MC toy model calculation assuming net-proton distribution

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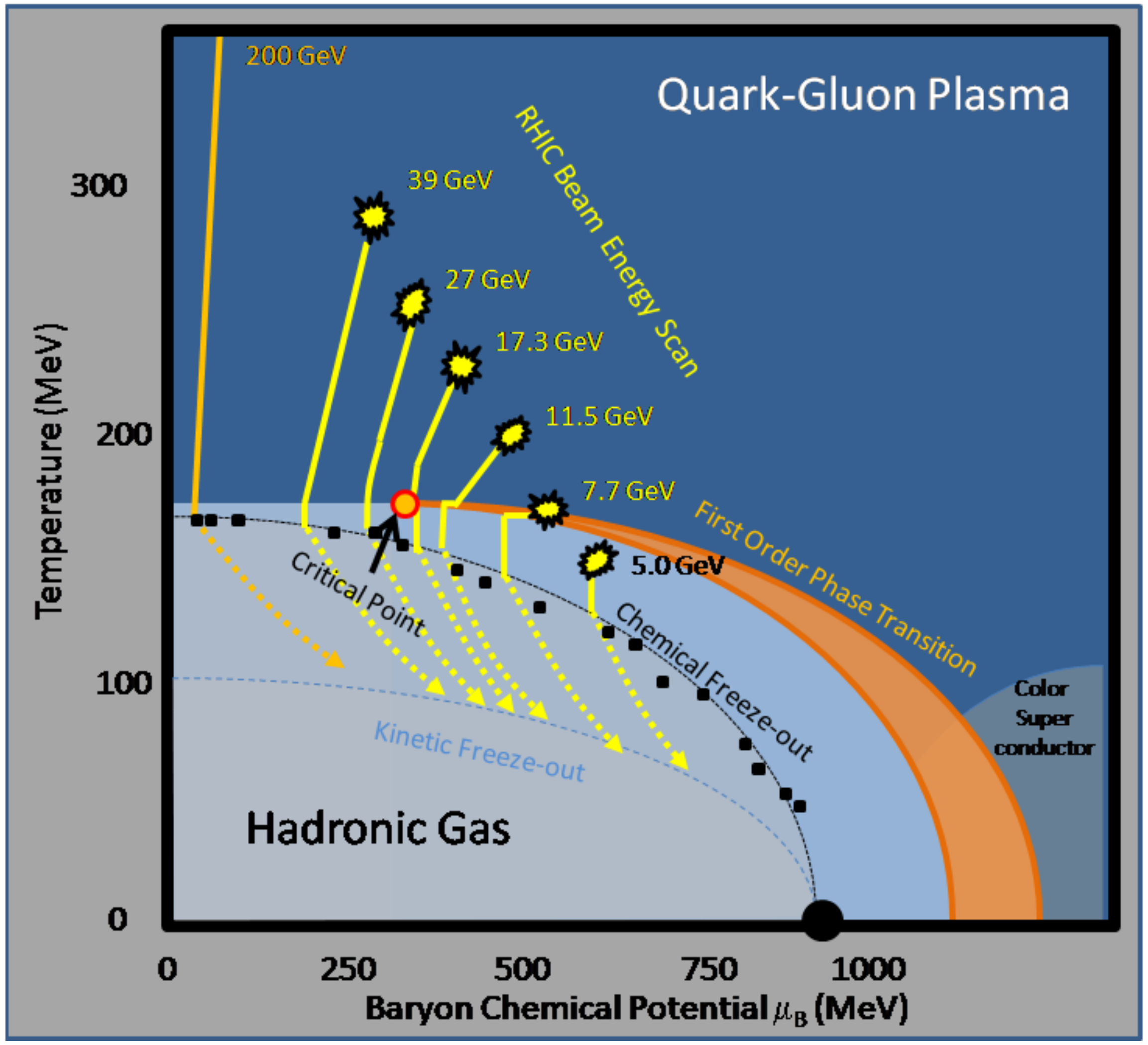
Outline

- Introduction
- Comparison with published net-proton results
- Importance of separated efficiencies
- Efficiency correction in case of arbitrary number of phase spaces
- Summary

Search for the critical point



- Beam Energy Scan program has been carried out at RHIC for searching the QCD critical point.
- One of observables is **cumulant of conserved quantities.**
- Net-charge, net-proton and net-kaon have been analyzed at STAR experiment.
- Cumulants are extensive variables, so the cumulant ratios have been measured as a function of beam energy.

