symbols in each panel show the $v_2$ in minimum bias event.

Figure 5.21: Ratio $v_2$(Data)/$v_2$(Fit) as a function of $KE_T/n_q$ in different centrality classes. Yellow bands around $v_2/n_q = 0$ denote the systematic error on $v_2(\pi)$.  

$$(KE_T/n_q) = (m_T - m_0)/n_q (GeV)$$
Figure 5.22: \( v_2/n_q \) vs \((m_T - m_0)/n_q\) (top), and \(v_2(\text{Data})/v_2(\text{Fit})\) vs \((m_T - m_0)/n_q\) (bottom) for \(d\) and \(\phi\) in different centrality classes. Dashed lines represent the simultaneous fitting results by the 4\textsuperscript{th} polynomial functions.

### 5.4 Summary

From the discussions, we conclude that

- **Eccentricity Scaling**
  
  - \( \langle \varepsilon_{\text{var}} \rangle \), which is taking into account the position fluctuations of participant nucleons, is the relevant quantity to explain the relation between initial geometry overlap and \(v_2\).
  
  - Eccentricity scaling works in both Au + Au and Cu + Cu collisions with \( \langle \varepsilon_{\text{var}} \rangle \). This result suggest that the ratio \( \langle v_2 \rangle / \langle \varepsilon \rangle \) is determined by the number density even in the different systems.

- **Blast-wave fit**
  
  - For the initial density profile, \( T \) from \(v_2\) fit is 100 – 200 MeV larger than that from spectra fit, whereas \( \langle \beta_T \rangle \) is almost unchanged between \(v_2\) and spectra fit.
  
  - Better fitting results, i.e. smaller \( \chi^2 \), are obtained from \( N_{\text{coll}} \) density profile than \( N_{\text{part}} \) density profile for both \(v_2\) and spectra.
  
  - By considering in-plane 1D expansion, \( T \) from \(v_2\) fit strongly decreases with the expansion while that from spectra fit remains constant. \( \langle \beta_T \rangle \) does not changed so much with the expansion for both spectra and \(v_2\).
- If the kinetic freeze-out is defined at the $\langle \varepsilon \rangle$ estimated by azimuthal HBT, $T$ from $v_2$ fit is as large as the chemical freeze-out temperature ($\sim 150$ MeV).
- The results from the simple expansion are consistent with the picture of the collective in-plane expansion, where the initial eccentricity is decreasing with time and the magnitude of $v_2$ is developed through the expansion of the system.

- **Quark number scaling of $v_2$**
  - Scaling of $v_2$ with $p_T$ holds above $p_T = 1$ GeV/$c$, but it breaks for $p_T < 1$ GeV/$c$.
  - Scaling of $v_2$ with $KE_T$ holds for all centrality bins within systematic errors, except for $KE_T/n_q < 0.3$ GeV.
  - $\phi$ mesons are also follow the quark number scaling of $v_2$. This result support the picture that the $v_2$ for $\phi$ meson is established at the pre-hadronic stage where the partonic degrees of freedom is relevant.
Chapter 6

Conclusions

We have measured the elliptic flow $v_2$ for identified $\pi^+ + \pi^-$, $K^+ + K^-$, $p + \bar{p}$, $d + \bar{d}$ and $\phi$ in a broad range of $p_T$ (up to 4 GeV/c) and in detailed centrality in Au + Au collisions at $\sqrt{s_{NN}} = 200$ GeV.

The main goal of ultra-relativistic heavy ion collisions is to find the quark-gluon plasma (QGP) and to study its properties under extreme conditions. Elliptic flow is one of the observables to probe the properties of QGP possibly created in the early stage of heavy ion collisions. If the local thermal equilibrium is reached, elliptic flow is determined through the initial geometry overlap (eccentricity) and the initial density profile. Therefore, elliptic flow could shed light on the possible local thermal equilibrium in the produced matter.

Using the data set taken in Run-4 period by the PHENIX experiment, the statistics increase by a factor 20 compared to the previous Run-2 data set. This allows us to extend the $p_T$ reach for $\pi$ and $p$ with the careful estimate of contamination, and to study the detailed centrality dependence of $v_2$. With the excellent capability of particle identification by Time-Of-Flight Counter ($\sigma_t \sim 120$ ps) and Electro-Magnetic Calorimeter ($\sigma_t \sim 400$ ps), and also with the good resolution of event plane at Beam-Beam Counter, we found the following important features:

- For $p_T < 2$ GeV/c, $v_2$ shows the mass ordering, i.e. $(v_2(\pi) > v_2(K) > v_2(p) \geq v_2(\phi) > v_2(d))$. This characteristic $p_T$ dependence of $v_2$ is consistent with hydrodynamical model calculations.

- For $p_T > 2$ GeV/c, $v_2$ for mesons saturate earlier than that for baryons, i.e. $(v_2(\pi) \approx v_2(K) \approx v_2(\phi) < v_2(p) \approx v_2(d))$.

- $v_2$ increase from central to peripheral collisions. This behavior of $v_2$ is qualitatively consistent with the centrality dependence of the eccentricity.

The relation between $\langle \varepsilon \rangle$ (initial eccentricity) and final elliptic flow has been studied by eccentricity scaling of $v_2$. We have used $\langle \varepsilon_{var} \rangle$ (participant eccentricity) which is taking into account the position fluctuations for the participant nucleons, instead of the standard eccentricity. The average $v_2$, $\langle v_2 \rangle$, among Au + Au and Cu + Cu collisions are scaled together with $\langle \varepsilon_{var} \rangle$. This result suggests that the $\langle v_2 \rangle/\langle \varepsilon \rangle$ is determined by the number density of participant nucleons even in the different colliding system.
We have developed an extended Blast-wave model in which the effect of initial density profile has been investigated for both number of participant \((N_{\text{part}})\) and number of collision \((N_{\text{coll}})\) density distributions. Thermodynamic quantities, such as the freeze-out temperature \(T\) and the radial flow velocity \(\beta_T\), have been extracted for both single particle spectra and \(v_2\) independently. Extracted \(T\)'s from \(v_2\) fit are 100–200 MeV larger than that from spectra fit, and the results obtained from \(N_{\text{coll}}\) density profile have always smaller \(\chi^2/\text{NDF}\) for both spectra and \(v_2\). This result could be attributed that the \(v_2\) is developed by the number of collisions among the constituents not the number density of participant nucleons since the number of collisions are closely related to the degrees of thermalization. By studying the 1D in-plane expansion of the system, i.e. changing the eccentricity of the system, we have found that \(T\) from \(v_2\) fit significantly decreases with the eccentricity, while those from spectra fit are unchanged. \(\langle \beta_T \rangle\) for both spectra and \(v_2\) remain constant with respect to the eccentricity. \(T\) from \(v_2\) fit is as large as the chemical freeze-out temperature if we assume that the kinetic freeze-out takes place at the \(\langle \varepsilon \rangle\) obtained by the azimuthal HBT analysis. Larger \(T\) from \(v_2\) fit than that from spectra fit at the kinetic freeze-out may suggest that the freeze-out of \(v_2\) could be earlier than that of spectra. These results are consistent with the picture of the collective in-plane expansion, where the initial eccentricity is quenched and the magnitude of \(v_2\) is developed through the expansion with time.

