Importance of separated efficiencies between positively and negatively charged particles for cumulant calculations

Toshihiro Nonaka, University of Tsukuba
ATHIC2016 @New Delhi, India
Feb.17, 2016

筑波大学
University of Tsukuba

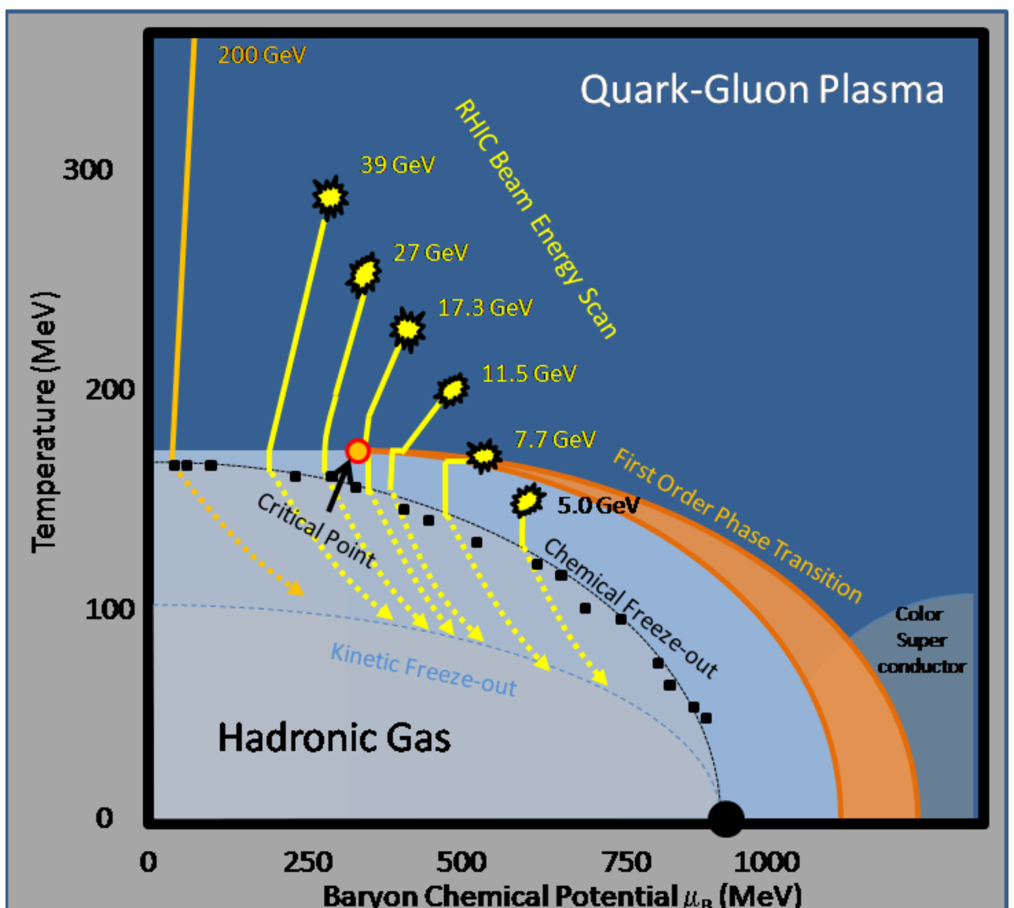
Outline

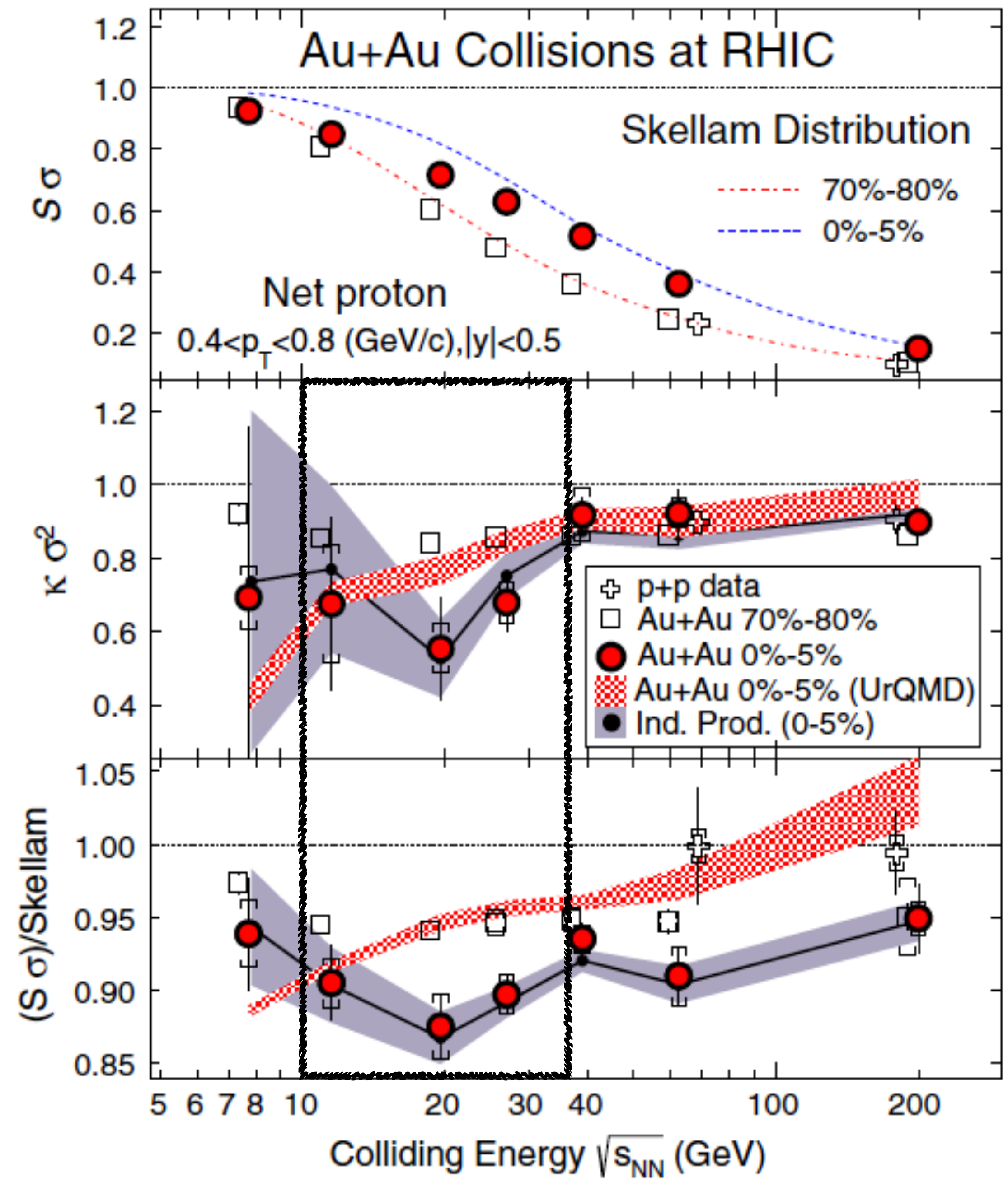
- Introduction
- Cumulants of conserved quantities
- Efficiency correction
- MC toy model
- Analytical calculation
- Beam energy dependence
- Summary

Search for the critical point

- ✓ Beam energy scan phase I
- ✓ $\sqrt{s_{NN}}$ = 7.7, 11.5, 19.6, 27, 39, 62.4, 200GeV
- ✓ Observables are measured as a function of beam energy
- ✓ Fluctuation of conserved quantities

