

**Measurements of Two-Particle Correlations
with respect to Higher-Order Event Planes
in $\sqrt{s_{NN}}=200$ GeV Au+Au collisions
at RHIC-PHENIX**

Pre-Defense Session

Nov 6th 2013

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High Energy Nuclear Physics Group

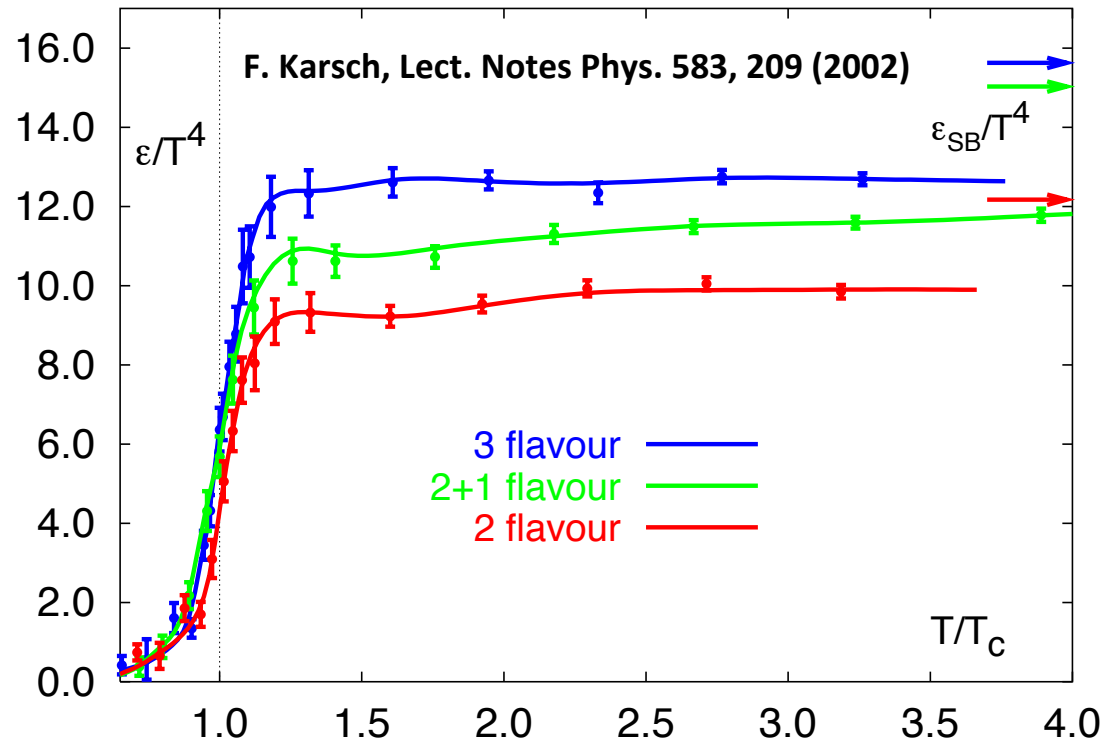
Outline

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- Analysis
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 - Flow and Two-Particle Correlation Measurements
 - Correlations with respect to Event Planes
- Results & Discussion
 - Inclusive Trigger Correlations
 - Event Plane Dependent Correlations
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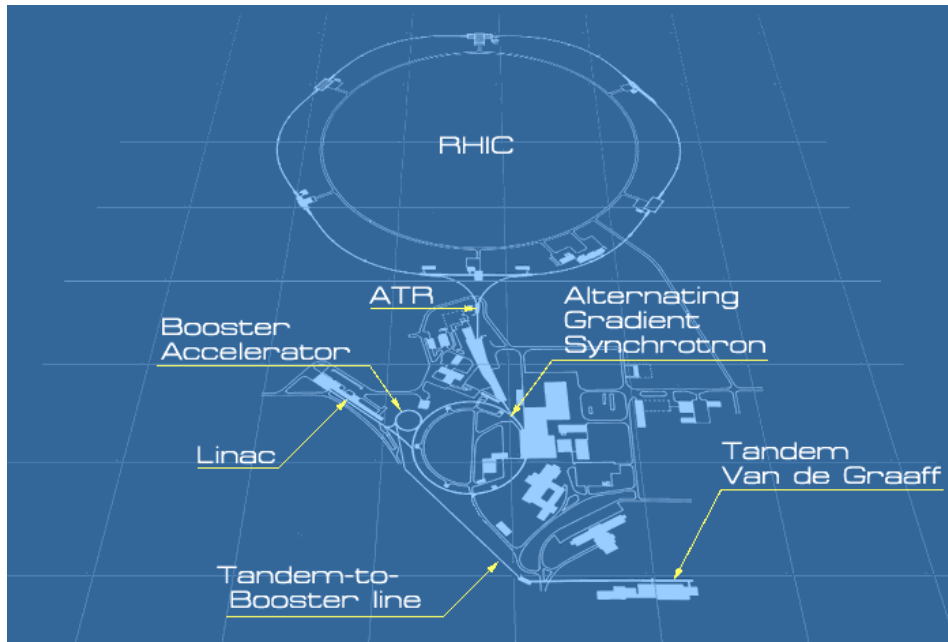
INTRODUCTION

Quark Gluon Plasma (QGP)

- A fluid of quark and gluons deconfined from hadrons at high energy-density ε & temperature T
- Predicted transition ε & T by Lattice-QCD
 - $\varepsilon_c \sim 1.0$ [GeV/fm³]
 - $T_c \sim 170$ [MeV]
- Relativistic Heavy Ion Collisions at RHIC
 - $\varepsilon \sim 5.0 - 15.0$ [GeV/fm³]



Relativistic Heavy Ion Collider



PHENIX Run Summary

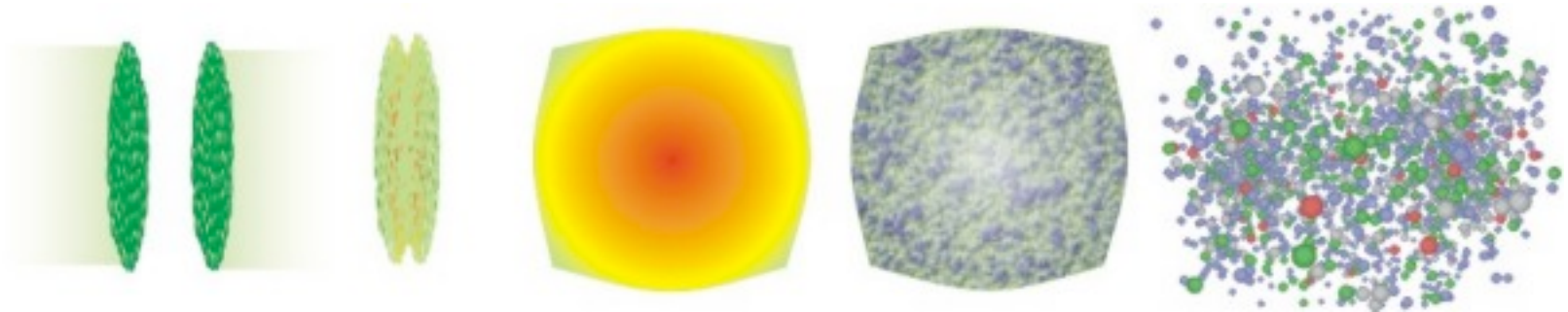
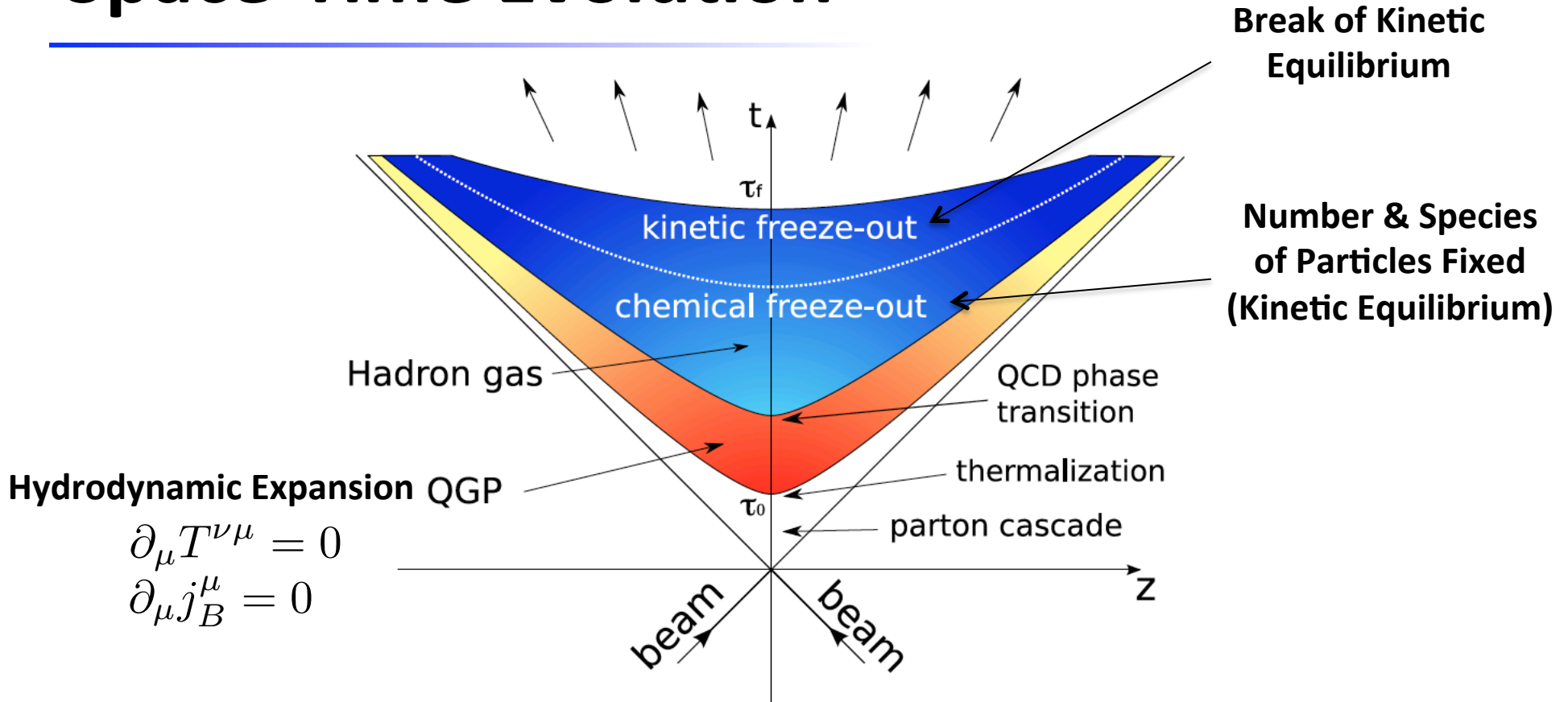
	Year	Species	\sqrt{s} [GeV]	$\int \text{Ldt}$	N_{tot} (sampled)	Data Size
Run1	2000	Au - Au	130	1 μb^{-1}	10 M	3 TB
Run2	2001/02	Au - Au	200	24 μb^{-1}	170 M	10 TB
		Au - Au	19		< 1 M	
Run3	2002/03	p - p	200	0.15 pb^{-1}	3.7 B	20 TB
		d - Au	200	2.74 nb^{-1}	5.5 B	46 TB
Run4	2003/04	p - p	200	0.35 pb^{-1}	6.6 B	35 TB
		Au - Au	200	241 μb^{-1}	1.5 B	270 TB
Run5	2005	Au - Au	62.4	9 μb^{-1}	58 M	10 TB
		Cu - Cu	200	3 nb^{-1}	8.6 B	173 TB
Run6	2006	Cu - Cu	62.4	0.19 nb^{-1}	0.4 B	48 TB
		Cu - Cu	22.4	2.7 μb^{-1}	9 M	1 TB
Run7	2007	p - p	200	3.8 pb^{-1}	85 B	262 TB
		p - p	200	10.7 pb^{-1}	233 B	310 TB
Run8	2007/08	p - p	62.4	0.1 pb^{-1}	28 B	25 TB
		Au - Au	200	813 μb^{-1}	5.1 B	650 TB
Run8	2007/08	d - Au	200	80 nb^{-1}	160 B	437 TB
		Au - Au	9.2		few k	

- Accelerators at Brookhaven National Laboratory

- Tandem van de Graaff
- Linear Accelerator
- Booster Synchrotron
- Alternating Gradient Synchrotron
- Relativistic Heavy Ion Collider

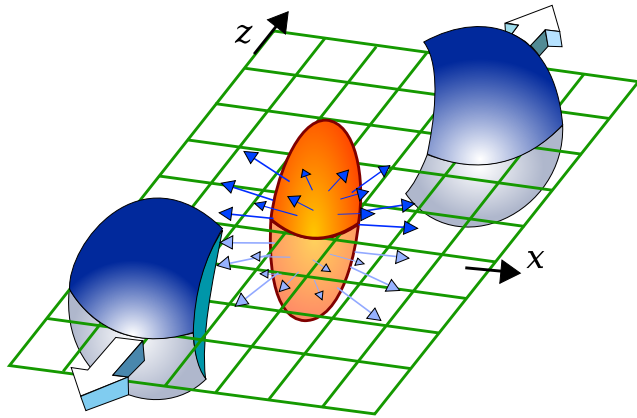
- p+p : 510 GeV
- d+Au, Cu+Cu, Cu+Au, Au+Au: 200GeV
- U+U : 193 GeV

Space-Time Evolution



Collective Expansion

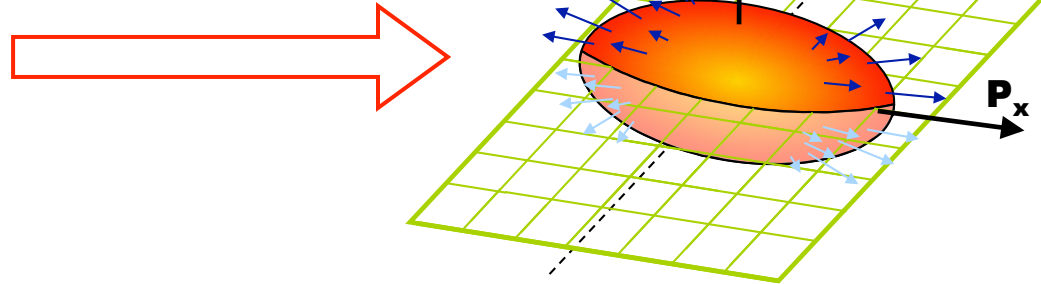
Smooth Parton Density



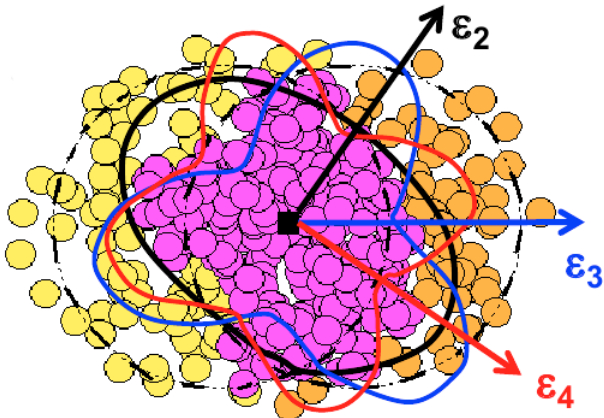
- Pressure Gradient due to small $\lambda/R \ll 1$

$$E \frac{d^3 N}{dp^3} = \frac{d^2 N}{2\pi dp_T d\eta} \left\{ 1 + \sum 2v_n \cos n(\phi - \Psi_2) \right\}$$

$$v_n = \langle \cos n(\phi - \Psi_2) \rangle$$



Fluctuating Parton Density



- Expansion with respect to Ψ_n

$$E \frac{d^3 N}{dp^3} = \frac{d^2 N}{2\pi dp_T d\eta} \left\{ 1 + \sum 2v_n \cos n(\phi - \Psi_n) \right\}$$

$$v_n = \langle \cos n(\phi - \Psi_n) \rangle$$

ϕ : azimuthal angle of emitted particles

Ψ : azimuthal angle of event plane

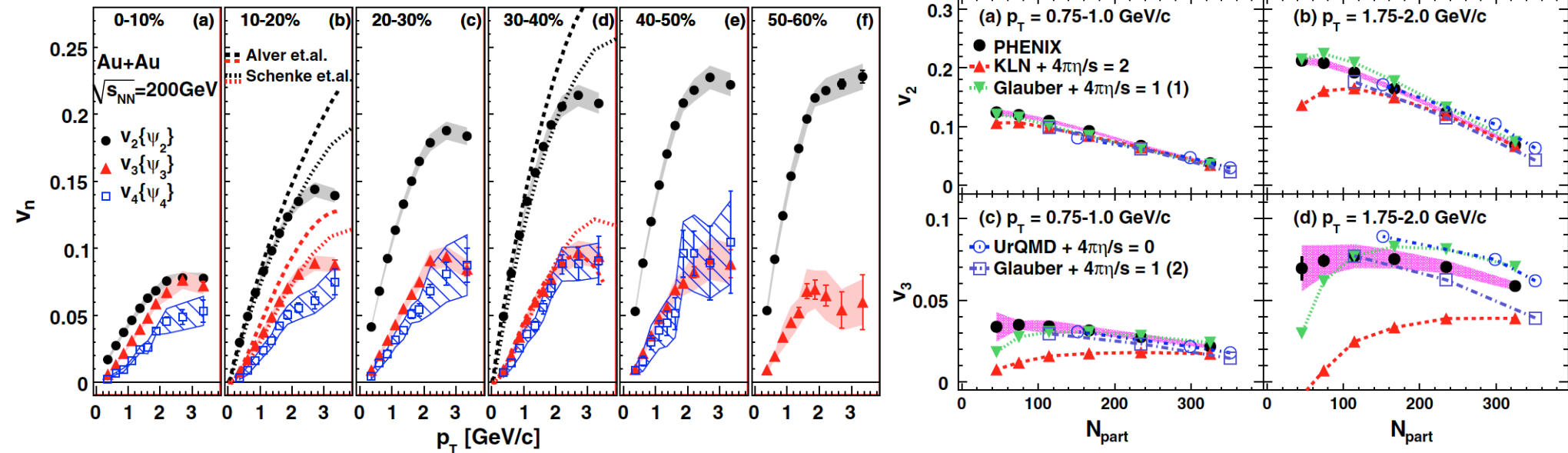
PRC81.054905 (2010)

2013/11/06

pre-Defense

Higher-Order Flow Harmonics

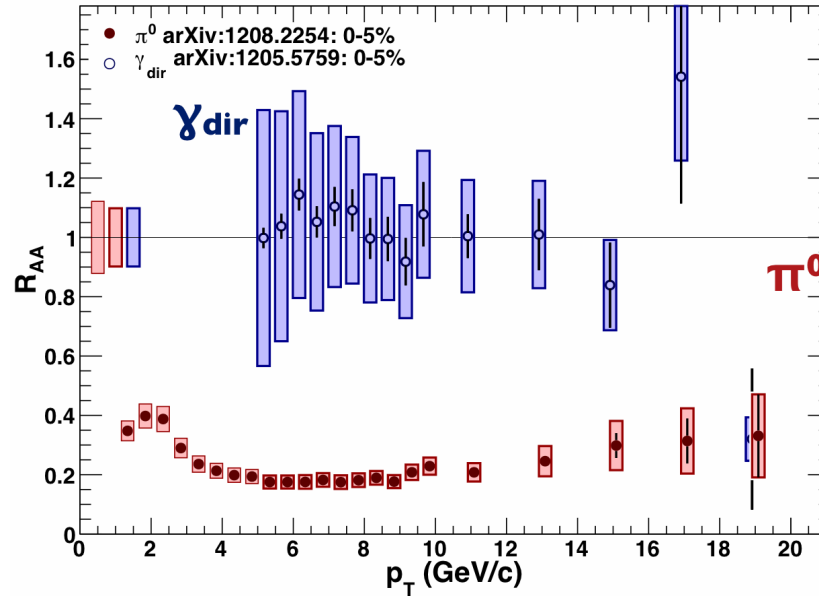
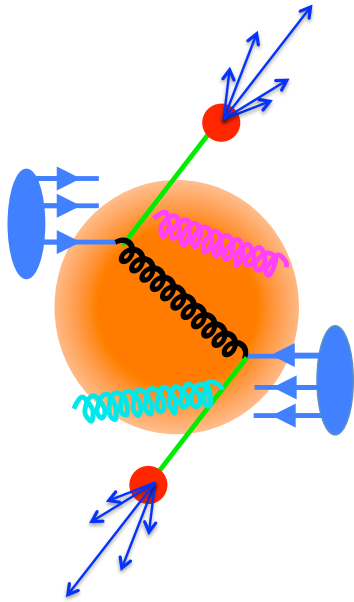
PRL107.252301 (2011)



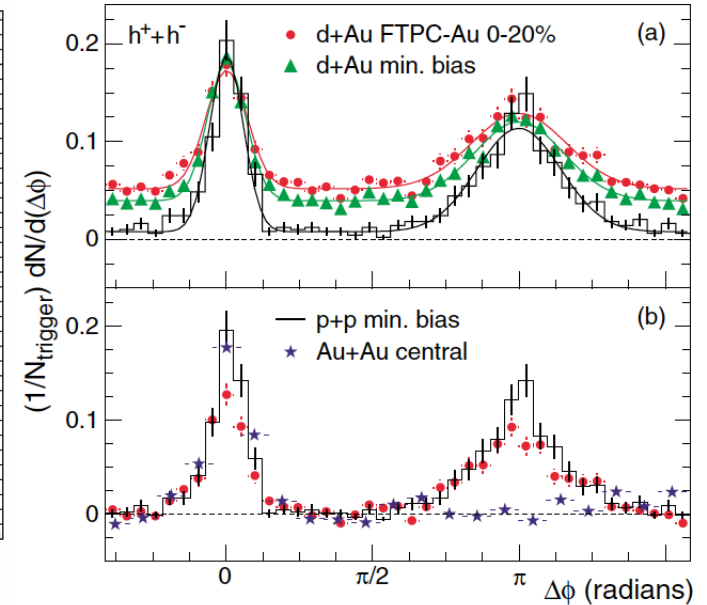
- None-zero v_n ($n>2$)
- Disentanglement of a degeneracy among models
 - Initial Condition, Shear Viscosity in Hydrodynamics
- Backgrounds in correlation studies

Jet-Quenching

Au+Au



PRL91.072304 (2003)

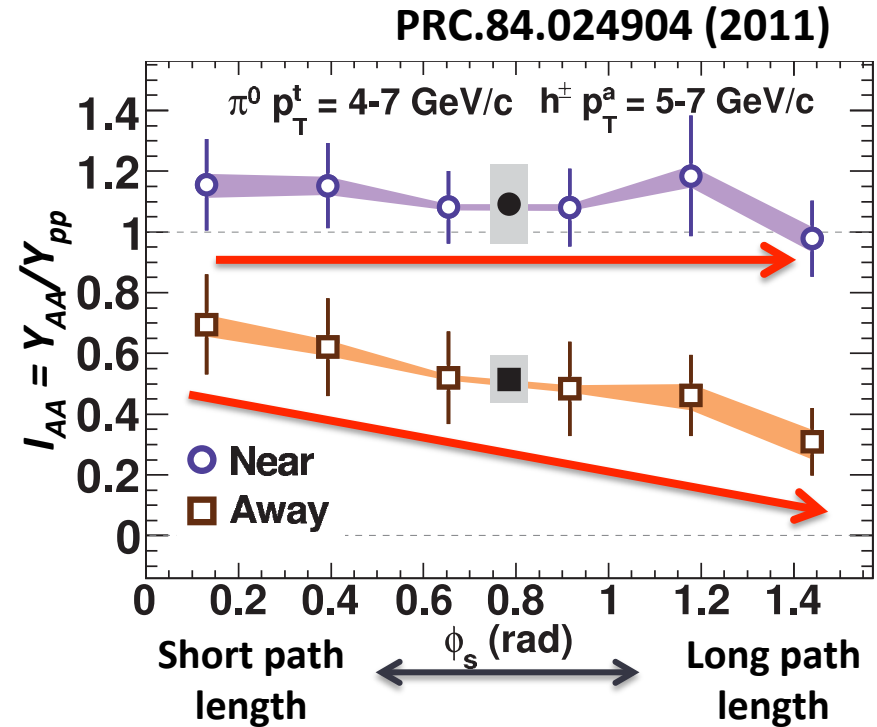
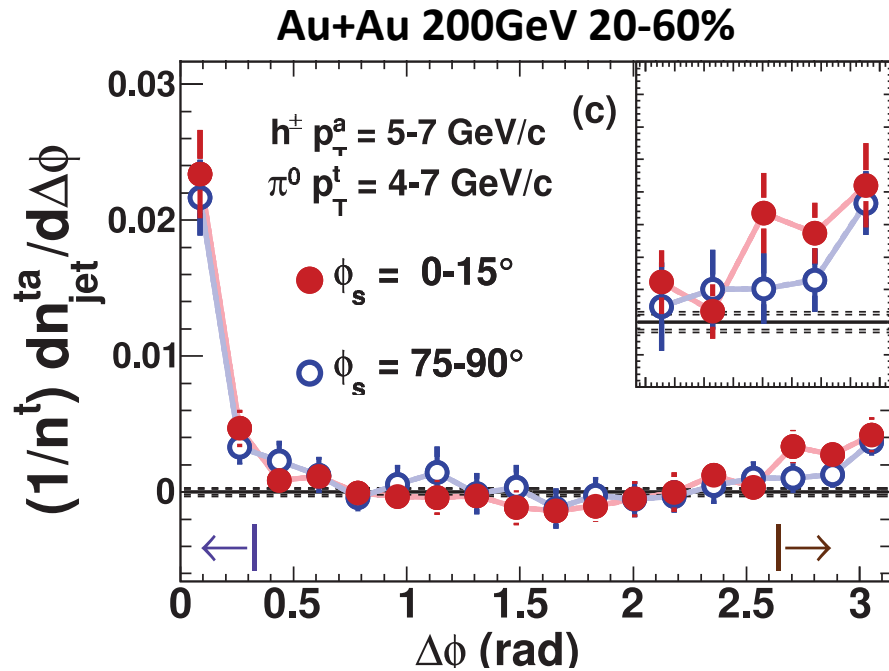