statistical baseline

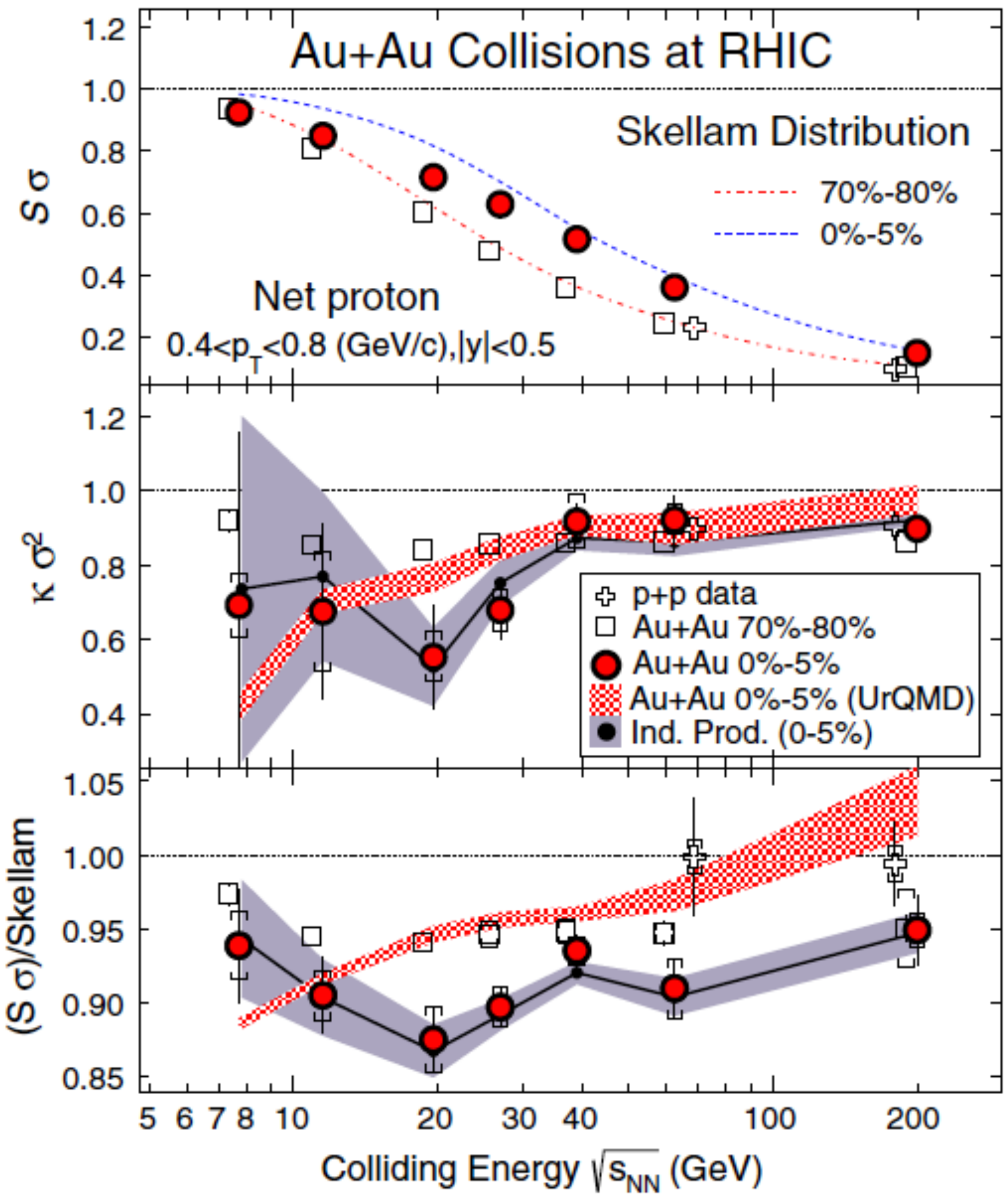
- Two independent Poisson distributions for positively and negatively charged particles.
- Observe deviations from Poisson baseline as a function of beam energy.

$$C_1 = \mu_1 - \mu_2$$
$$C_2 = \mu_1 + \mu_2$$
$$C_3 = \mu_1 - \mu_2$$
$$C_4 = \mu_1 + \mu_2$$