The quark number scaling of \(v_2\) has been examined for measured centrality range. The scaling works well for all particle species at \(p_T/n_q > 1\) GeV/\(c\), however, it breaks at lower \(p_T\). By assuming the \(v_2\) is determined by the transverse kinetic energy \(KE_T = m_T - m_0\), the quark number scaling of \(v_2\) with \(KE_T\) lie on the universal curve for \(\pi, K\) and \(p\) in all centrality bins within the systematic errors, whereas the scaling of \(v_2\) breaks for \(KE_T/n_q < 0.3\) GeV. Since the pressure gradient is directly linked to the transverse kinetic energy, this results could suggest that the collective pressure gradient is the driving force of elliptic flow. We have also observed that the quark number scaling of \(v_2\) with \(KE_T\) holds for \(\phi\) and \(d\) in different centrality selections. The cross section (mean free path) of \(\phi\) meson with the system is small (large) compared to the non-strange hadrons, so that \(\phi\) mesons do not suffer from the hadronic interactions. Therefore, the observation of the quark number scaling of \(v_2\) for \(\phi\) mesons could indicate the partonic collectivity in the pre-hadronic phase of heavy ion collisions.
Appendix A

Glauber Model

A.1 Parameterization

The Glauber model, which is a semi-classical model treating the nucleus-nucleus collisions as the superposition of the nucleon-nucleon collisions, has been successfully applied in the description of high-energy nuclear reactions. Nucleons are assumed to travel in straight lines, and are not deflected after the collisions, which holds as a good approximation at very high energies. Also, the nucleon-nucleon inelastic cross-section \( \sigma_{NN} \), is assumed to be the same as that in the vacuum. In other words, secondary particle production and possible excitation of nucleons are not considered in this model.

The density distribution of the two nuclei with mass number \( A \) (here we consider Au nucleus \( A = 197 \)), is described by a Woods-Saxon parameterization

\[
\rho^{Au}(r) = \frac{\rho_0^{Au}}{1 + e^{(r-R_{Au})/a^{Au}}} \tag{A.1}
\]

where \( R_{Au} = 6.38 \text{ fm} \) is the radius of Au nucleus and \( a^{Au} = 0.54 \text{ fm} \) is the surface diffuseness parameter. The normalization factor \( \rho_0^{Au} = 0.17 \text{ fm}^{-3} \) is set to give \( \int d^3 r \rho^{Au}(r) = A = 197 \).

The relevant quantity for the following considerations is the nuclear thickness function, which integrates the nuclear density function over the longitudinal coordinate \( z \);

\[
T_A(x, y) = \int_{-\infty}^{\infty} dz \rho^{Au}(x, y, z) \tag{A.2}
\]

The opacity of the nucleus is obtained simply by multiplying the thickness function with the total inelastic cross section \( \sigma_0 \) of a nucleon-nucleon collisions. We use \( \sigma_0 = 42 \text{ mb} \) at \( \sqrt{s_{NN}} = 200 \text{ GeV} \) [60].

Number of participant nucleons \( (N_{part}) \), which is defined by the number of nucleons participate in inelastic collisions at least once, for two colliding nuclei with mass number
$A$ and $B$ is given by

$$n_{\text{part}}(x, y; b) = T_A(x + b/2, y) \left( 1 - \frac{\sigma_0 T_B(x - b/2, y)}{B} \right)^B + T_B(x - b/2, y) \left( 1 - \frac{\sigma_0 T_A(x + b/2, y)}{A} \right)^A$$  \hspace{1cm} (A.3)$$

$$N_{\text{part}}(b) = \int dxdy \ n_{\text{part}}(x, y; b)$$  \hspace{1cm} (A.4)$$

where $A$ and $B$ is the mass number, $b$ is the impact parameter. The thickness function of each nucleus is simply shifted by \(\pm b/2\) to the left or to the right along the $x$-axis to meet the thickness function of the other nucleus shifted in the other direction. Integrating the Eq. (A.3) over the transverse plane yields the total number of participant nucleons as a function of the impact parameter as shown in Eq. (A.4).

Number of nucleon-nucleon collisions ($N_{\text{coll}}$) in non-central $A + B$ collisions is expressed by the product of the thickness function of one nucleus with the encountered opacity of the other nucleus

$$n_{\text{coll}}(x, y; b) = \sigma_0 T_A(x + b/2, y) T_B(x - b/2, y)$$  \hspace{1cm} (A.5)$$

$$N_{\text{coll}}(b) = \int dxdy \ n_{\text{coll}}(x, y; b)$$  \hspace{1cm} (A.6)$$

Figure A.1: Contours of constant densities of $N_{\text{part}}$ in Au + Au collisions with $b = 7.4$ fm at $\sqrt{s_{\text{NN}}} = 200$ GeV. The contours are given by 5% step of the maximum value of the density. The dashed line indicate the radius of the colliding nuclei.
Fig. A.1 show the contours of overlap density distribution weighted by number of nucleon-nucleon collisions at $b = 7.4$ fm obtained from Monte Carlo simulation by Glauber model.

$N_{\text{part}}$ and $N_{\text{coll}}$ value from Glauber Monte Carlo simulation are plotted as a function of centrality in Fig. A.2.

Figure A.2: Number of participant nucleons ($N_{\text{part}}$) and number of nucleon-nucleon collisions ($N_{\text{coll}}$) as a function of centrality from Glauber Monte Carlo simulation.

### A.2 Participant Eccentricity ($\varepsilon_{\text{var}}$)

Initial spatial anisotropy (eccentricity) for a given impact parameter is also calculated. In the Glauber Monte Carlo simulations, the standard eccentricity is calculated in a reference frame that is defined by the center of the two colliding nuclei, and is given by

\begin{align}
\varepsilon_{\text{part}}^{\text{std}} & \equiv \frac{\{n_{\text{part}} \times (y_i^2 - x_i^2)\}}{\{n_{\text{part}} \times (y_i^2 + x_i^2)\}} = \frac{\sum_i n_{\text{part}}^i \times (y_i^2 - x_i^2)}{\sum_i n_{\text{part}}^i \times (y_i^2 + x_i^2)} \quad (A.7) \\
\varepsilon_{\text{coll}}^{\text{std}} & \equiv \frac{\{n_{\text{coll}} \times (y_i^2 - x_i^2)\}}{\{n_{\text{coll}} \times (y_i^2 + x_i^2)\}} = \frac{\sum_i n_{\text{coll}}^i \times (y_i^2 - x_i^2)}{\sum_i n_{\text{coll}}^i \times (y_i^2 + x_i^2)} \quad (A.8)
\end{align}

where $(x, y)$ is the position of a participant nucleon in the coordinate system ($x$ and $y$ axes are defined as shown in Fig. A.1), and superscript of part and coll represent that average is taken by weighting with $n_{\text{part}}$ and $n_{\text{coll}}$ density, respectively. Brackets $\{\ldots\}$ denotes an sample average, it means an average over all participant nucleons in one collision event.
Because of the event-by-event fluctuations in the participant nucleon positions [61],
the eccentricity in a given event is shifted and tilted with respect to the (x, y) frame.
The center-of-gravity of the participant nucleons is given by

\[ \{x\} \equiv x_c = \frac{1}{N_{part}} \sum_i x_i \]  
(A.9)

\[ \{y\} \equiv y_c = \frac{1}{N_{part}} \sum_i y_i \]  
(A.10)