PRL112 032302 (2014)
→ net-proton





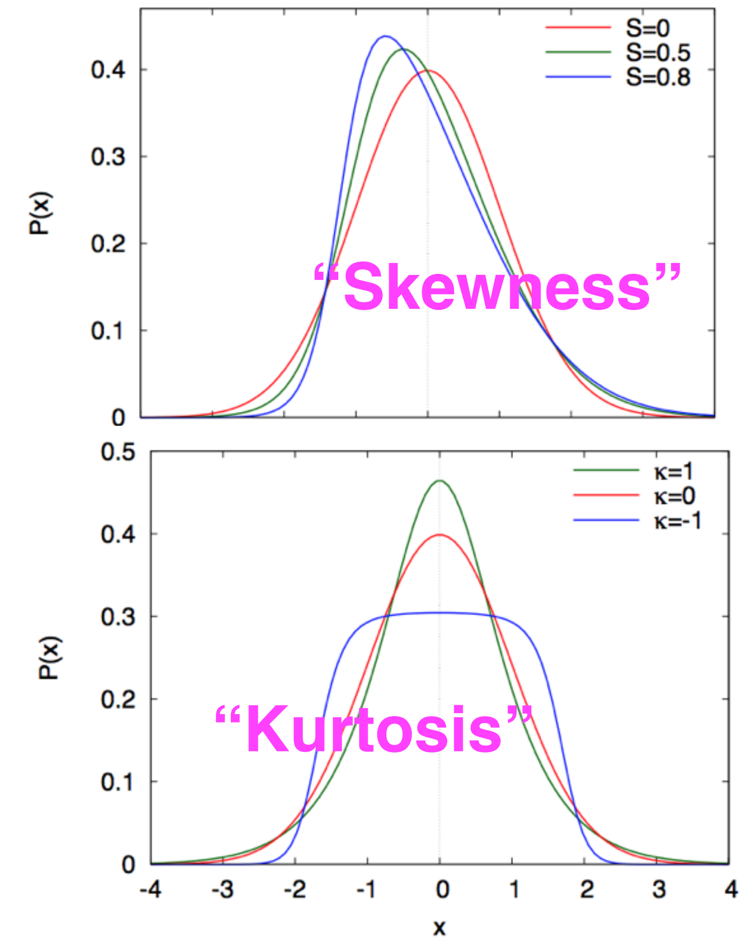
Cumulants of conserved quantities

✓ Extensive variable
✓ Proportional to the power of correlation length
✓ Directly connected to susceptibilities

$$\langle \delta N \rangle = N - \langle N \rangle$$
$$C_1 = M = \langle N \rangle$$
$$C_2 = \sigma^2 = \langle \delta N^2 \rangle$$
$$C_3 = S\sigma^3 = \langle \delta N^3 \rangle \sim \xi^{4.5}$$
$$C_4 = \kappa\sigma^4 = \langle \delta N^4 \rangle - 3\langle \delta N^2 \rangle^2 \sim \xi^7$$

$$S\sigma = \frac{C_3}{C_2} = \frac{\chi_3}{\chi_2}$$
$$\kappa\sigma^2 = \frac{C_4}{C_2} = \frac{\chi_4}{\chi_2}$$

arXiv:1512.05038



Poisson baseline

Poisson - Poisson = "Skellam" distribution

$$C_{odd} = \mu_1 - \mu_2$$
$$C_{even} = \mu_1 + \mu_2$$

$$S\sigma = \frac{C_3}{C_2} = \frac{\mu_1 - \mu_2}{\mu_1 + \mu_2}$$
$$\kappa\sigma^2 = \frac{C_4}{C_2} = 1$$

Efficiency correction

- ✓ Effect of tracking efficiency must be corrected for measured cumulants.
- ✓ Averaged efficiency $(\varepsilon_{+}+\varepsilon_{-})/2$ between positively and negatively charged particles were used in published net-charge and net-proton results.

PRC 86(2012) 044904

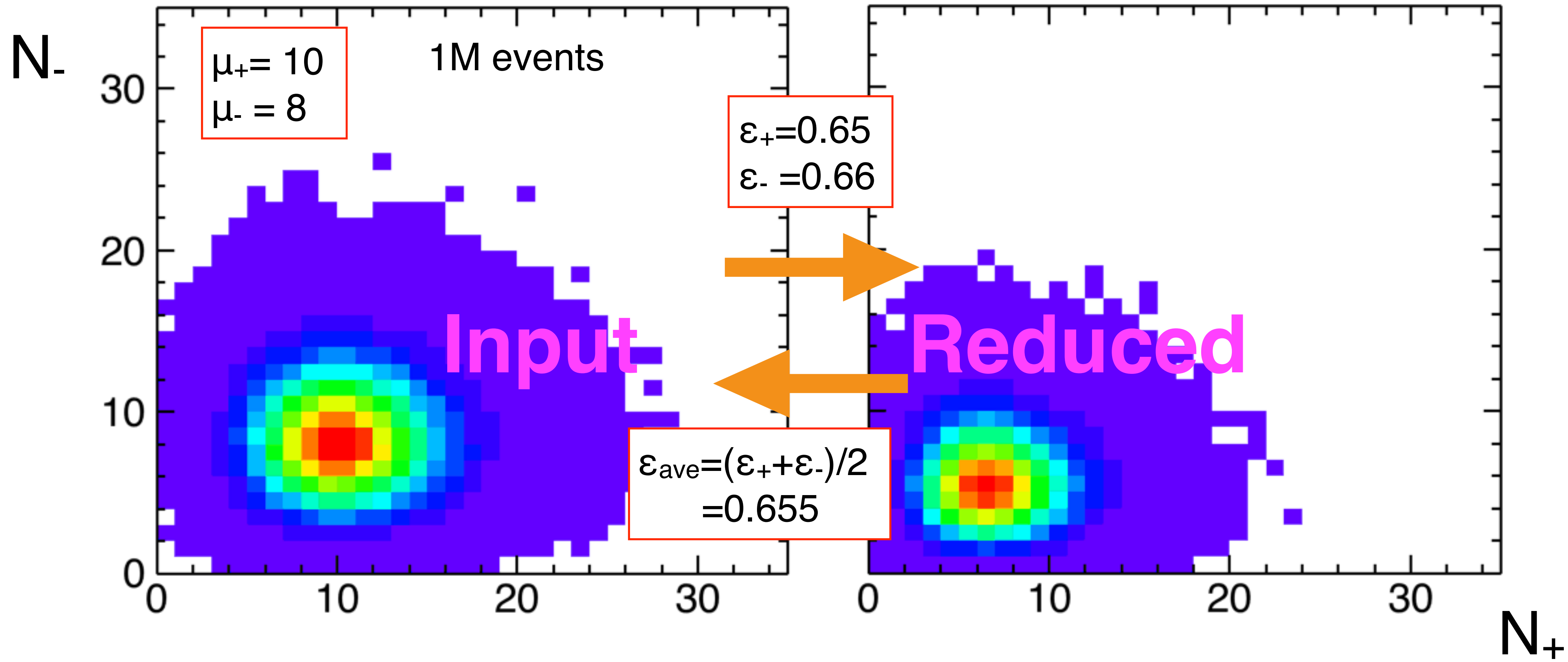
p : tracking efficiency
f : factorial moment
c : measured cumulant
K: corrected cumulant

$$
\begin{aligned}
pK_1 &= c_1, \\
p^2K_2 &= c_2 - n(1-p), \\
p^3K_3 &= c_3 - c_1(1-p^2) - 3(1-p)(f_{20} - f_{02} - nc_1), \\
p^4K_4 &= c_4 - np^2(1-p) - 3n^2(1-p)^2 - 6p(1-p)(f_{20}+f_{02}) + 12c_1(1-p)(f_{20}-f_{02}) \\
&\quad -(1-p^2)(c_2 - 3c_1^2) - 6n(1-p)(c_1^2 - c_2) \\
&\quad -6(1-p)(f_{03} - f_{12} + f_{02} + f_{20} - f_{21} + f_{30}).
\end{aligned}
$$

- ✓ Actually there is finite difference of tracking efficiency between positively and negatively charged particles.
- ✓ How will the published results be changed if the separated efficiencies are used？
- ✓ Difference between two correction methods are studied by
  - ➡ MC toy model assuming net-proton distribution.
  - ➡ Analytical calculation from the correction formula.