- Suppression of particles compared to p+p collisions

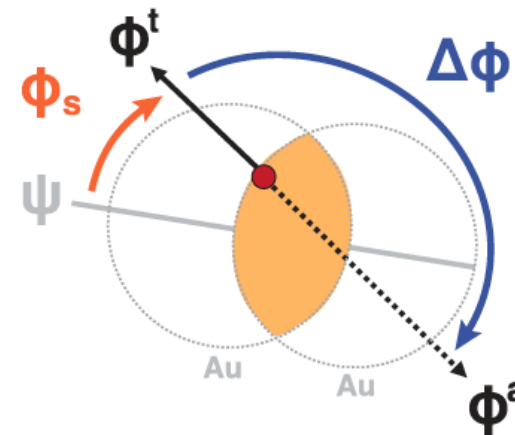
- Nuclear modification Factor $R_{AA} < 1$
- Away-Side suppression of Correlations

$$R_{AA} = \frac{d^2 N^{AA} / dp_T d\eta}{N_{coll} d^2 N^{pp} / dp_T d\eta}$$

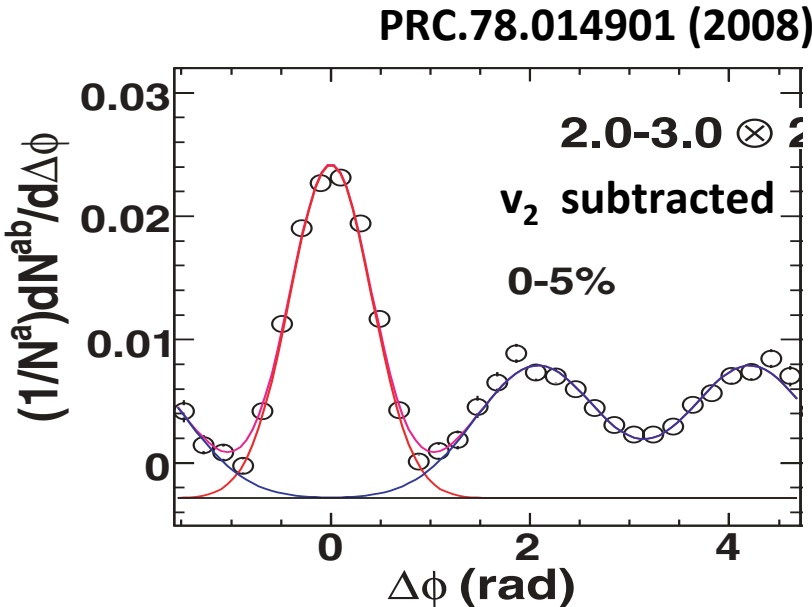
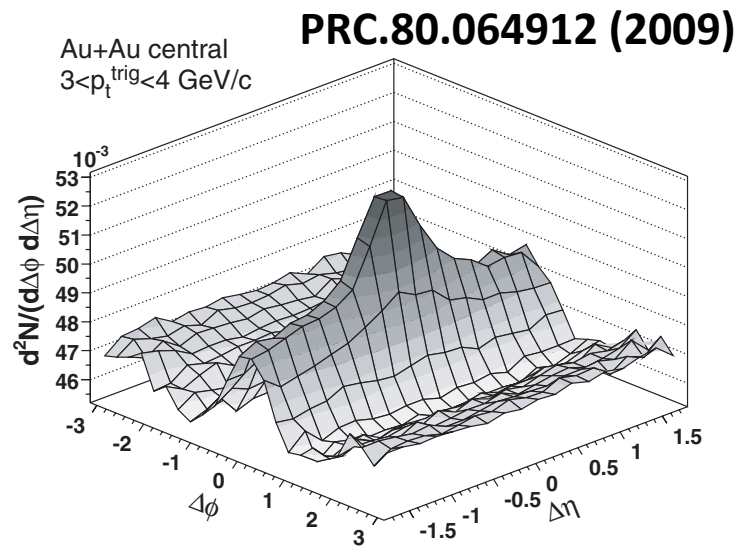
Path Length Dependence of High p_T Correlations



- v_2 and $v_4\{\Psi_2\}$ subtracted
- Monotonic suppression of away-side
- Parton Energy Loss depending on path length



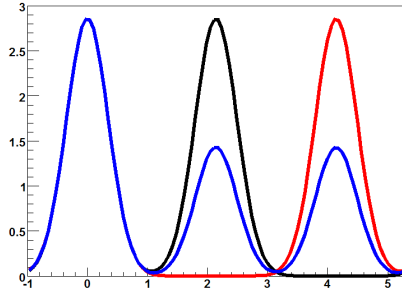
Double-Humps & Ridge of Interm. p_T Correlations



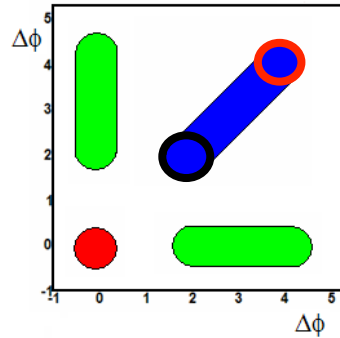
- Near-side long-range rapidity correlations (Ridge)
- Away-side double-humps of azimuthal correlations
 - v_2 contribution subtracted
- Additional Effect to Parton Energy-Loss
 - Lost Energy-Redistribution
 - Boost of jet by medium expansion

Conical Emission of Away-Side

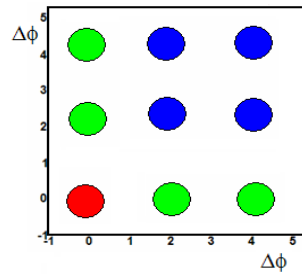
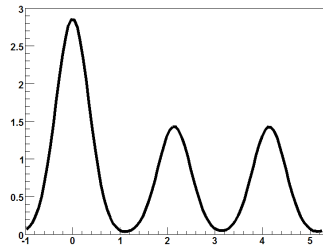
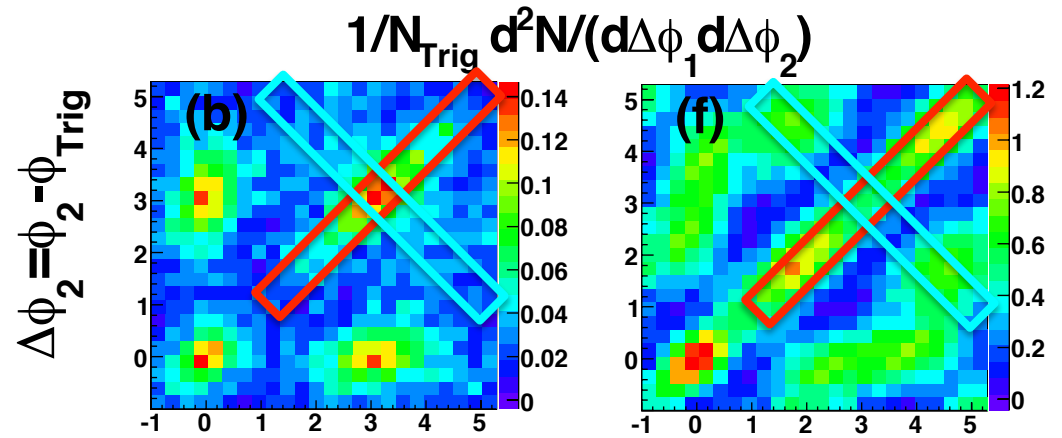
2-particle correlation



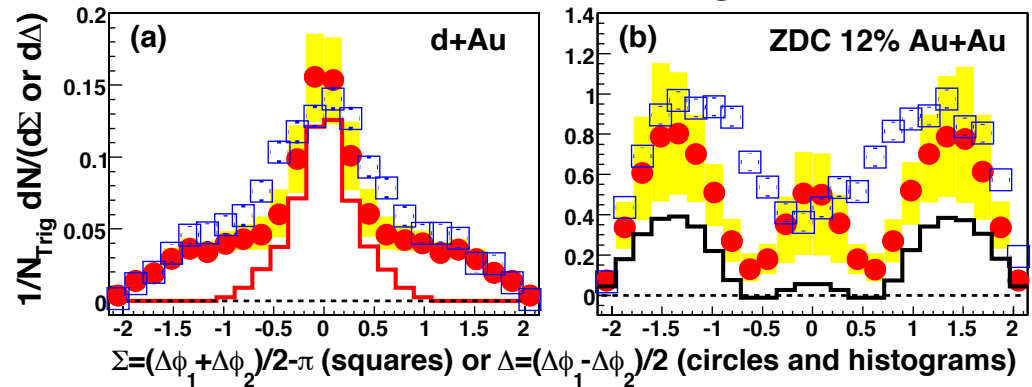
3-particle correlation



Experimental Data



$$\Delta\phi_1 = \phi_1 - \phi_{\text{Trig}}$$



PRL102.052302 (2009)

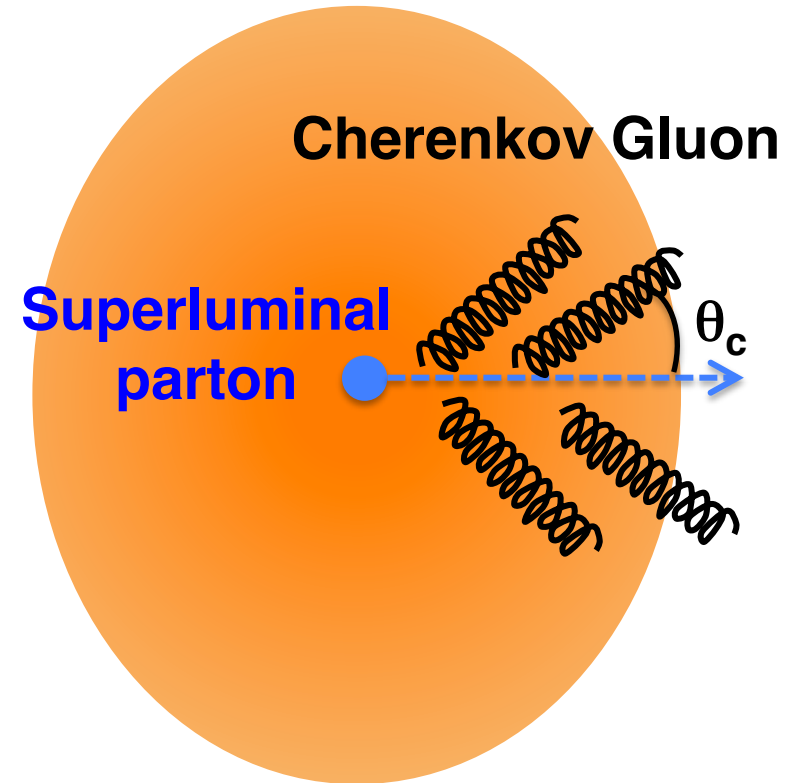
Cherenkov Gluon Radiation

- Gluon Radiation from a superluminal parton
- Momentum dependence of gluon opening angle

$$\cos \theta_c = c/n(p)v_{part} \simeq 1/n(p)$$

$v_{part} \sim c$ at GeV/c scale

$n(p) = p/p_0$: Index of Reflection



PRL96.172302 (2006)

Mach Cone Shock Wave

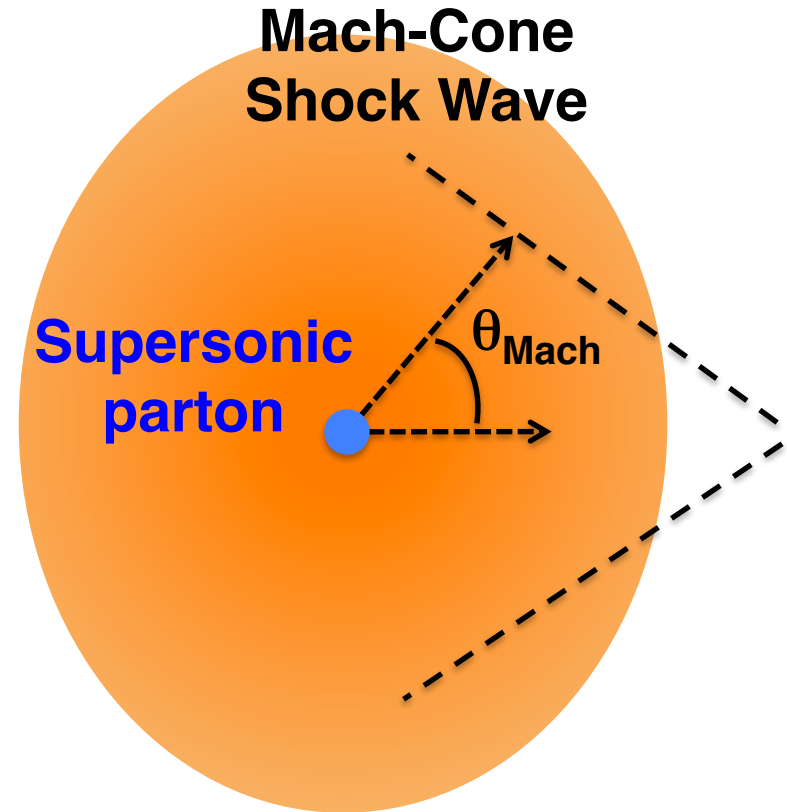
- Shock Wave from a supersonic parton

$$\cos \theta_{Mach} = c_s / v_{part}$$

c_s : speed of sound

- p_T independence of double-hump position
 - $v_{part} \sim$ constant at GeV scale
 - $c_s \sim$ constant at Au+Au 200 GeV

PRC73.011901(R) (2006)



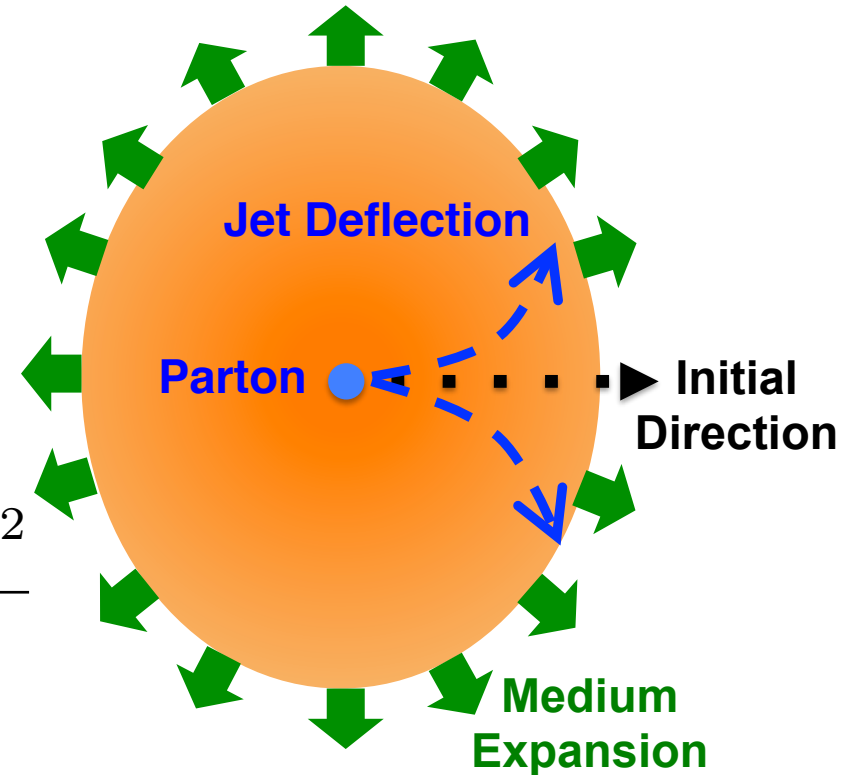
Jet Deflection

- Deflection from initial direction
 - Boost by medium expansion
 - Energy-momentum loss
- Hydro + Energy-Momentum Loss

$$\partial_\mu T^{\mu\nu} = S^\nu$$

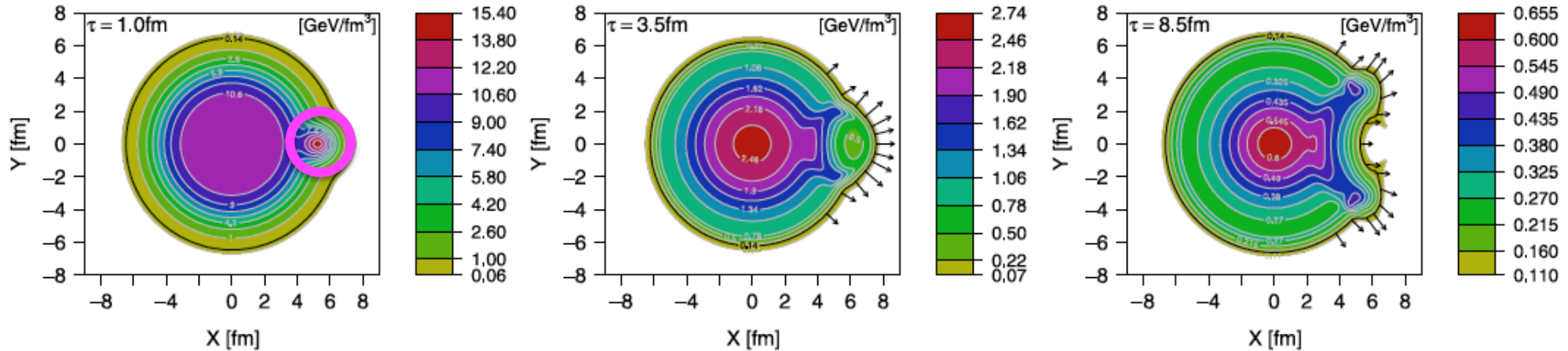
$$S^\nu(t, \vec{x}) = \frac{1}{(\sqrt{2\pi}\sigma)^3} \exp\left[-\frac{[\vec{x} - \vec{x}_{jet}(t)]^2}{2\sigma^2}\right] \times \left(\frac{dE}{dt}, \frac{dM}{dt}, 0, 0\right) \left[\frac{T(t, \vec{x})}{T_{max}}\right]^3$$

PRL105.222301 (2010)



Hot-Spot Model

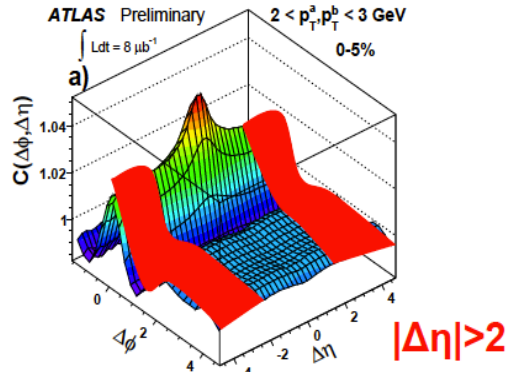
PLB712.226 (2012)



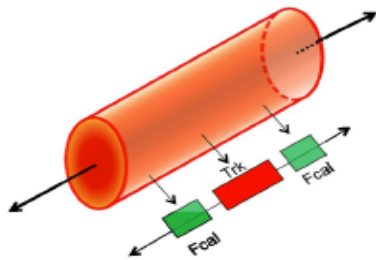
- Hot-Spot : domain of high parton density
- Hot-Spot + Hydrodynamic Expansion
- Double-humps without hard-scattered partons

Contributions of v_n ($n>2$) in correlations

2Par. Correlation



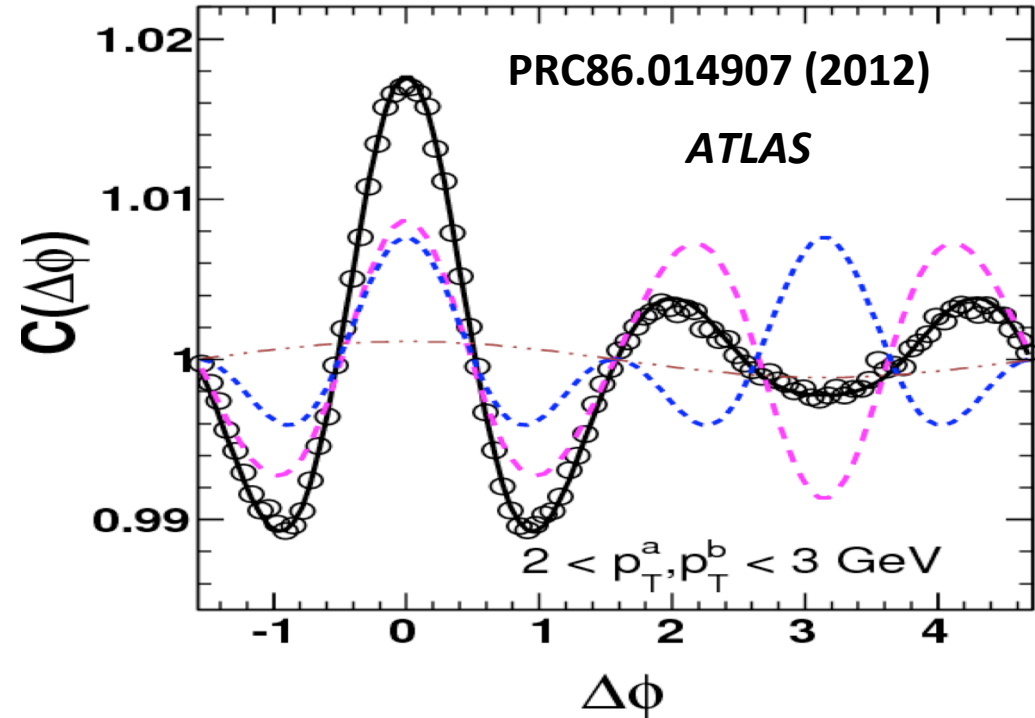
v_n with EP Method



Track at $|\eta| < 2.5$
 with EP from
 full FCAL $3.3 < |\eta| < 4.8$

$$C(\Delta\phi) = b^{2P} (1 + 2v_{1,1}^{2P} \cos \Delta\phi + 2 \sum_{n=2}^6 v_n^{EP} v_n^{EP} \cos n\Delta\phi)$$

↑ From 2PC method ↑ From EP method



- Double-hump & ridge of long-rapidity correlation explained
- Short-rapidity correlation with v_n subtraction to discuss parton behavior

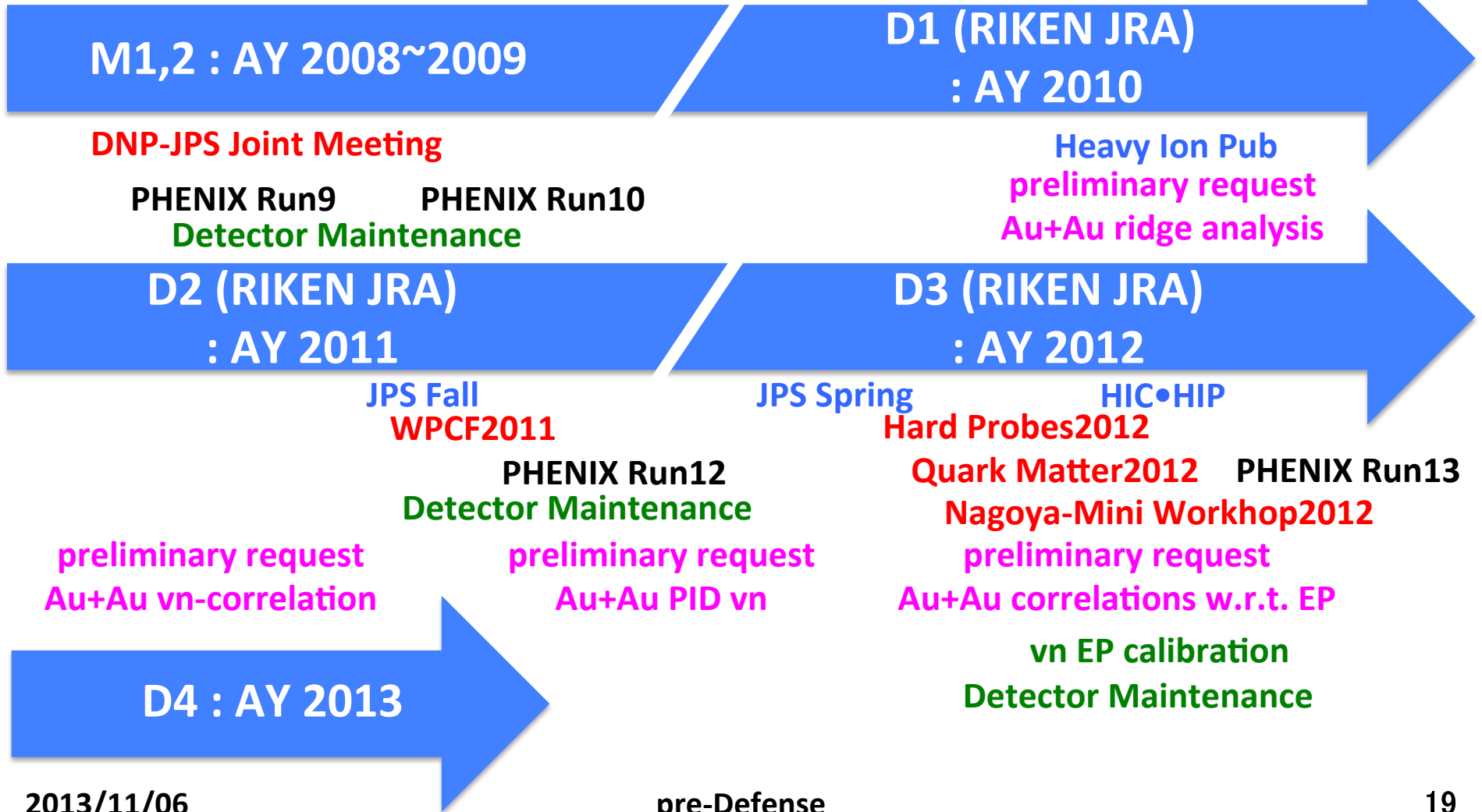
Motivation of this Dissertation

- Focus on azimuthal two-particle correlations
- Provide robust experimental results after v_n subtraction
 - Centrality & p_T dependence, double-humps etc.
 - Revisit of previous models
- Diagnose correlation phenomena at intermediate- p_T with control of parton path length with respect to event-planes (Ψ_2, Ψ_3)
 - Parton-Energy Loss
 - Re-distribution of lost energy from partons
 - Possible boost of Jet by medium expansion