We denote the position of participant i in the shifted reference frame \( S_S \), in which the
center-of-gravity is at the origin, as \((x_s^i, y_s^i) = (x_i - x_c, y_i - y_c)\). We can now determine
as reference frame \( S_S' \) which is rotated relative to \( S_S \) by an angle \( \Psi \) and in which the
eccentricity is maximal. The coordinate in the rotated system is given by

\[ \begin{pmatrix} x_r^i \\ y_r^i \end{pmatrix} = \begin{pmatrix} \cos \Psi & -\sin \Psi \\ \sin \Psi & \cos \Psi \end{pmatrix} \begin{pmatrix} x_s^i \\ y_s^i \end{pmatrix} \]  
(A.11)

The eccentricity in the rotated frame (participant eccentricity) is given by

\[ \varepsilon_{var}^{part} \equiv \frac{\{n_{part} \times (y_r^2 - x_r^2)\}}{\{n_{part} \times (y_r^2 + x_r^2)\}} \]  
(A.12)

\[ \varepsilon_{coll}^{var} \equiv \frac{\{n_{coll} \times (y_r^2 - x_r^2)\}}{\{n_{coll} \times (y_r^2 + x_r^2)\}} \]  
(A.13)

with

\[ x_r^2 = (x_s \cos \Psi - y_s \sin \Psi)^2 \]
\[ = x_s^2 \cos^2 \Psi - 2x_s y_s \cos \Psi \sin \Psi + y_s^2 \sin^2 \Psi \]  
(A.14)

\[ y_r^2 = (x_s \sin \Psi + y_s \cos \Psi)^2 \]
\[ = x_s^2 \sin^2 \Psi + 2x_s y_s \cos \Psi \sin \Psi + y_s^2 \cos^2 \Psi \]  
(A.15)

Participant eccentricity for a given event can be rewritten by using Eq. (A.14), (A.15),

\[ \varepsilon_{var} = \frac{\{(y_r^2 - x_r^2) \cos^2 \Psi + (x_r^2 - y_r^2) \sin^2 \Psi + 4x_s y_s \cos \Psi \sin \Psi\}}{\{x_r^2 + y_r^2\}} \]
\[ = \frac{\{y_r^2 - x_r^2\} \cos (2\Psi) + 2\{x_s y_s\} \sin (2\Psi)}{\{x_r^2 + y_r^2\}} \]
\[ = \frac{(\sigma_y^2 - \sigma_x^2) \cos (2\Psi) + 2\sigma_{xy} \sin (2\Psi)}{\sigma_y^2 + \sigma_x^2} \]  
(A.16)

where \( \sigma_x, \sigma_y \) and \( \sigma_{xy} \) are defined as follows

\[ \sigma_x^2 \equiv \{x^2\} - \{x\}^2 \]  
(A.17)

\[ \sigma_y^2 \equiv \{y^2\} - \{y\}^2 \]  
(A.18)

\[ \sigma_{xy} \equiv \{xy\} - \{x\} \{y\} \]  
(A.19)
\{y_s^2 - x_s^2\}, \{x_s y_s\} and \{y_s^2 + x_s^2\} in Eq. (A.16) can be expressed by making use of Eq. (A.17) - (A.19)

\[
\begin{align*}
\{y_s^2 - x_s^2\} &= \{(y_i - \{y\})^2 - (x_i - \{x\})^2\} \\
&= \{y_i^2 - 2y_i\{y\} + \{y\}^2 - (x_i^2 - 2x_i\{x\} + \{x\}^2)\} \\
&= \{y^2\} - \{y\}^2 - (\{x^2\} - \{x\}^2) = \sigma_y^2 - \sigma_x^2 \\
\{x_s y_s\} &= \{(x_i - \{x\})(y_i - \{y\})\} \\
&= \{x_i y_i - \{x\} y_i - x_i \{y\} + \{x\} \{y\}\} \\
&= \{xy\} - \{x\} \{y\} = \sigma_{xy} \\
\{y_s^2 + x_s^2\} &= \sigma_y^2 + \sigma_x^2
\end{align*}
\] (A.20)

The rotation angle \(\Psi\) which maximize the eccentricity is given by the following condition

\[
\frac{d\varepsilon_{\text{var}}}{d\Psi} = 0
\] (A.23)

\(\Psi\) can be determined by

\[
\frac{d\varepsilon_{\text{var}}}{d\Psi} = -2(\sigma_y^2 - \sigma_x^2) \sin (2\Psi) + 4\sigma_{xy} \cos (2\Psi) = 0
\]

\[
\therefore \tan (2\Psi) = \frac{2\sigma_{xy}}{\sigma_y^2 - \sigma_x^2}
\]
\[
\cos (2\Psi) = \frac{1}{\sqrt{1 + \tan^2 (2\Psi)}} = \frac{\sigma_y^2 - \sigma_x^2}{\sqrt{(\sigma_y^2 - \sigma_x^2)^2 + 4\sigma_{xy}^2}}
\] (A.24)
\[
\sin (2\Psi) = 1 - \cos^2 (2\Psi) = \frac{2\sigma_{xy}}{\sqrt{(\sigma_y^2 - \sigma_x^2)^2 + 4\sigma_{xy}^2}}
\] (A.25)

Therefore

\[
\varepsilon_{\text{var}} = \frac{1}{\sigma_y^2 + \sigma_x^2} \left(\frac{\sigma_y^2 - \sigma_x^2}{\sqrt{(\sigma_y^2 - \sigma_x^2)^2 + 4\sigma_{xy}^2}} + 2\sigma_{xy} \frac{2\sigma_{xy}}{\sqrt{(\sigma_y^2 - \sigma_x^2)^2 + 4\sigma_{xy}^2}}\right)
\]
\[
= \frac{\sqrt{(\sigma_y^2 - \sigma_x^2)^2 + 4\sigma_{xy}^2}}{\sigma_y^2 + \sigma_x^2}
\] (A.26)

If the event-by-event fluctuations in the position of participant nucleons are small, participant eccentricity become standard eccentricity. Since \(\sigma_x \sim \sqrt{\{x^2\}}\), \(\sigma_y \sim \sqrt{\{y^2\}}\) \((\{x\} \sim 0, \{y\} \sim 0)\) and \(\sigma_{xy} \sim 0\)

\[
\varepsilon_{\text{var}} \approx \frac{\sigma_y^2 - \sigma_x^2}{\sigma_y^2 + \sigma_x^2}
\]
\[
\approx \frac{\{y^2\} - \{x^2\}}{\{y^2\} + \{x^2\}} = \varepsilon_{\text{std}}
\] (A.27)
Figure A.3: Standard and participant eccentricity as a function of \( N_{\text{part}} \). Lines and symbols represent the results with \( N_{\text{part}} \) and \( N_{\text{coll}} \) weighted eccentricity, respectively. Yellow bands and solid lines around denote the absolute systematic error on \( \langle \varepsilon \rangle \) for standard and participant eccentricity.