Example

1. Generate two Poisson distributions.
2. Random sampling according to binomial efficiency.
3. Apply correction using averaged or separated efficiencies.




```
Poisson baseline : 2
Uncorrected mean : 1.22045
Corrected (average) : 1.86328
    uncorr/corr = 0.655
Corrected (separate) : 2.00064
    uncorr/corr = 0.610
```

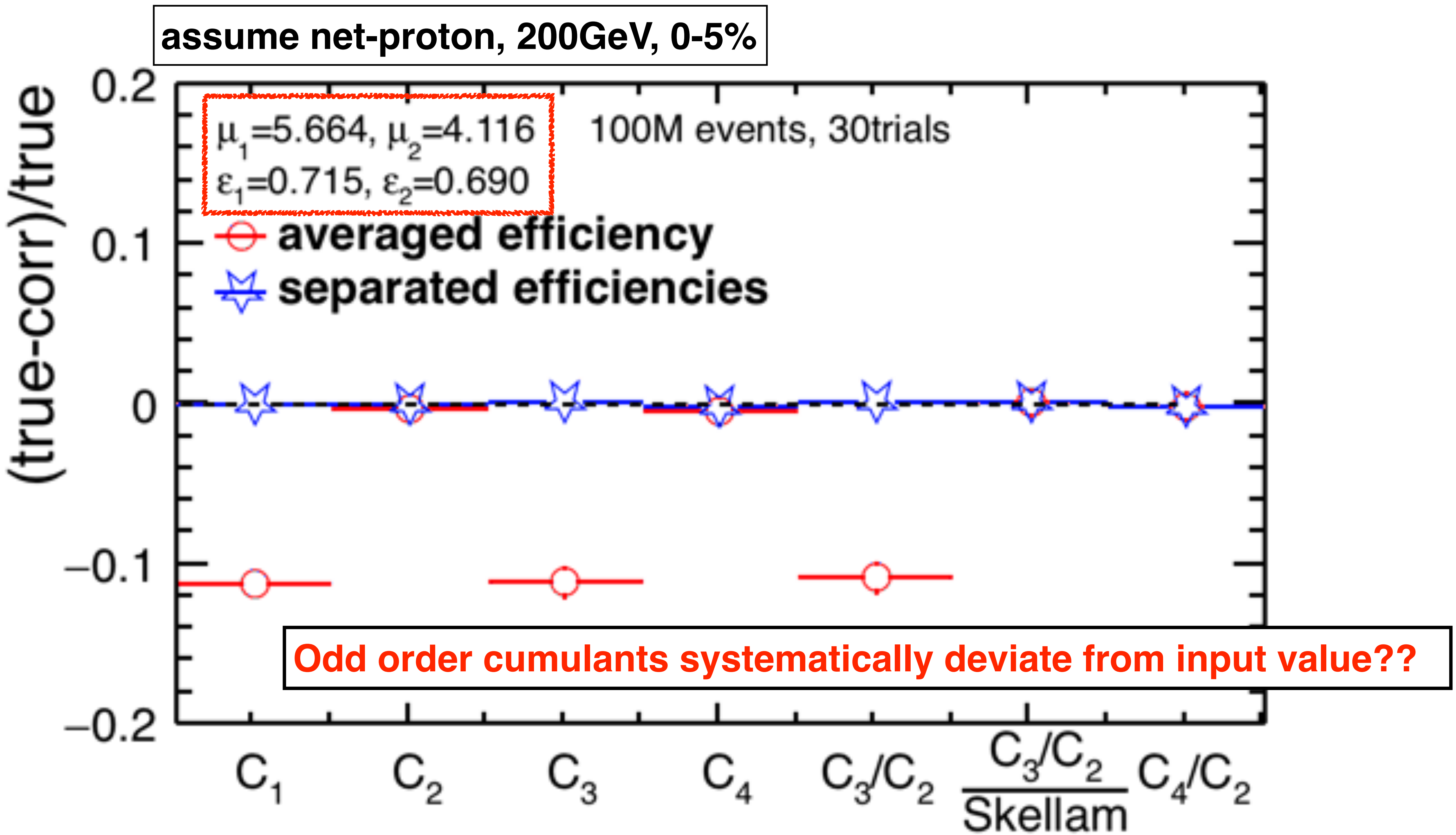
efficiency correction doesn't work

efficiency correction works well

✓ From next page, toy model assuming net-proton will be shown. Parameters were taken from published results.

Order dependence

- Relative difference from input value ***(true-corr)/true*** as a function of each order of cumulants.
- Assume net-proton distribution in the most central collisions at 200GeV.



Analytical calculation, C_1

N± : # of produced particles
M± : # of observed particles
ε± : efficiency for charged or anti-charged particles
ε : averaged efficiency
Δε : efficiency difference

$$\varepsilon_+ = \varepsilon + \Delta\varepsilon,$$
$$\varepsilon_- = \varepsilon - \Delta\varepsilon,$$
$$\varepsilon = \frac{\varepsilon_+ + \varepsilon_-}{2},$$
$$\Delta\varepsilon = \frac{\varepsilon_+ - \varepsilon_-}{2}.$$

$$\begin{aligned} K_{1,\mathrm{sep}} &= \langle N_+\rangle - \langle N_-\rangle = \frac{\langle M_+\rangle}{\varepsilon_+} - \frac{\langle M_-\rangle}{\varepsilon_-} \\ &= \frac{\langle M_+\rangle}{\varepsilon + \Delta\varepsilon} - \frac{\langle M_-\rangle}{\varepsilon - \Delta\varepsilon} \end{aligned}$$

Taylor expansion around Δε=0

$$\begin{aligned} K_{1,\mathrm{sep}}(\Delta\varepsilon) &\simeq K_{1,\mathrm{sep}}(0) + \left.\frac{\partial K_{1,\mathrm{sep}}}{\partial \Delta\varepsilon}\right|_{\Delta\varepsilon=0} \Delta\varepsilon + \mathcal{O}(\Delta\varepsilon^2) \\ &= \left(\frac{\langle M_+\rangle}{\varepsilon} - \frac{\langle M_-\rangle}{\varepsilon}\right) - \left(\frac{\langle M_+\rangle}{\varepsilon^2} + \frac{\langle M_-\rangle}{\varepsilon^2}\right)\Delta\varepsilon + \mathcal{O}(\Delta\varepsilon^2). \end{aligned}$$

Δε=0

$$\begin{aligned} K_{1,\mathrm{ave}} &= \langle N_+\rangle - \langle N_-\rangle \\ &= \frac{\langle M_+\rangle - \langle M_-\rangle}{\varepsilon}. \end{aligned}$$

$$\begin{aligned} \Delta K_1 &= \left|K_{1,\mathrm{ave}} - K_{1,\mathrm{sep}}\right| \\ &= \left|\left(\frac{\langle M_+\rangle}{\varepsilon} - \frac{\langle M_-\rangle}{\varepsilon}\right) - \left(\frac{\langle M_+\rangle}{\varepsilon} - \frac{\langle M_-\rangle}{\varepsilon}\right) + \left(\frac{\langle M_+\rangle}{\varepsilon^2} + \frac{\langle M_-\rangle}{\varepsilon^2}\right)\Delta\varepsilon\right| \\ &= \frac{\Delta\varepsilon}{\varepsilon^2}\left(\langle M_+\rangle + \langle M_-\rangle\right). \end{aligned}$$

Difference ΔK_1 is proportional to the sum of multiplicity.