My Contributions

Activity Categories

*Oral (intern.) *Oral (Domestic) *Experimental *Analysis *Service work

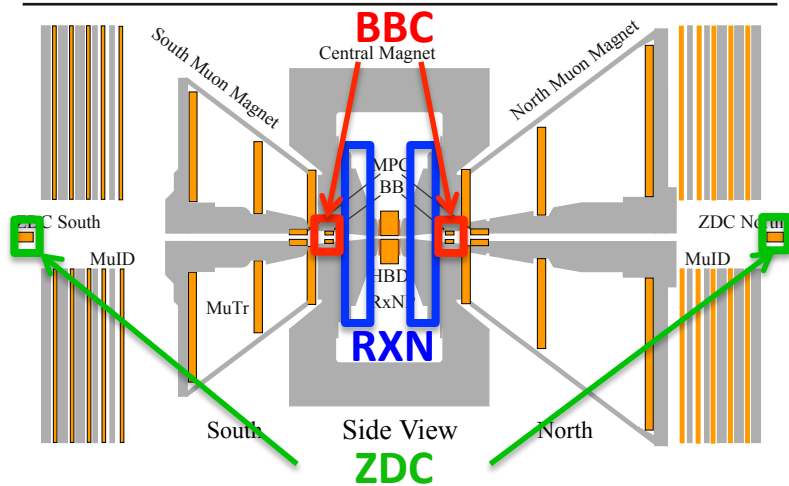
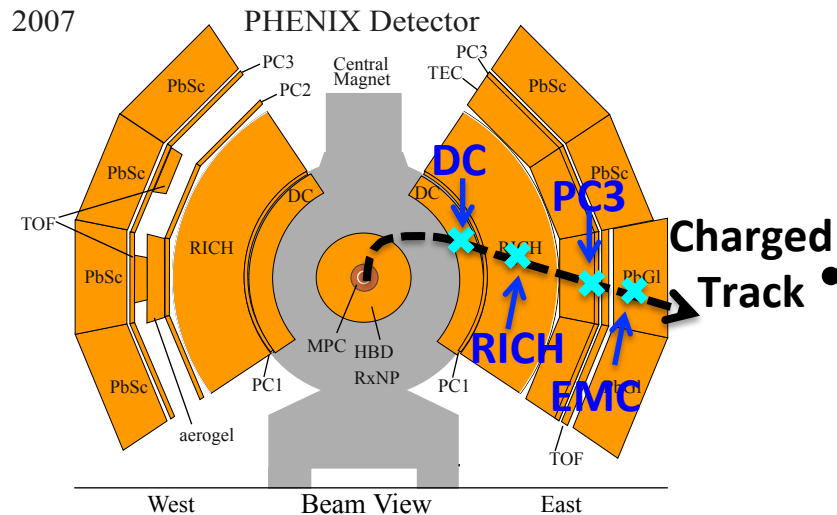


ANALYSYS

Data Set & Particle Selection

- PHENIX year 2007 Experiment
- Au+Au collisions at $\sqrt{s_{NN}}=200$ GeV
 - Minimum Bias Trigger
 - 4.4 billion events
- Charged Hadron Selection
 - 2σ Matching Cut
 - Electron Veto
 - Energy/Momentum cut for High p_T particles
 - Background rejection
 - Pair cut of miss-reconstructed hadron pairs

PHENIX Detector - 2007 Experiment

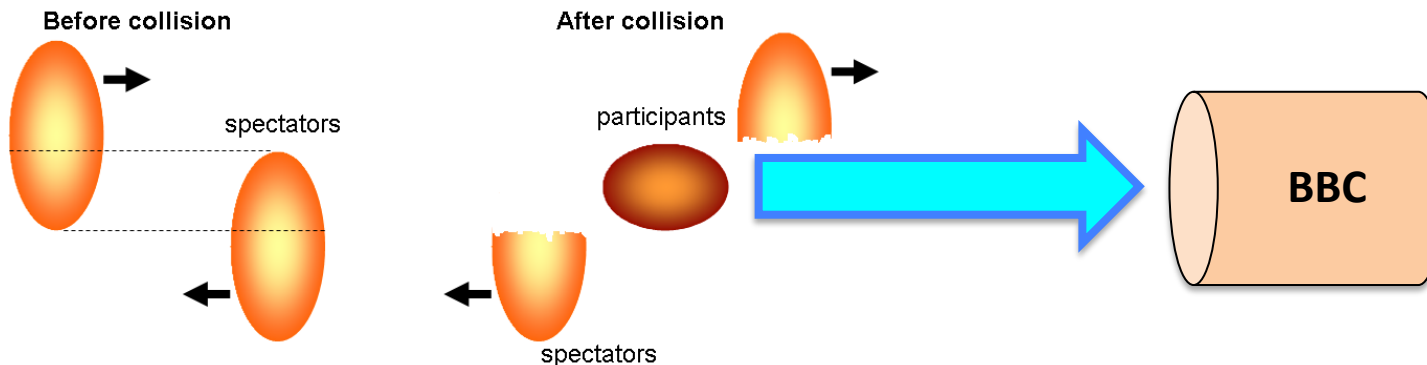
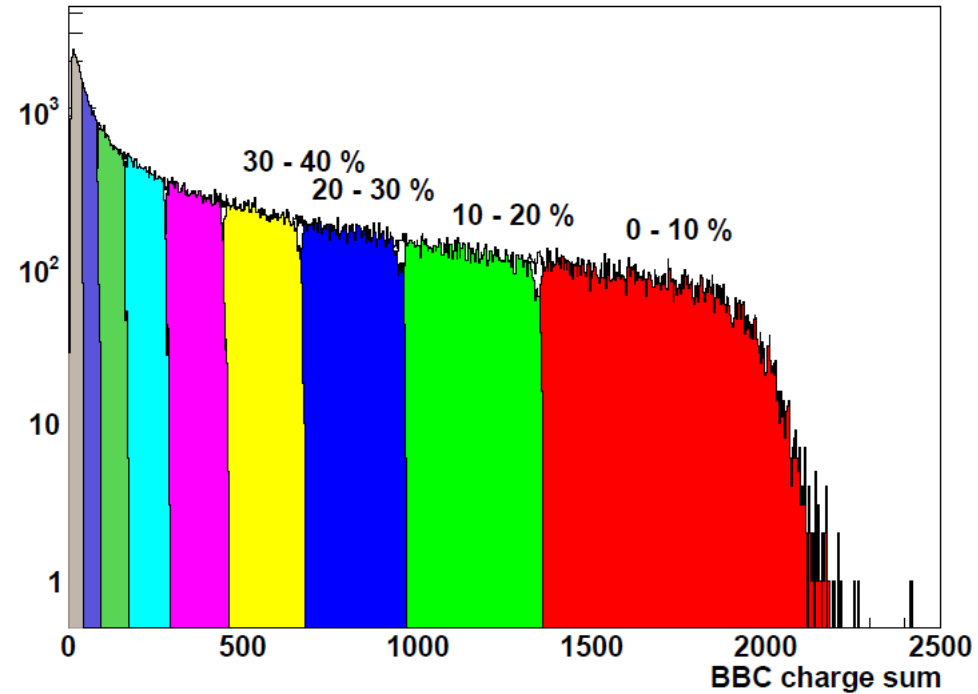


- Trigger, collision vertex, centrality
 - Beam-Beam-Counter (BBC)
 - Zero-Degree-Calorimeter(ZDC)
- Event Plane
 - BBC
 - Reaction-Plane-Detector(RXN)
- Central Arm, $\Delta\phi=\pi$ $|\eta|<0.35$
 - Momentum, Tracking, Electron Veto
 - Drift Chamber (DC)
 - Pad Chamber(PC)
 - Electromagnetic Calorimeter(EMC)
 - Ring Image Cherenkov Detector(RICH)

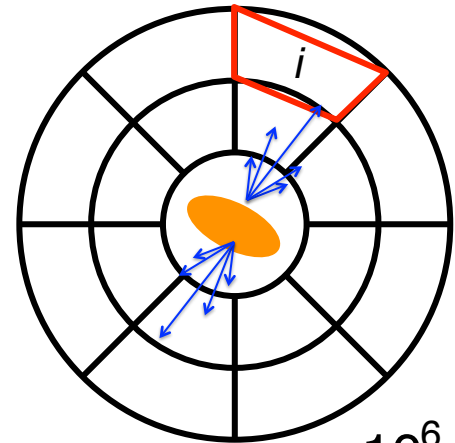
Centrality

- BBC charge sum divided in percentile
 - Same number of events in each bins

Centrality determination (Run7)



Event Plane Calibration



ϕ_i : Azimuthal angle

w_i : Weight (Charge etc.)

Raw distribution

$$Q_x = \frac{\sum_i w_i \cos(n\phi_i)}{\sum_i w_i}, Q_y = \frac{\sum_i w_i \sin(n\phi_i)}{\sum_i w_i}$$

$$\Psi_n = \frac{1}{n} \tan^{-1} \left(\frac{Q_y}{Q_x} \right)$$

Re-centering

$$Q_x^{Rec} = \frac{Q_x - \langle Q_x \rangle}{\sigma_x}, Q_y^{Rec} = \frac{Q_y - \langle Q_y \rangle}{\sigma_y}$$

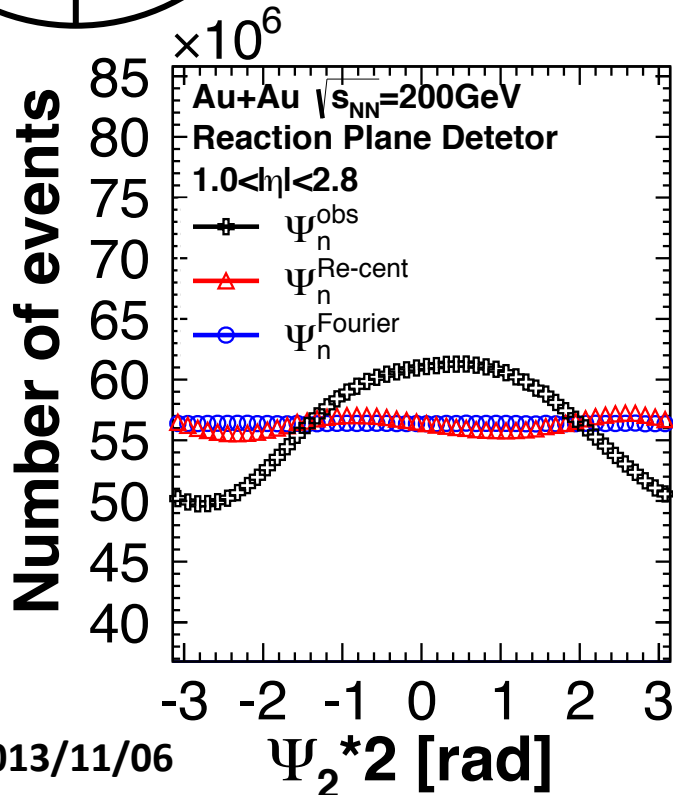
$$\Psi_n^{Rec} = \frac{1}{n} \tan^{-1} (Q_y^{Rec} / Q_x^{Rec})$$

Fourier correction

$$n\Psi_n^{Fourier} = n\Psi_n^{Rec} + n\Delta\Psi_n$$

$$n\Delta\Psi_n = \sum_k \{ A_k \cos(kn\Psi_n^{Rec}) + B_k \sin(kn\Psi_n^{Rec}) \}$$

$$A_k = -\frac{2}{k} \langle \cos(kn\Psi_n^{Rec}) \rangle, B_k = \frac{2}{k} \langle \sin(kn\Psi_n^{Rec}) \rangle$$



Event Plane Resolution & v_n Measurements

EP Resolution

PRC58.1671

- Resolution +/- η

$$\begin{aligned}\sigma_n^{EP} &= \langle \cos kn(\Psi_n^{EP\pm\eta} - \Psi_n) \rangle \\ &= \sqrt{\langle \cos kn(\Psi_n^{EP(+\eta)} - \Psi_n^{EP(-\eta)}) \rangle} \\ &= \frac{\pi}{8} \chi_n^2 \left[I_{(k-1)/2} \left(\frac{\chi_n^2}{4} \right) + I_{(k+1)/2} \left(\frac{\chi_n^2}{4} \right) \right]^2\end{aligned}$$

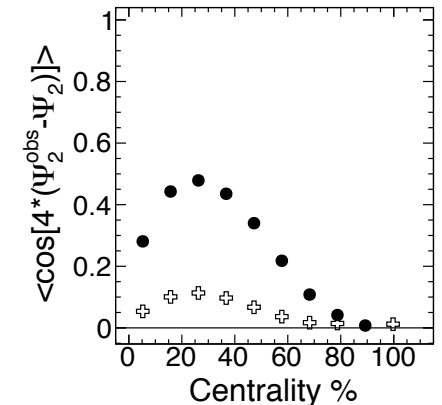
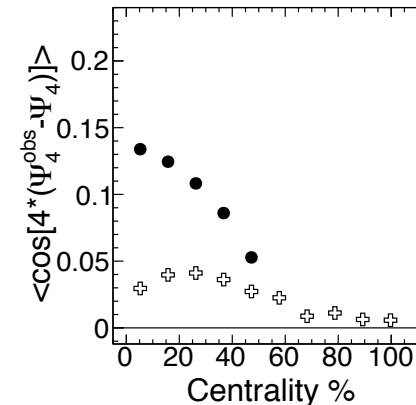
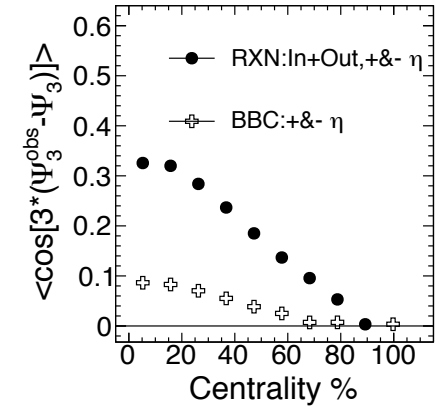
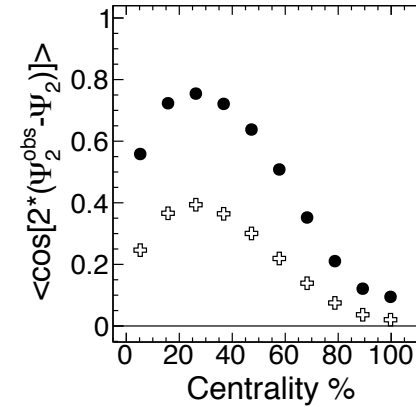
- Resolution +&- η

$$- \chi \rightarrow \sqrt{2}\chi$$

$$\sigma_n^{EP} = \frac{\pi}{8} 2\chi_n^2 \left[I_{(k-1)/2} \left(\frac{2\chi_n^2}{4} \right) + I_{(k+1)/2} \left(\frac{2\chi_n^2}{4} \right) \right]^2$$

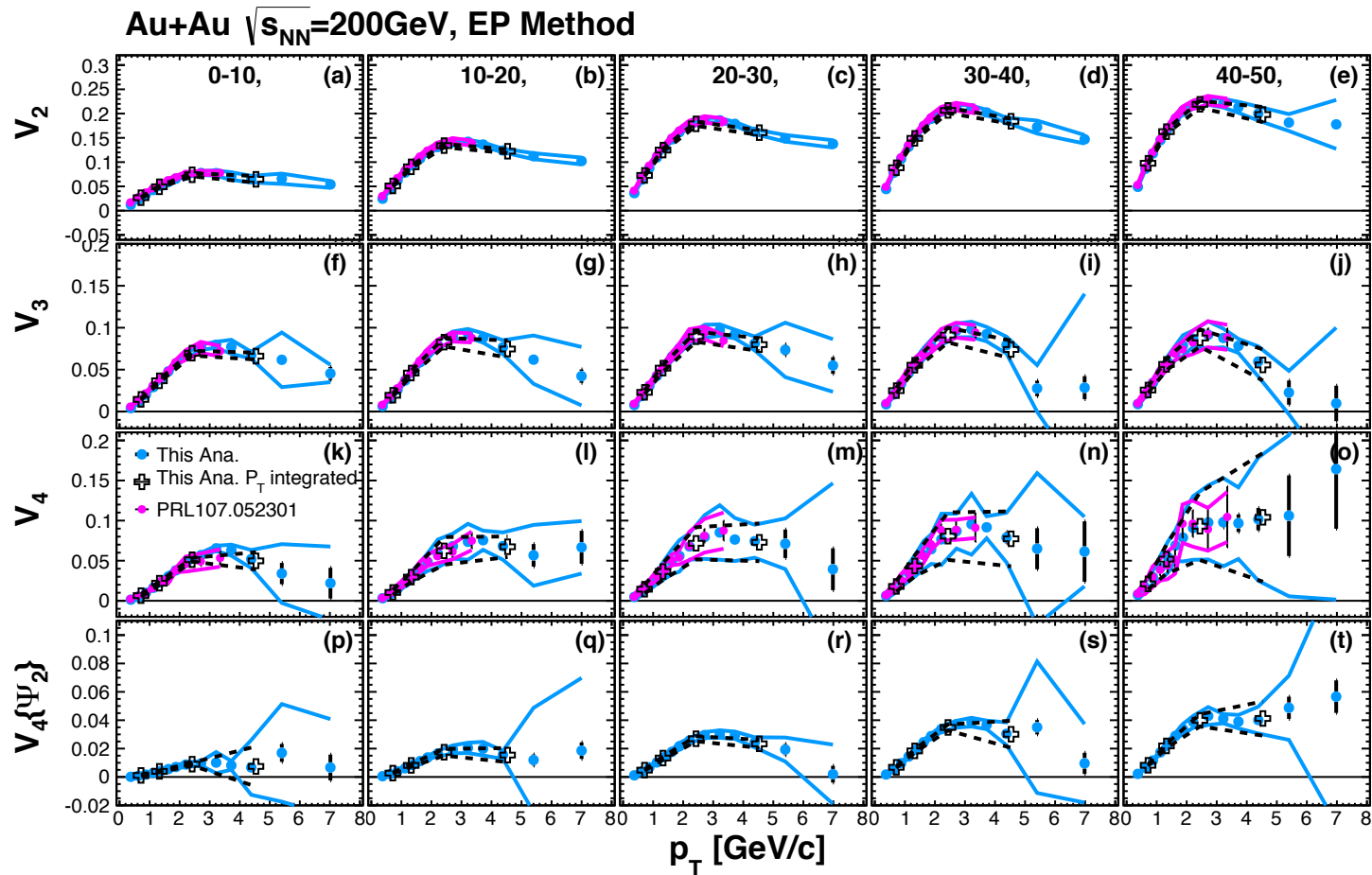
- v_n measurements

$$v_n = \frac{v_n^{obs}}{\sigma_n^{EP}} = \frac{\langle \cos n(\phi - \Psi_n^{EP}) \rangle}{\langle \cos n(\Psi_n^{EP} - \Psi_n) \rangle}$$



v_n Results

- Consistent results with previous measurements
 - Used for background subtraction

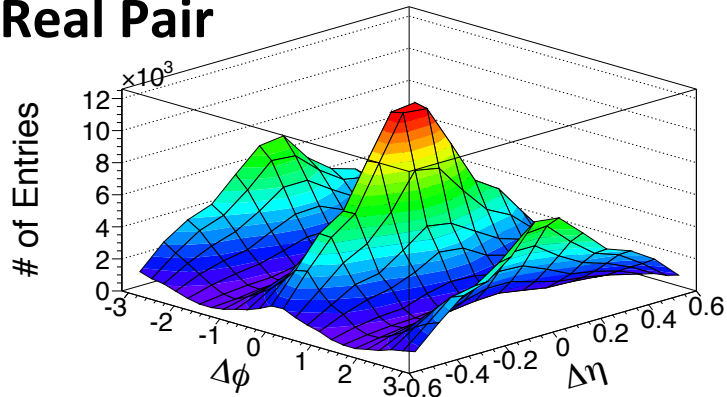


Two Particle Correlations

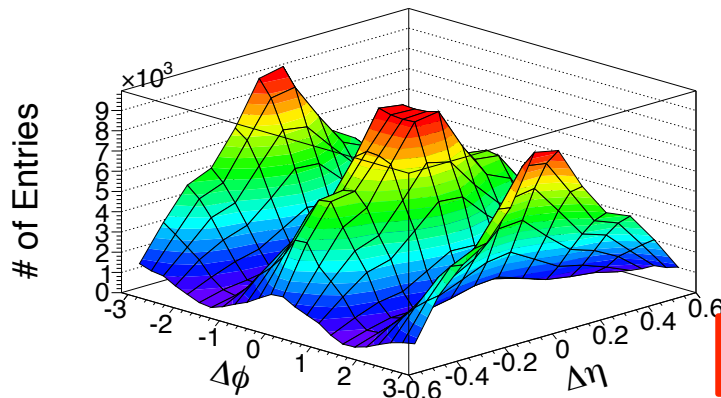
- Pair distributions in real and mixed events

$$\Delta\phi = \phi^a - \phi^t, \Delta\eta = \eta^a - \eta^t$$

Real Pair

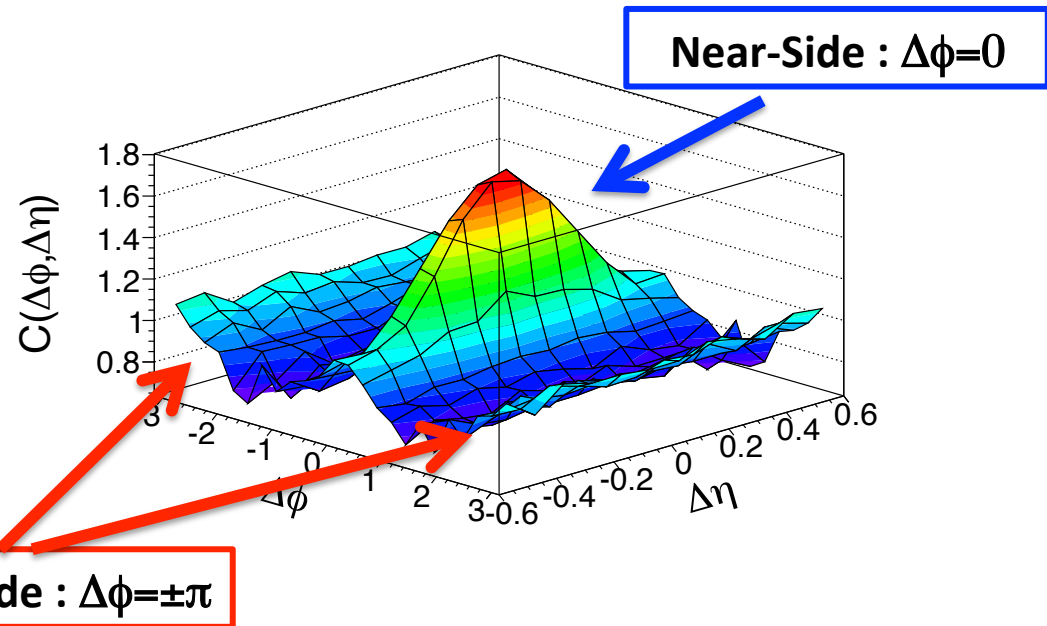


Mixed Pair



- Two-Particle Correlations

$$C(\Delta\phi, \Delta\eta) = \frac{N_{mix}^{ta}}{N_{real}^{ta}} \frac{d^2 N_{real}^{ta} / d\Delta\phi d\Delta\eta}{d^2 N_{mix}^{ta} / d\Delta\phi d\Delta\eta}$$



Flow Subtraction & Pair Yield per a Trigger (PTY)

- Analytical Formula of Pure Flow

$$\begin{aligned}
 F(\Delta\phi) &= 1 + \sum 2V_{\Delta n} \cos(n\Delta\phi) \\
 &= 1 + \sum 2v_n^t v_n^a \cos(n\Delta\phi)
 \end{aligned}$$

