Measurements of Two-Particle Correlations with respect to Higher-Order Event Planes in $\sqrt{s_{NN}}$=200 GeV Au+Au Collisions at RHIC-PHENIX

(RHIC-PHENIX $\sqrt{s_{NN}}$ = 200 GeV 金・金衝突実験における二粒子相関の反応平面依存性の測定)

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Outline

- ✧ **Introduction**
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  - Higher-order event-planes & flow harmonics
  - Previous two-particle correlation measurements and theoretical models
- ✧ **Analysis**
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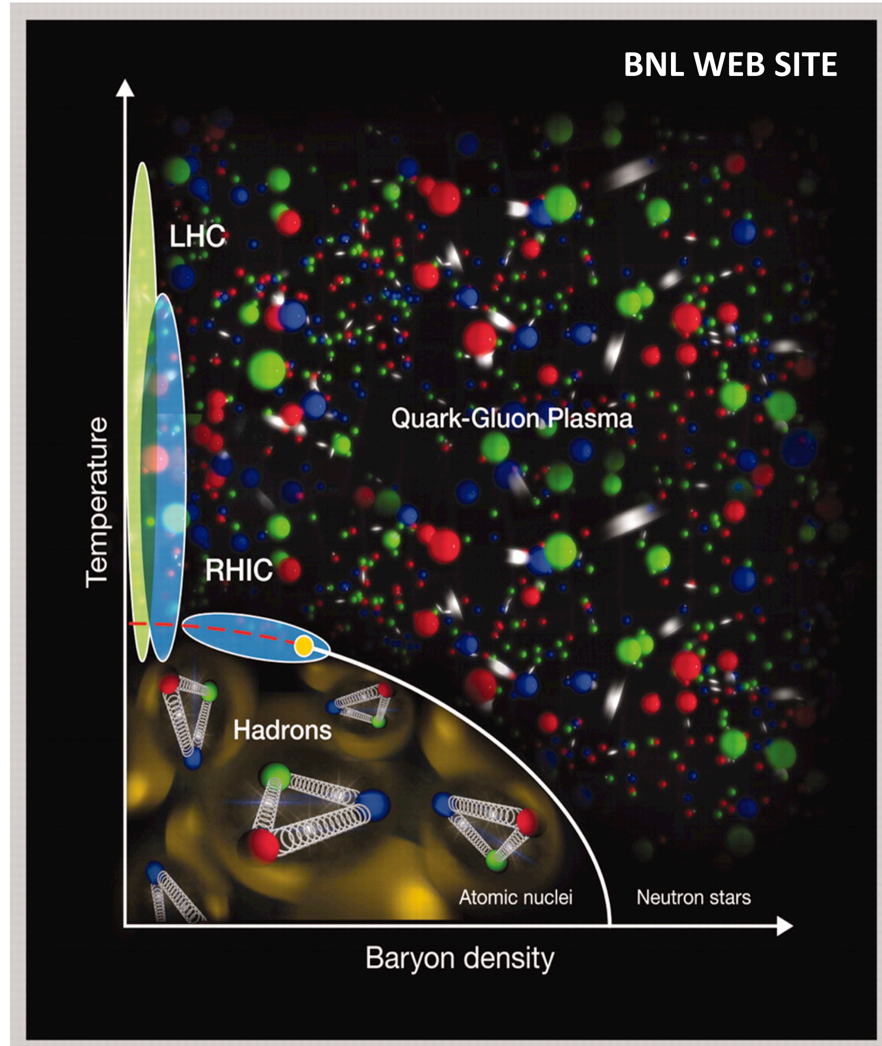
  - Flow and two-particle correlation measurements
  - Correlations with respect to event-planes
- ✧ **Results & Discussion**
  - Two-particle correlations
  - Event-plane dependent correlations
- ✧ **Conclusion**

INTRODUCTION

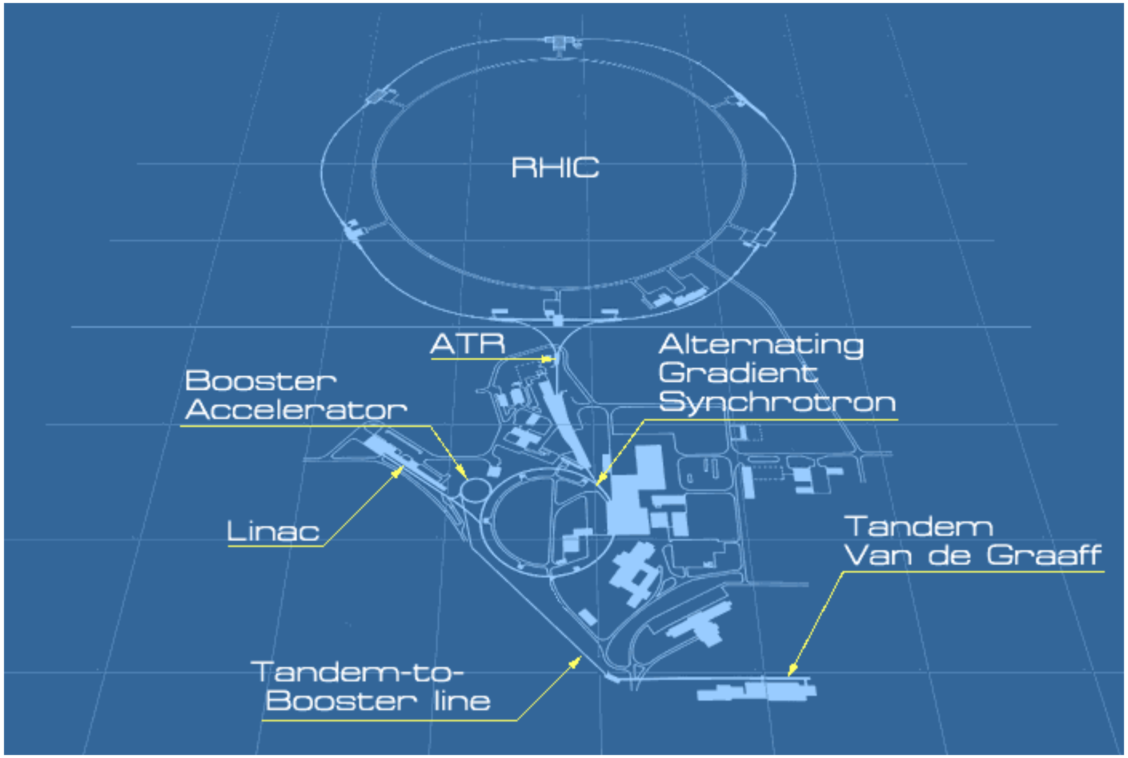
Quark Gluon Plasma (QGP)

- ✧ **A fluid in which quark and gluons are deconfined from hadrons at high energy-density ε & temperature T**
- ✧ **Predicted transition ε & T by Lattice-QCD**
  - $\varepsilon_c \sim 1.0$ [GeV/fm^3]
  - $T_c \sim 170$ [MeV]

F. Karsch, Lect. Notes Phys. 583, 209 (2002)



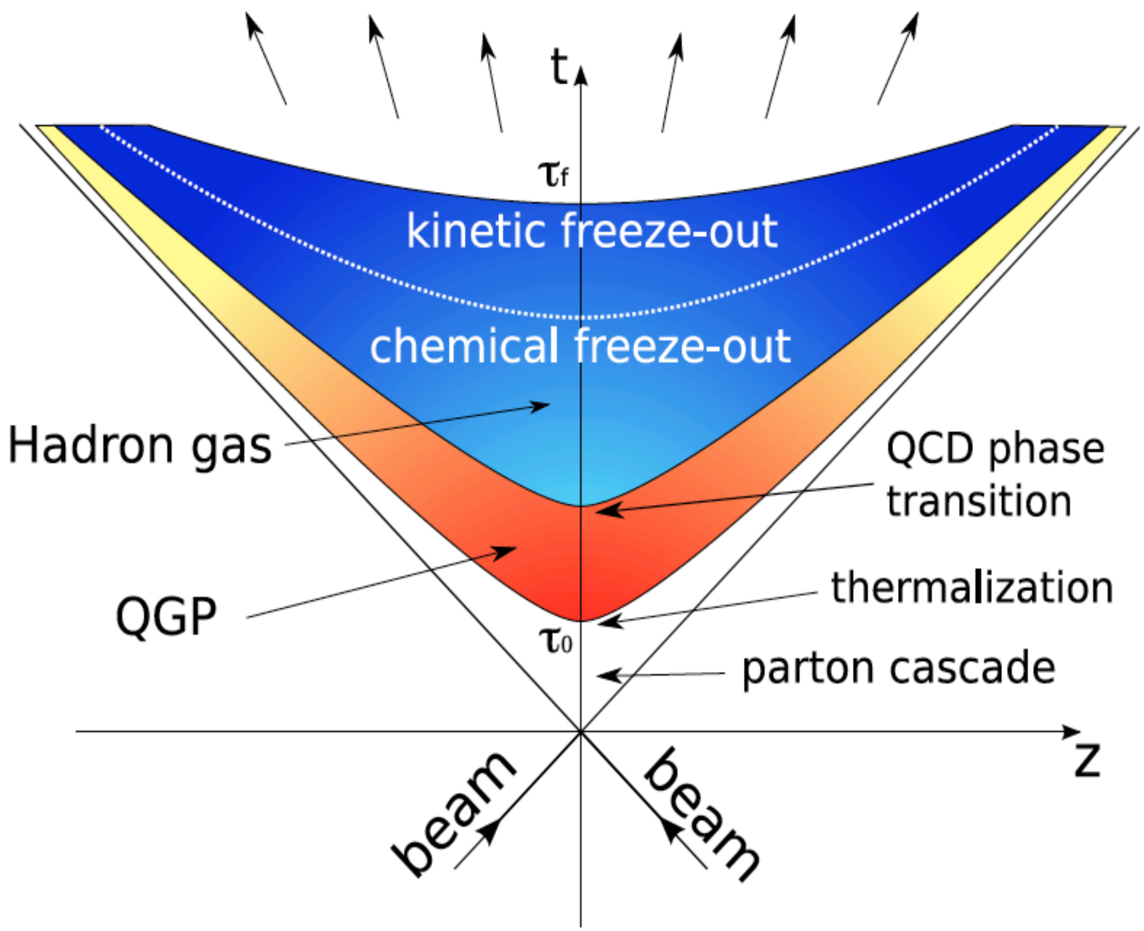
Relativistic Heavy Ion Collider (RHIC)



Year	Collisions System
2001	Au+Au 130 GeV
2002	p+p,Au+Au 200 GeV
2003	p+p, d+Au 200 GeV
2004	Au+Au 62.4, 200 GeV
2005	Cu+Cu 22.4, 62.4, 200 GeV
2006	p+p 62.4, 200 GeV
2007	Au+Au 200 GeV
2008	p+p, d+Au 200 GeV
2009	p+p 200 500 GeV
2010	Au+Au 7.7, 39, 62.4, 200 GeV
2011	Au+Au 19.6, 27, 200
2012	U+U 193GeV, Cu+Au 200 GeV

- **Relativistic heavy Ion collision is an unique tool to form QGP on the Earth**
- **Energy density at RHIC**
  - ε_{RHIC} ~5.0 - 15.0 [GeV/fm^3]
  - $\varepsilon_{RHIC} > \varepsilon_c$

Space-Time Evolution of Heavy Ion Collisions



5: Kinetic freeze-out

4: Chemical freeze-Out
No./spp. of particle fixed

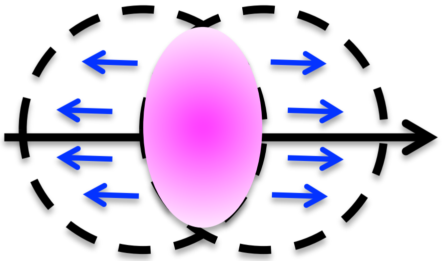
3: Phase transition
➔Hadron Gas

2: Partonic cascade &
thermalization
➔ QGP state

1: Initial collision

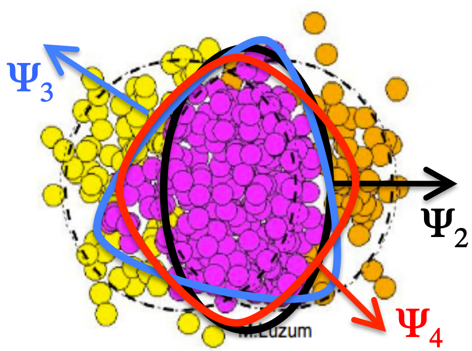
Higher-Order Event-Planes & Flow-Harmonics

Smooth participant density



Expansion to the short-axis direction by pressure gradient

Fluctuating participant density



Expansion to the short-axis directions of event-planes by pressure gradient

✧ **Azimuthal distribution of emitted particles**

$$\frac{dN}{d\phi} \propto 1 + 2v_2 \cos 2(\phi - \Psi_2) + 2v_3 \cos 3(\phi - \Psi_3) + 2v_4 \cos 4(\phi - \Psi_4) \ldots$$

$$v_n = < cosn(\phi - \Psi_n) >$$

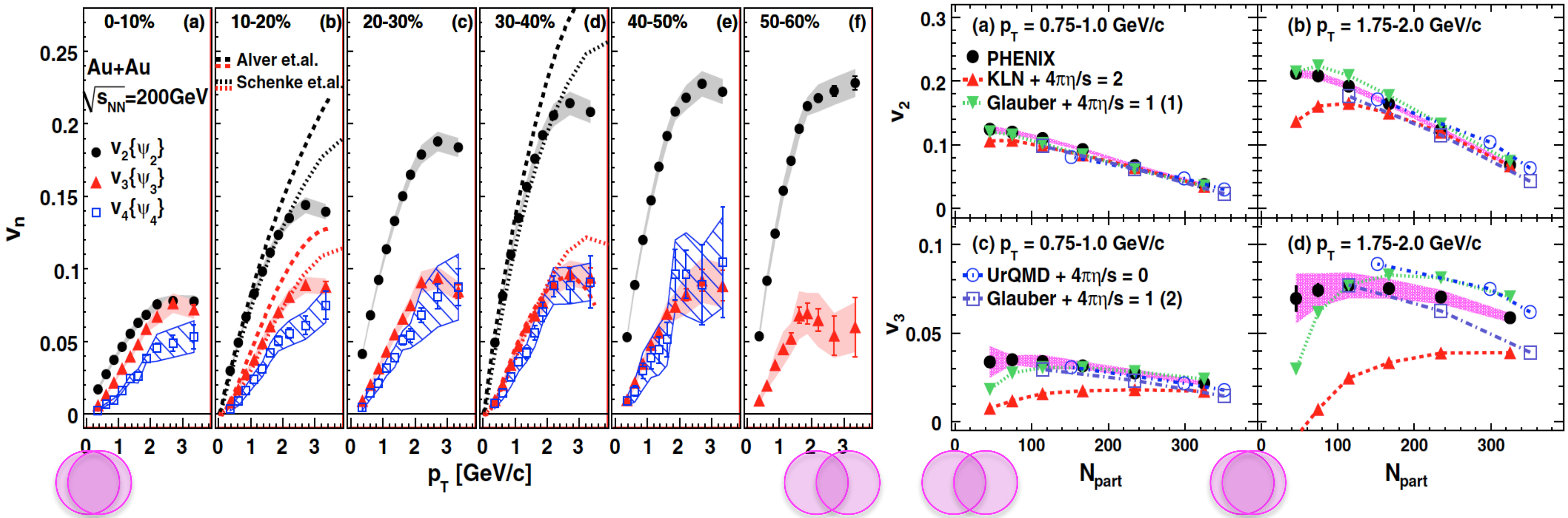
v_n : Higher-order flow harmonics

Ψ_n : Higher-order event planes

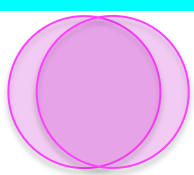
ϕ : Azimuthal angle of emitted particles

Higher-Order Flow Harmonics

PRL107.252301 (2011)



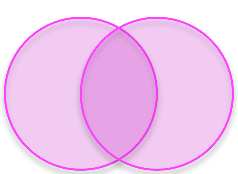
Central Collisions



N_{part} : # of participant nucleons in a collision

Centrality~0%
N_{part} ~394

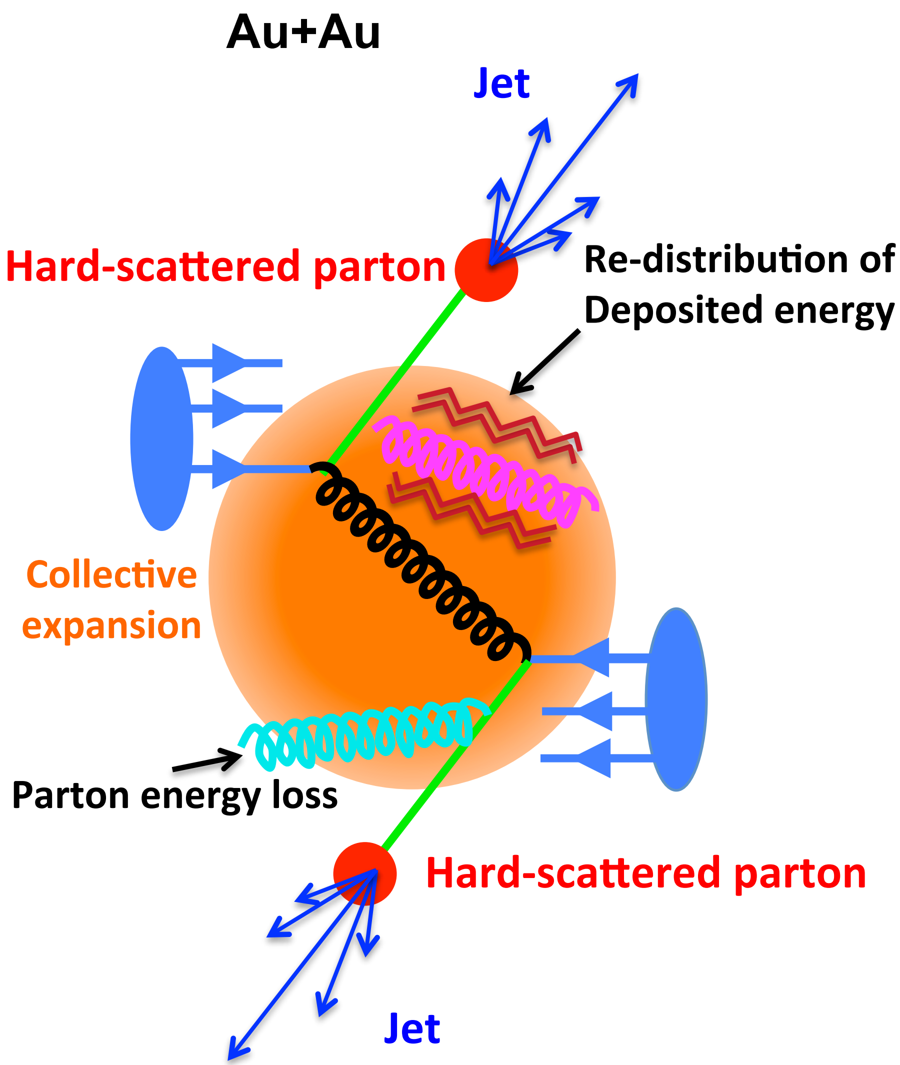
Peripheral Collisions



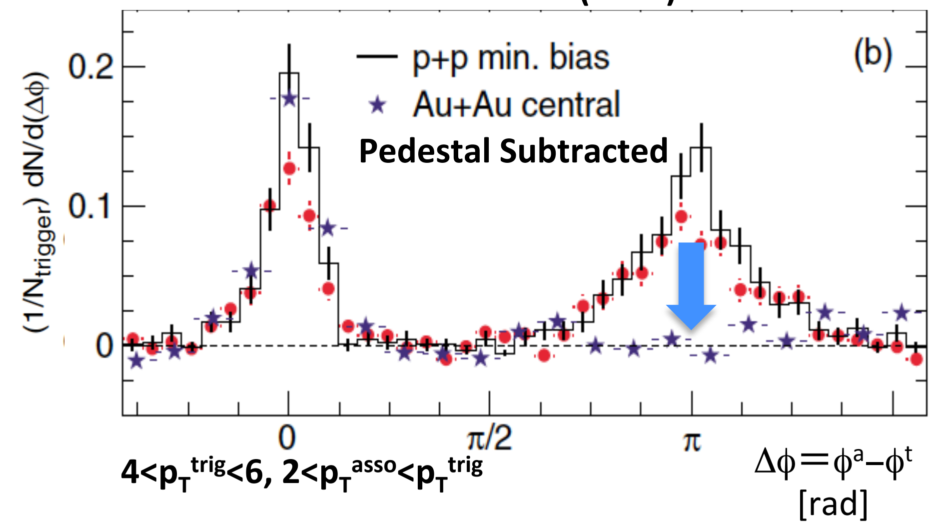
Centrality~100%
N_{part} ~2

- ✧ None-zero v_n(n>2) is observed
- ✧ Degeneracy of models disentangled
  - – Initial Condition, shear viscosity of QGP, different expansion mechanism between v_2 & v_3
- ✧ Backgrounds in correlation functions

Jet-Quenching

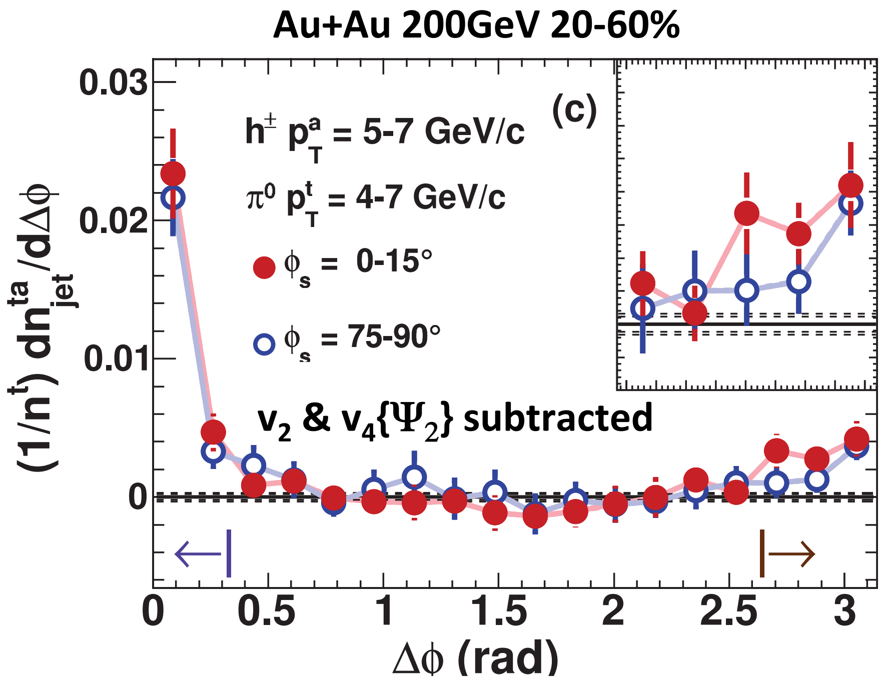


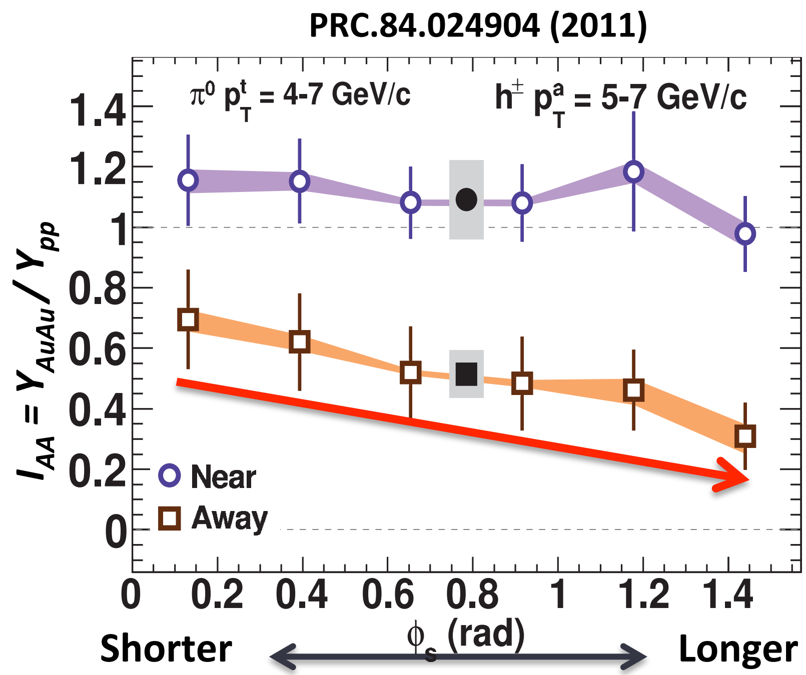
PRL91.072304 (2003)

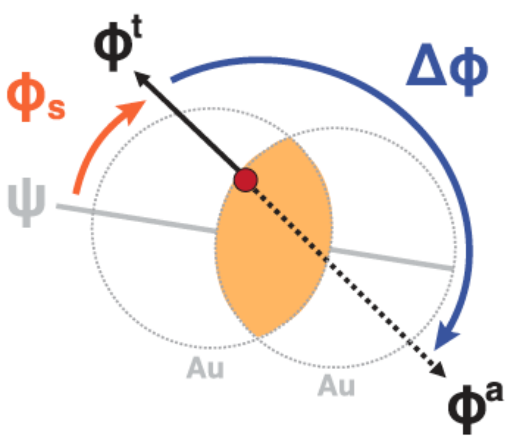


- ✧ **p+p collisions : no suppression**
- ✧ **Suppression in away-side of high p_T correlations**

Ψ_2 Dependence of Away-Side Suppression of high p_T corrs.







✧ **Monotonic suppression with increase of path length**

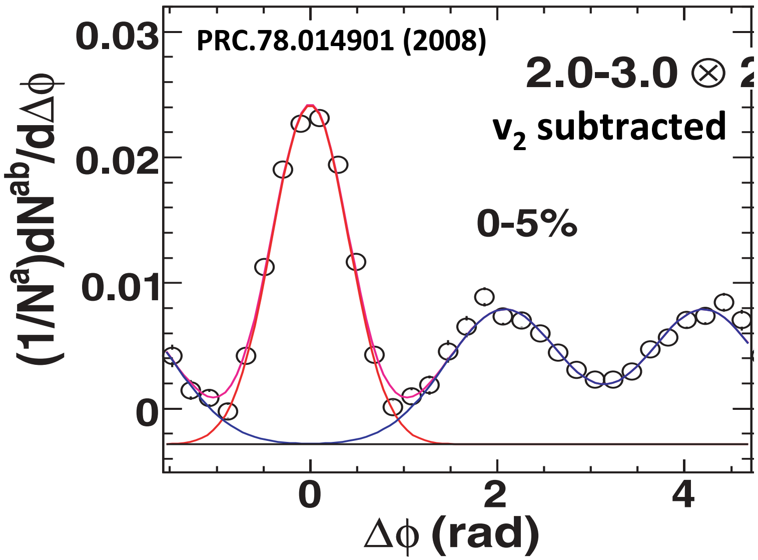
✧ **Parton energy loss depending on path-length**

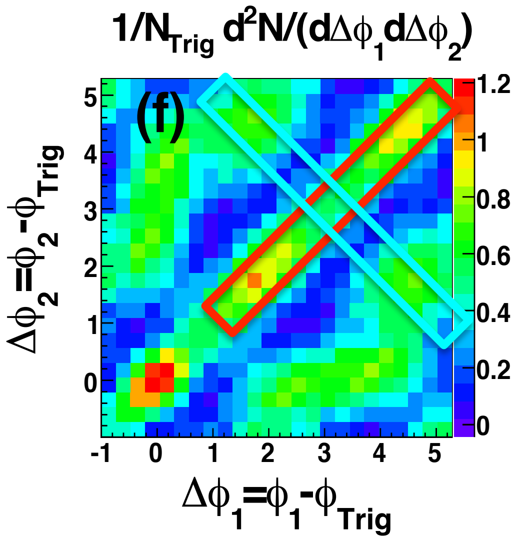
✧ **Similar trends seen in Ψ_3 dependence?**

Conical Emission of Intermediate p_T correlations

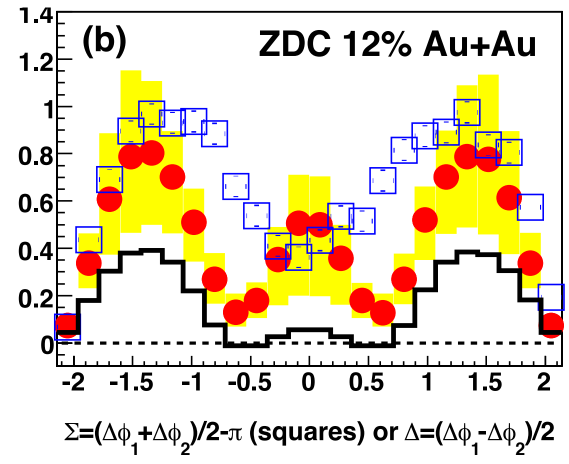
Two-Particle Correlations

Three-Particle Correlations





PRL102.052302 (2009)



- ✧ Away-side double hump in two-particle correlations
- ✧ Conical Emission confirmed via three-particle correlations

Models of Double-Hump : 1

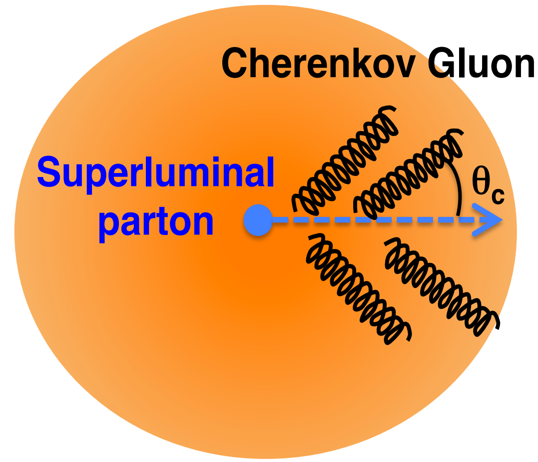
- Cherenkov gluon radiation by superluminal partons

$$\cos\theta_c = 1/n(p)$$

$n(p)$: Index of refraction

p : Gluon Momentum

PRL 96.172302 (2006)



- Shock-wave by supersonic partons

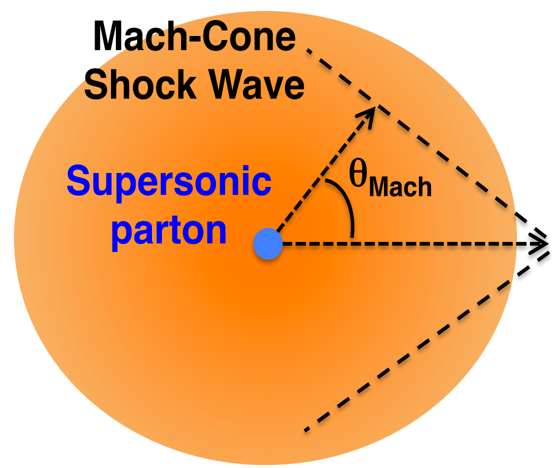
$$\cos\theta_{Mach} = c_s/v_{part}$$

c_s : Speed of sound

v_{part} : Speed of parton

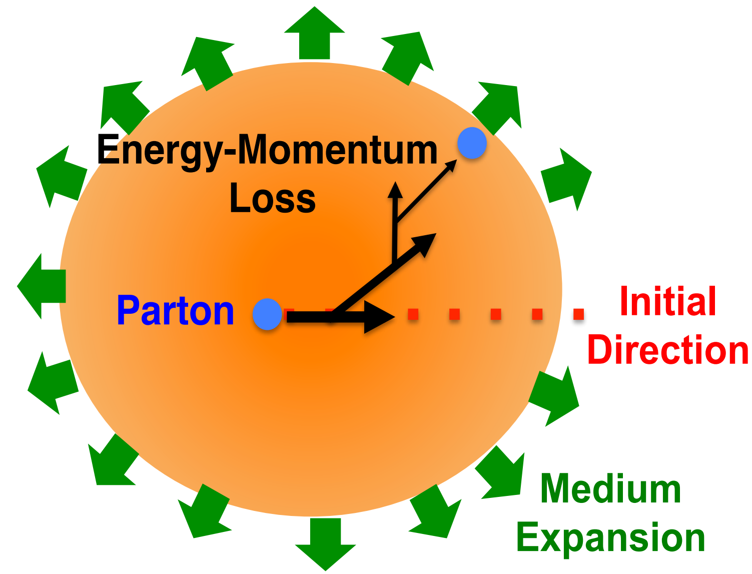
Phys. Rev. C 73, 011901(R), (2006)

PRL 105.222301 (2010)



Models of Double-Hump : 2

✧ **Energy-momentum loss + expanding medium**



$$\partial_\mu T^{\mu\nu} = S^\nu$$

$$S^\nu(t,\vec{x}) = \frac{1}{(\sqrt{2\pi}\sigma)^3}\exp\left[-\frac{[\vec{x}-\vec{x}_{jet}(t)]^2}{2\sigma^2}\right]$$

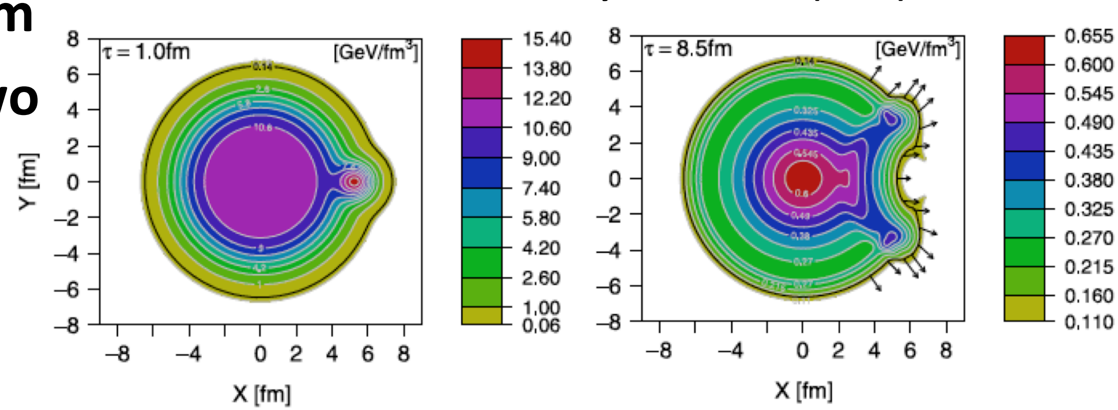
$$\times\left(\frac{dE}{dt},\frac{dM}{dt},0,0\right)\left[\frac{T(t,\vec{x})}{T_{max}}\right]^3$$