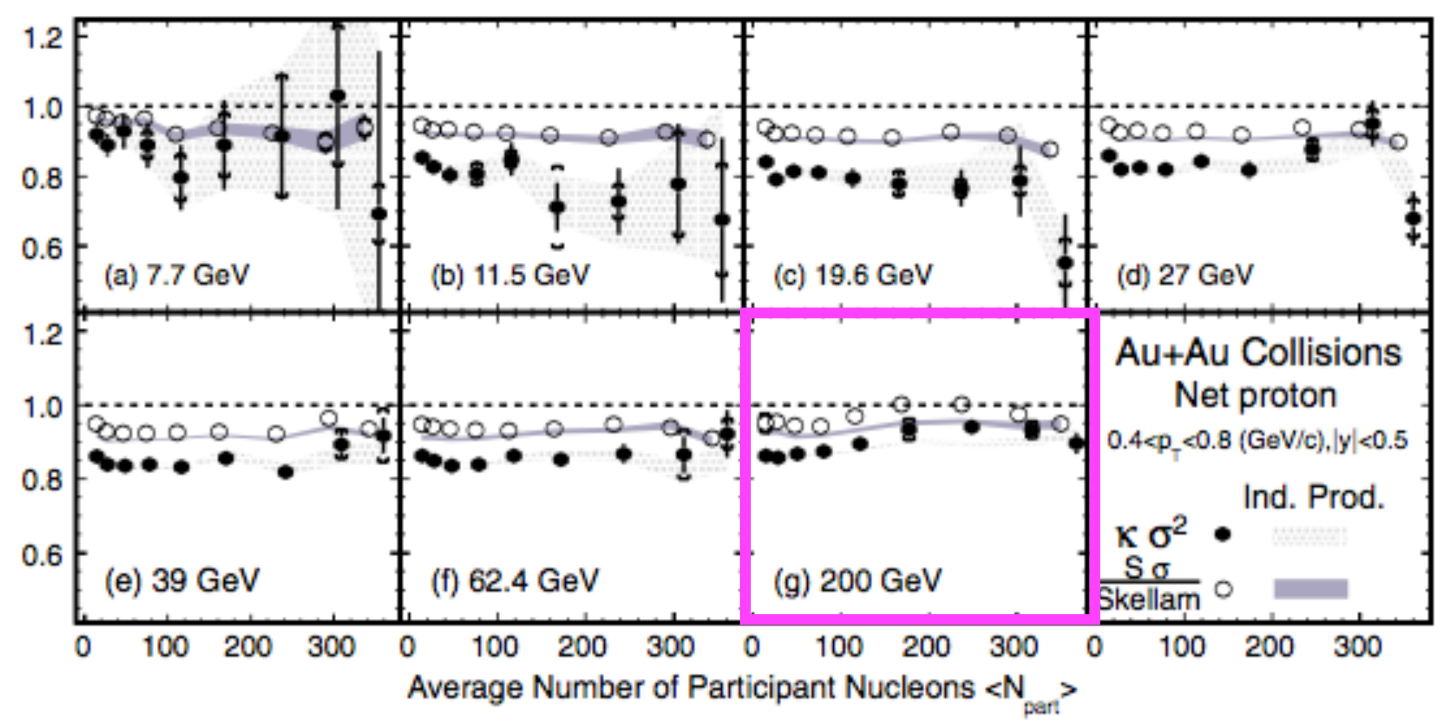
$$\boxed{S\sigma = \frac{C_3}{C_2} = \frac{\mu_1 - \mu_2}{\mu_1 + \mu_2} \qquad \kappa\sigma^2 = \frac{C_4}{C_2} = 1}$$