Figure A.4: Standard and participant eccentricity as a function of \( N_{\text{part}} \). Different definition of eccentricity \( \varepsilon\{2\} \equiv \sqrt{\langle \varepsilon^2 \rangle} \) is used.
Fig. A.3 and A.4 show \( N_{\text{part}} \) dependence of the standard and participant eccentricity calculated in Glauber MC model.

The participant eccentricity is defined by the principal axes (or event plane) which is determined by the position of all participating nucleons. It always includes auto-correlations between participants and the event plane, so that the \( \langle \varepsilon_{\text{var}} \rangle \) goes 1 in peripheral events due to such correlations. In order to take into account the effect of auto-correlations, we subtract the auto-correlation from participant eccentricity event-by-event basis. We define event plane eccentricity, \( \langle \varepsilon_2 \rangle \), and calculate it by using following redefined event plane angle \( \Psi^i \) for \( i \)-th event

\[
x^i_s = r^i_s \cos (2\phi^i_s) \tag{A.28}
y^i_s = r^i_s \sin (2\phi^i_s) \tag{A.29}
\phi^i_s = \tan^{-1} \left( \frac{y^i_s}{x^i_s} \right) \tag{A.30}
\]

\[
\tan (2\Psi^i) = -\frac{\sum_j (r^j_s)^2 \sin (2\phi^j_s) - (r^i_s)^2 \sin (2\phi^i_s)}{\sum_j (r^j_s)^2 \cos (2\phi^j_s) - (r^i_s)^2 \cos (2\phi^i_s)} \tag{A.31}
\]

Figure A.5: Comparison of eccentricity as a function of impact parameter in Au + Au collisions. Left (right) figure shows the eccentricity calculated with \( N_{\text{part}} (N_{\text{coll}}) \) density profile.

Fig. A.5 and A.6 show the comparison of eccentricity in Au + Au and Cu + Cu collisions (dashed lines). The event plane eccentricity takes almost intermediate value between standard and participant eccentricity. We also compare that the different averaging, \( \varepsilon \{2\} \equiv \sqrt{\langle \varepsilon^2 \rangle} \), to the standard calculation as shown in the data symbols in Fig.
Figure A.6: Comparison of eccentricity as a function of impact parameter in Cu + Cu collisions. Left (right) figure shows the eccentricity calculated with $N_{\text{part}}$ ($N_{\text{coll}}$) density profile.

A.5 and A.6. In the $\varepsilon\{2\}$, the difference of $\varepsilon$ between different definitions of eccentricity is relatively smaller than the $\varepsilon$.

Fig. A.7 and A.8 show the ratio of $\varepsilon^{\text{coll}}$ to $\varepsilon^{\text{part}}$ as a function of impact parameter in Au + Au and Cu + Cu collisions. $\varepsilon^{\text{coll}}$ is about 1.5 – 2 times larger in most central collision and close to unity in most peripheral collisions than $\varepsilon^{\text{part}}$. 
Figure A.7: $\langle \varepsilon^{\text{coll}} \rangle / \langle \varepsilon^{\text{part}} \rangle$ (left) and $\varepsilon^{\text{coll}}_2 / \varepsilon^{\text{part}}_2$ (right) as a function of impact parameter in Au + Au.

Figure A.8: $\langle \varepsilon^{\text{coll}} \rangle / \langle \varepsilon^{\text{part}} \rangle$ (left) and $\varepsilon^{\text{coll}}_2 / \varepsilon^{\text{part}}_2$ (right) as a function of impact parameter in Cu + Cu.
A.3 Results and Systematic Errors

Results of Glauber Monte Carlo simulation are summarized in Table A.1 and A.2. The variables are calculated under the default condition (see below). Centrality classes are defined as: (0) 0–5 %, (1) 5–10 %, (2) 10–15 %, (3) 15–20 %, (4) 20–30 %, (5) 30–40 %, (6) 40–50 %, (7) 50–60 %, and (8) 60–92 %.

Systematic errors for calculated quantities are evaluated by varying the input parameters

1. \( \sigma_{NN} = 42 \text{ mb}, R_{Au} = 6.38 \text{ fm}, a = 0.53 \text{ fm} \) \( (R_{Cu} = 4.27 \text{ fm}) \) : default

2. \( \sigma_{NN} = 37 \text{ mb}, R_{Au} = 6.38 \text{ fm}, a = 0.53 \text{ fm} \)

3. \( \sigma_{NN} = 47 \text{ mb}, R_{Au} = 6.38 \text{ fm}, a = 0.53 \text{ fm} \)

4. \( \sigma_{NN} = 42 \text{ mb}, R_{Au} = 6.08 \text{ fm}, a = 0.53 \text{ fm} \) \( (R_{Cu} = 4.07 \text{ fm}) \)

5. \( \sigma_{NN} = 42 \text{ mb}, R_{Au} = 6.68 \text{ fm}, a = 0.53 \text{ fm} \) \( (R_{Cu} = 4.47 \text{ fm}) \)

6. \( \sigma_{NN} = 42 \text{ mb}, R_{Au} = 6.38 \text{ fm}, a = 0.43 \text{ fm} \)

7. \( \sigma_{NN} = 42 \text{ mb}, R_{Au} = 6.38 \text{ fm}, a = 0.63 \text{ fm} \)

where the radius inside () represent the values in Cu + Cu. Relative systematic errors on each quantity are summarized in Table A.3 and A.4.

### Table A.1: Summary of results in Au + Au at \( \sqrt{s_{NN}} = 200 \text{ GeV} \)

<table>
<thead>
<tr>
<th>Centrality</th>
<th>(0)</th>
<th>(1)</th>
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<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N_{part} )</td>
<td>351.3</td>
<td>298.2</td>
<td>252.9</td>
<td>212.5</td>
<td>163.2</td>
<td>112.2</td>
<td>73.3</td>
<td>44.9</td>
<td>15.1</td>
</tr>
<tr>
<td>( N_{coll} )</td>
<td>1071.4</td>
<td>847.0</td>
<td>671.0</td>
<td>528.7</td>
<td>366.4</td>
<td>215.6</td>
<td>119.1</td>
<td>59.9</td>
<td>15.4</td>
</tr>
<tr>
<td>( \langle \varepsilon_{\text{std}} \rangle )</td>
<td>0.031</td>
<td>0.093</td>
<td>0.147</td>
<td>0.198</td>
<td>0.257</td>
<td>0.316</td>
<td>0.368</td>
<td>0.398</td>
<td>0.391</td>
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<td>( \langle \varepsilon_{\text{part}} \rangle )</td>
<td>0.084</td>
<td>0.132</td>
<td>0.183</td>
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<td>0.159</td>
<td>0.212</td>
<td>0.276</td>
<td>0.345</td>
<td>0.409</td>
<td>0.462</td>
<td>0.527</td>
</tr>
<tr>
<td>( \varepsilon_{\text{std}} {2} )</td>
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<td>0.123</td>
<td>0.174</td>
<td>0.224</td>
<td>0.285</td>
<td>0.351</td>
<td>0.414</td>
<td>0.463</td>
<td>0.556</td>
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<tr>
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<td>0.148</td>
<td>0.200</td>
<td>0.251</td>
<td>0.318</td>
<td>0.394</td>
<td>0.470</td>
<td>0.544</td>
<td>0.703</td>
</tr>
<tr>
<td>( \varepsilon_{2} {2} )</td>
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<td>0.140</td>
<td>0.187</td>
<td>0.238</td>
<td>0.304</td>
<td>0.378</td>
<td>0.450</td>
<td>0.517</td>
<td>0.651</td>
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<tr>
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<td>0.195</td>
<td>0.250</td>
<td>0.312</td>
<td>0.368</td>
<td>0.410</td>
<td>0.426</td>
<td>0.410</td>
</tr>
<tr>
<td>( \langle \varepsilon_{\text{coll}} \rangle )</td>
<td>0.119</td>
<td>0.172</td>
<td>0.228</td>
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<td>0.345</td>
<td>0.412</td>
<td>0.475</td>
<td>0.536</td>
<td>0.675</td>
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<td>0.149</td>
<td>0.207</td>
<td>0.262</td>
<td>0.325</td>
<td>0.386</td>
<td>0.436</td>
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<td>0.400</td>
<td>0.454</td>
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<tr>
<td>( \varepsilon_{\text{std}} {2} )</td>
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<td>0.190</td>
<td>0.246</td>
<td>0.299</td>
<td>0.365</td>
<td>0.436</td>
<td>0.503</td>
<td>0.569</td>
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<td>( \varepsilon_{\text{coll}} {2} )</td>
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<td>0.352</td>
<td>0.418</td>
<td>0.481</td>
<td>0.539</td>
<td>0.663</td>
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Table A.2: Summary of results in Cu + Cu at $\sqrt{s_{NN}} = 200$ GeV