PRL 105.222301 (2010)

✧ **Hot spot+ expanding medium**

– **Split of the hot spot into two directions**

Phys. Let. B 712 (2012) 226-230



Motivation of this Dissertation

- ✧ **Provide experimental results after v_n subtraction**
  - **Centrality & p_T dependence, double-humps etc.**
  - **Revisit of previous models for double-humps**

- ✧ **Examine the path length dependence of Ψ_2 dependent correlations after v_n subtraction**

- ✧ **Search for differences between Ψ_2 & Ψ_3 dependent correlations which may reflects possible different evolution processes between the 2nd- and 3rd-order geometry planes**

My Contributions

*Oral Presentation *Collaboration work *Analysis

M1,2 : AY 2008~2009

DNP-JPS Joint Meeting

PHENIX Run9 PHENIX Run10
Detector Maintenance

D1 (RIKEN JRA) : AY 2010

Heavy Ion Pub

preliminary approval
Au+Au ridge analysis

D2 (RIKEN JRA) : AY 2011

JPS Fall
WPCF2011

PHENIX Run12
Detector Maintenance

preliminary approval
Au+Au vn-correlation

preliminary approval
Au+Au PID vn

D3 (RIKEN JRA) : AY 2012

JPS Spring HIC/HIP
Hard Probes2012

Quark Matter2012

Nagoya-Mini Workhop2012

preliminary approval Au+Au
correlations with respect to EP

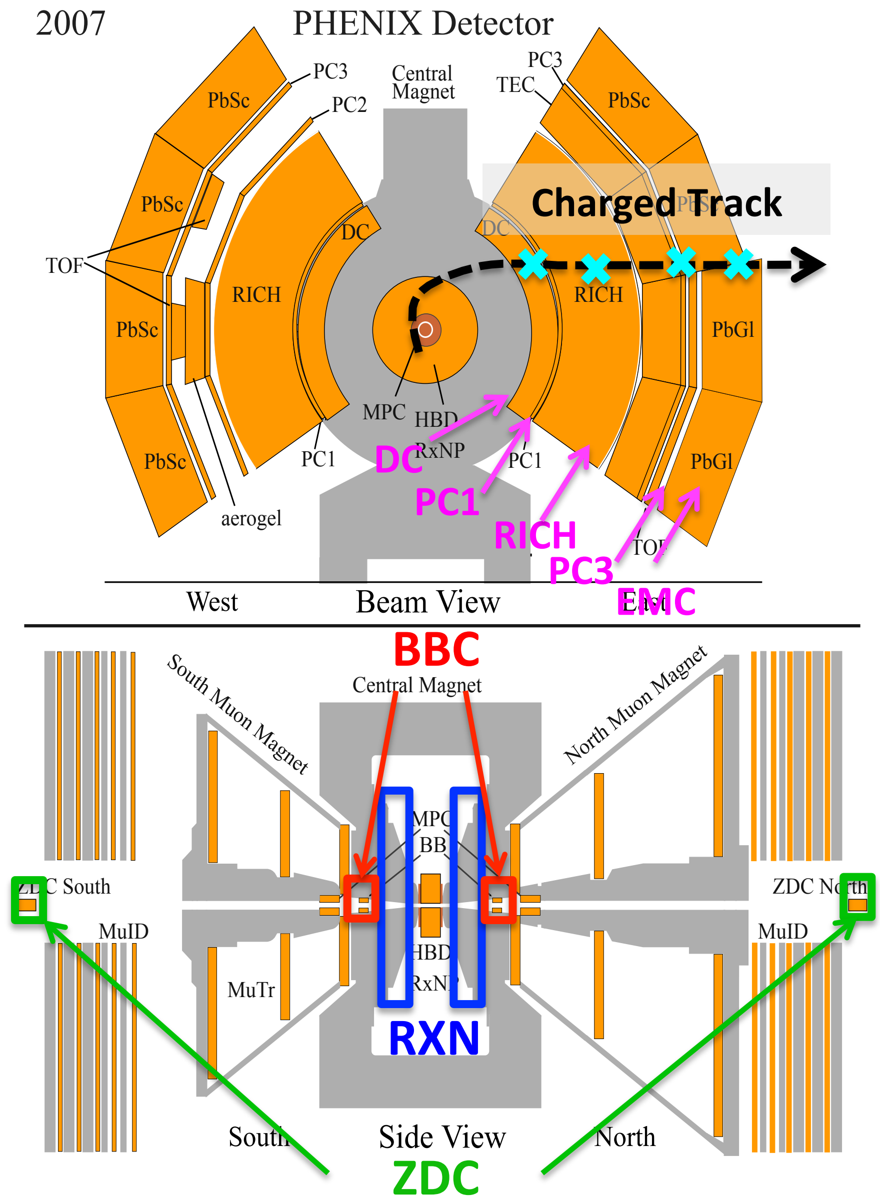
D3 (4) : AY 2013

PHENIX Run13
vn EP calibration
Detector Maintenance

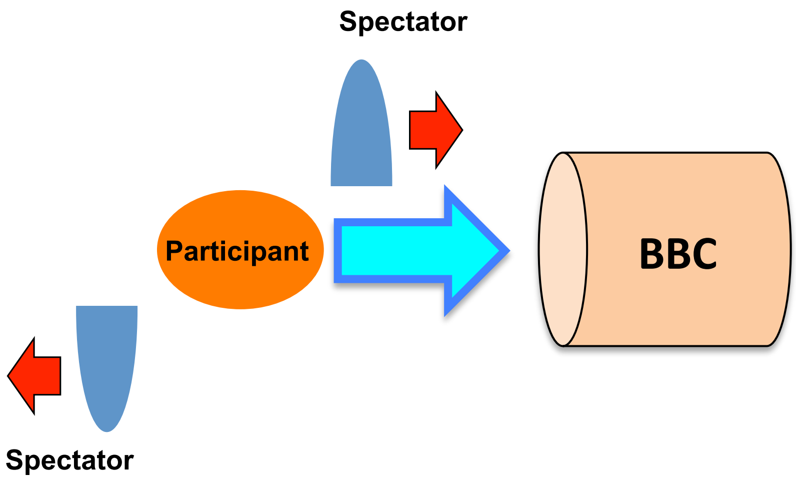
ANALYSYS

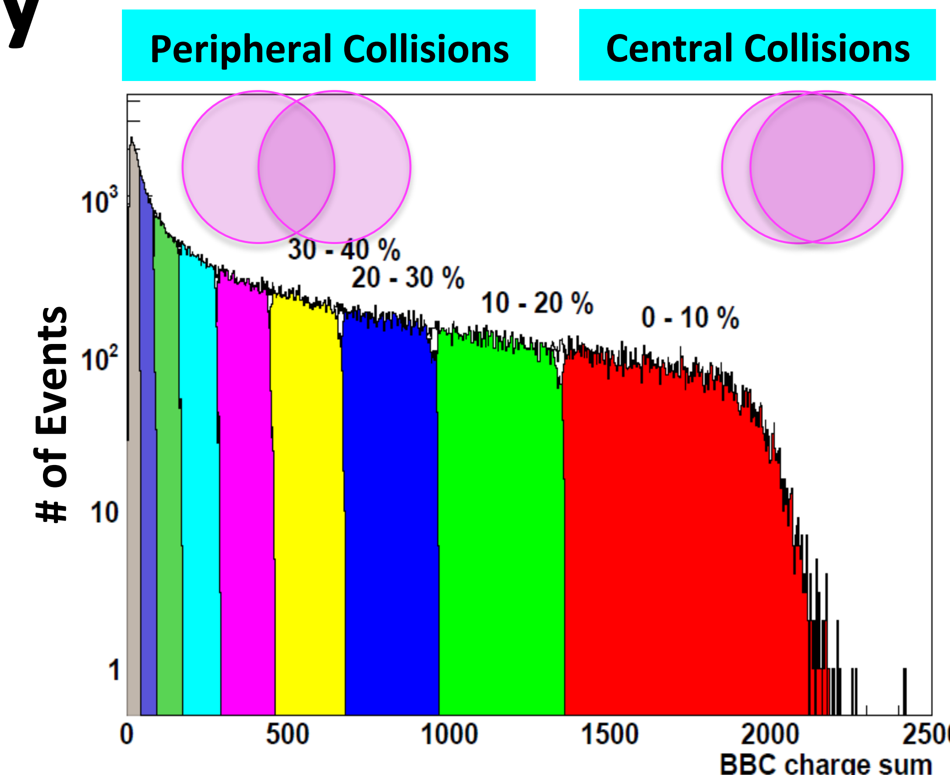
PHENIX 2007 Experiment: Au+Au 200 GeV Collisions

- ✧ Minimum Bias trigger : 4.4 billion events
- ✧ Trigger, collision vertex, centrality
  - Zero-Degree-Calorimeter(ZDC)
  - Beam-Beam-Counter (BBC)
- ✧ Event-plane
  - BBC
  - Reaction-Plane-Detector(RXN)
- ✧ Central Arm, $\Delta\phi=\pi$, $|\eta|<0.35$
  - Drift Chamber (DC)
  - Pad Chambers(PC)
  - Electromagnetic Calorimeter(EMC)
    - Momentum, charged particle tracking
  - Ring Image Cherenkov Detector(RICH)
    - Electron veto



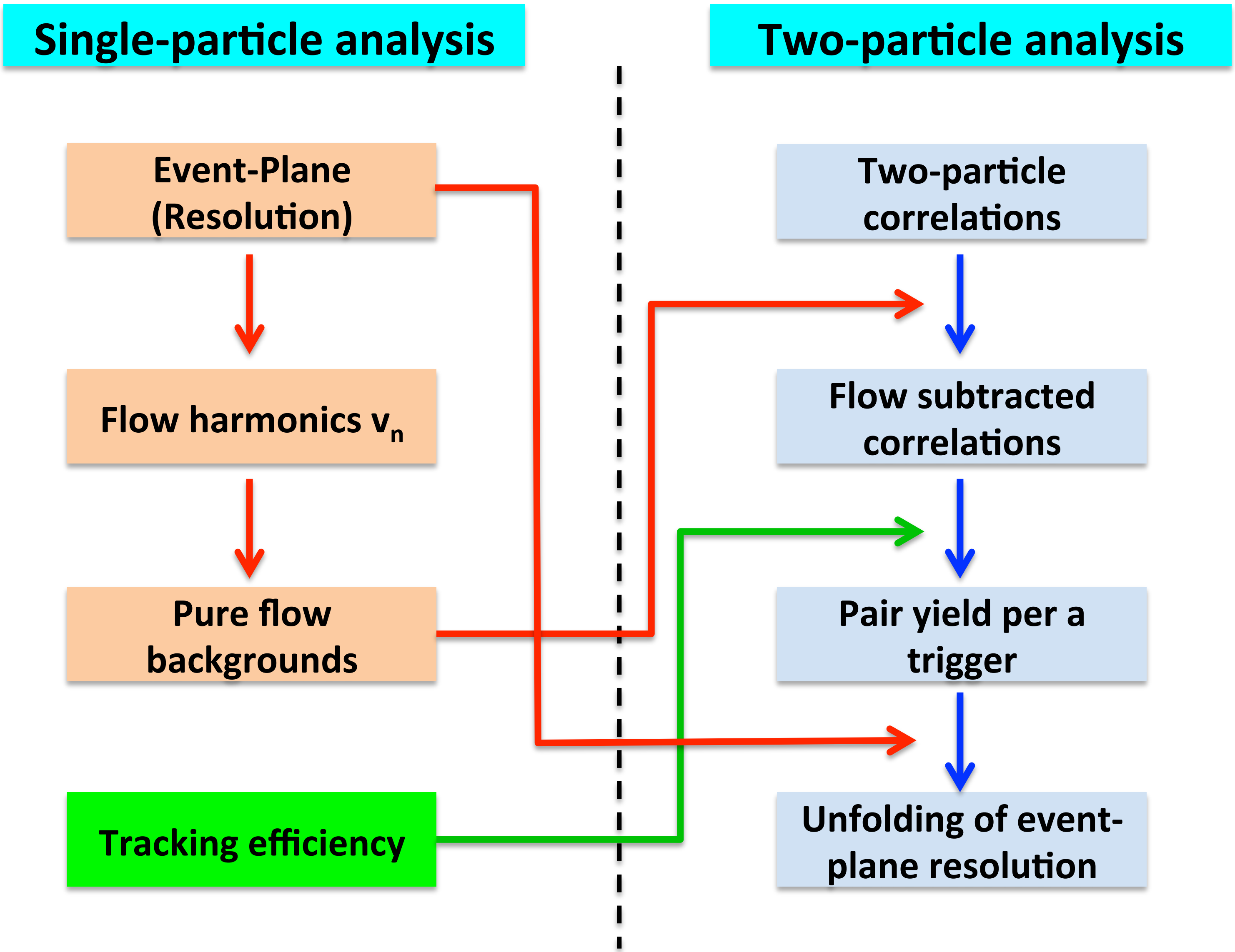
Collision Centrality



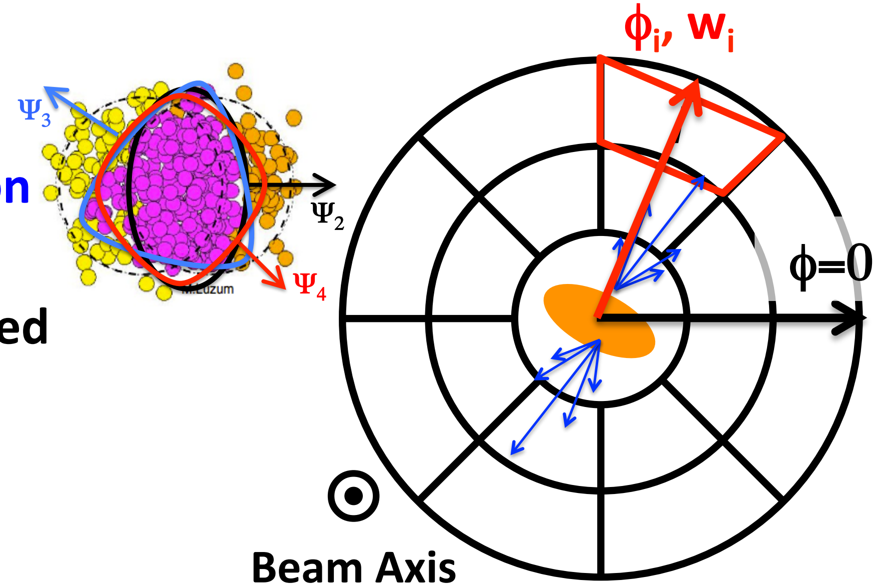


- ✧ **A degree of overlap of two colliding nuclei**
  - **Distance between center of the nuclei ➔ multiplicity ➔ charge deposited in BBC**
- ✧ **Require each percentile contains same # of events**
  - **Most-central Collision : 0%**
  - **Most-peripheral Collision : 100% (PHENIX determines it up to 92%)**

Analysis Flow-Chart



Event-Plane



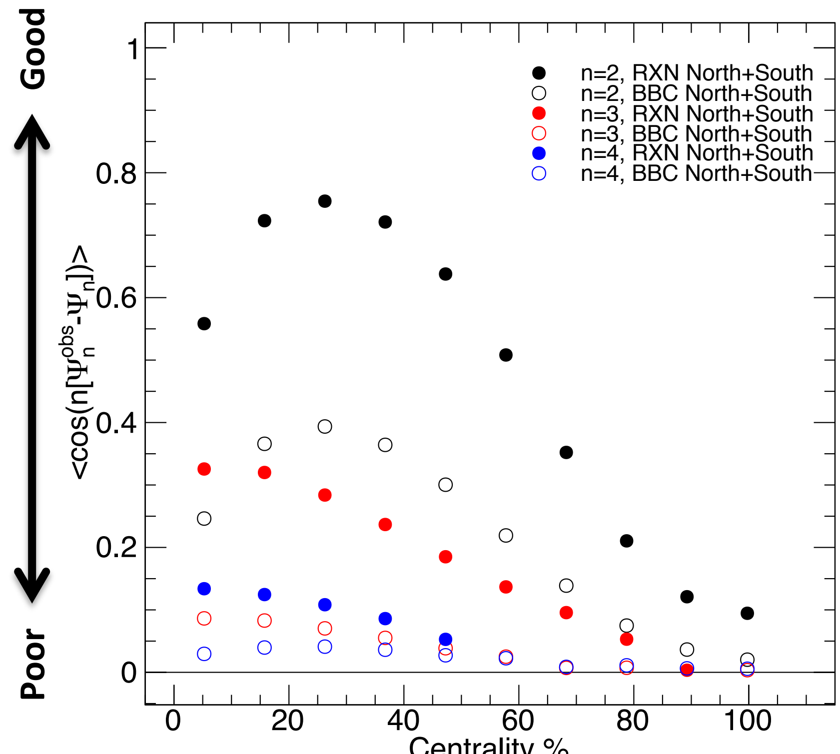
- ✧ **Expansion to the initial short-axis direction by pressure gradient**
  - **EP is a direction most particles are emitted after freeze-out**
  - **EP is determined by flow signal itself**

- ✧ **EP is determined by RXN and BBC detectors**
  - **RXN (1<|η|<2.8) : 24 segments x 2 sectors**
  - **BBC (3<|η|<3.9) : 64 segments x 2 sectors**

$$\Psi_n = \frac{1}{n}\tan^{-1}\left(\frac{\sum_i w_i \cos(n\phi_i)/\sum_i w_i}{\sum_i w_i \sin(n\phi_i)/\sum_i w_i}\right)$$

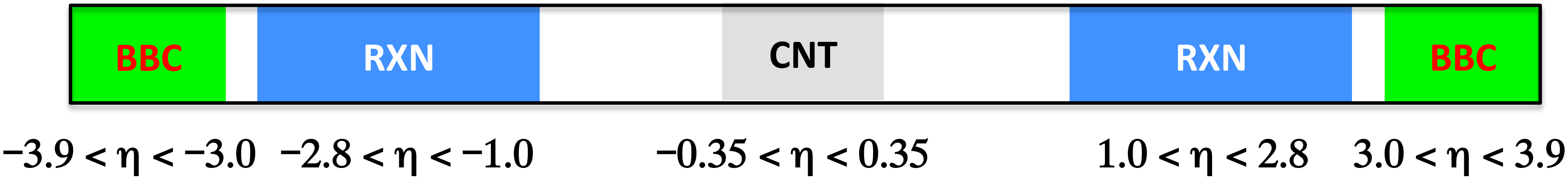
ϕ_i : **Azimuthal angle of i^{th} segments**

w_i : **Weight (Charge etc.) of i^{th} segments**



Flow Measurements

- Rapidity ranges of CNT, RXN, & BBC
  - Rapidity gap between particles & EP to avoid auto-correlations by jets



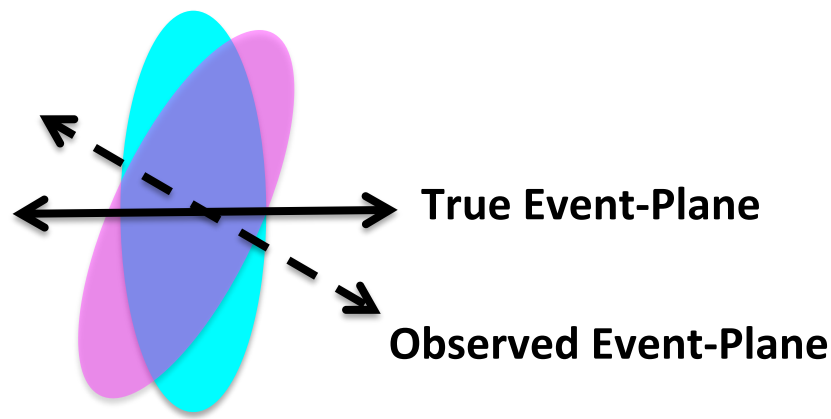
- Raw flow harmonics

$$v_n^{raw} = < cosn(\phi - \Psi_n^{obs}) >$$

- Resolution correction
  - Smearing due to limited resolution

$$v_n = \frac{< cosn(\phi - \Psi_n^{obs}) >}{< cosn(\Psi_n - \Psi_n^{obs}) >}$$

Event-Plane Resolution

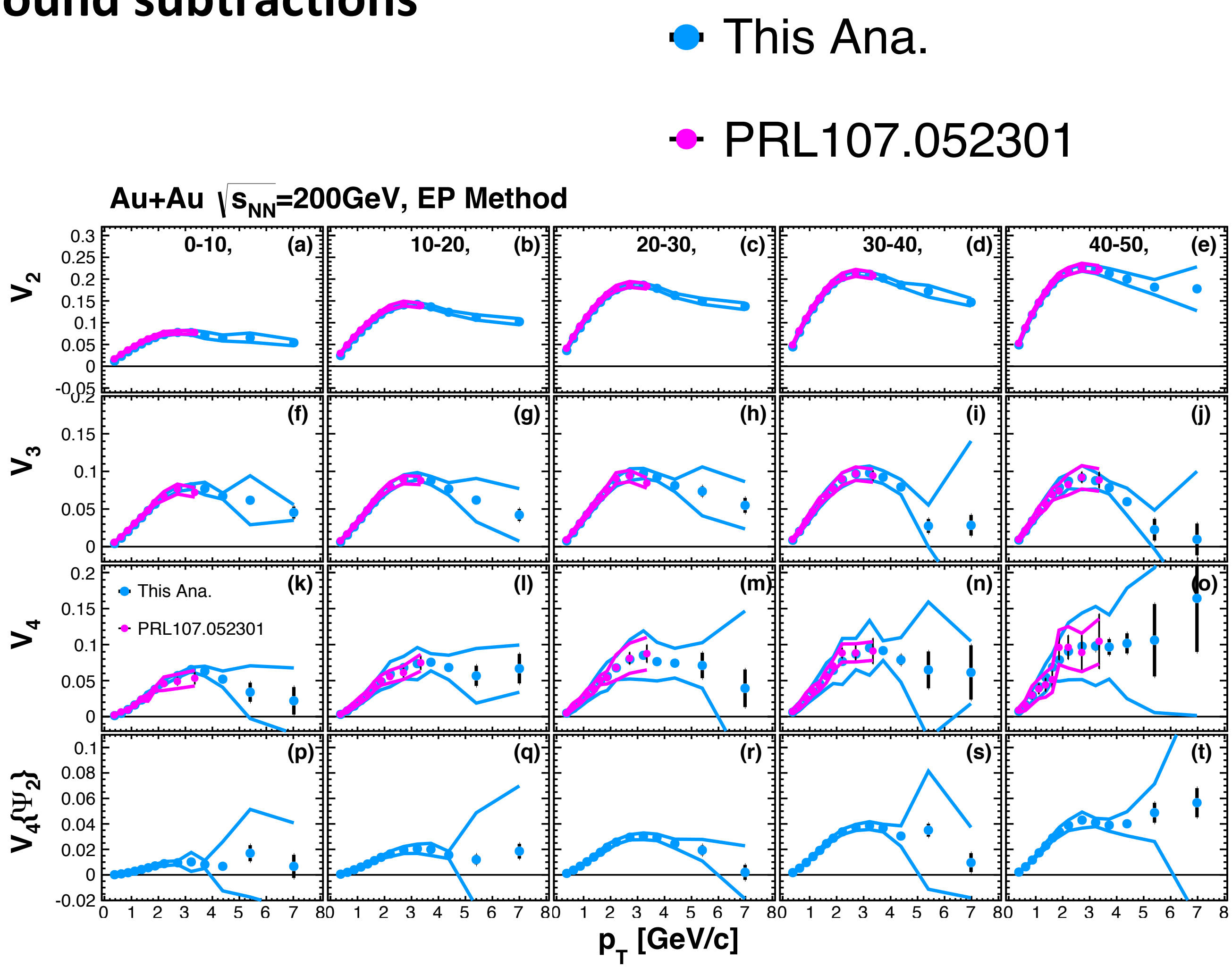


v_n Results

- ✧ **Consistent results with previous PHENIX measurements**
  - – **Used for background subtractions**

Total systematics (%) at p_T=1-2 GeV/c

Centrality	0-10%	40-50%
v_2	4.3 %	2.7%
v_3	4.9%	12%
v_4	10%	34%
$v_4\{\Psi_4\}$	15%	6.5%



Two-Particle Correlations

Definition

Ratio of two-particle probability over single-particle ones

$$C(\Delta\phi, \Delta\eta) = \frac{P(\phi^a, \phi^t|\eta^a, \eta^t)}{P(\phi^a|\eta^a)P(\phi^t|\eta^t)}$$

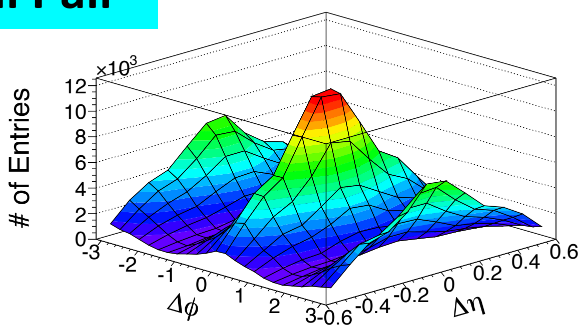
Experimental Def.

Ratio of real pair distribution over mixed one

$$C(\Delta\phi, \Delta\eta) = \frac{N^{ta}_{mix}}{N^{ta}_{real}} \frac{d^2N^{ta}_{real}/d\Delta\phi d\Delta\eta}{d^2N^{ta}_{mix}/d\Delta\phi d\Delta\eta}$$

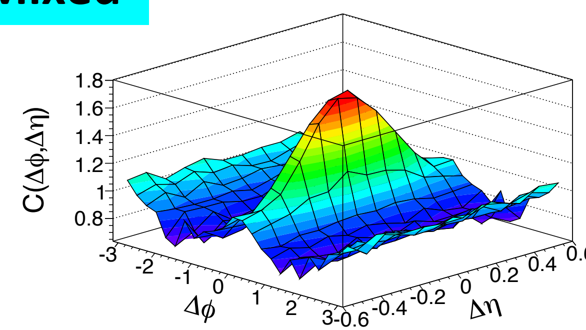
$$\Delta\phi = \phi^a - \phi^t, \Delta\eta = \eta^a - \eta^t$$

Real Pair

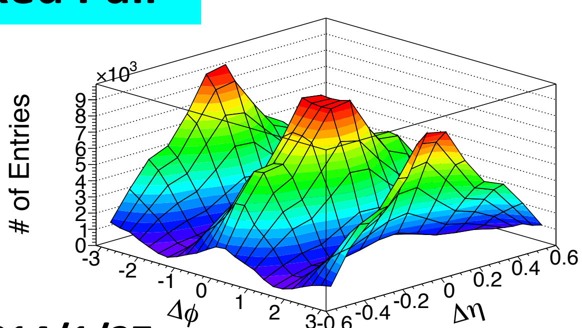


Correlations = Real/Mixed

Event mixing also corrects acceptance effects by choosing similar events: centrality, collision points



Mixed Pair

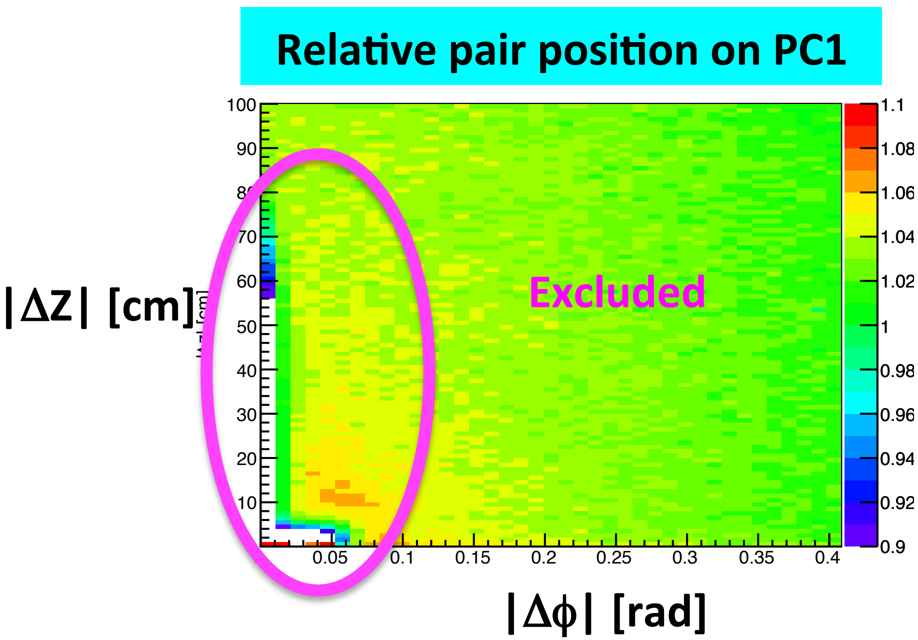


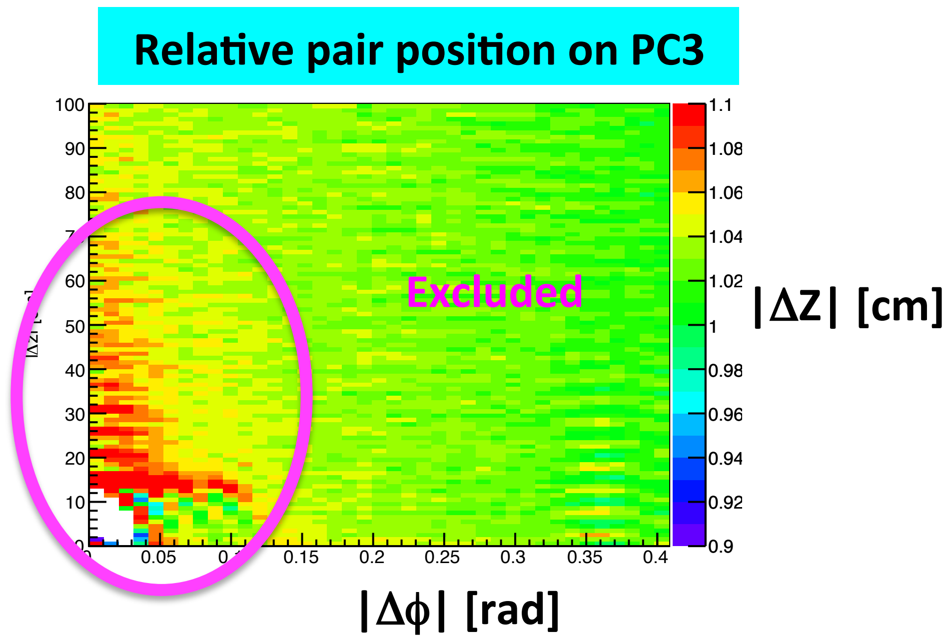
Pair Yield Per a Trigger

Dimension : Number of Particles

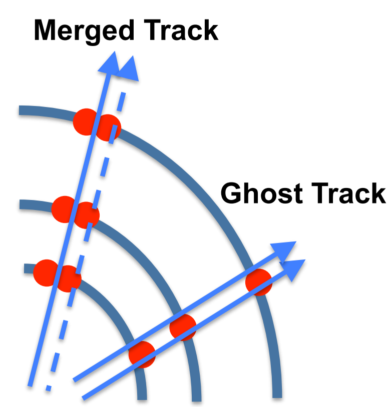
$$\frac{1}{N^t}\frac{d^2N^{ta}}{d\Delta\phi d\Delta\eta} = \frac{1}{2\pi\varepsilon}\frac{N^{ta}}{N^t}C(\Delta\phi, \Delta\eta)$$

Pair Selection on Tracking Detectors





- ✧ **Ghost track**
  - – **A single particle is counted as two tracks**
- ✧ **Merged tracks**
  - – **Two particles are counted as one track**
- ✧ **Real/Mix pair ratio should be 1 if an ideal detector**



Flow Subtraction & Pair Yield per a Trigger (PTY)

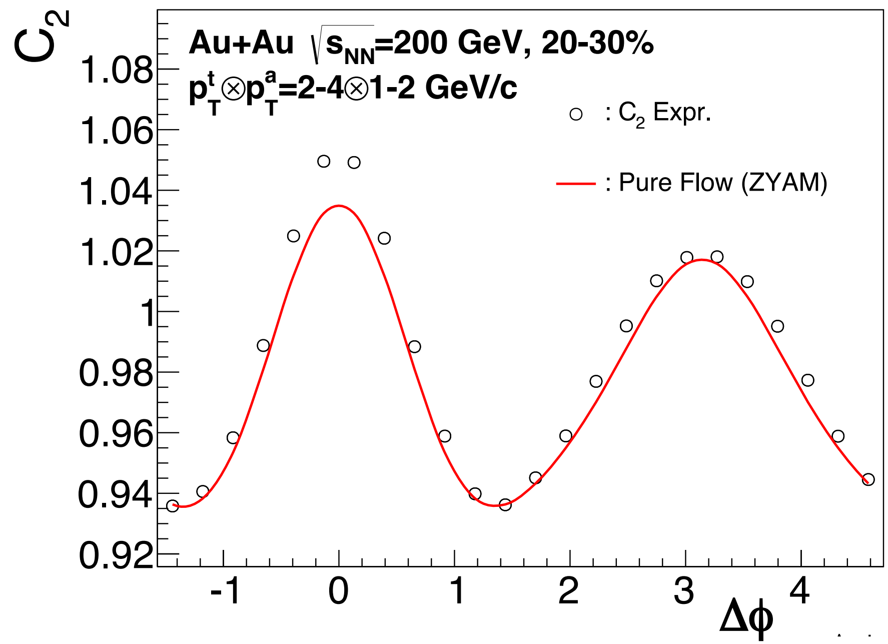
✧ **Pure flow background**

$$F(\Delta\phi) = 1 + \sum 2v_n^t v_n^a \cos(n\Delta\phi)$$

✧ **Flow subtractions by ZYAM**

– **Zero Yield At Minimum Assumption**

$$j(\Delta\phi) = C(\Delta\phi) - b_0 \left[1 + \sum_{n=1} 2v_n^t v_n^a \cos(n\Delta\phi)\right]$$