Published net-proton results

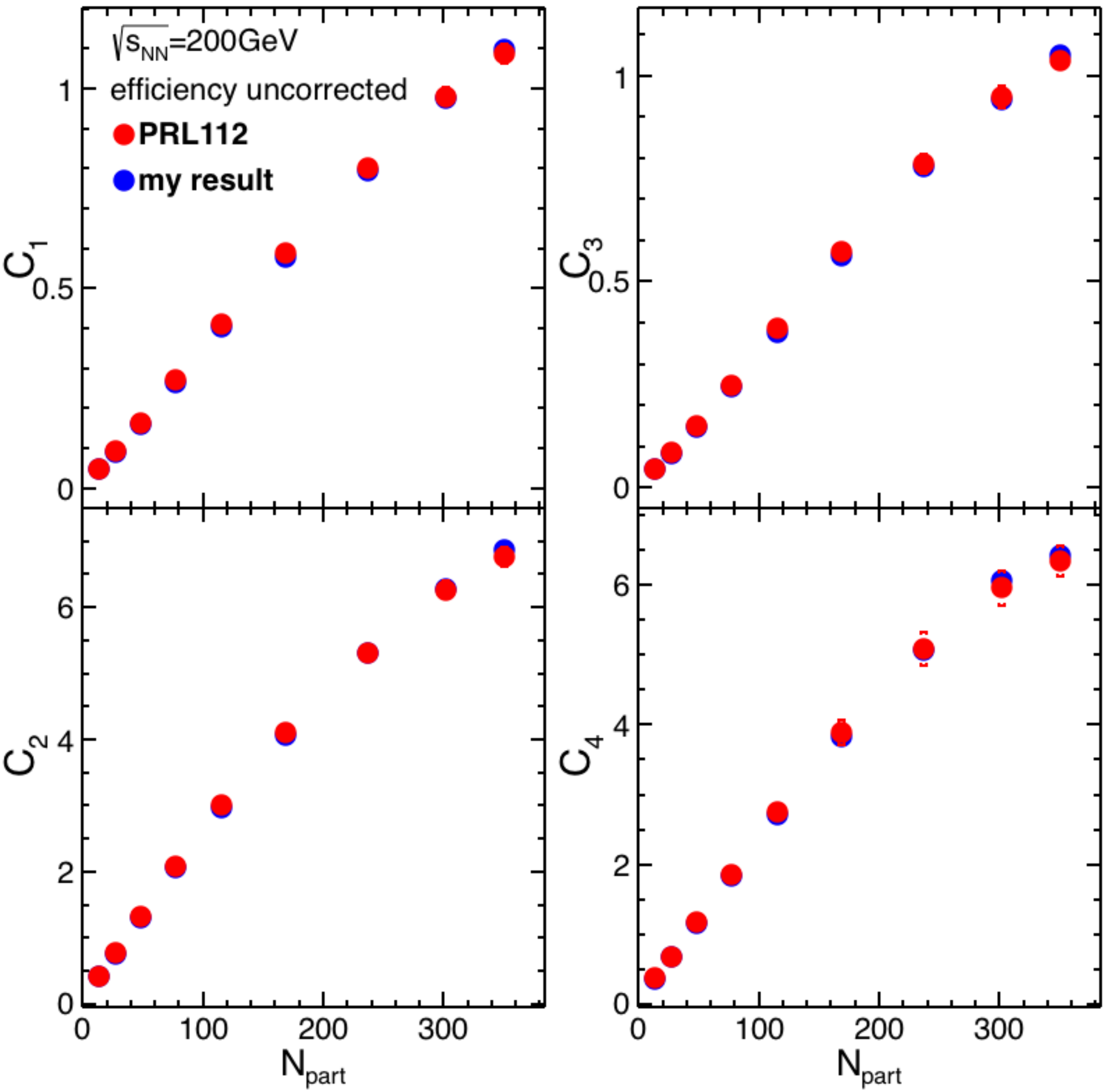
PRL112 032302 (2014)



- In order to start fluctuation analysis at STAR, published net-proton results have been reproduced at 200GeV.
- Difference of correction method with published analysis will be shown by MC toy models and analytical calculations.