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<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
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<td>79.0</td>
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<td>34.8</td>
<td>23.0</td>
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<tr>
<td>$N_{\text{coll}}$</td>
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<td>174.1</td>
<td>137.6</td>
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<td>74.7</td>
<td>44.3</td>
<td>25.5</td>
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<td>$\langle \varepsilon_{\text{part}} \rangle$</td>
<td>0.021</td>
<td>0.062</td>
<td>0.102</td>
<td>0.138</td>
<td>0.188</td>
<td>0.232</td>
<td>0.259</td>
<td>0.275</td>
<td>0.240</td>
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<td>$\langle \varepsilon_{\text{var}} \rangle$</td>
<td>0.151</td>
<td>0.191</td>
<td>0.236</td>
<td>0.280</td>
<td>0.348</td>
<td>0.434</td>
<td>0.523</td>
<td>0.611</td>
<td>0.737</td>
</tr>
<tr>
<td>$\langle \varepsilon_{2\text{part}} \rangle$</td>
<td>0.075</td>
<td>0.120</td>
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<td>0.276</td>
<td>0.353</td>
<td>0.421</td>
<td>0.471</td>
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</tr>
<tr>
<td>$\langle \varepsilon_{\text{std}} \rangle$</td>
<td>0.021</td>
<td>0.062</td>
<td>0.102</td>
<td>0.138</td>
<td>0.188</td>
<td>0.232</td>
<td>0.259</td>
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<td>$\langle \varepsilon_{\text{var}} \rangle$</td>
<td>0.151</td>
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<td>0.737</td>
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<td>$\langle \varepsilon_{2\text{part}} \rangle$</td>
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Table A.3: Summary of relative systematic errors (%) in Au + Au at $\sqrt{s_{NN}} = 200$ GeV

<table>
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<td>$N_{\text{part}}$</td>
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<td>23.9</td>
<td>34.2</td>
<td>59.4</td>
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<tr>
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<td>18.6</td>
<td>17.7</td>
<td>16.7</td>
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<td>20.8</td>
<td>29.5</td>
<td>42.1</td>
<td>69.1</td>
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<td>16.8</td>
<td>17.4</td>
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Table A.4: Summary of relative systematic errors (%) in Cu + Cu at $\sqrt{s_{\text{NN}}} = 200$ GeV

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Appendix B

Event Plane Resolution

In this chapter, we derive the formula of the event plane resolution under the assumptions introduced in the Section 3.2.4.

Starting from the assumptions in Section 3.2.4, the probability distribution can be given by the gaussian as

\[
\frac{dP}{dn d\theta_n} = \frac{v_n}{2\pi\sigma_n^2} \exp \left( -\frac{|v_n - \bar{v}_n|^2}{2\sigma_n^2} \right) = \frac{v_n}{2\pi\sigma_n^2} \exp \left( -\frac{v_n^2 + \bar{v}_n^2 - 2v_n\bar{v}_n\cos(\theta_n)}{2\sigma_n^2} \right)
\]

\[
= \frac{\chi_n}{\sqrt{2\pi}\sigma_n} \exp \left( -\chi_n^2 - \bar{\chi}_n^2 + 2\chi_n\bar{\chi}_n\cos(\theta_n) \right)
\]

(B.1)

where \( \theta_n = n\Delta\Psi = n(\Psi_n - \Psi) \), and \( \chi_n = v_n/(\sqrt{2}\sigma_n) \), \( \bar{\chi}_n = \bar{v}_n/(\sqrt{2}\sigma_n) \). For isotropic probability distribution (\( \bar{v}_n = 0 \)), the variance \( \sigma_n \) is same for any \( n \)-th moment,

\[
\sigma^2 \equiv \sigma_n^2 = \langle (v_n - \bar{v}_n)^2 \rangle = \langle v_n^2 \rangle \quad (\because \bar{v}_n = 0)
\]

\[
= \left\langle \frac{x_n^2}{x_0^2} \right\rangle = M \langle w^2 \rangle \langle \cos^2(n\phi) \rangle
\]

\[
= \frac{M^2 \langle w^2 \rangle}{2M \langle w^2 \rangle}
\]

(B.2)

where \( \langle \cos(n\phi) \rangle = 1/2 \) for isotropic probability distribution.
Eq. (B.1) can be integrated over dimensionless parameter $\chi_n$.

\[
\frac{dP}{d\theta_n} = \frac{1}{\pi} \int_0^{\infty} \chi_n d\chi_n \exp (-\chi_n^2 - \tilde{\chi_n}^2 + 2\chi_n \tilde{\chi_n} \cos (\theta_n))
\]

\[
= \frac{e^{-\tilde{\chi_n}^2}}{\pi} \int_0^{\infty} \chi_n d\chi_n \exp (-\chi_n^2 + 2\chi_n \tilde{\chi_n} \cos (\theta_n))
\]

\[
= \frac{e^{-\tilde{\chi_n}^2}}{\pi} \int_0^{\infty} \chi_n d\chi_n \exp (- \tilde{\chi_n}^2) \times \exp (a_n^2) \quad (a_n \equiv \tilde{\chi_n} \cos (\theta_n))
\]

\[
= \frac{e^{-\tilde{\chi_n}^2} e^{a_n^2}}{\pi} \int_{-\infty}^{\infty} (y_n + a_n) dy_n \exp (-y_n^2) \quad (y_n = \chi_n - a_n)
\]

\[
= \frac{e^{-\tilde{\chi_n}^2} e^{a_n^2}}{\pi} \left[ \int_{-\infty}^{\infty} y_n dy_n \exp (-y_n^2) + a_n \int_{-\infty}^{\infty} dy_n \exp (-y_n^2) \right]
\]

\[\text{Eq. (B.3)}\]