✧ **Pair yield per a trigger (PTY)**

— **Dimension : number of particles**

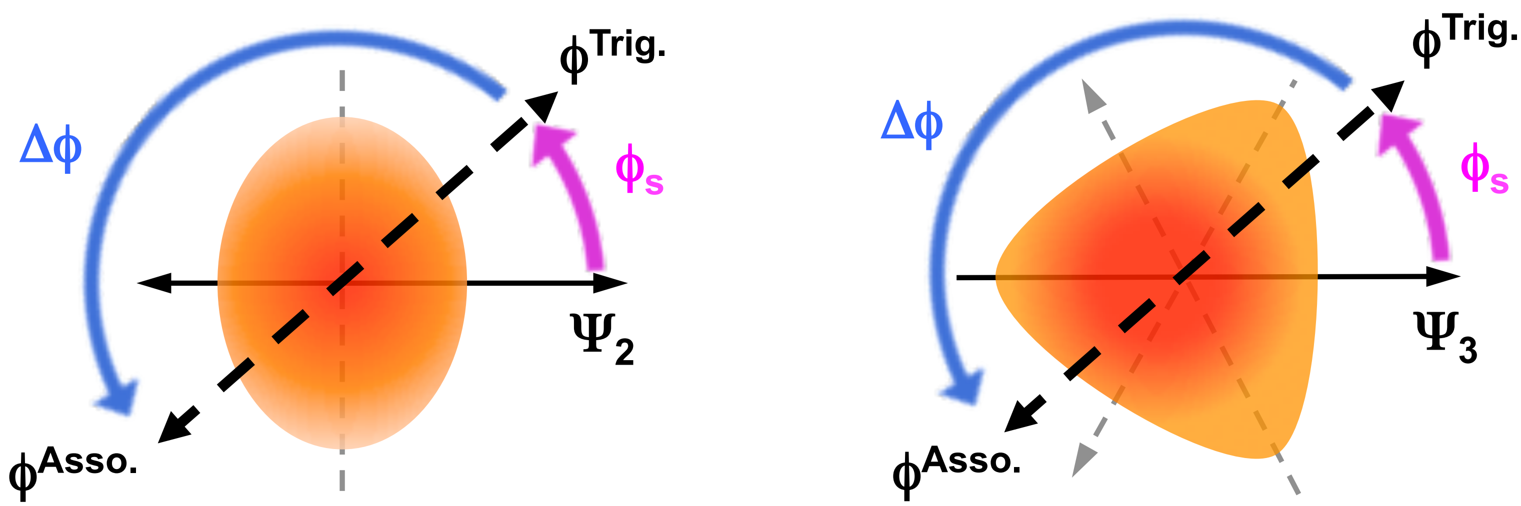
$$\frac{1}{N^t}\frac{dN^{ta}}{d\Delta\phi} = \frac{1}{2\pi\varepsilon}\frac{N^{ta}}{N^t} j(\Delta\phi)$$

ε : **Tracking efficiency of associate particles**

N^t : **Number of triggers**

N^{ta} : **Number of pairs**

Trigger Selection with respect to Event-Plane



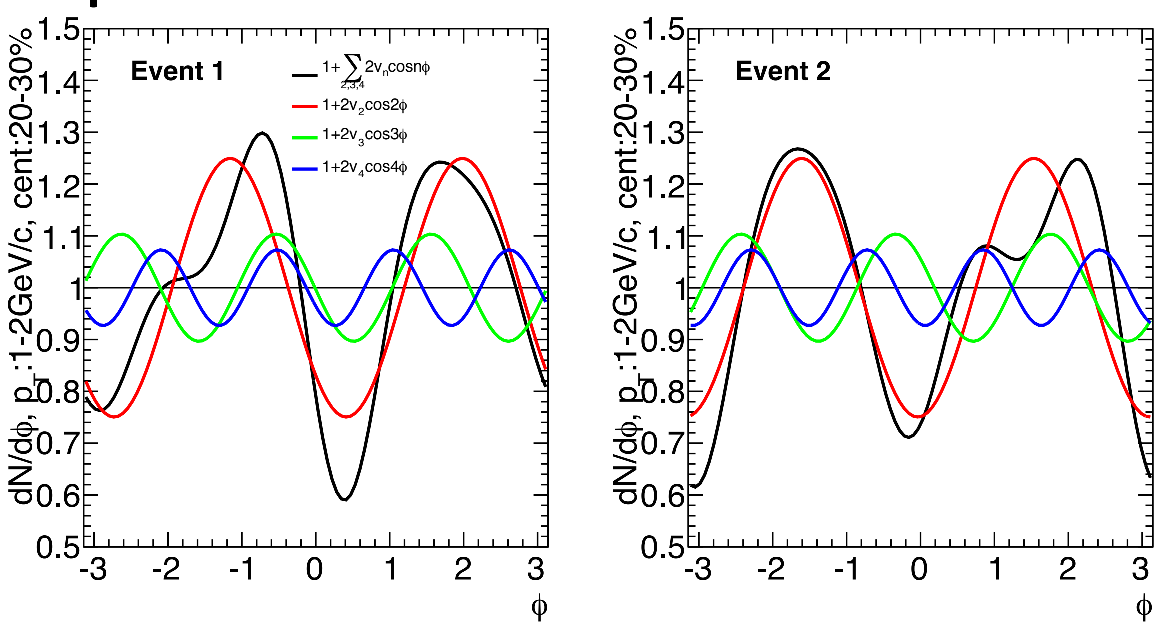
- ✧ **Expansion to the short-axis direction by pressure gradient**
  - **EP : direction most particles are emitted after freeze-out**
- ✧ **Selecting trigger particles with respect to Ψ_2 & Ψ_3**
  - **8 bins :** $\phi^{trig} - \Psi_n : [-\pi/n, \pi/n]$
- ✧ **Control of path length of trigger and associate particles**
- ✧ **Three p_T combinations: 2-4x1-2, 2-4x2-4, 4-10x1-2 GeV/c**

Flow Backgrounds with respect to EP

- ✧ **A Monte Carlo simulation employed**
- ✧ **Azimuthal distribution using**
  - **Measured v_n**
  - **Observed correlation between EP**
    - **$\langle 4(\Psi_2-\Psi_4)\rangle = v_4\{\Psi_2\}/v_4\{\Psi_4\}$**
    - **$\langle 6(\Psi_2-\Psi_3)\rangle = 0$**

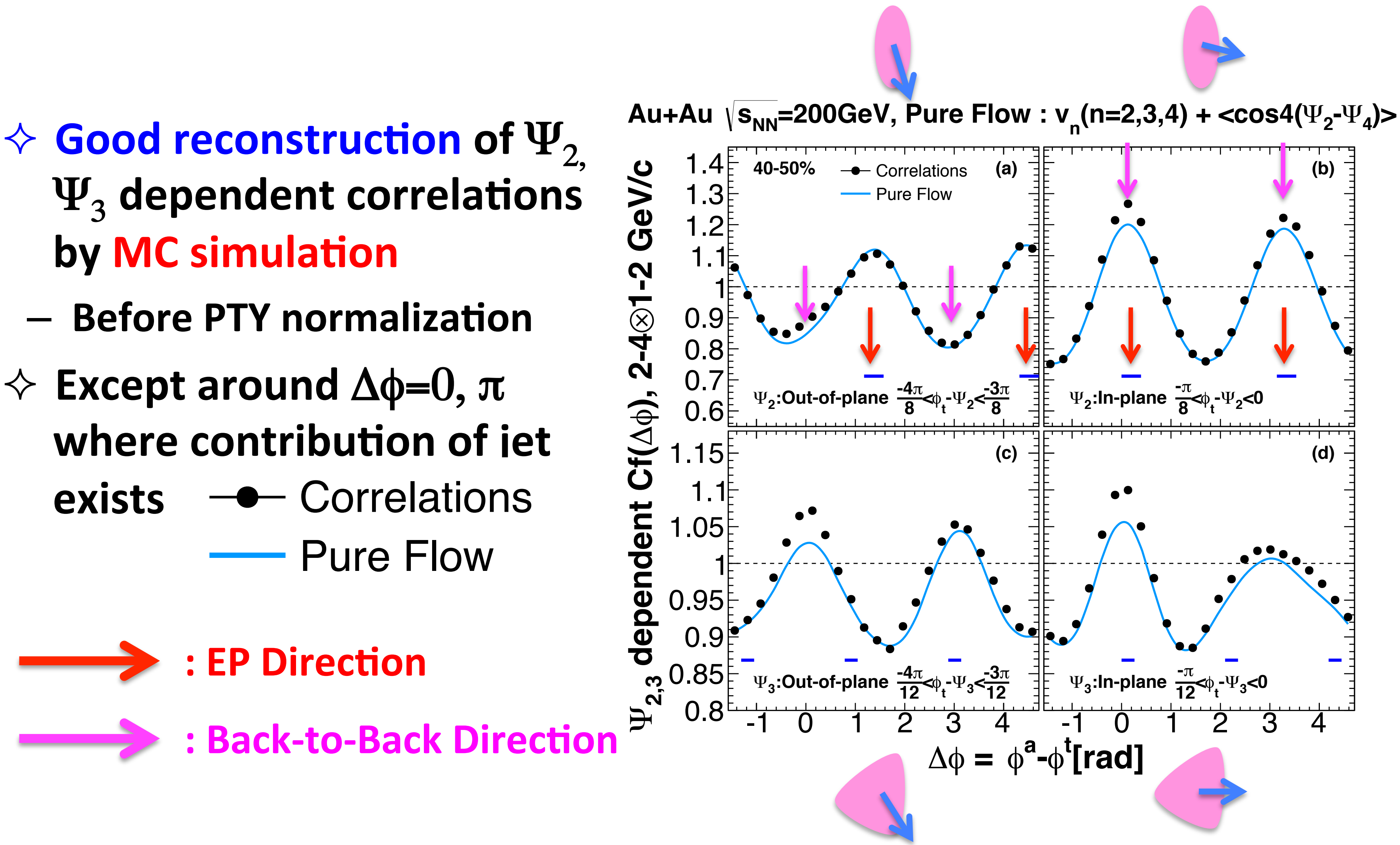
$$\frac{dN}{d\phi} \propto 1 + \sum_{n=2,3,4} 2v_n \cos n(\phi - \Psi_n)$$

- ✧ **Determine trigger particle relative to EP taking into account EP resolutions**
- ✧ **Calculate two-particle correlations**



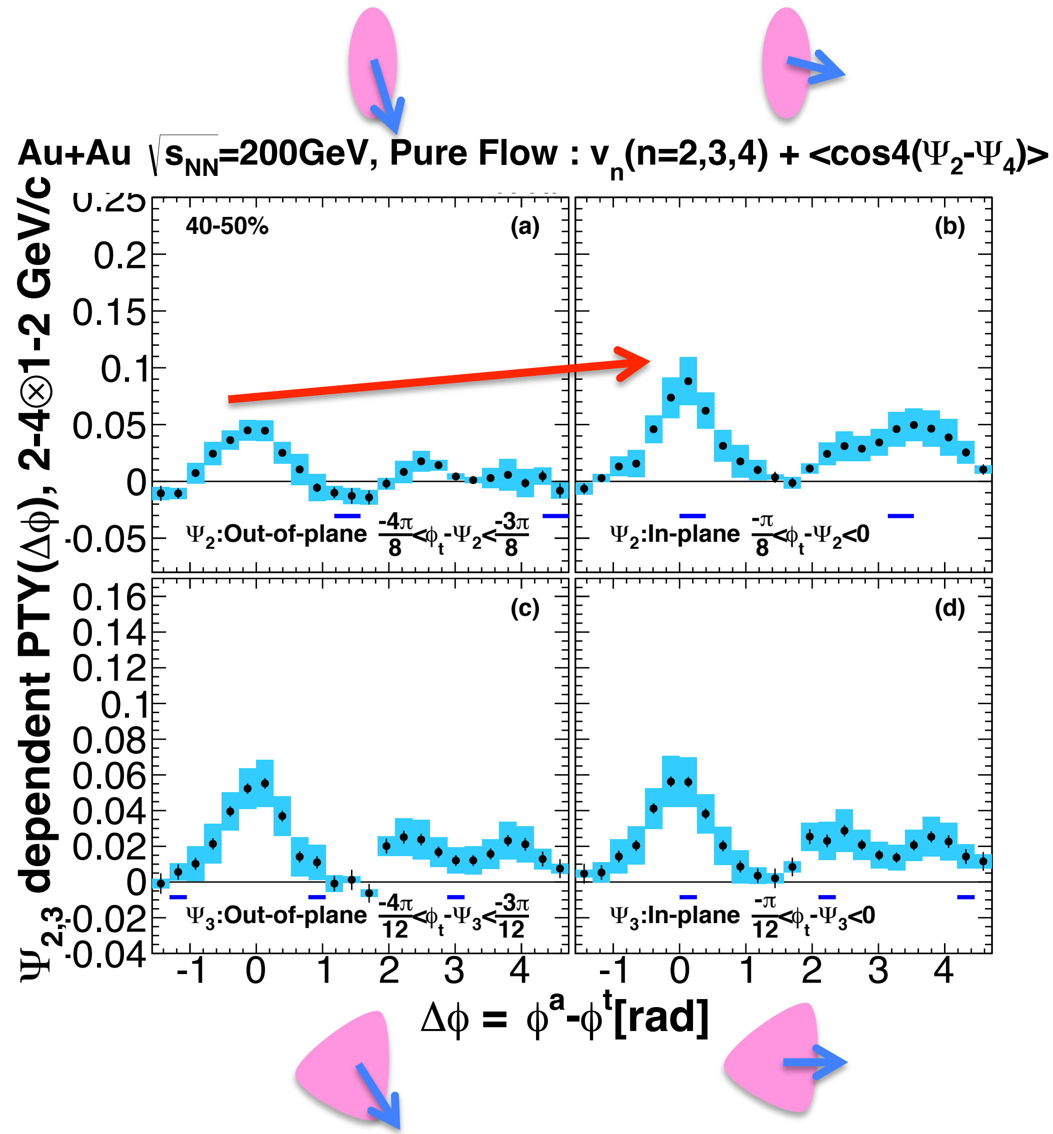
Flow Backgrounds with respect to EP

- ✧ **Good reconstruction** of Ψ_2, Ψ_3 dependent correlations by **MC simulation**
  - – Before PTY normalization
- ✧ Except around $\Delta\phi=0, \pi$ where contribution of jet exists



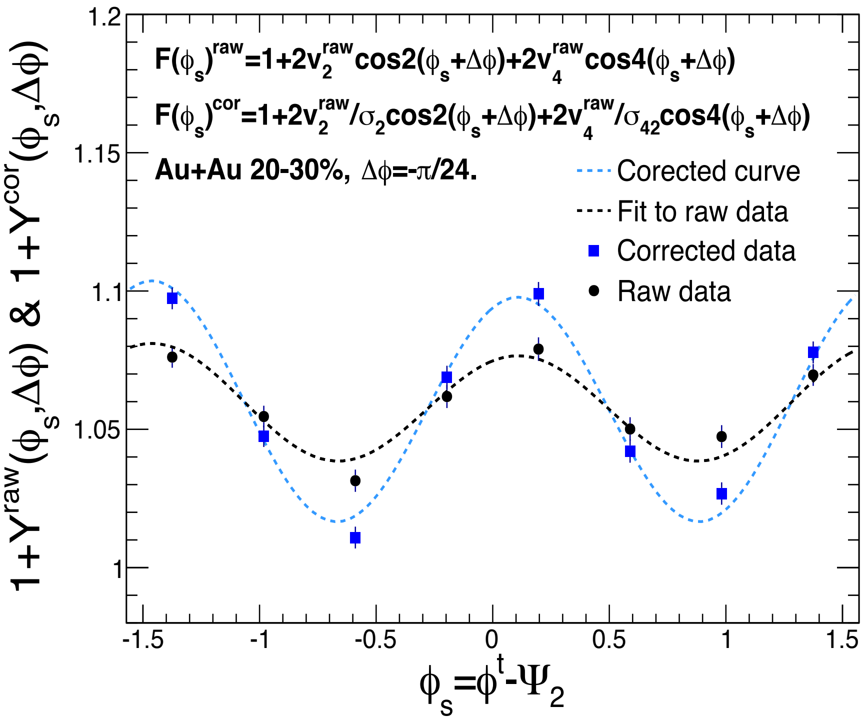
Two-Particle Correlations with respect to EP

- ✧ Flow subtracted Ψ_2, Ψ_3 dependent correlations
- ✧ Clear Ψ_2 dependence
- ✧ No Ψ_3 dependence?
- ✧ Smearing by neighboring trigger bins due to limited EP resolution
  – Needs unfolding !!



Unfolding Methods of EP Resolution

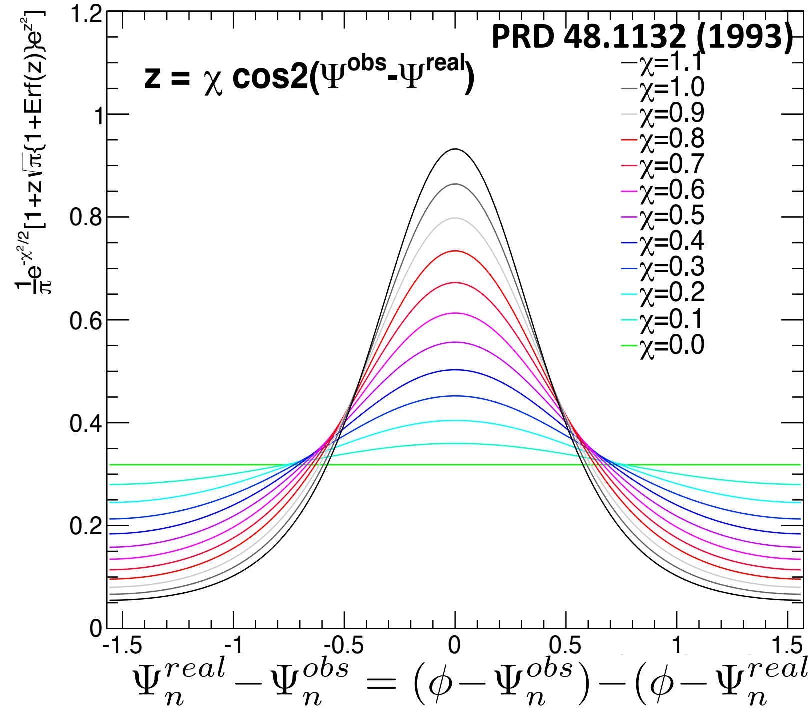
Fitting Method



- Azimuthal anisotropy of correlation yield corrected by the event-plane resolution

Method by PRC.84.024904 (2011)

Iteration Method



- Trigger smearing matrix "*S*"
- True & Observed Correlations "A" & "B"
  - Vector elements : Trigger bin
- Solve simultaneous equations via iteration

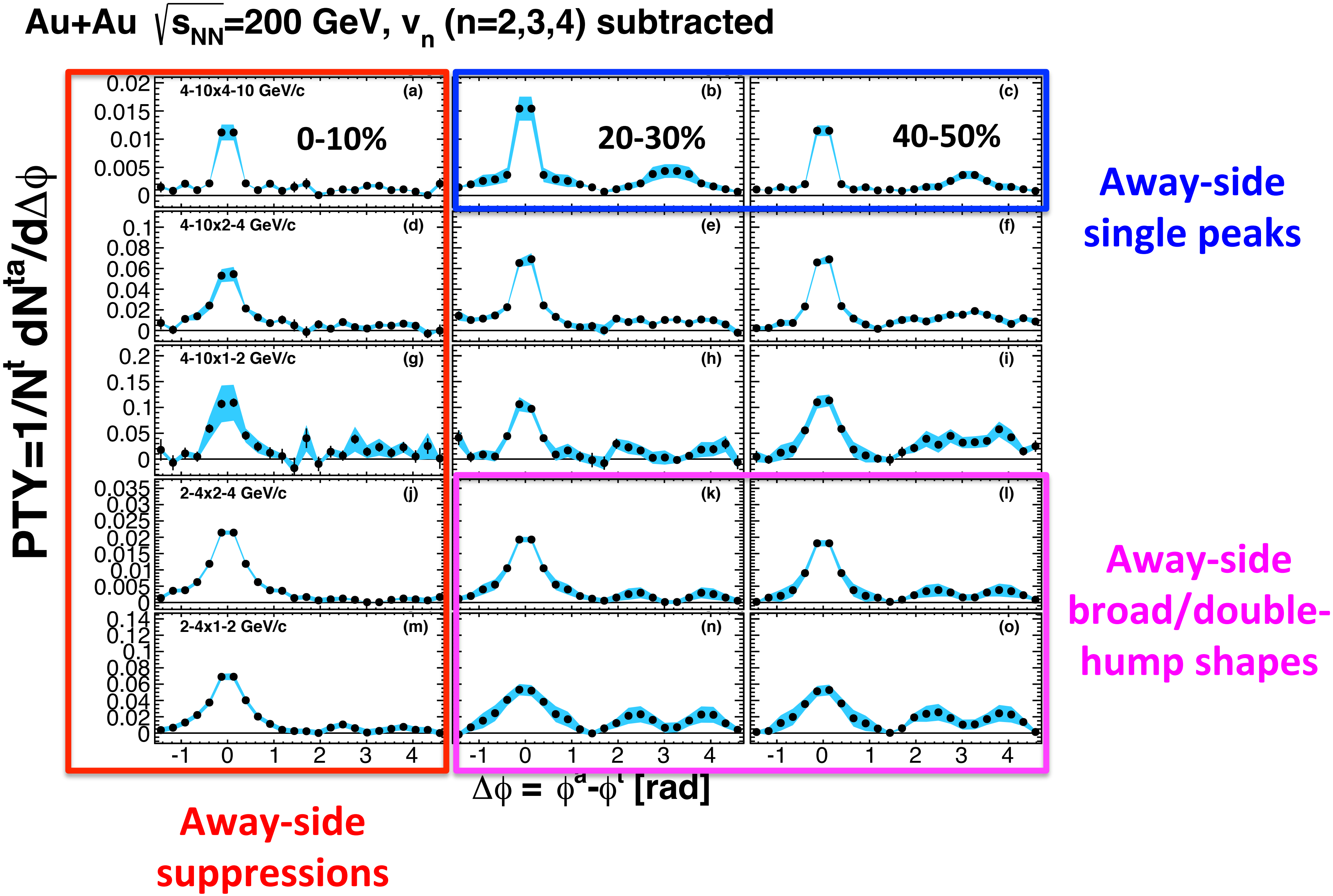
$$\mathbf{B} = \mathbf{S}\mathbf{A} \quad \Rightarrow \quad \mathbf{A} = \mathbf{S}^{-1}\mathbf{B}$$

Systematic Uncertainties

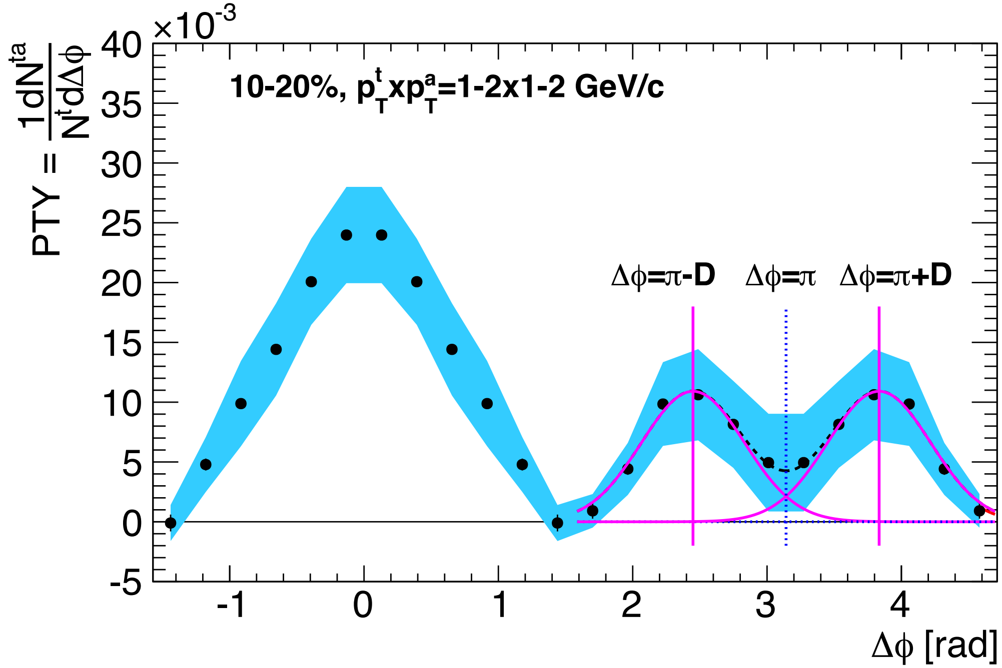
- ✧ **Flow v_n measurements**
  - Systematic difference within RXN segments
  - Rapidity dependence of EP : RXN-BBC difference
  - Matching cut of CNT particles
- ✧ **Two-particle correlations**
  - Systematics from v_n
  - Matching cut of CNT particles
- ✧ **Unfolding of event plane dependent correlations**
  - Difference of two methods : Fit & Iteration Methods
  - Parameter in the iteration method

Results & Discussion

v_n (n=2,3,4) subtracted correlations



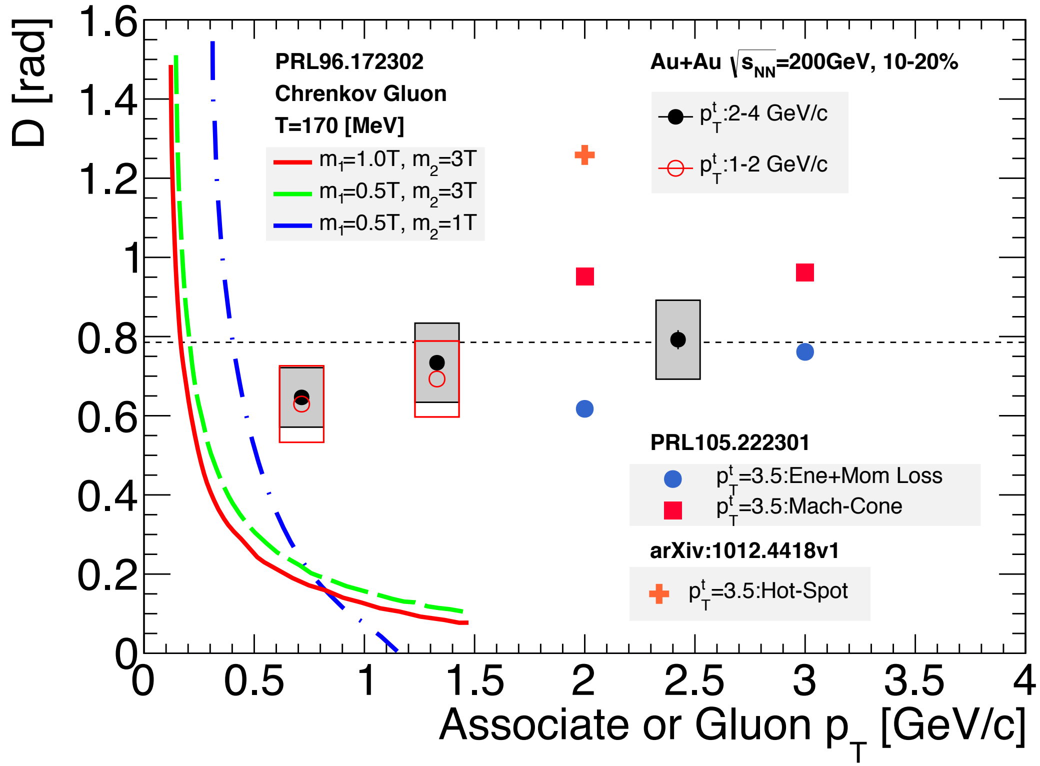
Extraction of Double-Hump Position



✧ **Extraction of double-hump position via two-Gaussian fitting to away-side (|Δφ−π|<π) at centrality 10%>, where double-humps seen**

$$F(\Delta\phi) = Ae^{-\frac{(\Delta\phi-\pi-D)^2}{\sigma^2}} + Ae^{-\frac{(\Delta\phi-\pi+D)^2}{\sigma^2}}$$

Comparison with Models

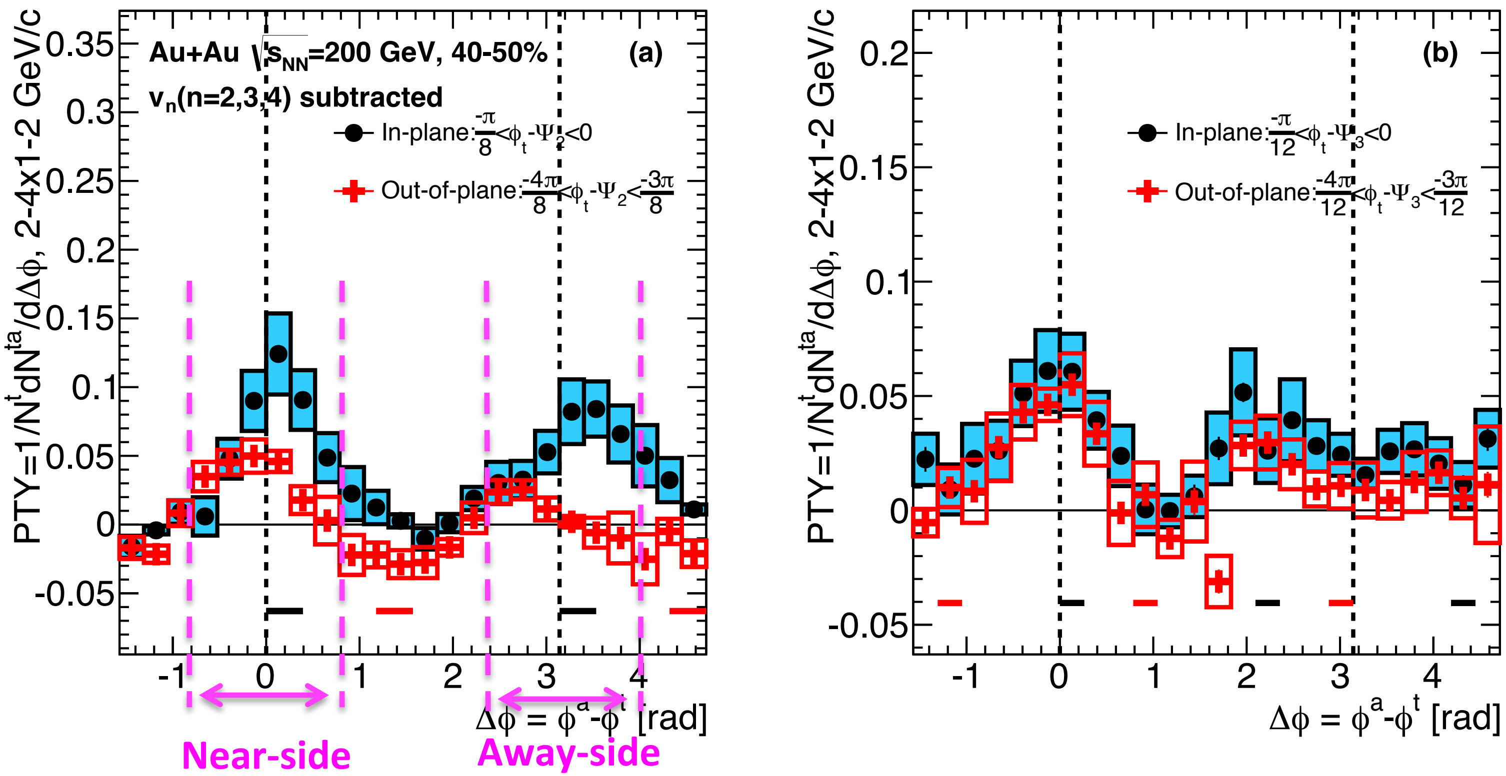


- ✧ **Cherenkov gluon : <25 % of experimental data at p_T =1 GeV/c**
- ✧ **Mach-cone & Energy-momentum loss :**
  - **Independence of p_T is similar to the experimental data**
  - **20 % larger/smaller than experimental data at p_T =2 GeV/c**
- ✧ **Hot-spot : 50% larger than experimental data**