- Flow subtractions

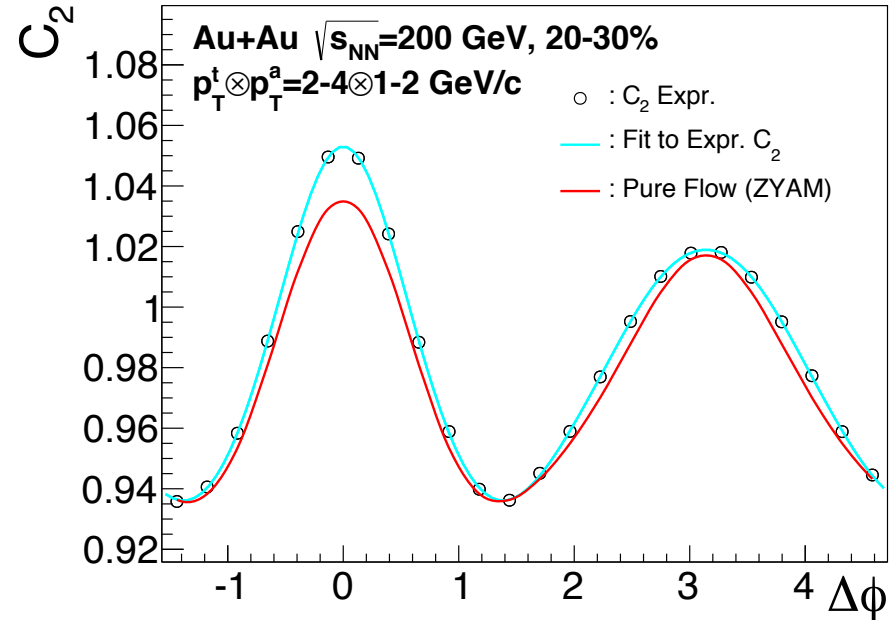
– Zero Yield At Minimum Assumption

$$j(\Delta\phi) = C(\Delta\phi) - b_0 \left[1 + \sum_{n=1} 2v_n^t v_n^a \cos(n\Delta\phi) \right]$$

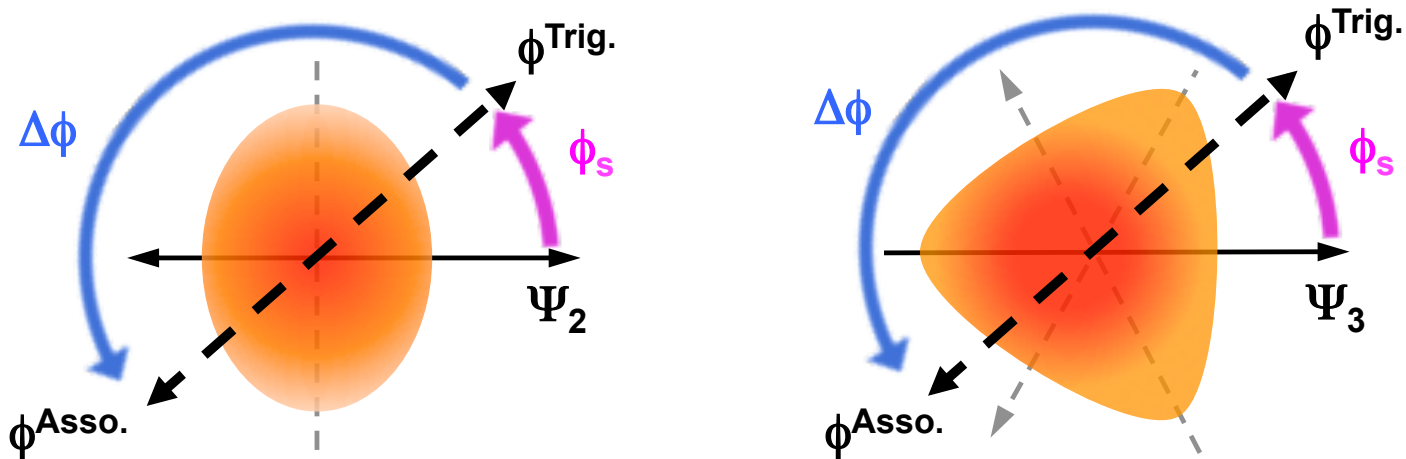
- Pair yield per a trigger

$$\frac{1}{N^t} \frac{dN^{ta}}{d\Delta\phi} = \frac{1}{2\pi\varepsilon} \frac{N^{ta}}{N^t} j(\Delta\phi)$$

ε : Tracking efficiency of associate particles



Event-Plane Dependence



- Selecting trigger particle w.r.t. Ψ_2 & Ψ_3
- Control of parton path length inside medium
- Sensitivity to each harmonic plane

Flow Contributions with respect to EP

- No analytical formula of pure flow w.r.t EP
- Run a Monte Carlo simulation
- Azimuthal distribution
 - Using Measured v_n

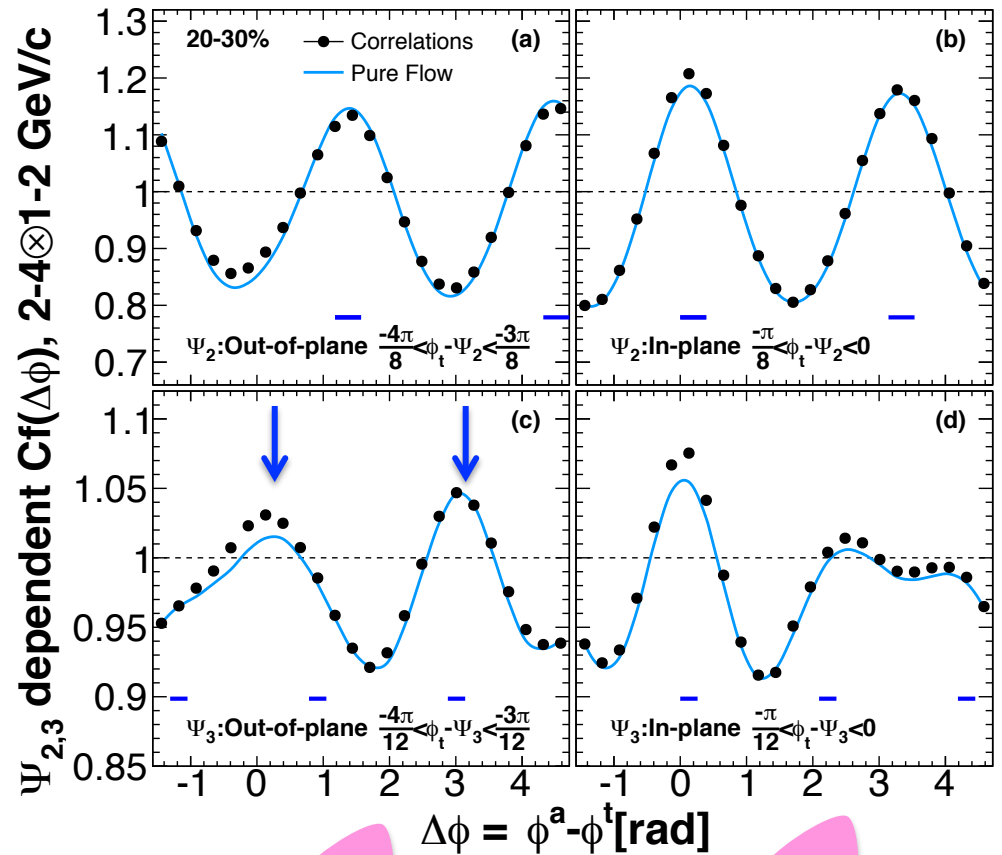
$$E \frac{d^3 N}{dp^3} = \frac{d^2 N}{2\pi dp_T d\eta} \left\{ 1 + \sum_{n=2,3,4} 2v_n \cos n(\phi - \Psi_n) \right\}$$

- Observed Event Plane Resolution
- Observed correlation between EP
 - $\langle 4(\Psi_2 - \Psi_4) \rangle = v_4\{\Psi_2\} / v_4\{\Psi_4\}$
 - $\langle 6(\Psi_2 - \Psi_3) \rangle = 0$

Flow Contributions with respect to EP

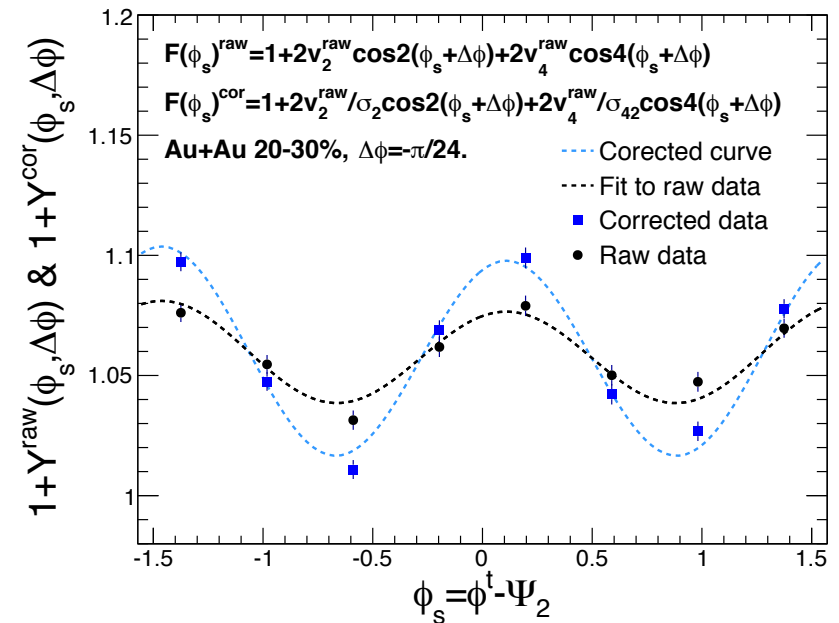
- p_T^T : 2-4, p_T^a :1-2 GeV/c
- Good reconstruction of correlation shape by MC simulation for both Ψ_2, Ψ_3 dependence
- Except around $\Delta\phi=0, \pi$ where affected by jet

Au+Au $\sqrt{s_{NN}}=200\text{GeV}$, Pure Flow : $v_n (n=2,3,4) + \langle \cos 4(\Psi_2 - \Psi_4) \rangle$



Resolution Correction for Trigger Selection

- Fitting Method
 - Azimuthal anisotropy of correlations yield
 - Resolution correction in analogous to v_n measurements
- Iteration Method
 - Trigger smearing effect by measured resolution S
 - Correlation Yield A
 - Smearred Correlations B
 - Effective correction coefficient C
 - Iteration until convergence



$$\begin{aligned}
 \mathbf{B}^{(n)} &= S \mathbf{A}^{(n)} \\
 \mathbf{C}^{(n)} &= \mathbf{A}^{(n)} / \mathbf{B}^{(n)} \\
 \mathbf{C} \mathbf{A}^{(n)} &= \mathbf{A}^{(n+1)}
 \end{aligned}$$

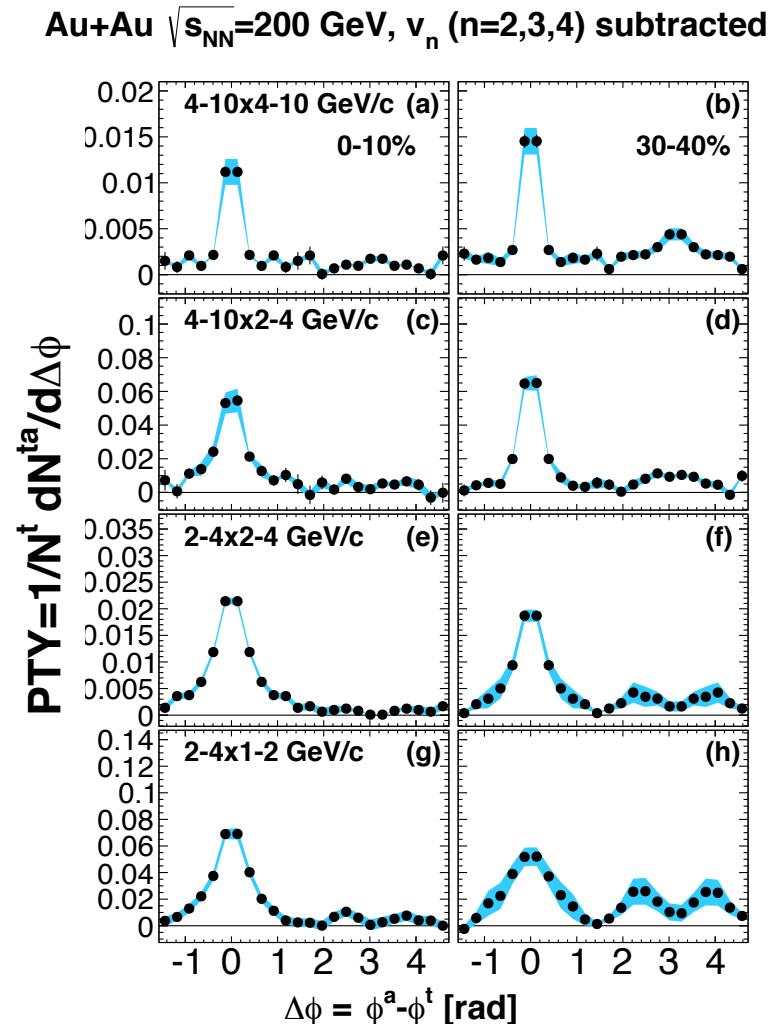
Systematic Uncertainties

- Flow v_n Measurements
 - Fluctuation within RXN EP
 - Rapidity dependence of EP : RXN-BBC difference
 - Matching cut of CNT Particles
- Two-Particle Correlations
 - Systematics from v_n
 - Matching cut of CNT Particles
- Event Plane Dependent Correlations
 - Unfolding Method : Fit & Iteration
 - Parameter in iteration method

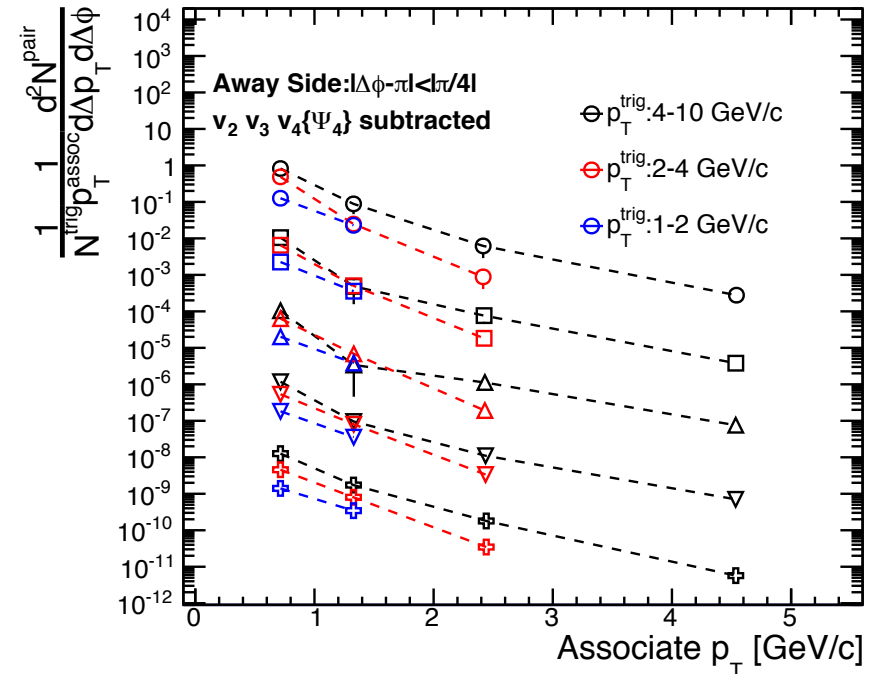
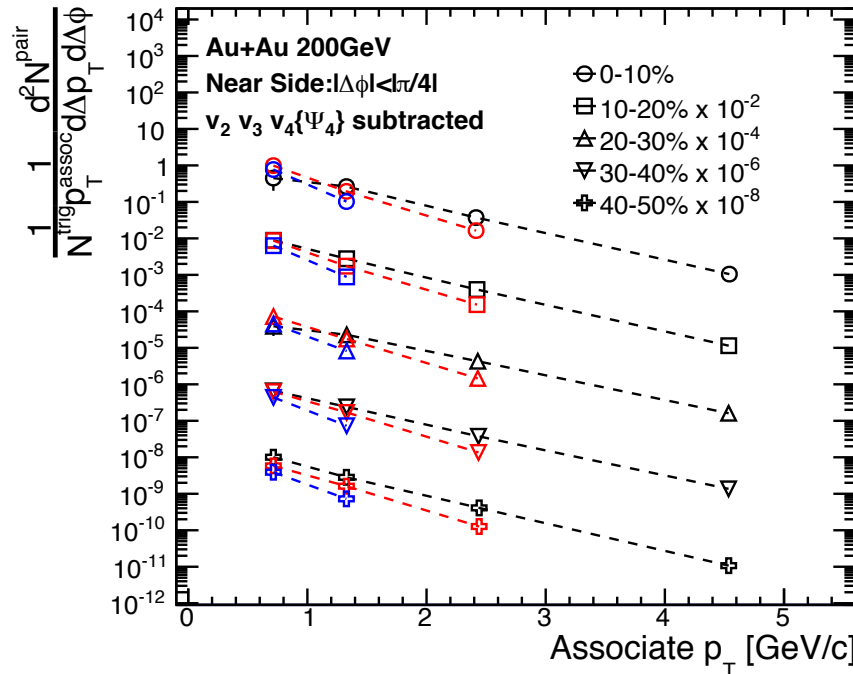
Results & Discussion

Two-Particle Correlations

- Suppression of Away-Side in most central collisions
- Away-side shape in peripheral collisions
 - Single peak at high p_T
 - Double hump at intm. p_T
- p_T independence of double hump position below $p_T < 4$ GeV/c



Hardness of Correlation Yields

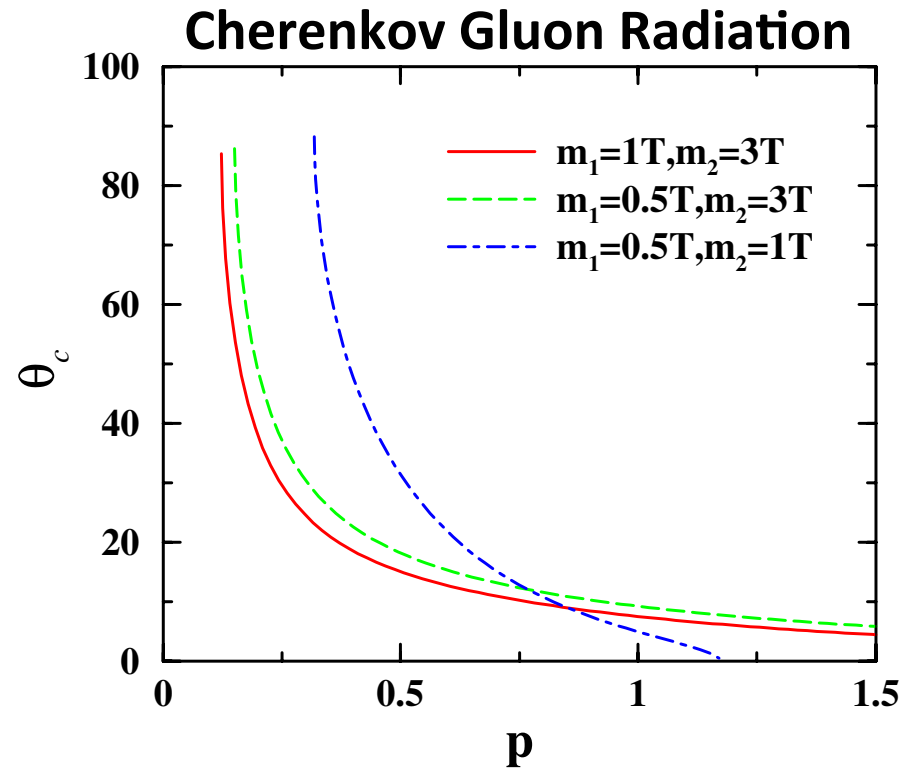


- p_T spectra of correlation yield

$$\frac{1}{p_T^a} \frac{dY}{d\Delta p_T} = \frac{1}{p_T^a} \frac{1}{(p_T^{a,max} - p_T^{a,min})} \int d\Delta\phi \frac{1}{N_{trig}} \frac{dN}{d\Delta\phi}$$

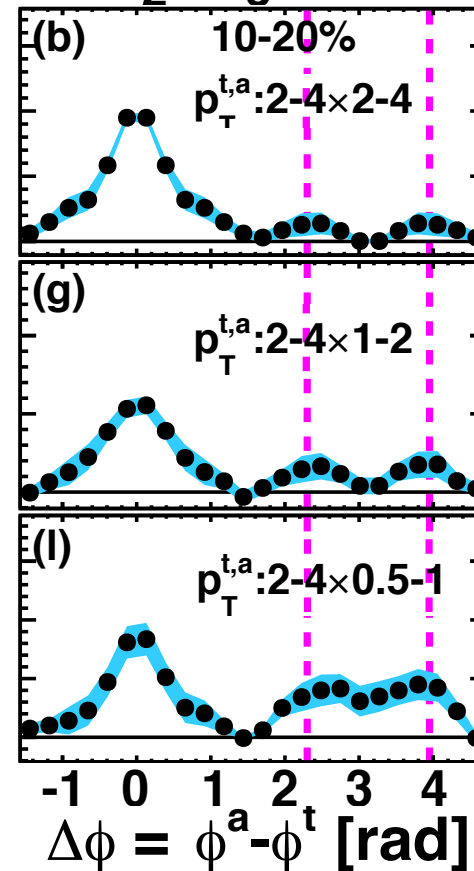
- Hardness increase with trigger and associate p_T
 - Different Physics depending on p_T

Comp. w/ Model : Cherenkov Gluon Radiation



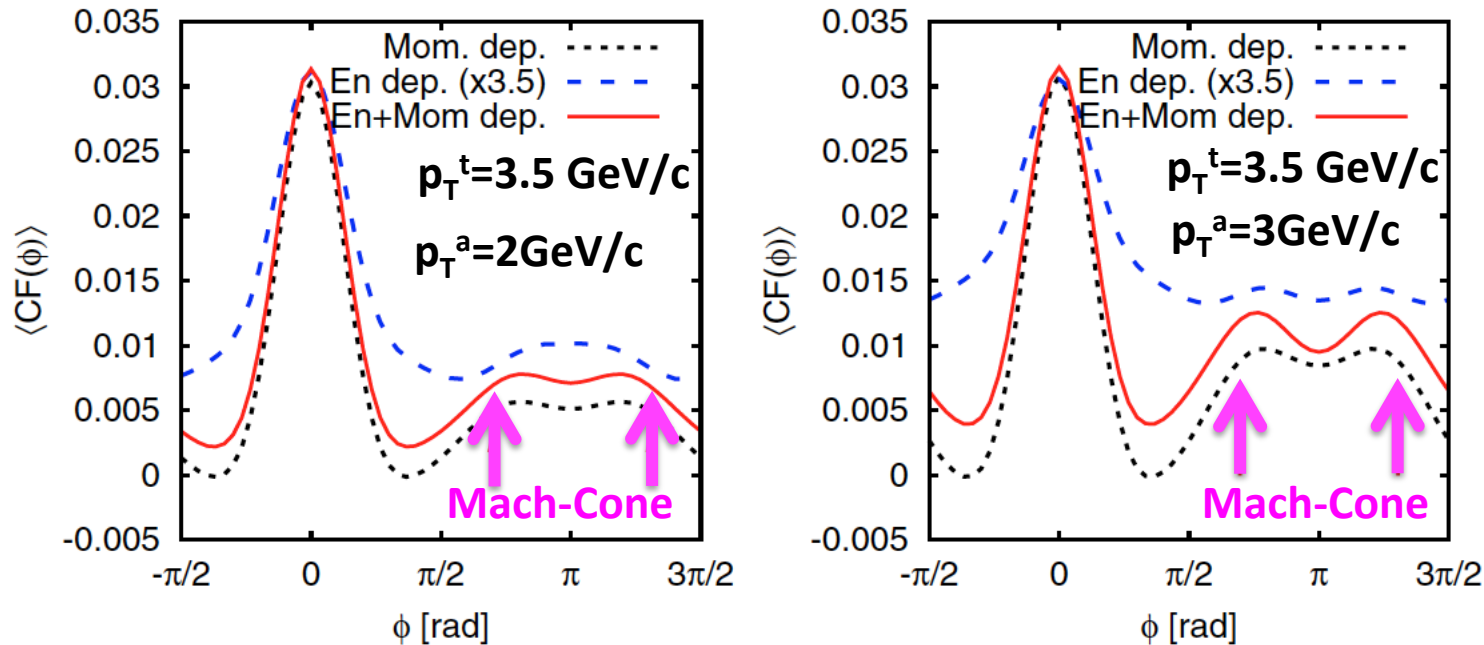
- Strong momentum dependence
- Disfavored by p_T independence of experimental data

Experimental Results



Comp. w/ Model : Jet-Deflection & Mach-Cone

PRL105.222301 (2010)