Analytical calculation, C_2

- **Difference of C_2 can be calculated by similar approach**

$$
\begin{aligned}
K_{2,\mathrm{sep}} &= \left(\frac{\langle M_+^2\rangle}{(\varepsilon+\Delta\varepsilon)^2}+\frac{\langle M_-^2\rangle}{(\epsilon-\Delta\varepsilon)^2}\right)-\left(\frac{\langle M_+\rangle}{(\varepsilon+\Delta\varepsilon)^2}+\frac{\langle M_-\rangle}{(\epsilon-\Delta\varepsilon)^2}\right)-\left(\frac{\langle M_+\rangle^2}{(\varepsilon+\Delta\varepsilon)^2}+\frac{\langle M_-\rangle^2}{(\epsilon-\Delta\varepsilon)^2}\right) \\
&+\left(\frac{\langle M_+\rangle}{\varepsilon+\Delta\varepsilon}+\frac{\langle M_-\rangle}{\epsilon-\Delta\varepsilon}\right)-2\frac{\langle M_+M_-\rangle}{(\varepsilon+\Delta\varepsilon)(\epsilon-\Delta\varepsilon)}+2\frac{\langle M_+\rangle\langle M_-\rangle}{(\varepsilon+\Delta\varepsilon)(\varepsilon-\Delta\varepsilon)}.
\end{aligned}
$$

$$
\begin{aligned}
K_{2,\mathrm{ave}} &= \left(\frac{\langle M_+^2\rangle}{\varepsilon^2}+\frac{\langle M_-^2\rangle}{\epsilon^2}\right)-\left(\frac{\langle M_+\rangle}{\varepsilon^2}+\frac{\langle M_-\rangle}{\epsilon^2}\right)-\left(\frac{\langle M_+\rangle^2}{\varepsilon^2}+\frac{\langle M_-\rangle^2}{\epsilon^2}\right)+\left(\frac{\langle M_+\rangle}{\varepsilon}+\frac{\langle M_-\rangle}{\epsilon}\right) \\
&-2\frac{\langle M_+M_-\rangle}{\varepsilon^2}+2\frac{\langle M_+\rangle\langle M_-\rangle}{\varepsilon^2}.
\end{aligned}
$$

$$
\begin{aligned}
K_{2,\mathrm{sep}}(\Delta\varepsilon) &\simeq K_{2,\mathrm{sep}}(0)+\left.\frac{\partial K_{2,\mathrm{sep}}}{\partial\Delta\varepsilon}\right|_{\Delta\varepsilon=0}\Delta\varepsilon+\mathcal{O}(\Delta\varepsilon^2). \\
&\simeq \frac{\langle M_+^2\rangle+\langle M_-^2\rangle}{\varepsilon^2}-2\left(\frac{\langle M_+^2\rangle-\langle M_-^2\rangle}{\epsilon^3}\right)\Delta\epsilon-\frac{\langle M_+\rangle+\langle M_-\rangle}{\varepsilon^2}+2\left(\frac{\langle M_+\rangle-\langle M_-\rangle}{\epsilon^3}\right)\Delta\epsilon \\
&-\frac{\langle M_+\rangle^2+\langle M_-\rangle^2}{\varepsilon^2}+2\left(\frac{\langle M_+\rangle^2-\langle M_-\rangle^2}{\epsilon^3}\right)\Delta\epsilon+\frac{\langle M_+\rangle+\langle M_-\rangle}{\varepsilon}-2\left(\frac{\langle M_+\rangle-\langle M_-\rangle}{\epsilon^2}\right)\Delta\epsilon \\
&-2\frac{\langle M_+M_-\rangle}{\varepsilon^2}+2\frac{\langle M_+\rangle\langle M_-\rangle}{\epsilon^2}+\mathcal{O}(\Delta\varepsilon^2).
\end{aligned}
$$

$$
\begin{aligned}
\Delta K_2 &= \left|K_{2,\mathrm{ave}}-K_{2,\mathrm{sep}}\right| \\
&\simeq \frac{2\Delta\varepsilon}{\epsilon^2}\left[\frac{\underline{(\langle M_+\rangle-\langle M_-\rangle)}-(\sigma_+^2-\sigma_-^2)}{\epsilon}-\frac{1}{2}\Big(\underline{\langle M_+\rangle-\langle M_-\rangle}\Big)\right].
\end{aligned}
$$

represented by “net-charge” terms

Difference ΔK_2 is proportional to the difference of multiplicity.

Analytical calculation, C_3

- **Difference of C_3 can be also calculated by similar approach**

$$\begin{aligned} K_{3,\mathrm{sep}}(\Delta\varepsilon) &= \frac{A_+}{(\varepsilon+\Delta\varepsilon)^3} - \frac{A_-}{(\varepsilon-\Delta\varepsilon)^3} - \frac{B_+}{(\varepsilon+\Delta\varepsilon)^2(\varepsilon-\Delta\varepsilon)} + \frac{B_-}{(\varepsilon+\Delta\varepsilon)(\varepsilon-\Delta\varepsilon)^2} \\ &\quad + \frac{C_+}{(\varepsilon+\Delta\varepsilon)^2} - \frac{C_-}{(\varepsilon-\Delta\varepsilon)^2} + \frac{D_+}{\varepsilon+\Delta\varepsilon} - \frac{D_-}{\varepsilon-\Delta\varepsilon}, \end{aligned}$$

where constant terms are defined as

$$\begin{aligned} A_\pm &= \langle M_\pm^3\rangle + 2\langle M_\pm\rangle - 3\langle M_\pm^2\rangle - 3\langle M_\pm\rangle\big(\langle M_\pm^2\rangle - \langle M_\pm\rangle\big) + 2\langle M_\pm\rangle^3, \\ B_\pm &= 3\langle M_\pm^2 M_\mp\rangle - 3\langle M_\pm M_\mp\rangle - 3\langle M_\mp\rangle\big(\langle M_\pm^2\rangle - \langle M_\pm\rangle\big) - 6\langle M_\pm\rangle\langle M_\pm M_\mp\rangle + 6\langle M_\pm\rangle^2\langle M_\mp\rangle, \\ C_\pm &= 3\big(\langle M_\pm^2\rangle - \langle M_\pm\rangle^2 - \langle M_\pm\rangle\big), \\ D_\pm &= \langle M_\pm\rangle. \end{aligned}$$

$$\begin{aligned} \Delta K_3 &= K_{3,\mathrm{ave}} - K_{3,\mathrm{sep}} \\ &\simeq K_{3,\mathrm{ave}} - \left[K_{3,\mathrm{sep}}(0) + \left.\frac{\partial K_{3,\mathrm{sep}}}{\partial\Delta\varepsilon}\right|_{\Delta\varepsilon=0}\Delta\varepsilon + \mathcal{O}(\Delta\varepsilon^2)\right]. \\ &= -\left.\frac{\partial K_{3,\mathrm{sep}}}{\partial\Delta\varepsilon}\right|_{\Delta\varepsilon=0}\Delta\varepsilon + \mathcal{O}(\Delta\varepsilon^2) \\ &= \left[\frac{1}{\varepsilon^4}\left[3\underline{(A_+ + A_-)} - \underline{(B_+ + B_-)}\right] + \frac{2}{\varepsilon^3}\underline{(C_+ + C_-)} + \frac{2}{\varepsilon^2}\underline{(D_+ + D_-)}\right]\Delta\varepsilon. \end{aligned}$$

