Event shape dependence of jet correlations at RHIC

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2014/July/19

Heavy Ion Café @ University of Tokyo
Jet-Quenching

Au+Au

Hard-scattered parton

Jet

Re-distribution of Deposited energy

Collective expansion

Parton energy loss

Jet

Hard-scattered parton

Suppression of away-side

 Pedestal Subtracted

PRL91.072304 (2003)

\[
\Delta \phi = \phi^a - \phi^t \quad \text{[rad]}
\]

\[
\frac{1}{N_{\text{trigger}}} \frac{dN}{d\Delta \phi}
\]

\[4<p_T^{\text{trig}}<6, \ 2<p_T^{\text{asso}}<p_T^{\text{trig}}\]
$\Psi_2$ Dependence of Suppression in High $p_T$ correlations

![Graph showing $h^+ p_T^a = 5$-7 GeV/c and $\pi^0 p_T^l = 4$-7 GeV/c]  

- Figure 6. (Color online) Per-trigger azimuthal jet yields for the most central events. The near-side suppression is consistent through the collision zone. The same set of representative particle orientations are indicated with arrows.

- Figure 7. (Color online) Nuclear jet suppression factor $I_{AA} = Y_{AuAu}/Y_{pp}$

- Monotonic suppression with increase of path length, which can be taken as “parton energy loss”

- Where deposited energy goes?
Conical Emission of Intermediate $p_T$ correlations

Two-Particle Correlations

Three-Particle Correlations

✧ Away-side double hump in two-particle correlations
✧ Conical Emission confirmed by three-particle correlations
✧ Seems parton-medium interactions

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HIC
Models for Double-Hump : 1

✧ Cherenkov gluon radiation by superluminal partons

\[ \cos \theta_c = 1/n(p) \]

\[ n(p) \text{ : Index of refraction} \]
\[ p \text{ : Gluon Momentum} \]

PRL 96.172302 (2006)

✧ Shock-wave by supersonic partons

\[ \cos \theta_{Mach} = c_s / v_{part} \]

\[ c_s \text{ : Speed of sound} \]
\[ v_{part} \text{ : Speed of parton} \]

PRL 105.222301 (2010)
Models for Double-Hump : 2

✧ Energy-momentum loss + expanding medium

\[ \partial_\mu T^{\mu\nu} = S^\nu \]

\[ S^\nu(t, \vec{x}) = \frac{1}{(\sqrt{2\pi}\sigma)^3} \exp \left[ -\frac{[\vec{x} - \vec{x}_{jet}(t)]^2}{2\sigma^2} \right] \times \left( \frac{dE}{dt}, \frac{dM}{dt}, 0, 0 \right) \left( \frac{T(t, \vec{x})}{T_{\text{max}}} \right)^3 \]

PRL 105.222301 (2010)

✧ Hot spot + expanding medium

– Split of the hot spot into two directions

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Higher-Order Event-Planes & Flow-Harmonics

Smooth participant density

Expansion to the short-axis direction by pressure gradient

Fluctuating participant density

Expansion to the short-axis directions of event-planes by pressure gradient

✧ Azimuthal distribution of emitted particles

\[ \frac{dN}{d\phi} \propto 1 + 2v_2 \cos 2(\phi - \Psi_2) \]

\[ + 2v_3 \cos 3(\phi - \Psi_3) \]

\[ + 2v_4 \cos 4(\phi - \Psi_4) \ldots \]

\[ \nu_n = \langle \cos n(\phi - \Psi_n) \rangle \]

\( \nu_n \) : Higher-order flow harmonics

\( \Psi_n \) : Higher-order event planes

\( \phi \) : Azimuthal angle of emitted particles
Higher-Order Flow Harmonics

None-zero $v_n(n>2)$ is observed

Degeneracy of models disentangled

- Initial Condition, shear viscosity of QGP, different expansion mechanism between $v_2$ & $v_3$

Backgrounds in correlation functions

Central Collisions

$N_{\text{part}}$: # of participant nucleons in a collision

Centrality~0%

$N_{\text{part}}$~394

Peripheral Collisions

Centrality~100%

$N_{\text{part}}$~2

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Motivation of analysis

✧ Providing experimental results of two-particle correlations after $v_n$ background subtractions

✧ Examine the path length dependence of $\Psi_2$ dependent intermediate-$p_T$ correlations in order to search for deposited energy from high $p_T$ partons

✧ Search for differences between $\Psi_2$ & $\Psi_3$ dependent correlations which may reflects possible different evolution processes between the 2nd- and 3rd-order geometry planes
Analysis Flow-Chart

Single-particle analysis

- Event-Plane (Resolution)
- Flow harmonics $v_n$
- Pure flow backgrounds
- Tracking efficiency

Two-particle analysis

- Two-particle correlations
- Flow subtracted correlations
- Pair yield per a trigger
- Unfolding of event-plane resolution

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PHENIX 2007 Experiment: Au+Au 200 GeV Collisions

- Minimum Bias trigger: 4.4 billion events
- Trigger, collision vertex, centrality
  - Zero-Degree-Calorimeter (ZDC)
  - Beam-Beam-Counter (BBC)
- Event-plane
  - BBC
  - Reaction-Plane-Detector (RXN)
- Central Arm, $\Delta \phi = \pi$, $|\eta| < 0.35$
  - Drift Chamber (DC)
  - Pad Chambers (PC)
  - Electromagnetic Calorimeter (EMC)
    - Momentum, charged particle tracking
    - Ring Image Cherenkov Detector (RICH)
    - Electron rejection
Event-Plane

Expansion to the initial short-axis direction by pressure gradient

- EP is a direction most particles are emitted after freeze-out
- EP is determined by flow signal itself

EP is determined by RXN and BBC detectors

- RXN (1<|η|<2.8) : 24 segments x 2 sectors
- BBC (3<|η|<3.9) : 64 segments x 2 sectors

\[ \Psi_n = \frac{1}{n} \tan^{-1} \left( \frac{\sum_i w_i \cos(n \phi_i)}{\sum_i w_i \sin(n \phi_i)} \right) \]

\( \phi_i \) : Azimuthal angle of \( i^{\text{th}} \) segments

\( w_i \) : Weight (Charge etc.) of \( i^{\text{th}} \) segments

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Rapidity Selections in Analysis