1\textsuperscript{st} term of Eq. (B.3) can be calculated by using integration by parts:

\[
\int_{-\infty}^{\infty} y_n dy_n \exp (-y_n^2) = \left[ -\frac{1}{2} e^{-y_n^2} \right]_{-\infty}^{\infty}
\]

\[
= \frac{1}{2} e^{-a_n^2}
\]

\[\text{Eq. (B.4)}\]

2\textsuperscript{nd} term of Eq. (B.3) can be expressed by using standard error function $\text{erf}(x)$:

\[
\int_{-\infty}^{\infty} dy_n \exp (-y_n^2) = \int_{-\infty}^{0} dy_n \exp (-y_n^2) + \int_{0}^{\infty} dy_n \exp (-y_n^2)
\]

\[
= \frac{\sqrt{\pi}}{2} \text{erf}(a_n) + \frac{\sqrt{\pi}}{2}
\]

\[\text{Eq. (B.5)}\]

Therefore, Eq. (B.3) reduces to:

\[
\frac{dP}{d\theta_n} = \frac{e^{-\tilde{\chi_n}^2} e^{a_n^2}}{\pi} \left[ \frac{1}{2} e^{-a_n^2} + \frac{a_n \sqrt{\pi}}{2} (1 + \text{erf}(a_n)) \right]
\]

\[
= \frac{e^{-\tilde{\chi_n}^2}}{2\pi} \left[ 1 + \sqrt{\pi} \tilde{\chi_n} \cos \theta_n e^{\tilde{\chi_n}^2 \cos^2(\theta_n)} [1 + \text{erf}(\tilde{\chi_n} \cos (\theta_n))] \right]
\]

\[\text{Eq. (B.6)}\]

Fig. B.1 shows the distribution of $\theta_n$ for different value of $\chi$ ($\chi = 0.5, 1, 1.5, 2$). As one can see in Eq. (B.6), the distribution of $\theta_n$ is symmetric under the transformation $\theta_n \rightarrow -\theta_n$. If $\chi_n \ll 1$, statistical fluctuations are large compared to the dynamical anisotropy, Eq. (B.6) becomes:

\[
\frac{dP}{d\theta_n} \approx \frac{1}{2\pi} [1 + \sqrt{\pi} \tilde{\chi_n} \cos (\theta_n)]
\]

\[\text{Eq. (B.7)}\]

On the other hand, if $\chi_n \gg 1$, one can expand the cosine and sine as $\cos \theta_n \approx 1$ and
\[
\frac{dP}{d\theta_n} = e^{-\frac{\chi_n^2}{2}} + \frac{1}{2\sqrt{\pi}} \chi_n \cos \theta_n e^{-\frac{\chi_n^2}{2} \sin^2(\theta_n)} [1 + \text{erf}(\chi_n \cos(\theta_n))]
\approx \frac{e^{-\frac{\chi_n^2}{2}}}{2\pi} + \frac{1}{2\sqrt{\pi}} \chi_n e^{-\frac{\chi_n^2}{2} \theta_n^2} [1 + \text{erf}(\chi_n)]
\approx \frac{\chi_n}{\sqrt{\pi}} e^{-\frac{\chi_n^2}{2} \theta_n^2} \quad (\because e^{-\frac{\chi_n^2}{2}} \to 0, \ \text{erf}(\chi_n) \to 1)
\]

We can measure \( n \)-th Fourier coefficient \( v_n \) by using the event planes determined from any harmonic \( k \), with \( n \geq k \). Thus, it is convenient to introduce the event plane resolution by \( \langle \cos k\theta_n \rangle \) [13]:

\[
\langle \cos k\theta_n \rangle = \int_0^\infty d\chi_n \int_0^{2\pi} d\theta_n \cos (k\theta_n) \frac{dP}{d\chi_n d\theta_n}
= \frac{e^{-\frac{\chi_n^2}{2}}}{\pi} \int_0^\infty d\chi_n \chi_n e^{-\frac{\chi_n^2}{2}} \int_0^{2\pi} d\theta_n \cos (k\theta_n) e^{2\chi_n \tilde{\chi}_n \cos(\theta_n)}
= 2e^{-\frac{\chi_n^2}{2}} \int_0^\infty d\chi_n \chi_n e^{-\frac{\chi_n^2}{2} I_k(2\chi_n \tilde{\chi}_n)}
= 2e^{-\frac{\chi_n^2}{2}} \left[ \left[-\frac{1}{2} e^{-\frac{\chi_n^2}{2} I_k(2\chi_n \tilde{\chi}_n)} \right]_0^\infty + \frac{1}{2} \int_0^\infty d\chi_n e^{-\frac{\chi_n^2}{2}} \frac{dI_k}{d\chi_n}(2\chi_n \tilde{\chi}_n) \right]
= e^{-\frac{\chi_n^2}{2}} \int_0^\infty d\chi_n e^{-\frac{\chi_n^2}{2}} \frac{dI_k}{d\chi_n}(2\chi_n \tilde{\chi}_n)
\]
where 1st term becomes 0, and \( dI_k/d\chi_n \) is given by the following recurrence relation:

\[
\frac{dI_k}{d\chi_n} = \frac{1}{2\pi} \frac{d}{d\chi_n} \left( \int_0^{2\pi} d\theta_n \ \cos (k\theta_n)e^{2\chi_n\tilde{\chi}_n\cos (\theta_n)} \right)
\]

\[
= \frac{\tilde{\chi}_n}{\pi} \int_0^{2\pi} d\theta_n \ \cos (k\theta_n) \cos (\theta_n)e^{2\chi_n\tilde{\chi}_n\cos (\theta_n)}
\]

\[
= \frac{\tilde{\chi}_n}{2\pi} \int_0^{2\pi} d\theta_n \ \cos ((k+1)\theta_n) + \cos ((k-1)\theta_n) \times e^{2\chi_n\tilde{\chi}_n\cos (\theta_n)}
\]

\[
= \tilde{\chi}_n [I_{k+1}(2\chi_n\tilde{\chi}_n) + I_{k-1}(2\chi_n\tilde{\chi}_n)]
\]

Therefore

\[
\langle \cos (k\theta_n) \rangle = \tilde{\chi}_n e^{-\tilde{\chi}_n^2} \int_0^{\infty} d\chi_n \ e^{-\chi_n^2} \ [I_{k+1}(2\chi_n\tilde{\chi}_n) + I_{k-1}(2\chi_n\tilde{\chi}_n)]
\]

\[
= \frac{\sqrt{\pi}}{2} \tilde{\chi}_n e^{-\tilde{\chi}_n^2/2} \left[ I_{(k-1)/2} \left( \frac{\tilde{\chi}_n^2}{2} \right) + I_{(k+1)/2} \left( \frac{\tilde{\chi}_n^2}{2} \right) \right]
\]

\[
\left( \int_0^{\infty} e^{-\tilde{\chi}_n^2} I_k(2\chi_n\tilde{\chi}_n)d\chi_n = \frac{\sqrt{\pi}}{2} e^{\tilde{\chi}_n^2/2} I_{k/2} \left( \frac{\tilde{\chi}_n^2}{2} \right) \right) \quad (B.9)
\]