represented by “multiplicity” terms

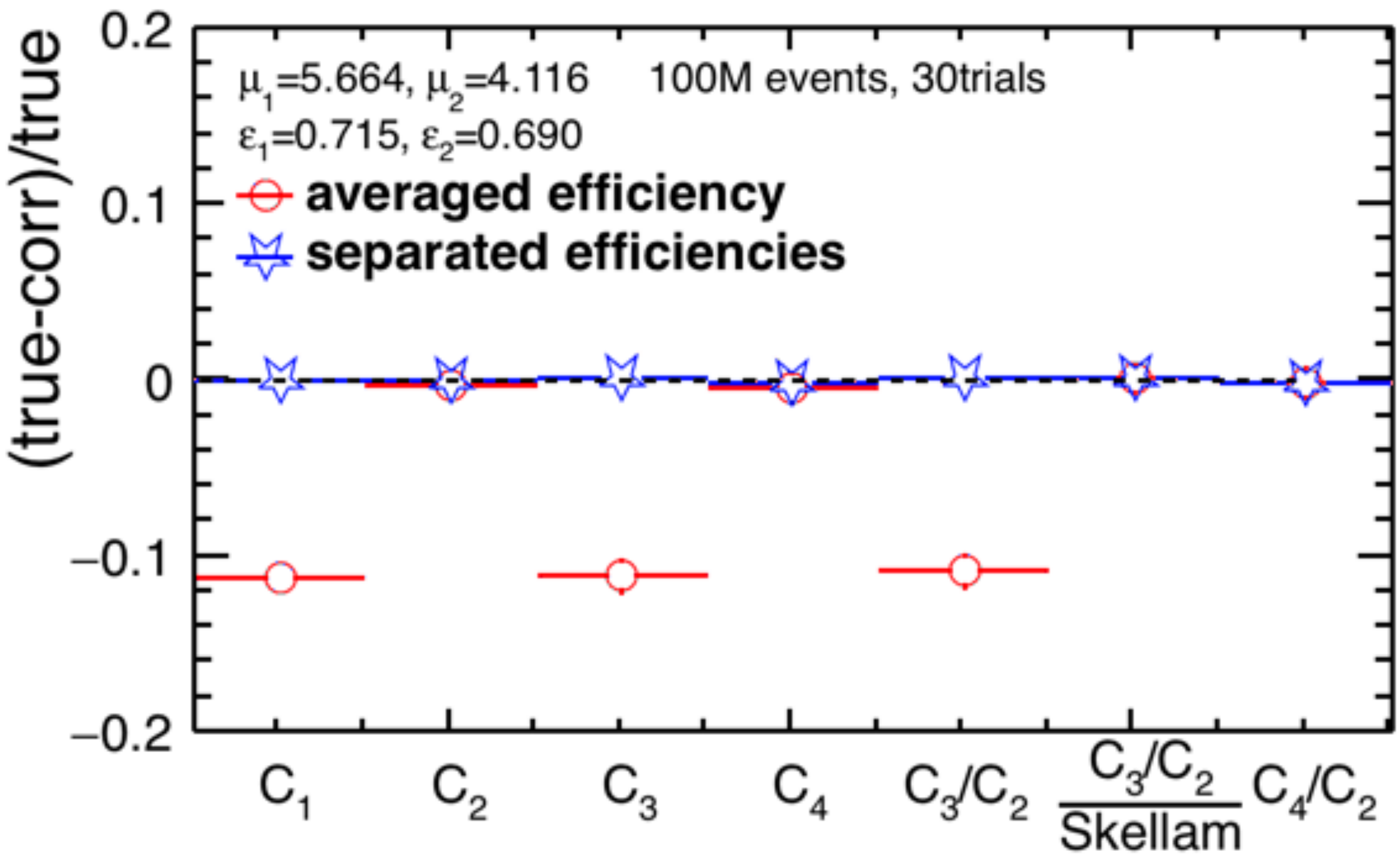
Difference ΔK_3 is proportional to the sum of multiplicity.

$$\boxed{\begin{aligned} \Delta K_{odd} &\propto \langle M_+\rangle + \langle M_-\rangle \\ \Delta K_{even} &\propto \langle M_+\rangle - \langle M_-\rangle \end{aligned}}$$

<u>net-proton, 200GeV, 0-5%</u>

$$\langle M_+\rangle = 5.664,\ \langle M_-\rangle = 4.116$$
$$\langle M_+\rangle + \langle M_-\rangle = 9.78$$
$$\langle M_+\rangle - \langle M_-\rangle = 1.55$$

$$\Delta K_{odd} \propto \langle M_+ \rangle + \langle M_- \rangle$$
$$\Delta K_{even} \propto \langle M_+ \rangle - \langle M_- \rangle$$



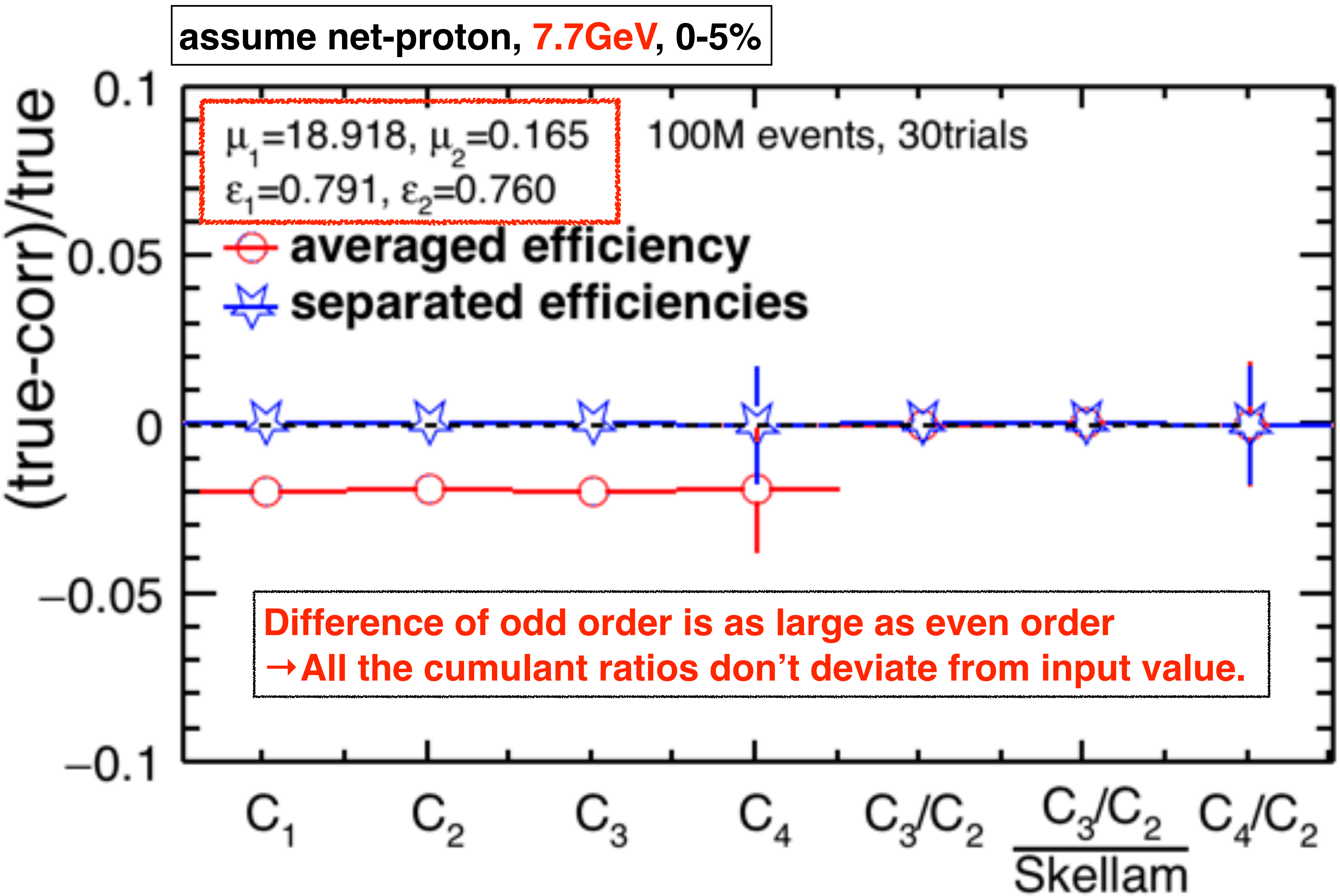
$$\boxed{\begin{aligned} \Delta K_{odd} &\propto \langle M_+\rangle + \langle M_-\rangle \\ \Delta K_{even} &\propto \langle M_+\rangle - \langle M_-\rangle \end{aligned}}$$

<u>net-proton, 200GeV, 0-5%</u>

$$\langle M_+\rangle = 5.664,\ \langle M_-\rangle = 4.116$$
$$\langle M_+\rangle + \langle M_-\rangle = 9.78$$
$$\langle M_+\rangle - \langle M_-\rangle = 1.55$$