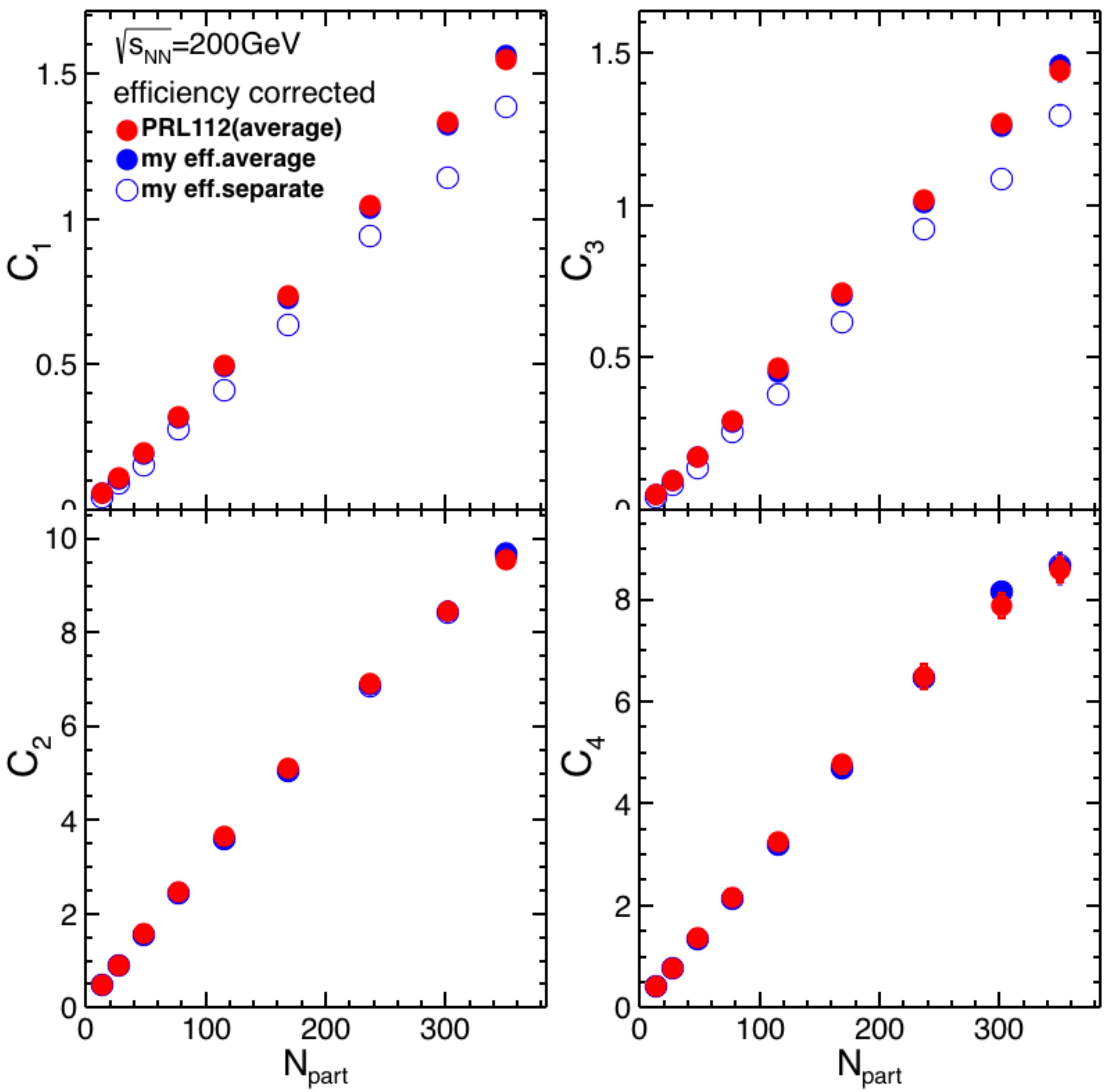


Cumulant (uncorrected)



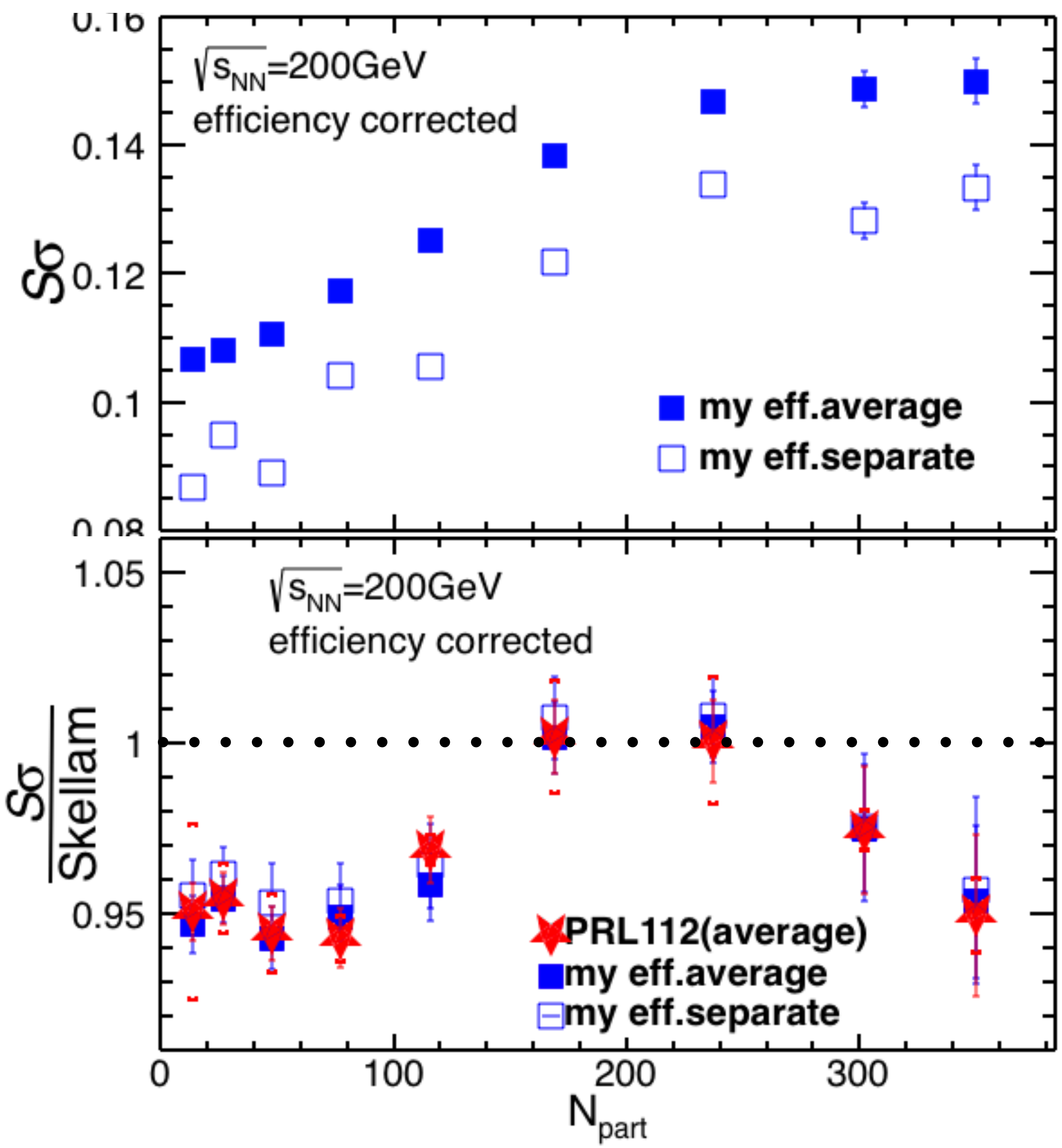
- Agree with published results within statistical errors.