✧ Rapidity ranges of CNT, RXN, & BBC
  
  - 2PC at $|\Delta \eta|<0.35$
  
  - Rapidity gap between particles & EP to avoid auto-correlations by jets

-3.9 < $\eta$ < -3.0  -2.8 < $\eta$ < -1.0  -0.35 < $\eta$ < 0.35  1.0 < $\eta$ < 2.8  3.0 < $\eta$ < 3.9

✧ Raw flow harmonics

$$v_{n}^{raw} = \langle \cos n(\phi - \Psi_{n}^{obs}) \rangle$$

✧ Resolution correction

- Smearing due to limited resolution

$$v_{n} = \frac{\langle \cos n(\phi - \Psi_{n}^{obs}) \rangle}{\langle \cos n(\Psi_{n} - \Psi_{n}^{obs}) \rangle}$$

Event-Plane Resolution
\( v_n \) Results

✧ Consistent results with previous PHENIX measurements
  – Used for background subtractions

**Total systematics (%) at \( p_T = 1-2 \) GeV/c**

<table>
<thead>
<tr>
<th>Centrality</th>
<th>0-10%</th>
<th>40-50%</th>
</tr>
</thead>
<tbody>
<tr>
<td>( v_2 )</td>
<td>4.3%</td>
<td>2.7%</td>
</tr>
<tr>
<td>( v_3 )</td>
<td>4.9%</td>
<td>12%</td>
</tr>
<tr>
<td>( v_4 )</td>
<td>10%</td>
<td>34%</td>
</tr>
<tr>
<td>( v_4^{(\Psi_4)} )</td>
<td>15%</td>
<td>6.5%</td>
</tr>
</tbody>
</table>

**Au+Au \( \sqrt{s_{NN}} = 200 \) GeV, EP Method**

![Graph showing \( v_n \) results for different centrality classes and \( p_T \) values](image-url)
Two-Particle Correlations

**Definition**

Ratio of two-particle probability over single-particle ones

\[ C(\Delta \phi, \Delta \eta) = \frac{P(\phi^a, \phi^t | \eta^a, \eta^t)}{P(\phi^a | \eta^a)P(\phi^t | \eta^t)} \]

**Real Pair**

Event mixing also corrects acceptance effects by choosing similar events: centrality, collision points

**Correlations = Real/Mixed**

**Experimental Def.**

Ratio of real pair distribution over mixed one

\[ C(\Delta \phi, \Delta \eta) = \frac{N_{mix}^{ta} d^2 N_{real}^{ta} / d\Delta \phi d\Delta \eta}{N_{real}^{ta} d^2 N_{mix}^{ta} / d\Delta \phi d\Delta \eta} \]

\[ \Delta \phi = \phi^a - \phi^t, \Delta \eta = \eta^a - \eta^t \]

**Pair Yield Per a Trigger**

Dimension: Number of Particles

\[ \frac{1}{N^t} \frac{d^2 N^{ta}}{d\Delta \phi d\Delta \eta} = \frac{1}{2\pi \varepsilon} \frac{N^{ta}}{N^t} C(\Delta \phi, \Delta \eta) \]
Flow Subtraction & Pair Yield per a Trigger (PTY)

✧ Pure flow background

\[ F(\Delta \phi) = 1 + \sum_{n=1}^{\Delta \phi/\pi} 2v_t^n v_a^n \cos(n\Delta \phi) \]

✧ Flow subtractions by ZYAM

– Zero Yield At Minimum Assumption

\[ j(\Delta \phi) = C(\Delta \phi) - b_0 \left[ 1 + \sum_{n=1}^{\Delta \phi/\pi} 2v_t^n v_a^n \cos(n\Delta \phi) \right] \]

✧ Pair yield per a trigger (PTY)

– Dimension: number of particles

\[ \frac{1}{N_t} \frac{dN^{ta}}{d\Delta \phi} = \frac{1}{2\pi \varepsilon} \frac{N^{ta}}{N_t} j(\Delta \phi) \]

\[ \varepsilon \]: Tracking efficiency of associate particles

\[ N_t \]: Number of triggers

\[ N^{ta} \]: Number of pairs

\[ Au+Au, \sqrt{s_{NN}}=200 \text{ GeV, 20-30\%} \]

\[ p_t^T \otimes p_t^a = 2-4 \otimes 1-2 \text{ GeV/c} \]
Expansion to the short-axis direction by pressure gradient
- EP: direction most particles are emitted after freeze-out

Selecting trigger particles with respect to $\Psi_2$ & $\Psi_3$
- 8 bins: $\phi^{trig} - \Psi_n : [-\pi/n, \pi/n]$

Control of path length of trigger and associate particles

Three $p_T$ combinations: 2-4x1-2, 2-4x2-4, 4-10x2-4 GeV/c
Flow Backgrounds with respect to EP

✧ A Monte Carlo simulation employed
✧ Azimuthal distribution using
  – Measured $v_n$
  – Observed correlation between EP
    • $<4(\Psi_2-\Psi_4)>=\frac{v_4\{\Psi_2\}}{v_4\{\Psi_4\}}$
    • $<6(\Psi_2-\Psi_3)>=0$
✧ Determine trigger particle relative to EP taking into account EP resolutions
✧ Calculate two-particle correlations

\[
\frac{dN}{d\phi} \propto 1 + \sum_{n=2,3,4} 2v_n \cos n(\phi - \Psi_n)
\]
Flow Backgrounds with respect to EP

(web)

- Good reconstruction of $\Psi_2$, $\Psi_3$ dependent correlations by MC simulation
  - Before PTY normalization
- Except around $\Delta \phi = 0, \pi$ where contribution of jet exists

\[
\Psi_2: \text{Out-of-plane } \Psi_2 < 0
\]

\[
\Psi_2: \text{In-plane } \Psi_2 < 0
\]

\[
\Psi_3: \text{Out-of-plane } \Psi_3 < 0
\]

\[
\Psi_3: \text{In-plane } \Psi_3 < 0
\]

$\Delta \phi = \phi^a - \phi^t [\text{rad}]$

40-50% — Correlations

Pure Flow

: EP Direction

: Back-to-Back Direction

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Two-Particle Correlations with respect to EP

✧ Flow subtracted $\Psi_2$, $\Psi_3$ dependent correlations
✧ Clear $\Psi_2$ dependence
✧ No $\Psi_3$ dependence?
✧ Smearing by neighboring trigger bins due to limited EP resolution
  – Needs unfolding !!
Unfolding Methods of EP Resolution