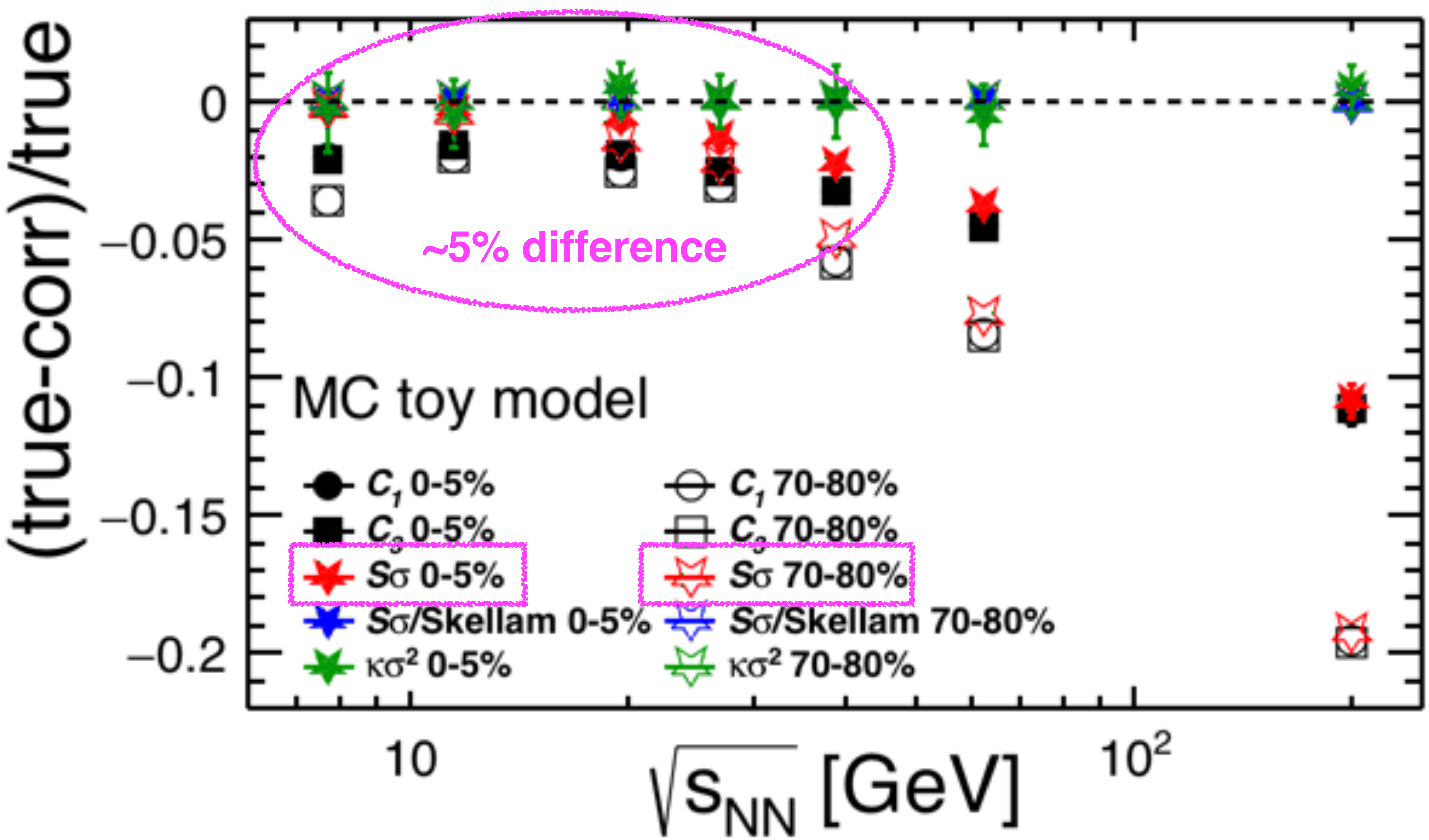
→ **How about low beam energies？**

Order dependence (7.7GeV)



Beam energy dependence

- ✓ Relative difference is calculated for each BES energy.
- ✓ There is ~5% difference for *Sσ* at experimental interest region(~39GeV).
- ✓ **Conclusions in net-proton published paper won't be changed.**



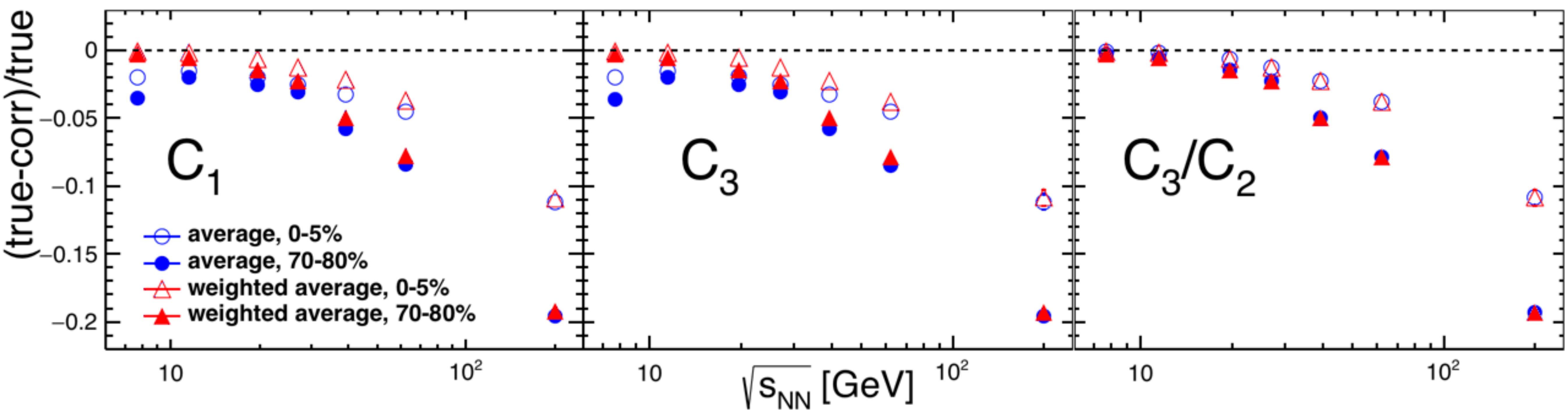
Weighted average?

$$\varepsilon_w = \frac{\sum_i^N (M_{1,i}\varepsilon_1 + M_{2,i}\varepsilon_2)}{\sum_i^N (M_{1,i} + M_{2,i})}$$

N : # of events
M : # of particles
ε : efficiency

✓ At lower energies, weighted averaged efficiency gives a better results(C_1,C_3) than averaged efficiency.

✓ At higher energies, however, the difference is as large as averaged efficiency.



➡ Separated efficiencies should be used.

Summary

- The results of MC toy model calculations indicate that
  - ➡ odd order cumulants and C3/C2 systematically deviate from input value in case of averaged efficiency @200GeV.
  - ➡ deviation of even order cumulants is as large as odd order cumulants @7.7GeV.
- According to analytical calculations, this is because
  - ➡ deviation of odd order cumulants is proportional to the sum of multiplicity.
  - ➡ deviation of even order cumulants is proportional to the difference of multiplicity.
- Beam energy dependence indicates that
  - ➡ conclusions in published net-proton paper won't be changed.
- Please note that current fluctuation analysis at STAR is being done by using separated efficiencies.