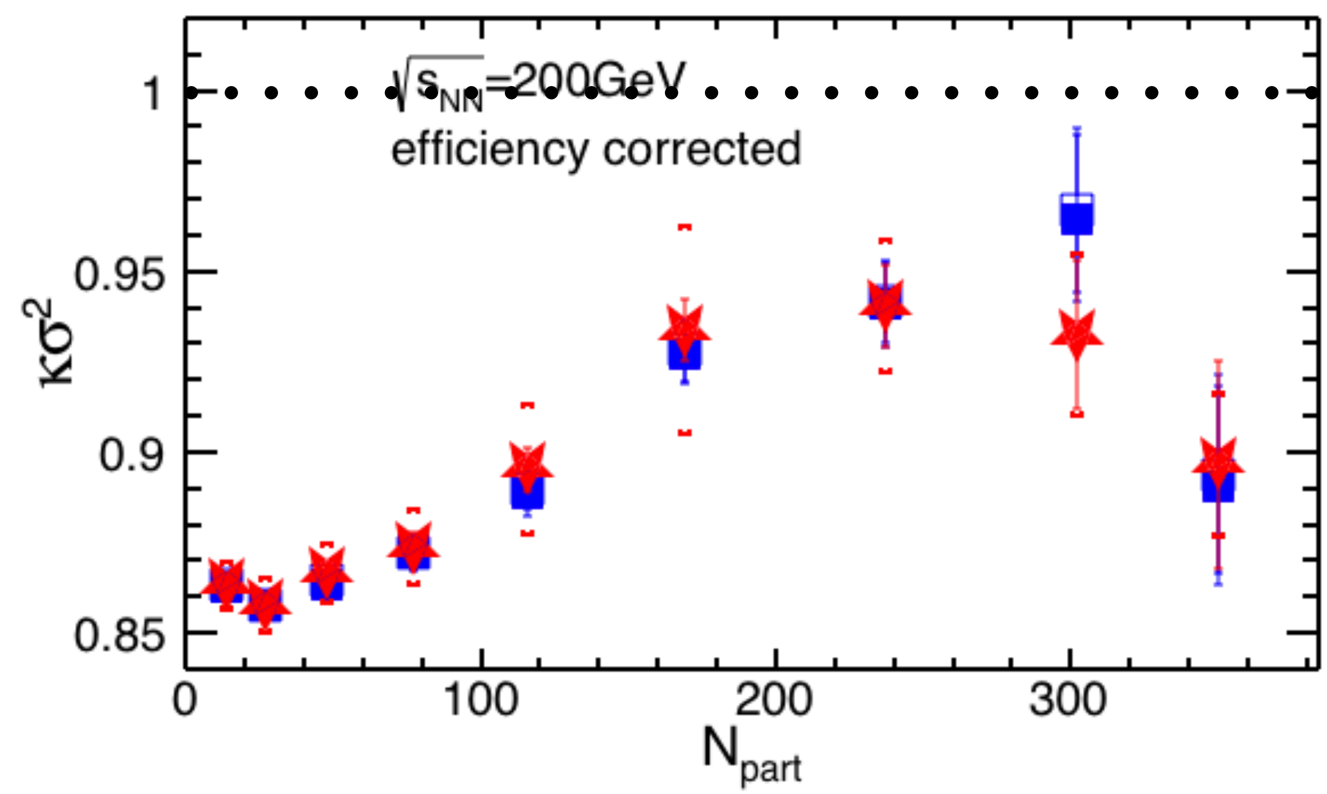
Cumulant (eff.corrected)



- In published analysis, **average efficiency** between positively and negatively charged particles are used.
- In case of odd order cumulants, the separate efficiency gives a different result (~10% smaller) from the average efficiency correction.
- However, in case of even order cumulants, they have little difference.