Fitting Method

\[ F(\psi_s)_{\text{raw}} = 1 + 2v_2 \cos(2(\psi_s + \Delta\phi)) + 2v_4 \cos(4(\psi_s + \Delta\phi)) \]

\[ F(\psi_s)_{\text{cor}} = 1 + 2v_2 \cos(2(\psi_s + \Delta\phi)) + 2v_4 \cos(4(\psi_s + \Delta\phi)) \]

Au+Au 20-30%, \( \Delta\phi = \pi/24. \)

Iteration Method

\[ z = \chi \cos(2(\Psi^{\text{obs}} - \Psi^{\text{real}})) \]

Azimuthal anisotropy of correlation yield corrected by the event-plane resolution

Method by PRC.84.024904 (2011)

✧ Trigger smearing matrix “S”
✧ True & Observed Correlations “A” & “B”
  – Vector elements: Trigger bin
✧ Solve simultaneous equations via iteration

\[ B = SA \quad \Rightarrow \quad A = S^{-1}B \]
$v_n$ ($n=2,3,4$) subtracted correlations

Au+Au $\sqrt{s_{NN}}=200$ GeV, $v_n$ ($n=2,3,4$) subtracted

Away-side suppressions

Away-side single peaks

Away-side broad/double-hump shapes

$\Delta \phi = \phi^a - \phi^i$ [rad]
\( p_T \) spectra of Per Trigger Yields

- Hardness increases with trigger and associate \( p_T \)
- Existence of high \( p_T \) particles enhances lot \( p_T \) particles

\[ \Phi \quad \Psi \quad 4 \quad \frac{1}{4} \quad |<| \quad \pi \quad - \quad \phi \quad \Delta \quad \text{subtracted} \]

\[ \text{Near Side:} |\phi| < \pi/4 \]
\[ v_2 \ v_3 \ v_4 \{ \Psi_4 \} \text{ subtracted} \]

\[ \text{Away Side:} |\Delta \phi - \pi| < \pi/4 \]
\[ v_2 \ v_3 \ v_4 \{ \Psi_4 \} \text{ subtracted} \]

\[ \text{Au+Au 200GeV} \]

\[ \text{\( N \)} \frac{1}{p_T} \frac{d^2N_{\text{pair}}}{dp_T d\phi} \]

\[ \text{Associate } p_T \text{ [GeV/c]} \]

\[ \Phi \quad \Psi \quad 4 \quad \frac{1}{4} \quad |<| \quad \pi \quad - \quad \phi \quad \Delta \quad \text{subtracted} \]

\[ \text{\( p_T \)}^{\text{hig.}} 4-10 \text{ GeV/c} \]
\[ \text{\( p_T \)}^{\text{hig.}} 2-4 \text{ GeV/c} \]
\[ \text{\( p_T \)}^{\text{hig.}} 1-2 \text{ GeV/c} \]
Extraction of Double-Hump Position

寝室 of double-hump position via two-Gaussian fitting to away-side (|Δφ−π|<π) at centrality 10%, where double-humps seen

\[ F(\Delta \phi) = Ae^{-\frac{(\Delta \phi - \pi - D)^2}{\sigma^2}} + Ae^{-\frac{(\Delta \phi - \pi + D)^2}{\sigma^2}} \]
Two-Gaussian Height

Double-hump height more than one sigma of systematic uncertainties

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**Comparison with Models**

✧ Cherenkov gluon: <25% of experimental data at $p_T = 1$ GeV/c

✧ Mach-cone & Energy-momentum loss:
  
  – Independence of $p_T$ is similar to the experimental data
  
  – 20% larger/smaller than experimental data at $p_T = 2$ GeV/c

✧ Hot-spot: 50% larger than experimental data
Realistic Model Calculation


Au+Au \( \sqrt{s} = 200 \text{ GeV/n} \)

\[ p_{T}^{\text{trig}} \in [2,4] \text{ GeV/c}, p_{T}^{\text{asso}} \in [1,2] \text{ GeV/c} \]

\[
\begin{align*}
(\Delta\eta < 2.8) \text{ at PHENIX} \\
(1 < |\Delta\eta| < 2.8) \text{ at PHENIX}
\end{align*}
\]

\[
\begin{align*}
(1 < |\Delta\eta| < 2.8) \text{ at PHENIX} \\
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\[
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(1 < |\Delta\eta| < 2.8) \text{ at PHENIX} \\
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\end{align*}
\]

✧ Fluctuations of initial parton energy density
✧ Parton cascade
✧ Event-by-event (3+1)D hydrodynamics
✧ Parton Energy Momentum Loss

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$\Psi_2$ & $\Psi_3$ Dependent Correlations at $p_T$: 2-4x1-2 GeV/c

$\Psi_2$ dependence

$\Psi_3$ dependence

$\Psi_2$ dependence is observed at intermediate-$p_T$ correlations
Near-Side Integrated Yield vs Associate Angle from $\Psi_2$

**Au+Au 200GeV**

$\Psi_2$ dependence
Near Side : $|\Delta \phi| < \pi/4$

- 2-4 $\otimes$ 1-2GeV/c
- 2-4 $\otimes$ 2-4GeV/c
- 4-10 $\otimes$ 2-4GeV/c

✧ Similar near and away-side trends
✧ $p_T$ 2-4x2-4, 4-10x2-4 GeV/c: in-plane $\geq$ out-of-plane
  - Consistent with the parton energy loss picture
✧ $p_T$ 2-4x1-2 GeV/c
  - 0-10%: Out-of-plane > In-plane
  - 40-50%: In-plane > Out-of-plane
  - More than 1σ significance of total systematics
STAR Result of $\Psi_2$ Dependent Correlations

Au+Au 200 GeV, 20-60%, $3<p_T^{(t)}<4$ GeV/c, $1<p_T^{(a)}<2$ GeV/c, $|\eta|<1$  $v_n(n=2,3,4)$ subtracted

Consistent with the results in mid-central collisions by PHENIX
Interpretation of $\Psi_2$ Dependent Correlations

Central

Mid-central

Energy Deposit
Energy Re-distribution

Fragmentation
Penetration
Re-distribution Dominance
Penetration Dominance

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Hydro + energy redistribution