Back up

Skellam baseline of Sσ

$$\varepsilon = \frac{\varepsilon_1 + \varepsilon_2}{2}$$

efficiency vanishes!!

$$S\sigma_{skellam,ave} = \frac{(\mu_1 - \mu_2)/\varepsilon}{(\mu_1 + \mu_2)/\varepsilon} = \frac{\mu_1 - \mu_2}{\mu_1 + \mu_2}$$

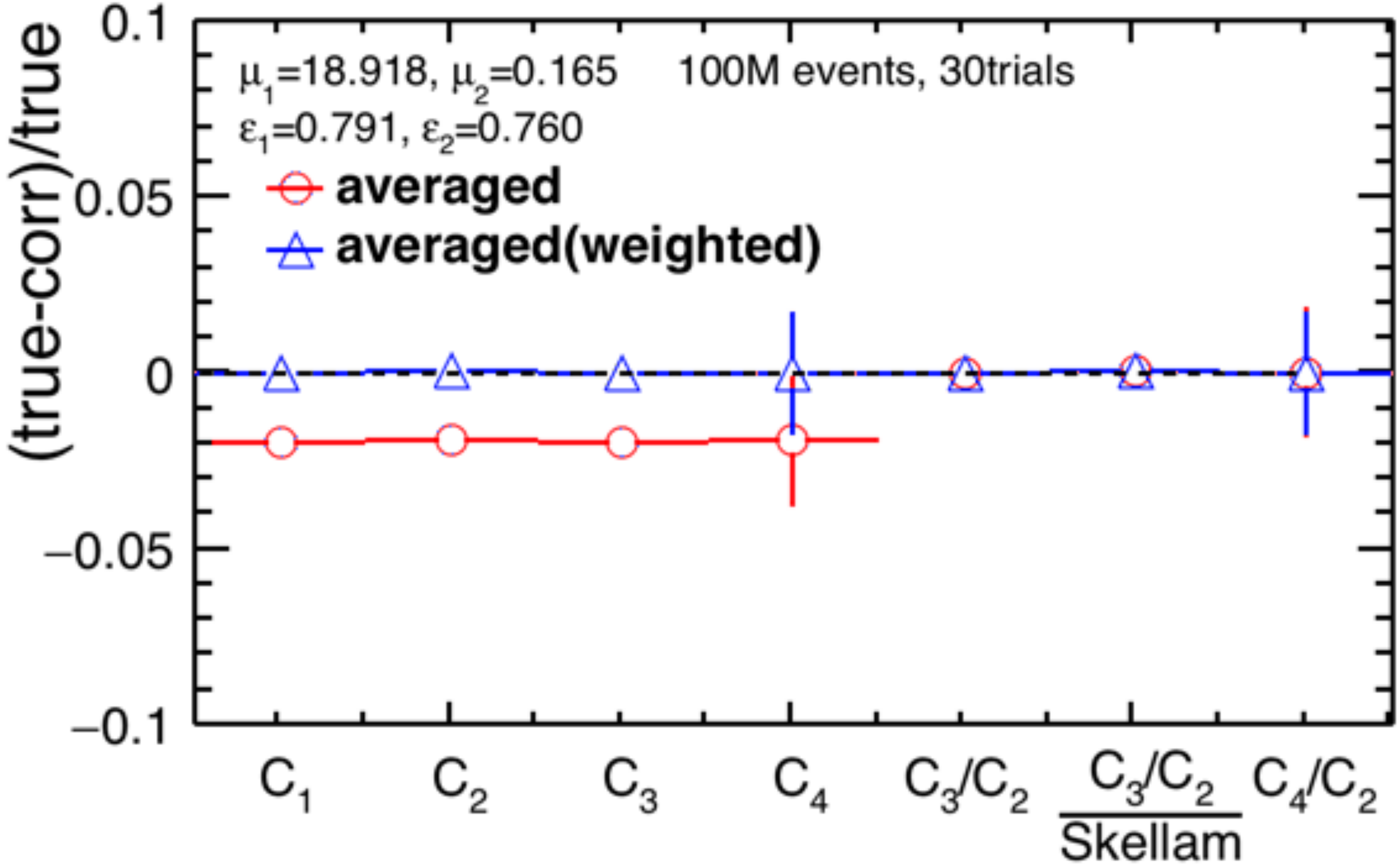
$$S\sigma_{skellam,sep} = \frac{\frac{\mu_1}{\varepsilon_1} - \frac{\mu_2}{\varepsilon_2}}{\frac{\mu_1}{\varepsilon_1} + \frac{\mu_2}{\varepsilon_2}} = \frac{\varepsilon_2\mu_1 - \varepsilon_1\mu_2}{\varepsilon_2\mu_1 + \varepsilon_1\mu_2}$$

➡ Baseline is also changed

Weighted average @7.7GeV

$$\varepsilon_w = \frac{\sum_i^N \left(M_{1,i}\varepsilon_1 + M_{2,i}\varepsilon_2\right)}{\sum_i^N \left(M_{1,i} + M_{2,i}\right)}$$

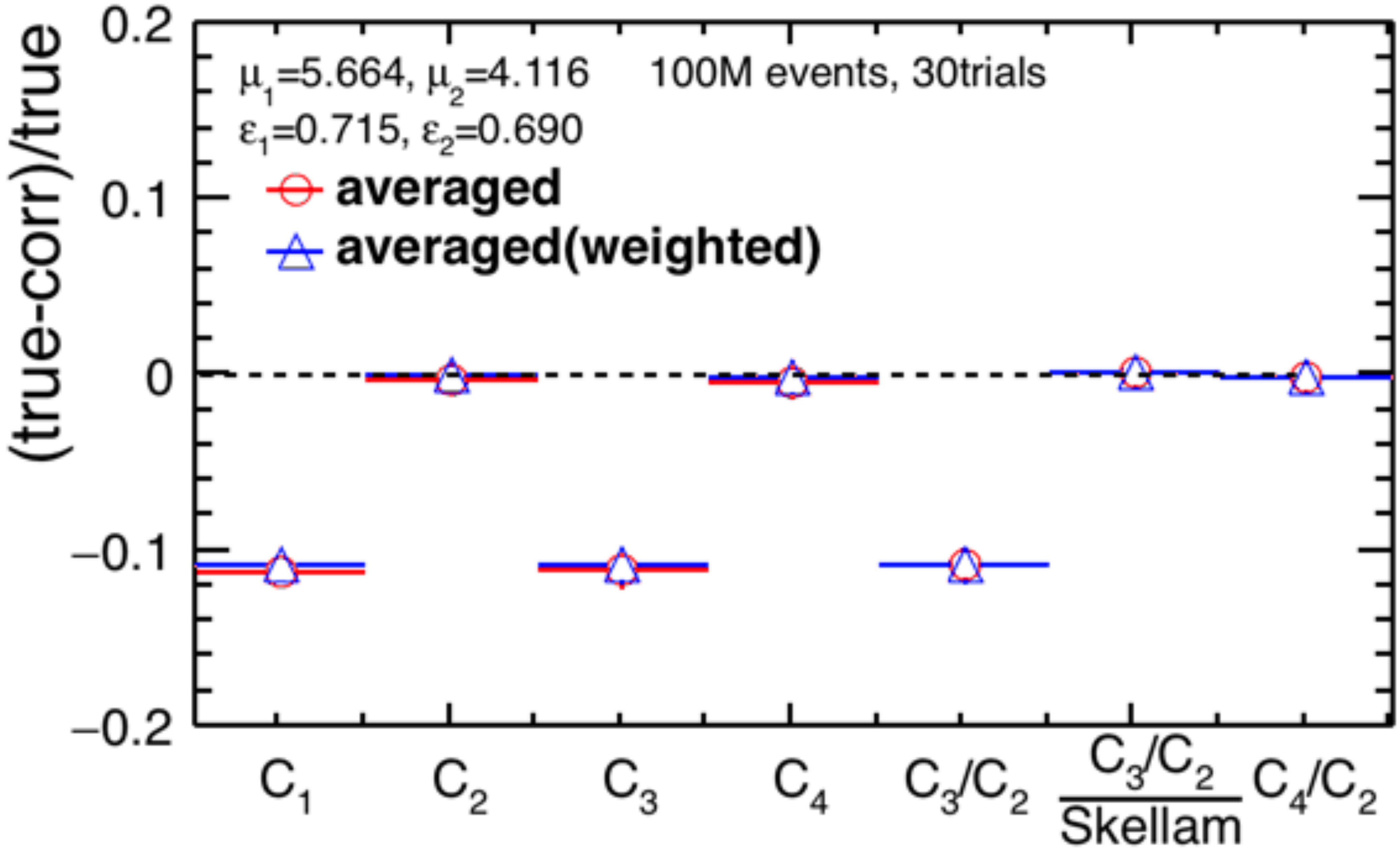
N : # of events
M : # of particles
ε : efficiency