Cumulant ratios



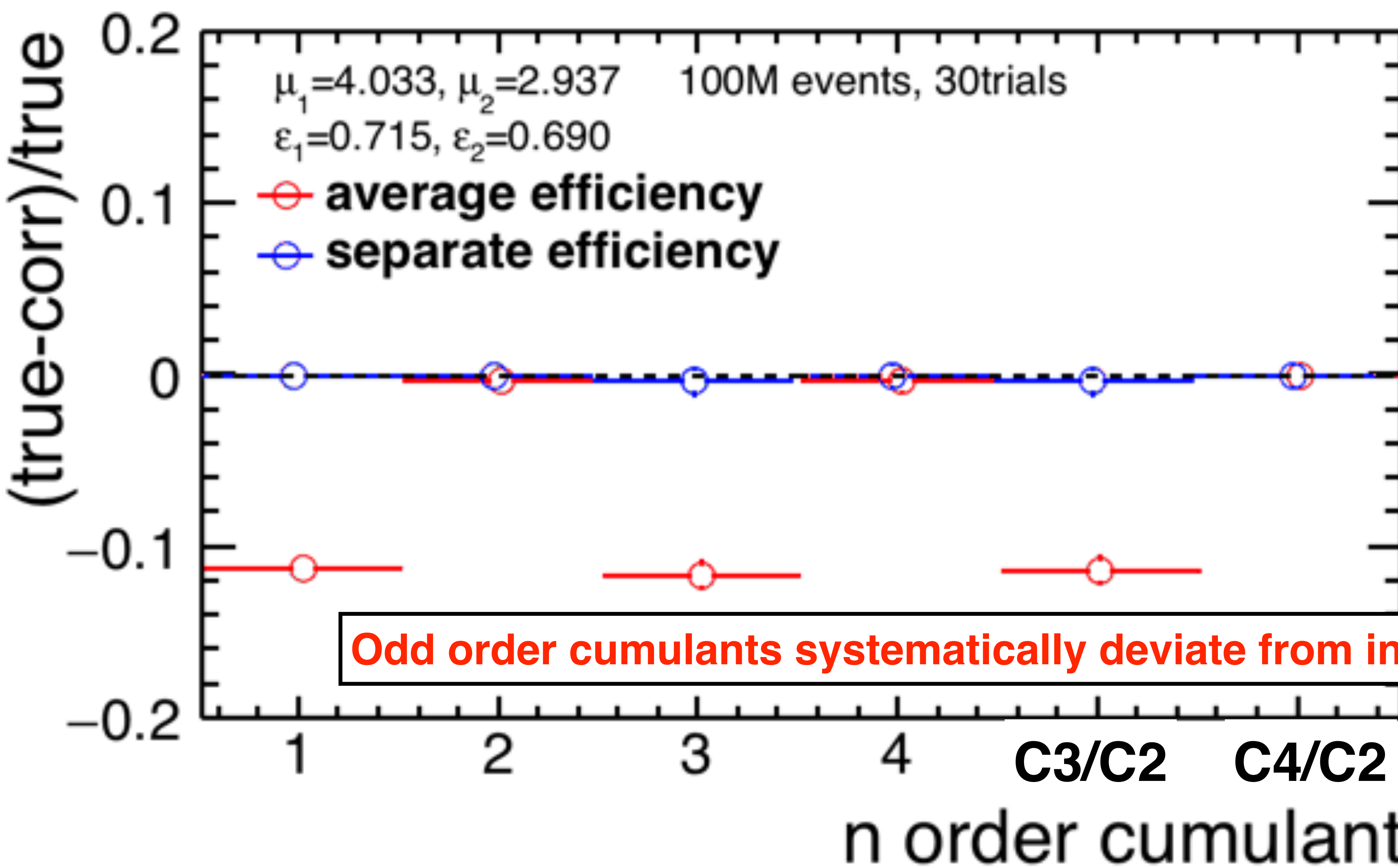


Difference between two correction methods is very small except for *S*σ.