Ψ_2 & Ψ_3 Dependent Correlations at p_T:2-4x1-2 GeV/c

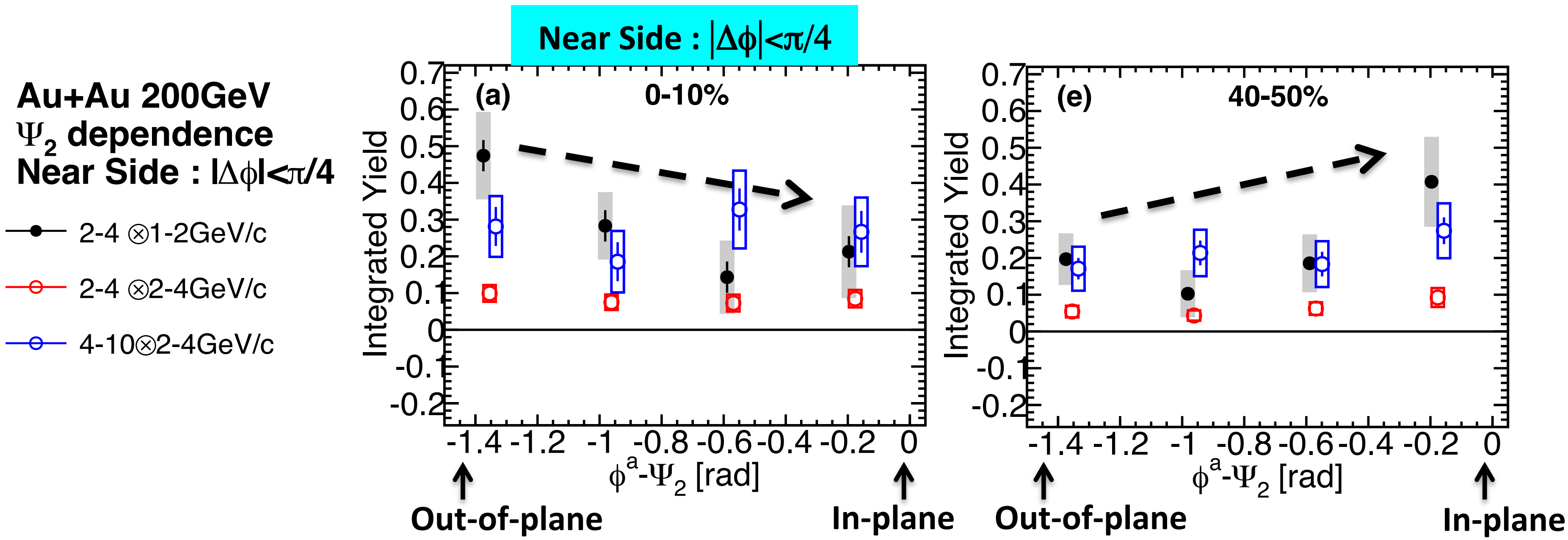
Ψ_2 dependence

Ψ_3 dependence

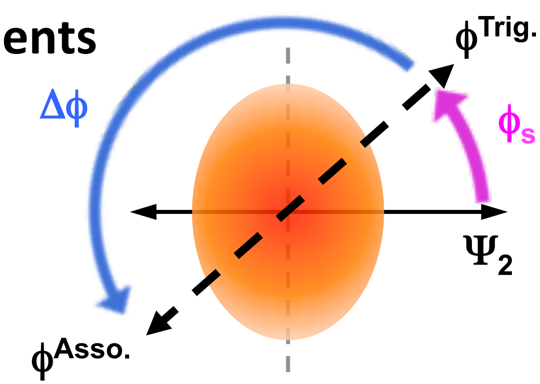


✧ **Event-plane dependence is discussed via integrated near ($|\Delta\phi|<\pi/4$) and away ($|\Delta\phi-\pi|<\pi/4$) side yields vs associate angle from Ψ_2 and Ψ_3**

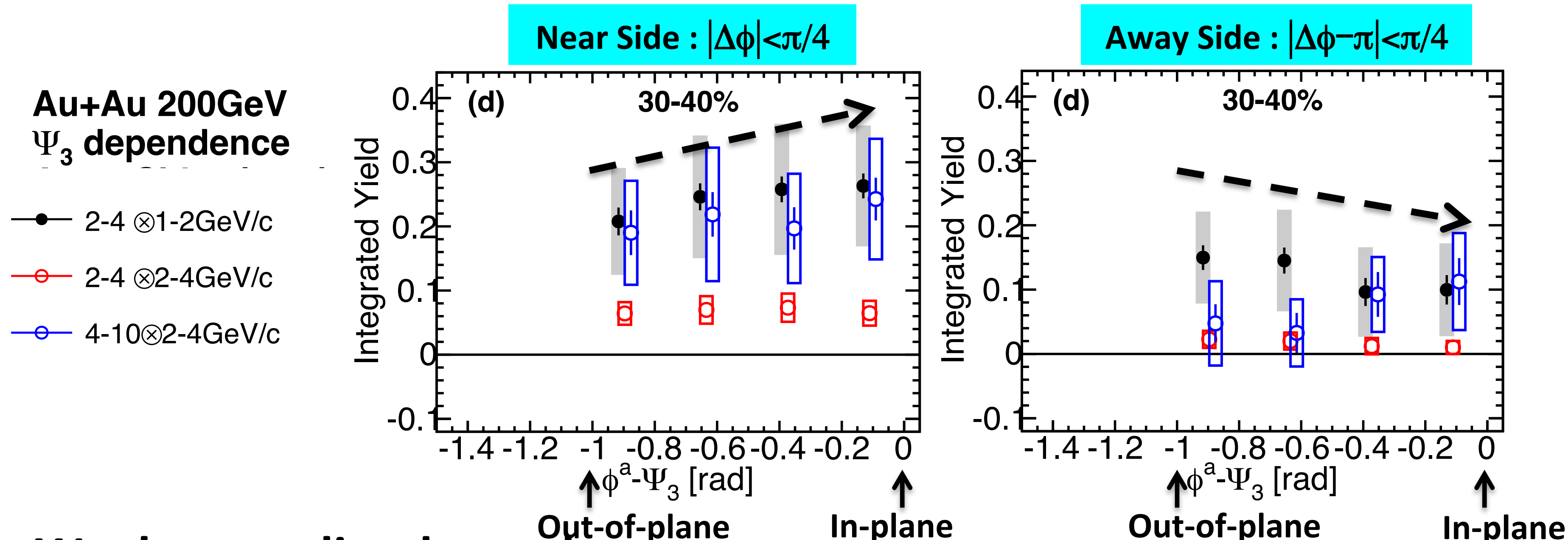
Near-Side Integrated Yield vs Associate Angle from Ψ_2



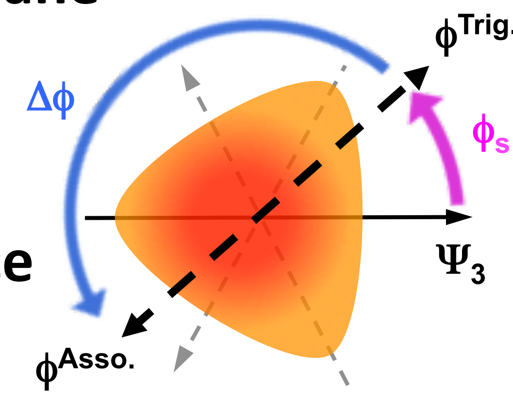
- ✧ Similar near and away-side trends
- ✧ p_T 2-4x2-4, 4-10x2-4 GeV/c : in-plane >= out-of-plane
  - Qualitatively consistent with previous measurements
- ✧ p_T 2-4x1-2 GeV/c
  - 0-10% : Out-of-plane > In-plane
  - 40-50% : In-plane > Out-of-plane
  - More than 1σ significance of total systematics



Integrated Yield vs Associate Angle from Ψ_3



- ✧ Weak centrality dependence
- ✧ p_T 2-4x2-4, 4-10x2-4 GeV/c : in-plane >= out-of-plane
- ✧ Event-plane dependence is not clearly seen
  – Flat within systematic uncertainties
- ✧ Centroids show different event-plane dependence between near and away-side



Azimuthal Anisotropy of PTY

- ✧ Integrated yield vs associate angle from EP is translated into azimuthal anisotropy v_n^{PTY}
- ✧ v_n^{PTY} can be compared with single particle v_n because the dimension of PTY is "# of particles"

- ✧ v_n^{PTY} is extracted via Fourier fitting

Ψ_2 dependence $F(\phi^a - \Psi_2) = a\{1 + 2v_2^{PTY}\cos 2(\phi^a - \Psi_2) + 2v_4^{PTY}\cos 4(\phi^a - \Psi_2)\}$

Ψ_3 dependence $F(\phi^a - \Psi_3) = a\{1 + 2v_3^{PTY}\cos 3(\phi^a - \Psi_3)\},$

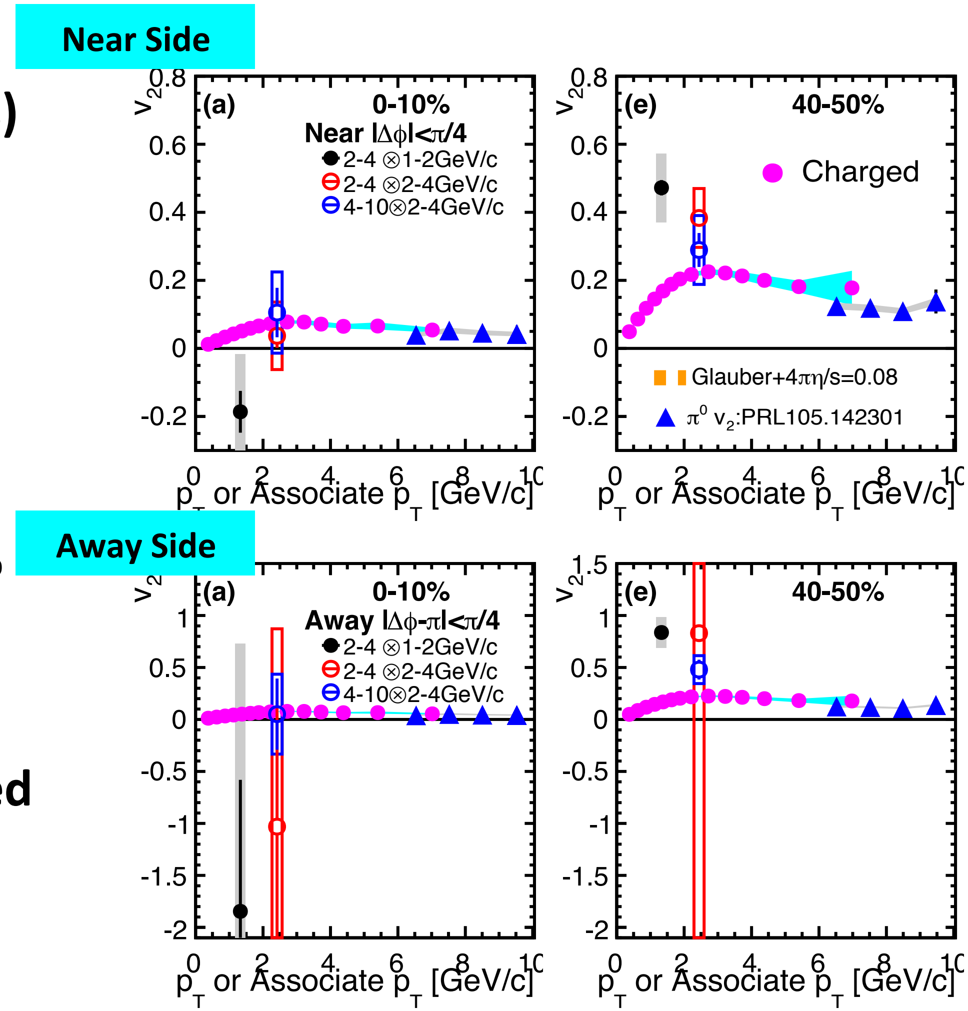
- ✧ Anisotropy of associate particles per a trigger → Anisotropy of associate particles per a event

$$v_n^{PTY,cor} = v_n^{PTY} + v_n^{trig}\cos n(\phi^t - \phi^a)$$

v_2^{PTY}

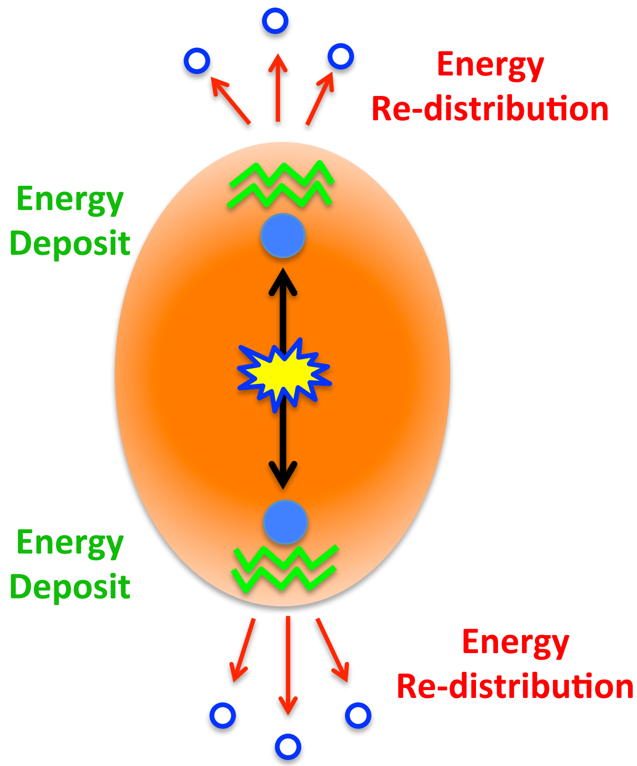
- ✧ Positive hadron v_2 (Hydrodynamics)
- ✧ Positive π^0 v_2 (Parton energy-loss)
  - – Superposition of those assembles only positive v_2
- ✧ Near & away-side v_2^{PTY}
  - – Positive value at 40-50%
  - – Near-side negative value at 0-10%

- ✧ New effects need to be considered
- ✧ Possible re-distribution of deposited energy in longer path direction



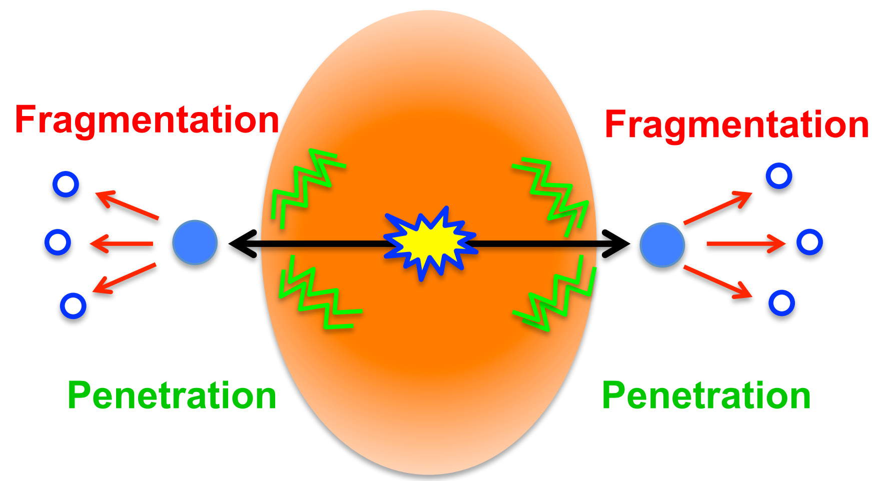
Interpretation of Ψ_2 Dependent Correlations

Central



Re-distribution Dominance

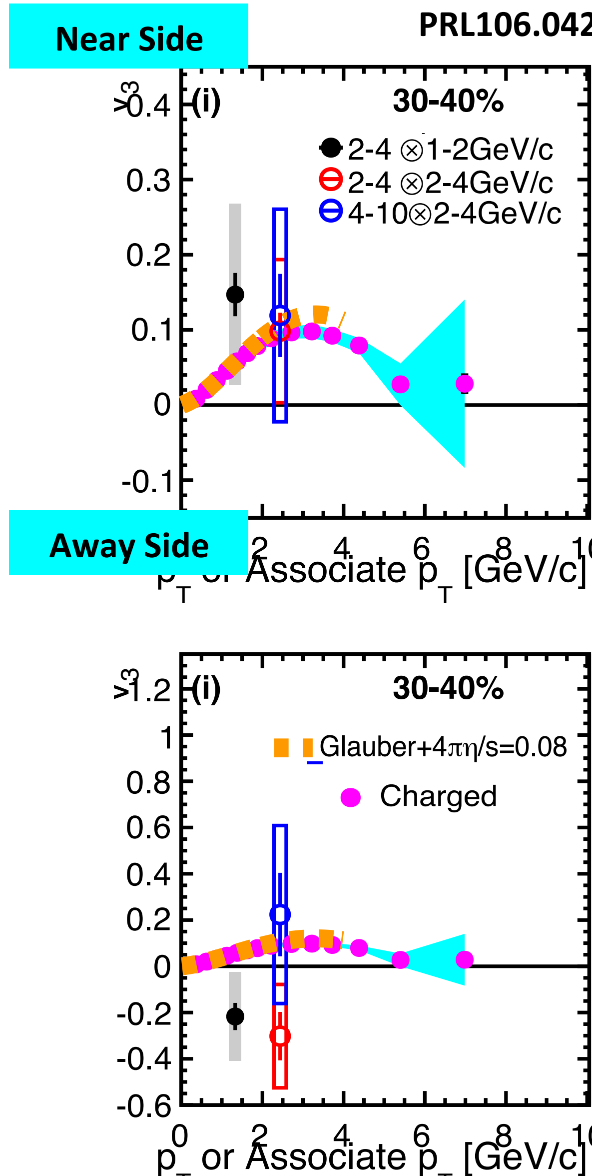
Mid-central



Penetration Dominance

v_3^{PTY}

Hydrodynamics calculation
PRL106.042301 (2011)

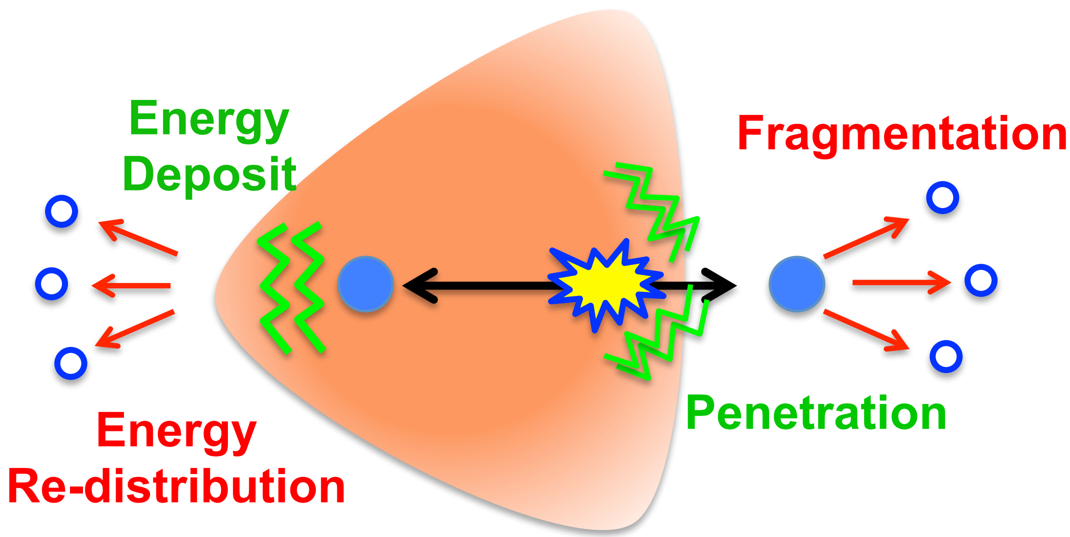


- ✧ Positive hadron v_3 (Hydrodynamics)
- ✧ Near & away-side v_3^{PTY} at 30-40%
  - – Positive near-side
  - – Negative away-side
- ✧ Weak centrality dependence

- ✧ Different near & away-side, as well as centrality dependences from those of v_2^{PTY}
- ✧ Possible different evolution processes between the 2nd- and 3rd-order geometry planes

Interpretation of Ψ_3 Dependent Correlations

Away-Side | **Near-Side**



Re-distribution Dominance

Penetration Dominance

Conclusion - I

- Two-particle correlations with v_n (n=2,3,4) subtractions are measured in Au+Au $\sqrt{s_{NN}}$=200 GeV collisions

✧ Away-side suppression in 0-10% independent of trigger and associate p_T combinations

✧ Single away-side peak of high p_T correlations in mid-central collisions

✧ Broad/double-peak structure at away-side of intermediate p_T correlations in mid-central collisions

✧ New experimental data to be compared among theoretical models

Conclusion - II

- Two-particle correlations with respect to the event-planes are also measured in Au+Au $\sqrt{s_{NN}}$=200 GeV collisions

- Path length dependence of high p_T Ψ_2 dependent correlations
  - qualitatively consistent with previous PHENIX measurements

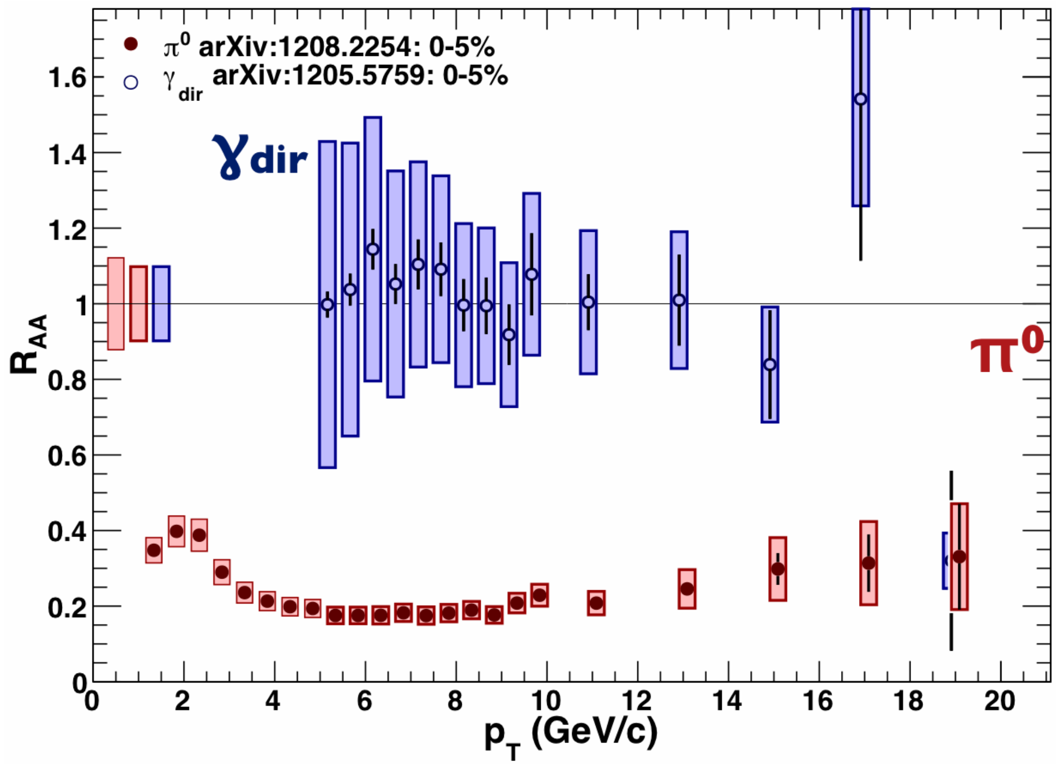
- Intermediate p_T Ψ_2 dependence
  - Enhance of correlation yield in out-of-plane direction in central collisions
    - Re-distribution of deposited energy by hard-scattered partons
  - Enhance of correlation yield in in-plane direction in mid-central collisions
    - Penetration of hard-scattered partons via parton energy loss

- Intermediate p_T Ψ_3 dependence
  - Different path near and away-side as well as centrality dependences of correlations from those of Ψ_2 dependent case, which may suggest possible different evolution processes between the 2nd- and 3rd-order geometry planes

BACK UP

Nuclear Modification Factor R_{AA}

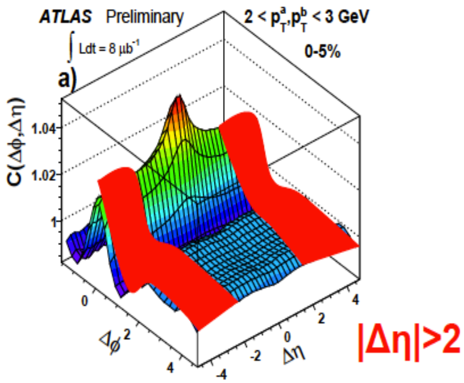
$$R_{AA} = \frac{d^2N^{AA}/dp_T d\eta}{N_{coll} d^2N^{pp}/dp_T d\eta}$$



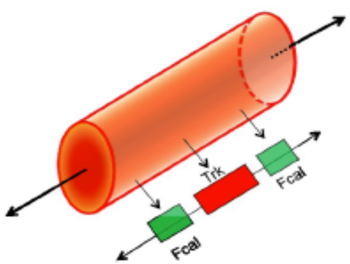
- ✧ Ratio of invariant yield scaled by that in p+p collision with scale
  - – R_{AA}<1(suppression), R_{AA}=1(no change), R_{AA}>1(enhance)
- ✧ Suppression of hadron production
- ✧ No suppression of direct photon

Contributions of v_n (n>2) in correlations

2Par. Correlation

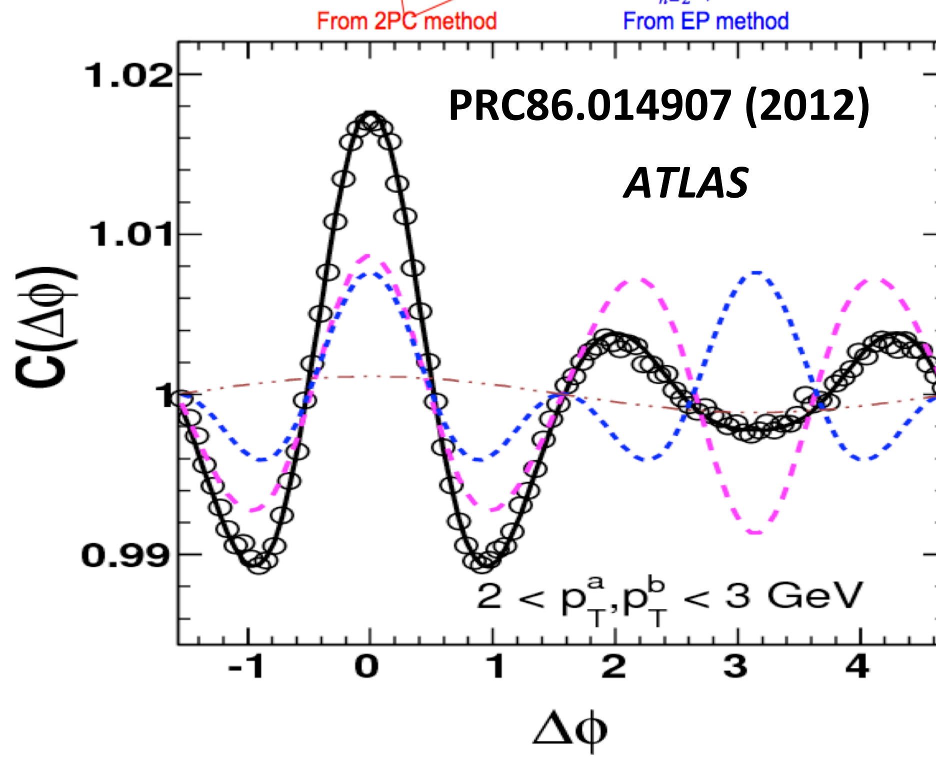


v_n with EP Method



Track at |η| <2.5 with EP from full FCAL 3.3<|η|<4.8

$$C(\Delta\phi) = b^{2P}(1 + 2v_{1,1}^{2P}\cos\Delta\phi + 2\sum_{n=2}^{6} v_n^{EP} v_n^{EP} \cos n\Delta\phi)$$



- ✧ Double-hump & ridge of long-rapidity correlation explained
- ✧ Short-rapidity correlation with v_n subtraction to discuss parton behavior

Data Set & Particle Selection

- ✧ PHENIX year 2007 Experiment
- ✧ Au+Au collisions at $\sqrt{s_{NN}}$=200 GeV
  - Minimum Bias trigger 4.4 billion events
- ✧ Charged hadron selection
  - 2σ cut of track-hit matching
  - Electron veto
  - Energy/momentum cut of high p_T particles for background rejection
    - $E^{EMC}<0.30+0.20*p_T$ rejected for $p_T>5.0$ GeV/c
  - Pair cut of miss-reconstructed hadron pairs

Tracking Efficiency

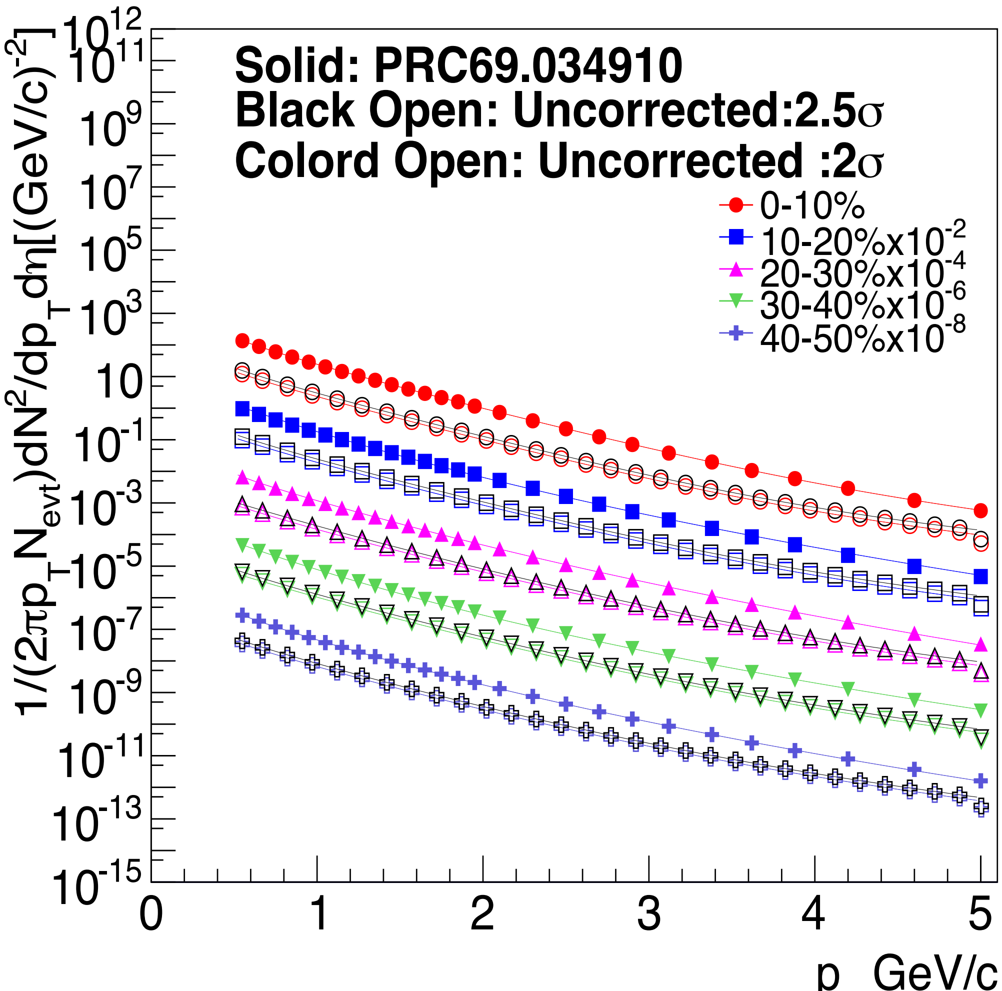
✧ **Efficiency correction by ratio of uncorrected invariant yield over fully corrected ones**

$$\varepsilon = \frac{\sigma^{uncor}}{\sigma^{cor}}$$

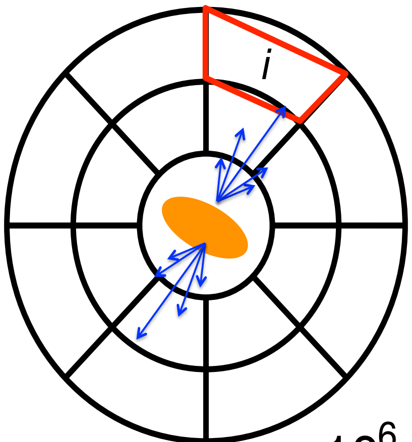
✧ **Ratio calculated by fitting functions to the invariant yields**

Fit Function

$$F(p_T) = p_0 * \left(\frac{p_1}{p_1 + p_T}\right)^{p_2}$$

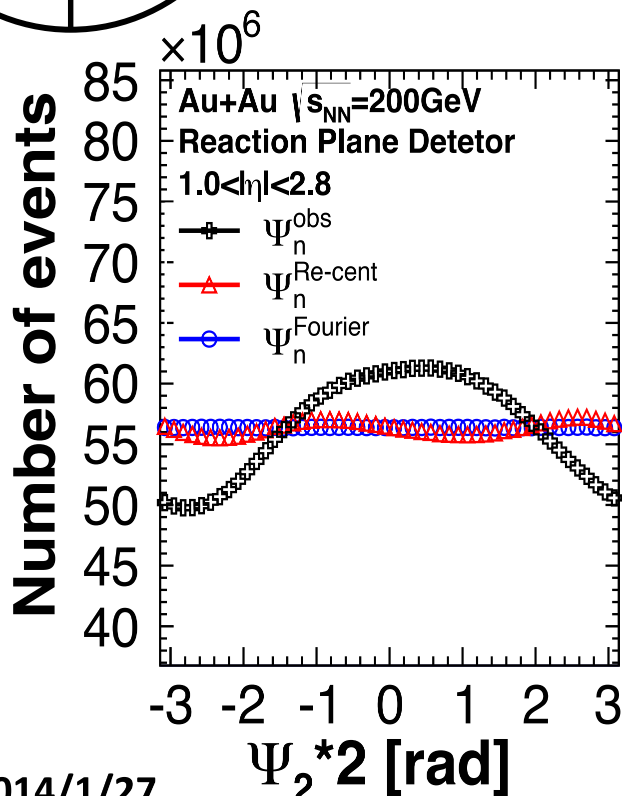


Event Plane Calibration



ϕ_i : Azimuthal angle

w_i : Weight (Charge etc.)