Purposes and Methods

- **Purpose of this study**
  
  Study the collective response to jet propagation in QGP transport of the jet's lost energy

- **Method**
  
  Hydrodynamical simulations of di-jet asymmetric events in heavy ion collisions

- **Relativistic hydrodynamic equations with source terms**
  
  Hydrodynamic equations with incoming energy and momentum
  
  $\partial_\mu T^{\mu\nu} = J^\nu$

  $T^{\mu\nu}$: energy-momentum tensor of the QGP fluid
  
  $J^\nu$: source term (energy-momentum deposit from jets)

- **Source terms**
  
  Assume sudden thermalization of deposited momentum inside a fluid cell:
  
  $$J^\mu(x) = -\sum_a \frac{dp_a^\mu}{dt} \delta^{(3)}(x - x_a(t))$$

  $a$: index for each jet particle

- **Collective flow induced by a jet**
  
  Test study result in the case of 1-jet traveling through a uniform fluid

  - Mach cone
    
    Interference of sound waves induced by a source moving at supersonic speed
  
  - Vortex
    
    Vortex ring around the jet passage in 3-D space

  Mach cone carries information about jet energy loss and properties of QGP
Integrated Yield vs Associate Angle from $\Psi_3$

Au+Au 200GeV $\Psi_3$ dependence

- 2-4 $\otimes$ 1-2GeV/c
- 2-4 $\otimes$ 2-4GeV/c
- 4-10 $\otimes$ 2-4GeV/c

Near Side: $|\Delta \phi|<\pi/4$

Away Side: $|\Delta \phi-\pi|<\pi/4$

✧ Weak centrality dependence
✧ Event-plane dependence is not clearly seen
  – Flat within systematic uncertainties

$20\text{-}30\%$
Out Look : Event-Shape Engineering

✧ Q-vector

\[ Q_{n,x} = \sum_i^{M} \cos(n\phi_i); \quad Q_{n,y} = \sum_i^{M} \sin(n\phi_i); \]

✧ Selection of flow rich events

✧ Differential analysis of medium response

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Summary

✧ $v_n$ subtracted correlations are presented

✧ Non-monotonic path-length dependence is seen in $\Psi_2$ dependent correlations at low $p_T$
  – Can be taken as re-distribution of deposited energy?

✧ Non-$\Psi_3$ dependence is observed due to large systematics
BACK UP
Significance of Double-Hump

✧ Examined the significance of Double-Hump in terms of $v_4$ systematics
✧ $v_2$ and $v_3$ are fixed in flow subtractions but $v_4$ is varied $\pm 1\sigma$
✧ Lower boundary of yellow band covers that of green band
✧ Significance is $\pm 1\sigma$ level of $v_4$ systematics

$\Delta \phi = \phi^a - \phi^t [\text{rad}]$
Zero Yield at Near-Side

- Correlation and Pure Flow is fitted at $\Delta \phi = 0$
- Double-hump is not so sensitive to flow subtraction
Consistency check : high-p_T trigger

✧ Three-Centralities
  – 0-20, 20-40, 40-60%
✧ Particle Selections
  – Trigger p_T: 5-10 GeV/c
  – Associate p_T : 1-10 GeV/c
✧ Subtracted Backgrounds
  – Only v_2
✧ Consistent with previous PHENIX results
  (PRC78.014901)

○ : This Analysis
○ : PRC78.014901
Consistency check: mid-\( p_T \) trigger

- Three-Centralities
  - 0-20, 20-40, 40-60%
- Particle Selections
  - Trigger \( p_T \): 4-5 GeV/c
  - Associate \( p_T \): 1-5 GeV/c
- Subtracted Backgrounds
  - Only \( v_2 \)
- Consistent with previous PHENIX results (PRC78.014901)

- This Analysis
- PRC78.014901

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Consistency check: low-$p_T$ trigger

✧ Three-Centralities
  – 0-20, 20-40, 40-60%
✧ Particle Selections
  – Trigger $p_T$: 2-4 GeV/c
  – Associate $p_T$: 1-4 GeV/c
✧ Subtracted Backgrounds
  – Only $v_2$
✧ Consistent with previous PHENIX results (PRC78.014901)

This Analysis

: PRC78.014901

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High $p_T$ Trigger Two-Particle Correlations

Au+Au 200GeV, $v_2$, $v_3$ & $v_{4\{\Psi\}}$ subtracted

1/N \, dN/d(Δφ)

Δφ = φ_{asso} - φ_{trig} [rad]

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Intermediate $p_T$ Two-Particle Correlations

$\text{Au+Au } 200\text{GeV, } v_2, v_3 \text{ & } v_4^{\Psi}$ subtracted

$\Delta \phi = \phi_{\text{asso}} - \phi_{\text{trig}} \text{ [rad]}$

$0-10\%$

$p_T^{t,a}:4-10 \times 4-10$

$10-20\%$

$p_T^{t,a}:2-4 \times 2-4$

$20-30\%$

$p_T^{t,a}:2-4 \times 1-2$

$30-40\%$

$p_T^{t,a}:2-4 \times 0.5-1$

$40-50\%$

$p_T^{t,a}:1-2 \times 1-2$

$0-10\%$

$p_T^{t,a}:1-2 \times 0.5-1$

$10-20\%$

$p_T^{t,a}:1-2 \times 0.5-1$

$20-30\%$

$p_T^{t,a}:1-2 \times 0.5-1$

$30-40\%$

$p_T^{t,a}:1-2 \times 0.5-1$

$40-50\%$

$p_T^{t,a}:1-2 \times 0.5-1$
$\Psi_2$ Dependent Correlations: $p_T$ 2-4x1-2 GeV/c

$\text{Au+Au } \sqrt{s_{NN}}=200\text{GeV, Pure Flow: } v_n (n=2,3,4) + \langle \cos 4(\Psi_2'\Psi_4') \rangle \text{ by ZYAM}$

$\Delta \phi = \phi^A - \phi^t [\text{rad}]$

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$\Psi_3$ Dependent Correlations : $p_T$ 2-4x1-2 GeV/c

Au+Au $\sqrt{s_{\text{NN}}}=200$GeV, Pure Flow: $v_n$ (n=2,3,4) + $\langle \cos 4(\Psi^2-\Psi^3) \rangle$ by ZYAM

![Graph showing $\Psi_3$ dependent correlations](image)

$\Delta \phi = \phi^a - \phi^t$ [rad]