If \( \chi \ll 1 \), Eq. (B.9) reduces to

\[
\langle \cos (k\theta_n) \rangle \approx \frac{\sqrt{\pi}}{2} \tilde{\chi}_n \left[ \frac{1}{\Gamma(k+1/2)} \left( \frac{\tilde{\chi}_n^2}{2} \right)^{(k-1)/2} \right. \left. + \frac{1}{\Gamma(k+3/2)} \left( \frac{\tilde{\chi}_n^2}{4} \right)^{(k+1)/2} \right]
\]

\[
= \frac{\sqrt{\pi}}{2} \tilde{\chi}_n \frac{1}{\Gamma(k+1/2)} \left( \frac{\tilde{\chi}_n^2}{4} \right)^{(k-1)/2} \left[ 1 + \frac{2}{k+1} \left( \frac{\tilde{\chi}_n^2}{4} \right)^2 \right]
\]

\[
\approx \frac{\sqrt{\pi}}{2^k} \tilde{\chi}_n^k \frac{1}{\Gamma(k+1/2)} \quad (B.10)
\]

If \( \chi \gg 1 \), Eq. (B.9) reduces to

\[
\langle \cos (k\theta_n) \rangle = \int_{-\pi}^{\pi} d\theta_n \ \cos k\theta_n \frac{dP}{d\theta_n}
\]

\[
\approx \frac{\tilde{\chi}_n}{\sqrt{\pi}} \int_{-\pi}^{\pi} d\theta_n \ \cos k\theta_n e^{-\tilde{\chi}_n^2\theta_n^2}
\]

\[
= \frac{1}{\sqrt{\pi}} \int_{-\chi_n\pi}^{\chi_n\pi} da_n \ \cos \left( \frac{ka_n}{\tilde{\chi}_n} \right) e^{-a_n^2} \ (a_n = \tilde{\chi}_n\theta_n)
\]

\[
= \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} da_n \ \cos \left( \frac{ka_n}{\tilde{\chi}_n} \right) e^{-a_n^2} \ (\tilde{\chi}_n \gg 1)
\]

\[
= \exp \left( -\frac{k^2}{4\tilde{\chi}_n^2} \right) \quad (B.11)
\]

in the last line we use the following integral formula

\[
\int_{-\infty}^{\infty} e^{-ax^2} \cos (kx)dx = \sqrt{\frac{\pi}{a}} e^{-k^2/(4a)} \quad (B.12)
\]
Appendix C

Blast-wave Parameterization

C.1 Transverse Expansion

The invariant momentum spectrum of hadrons emitted at freeze-out is given by a local thermal distribution $f(x, p)$, with the freeze-out temperature $T$, boosted by a local velocity field $u^\mu$, at the freeze-out hyper-surface $\Sigma$ [59].

\[
E \frac{dN}{d^3p} = \frac{dN}{m_T dm_T dy d\phi_p} = \int_\Sigma f(x, p) \Omega(r, \phi) p^\mu d\Sigma_\mu
\]  

(C.1)

\[
p^\mu = (E, p_x, p_y, p_z)
\]

\[
= (m_T \cosh y, p_T \cos \phi_p, p_T \sin \phi_p, m_T \sinh y)
\]  

(C.2)

where $p^\mu$ is a four-momentum of the emitted hadrons, $d\Sigma_\mu$ is a normal vector to the hyper-surface $\Sigma$, and where transverse momentum ($p_T$), transverse mass ($m_T$), rapidity ($y$), azimuthal angle ($\phi_p$) refer to the momentum of the emitted hadrons. and $\Omega(r, \phi)$ is density distributions from Glauber model. The local thermal distribution $f(x, p)$ is given by

\[
f(x, p) = \frac{1}{(2\pi)^3} \frac{1}{e^{(p_\mu u^\mu(x) - \mu(x))/T(x)} + 1} \]  

(C.3)

where $\mu(x)$ is a local chemical potential, $T(x)$ is a local temperature, $u^\mu(x)$ is a local flow velocity with $u_\mu u^\mu = 1$, and upper (lower) sign is for fermions (bosons). We can omit $\pm 1$ in the denominator in Eq. (C.3) if we assume a Boltzmann distribution for all particles. Freeze-out hyper surface is parameterized as follows:

\[
\Sigma^\mu = (t, x, y, z) = (\tau \cosh \eta, r \cos \phi, r \sin \phi, \tau \sinh \eta)
\]  

(C.4)

\[
\eta = \frac{1}{2} \log \left( \frac{t + z}{t - z} \right)
\]  

(C.5)

where $r$ and $\phi$ are the usual cylindrical coordinates, and $\eta$ is space-time rapidity. A normal vector $d\Sigma_\mu$ in Eq. (C.1) to the surface is then given by

\[
d\Sigma_\mu = \epsilon_{\mu\nu\lambda\rho} \frac{\partial \Sigma^\nu}{\partial r} \frac{\partial \Sigma^\lambda}{\partial \phi} \frac{\partial \Sigma^\rho}{\partial \eta} dr d\phi d\eta = d\sigma_\mu dr d\phi d\eta
\]  

(C.6)
We explicitly calculate the components of $d\sigma_\mu$:

\[
d\sigma_0 = \epsilon_{0\nu\lambda\rho} \frac{\partial \Sigma_\nu}{\partial r} \frac{\partial \Sigma_\lambda}{\partial \phi} \cosh \eta
\]

\[
d\sigma_1 = \epsilon_{1\nu\lambda\rho} \frac{\partial \Sigma_\nu}{\partial r} \frac{\partial \Sigma_\lambda}{\partial \phi} \frac{\partial \Sigma_\rho}{\partial \eta} = 0
\]

\[
d\sigma_2 = \epsilon_{2\nu\lambda\rho} \frac{\partial \Sigma_\nu}{\partial r} \frac{\partial \Sigma_\lambda}{\partial \phi} \frac{\partial \Sigma_\rho}{\partial \eta} = 0
\]

\[
d\sigma_3 = \epsilon_{3\nu\lambda\rho} \frac{\partial \Sigma_\nu}{\partial r} \frac{\partial \Sigma_\lambda}{\partial \phi} \frac{\partial \Sigma_\rho}{\partial \eta}
\]

\[
= (\cos \phi r \cos \phi - \sin \phi (-r \sin \phi)) \cosh \eta = -r \tau \cosh \eta
\]

(C.10)

From Eq. (C.6) - (C.10) $d\Sigma_\mu$ becomes

\[
d\Sigma_\mu = (\cosh \eta, 0, 0, -\sinh \eta) \tau \tau d\tau d\phi d\eta
\]

(C.11)

Thus, we obtain

\[
p^\mu d\Sigma_\mu = (m_T \cosh y \cosh \eta - m_T \sinh y \sinh \eta) \tau \tau d\tau d\phi d\eta
\]

\[
= m_T \cosh (y - \eta) \tau \tau d\tau d\phi d\eta
\]

(C.12)

Local velocity field $u^\mu$ can be parameterized as

\[
u^\mu = (\cosh \eta_L \cosh \rho, \sinh \rho \cos \phi_b, \sinh \rho \sin \phi_b, \sinh \eta_L \cosh \rho)
\]

(C.13)

in cylindrical coordinates, where $\phi_b$ is azimuthal angle of radial boost velocity, $\eta_L$ is longitudinal flow rapidity, and $\rho = \rho(r, \phi s)$ is transvers flow rapidity defined as

\[
\rho(r, \phi s) = \tanh(\beta_T \cdot g(r, \phi s))
\]