Raw distribution

$$Q_x = \sum_i w_i \cos(n\phi_i), Q_y = \sum_i w_i \sin(n\phi_i)$$

$$\Psi_n = \frac{1}{n}\tan^{-1}\left(\frac{Q_y}{Q_x}\right)$$

Re-centering

$$Q_x^{Rec} = \frac{Q_x - \langle Q_x \rangle}{\sigma_x}, Q_y^{Rec} = \frac{Q_y - \langle Q_y \rangle}{\sigma_y}$$

$$\Psi_n^{Rec} = \frac{1}{n}\tan^{-1}\left(Q_y^{Rec}/Q_x^{Rec}\right)$$

Fourier correction

$$n\Psi_n^{Fourier} = n\Psi_n^{Rec} + n\Delta\Psi_n$$

$$n\Delta\Psi_n = \sum_k \left\{A_k \cos\left(kn\Psi_n^{Rec}\right) + B_k \sin\left(kn\Psi_n^{Rec}\right)\right\}$$

$$A_k = -\frac{2}{k}\left\langle \cos\left(kn\Psi_n^{Rec}\right)\right\rangle, B_k = \frac{2}{k}\left\langle \sin\left(kn\Psi_n^{Rec}\right)\right\rangle$$

Event Plane Resolution

EP Resolution **PRC 58.1671 (1998)**

✧ **Resolution +/-η**

$$\sigma_n^{EP} = \sqrt{\left\langle \cos kn(\Psi_n^{EP(+\eta)} - \Psi_n^{EP(-\eta)}) \right\rangle}$$

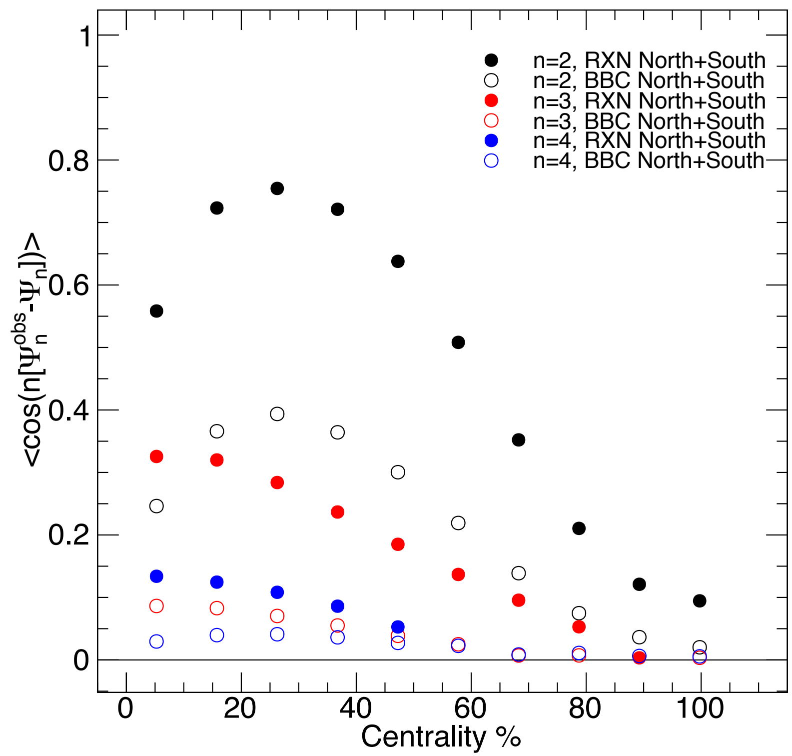
$$= \left\langle \cos kn(\Psi_n^{EP+/-\eta} - \Psi_n) \right\rangle$$

$$= \frac{\pi}{8}\chi_n^2 \left[I_{(k-1)/2}\left(\frac{\chi_n^2}{4}\right) + I_{(k+1)/2}\left(\frac{\chi_n^2}{4}\right) \right]^2$$

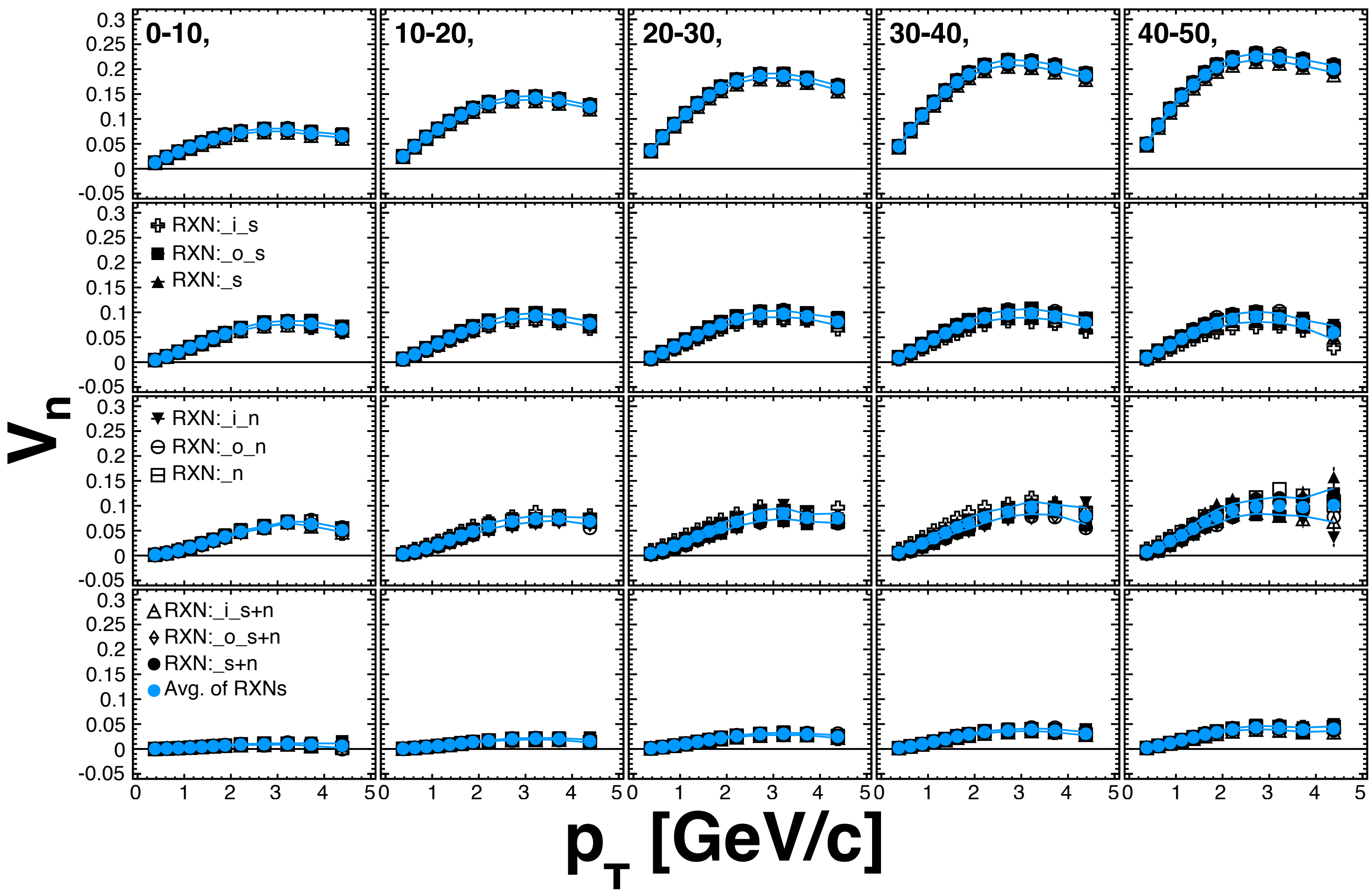
✧ **Resolution +&-η**

– χ_n ➔ $\sqrt{2}\chi_n$

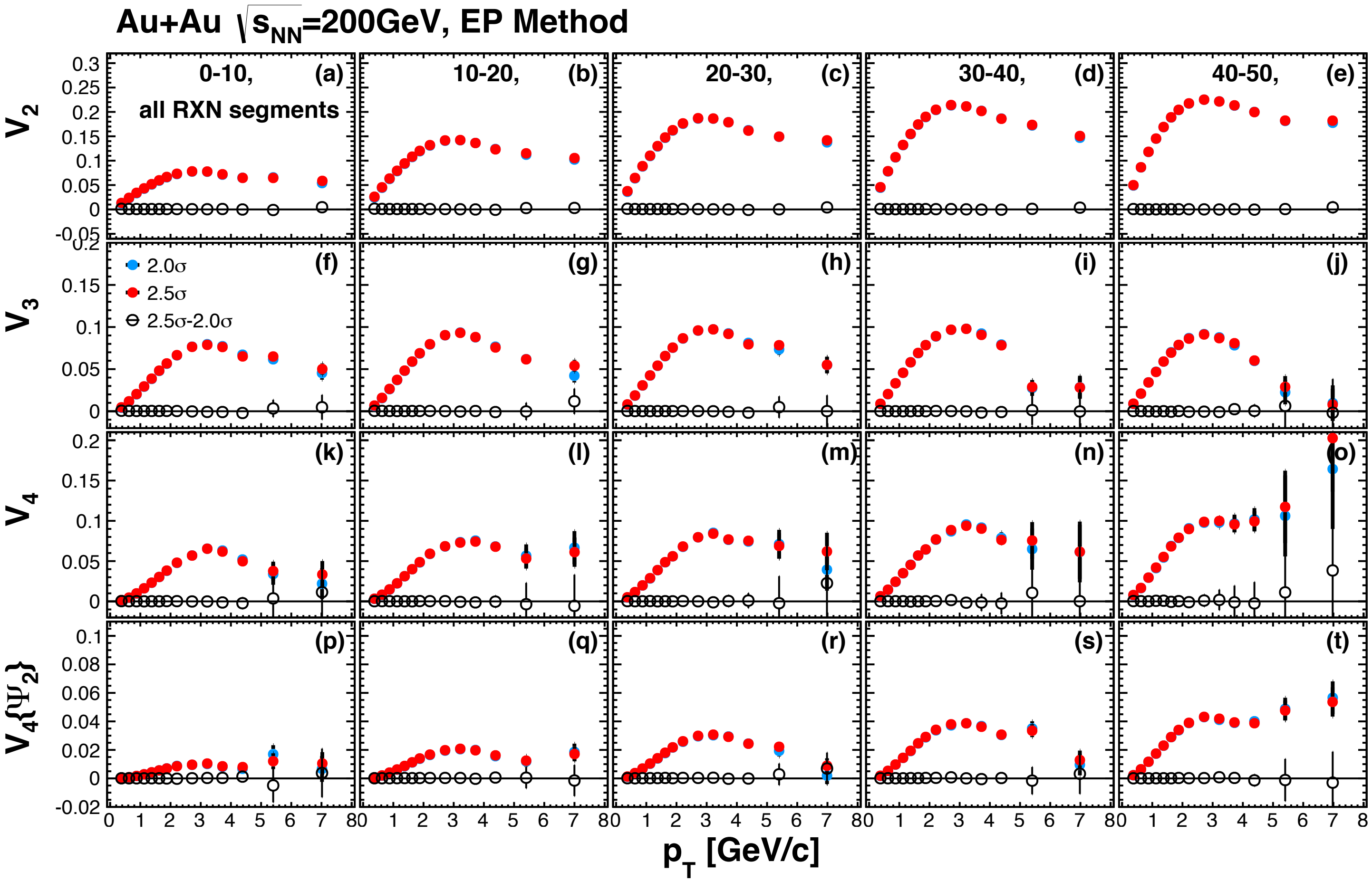
$$\sigma_n^{EP} = \frac{\pi}{8}2\chi_n^2 \left[I_{(k-1)/2}\left(\frac{2\chi_n^2}{4}\right) + I_{(k+1)/2}\left(\frac{2\chi_n^2}{4}\right) \right]^2$$



v_n systematics : RXN segments



v_n systematics : Matching Cut



v_n systematics : RXN-BBC Difference

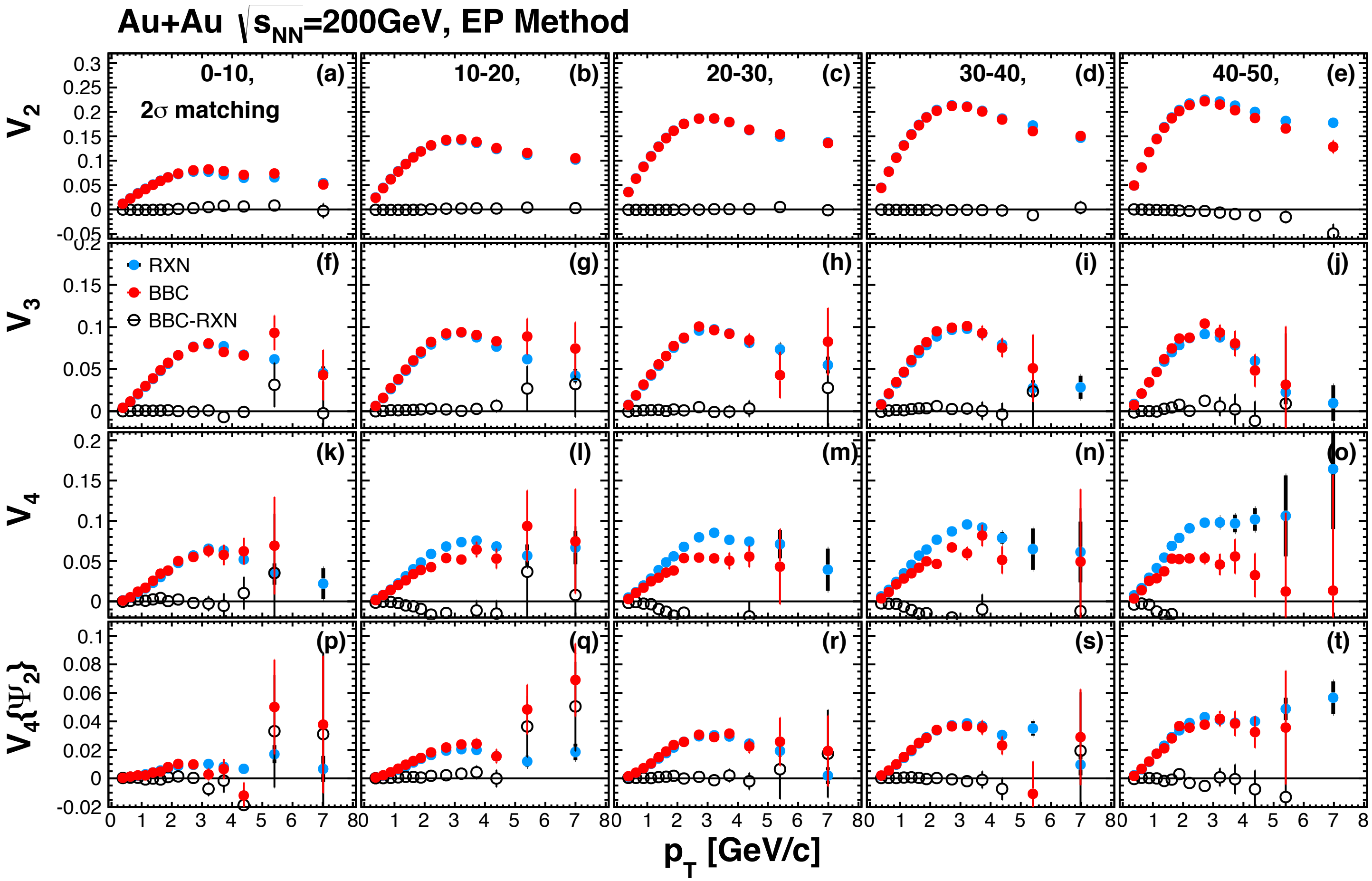


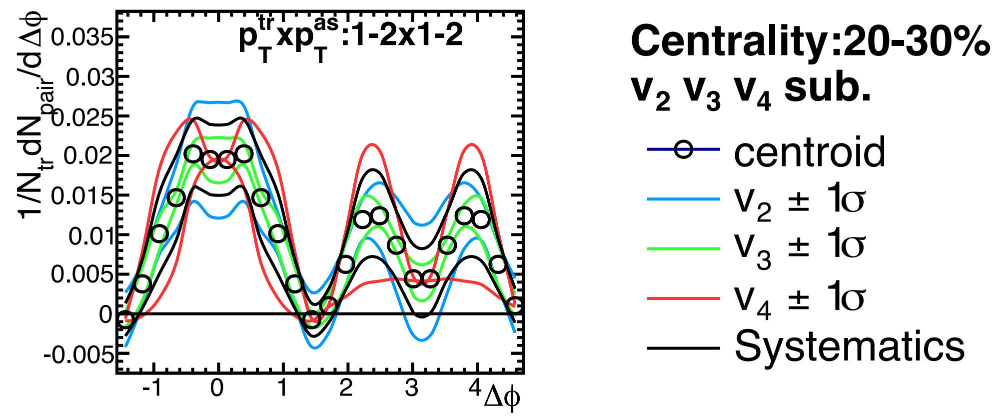
Table of total v_n systematic uncertainties

Table 3.8: Summary of percentile ratio of v_n systematic uncertainties

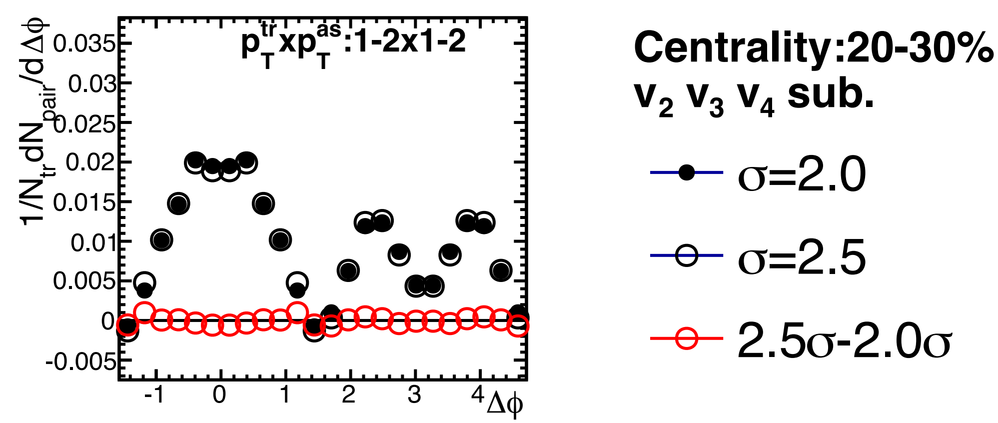
Centrality %	p_T GeV/c	v_2 sys. %	v_3 sys. %	v_4 sys. %	$v_4\{\Psi_2\}$ sys. %
0-10	0.5-1.0	5.449	6.387	24.87	48
	1.0-2.0	4.32	4.911	10.1	14.66
	2.0-4.0	4.536	4.131	4.412	11.39
	4.0-10.0	10.43	6.184	21.67	191.3
10-20	0.5-1.0	3.658	7.992	28.53	12.17
	1.0-2.0	2.891	6.431	20.16	12.27
	2.0-4.0	2.69	6.163	27.64	13.72
	4.0-10.0	3.124	13.62	19.09	32.09
20-30	0.5-1.0	2.811	9.469	35.48	9.633
	1.0-2.0	2.485	7.818	28.85	8.422
	2.0-4.0	2.391	6.822	28.03	6.577
	4.0-10.0	2.98	9.503	32.24	12.21
30-40	0.5-1.0	2.506	12.42	35.81	7.385
	1.0-2.0	2.462	9.695	29.88	6.509
	2.0-4.0	2.556	9.673	36.75	5.913
	4.0-10.0	2.934	14.18	44.32	31.73
40-50	0.5-1.0	2.575	13.8	32.96	6.338
	1.0-2.0	2.688	12.06	34.44	6.479
	2.0-4.0	3.224	11.7	45.4	10.71
	4.0-10.0	7.877	33.53	77.07	29.33

Systematics of Correlations

- ✧ Systematics propagated from v_n measurements
  - – Varying v_n value ± 1σ (# of harmonics 3 x ± 1σ 2 = 6 combinations)
  - – Systematics : RMS of above 6 combinations



- ✧ Systematics from matching cut
  - – Systematics : Difference between 2.5σ-2.0σ (main)



- ✧ Total Systematics
  - – Quadrature-sum of above two systematics

EP Resolution in Monte Carlo

- ✧ **Analytical formula of EP Resolution (RXN:S+N) as a function of χ_n**
  - **Convert Resolution to χ_n**

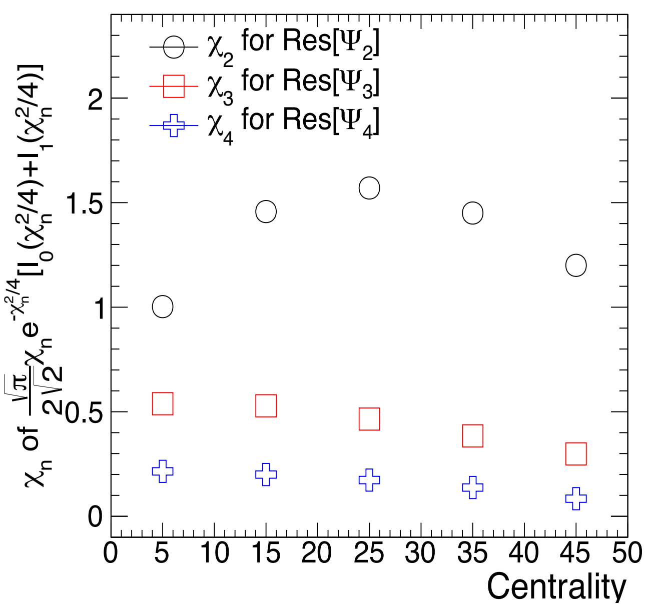
PRC 58.1671 (1998)

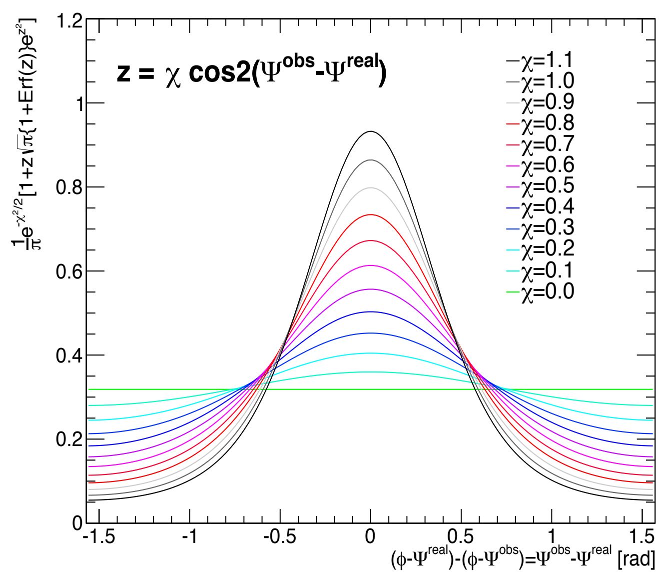
$$\left\langle \cos\left[kn(\Psi_n^{obs} - \Psi_n^{real})\right]\right\rangle = \frac{\sqrt{\pi}}{2\sqrt{2}}\chi_n e^{-\chi_n^2/4}\left[I_{(k-1)/2}\left(\frac{\chi_n^2}{4}\right) + I_{(k+1)/2}\left(\frac{\chi_n^2}{4}\right)\right].$$

- ✧ **Relative distribution between real and observed EP calculated using χ_n**

$$\frac{dN^{eve}}{d[kn(\Psi_n^{obs} - \Psi_n^{real})]} = \frac{1}{\pi}e^{-\chi_n^2/2}\left[1 + z\sqrt{\pi}[1+\mathrm{erf}(z)]e^{z^2}\right] \qquad z = \frac{1}{\sqrt{2}}\chi_n \cos n(\Psi_n^{obs} - \Psi_n^{real})$$

PRD 48.1132 (1993)





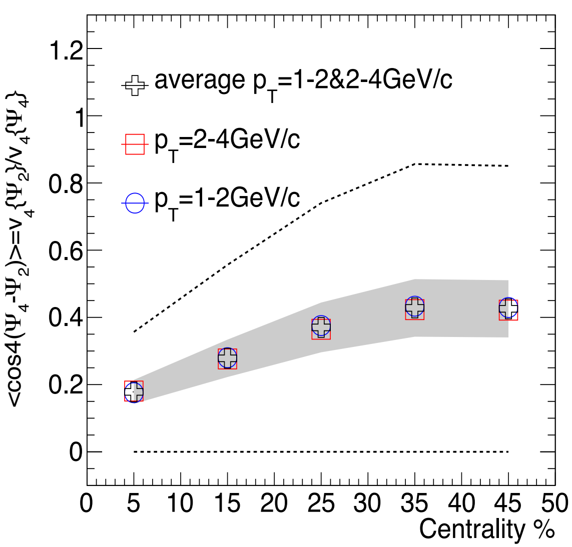
Ψ_2-Ψ_4 correlation in Monte Carlo

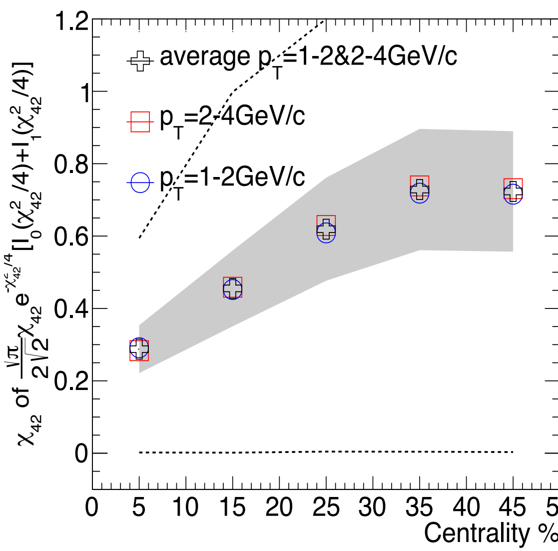
- ✧ **Ψ_2-Ψ_4 correlation at p_T 1-2&2-4GeV :** $\langle \cos[4(\Psi_2 - \Psi_4)] \rangle = v_4\{\Psi_2\}/v_4\{\Psi_4\}$
  - **To avoid jet contribution to the Ψ_2-Ψ_4 correlation**

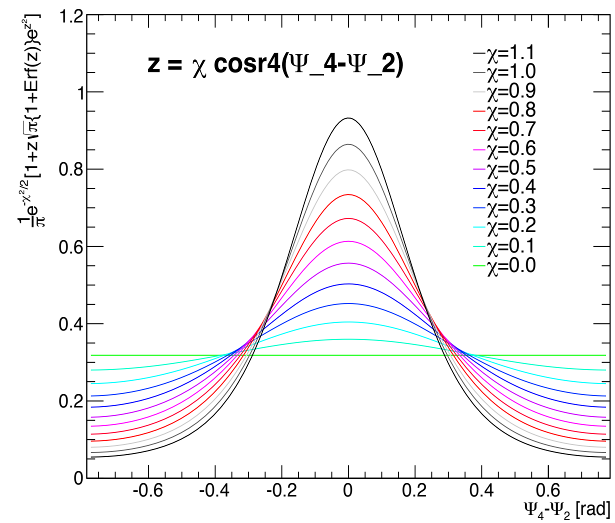
- ✧ **Obtain χ_{42} & reconstruct relative distribution between Ψ_2 & Ψ_4**

$$\langle \cos[4(\Psi_2 - \Psi_4)] \rangle = \frac{\sqrt{\pi}}{2\sqrt{2}} \chi_{42} e^{-\chi_{42}^2/4} \left[I_0\left(\frac{\chi_{42}^2}{4}\right) + I_1\left(\frac{\chi_{42}^2}{4}\right) \right]$$

$$\frac{dN^{eve}}{d[kn(\Psi_n^{obs} - \Psi_n^{real})]} = \frac{1}{\pi} e^{-\chi_n^2/2} \left[1 + z\sqrt{\pi}[1 + \mathrm{erf}(z)] e^{z^2} \right] \quad z = \frac{1}{\sqrt{2}} \chi_n \cos n(\Psi_n^{obs} - \Psi_n^{real})$$







Ψ_2-Ψ_3 correlation

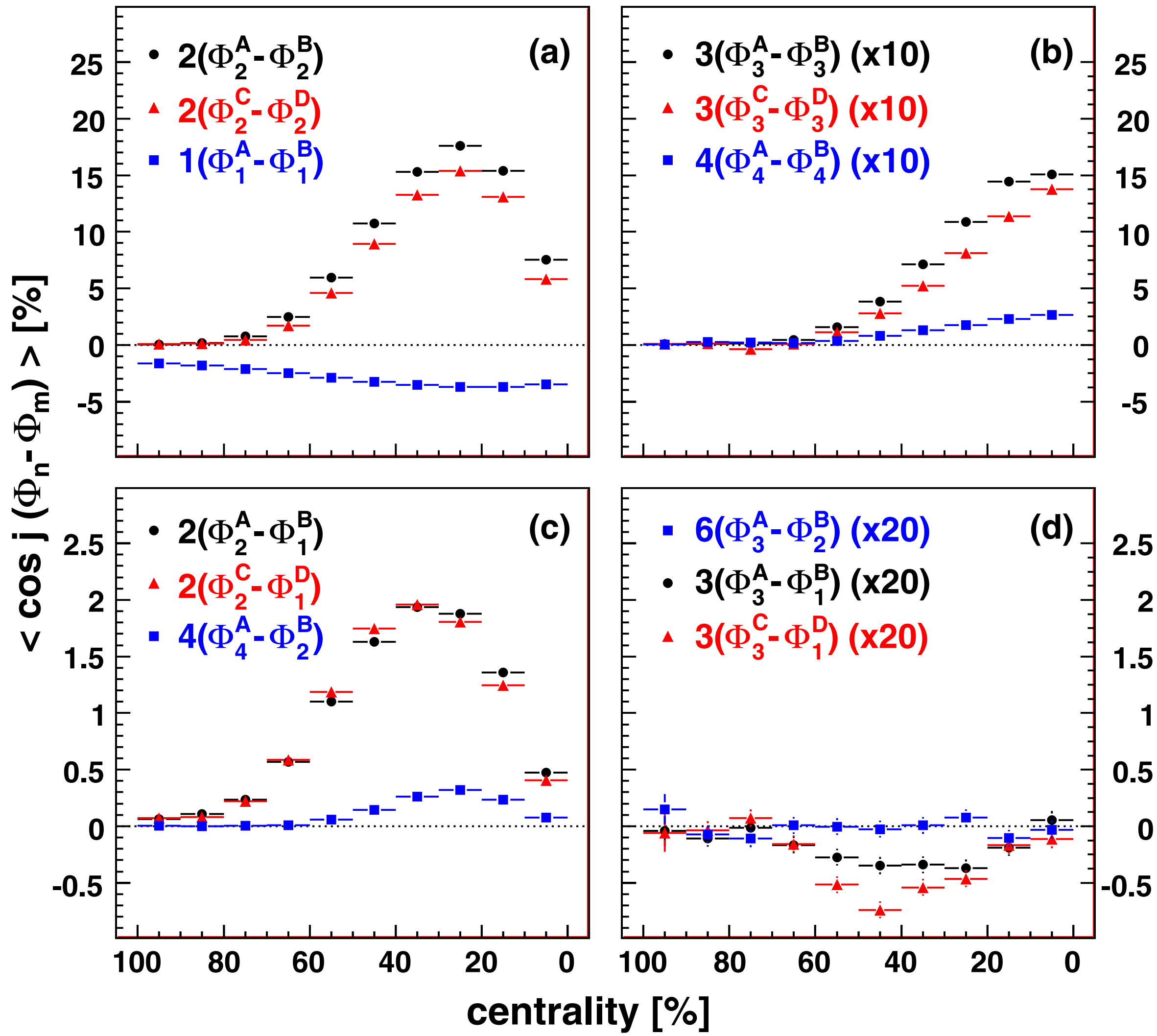
PRL107.252301 (2011)

A : RXN North

B : BBC South

C : MPC North

D : MPC South



EP Resolution Correction : Iteration-1

✧ Trigger bin is also smeared due to limited EP resolution as v_n

– Add an offset λ=1.0 to correlation *Y* to avoid possible divisions by zero

Raw Correlation

Offset Trigger Bin

$$\mathbf{A}(k) = \begin{pmatrix} 1+Y(0,k) \\ 1+Y(1,k) \\ 1+Y(2,k) \\ 1+Y(3,k) \\ 1+Y(4,k) \\ 1+Y(5,k) \\ 1+Y(6,k) \\ 1+Y(7,k) \end{pmatrix}, \quad k = 0, \cdots, 23$$

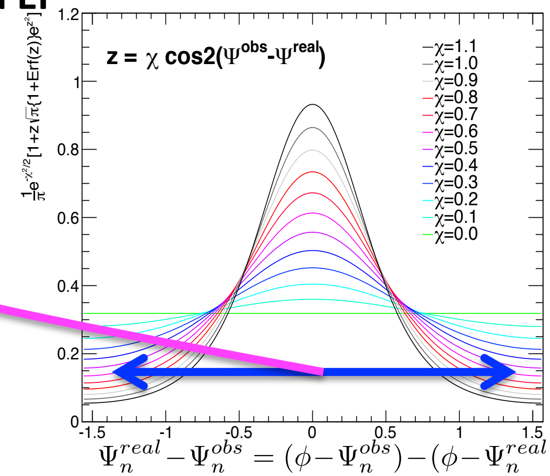
Δϕ Bin

Smearing Effect : Calculated by relative distribution between real and observed EP

Trigger Bin

$$S = \begin{pmatrix} s_0 & s_1 & s_2 & s_3 & s_4 & s_3 & s_2 & s_1 \\ s_1 & s_0 & s_1 & s_2 & s_3 & s_4 & s_3 & s_2 \\ s_2 & s_1 & s_0 & s_1 & s_2 & s_3 & s_4 & s_3 \\ s_3 & s_2 & s_1 & s_0 & s_1 & s_2 & s_3 & s_4 \\ s_4 & s_3 & s_2 & s_1 & s_0 & s_1 & s_2 & s_3 \\ s_3 & s_4 & s_3 & s_2 & s_1 & s_0 & s_1 & s_2 \\ s_2 & s_3 & s_4 & s_3 & s_2 & s_1 & s_0 & s_1 \\ s_1 & s_2 & s_3 & s_4 & s_3 & s_2 & s_1 & s_0 \end{pmatrix},$$