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$\Psi_2$ Dependent Correlations : $p_T$ 2-4x2-4 GeV/c

Au+Au $\sqrt{s_{NN}}=200$ GeV, Pure Flow: $v_n (n=2,3,4) + \langle \cos 4(\Psi_2 - \Psi_4) \rangle$ by ZYAM

$\Delta \phi = \phi^a - \phi^t$ [rad]
$\Psi_3$ Dependent Correlations: $p_T$ 2-4x2-4 GeV/c

Au+Au $\sqrt{s_{NN}}=200$ GeV, Pure Flow: $v_n(n=2,3,4) + \cos 4(\Psi_2^{T}\Psi_3)$ by ZYAM

\[ \Delta \phi = \phi^a - \phi^t [\text{rad}] \]
$\Psi_2$ Dependent Correlations : $p_T$ 4-10x2-4 GeV/c

Au+Au $\sqrt{s_{NN}}=200$ GeV, Pure Flow: $v_n (n=2,3,4) + <\cos 4(\Psi_2-\Psi_4)>$ by ZYAM

$\Delta \phi = \phi^a - \phi^t$ [rad]

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$\Psi_3$ Dependent Correlations: $p_T$ 4-10x2-4 GeV/c

Au+Au $\sqrt{s_{NN}}$=200GeV, Pure Flow: $v_n(n=2,3,4) + <\cos(\Psi_2^\pm \Psi_4)>$ by ZYAM

$\Delta\phi = \phi^a - \phi^t [\text{rad}]$
$\Psi_2$ Dependent Correlations : $p_T$ 2-4x1-2 GeV/c
$\Psi_3$ Dependent Correlations: $p_T$ 2-4x1-2 GeV/c

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Gravity Position of Two-Particle Correlations

**Definition**

\[ A_{LR} = \frac{\int d\Delta \phi \Delta \phi Y(\Delta \phi)}{\int d\Delta \phi Y(\Delta \phi)} - \begin{cases} 0 & \text{if near} - \text{side} \\ \pi & \text{if away} - \text{side} \end{cases} \]

**Integral Ranges**

Near – Side : \[ |\Delta \phi| < \pi / 3 \]

Away – Side : \[ |\Delta \phi - \pi| < \pi / 3 \]
Gravity position vs trigger angle from $\Psi_2$
Gravity position vs trigger angle from $\Psi_3$

- **Near:** $|\Delta \phi| < \pi/3$
  - 0-10%
  - 10-20%
  - 20-30%
  - 30-40%
  - 40-50%

- **Away:** $|\Delta \phi| > \pi/3$
  - 0-10%
  - 10-20%
  - 20-30%
  - 30-40%
  - 40-50%

$2014/7/19$
Ghost track
- A single particle is counted as \textbf{two} tracks
Merged tracks
- \textbf{Two} particles are counted as \textbf{one} track
Real/Mix pair ratio should be 1 if an ideal detector
Systematic Uncertainties

✧ Flow $v_n$ measurements
  – Systematic difference within RXN segments
  – Rapidity dependence of EP : RXN-BBC difference
  – Matching cut of CNT particles

✧ Two-particle correlations
  – Systematics from $v_n$
  – Matching cut of CNT particles

✧ Unfolding of event plane dependent correlations
  – Difference of two methods : Fit & Iteration Methods
  – Parameter in the iteration method
Azimuthal Anisotropy of PTY

- Integrated yield vs associate angle from EP is translated into azimuthal anisotropy $v_n^{PTY}$
- $v_n^{PTY}$ can be compared with single particle $v_n$ because the dimension of PTY is “# of particles”

- $v_n^{PTY}$ is extracted via Fourier fitting

\[ F(\phi^a - \Psi_2) = a \{ 1 + 2v_2^{PTY} \cos 2(\phi^a - \Psi_2) + 2v_4^{PTY} \cos 4(\phi^a - \Psi_2) \} \]

\[ F(\phi^a - \Psi_3) = a \{ 1 + 2v_3^{PTY} \cos 3(\phi^a - \Psi_3) \}, \]

- Anisotropy of associate particles per a trigger $\rightarrow$ Anisotropy of associate particles per a event

\[ v_{n, cor}^{PTY} = v_n^{PTY} + v_n^{trig} \cos n(\phi^t - \phi^a) \]
\( v_2^{PTY} \)

✧ Positive hadron \( v_2 \) (Hydrodynamics)

✧ Positive \( \pi^0 \) \( v_2 \) (Parton energy-loss)
  – Superposition of those assembles only positive \( v_2 \)

✧ Near & away-side \( v_2^{PTY} \)
  – Positive value at 40-50%
  – Near-side negative value at 0-10%

✧ New effects need to be considered

✧ Possible re-distribution of deposited energy in longer path direction
\(\mathbf{V}_3^{PY}\)

- Positive hadron \(v_3\) (Hydrodynamics)
- Near & away-side \(v_3^{PY}\) at 30-40%
  - Positive near-side
  - Negative away-side
- Weak centrality dependence

- Different near & away-side, as well as centrality dependences from those of \(v_2^{PY}\)
- Possible different evolution processes between the 2nd- and 3rd-order geometry planes
Interpretation of $\Psi_3$ Dependent Correlations

Away-Side

- Energy Deposit
- Energy Re-distribution

Near-Side

- Fragmentation
- Penetration

Redistribution Dominance  Penetration Dominance

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Collision Centrality

- A degree of overlap of two colliding nuclei
  - Distance between center of the nuclei $\Rightarrow$ multiplicity $\Rightarrow$ charge deposited in BBC
- Require each percentile contains same # of events
  - Most-central Collision : 0%
  - Most-peripheral Collision : 100% (PHENIX determines it up to 92%)

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Nuclear Modification Factor $R_{AA}$

$R_{AA} = \frac{d^2N^{AA}/dp_Td\eta}{N_{coll}d^2N^{pp}/dp_Td\eta}$

✧ Ratio of invariant yield scaled by that in p+p collision with scale
  
  – $R_{AA} < 1$ (suppression), $R_{AA} = 1$ (no change), $R_{AA} > 1$ (enhance)

✧ Suppression of hadron production

✧ No suppression of direct photon
Contributions of $v_n$ ($n > 2$) in correlations