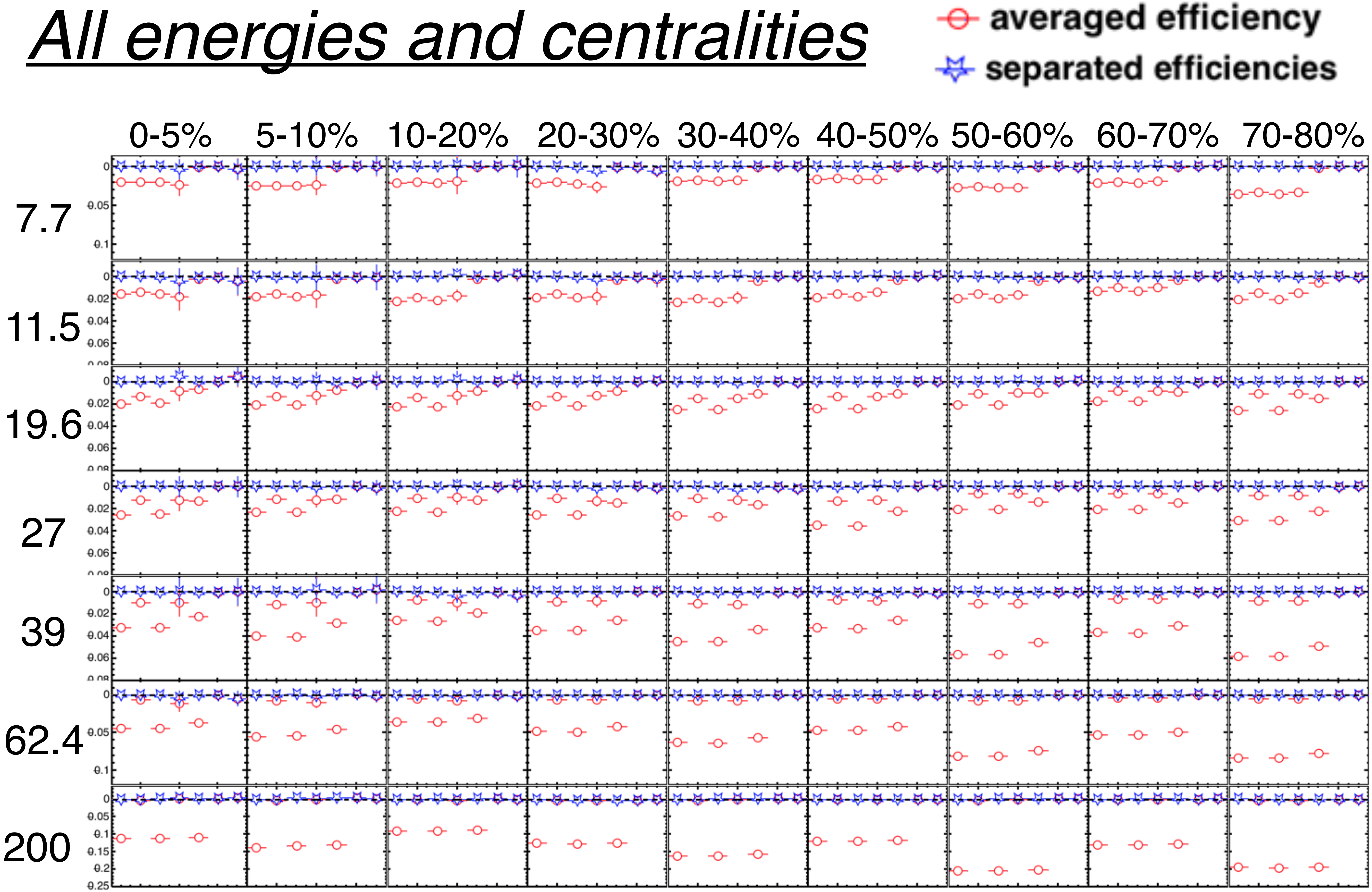
Weighted average @200GeV

$$\varepsilon_w = \frac{\sum_i^N \left(M_{1,i}\varepsilon_1 + M_{2,i}\varepsilon_2\right)}{\sum_i^N \left(M_{1,i} + M_{2,i}\right)}$$

N : # of events
M : # of particles
ε : efficiency

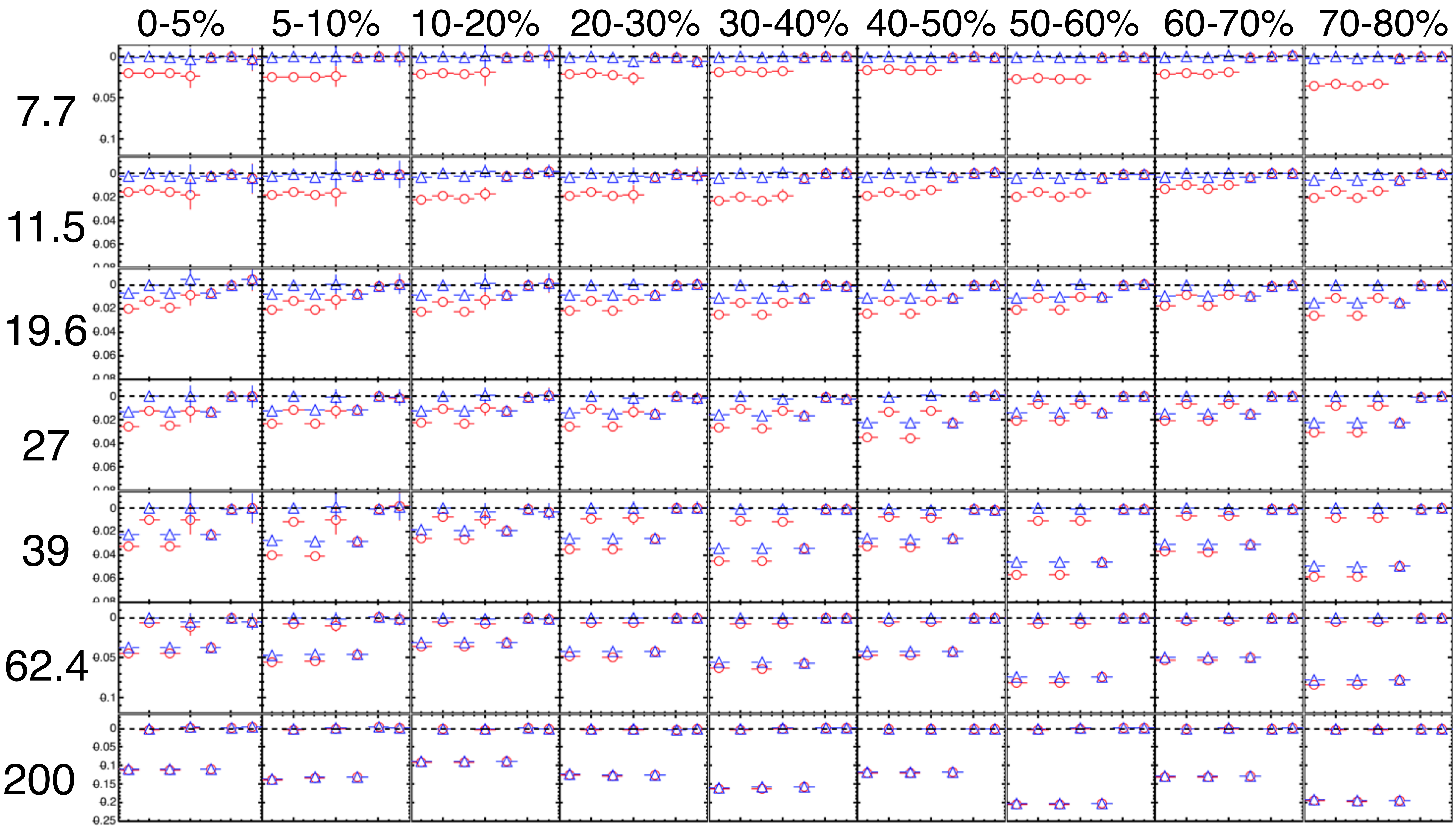


All energies and centralities



Each bin : C_1, C_2, C_3, C_4, *Sσ*, *Sσ/Skellam* and $\kappa\sigma^2$ from left to right.

All energies and centralities



Each bin : C_1, C_2, C_3, C_4, *Sσ*, *Sσ/Skellam* and *κσ²* from left to right.