$\sum_n s_n = 1$, s_n : Ratio from n[th] away-bin



Smeared Correlation

$$\mathbf{B}(k) = \mathbf{S}\mathbf{A}(k)$$

Correction Matrix

$$\mathbf{C}(\mathbf{k}) = (c_{ij})$$

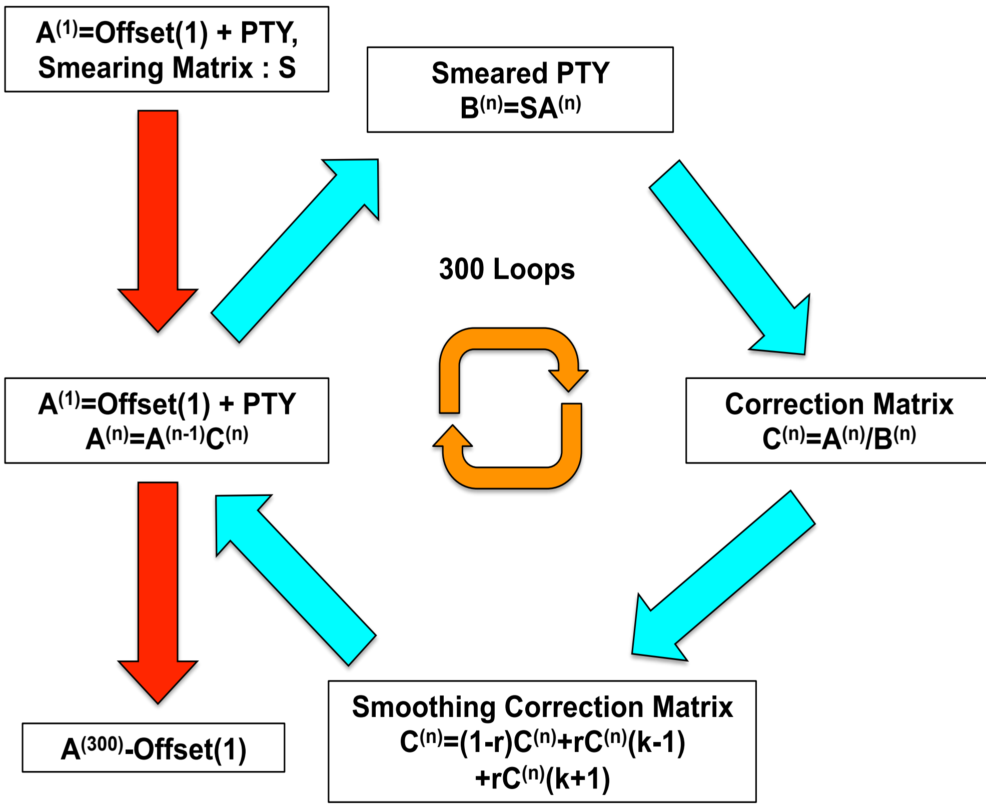
$$c_{ij} = \begin{cases} \frac{A(i,k)}{B(i,k)} & (i = j) \\ 0 & (i \neq j) \end{cases}$$

Corrected Correlation

$$\mathbf{A}^{\mathbf{cor}}(k) = \mathbf{C}(k)\mathbf{A}(k)$$

EP Resolution Correction : Iteration-2

- ✧ Start of iteration : experimental results (already smeared once)
- ✧ Obtained correction is not true
- ✧ Iteration until conversions of each coefficients
  - – 300 Loops



Notation in Iteration

$$\mathbf{A} \rightarrow \mathbf{A}^{(\mathbf{n})}$$

$$\mathbf{B} \rightarrow \mathbf{B}^{(\mathbf{n})}$$

$$\mathbf{C} \rightarrow \mathbf{C}^{(\mathbf{n})}$$

$$\mathbf{A}^{\mathbf{cor}} \rightarrow \mathbf{A}^{(\mathbf{n+1})}$$

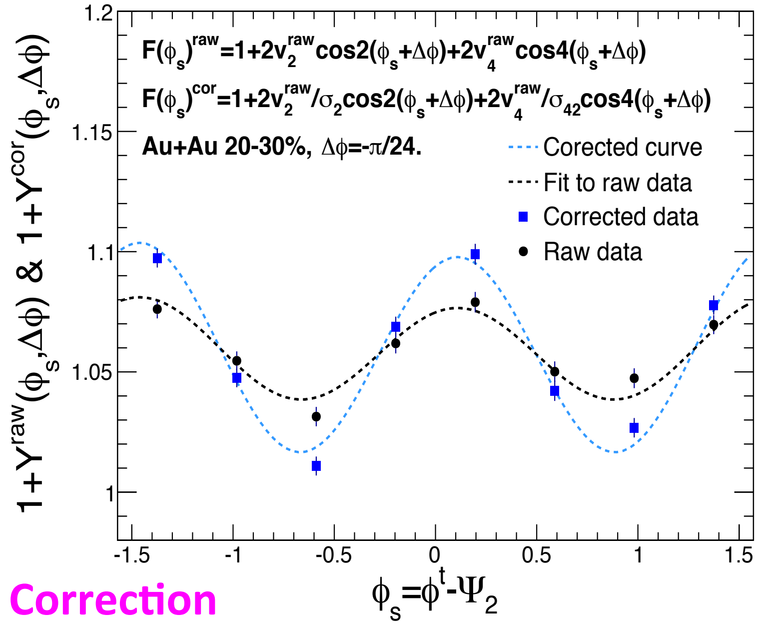
Smoothing

- ✧ Preventing a divergence of statistical fluctuations among Δϕ bins
- ✧ 2r=0.20 & 0.30

$$c_{ii}^{(n)}(k) = (1-r)c_{ii}^{(n)}(k) + (r/2)\, c_{ii}^{(n)}(k-1) + (r/2)\, c_{ii}^{(n)}(k+1)$$

EP Resolution Correction : Fitting Method

- ✧ Assuming correlation yield has anisotropy with respect to EP
- ✧ Correction by EP resolution as done in v_n measurements
  - – Method by PRC.84.024904(2011)
- ✧ Offset λ=1.0 to avoid possible division by zero



EP Resolution Correction

Ψ_2 dependent case

$$\lambda + Y^{cor}(\phi_s, \Delta\phi) = \frac{\lambda + b_0\left[1 + 2v_2^Y/\sigma\cos 2(\phi_s+\Delta\phi) + 2v_4^Y/\sigma_{42}\cos 4(\phi_s+\Delta\phi)\right]}{\lambda + b_0\left[1 + 2v_2^Y\cos 2(\phi_s+\Delta\phi) + 2v_4^Y\cos 4(\phi_s+\Delta\phi)\right]}(\lambda + Y(\phi_s,\Delta\phi))$$

Fitting

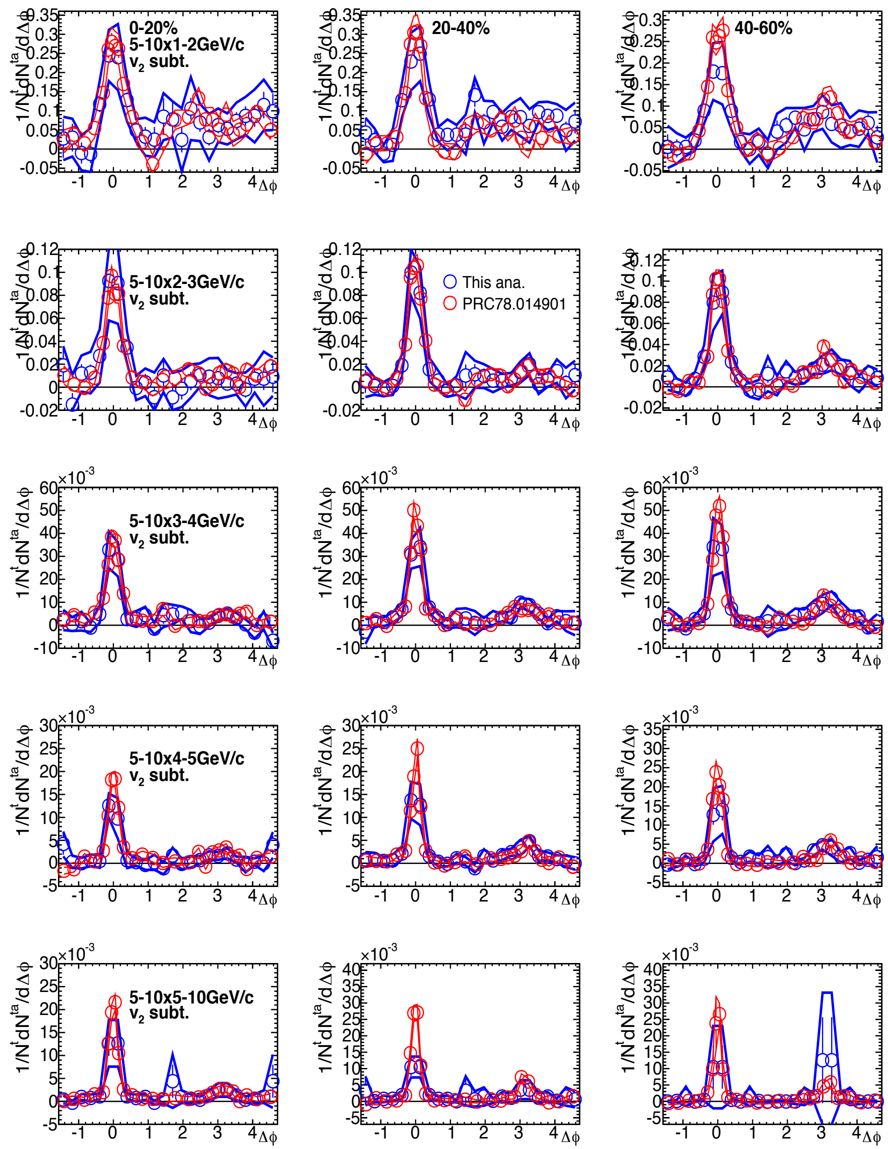
Ψ_3 dependent case

$$\lambda + Y^{cor}(\phi_s, \Delta\phi) = \frac{\lambda + b_0\left[1 + 2v_3^Y/\sigma_3\cos 3(\phi_s+\Delta\phi)\right]}{\lambda + b_0\left[1 + 2v_3^Y\cos 3(\phi_s+\Delta\phi)\right]}(\lambda + Y(\phi_s,\Delta\phi))$$

Fitting

Consistency check : high-p_T trigger

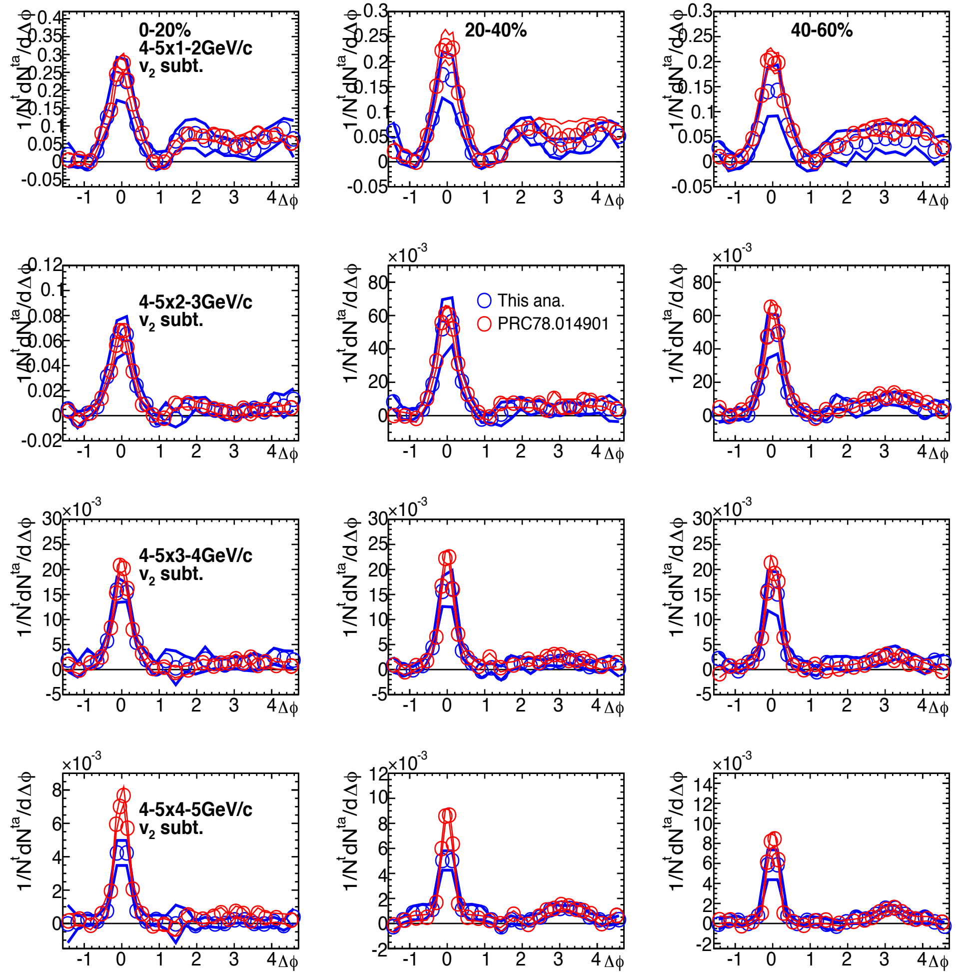
- ✧ **Three-Centralities**
  - – **0-20, 20-40,40-60%**
- ✧ **Particle Selections**
  - – **Trigger p_T:5-10 GeV/c**
  - – **Associate p_T : 1-10 GeV/c**
- ✧ **Subtracted Backgrounds**
  - – **Only v_2**
- ✧ **Consistent with previous PHENIX results (PRC78.014901)**



○ : This Analysis
○ : PRC78.014901

Consistency check : mid-p_T trigger

- ✧ Three-Centralities
  - 0-20, 20-40,40-60%
- ✧ Particle Selections
  - Trigger p_T: 4-5 GeV/c
  - Associate p_T : 1-5 GeV/c
- ✧ Subtracted Backgrounds
  - Only v_2
- ✧ Consistent with previous PHENIX results (PRC78.014901)

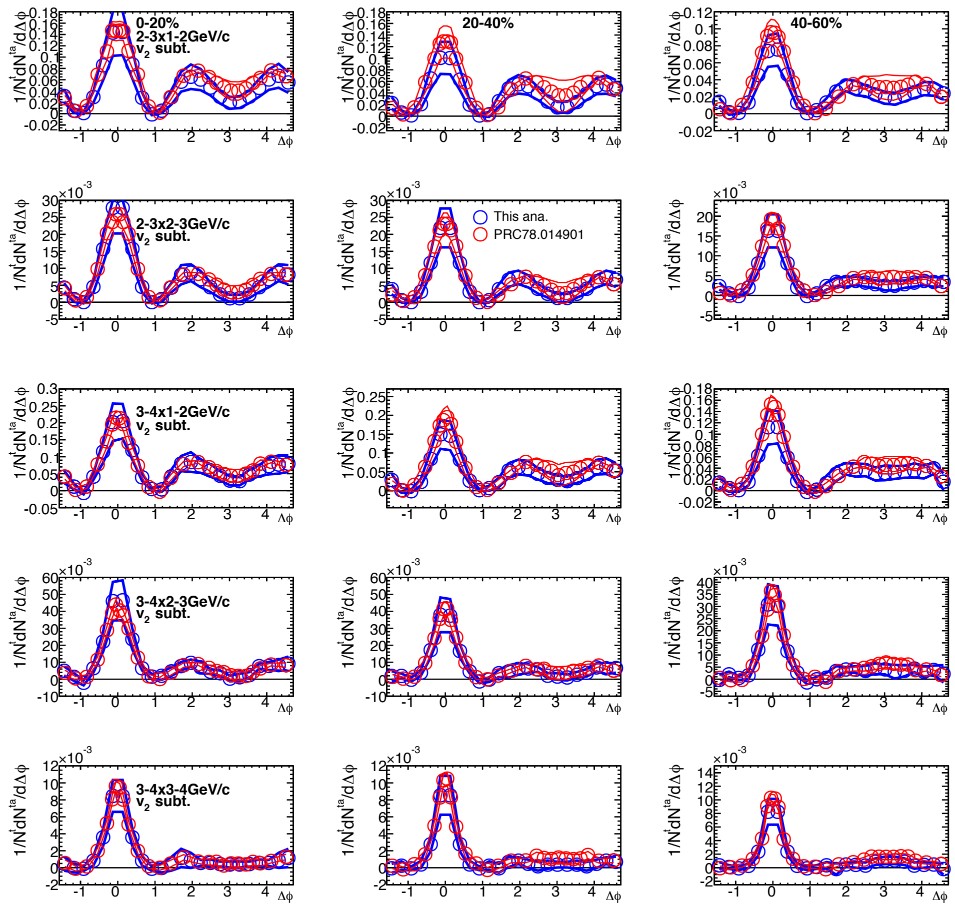


○ : This Analysis

○ : PRC78.014901

Consistency check : low-p_T trigger

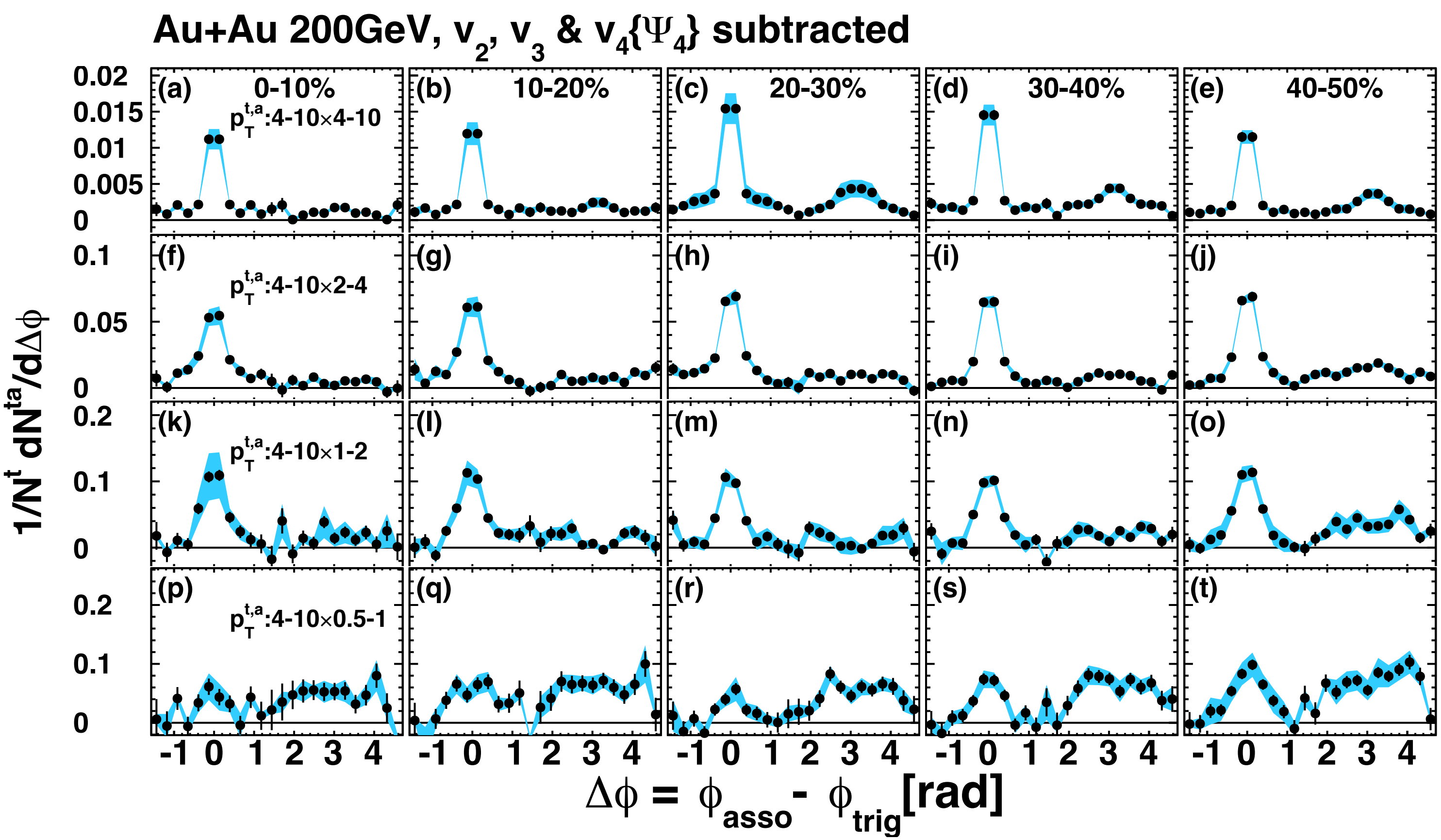
- ✧ Three-Centralities
  - – 0-20, 20-40,40-60%
- ✧ Particle Selections
  - – Trigger p_T: 2-4 GeV/c
  - – Associate p_T : 1-4 GeV/c
- ✧ Subtracted Backgrounds
  - – Only v_2
- ✧ Consistent with previous PHENIX results (PRC78.014901)



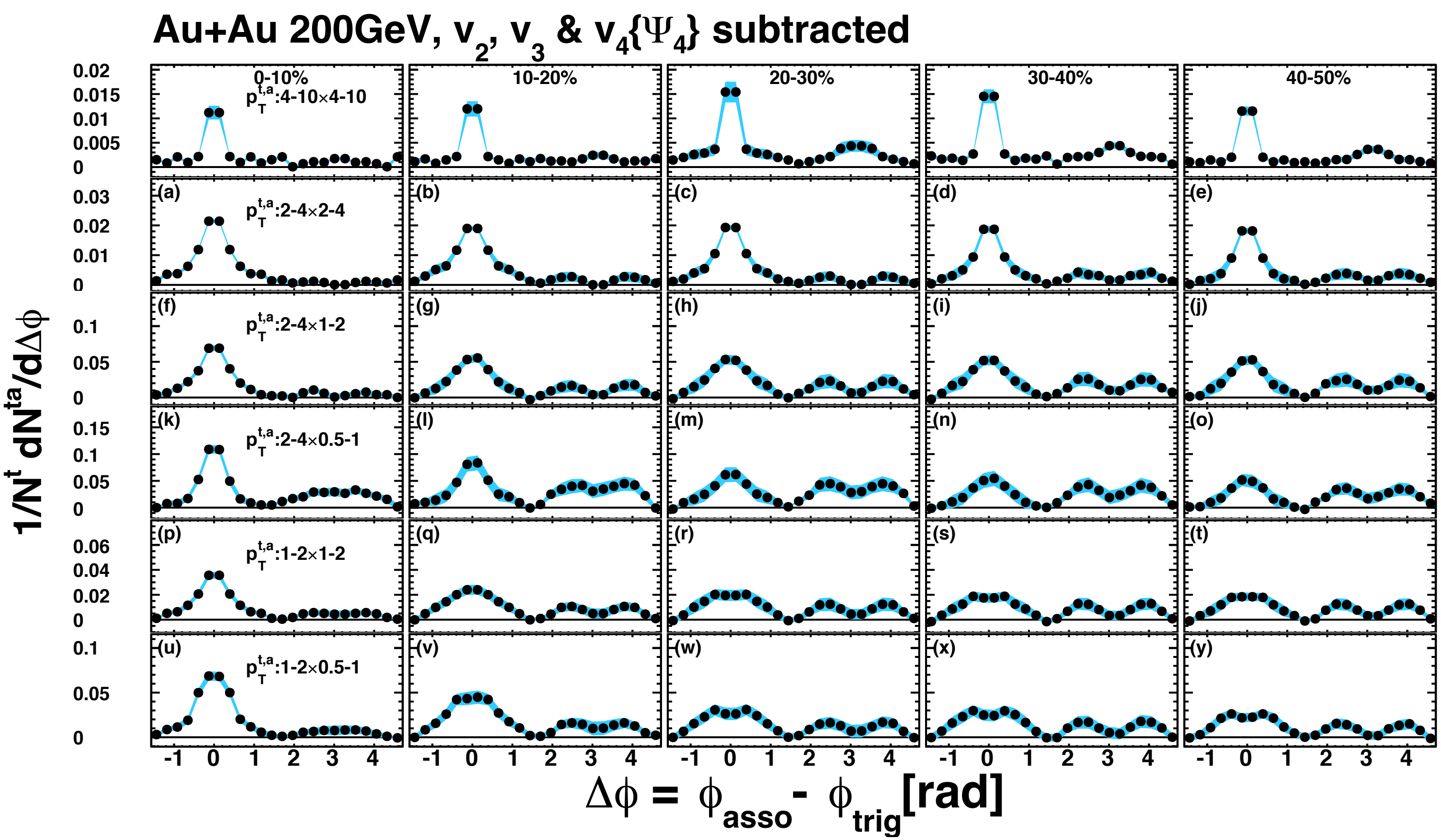
○ : This Analysis

○ : PRC78.014901

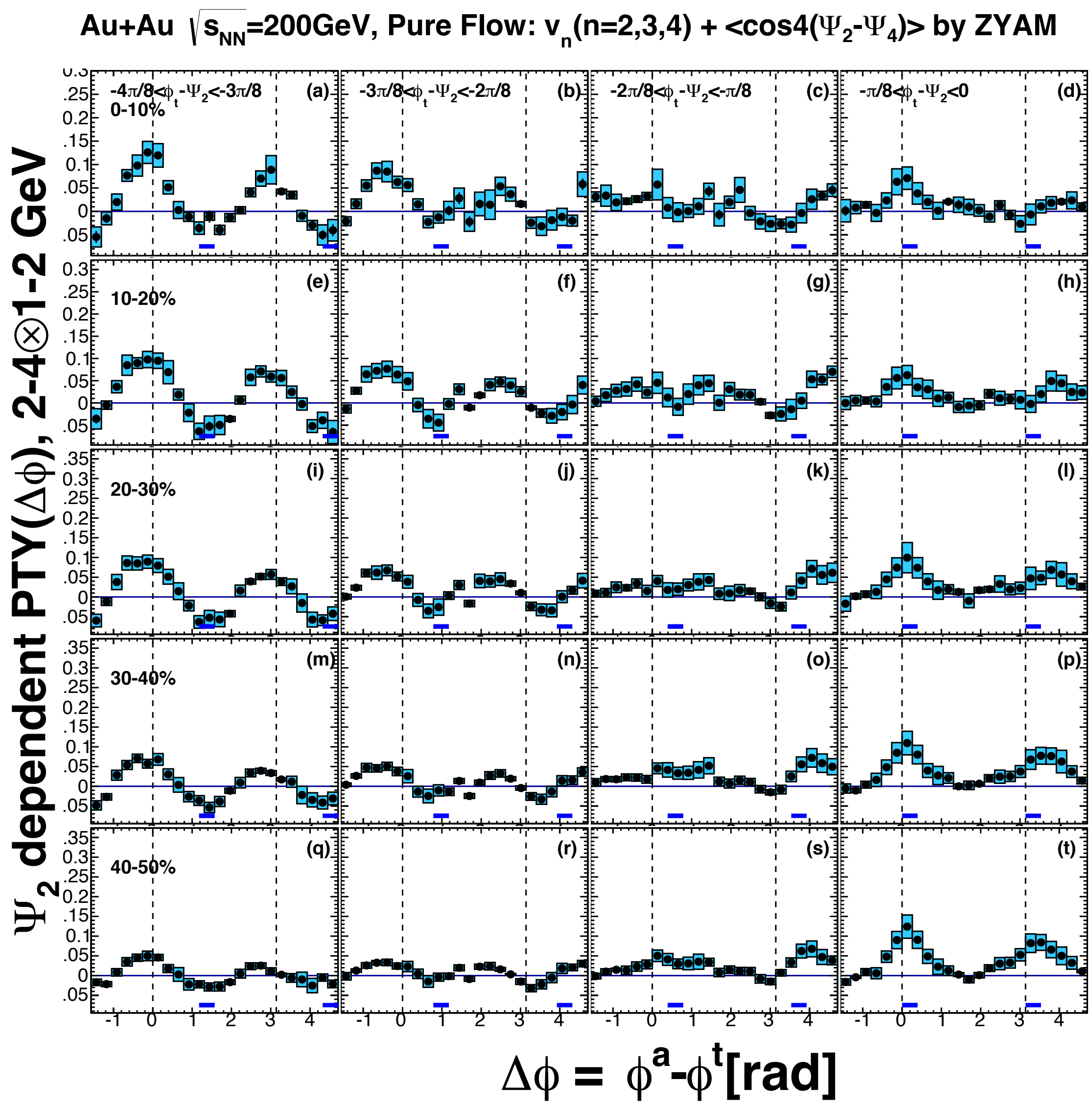
High p_T Trigger Two-Particle Correlations



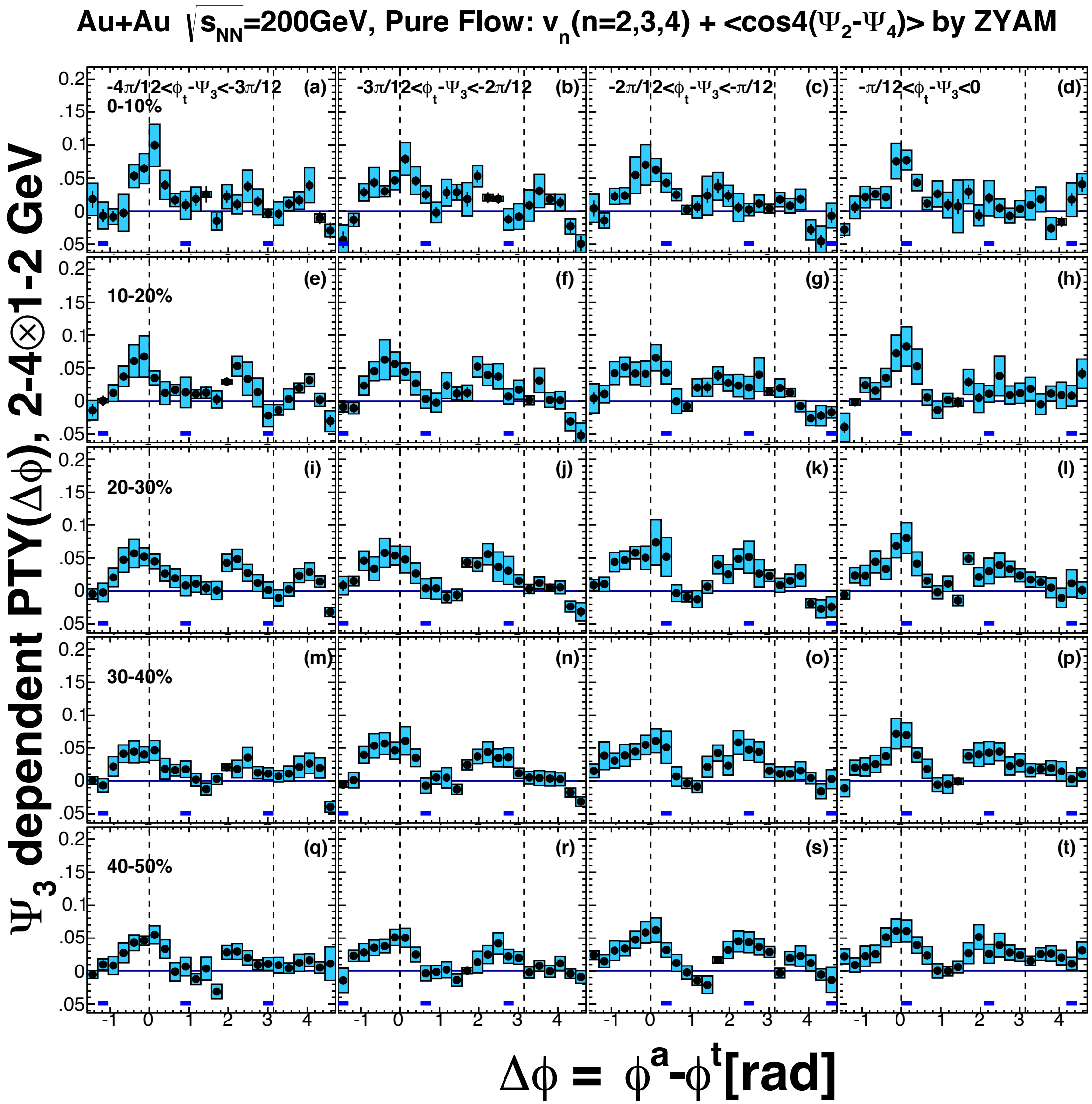
Intermediate p_T Two-Particle Correlations