2Par. Correlation

$C(\Delta\phi) = b^{2P} (1 + 2v^{2P}_{1,1} \cos \Delta\phi + 2\sum_{n=2}^{6} v^{EP}_{n} \cos n\Delta\phi)$

From 2PC method
From EP method

Double-hump & ridge of long-rapidity correlation explained
Short-rapidity correlation with $v_n$ subtraction to discuss parton behavior

Track at $|\eta| < 2.5$ with EP from full FCAL
3.3 < $|\eta|$ < 4.8
Data Set & Particle Selection

✧ PHENIX year 2007 Experiment
✧ Au+Au collisions at $\sqrt{s_{NN}}=200$ GeV
  – Minimum Bias trigger 4.4 billion events
✧ Charged hadron selection
  – $2\sigma$ cut of track-hit matching
  – Electron veto
  – Energy/momentum cut of high $p_T$ particles for background rejection
    • $E^{EMC}<0.30+0.20*p_T$ rejected for $p_T>5.0$ GeV/$c$
  – Pair cut of miss-reconstructed hadron pairs
Tracking Efficiency

Efficiency correction by ratio of uncorrected invariant yield over fully corrected ones

\[ \varepsilon = \frac{\sigma^{uncor}}{\sigma^{cor}} \]

Ratio calculated by fitting functions to the invariant yields

Fit Function

\[ F(p_T) = p_0 \times \left( \frac{p_1}{p_1 + p_T} \right)^{p_2} \]
Event Plane Calibration

$\phi_i$ : Azimuthal angle

$\mathcal{W}_i$ : Weight (Charge etc.)

## Raw distribution

$$Q_x = \sum_i w_i \cos(n\phi_i), \quad Q_y = \sum_i w_i \sin(n\phi_i)$$

$$\Psi_n = \frac{1}{n} \tan^{-1} \left( \frac{Q_y}{Q_x} \right)$$

## Re-centering

$$Q_x^{\text{Rec}} = \frac{Q_x - \langle Q_x \rangle}{\sigma_x}, \quad Q_y^{\text{Rec}} = \frac{Q_y - \langle Q_y \rangle}{\sigma_y}$$

$$\Psi_n^{\text{Rec}} = \frac{1}{n} \tan^{-1} \left( \frac{Q_y^{\text{Rec}}}{Q_x^{\text{Rec}}} \right)$$

## Fourier correction

$$n \Psi_n^{\text{Fourier}} = n \Psi_n^{\text{Rec}} + n \Delta \Psi_n$$

$$n \Delta \Psi_n = \sum_k \left\{ A_k \cos(kn \Psi_n^{\text{Rec}}) + B_k \sin(kn \Psi_n^{\text{Rec}}) \right\}$$

$$A_k = -\frac{2}{k} \langle \cos(kn \Psi_n^{\text{Rec}}) \rangle, \quad B_k = \frac{2}{k} \langle \sin(kn \Psi_n^{\text{Rec}}) \rangle$$
Event Plane Resolution

EP Resolution

PRC 58.1671 (1998)

 Resolution +/-\(\eta\)

\[
\sigma_{EP}^n = \sqrt{\left\langle \cos kn (\Psi_n^{EP+\eta} - \Psi_n^{EP-\eta}) \right\rangle}
\]

= \left\langle \cos kn (\Psi_n^{EP+/-\eta} - \Psi_n) \right\rangle

= \frac{\pi}{8} \chi_n^2 \left[ I_{(k-1)/2} \left( \frac{\chi_n^2}{4} \right) + I_{(k+1)/2} \left( \frac{\chi_n^2}{4} \right) \right]^2

 Resolution +&/-\(\eta\)

\[\chi_n \rightarrow \sqrt{2} \chi_n\]

\[
\sigma_{EP}^n = \frac{\pi}{8} 2 \chi_n^2 \left[ I_{(k-1)/2} \left( \frac{2\chi_n^2}{4} \right) + I_{(k+1)/2} \left( \frac{2\chi_n^2}{4} \right) \right]^2
\]
$v_n$ systematics : RXN segments

![Graph showing $v_n$ versus $p_T$ for different RXN segments.](image-url)
$v_n$ systematics: Matching Cut

**Au+Au $\sqrt{s_{NN}}=200$GeV, EP Method**

- **V2**
  - 0-10, all RXN segments
  - 10-20, (a)
  - 20-30, (b)
  - 30-40, (c)
  - 40-50, (d)

- **V3**
  - (f)
  - (g)
  - (h)
  - (i)
  - (j)

- **V4**
  - (k)
  - (l)
  - (m)
  - (n)
  - (o)

- **V4(Ψ2)**
  - (p)
  - (q)
  - (r)
  - (s)
  - (t)