(C.14)

where $\beta_T$ is the surface radial flow velocity, and $g(r, \phi s)$ is the density gradient distributions given by Glauber model. In Bjorken scenario $\eta_L$ is identical to the space-time rapidity $\eta$. From Eq. (C.2) and (C.13)

\[
p^\mu u^\mu = m_T \cosh y \cosh \eta \cosh \rho - p_T \cos \phi_p \sinh \rho \cos \phi_b
\]

\[
- p_T \sin \phi_p \sinh \rho \sin \phi_b - m_T \sinh y \sinh \eta \cosh \rho
\]

\[
= m_T \cosh \rho \cosh (y - \eta) - p_T \sinh \rho \cos (\phi_b - \phi_p)
\]

(C.15)

We can write the invariant momentum distribution by assuming a Boltzman distribution for all particles and Bjorken scenario as

\[
E \frac{dN}{d^3p} = \frac{1}{(2\pi)^3} \int_{\Sigma} e^{-(p^\mu u^\mu - \mu)/T} \Omega(r, \phi s) p^\mu d\Sigma_\mu
\]

\[
= m_T e^{\mu/T} \int_0^\infty r dr \int_0^{2\pi} d\phi_s \int_{-\infty}^\infty d\eta \cosh (y - \eta)
\]

\[
\times e^{-\beta(r, \phi_s) \cosh (y - \eta)} e^{\alpha(r, \phi_s) \cos (\phi_b - \phi_p)} \Omega(r, \phi s)
\]

(C.16)
where $\alpha(r, \phi_s)$ and $\beta(r, \phi_s)$ are defined as

$$\alpha(r, \phi_s) = \frac{p_T}{T} \sinh \rho(r, \phi_s) \quad (C.17)$$

$$\beta(r, \phi_s) = \frac{m_T}{T} \cosh \rho(r, \phi_s) \quad (C.18)$$

### C.2 Transverse Momentum Spectra and $v_2$

Azimuthally integrated transverse momentum spectra is obtained by integrating over $\phi_p$. We can integrate over $\eta$ with the help of modified Bessel functions of the second kind by assuming $y = 0$ because we only interested in mid-rapidity particles in the center-of-mass frame.

$$\frac{dN}{p_T dp_T} = \int_0^{2\pi} d\phi_p \, E \frac{dN}{d^3p}$$

$$= \frac{m_T \tau e^{\mu/T}}{(2\pi)^3} \int_0^{\infty} r dr \int_0^{2\pi} d\phi_p \int_0^{2\pi} d\phi_s \times \left( \int_{-\infty}^{\infty} d\eta \cosh (\eta) \, e^{-\beta(r, \phi_s) \cosh (\eta)} \right) e^{\alpha(r, \phi_s) \cos (\phi_s - \phi_p)} \Omega(r, \phi_s)$$

$$= \frac{2m_T \tau e^{\mu/T}}{(2\pi)^3} \int_0^{\infty} r dr \int_0^{2\pi} d\phi_p \int_0^{2\pi} d\phi_s \, K_1(\beta(r, \phi_s)) \times e^{\alpha(r, \phi_s) \cos (\phi_s - \phi_p)} \Omega(r, \phi_s) \quad (C.19)$$

$$K_n(z) = \frac{1}{2} \int_{-\infty}^{\infty} d\eta \cos (n \eta) e^{-z \cosh \eta} \quad (C.20)$$

We can exchange the order of integration in $\phi_p$ and $\phi_s$, and perform a transformation $\psi \equiv \phi_s - \phi_p$. Then the integral in $\psi$ can be performed analytically and lead to the modified Bessel functions of 1st kind

$$\frac{dN}{p_T dp_T} = \frac{2m_T \tau e^{\mu/T}}{(2\pi)^3} \int_0^{\infty} r dr \int_0^{2\pi} d\phi_s \left( \int_0^{2\pi} d\psi \, e^{\alpha(r, \phi_s) \cos (\psi)} \right) \times K_1(\beta(r, \phi_s)) \Omega(r, \phi_s)$$

$$= \frac{2m_T \tau e^{\mu/T}}{(2\pi)^2} \int_0^{\infty} r dr \int_0^{2\pi} d\phi_s \times I_0(\alpha(r, \phi_s)) \, K_1(\beta(r, \phi_s)) \Omega(r, \phi_s) \quad (C.21)$$

where $I_0$ is the modified Bessel functions of the first kind

$$I_n(z) = \int_{0}^{2\pi} d\phi \cos (n \phi) e^{z \cos \phi} \quad (C.22)$$

Elliptic flow can be calculated as

$$v_2(p_T) = \frac{\int_0^{2\pi} d\phi_p \cos (2\phi_p) \, dN/(p_T dp_T d\phi_p)}{\int_0^{2\pi} d\phi_p \, dN/(p_T dp_T d\phi_p)} \quad (C.23)$$
The denominator is already calculated as shown in Eq. (C.21). The numerator is obtained in similar way to integrate transverse momentum spectra. Thus,

\[
\int_0^{2\pi} d\phi_p \cos (2\phi_p) \frac{dN}{(p_T dp_T d\phi_p)} = \frac{2m_T T e^{\mu/T}}{(2\pi)^3} \int_0^\infty rdr \int_0^{2\pi} d\phi_s \ K_1(\beta(r, \phi_s)) \Omega(r, \phi_s) \times \int_0^{2\pi} d\phi_p \cos (2\phi_p) e^{\alpha(r, \phi_s) \cos (\phi_s - \phi_p)} \quad \text{(C.24)}
\]

Again, we write \(\psi = \phi_s - \phi_p\) and decompose

\[
\cos (2\phi_p) = \cos (2\phi_s - 2\psi) = \cos (2\phi_s) \cos (2\psi) + \sin (2\phi_s) \sin (2\psi) \quad \text{(C.25)}
\]

The \(\psi\) integration with the term proportional to \(\sin (2\psi)\) vanishes because of the symmetry in azimuthal direction of \(\psi\). Thus, the numerator becomes

\[
\int_0^{2\pi} d\phi_p \cos (2\phi_p) \frac{dN}{(p_T dp_T d\phi_p)} = \frac{2m_T T e^{\mu/T}}{(2\pi)^3} \int_0^\infty rdr \int_0^{2\pi} d\phi_s \ K_1(\beta(r, \phi_s)) \Omega(r, \phi_s) \times \cos (2\phi_s) \left( \int_0^{2\pi} d\psi \cos (2\psi) e^{\alpha(r, \phi_s) \cos (\psi)} \right) \quad \text{(C.26)}
\]

The integral of \(\psi\) leads to a modified Bessel functions of 1st kind \(I_2\) (see, Eq. (C.22)). We finally obtain the expression of \(v_2(p_T)\) as

\[
v_2(p_T) = \frac{\int_0^\infty rdr \int_0^{2\pi} d\phi_s K_1(\beta(r, \phi_s)) \Omega(r, \phi_s) \cos (2\phi_s) I_2(\alpha(r, \phi_s))}{\int_0^\infty rdr \int_0^{2\pi} d\phi_s K_1(\beta(r, \phi_s)) \Omega(r, \phi_s) I_0(\alpha(r, \phi_s))} \quad \text{(C.27)}
\]
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