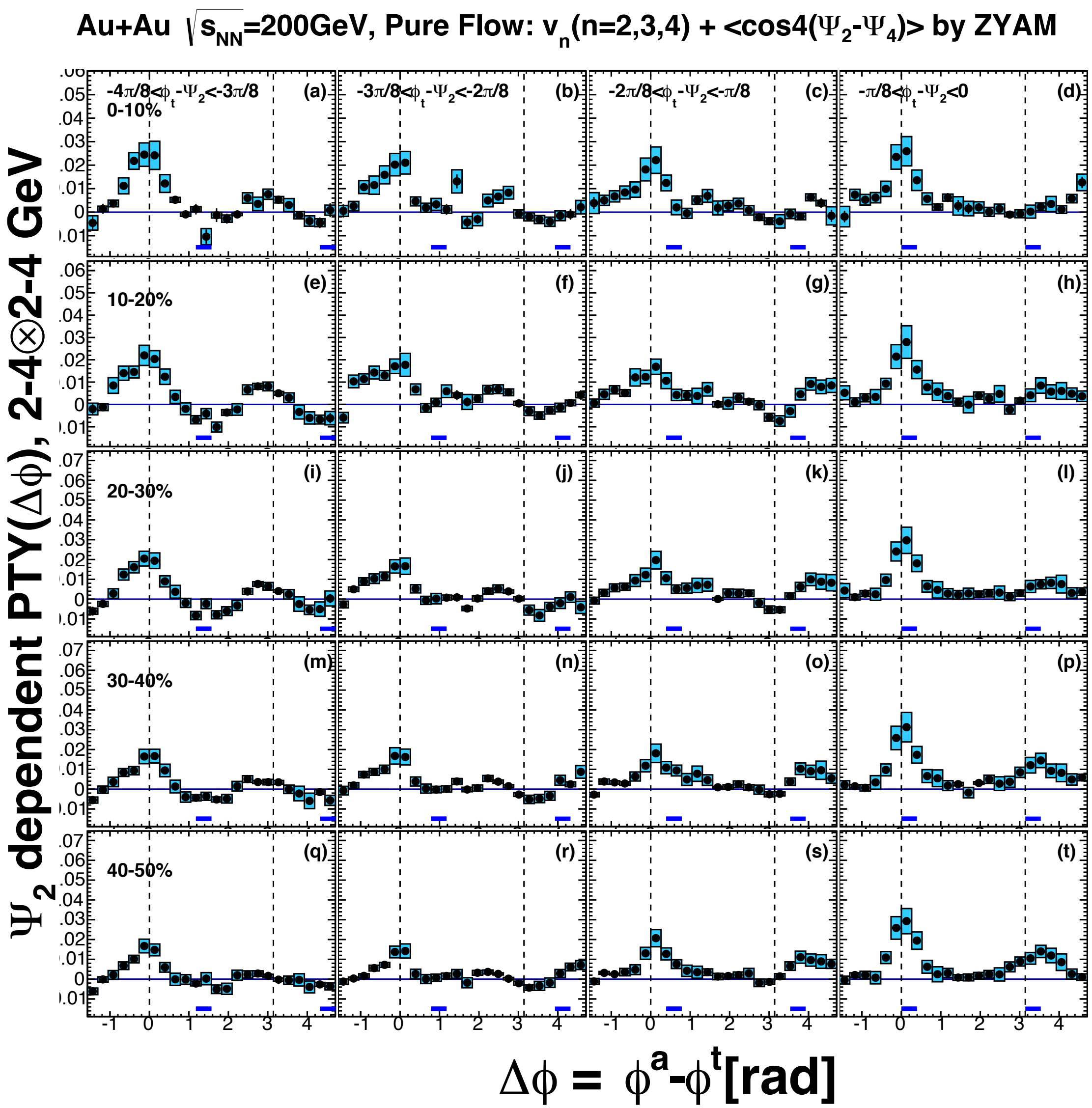
Ψ_2 Dependent Correlations : p_T 2-4x1-2 GeV/c



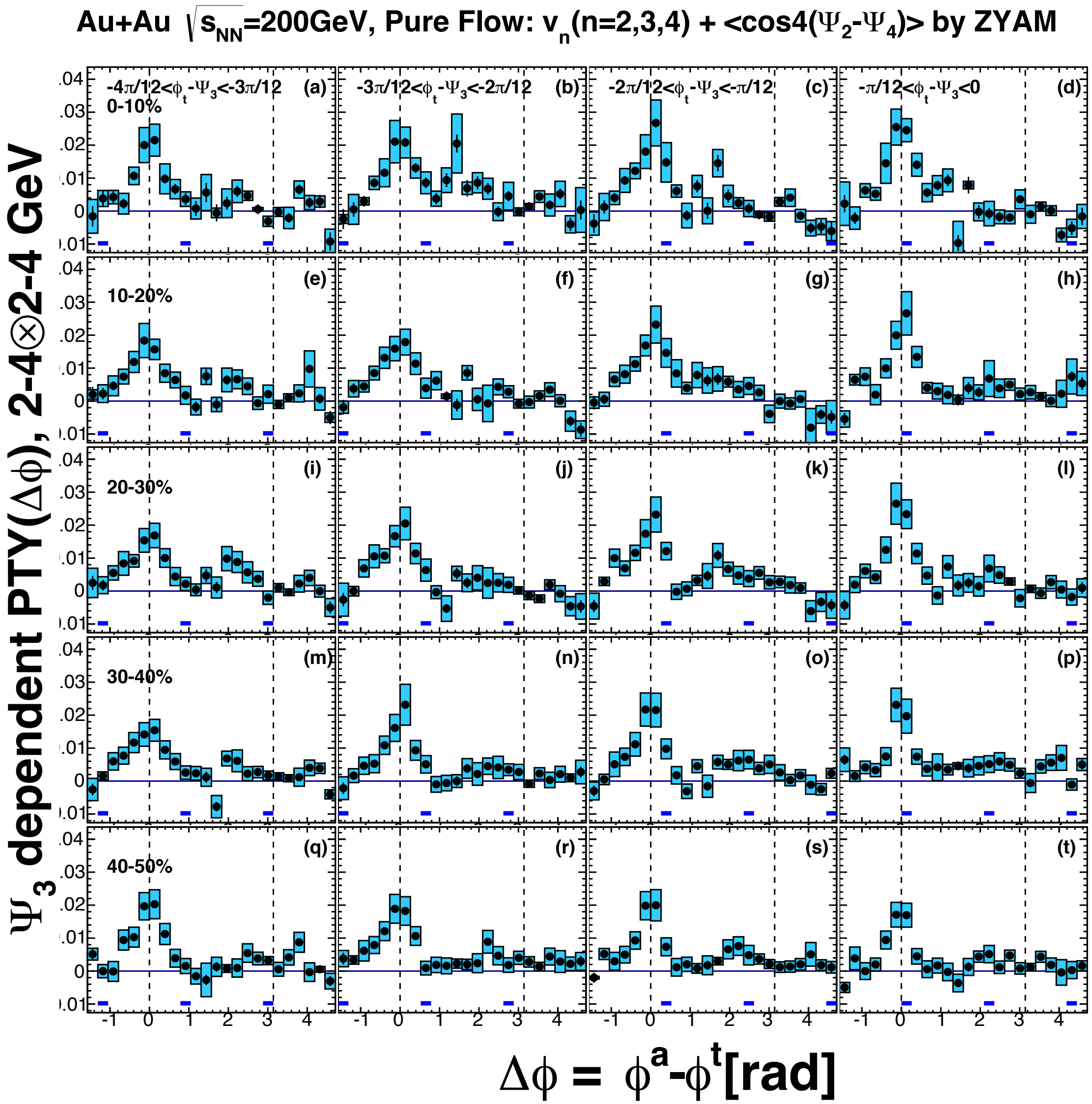
Ψ_3 Dependent Correlations : p_T 2-4x1-2 GeV/c



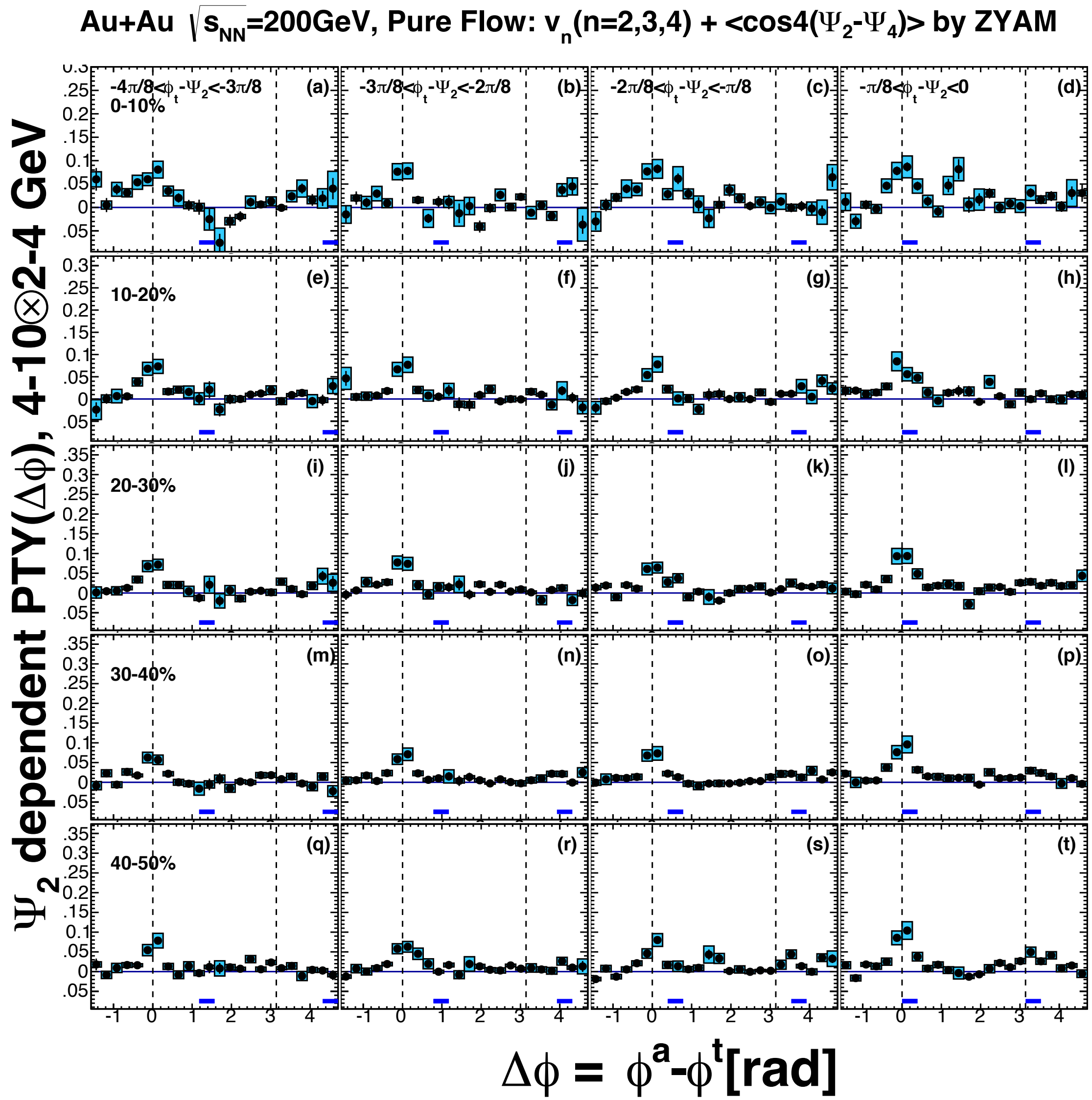
Ψ_2 Dependent Correlations : p_T 2-4x2-4 GeV/c



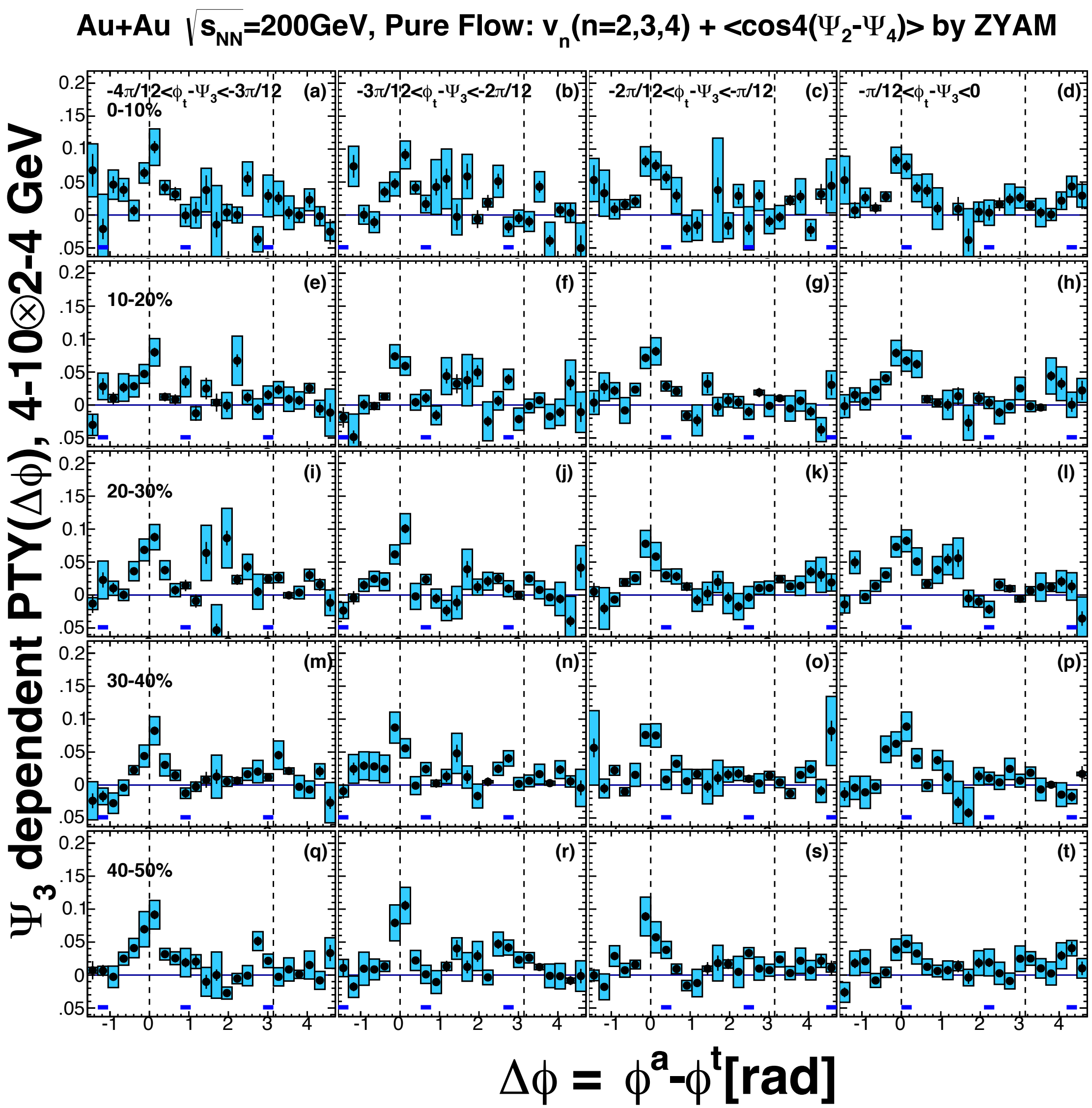
Ψ_3 Dependent Correlations : p_T 2-4x2-4 GeV/c



Ψ_2 Dependent Correlations : p_T 4-10x2-4 GeV/c

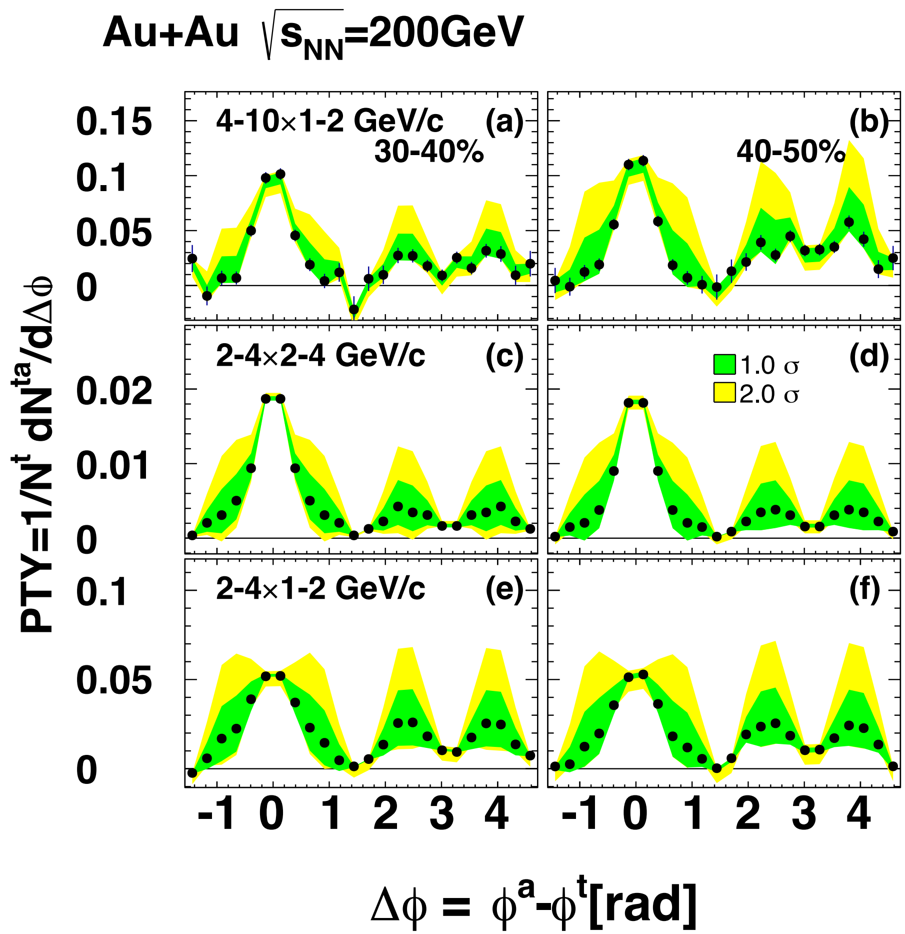


Ψ_3 Dependent Correlations : p_T 4-10x2-4 GeV/c



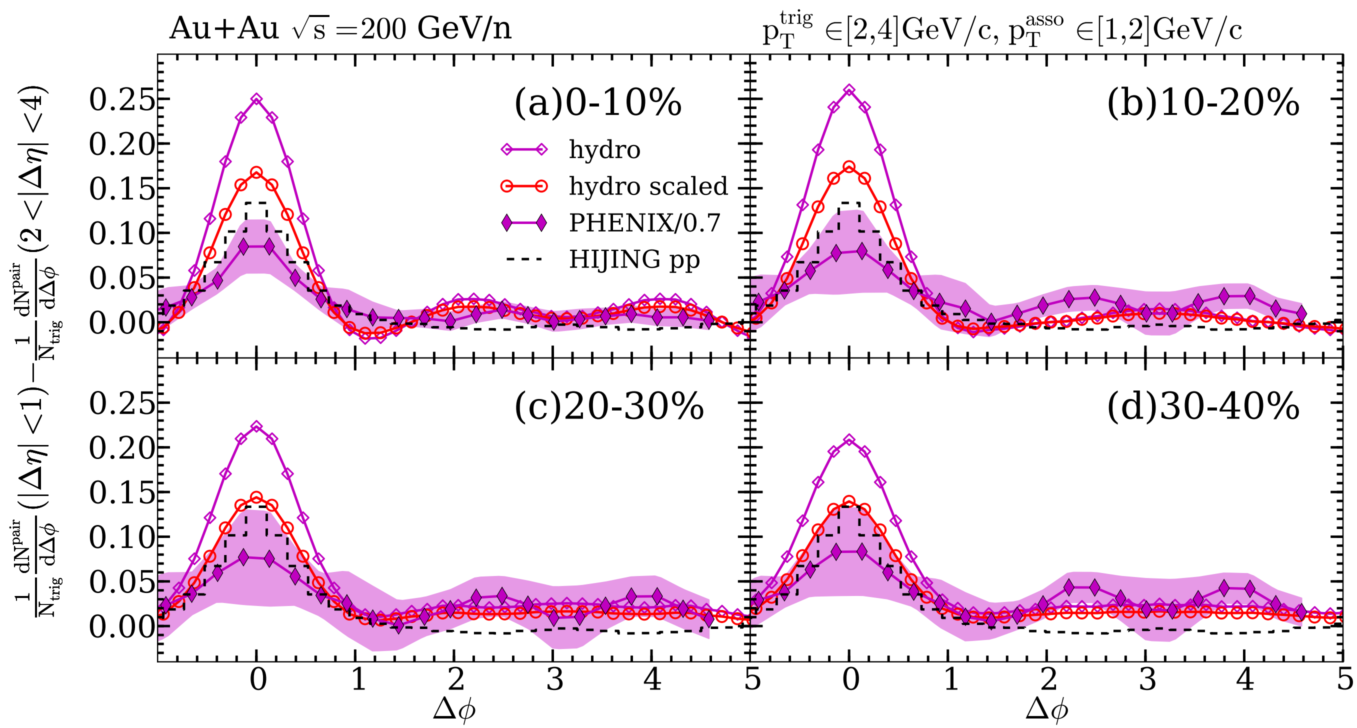
Significance of Double-Hump

- ✧ Examined the significance of Double-Hump in terms of v_4 systematics
- ✧ v_2 and v_3 are fixed in flow subtractions but v_4 is varied $\pm 1\sigma$
- ✧ Lower boundary of yellow band covers that of green band
- ✧ Significance is $\pm 1\sigma$ level of v_4 systematics



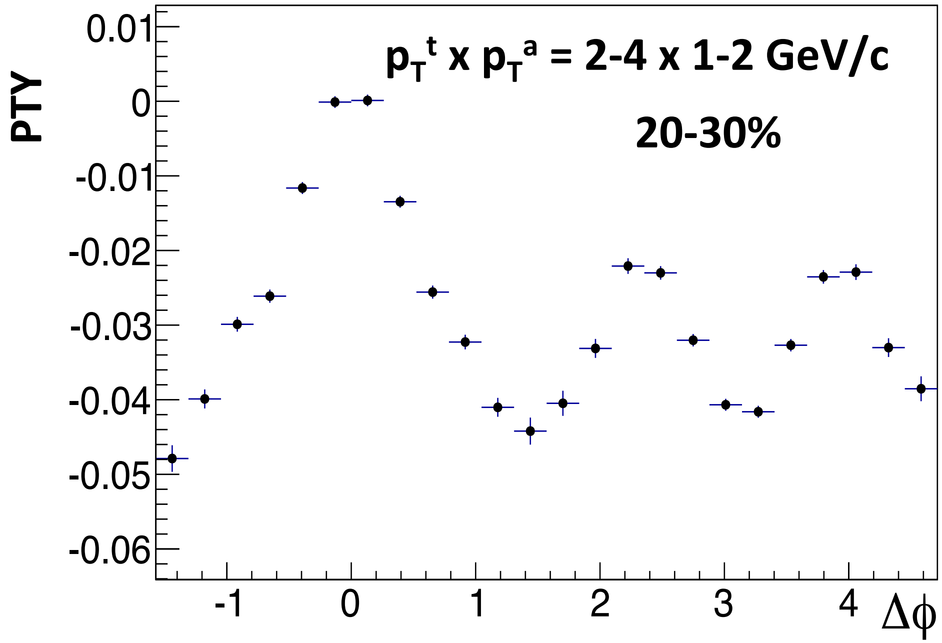
Latest Model for Two-Particle Correlations

arXiv:nucl-th/1309.6735v2 (2013)



- ✧ **Fluctuations of initial parton energy density**
- ✧ **Parton cascade**
- ✧ **Event-by-event (3+1)D hydrodynamics**
- ✧ **Parton Energy Momentum Loss**

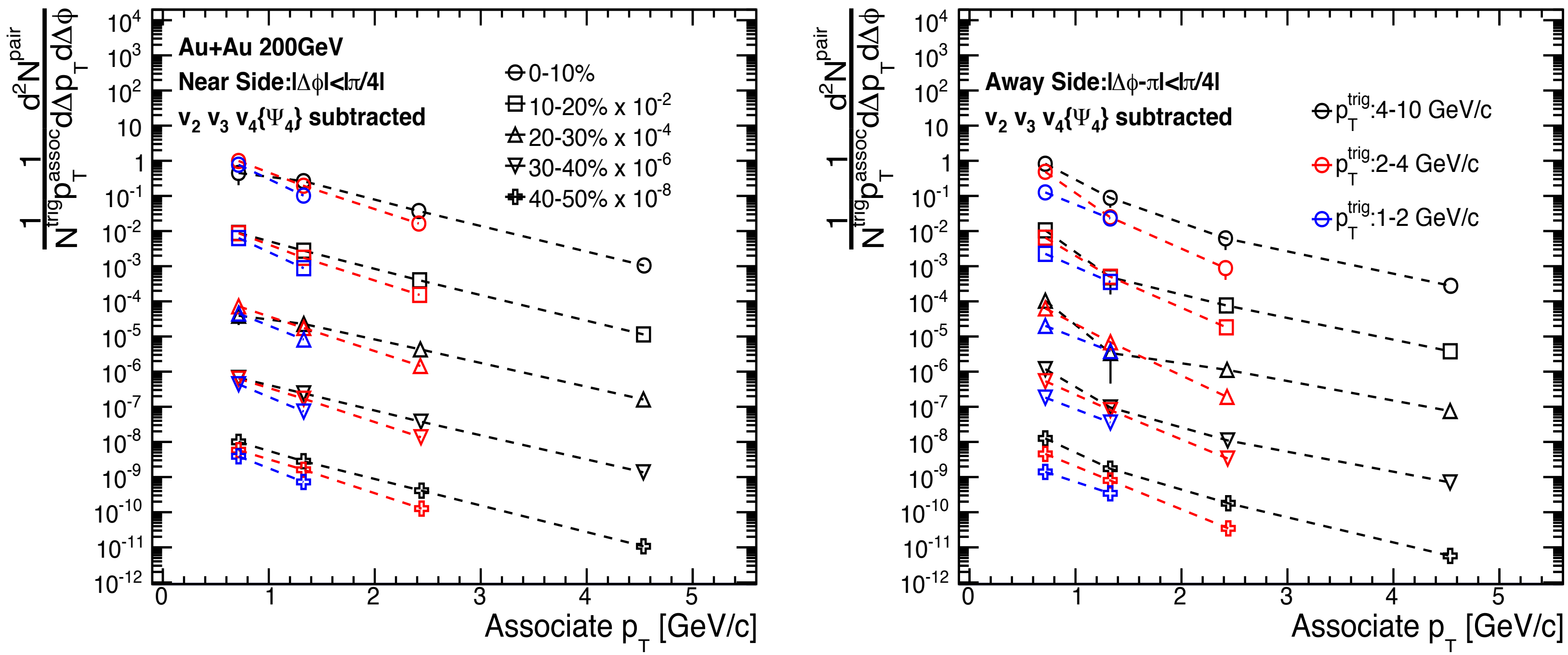
Zero Yield at Near-Side



✧ **Correlation and Pure Flow is fitted at $\Delta\phi=0$**

✧ **Double-hump is not so sensitive to flow subtraction**

Hardness of Correlation Yields



- p_T spectra of correlation yield

$$\frac{1}{p_T^a}\frac{dY}{d\Delta p_T} = \frac{1}{p_T^a}\frac{1}{(p_T^{a,max} - p_T^{a,min})}\int d\Delta\phi \frac{1}{N_{trig}}\frac{dN}{d\Delta\phi}$$

✧ Hardness increase with trigger and associate p_T

– Different Physics depending on p_T

Glauber Model

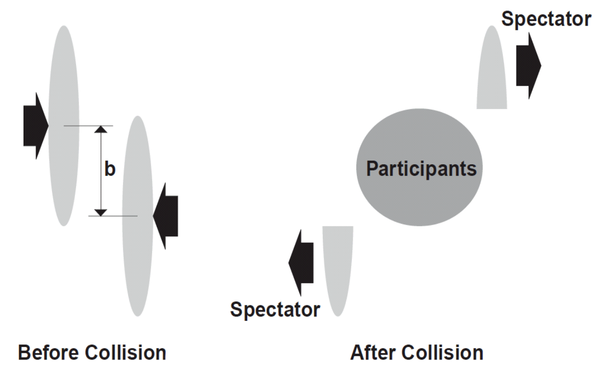
- ✧ Woods-Saxon Distribution
  - – Nucleon density distribution
- ✧ Collision range
- ✧ Number of nucleons participated in a collision
- ✧ Number of binary nucleons collisions in a collision

$$\rho(r) = \frac{1}{1 + \exp \frac{r - r_a}{a}}$$

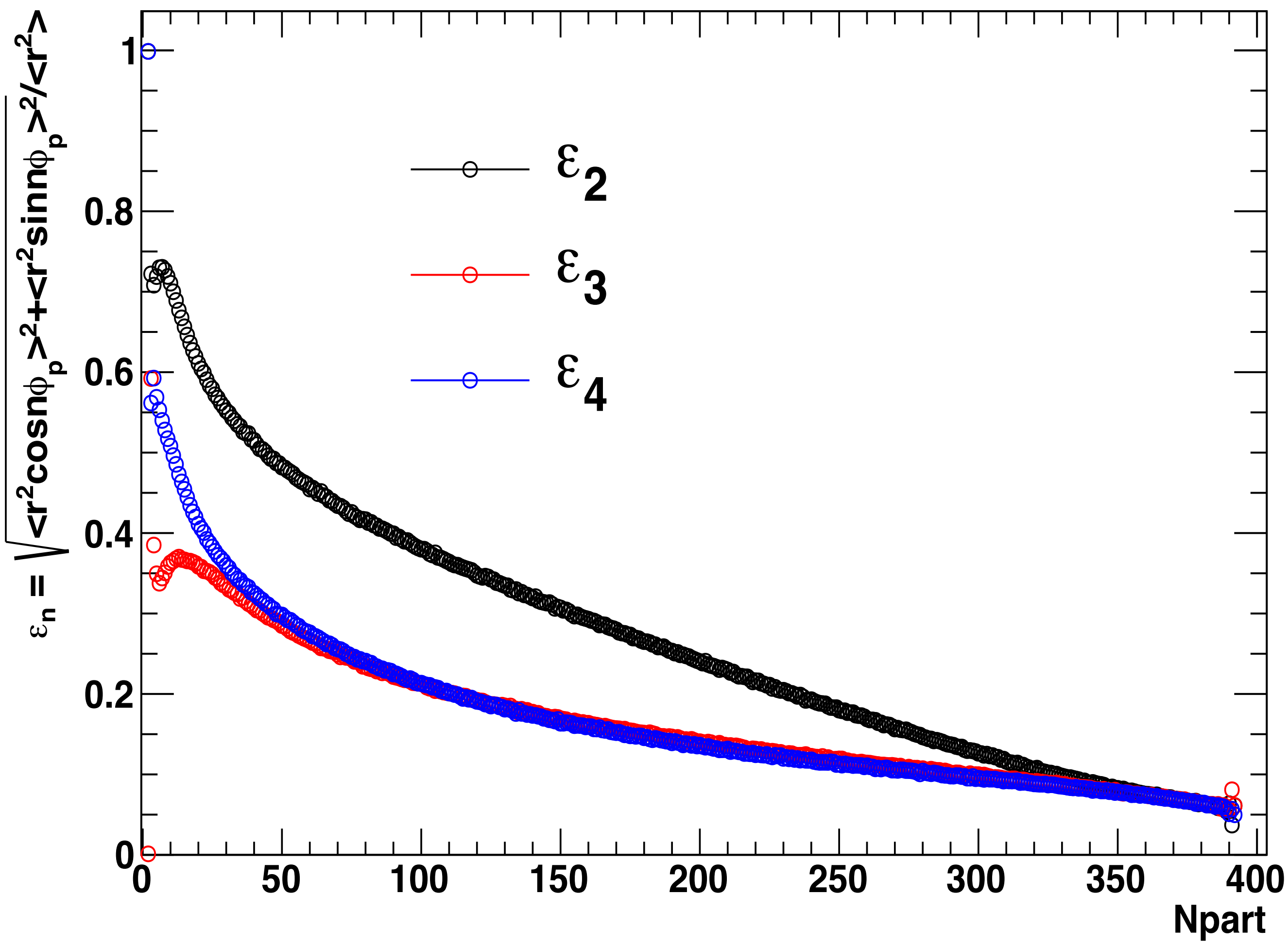
$$d < \sqrt{\frac{\sigma_{nn}}{\pi}}, \sigma_{nn} = 42mb$$

$$N_{part}$$

$$N_{coll}$$



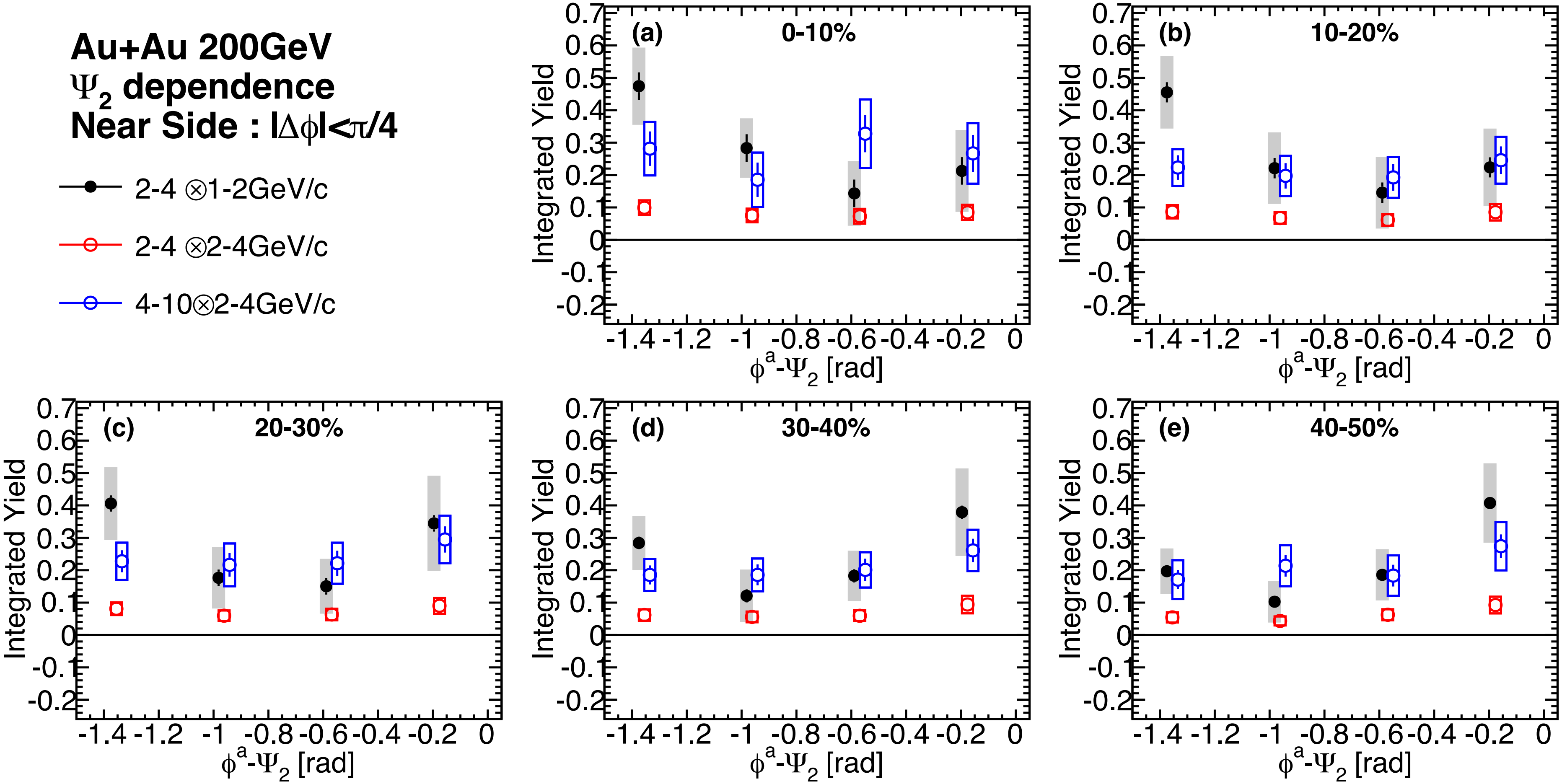
Participant Eccentricity by Glauber Model