$p_T$ [GeV/c]
$v_n$ systematics: RXN-BBC Difference

Au+Au $\sqrt{s_{NN}}=200\text{GeV}$, EP Method

2$\sigma$ matching

$p_T$ [GeV/c]
## Table of total $v_n$ systematic uncertainties

### Table 3.8: Summary of percentile ratio of $v_n$ systematic uncertainties

<table>
<thead>
<tr>
<th>Centrality %</th>
<th>$p_T$ GeV/c</th>
<th>$v_2$ sys. %</th>
<th>$v_3$ sys. %</th>
<th>$v_4$ sys. %</th>
<th>$v_4{\Psi_2}$ sys. %</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-10</td>
<td>0.5-1.0</td>
<td>5.449</td>
<td>6.387</td>
<td>24.87</td>
<td>48</td>
</tr>
<tr>
<td></td>
<td>1.0-2.0</td>
<td>4.32</td>
<td>4.911</td>
<td>10.1</td>
<td>14.66</td>
</tr>
<tr>
<td></td>
<td>2.0-4.0</td>
<td>4.363</td>
<td>4.131</td>
<td>4.412</td>
<td>11.39</td>
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<tr>
<td></td>
<td>4.0-10.0</td>
<td>10.43</td>
<td>6.184</td>
<td>21.67</td>
<td>191.3</td>
</tr>
<tr>
<td>10-20</td>
<td>0.5-1.0</td>
<td>3.658</td>
<td>7.992</td>
<td>28.53</td>
<td>12.17</td>
</tr>
<tr>
<td></td>
<td>1.0-2.0</td>
<td>2.891</td>
<td>6.431</td>
<td>20.16</td>
<td>12.27</td>
</tr>
<tr>
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<td>2.0-4.0</td>
<td>2.69</td>
<td>6.163</td>
<td>27.64</td>
<td>13.72</td>
</tr>
<tr>
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<td>4.0-10.0</td>
<td>3.124</td>
<td>13.62</td>
<td>19.09</td>
<td>32.09</td>
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<tr>
<td>20-30</td>
<td>0.5-1.0</td>
<td>2.811</td>
<td>9.469</td>
<td>35.48</td>
<td>9.633</td>
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<td>7.818</td>
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<td>6.163</td>
<td>28.03</td>
<td>6.577</td>
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<td>4.0-10.0</td>
<td>2.98</td>
<td>9.503</td>
<td>32.24</td>
<td>12.21</td>
</tr>
<tr>
<td>30-40</td>
<td>0.5-1.0</td>
<td>2.506</td>
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<td>35.81</td>
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<td>1.0-2.0</td>
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<td>29.88</td>
<td>6.509</td>
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<td>2.0-4.0</td>
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<td>9.673</td>
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<td>2.934</td>
<td>14.18</td>
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<tr>
<td>40-50</td>
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<td>2.575</td>
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<td>6.338</td>
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<td>12.06</td>
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<td>11.7</td>
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<td>10.71</td>
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<tr>
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<td>4.0-10.0</td>
<td>7.877</td>
<td>33.53</td>
<td>77.07</td>
<td>29.33</td>
</tr>
</tbody>
</table>
**Systematics of Correlations**

- **Systematics propagated from \( v_n \) measurements**
  - Varying \( v_n \) value \( \pm 1\sigma \) (# of harmonics \( 3 \times \pm 1\sigma \) \( 2 = 6 \) combinations)
  - Systematics: RMS of above 6 combinations

- **Systematics from matching cut**
  - Systematics: Difference between \( 2.5\sigma - 2.0\sigma \) (main)

- **Total Systematics**
  - Quadrature-sum of above two systematics

---

**Centrality:20-30%**

- \( v_2 \ v_3 \ v_4 \) sub.
  - **Centrality:20-30%**
    - \( v_2 \pm 1\sigma \)
    - \( v_3 \pm 1\sigma \)
    - \( v_4 \pm 1\sigma \)
    - Systematics

---

**Centrality:20-30%**

- \( v_2 \ v_3 \ v_4 \) sub.
  - \( \sigma=2.0 \)
  - \( \sigma=2.5 \)
  - \( 2.5\sigma-2.0\sigma \)
EP Resolution in Monte Carlo

- Analytical formula of EP Resolution (RXN:S+N) as a function of $\chi_n$
  
  \[ \langle \cos [kn(\Psi_n^{obs} - \Psi_n^{real})] \rangle = \frac{\sqrt{\pi}}{2\sqrt{2}} \chi_n e^{-\chi_n^2/4} \left[ I_{(k-1)/2} \left( \frac{\chi_n^2}{4} \right) + I_{(k+1)/2} \left( \frac{\chi_n^2}{4} \right) \right]. \]

- Relative distribution between real and observed EP calculated using $\chi_n$
  
  \[ \frac{dN^{\text{ev}}}{d[kn(\Psi_n^{obs} - \Psi_n^{real})]} = \frac{1}{\pi} e^{-\chi_n^2/2} \left[ 1 + z\sqrt{\pi}[1 + \text{erf}(z)]e^{z^2} \right] \quad z = \frac{1}{\sqrt{2}} \chi_n \cos n(\Psi_n^{obs} - \Psi_n^{real}) \]

PRD 48.1132 (1993)

PRC 58.1671 (1998)
$\Psi_2 - \Psi_4$ correlation in Monte Carlo

- $\Psi_2 - \Psi_4$ correlation at $p_T$ 1-2&2-4GeV: $\langle \cos [4(\Psi_2 - \Psi_4)] \rangle = v_4 \{\Psi_2\} / v_4 \{\Psi_4\}$
  - To avoid jet contribution to the $\Psi_2 - \Psi_4$ correlation

- Obtain $\chi_{42}$ & reconstruct relative distribution between $\Psi_2$ & $\Psi_4$

\[
\langle \cos [4(\Psi_2 - \Psi_4)] \rangle = \frac{\sqrt{\pi}}{2\sqrt{2}} \chi_{42} e^{-\chi_{42}^2/4} \left[ I_0 \left( \frac{\chi_{42}^2}{4} \right) + I_1 \left( \frac{\chi_{42}^2}{4} \right) \right]
\]

\[
\frac{dN^{eve}}{d[kn(\Psi_n^{obs} - \Psi_n^{real})]} = \frac{1}{\pi} e^{-\chi_n^2/2} \left[ 1 + z\sqrt{\pi}[1 + \text{erf}(z)]e^{-z^2} \right]
\]

$z = \frac{1}{\sqrt{2}} \chi_n \cos n(\Psi_n^{obs} - \Psi_n^{real})$
$\Psi_2 - \Psi_3$ correlation

A : RXN North
B : BBC South
C : MPC North
D : MPC South
EP Resolution Correction : Iteration-1

✧ Trigger bin is also smeared due to limited EP resolution as $v_n$

– Add an offset $\lambda=1.0$ to correlation $Y$ to avoid possible divisions by zero

Raw Correlation

Offset | Trigger Bin

| 0 | 1 - $Y(0,k)$ |
| 1 | 1 - $Y(1,k)$ |
| 2 | 1 - $Y(2,k)$ |
| 3 | 1 - $Y(3,k)$ |
| 4 | 1 - $Y(4,k)$ |
| 5 | 1 - $Y(5,k)$ |
| 6 | 1 - $Y(6,k)$ |
| 7 | 1 - $Y(7,k)$ |

$A(k) = \Delta\phi$ Bin

$A(k) = \begin{pmatrix} 1 - Y(0,k) \\ 1 - Y(1,k) \\ 1 - Y(2,k) \\ 1 - Y(3,k) \\ 1 - Y(4,k) \\ 1 - Y(5,k) \\ 1 - Y(6,k) \\ 1 - Y(7,k) \end{pmatrix}$