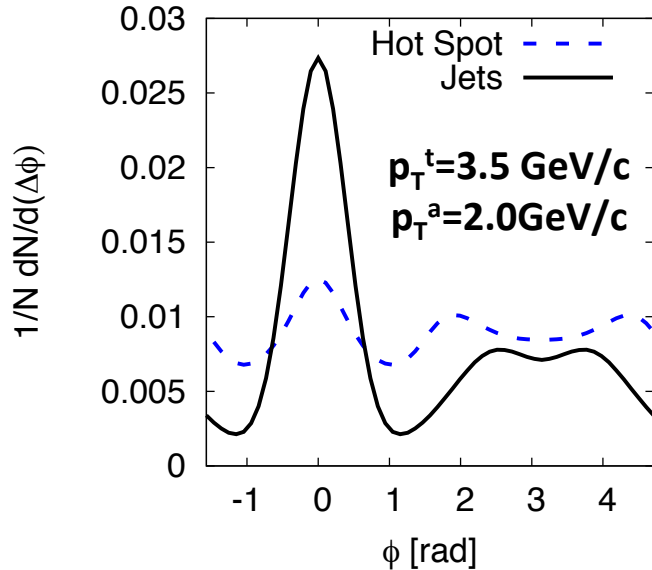
Energy-Momentum Loss Rate

$$dE/dt = 1 \text{ GeV/fm}$$
$$dM/dt = (1/v)(dE/dt)$$
$$(v = 0.999)$$

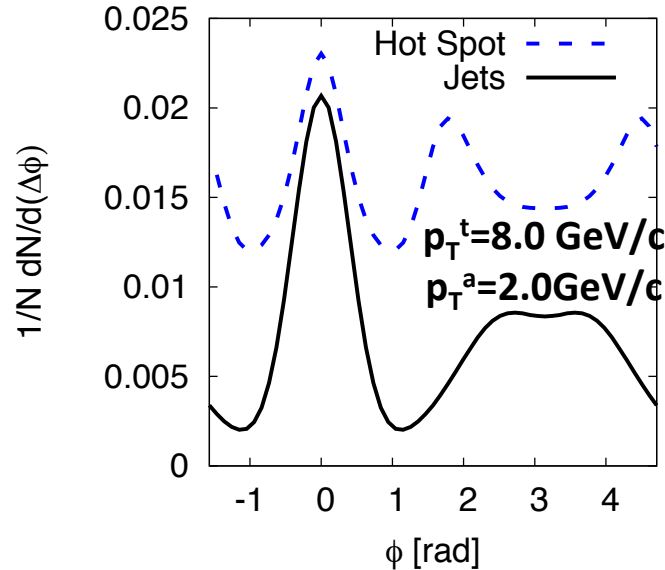
- Momentum Loss to obtain p_T independent Double-hump
 - Energy + Momentum Loss is most realistic
- Mach-Cone also shows p_T independence

Comp. w/ Model : Jet-Deflection & Hot-Spot

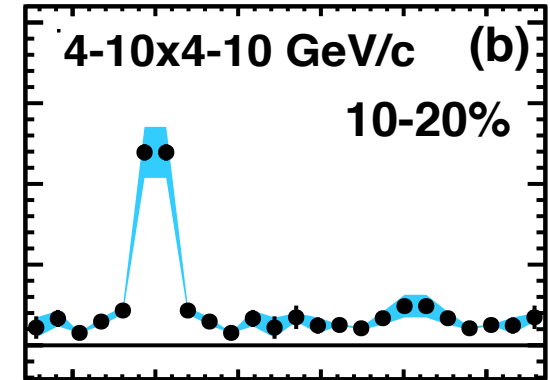
Jet : PRL105.222301 (2010)



Hot-Spot : PLB712.226 (2012)



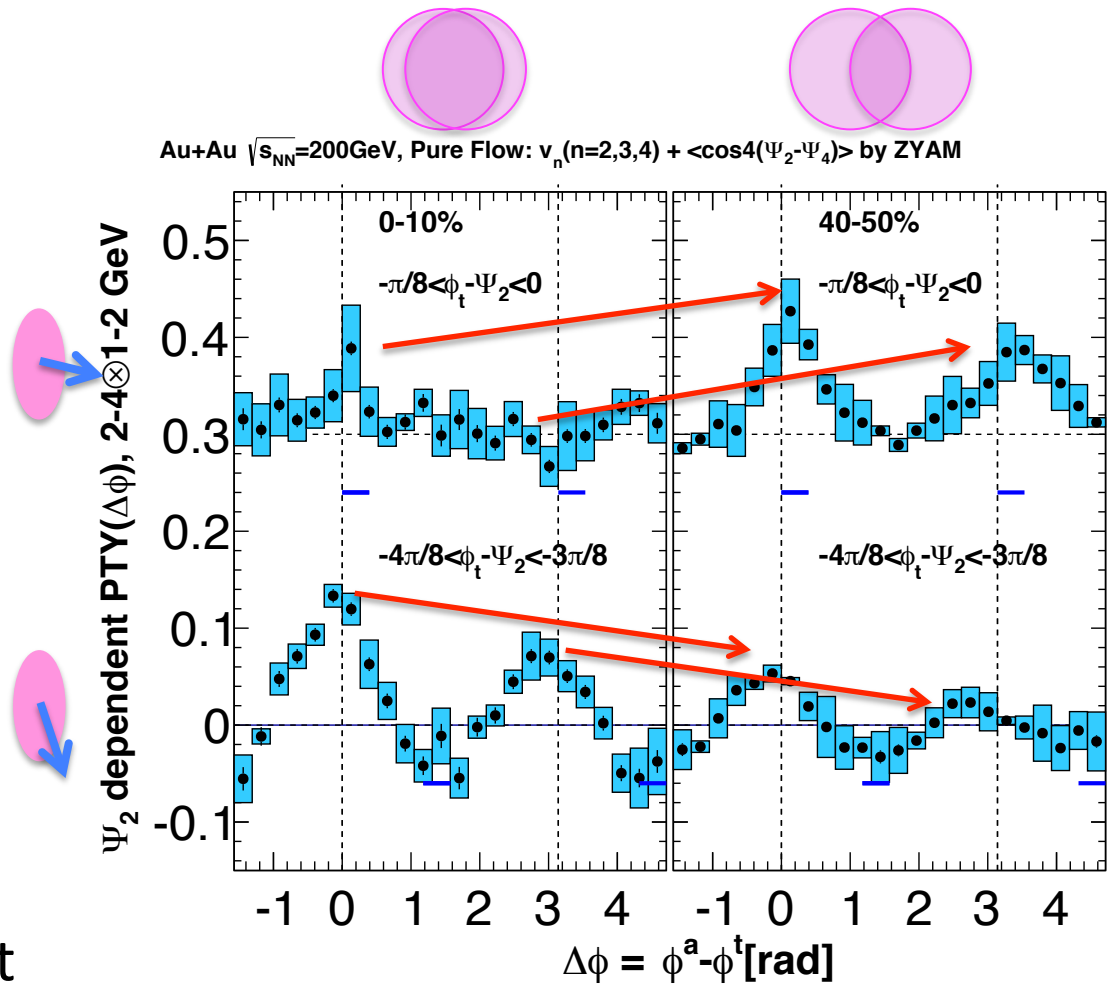
Experimental Results



- Intermediate p_T : Both models consistent with data
- Hot-Spot disfavored by the double-hump at high- p_T
- Consistent with the behavior of p_T spectra

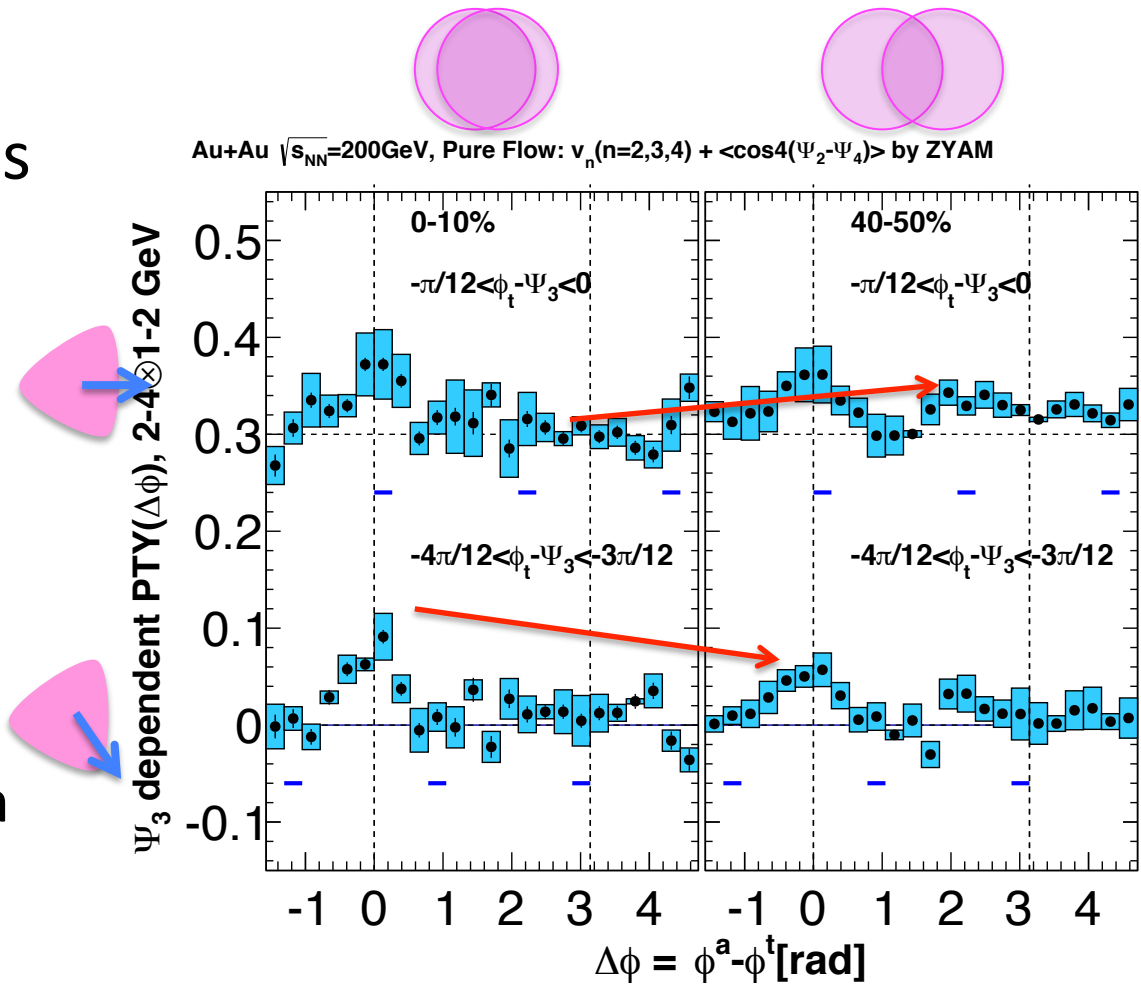
Ψ_2 dependence of PTY

- Mach-Cone disfavored
 - Double-hump must always appear in Mach-Cone
- In-plane trigger
 - Enhance in peripheral collisions: with thin medium
 - Parton Energy Loss
- Out-of-plane trigger
 - Enhance in central collisions: thick medium
 - Energy re-distribution & boost by collective expansion



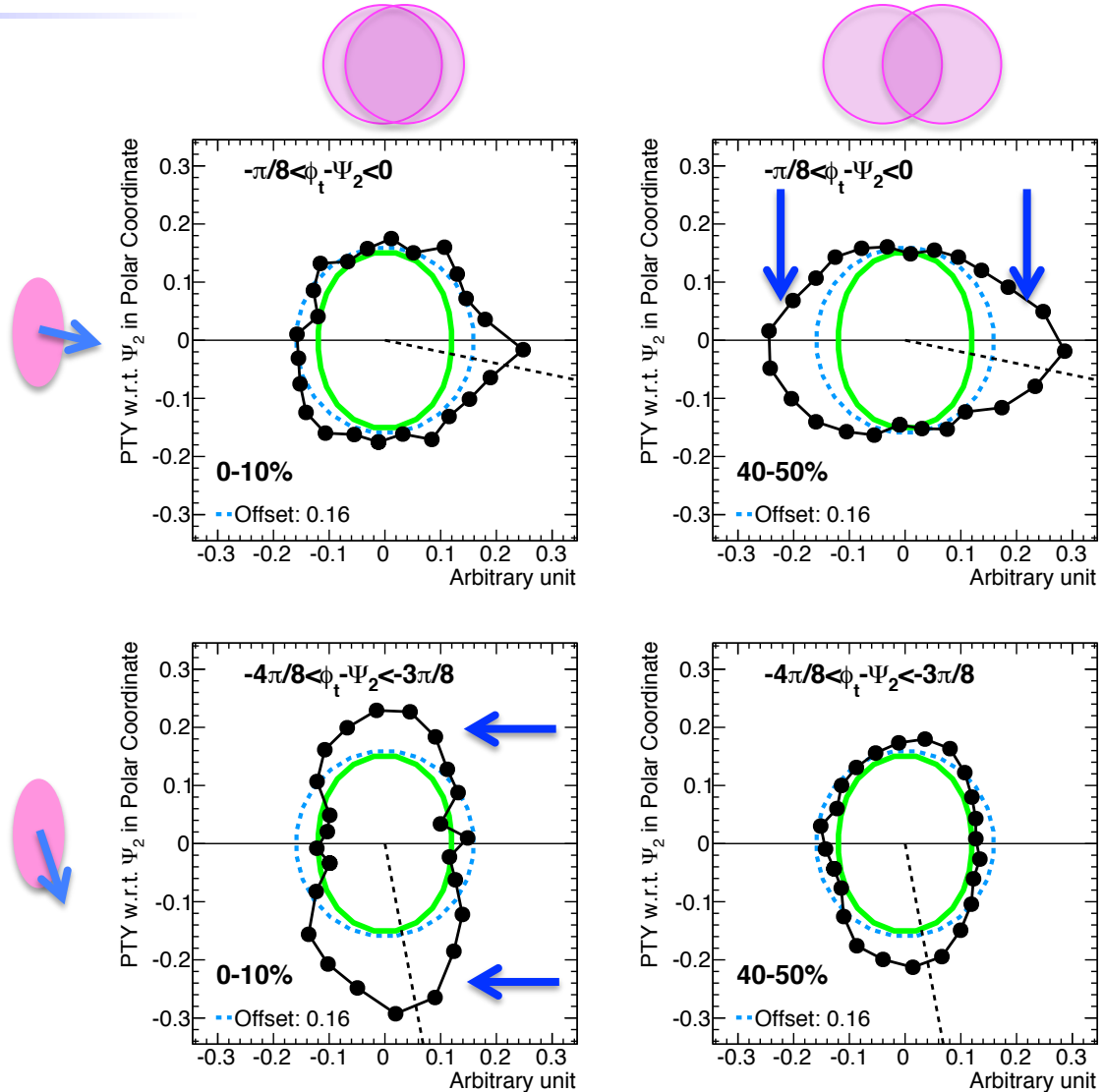
Ψ_3 dependence of PTY

- Similar but weaker trends as Ψ_2 dependence
- In-plane trigger
 - Enhance in peripheral collisions with thin medium
- Out-of-plane trigger
 - Possible enhance in central collisions with thick medium
- Weaker Ψ_3 dependence than Ψ_2



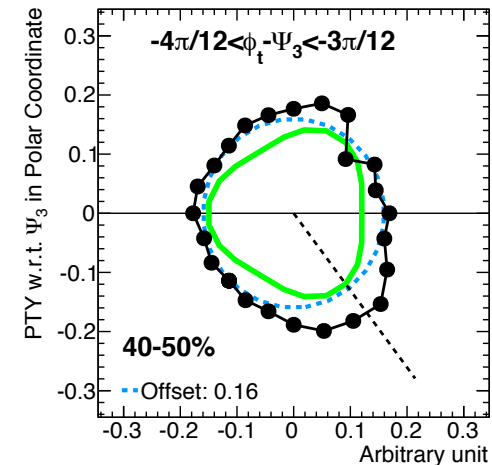
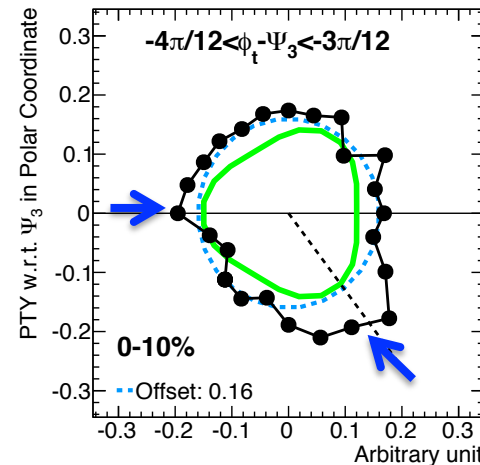
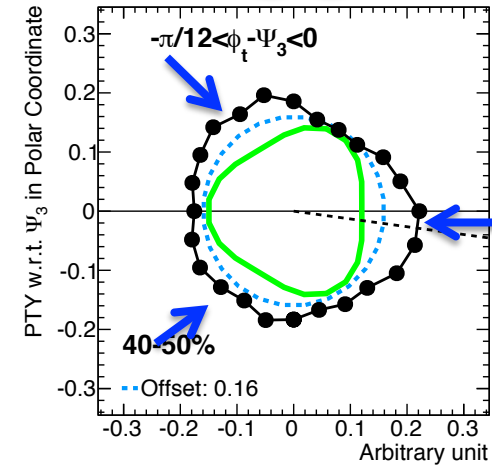
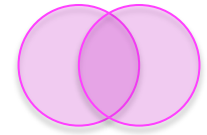
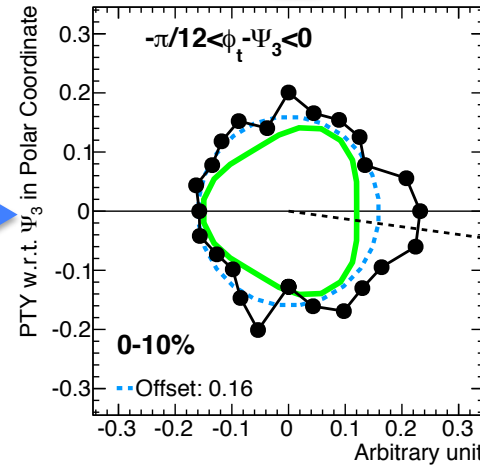
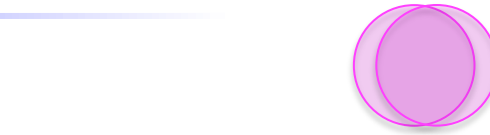
Ψ_2 dependence of PTY (Polar Coordinate)

- Yield correlates with geometry
 - Shortest average path length
- Enhance of In-plane yield with increase of centrality
 - Longest average path length
- Enhance of Out-of-plane yield with decrease of centrality
 - Longest average path length



Ψ_3 dependence of PTY (Polar Coordinate)

- Enhance of In-plane yield with increase of centrality
 - Shortest average path length
- Possible enhance of Out-of-plane yield with decrease of centrality



Azimuthal anisotropy of PTY : v_n^{PTY}

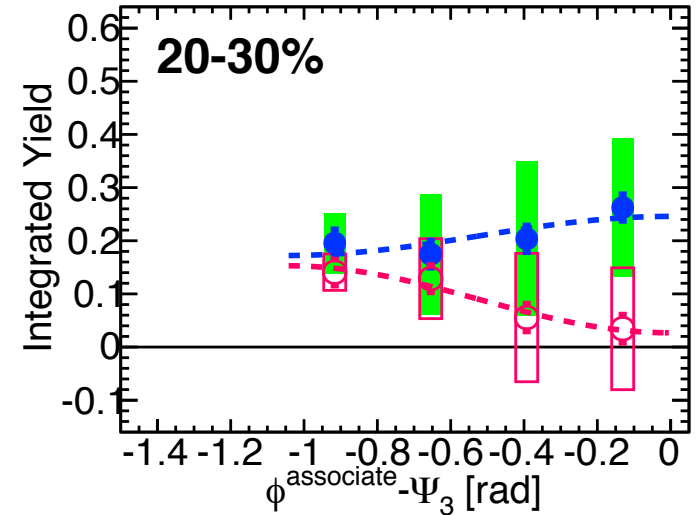
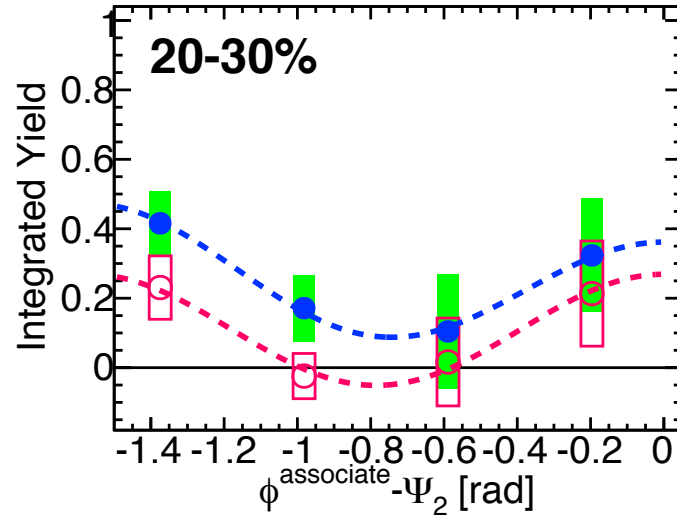
$$p_T^{\text{trig}} \otimes p_T^{\text{asso}} = 2-4 \otimes 1-2 \text{ GeV}/c$$



Near-Side, $\Delta\phi < |\pi/4|$



Away-Side, $\Delta\phi - \pi < |\pi/4|$



- Extraction of v_n^{PTY} by fitting

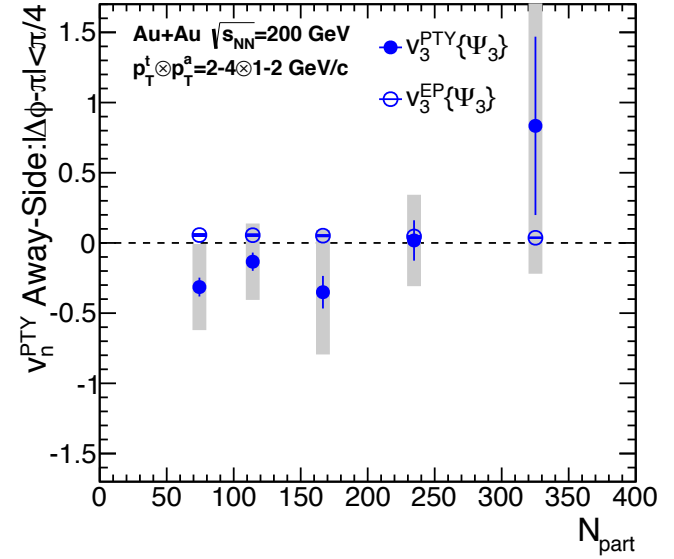
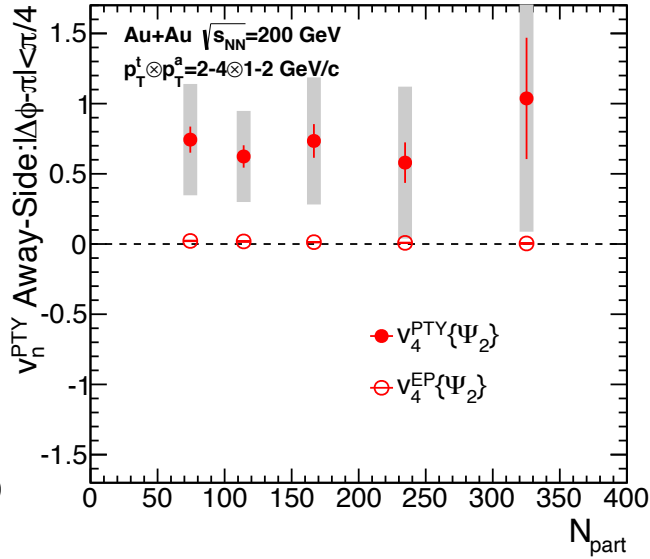
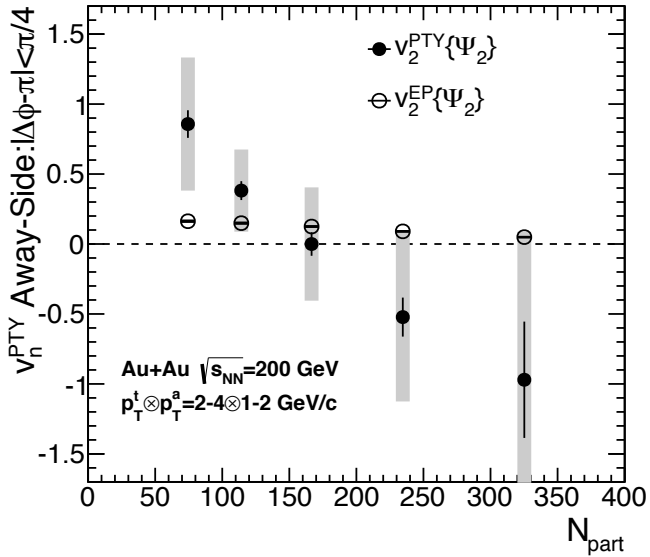
Ψ_2 dependence

$$F(\phi^a - \Psi_2) = a \{ 1 + 2v_2^{PTY} \cos 2(\phi^a - \Psi_2) + 2v_4^{PTY} \cos 4(\phi^a - \Psi_2) \},$$