Near-Side Integrated Yield vs Associate Angle from Ψ_2

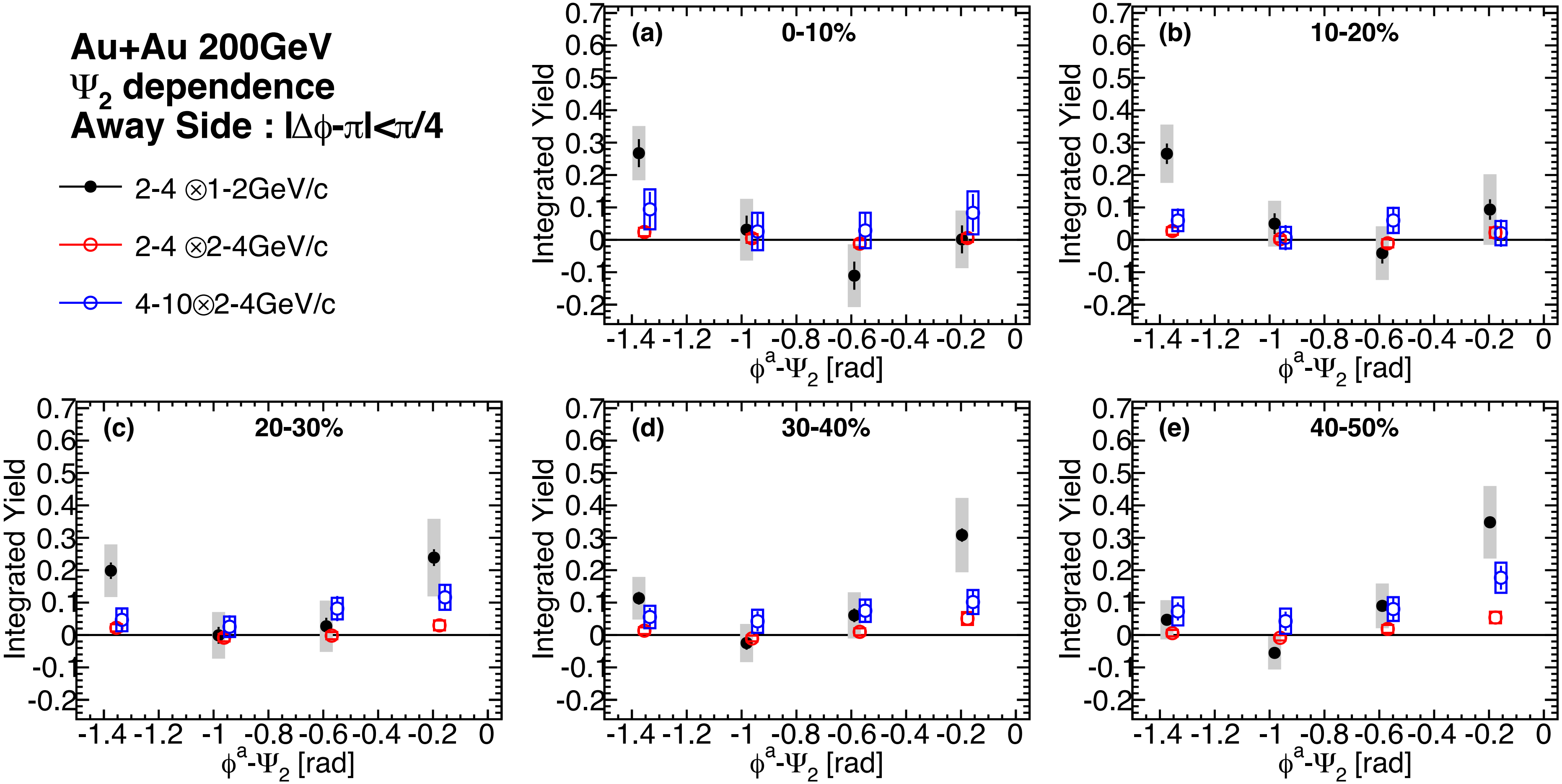
Au+Au 200GeV
Ψ_2 dependence
Near Side : $|\Delta\phi|<\pi/4$

- 2-4 ⊗1-2GeV/c
- 2-4 ⊗2-4GeV/c
- 4-10⊗2-4GeV/c



Away-Side Integrated Yield vs Associate Angle from Ψ_2

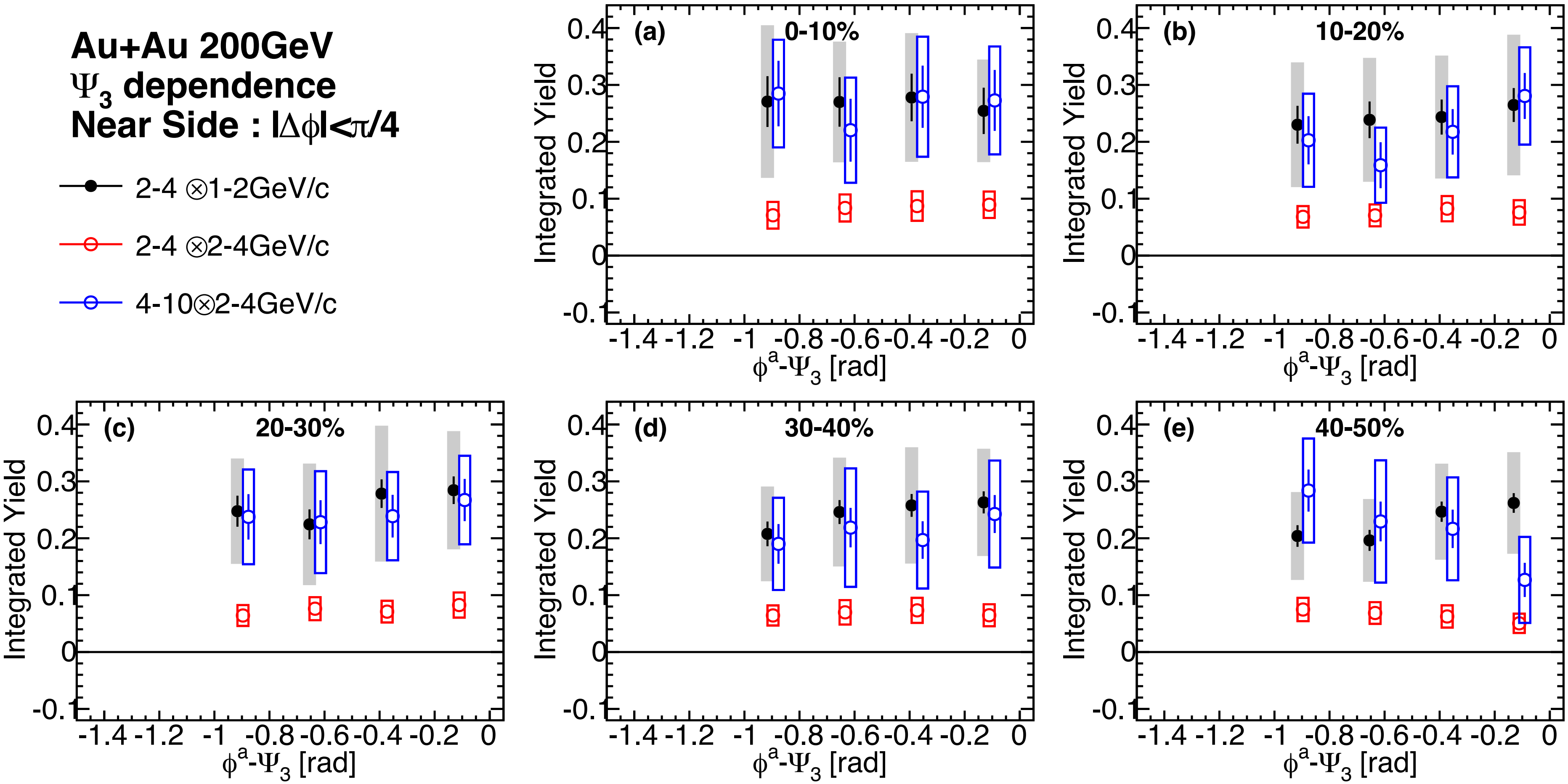
Au+Au 200GeV
Ψ_2 dependence
Away Side : $|\Delta\phi-\pi|<\pi/4$



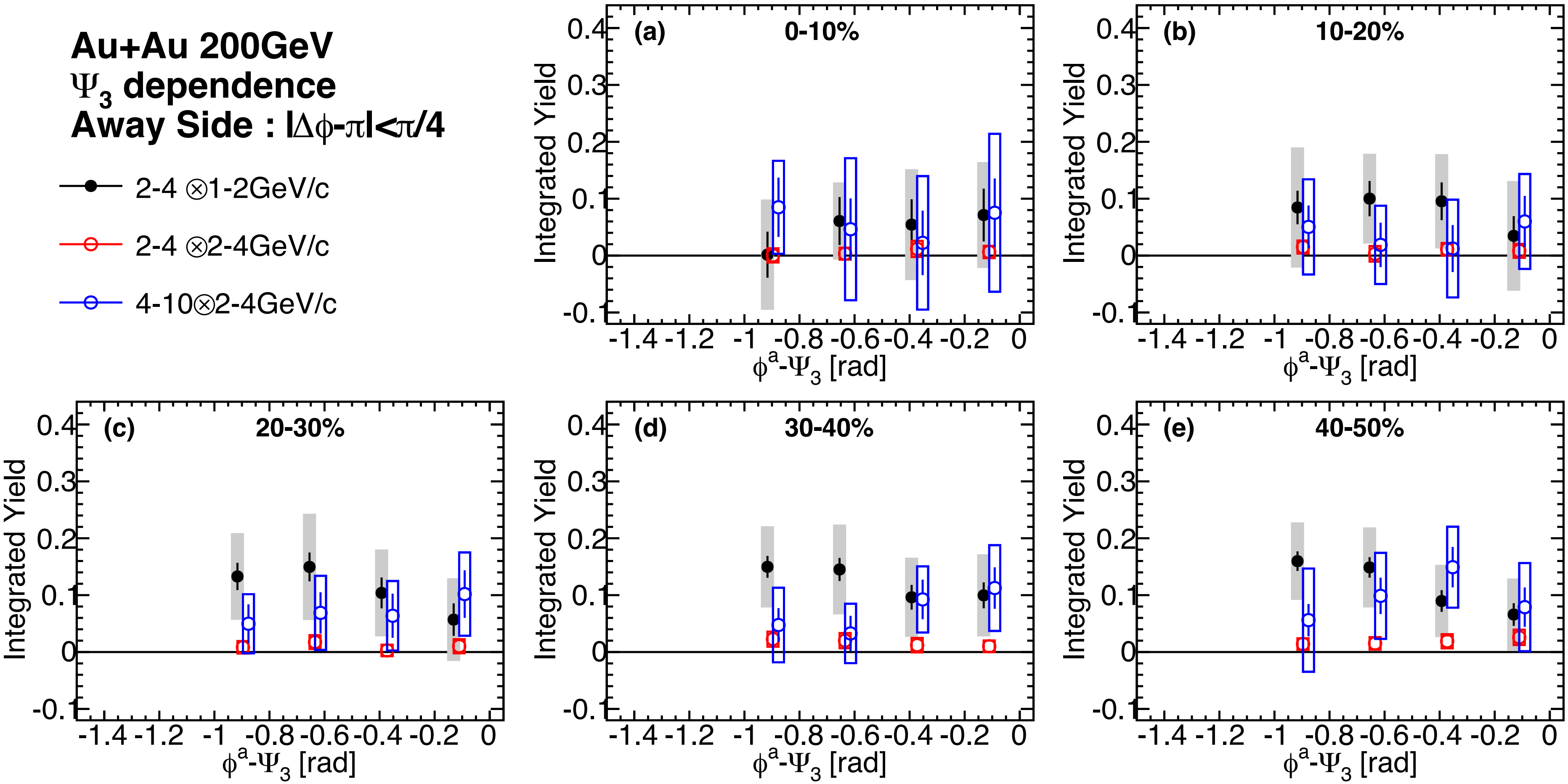
Near-Side Integrated Yield vs Associate Angle from Ψ_3

Au+Au 200GeV
Ψ_3 dependence
Near Side : $|\Delta\phi|<\pi/4$

- 2-4 ⊗1-2GeV/c
- 2-4 ⊗2-4GeV/c
- 4-10⊗2-4GeV/c



Away-Side Integrated Yield vs Associate Angle from Ψ_3

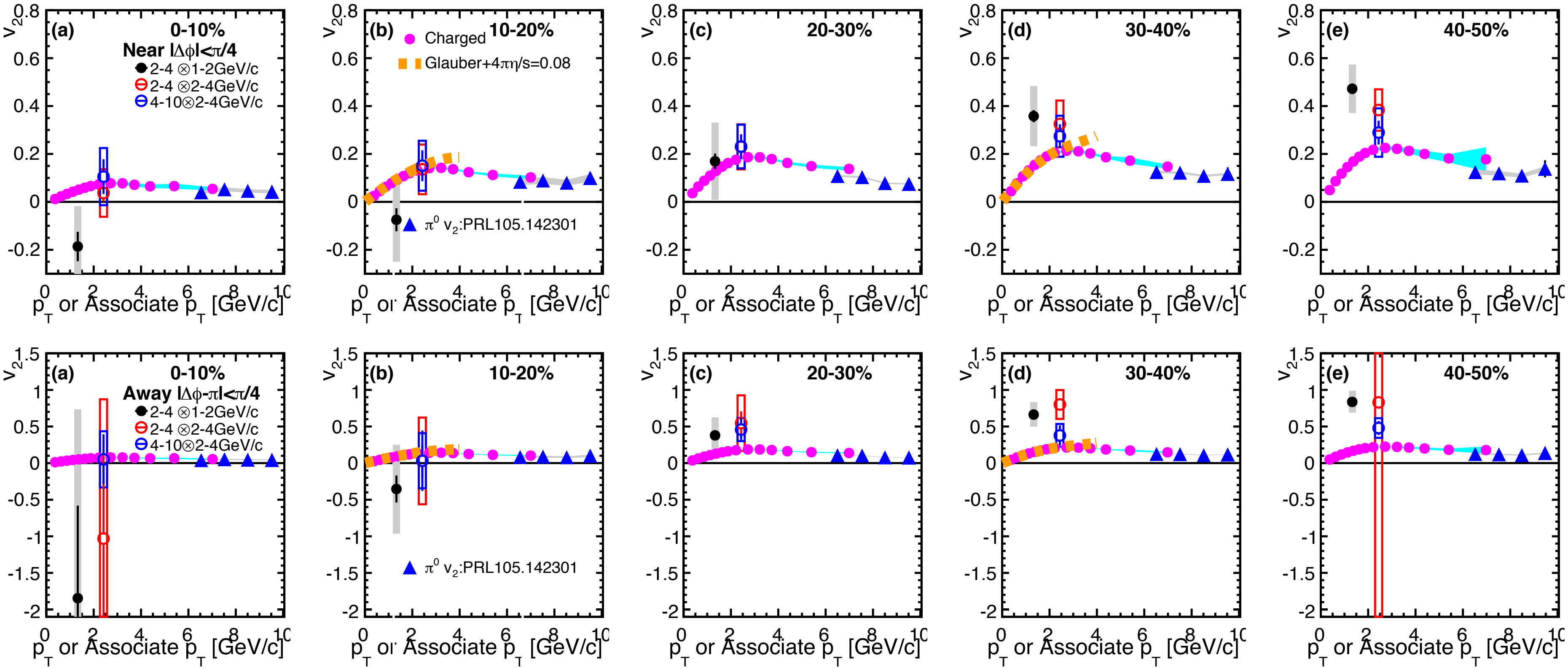


Anisotropy of particles per a jet → Anisotropy of particles per a event

$$
\begin{aligned}
&\{1+2v_n^{PTY}\cos n(\phi^a-\Psi_n)\}\times\{1+2v_n^t\cos n(\phi^t-\Psi_n)\}\\
=&\{1+2v_n^{PTY}\cos n(\phi^a-\Psi_n)\}\times\{1+2v_n^t\cos n(\phi^a-\phi^t)\cos n(\phi^a-\Psi_n)\}\\
\simeq&1+2v_n^{PTY}\cos n(\phi^a-\Psi_n)+2v_n^t\cos n(\phi^a-\phi^t)\cos n(\phi^a-\Psi_n)
\end{aligned}
$$

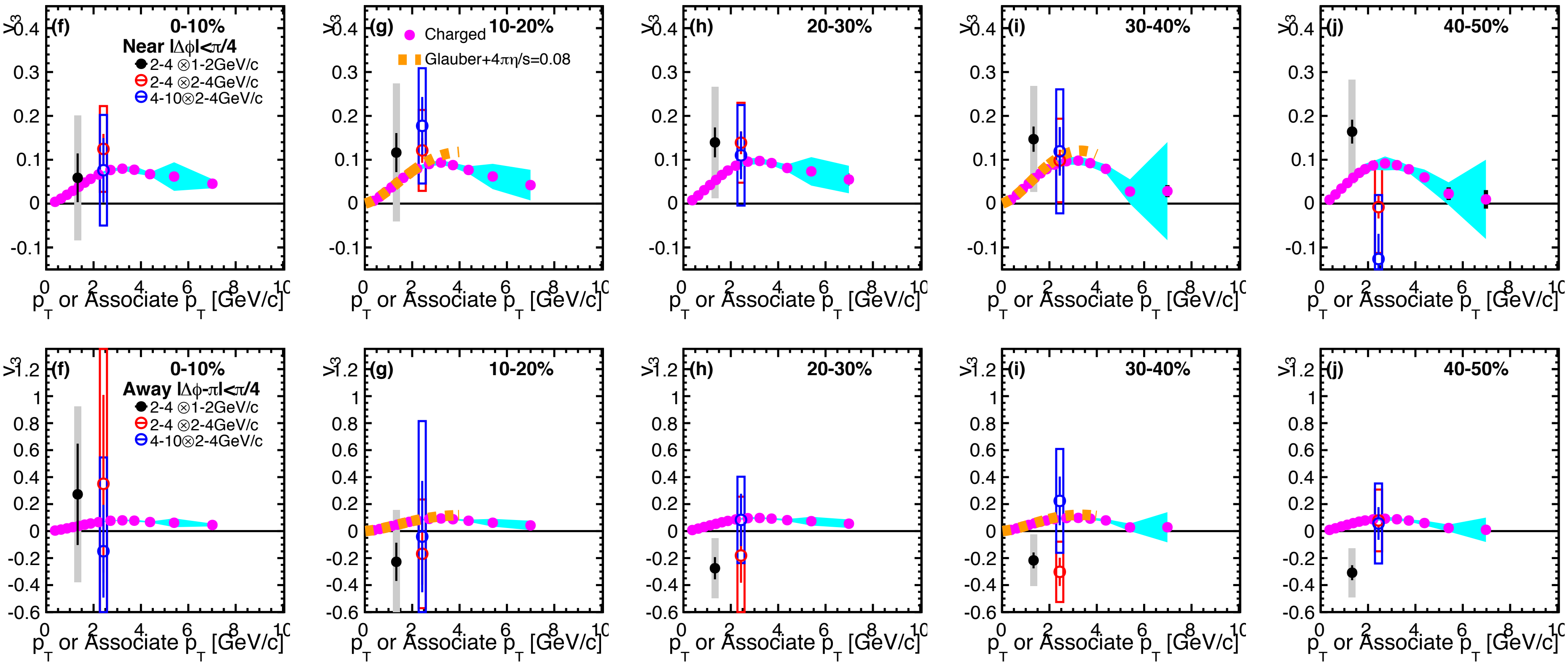
$$
v_n^{PTY,cor}=v_n^{PTY}+v_n^{trig}\cos n(\phi^t-\phi^a)
$$

v_2^{PTY}



(a) 0-10%
Near |Δφ|<π/4
2-4 ⊗1-2GeV/c
2-4 ⊗2-4GeV/c
4-10⊗2-4GeV/c
(b) 10-20%
Charged
Glauber+4πη/s=0.08
π0 v2:PRL105.142301
(c) 20-30%
(d) 30-40%
(e) 40-50%
pT or Associate pT [GeV/c]
v2
(a) 0-10%
Away |Δφ-π|<π/4
2-4 ⊗1-2GeV/c
2-4 ⊗2-4GeV/c
4-10⊗2-4GeV/c
(b) 10-20%
π0 v2:PRL105.142301
(c) 20-30%
(d) 30-40%
(e) 40-50%

v_3^{PTY}



(f) 0-10%
Near |Δφ|<π/4
2-4 ⊗1-2GeV/c
2-4 ⊗2-4GeV/c
4-10⊗2-4GeV/c
(g) 10-20%
Charged
Glauber+4πη/s=0.08
(h) 20-30%
(i) 30-40%
(j) 40-50%
(f) 0-10%
Away |Δφ-π|<π/4
2-4 ⊗1-2GeV/c
2-4 ⊗2-4GeV/c
4-10⊗2-4GeV/c
(g) 10-20%
(h) 20-30%
(i) 30-40%
(j) 40-50%
v_3
p_T or Associate p_T [GeV/c]

Gravity Position of Two-Particle Correlations

Definition

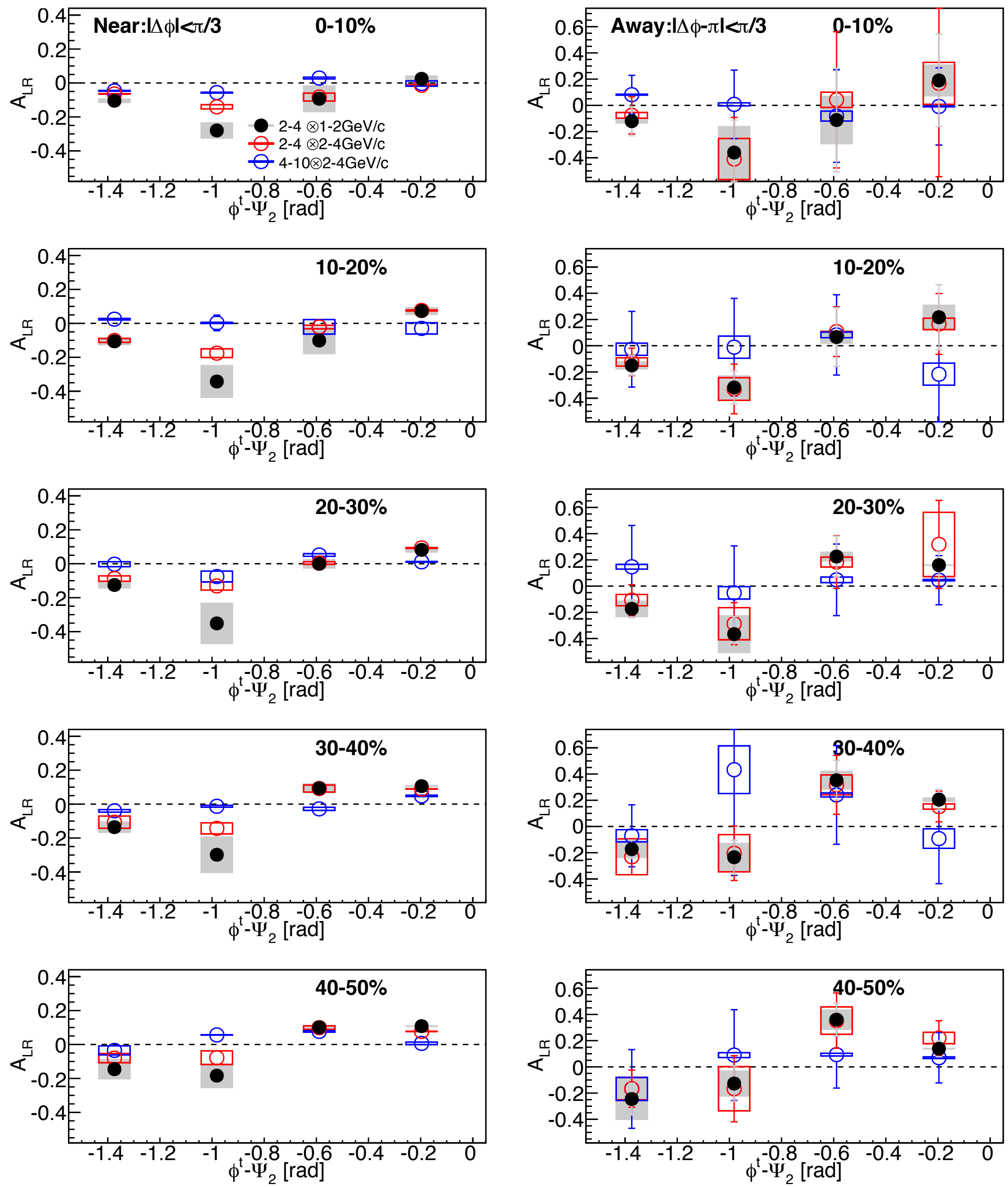
$$A_{LR} = \frac{\int d\Delta\phi \Delta\phi Y(\Delta\phi)}{\int d\Delta\phi Y(\Delta\phi)} - \begin{cases} 0 \text{ if near} - \text{side} \\ \pi \text{ if away} - \text{side} \end{cases}$$

Integral Ranges

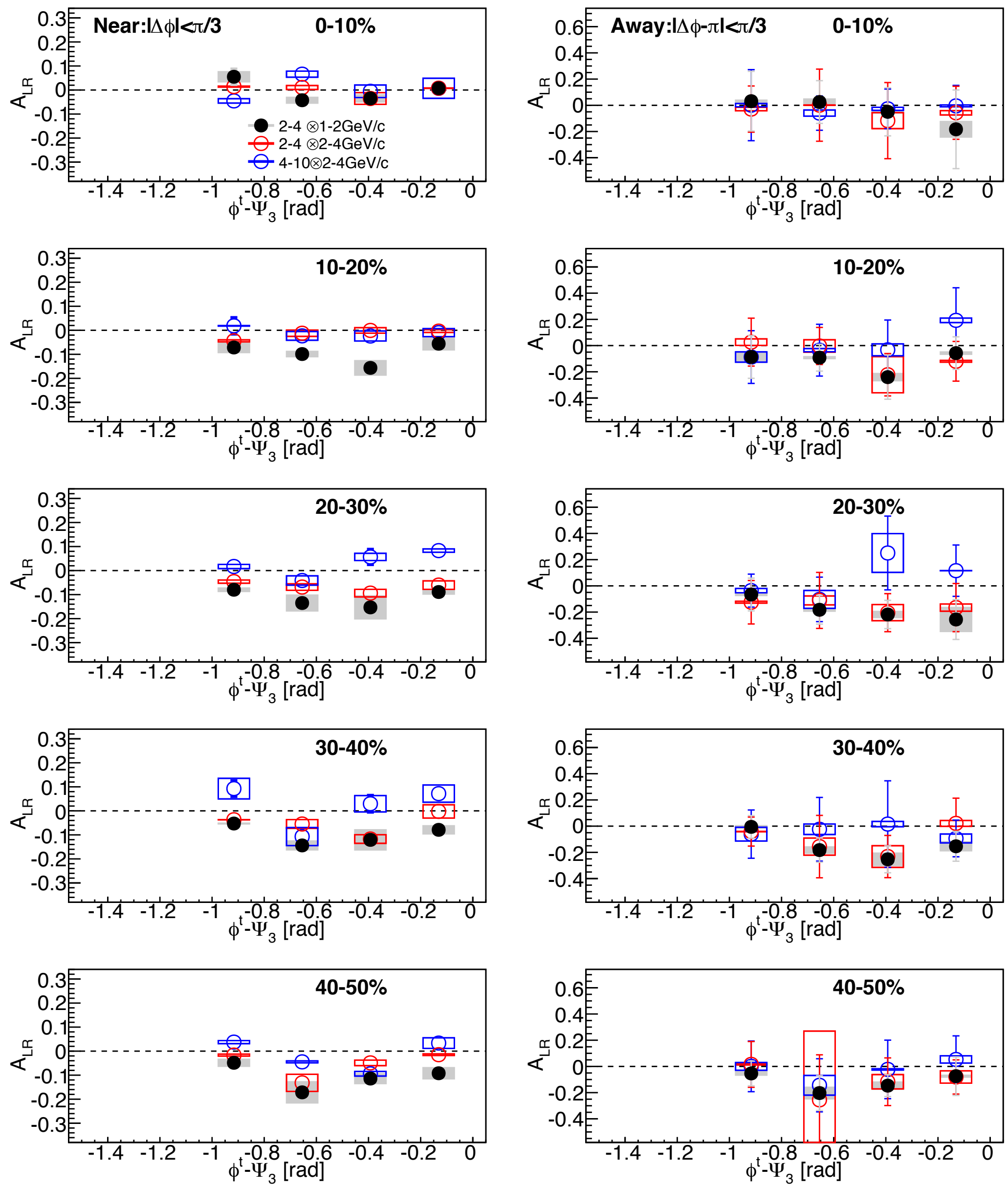
Near – Side : $|\Delta\phi| < \pi/3$

Away – Side : $|\Delta\phi - \pi| < \pi/3$

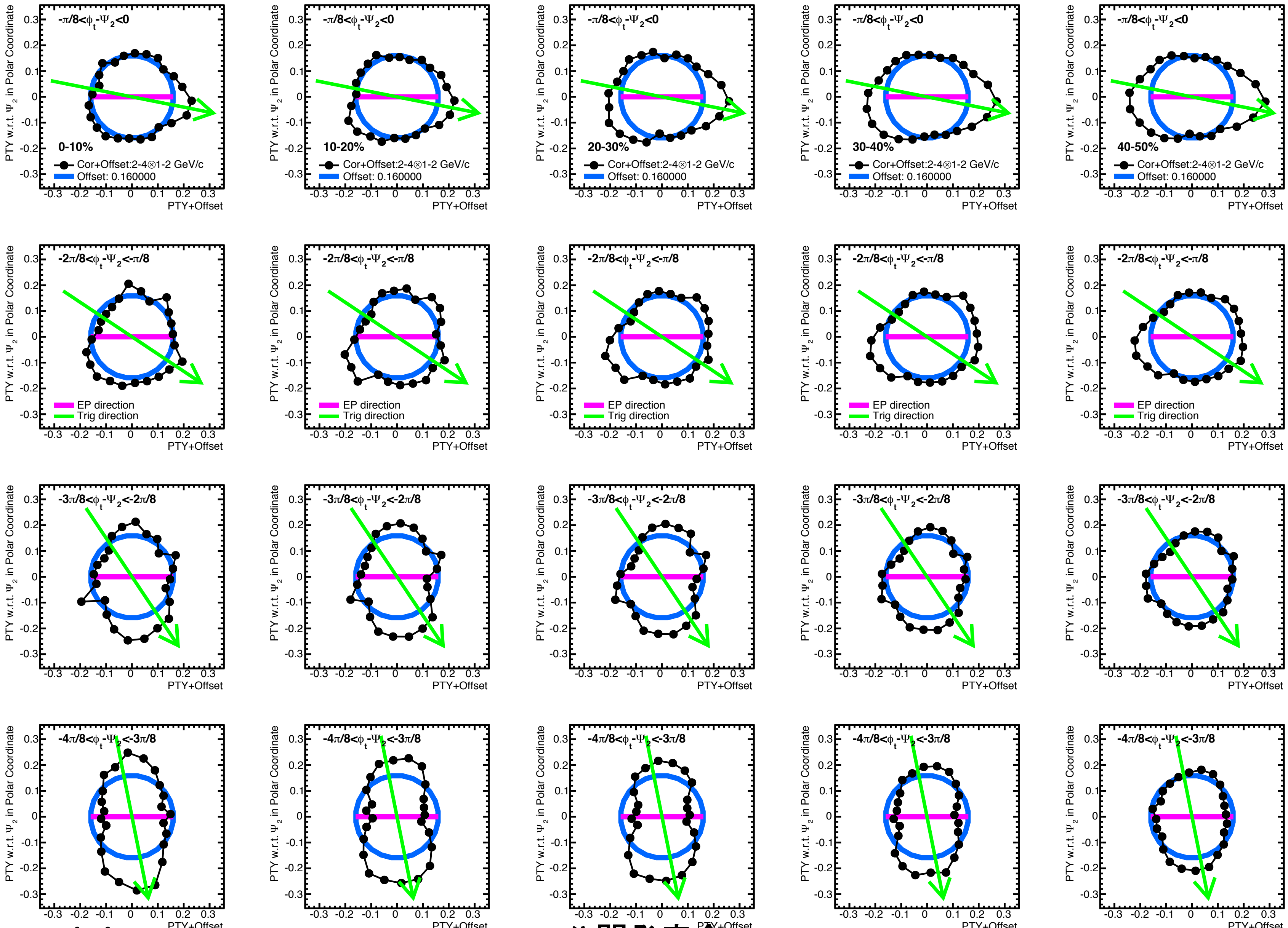
Gravity position vs trigger angle from Ψ_2



Gravity position vs trigger angle from Ψ_3



Ψ_2 Dependent Correlations : p_T 2-4x1-2 GeV/c



Ψ_3 Dependent Correlations : p_T 2-4x1-2 GeV/c