$k = 0, \cdots, 23$

Smearing Effect

Trigger Bin

$A(k) = \begin{pmatrix} s_0 & s_1 & s_2 & s_3 & s_4 & s_3 & s_2 & s_1 \\ s_1 & s_0 & s_1 & s_2 & s_3 & s_4 & s_3 & s_2 \\ s_2 & s_1 & s_0 & s_1 & s_2 & s_3 & s_4 & s_3 \\ s_3 & s_2 & s_1 & s_0 & s_1 & s_2 & s_3 & s_4 \\ s_4 & s_3 & s_2 & s_1 & s_0 & s_1 & s_2 & s_3 \\ s_3 & s_4 & s_3 & s_2 & s_1 & s_0 & s_1 & s_2 \\ s_2 & s_3 & s_4 & s_3 & s_2 & s_1 & s_0 & s_1 \\ s_1 & s_2 & s_3 & s_4 & s_3 & s_2 & s_1 & s_0 \end{pmatrix}$

$S = \sum_n s_n = 1, s_n$: Ratio from $n^{th}$ away-bin

Smeread Correlation

Correction Matrix

Corrected Correlation

$B(k) = SA(k)$

$C(k) = (c_{ij})$

$c_{ij} = \begin{cases} \frac{A(i,k)}{B(i,k)} & (i = j) \\ 0 & (i \neq j) \end{cases}$

$A_{cor}(k) = C(k)A(k)$
EP Resolution Correction : Iteration-2

- Start of iteration : experimental results (already smeared once)
- Obtained correction is not true
- Iteration until conversions of each coefficients
  - 300 Loops

**Notation in Iteration**

\[ A \rightarrow A^{(n)} \]

\[ B \rightarrow B^{(n)} \]

\[ C \rightarrow C^{(n)} \]

\[ A^{\text{cor}} \rightarrow A^{(n+1)} \]

**Smoothing**

- Preventing a divergence of statistical fluctuations among \( \Delta \phi \) bins
- \( 2r = 0.20 \& 0.30 \)

\[ c_{ii}^{(n)}(k) = (1 - r) c_{ii}^{(n)}(k) + \left( \frac{r}{2} \right) c_{ii}^{(n)}(k - 1) + \left( \frac{r}{2} \right) c_{ii}^{(n)}(k + 1) \]

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EP Resolution Correction : Fitting Method

✧ Assuming correlation yield has anisotropy with respect to EP
✧ Correction by EP resolution as done in \( v_n \) measurements
  — Method by PRC.84.024904(2011)
✧ Offset \( \lambda = 1.0 \) to avoid possible division by zero

\[ \Psi_2 \text{ dependent case} \]

\[
\lambda + Y^{\text{cor}}(\phi_s, \Delta \phi) = \frac{\lambda + b_0 \left[ 1 + 2v_2^{Y} \sigma_2 \cos 2(\phi_s + \Delta \phi) + 2v_4^{Y} \sigma_4 \cos 4(\phi_s + \Delta \phi) \right]}{\lambda + b_0 \left[ 1 + 2v_2^{Y} \cos 2(\phi_s + \Delta \phi) + 2v_4^{Y} \cos 4(\phi_s + \Delta \phi) \right]} \]

\[ \Psi_3 \text{ dependent case} \]

\[
\lambda + Y^{\text{cor}}(\phi_s, \Delta \phi) = \frac{\lambda + b_0 \left[ 1 + 2v_3^{Y} \sigma_3 \cos 3(\phi_s + \Delta \phi) \right]}{\lambda + b_0 \left[ 1 + 2v_3^{Y} \cos 3(\phi_s + \Delta \phi) \right]} \]
Near-Side Integrated Yield vs Associate Angle from $\Psi_2$

Au+Au 200GeV
$\Psi_2$ dependence
Near Side : $|\Delta \phi| < \pi/4$

- 2-4 $\otimes$ 1-2GeV/c
- 2-4 $\otimes$ 2-4GeV/c
- 4-10 $\otimes$ 2-4GeV/c

(a) 0-10%
(b) 10-20%
(c) 20-30%
(d) 30-40%
(e) 40-50%

2014/7/19
Away-Side Integrated Yield vs Associate Angle from $\Psi_2$

Au+Au 200GeV
$\Psi_2$ dependence
Away Side : $|\Delta \phi_{\pi\pi}| < \pi/4$

- **2-4 \times 1-2 GeV/c**
- **2-4 \times 2-4 GeV/c**
- **4-10 \times 2-4 GeV/c**

(a) 0-10%
(b) 10-20%
(c) 20-30%
(d) 30-40%
(e) 40-50%

Integrated Yield

2014/7/19
Near-Side Integrated Yield vs Associate Angle from $\Psi_3$

**Au+Au 200GeV**

$\Psi_3$ dependence
Near Side : $|\Delta \phi| < \pi/4$

- **2-4 $\otimes$ 1-2GeV/c**
- **2-4 $\otimes$ 2-4GeV/c**
- **4-10 $\otimes$ 2-4GeV/c**

---

[Graphs showing integrated yield vs $\phi^a - \Psi_3$ for different pseudorapidity intervals (a) 0-10%, (b) 10-20%, (c) 20-30%, (d) 30-40%, (e) 40-50%).]
Away-Side Integrated Yield vs Associate Angle from $\Psi_3$

**Au+Au 200GeV**

$\Psi_3$ dependence

Away Side: $|\Delta \phi - \pi| < \pi/4$

- $2-4 \otimes 1-2$GeV/c
- $2-4 \otimes 2-4$GeV/c
- $4-10 \otimes 2-4$GeV/c

(a) 0-10%
(b) 10-20%
(c) 20-30%
(d) 30-40%
(e) 40-50%

**2014/7/19**
Anisotropy of particles per a jet ➔
Anisotropy of particles per a event

\[
\left\{ 1 + 2v_n^{PTY} \cos n(\phi^a - \Psi_n) \right\} \times \left\{ 1 + 2v_n^t \cos n(\phi^t - \Psi_n) \right\} \\
= \left\{ 1 + 2v_n^{PTY} \cos n(\phi^a - \Psi_n) \right\} \times \left\{ 1 + 2v_n^t \cos n(\phi^a - \phi^t) \cos n(\phi^a - \Psi_n) \right\} \\
\approx 1 + 2v_n^{PTY} \cos n(\phi^a - \Psi_n) + 2v_n^t \cos n(\phi^a - \phi^t) \cos n(\phi^a - \Psi_n)
\]

\[
v_n^{PTY, cor} = v_n^{PTY} + v_n^{trig} \cos n(\phi^t - \phi^a)
\]