Ψ_3 dependence

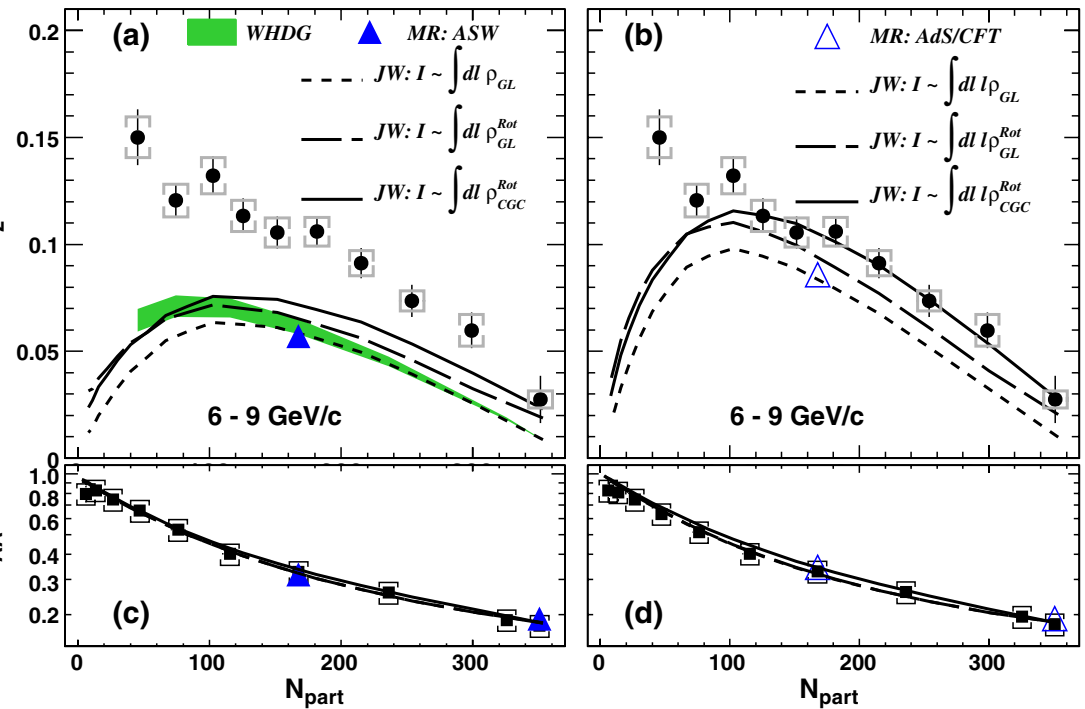
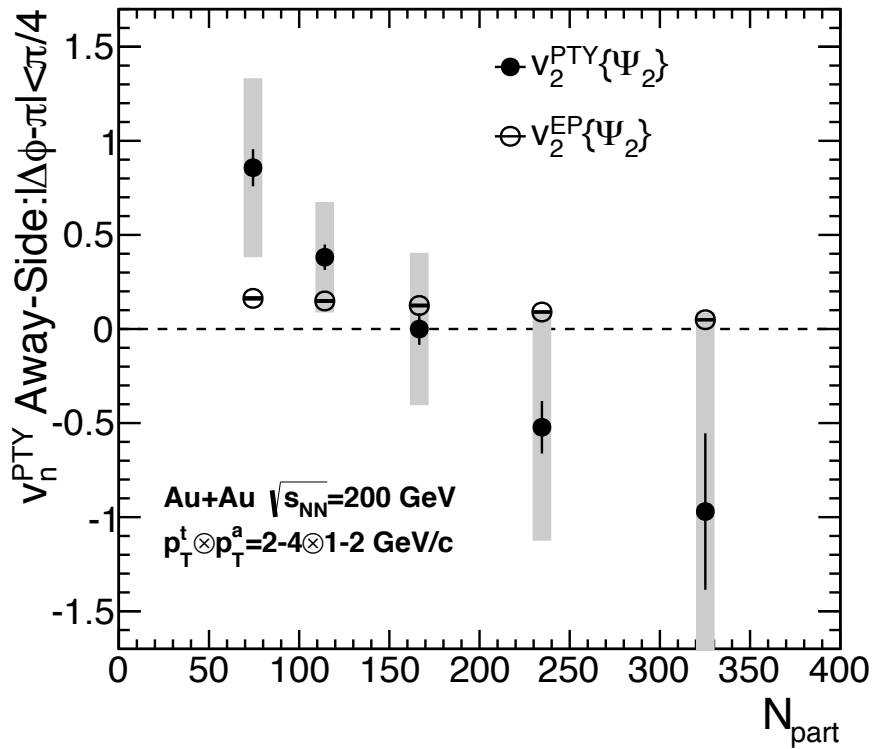
$$F(\phi^a - \Psi_3) = a \{ 1 + 2v_3^{PTY} \cos 3(\phi^a - \Psi_3) \},$$

$v_n^{\text{PTY}} & v_n^{\text{EP}}$



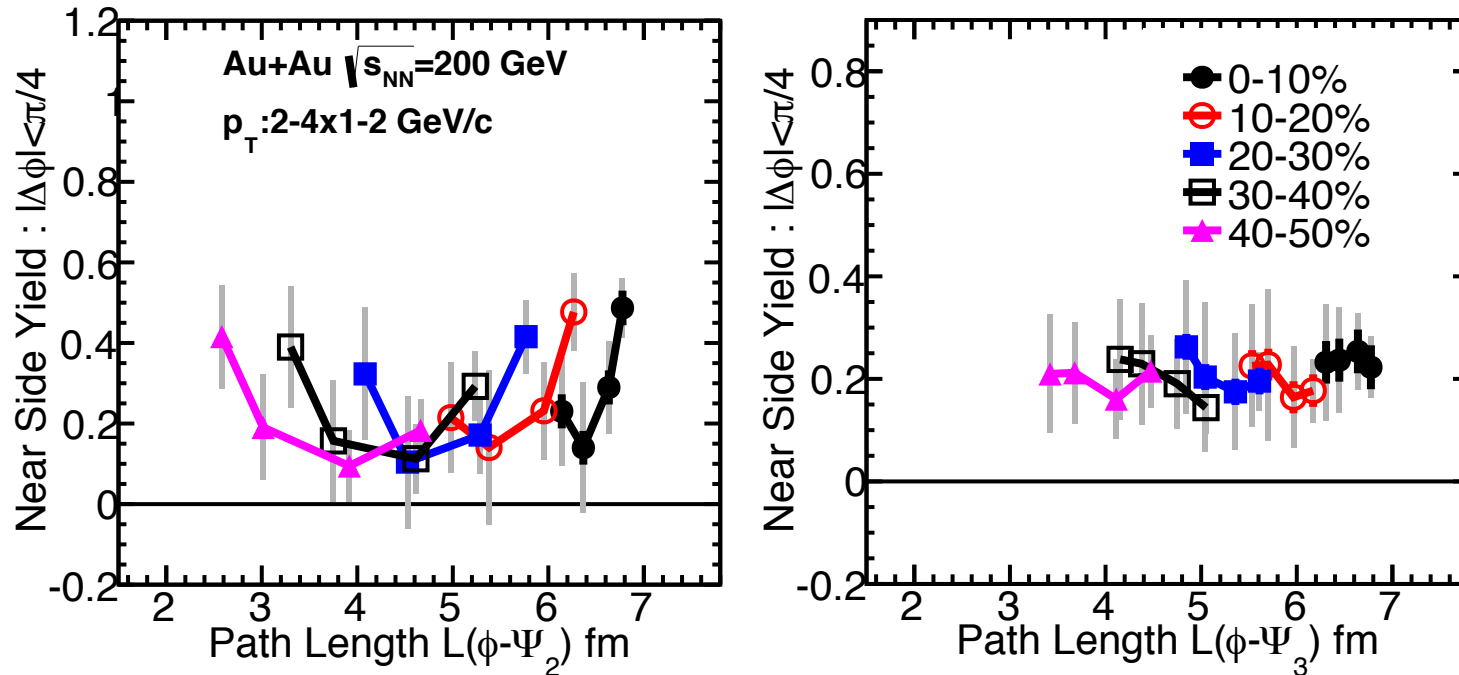
- v_n^{PTY} is a part of v_n^{EP}
- v_n^{PTY} amplitude is one order larger than v_n^{EP}

$v_2^{\text{PTY}} & \text{High-}p_T \pi^0 v_2$



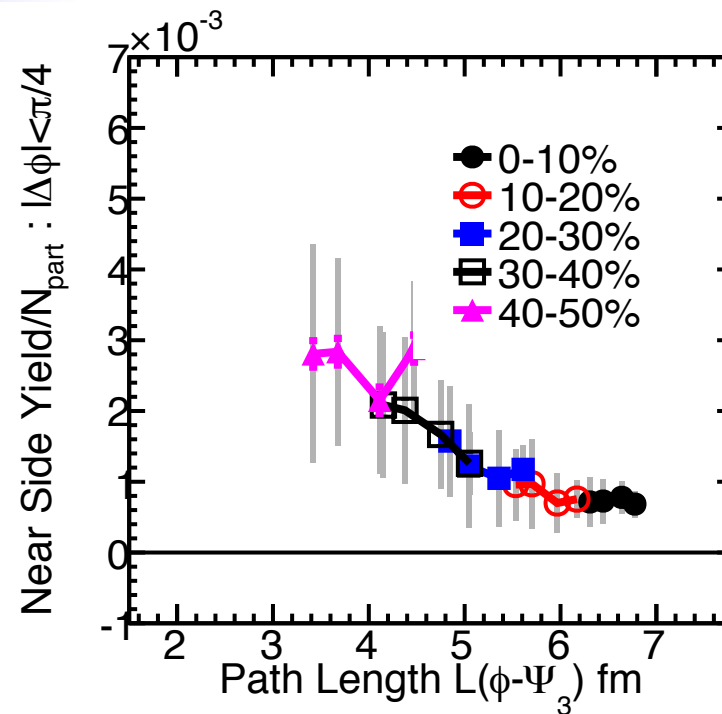
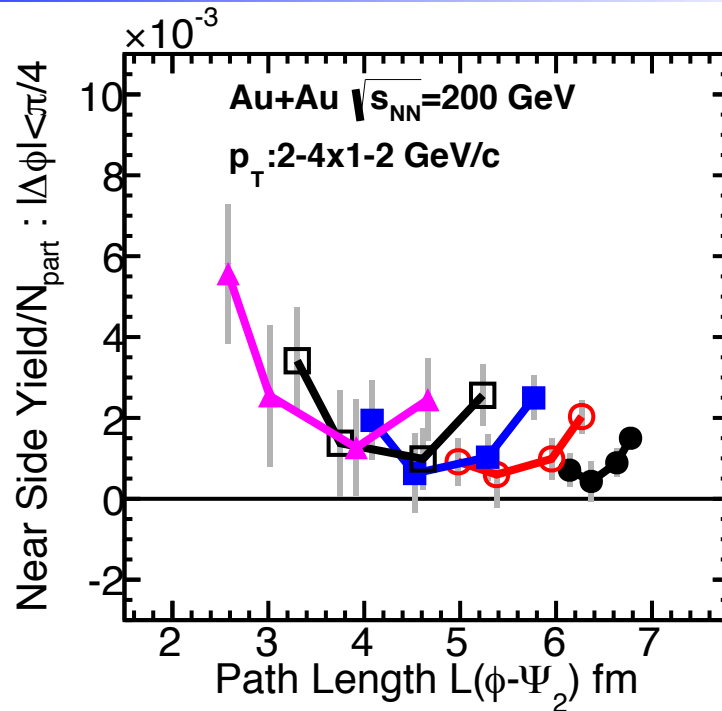
- Models with parton energy-loss provide positive $\pi^0 v_2$
- Additional effects required to explain negative v_2

Path length dependence of PTY



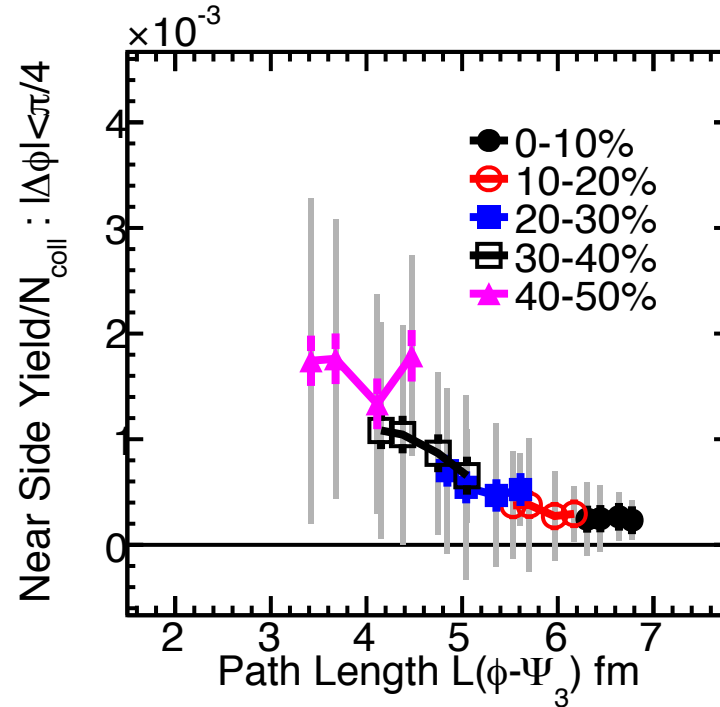
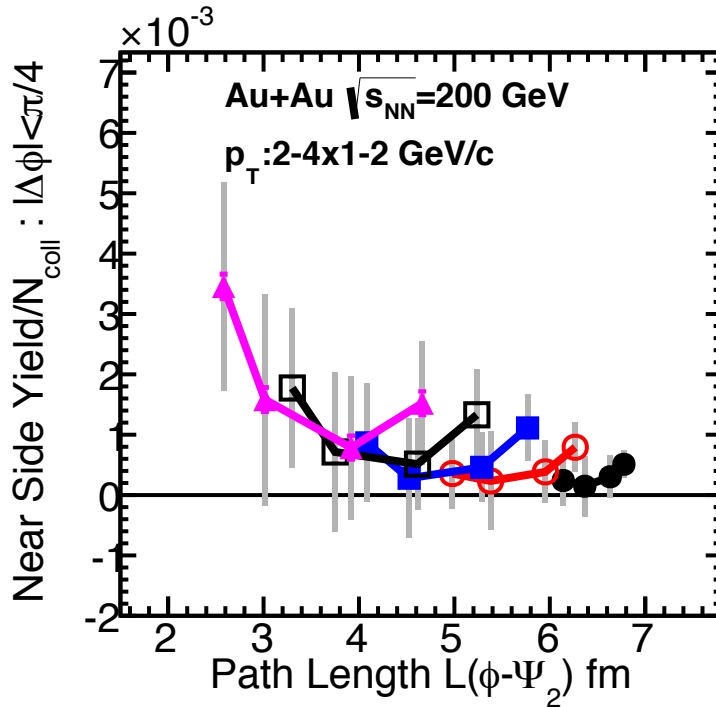
- Angle from EP converted to Path-Length by Glauber Model
- Non-monotonic behavior
- Yields does not scale on a universal curve

Path length dependence of PTY/N_{part}



- Approximately scaled on a curve
- Associate yields from QGP medium

Path length dependence of PTY/N_{coll}



- Also approximately scaled on a curve
- Associate yields from hard-scattering at initial collisions
- Coupling of jet with medium

Summary - I

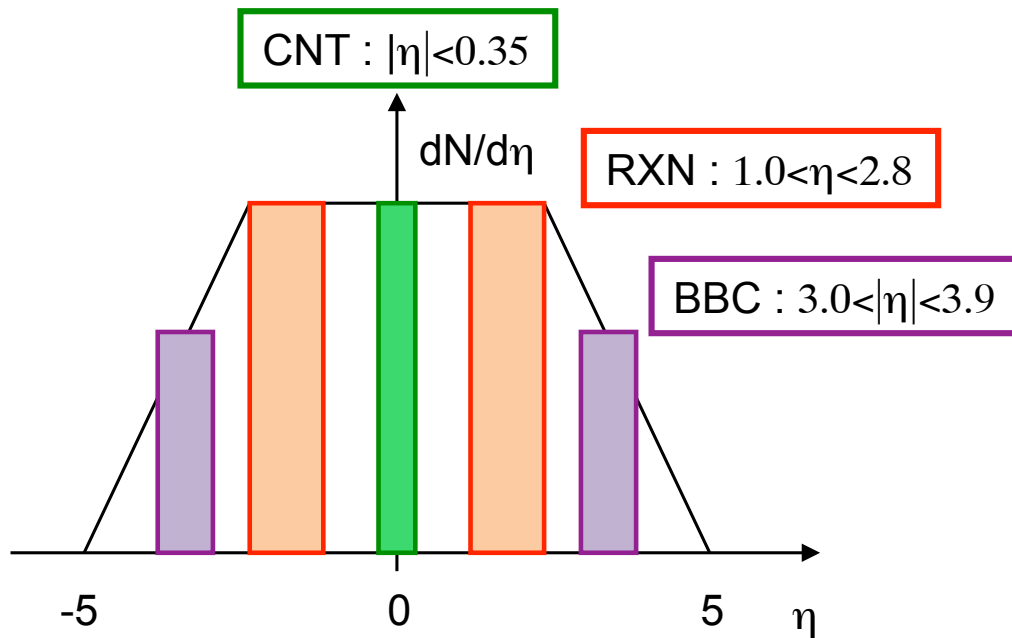
- Two-Particle correlations were measured with subtraction of backgrounds from $v_n(n=2,3,4)$
- New experimental data disfavored theoretical models
 - Cherenkov Gluon Radiation
 - Mach-Cone Shock Wave
 - Hot-Spot Models
- Jet-Deflection model is qualitatively consistent with data

Summary - II

- Event-Plane dependent correlations shows two competing effects
 - Increase / decrease of correlation yield with increase of path-length
- Anisotropy of correlation yield v_n^{PTY}
 - Additional Effect in addition to parton-energy loss
- Scale of Correlation Yield by N_{part} & N_{coll}
 - Jet-Medium Coupling
- Energy-Loss, Energy Re-distribution, and boost of jet by medium need to be considered in future theoretical models

BACK UP

Rapidity Selection in the Measurements



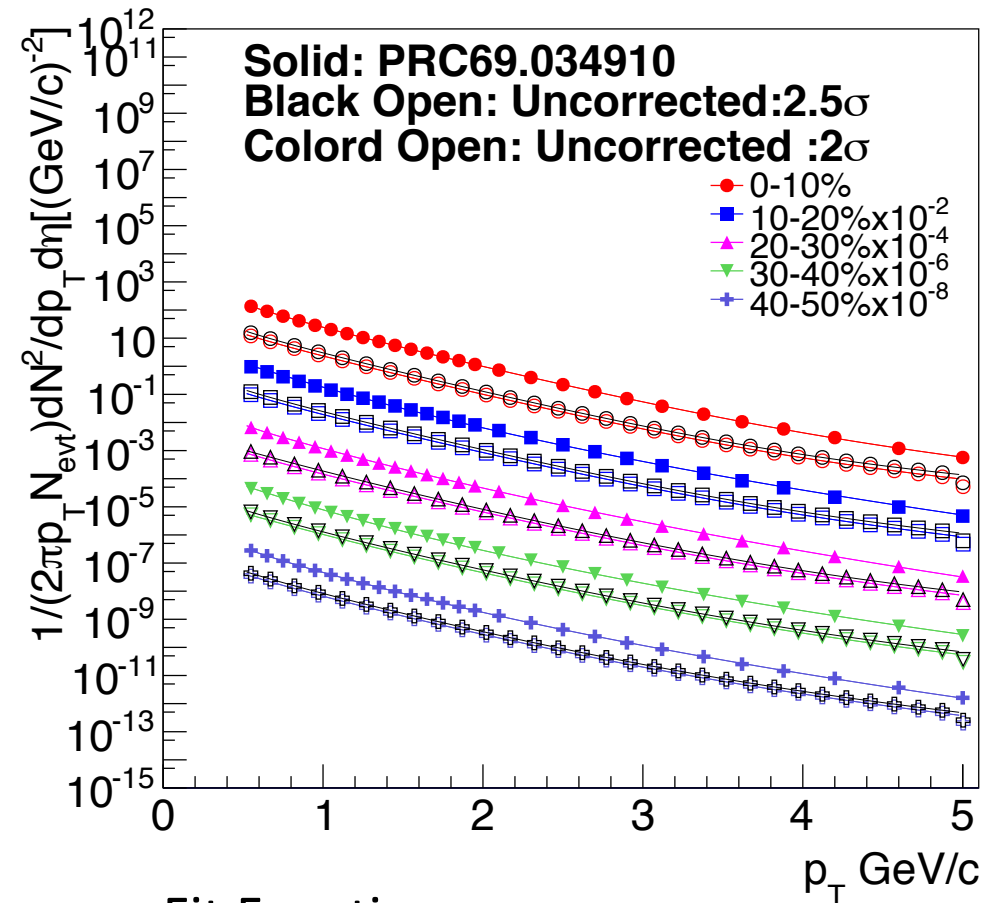
$$v_n = \langle \cos n(\phi - \Psi_n) \rangle$$

- Two-Particle Correlations at CNT
 - Without rapidity gap, where Jet contribution survives
- Flow Measurements
 - CNT Particle & Forward Event Plane at RXN, BBC
 - Jet contribution suppressed

Tracking Efficiency

- Efficiency correction by ratio of uncorrected invariant yield over corrected ones

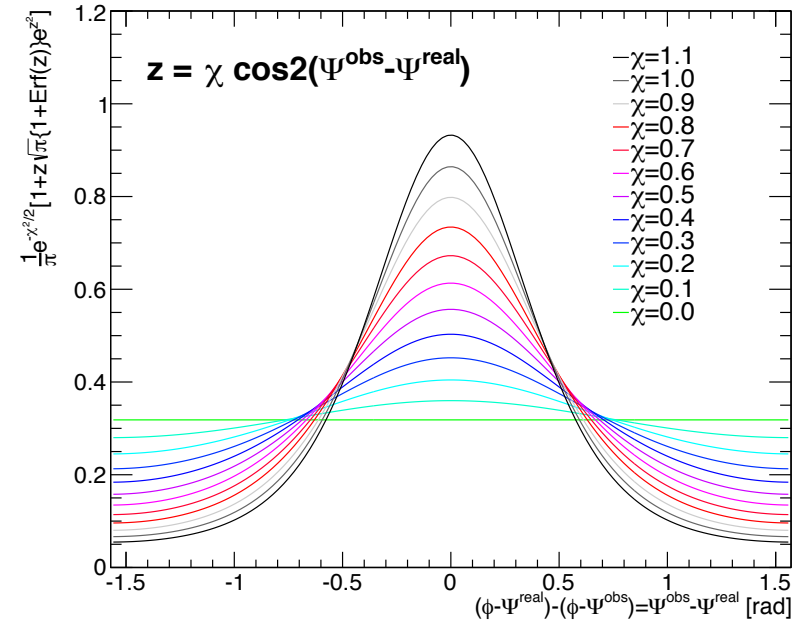
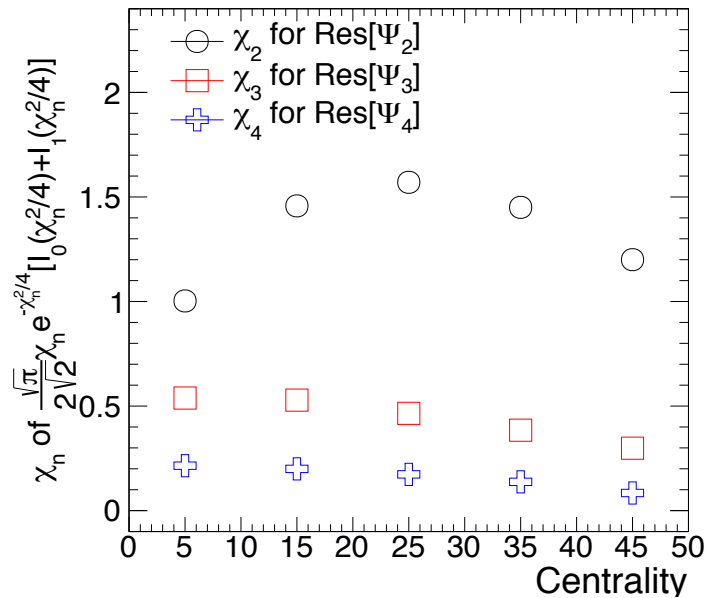
$$\varepsilon = \frac{\sigma^{uncor}}{\sigma^{cor}}$$



Fit Function

$$F(p_T) = p_0 * \left(\frac{p_1}{p_1 + p_T} \right)^{p_2}$$

EP Resolution in Monte Carlo



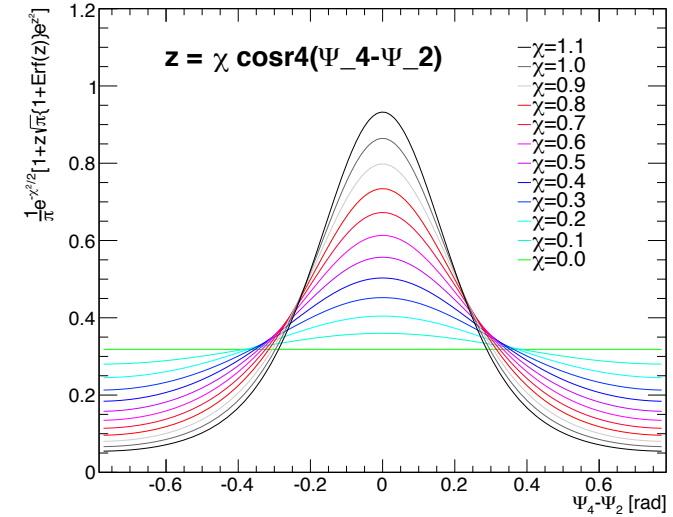
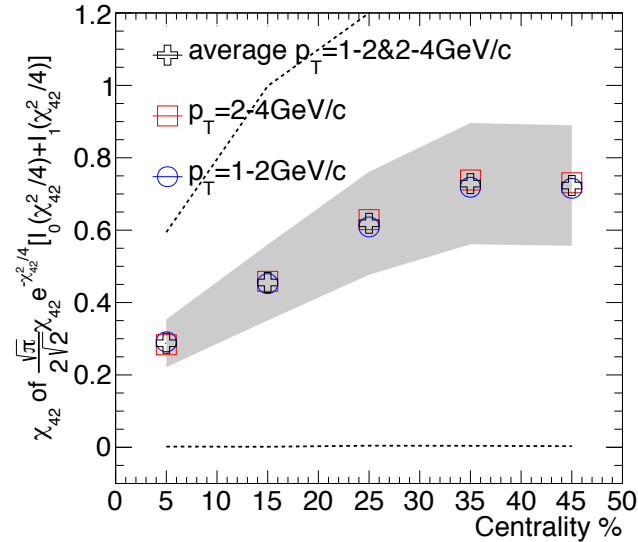
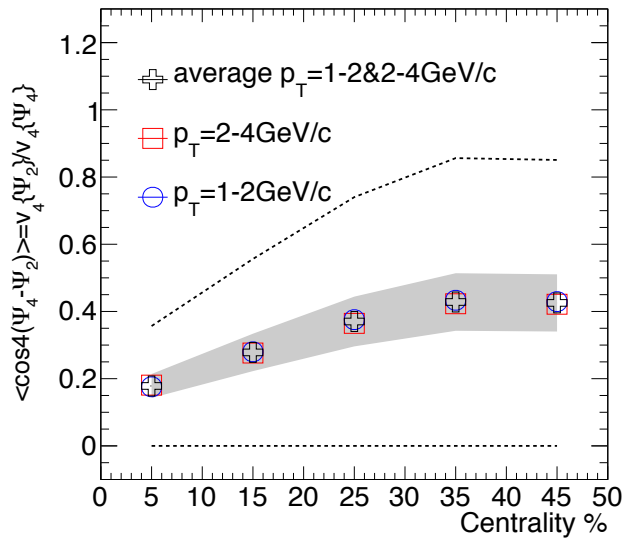
- Convert resolution to χ

$$\langle \cos [kn(\Psi_n^{obs} - \Psi_n^{real})] \rangle = \frac{\sqrt{\pi}}{2\sqrt{2}} \chi_n e^{-\chi_n^2/4} \left[I_{(k-1)/2} \left(\frac{\chi_n^2}{4} \right) + I_{(k+1)/2} \left(\frac{\chi_n^2}{4} \right) \right].$$

- Distribution between real and observed EP using χ

$$\frac{dN^{eve}}{d[kn(\Psi_n^{obs} - \Psi_n^{real})]} = \frac{1}{\pi} e^{-\chi_n^2/2} \left[1 + z\sqrt{\pi}[1 + \text{erf}(z)]e^{z^2} \right]$$

Ψ_2 - Ψ_4 correlation in Monte Carlo



- Ψ_2 - Ψ_4 correlation : $\langle \cos [4(\Psi_2 - \Psi_4)] \rangle = v_4 \{ \Psi_2 \} / v_4 \{ \Psi_4 \}$
- Obtain χ & reconstruct distribution

$$\langle \cos [4(\Psi_2 - \Psi_4)] \rangle = \frac{\sqrt{\pi}}{2\sqrt{2}} \chi_{42} e^{-\chi_{42}^2/4} \left[I_0 \left(\frac{\chi_{42}^2}{4} \right) + I_1 \left(\frac{\chi_{42}^2}{4} \right) \right]$$

$$\frac{dN^{eve}}{d[kn(\Psi_n^{obs} - \Psi_n^{real})]} = \frac{1}{\pi} e^{-\chi_n^2/2} \left[1 + z\sqrt{\pi}[1 + \text{erf}(z)]e^{z^2} \right]$$

Unfolding : Fitting Method

PRC.84.024904 (2011)

Ψ_2 dependent case

$$\lambda + Y^{cor}(\phi_s, \Delta\phi) = \frac{\lambda + b_0 [1 + 2v_2^Y / \sigma \cos 2(\phi_s + \Delta\phi) + 2v_4^Y / \sigma_{42} \cos 4(\phi_s + \Delta\phi)]}{\lambda + b_0 [1 + 2v_2^Y \cos 2(\phi_s + \Delta\phi) + 2v_4^Y \cos 4(\phi_s + \Delta\phi)]} (\lambda + Y(\phi_s, \Delta\phi))$$

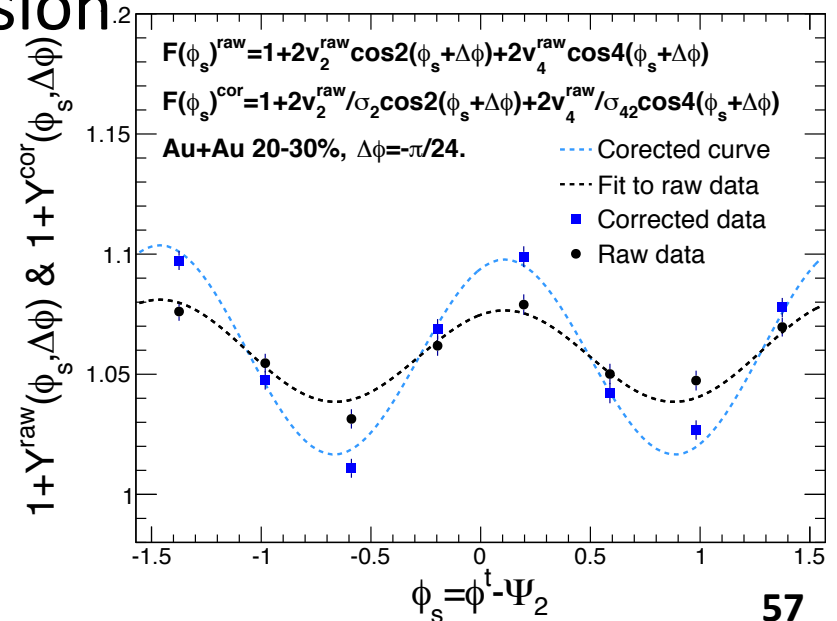
Fitting

Ψ_3 dependent case

$$\lambda + Y^{cor}(\phi_s, \Delta\phi) = \frac{\lambda + b_0 [1 + 2v_3^Y / \sigma_3 \cos 3(\phi_s + \Delta\phi)]}{\lambda + b_0 [1 + 2v_3^Y \cos 3(\phi_s + \Delta\phi)]} (\lambda + Y(\phi_s, \Delta\phi))$$

Fitting

- Offset $\lambda=1.0$ to avoid possible division by zero
- Assuming correlation yield has anisotropy w.r.t. EP
- Correction by EP resolution



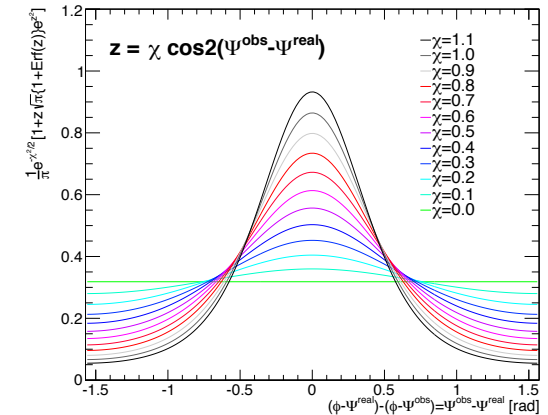
Unfolding : Iteration

Correlation + Offset

Smearing Effect

$$\mathbf{A}(k) = \begin{pmatrix} 1 + Y(0, k) \\ 1 + Y(1, k) \\ 1 + Y(2, k) \\ 1 + Y(3, k) \\ 1 + Y(4, k) \\ 1 + Y(5, k) \\ 1 + Y(6, k) \\ 1 + Y(7, k) \end{pmatrix},$$

$$\mathbf{S} = \begin{pmatrix} s_0 & s_1 & s_2 & s_3 & s_4 & s_3 & s_2 & s_1 \\ s_1 & s_0 & s_1 & s_2 & s_3 & s_4 & s_3 & s_2 \\ s_2 & s_1 & s_0 & s_1 & s_2 & s_3 & s_4 & s_3 \\ s_3 & s_2 & s_1 & s_0 & s_1 & s_2 & s_3 & s_4 \\ s_4 & s_3 & s_2 & s_1 & s_0 & s_1 & s_2 & s_3 \\ s_3 & s_4 & s_3 & s_2 & s_1 & s_0 & s_1 & s_2 \\ s_2 & s_3 & s_4 & s_3 & s_2 & s_1 & s_0 & s_1 \\ s_1 & s_2 & s_3 & s_4 & s_3 & s_2 & s_1 & s_0 \end{pmatrix},$$



- Smearing of measured correlations

$$\mathbf{B}(k) = \mathbf{S}\mathbf{A}(k)$$

- Correction factor

$$c_{ii} = A(i, k) / B(i, k)$$

$$c_{ij}(i \neq j) = 0$$

- Corrected yield

$$\mathbf{A}^{\text{cor}}(k) = \mathbf{C}(k)\mathbf{A}(k)$$

Unfolding : Iteration

Notation in iteration

$$\mathbf{A} \rightarrow \mathbf{A}^{(n)}$$

$$\mathbf{B} \rightarrow \mathbf{B}^{(n)}$$

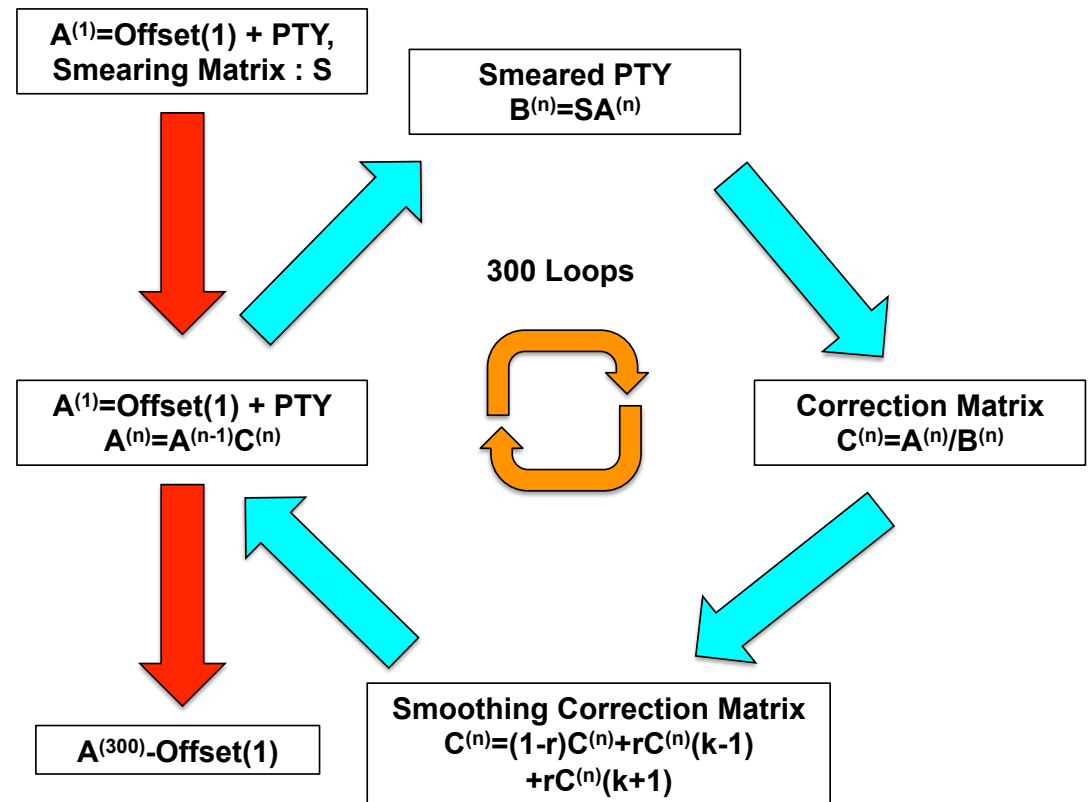
$$\mathbf{C} \rightarrow \mathbf{C}^{(n)}$$

$$\mathbf{A}^{\text{cor}} \rightarrow \mathbf{A}^{(n+1)}$$

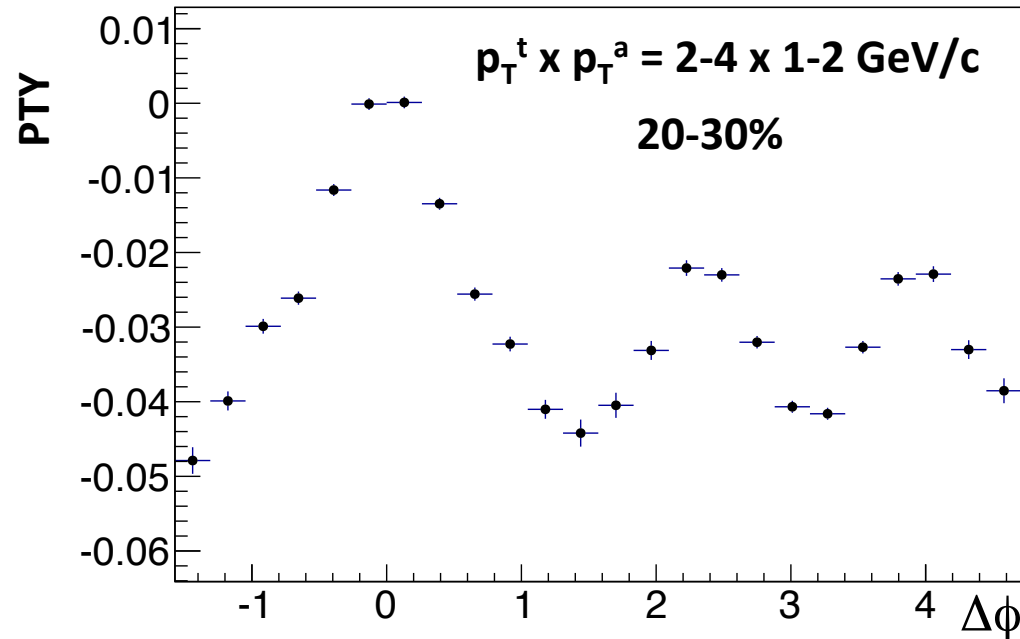
Smoothing

$$c_{ii}^{(n)}(k) = (1 - r)c_{ii}^{(n)}(k) + (r/2)c_{ii}^{(n)}(k - 1) + (r/2)c_{ii}^{(n)}(k + 1)$$

- Iteration until conversion



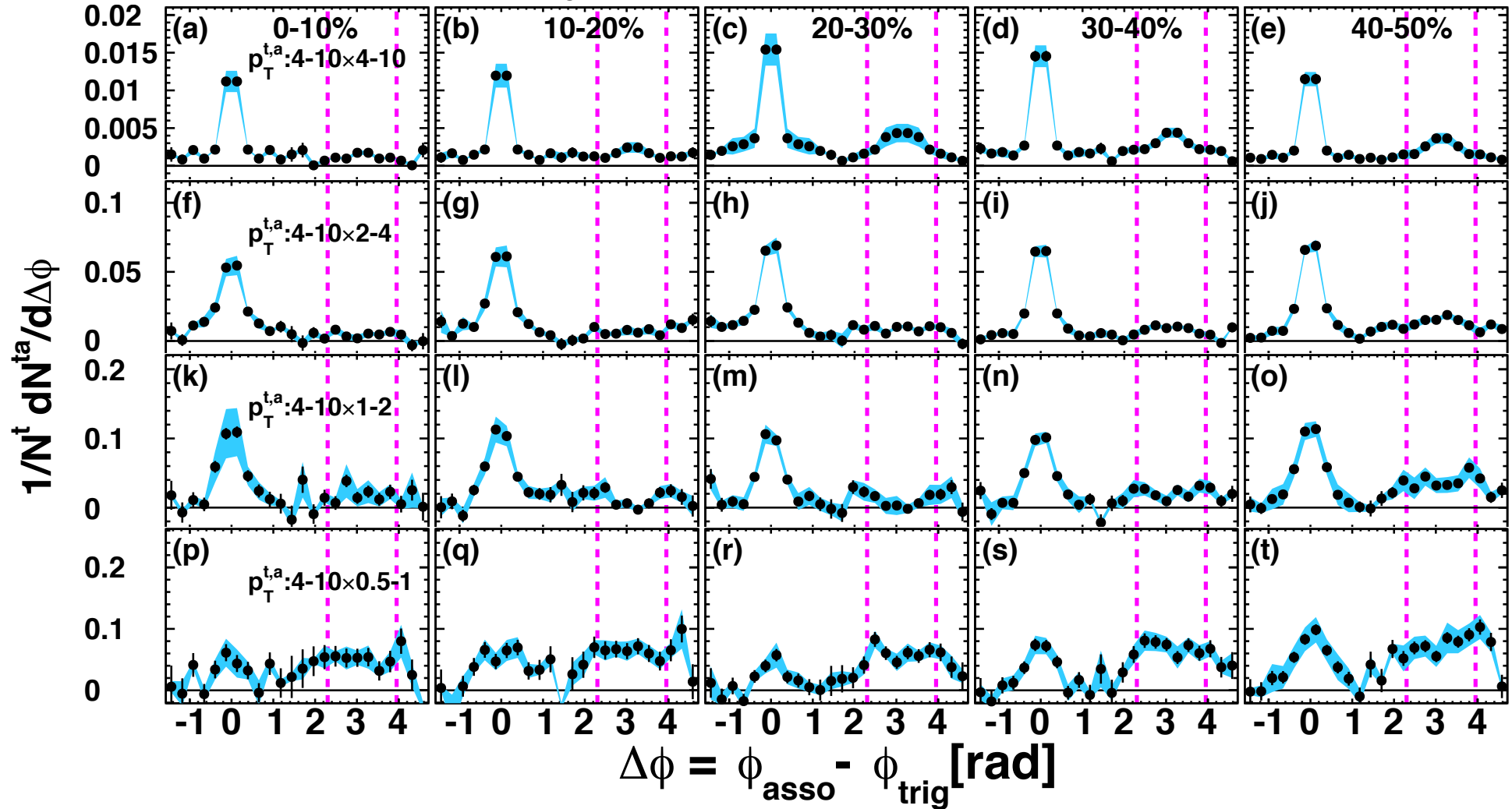
Zero Yield at Near-Side



- Correlation and Pure Flow is fitted at $\Delta\phi=0$
- Double-hump is not so sensitive to flow subtraction

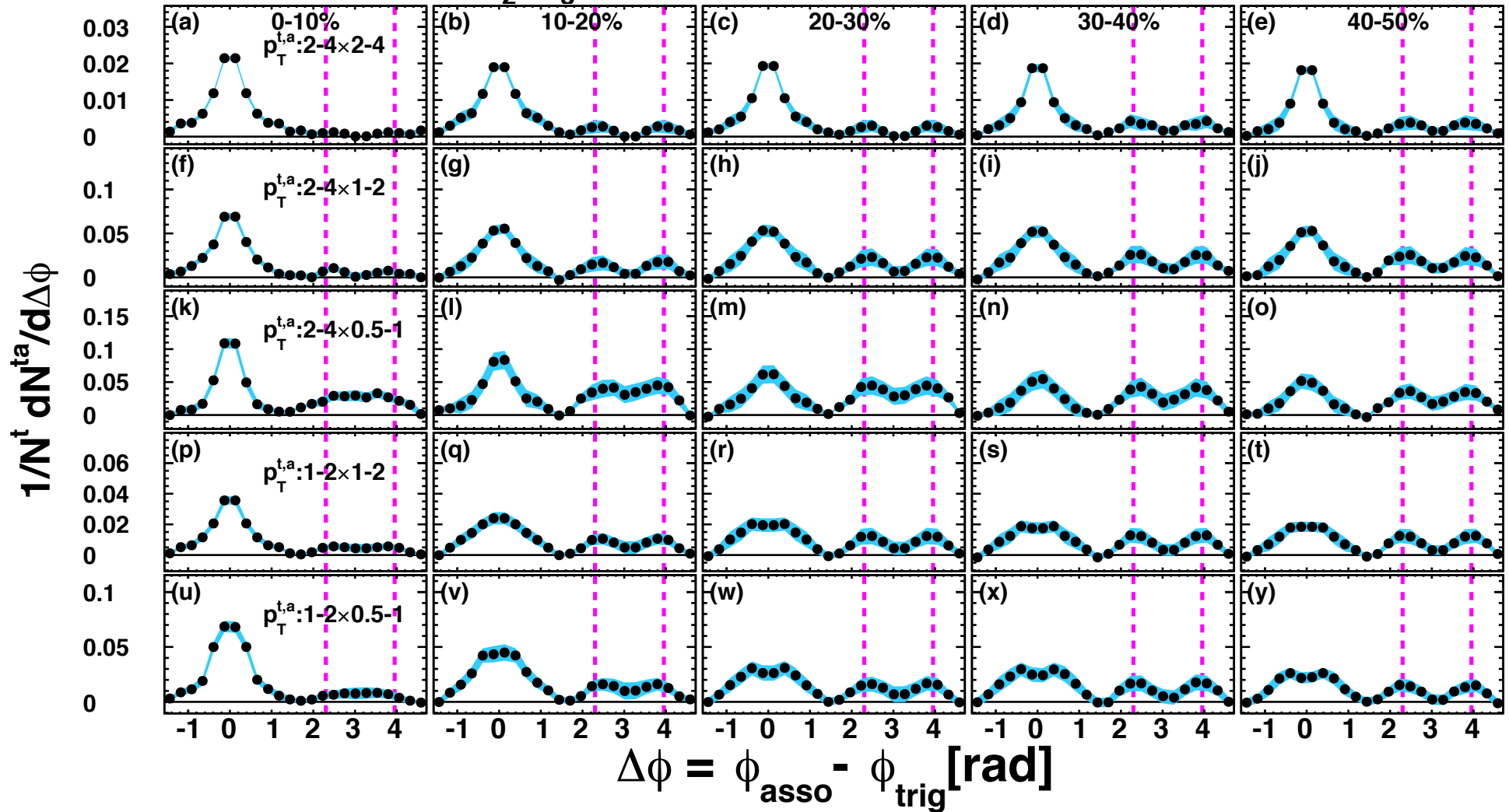
High p_T correlations

Au+Au 200GeV, v_2 , v_3 & $v_4(\Psi_4)$ subtracted



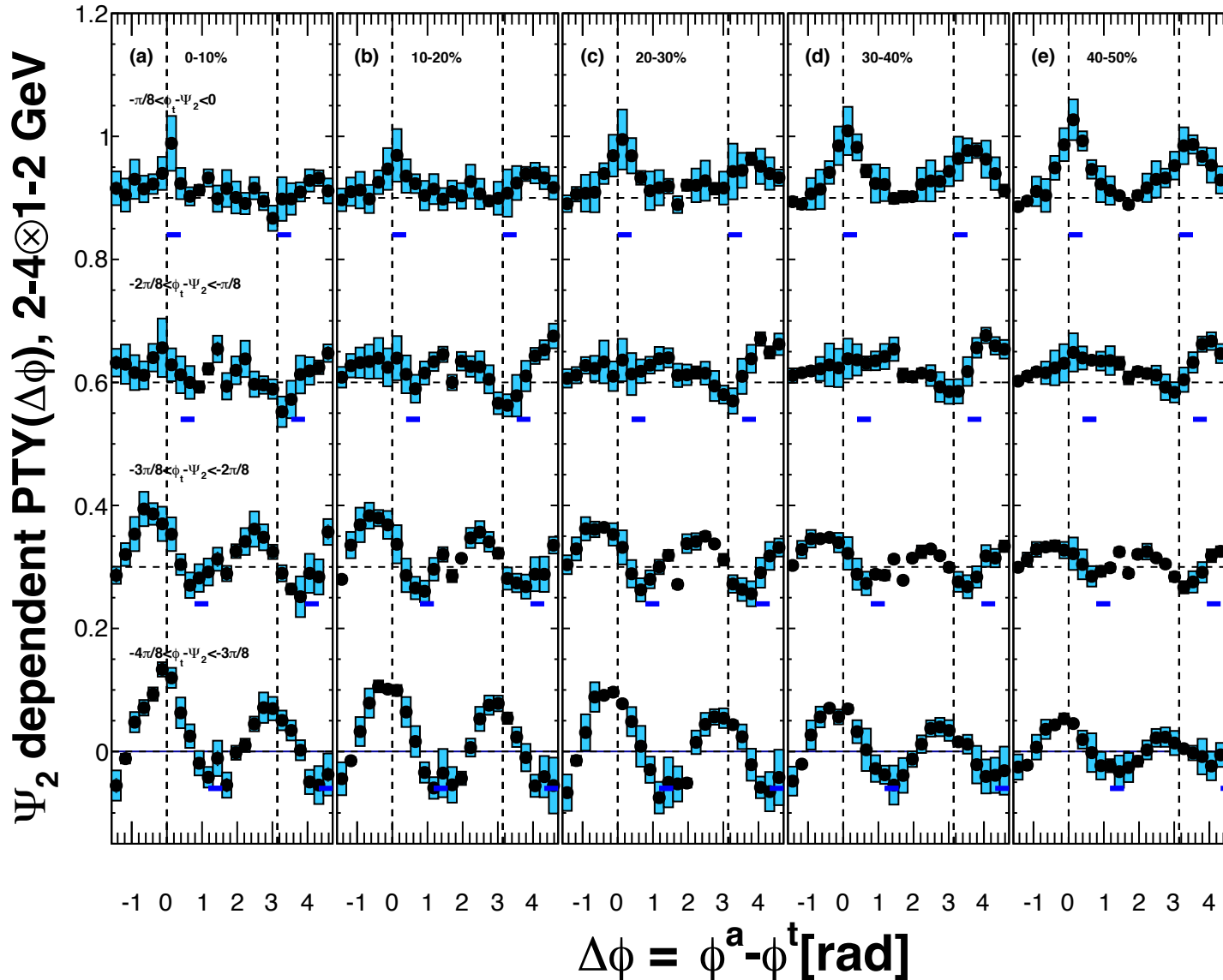
Intermediate p_T correlations

Au+Au 200GeV, v_2, v_3 & $v_4(\Psi_4)$ subtracted

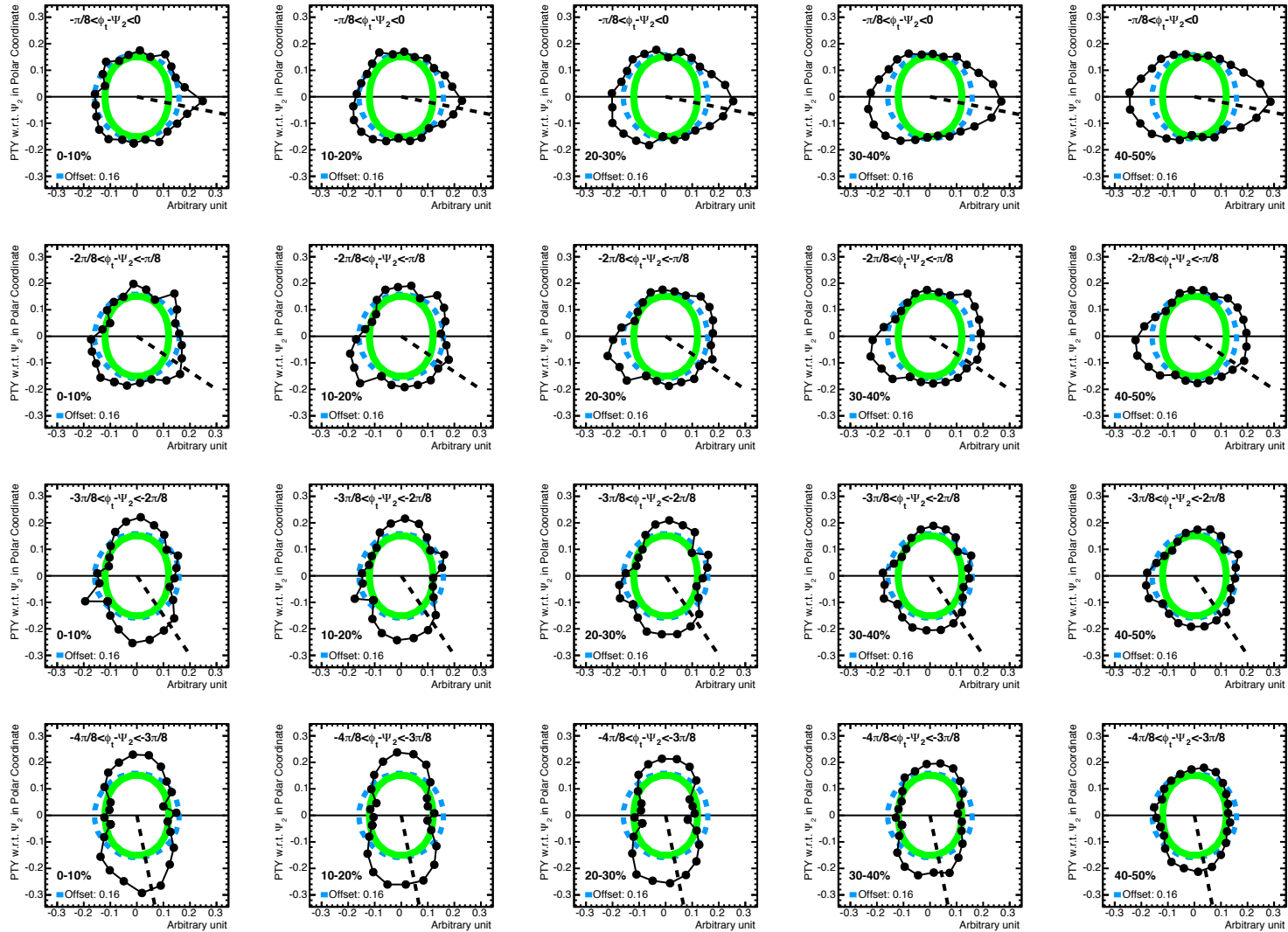


Ψ_2 dependence of PTY

Au+Au $\sqrt{s_{NN}}=200\text{GeV}$, Pure Flow: v_n ($n=2,3,4$) + $\langle \cos 4(\Psi_2 - \Psi_4) \rangle$ by ZYAM

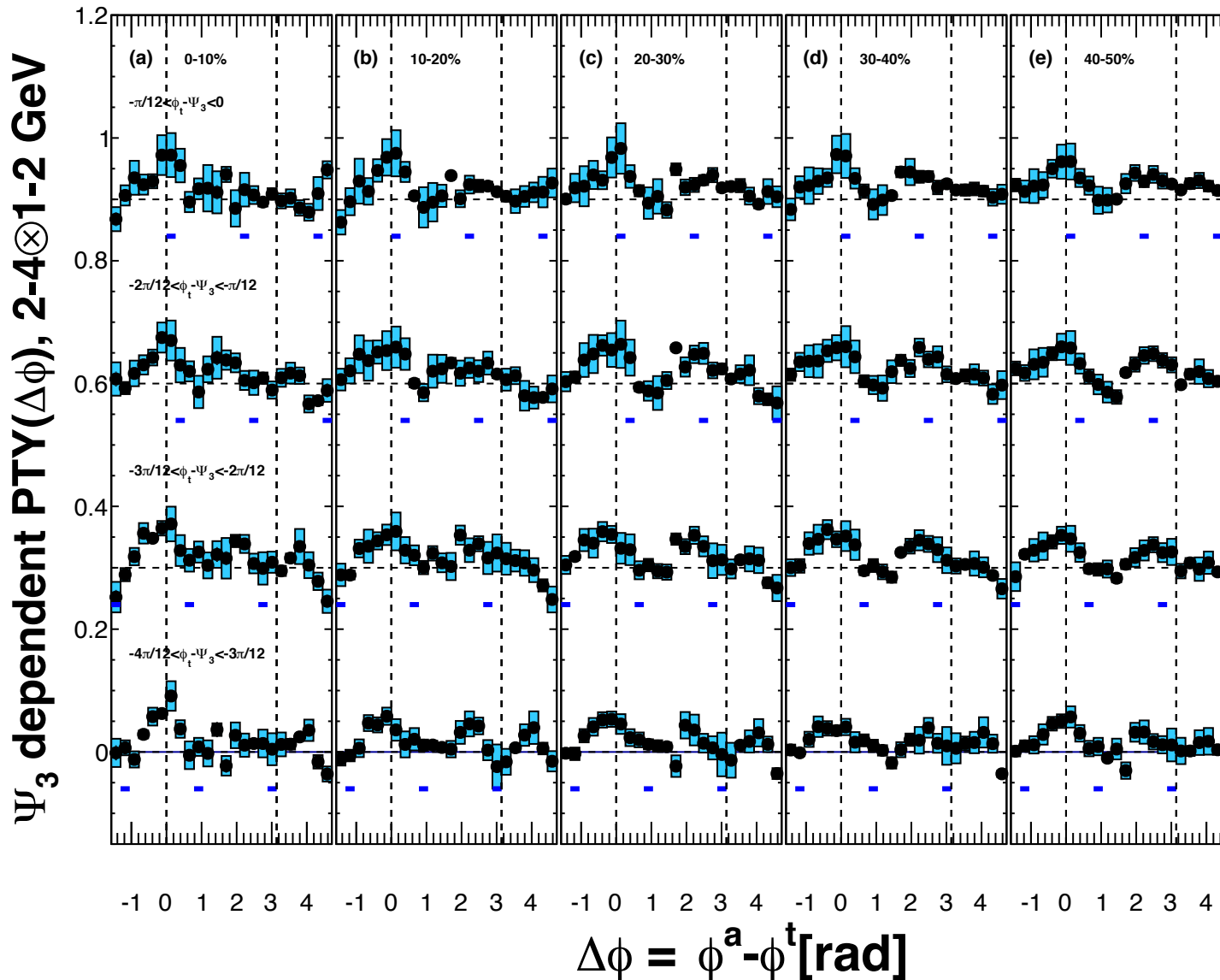


Ψ_2 dependence of PTY (Polar)

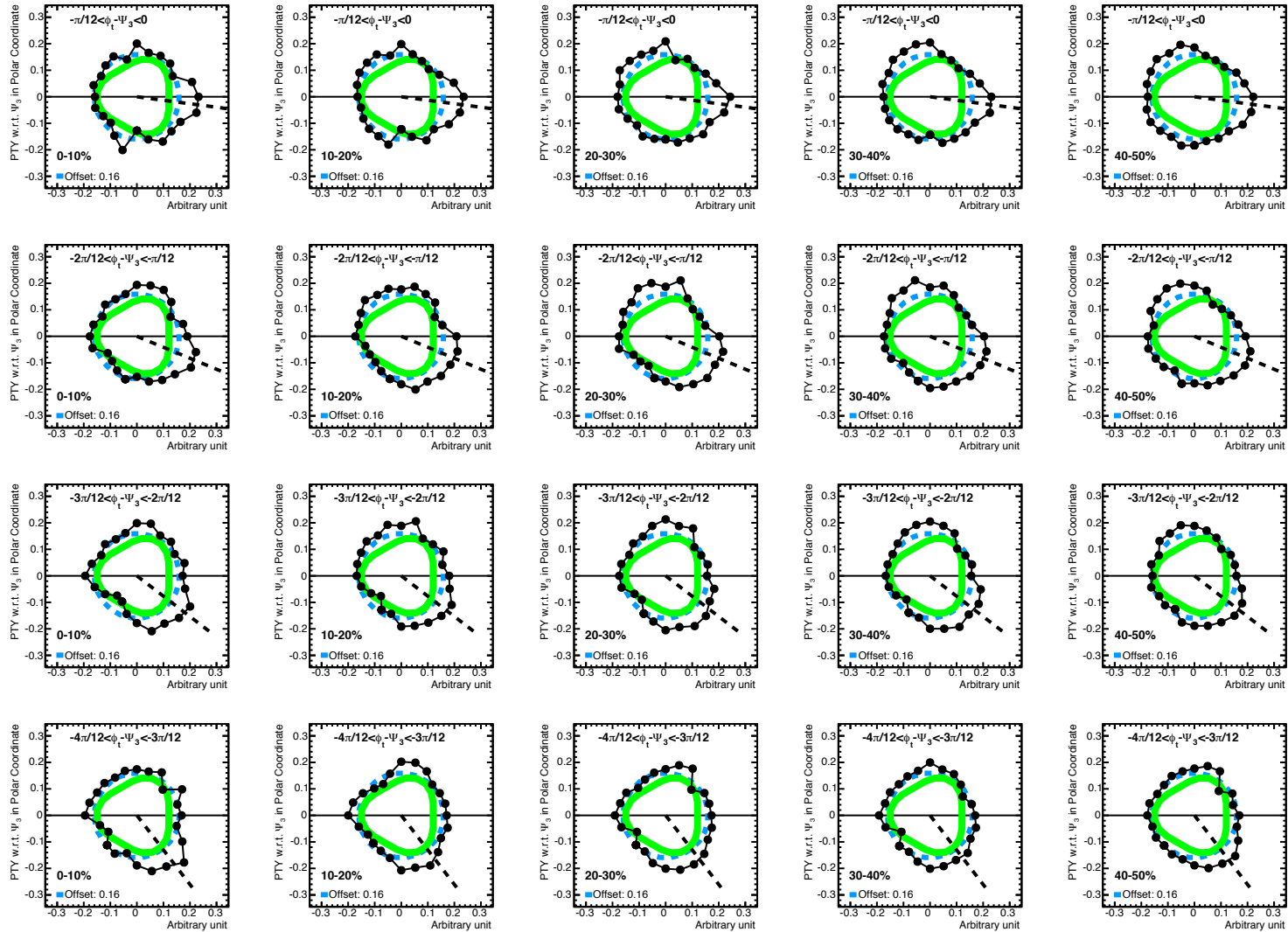


Ψ_3 dependence of PTY

Au+Au $\sqrt{s_{NN}}=200\text{GeV}$, Pure Flow: $v_n(n=2,3,4) + \langle \cos 4(\Psi_2 - \Psi_4) \rangle$ by ZYAM



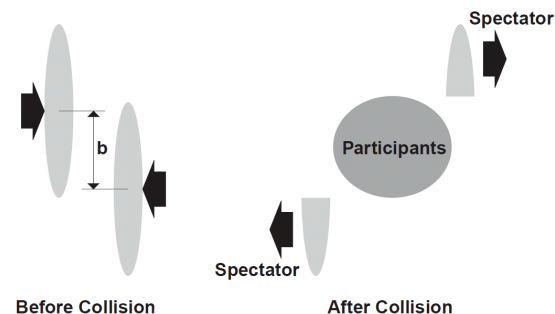
Ψ_3 dependence of PTY (Polar)



Glauber Model

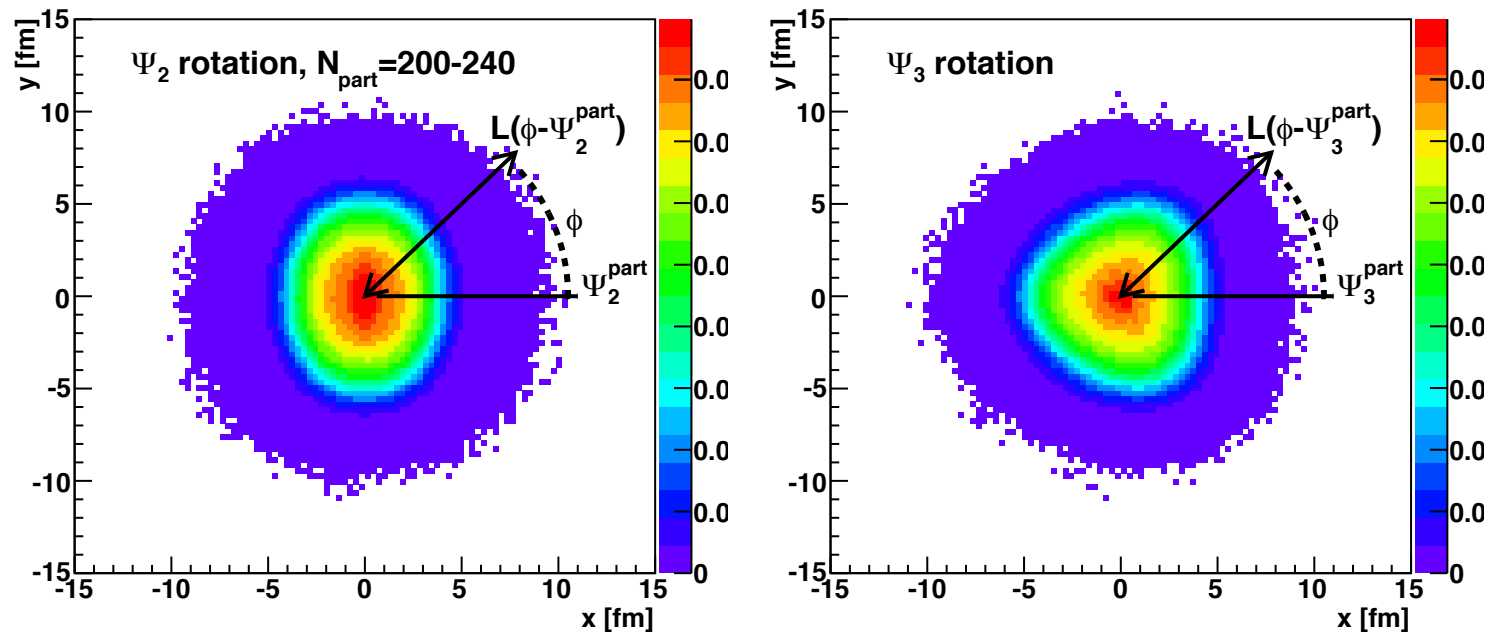
- Woods-Saxon Distribution
 - Nucleon density distribution
- Collision range
- Number of nucleons participated in a collision
- Number of binary nucleon collisions in a collision

$$\rho(r) = \frac{1}{1 + \exp \frac{r-r_a}{a}}$$
$$d < \sqrt{\frac{\sigma_{nn}}{\pi}}, \sigma_{nn} = 42mb$$
$$N_{part}$$
$$N_{coll}$$

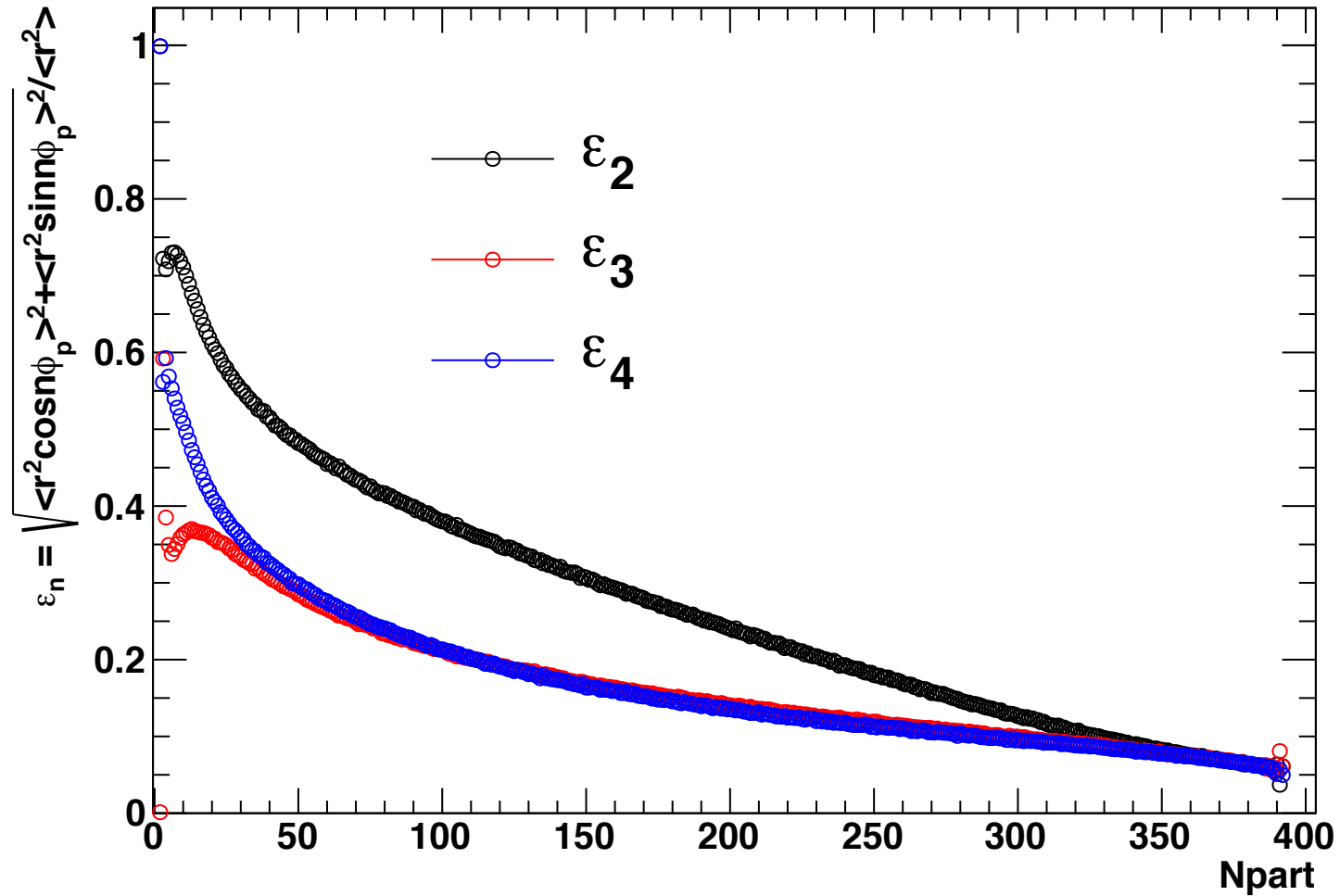


Path-Length based on Glauber MC

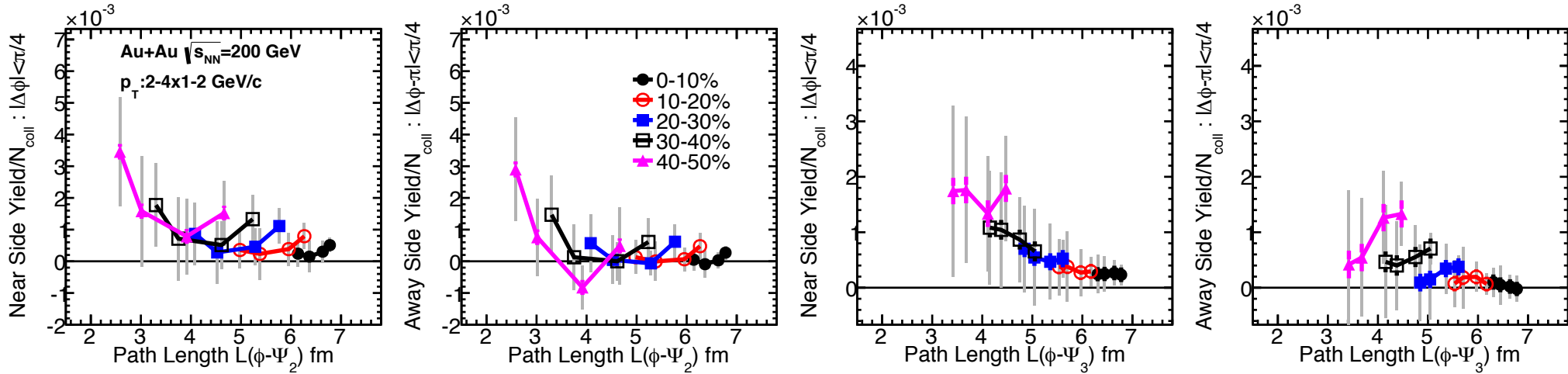
- Path-Length : distance from center to surface as a function of angle from the event-planes



Eccentricity by Glauber Model

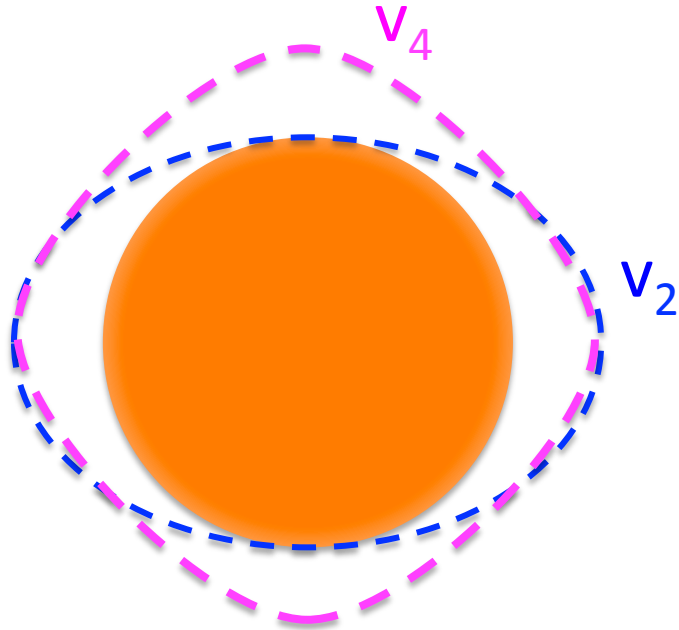


Path length dependence of PTY



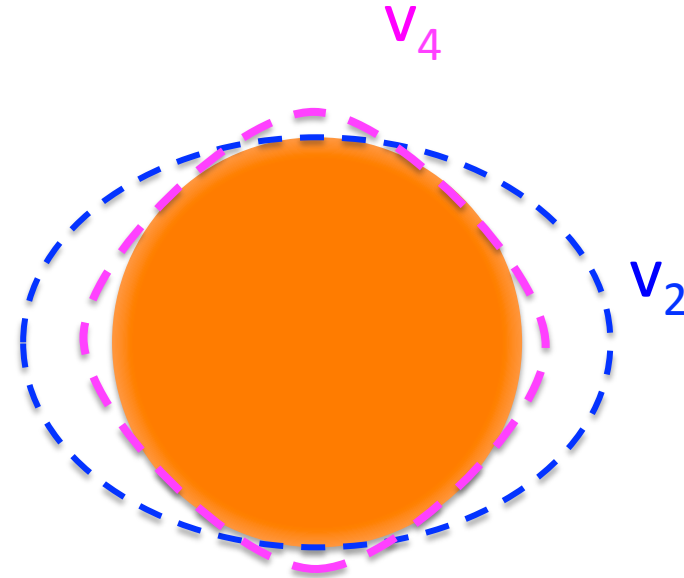
- Near & Away-Side of both Ψ_2 , Ψ_3 dependent correlations do not form a universal curve

Boost by v_4



Central Collisions

$$v_2 = v_4$$



Peripheral Collisions

$$v_2 > v_4$$

- Boost by v_4 to out-of-plane direction in central collisions
- Boost by v_2 to in-plane direction in peripheral collisions