MC toy model

- Also checked by MC toy model calculation assuming net-proton distribution in the most central collisions at 200GeV.
- This is roughly consistent with the results shown in earlier pages.

assume net-proton, 200GeV, 0-5%



Odd order cumulants systematically deviate from input value??

Analytical calculation, C_1

shown by X.Luo

N± : # of produced particles
M± : # of observed particles
ε± : efficiency for charged or anti-charged particles
ε : average efficiency
Δε : efficiency difference

$$\varepsilon_+ = \varepsilon + \Delta\varepsilon, \qquad \varepsilon = \frac{\varepsilon_+ + \varepsilon_-}{2},$$
$$\varepsilon_- = \varepsilon - \Delta\varepsilon, \qquad \Delta\varepsilon = \frac{\varepsilon_+ - \varepsilon_-}{2}.$$

$$K_{1,\mathrm{sep}} = \langle N_+\rangle - \langle N_-\rangle = \frac{\langle M_+\rangle}{\varepsilon_+} - \frac{\langle M_-\rangle}{\varepsilon_-}$$
$$= \frac{\langle M_+\rangle}{\varepsilon + \Delta\varepsilon} - \frac{\langle M_-\rangle}{\varepsilon - \Delta\varepsilon}$$

Δε=0

$$K_{1,\mathrm{ave}} = \langle N_+\rangle - \langle N_-\rangle$$
$$= \frac{\langle M_+\rangle - \langle M_-\rangle}{\varepsilon}.$$

Taylor expansion around Δε=0

$$K_{1,\mathrm{sep}}(\Delta\varepsilon) \simeq K_{1,\mathrm{sep}}(0) + \left.\frac{\partial K_{1,\mathrm{sep}}}{\partial \Delta\varepsilon}\right|_{\Delta\varepsilon=0} \Delta\varepsilon + \mathcal{O}(\Delta\varepsilon^2)$$
$$= \left(\frac{\langle M_+\rangle}{\varepsilon} - \frac{\langle M_-\rangle}{\varepsilon}\right) - \left(\frac{\langle M_+\rangle}{\varepsilon^2} + \frac{\langle M_-\rangle}{\varepsilon^2}\right)\Delta\varepsilon + \mathcal{O}(\Delta\varepsilon^2).$$

$$\Delta K_1 = \left|K_{1,\mathrm{ave}} - K_{1,\mathrm{sep}}\right|$$
$$= \left|\left(\frac{\langle M_+\rangle}{\varepsilon} - \frac{\langle M_-\rangle}{\varepsilon}\right) - \left(\frac{\langle M_+\rangle}{\varepsilon} - \frac{\langle M_-\rangle}{\varepsilon}\right) + \left(\frac{\langle M_+\rangle}{\varepsilon^2} + \frac{\langle M_-\rangle}{\varepsilon^2}\right)\Delta\varepsilon\right|$$
$$= \frac{\Delta\varepsilon}{\varepsilon^2}\left(\langle M_+\rangle + \langle M_-\rangle\right).$$

Difference ΔK1 is proportional to the sum of multiplicity.

Analytical calculation, C_2

- **Difference for C2 can be calculated by similar approach**

$$
\begin{aligned}
K_{2,\mathrm{sep}} &= \left(\frac{\langle M_+^2\rangle}{(\varepsilon+\Delta\varepsilon)^2}+\frac{\langle M_-^2\rangle}{(\epsilon-\Delta\varepsilon)^2}\right)-\left(\frac{\langle M_+\rangle}{(\varepsilon+\Delta\varepsilon)^2}+\frac{\langle M_-\rangle}{(\epsilon-\Delta\varepsilon)^2}\right)-\left(\frac{\langle M_+\rangle^2}{(\varepsilon+\Delta\varepsilon)^2}+\frac{\langle M_-\rangle^2}{(\epsilon-\Delta\varepsilon)^2}\right)\\
&\quad+\left(\frac{\langle M_+\rangle}{\varepsilon+\Delta\varepsilon}+\frac{\langle M_-\rangle}{\epsilon-\Delta\varepsilon}\right)-2\frac{\langle M_+M_-\rangle}{(\varepsilon+\Delta\varepsilon)(\epsilon-\Delta\varepsilon)}+2\frac{\langle M_+\rangle\langle M_-\rangle}{(\varepsilon+\Delta\varepsilon)(\varepsilon-\Delta\varepsilon)}.
\end{aligned}
$$

$$
\begin{aligned}
K_{2,\mathrm{ave}} &= \left(\frac{\langle M_+^2\rangle}{\varepsilon^2}+\frac{\langle M_-^2\rangle}{\epsilon^2}\right)-\left(\frac{\langle M_+\rangle}{\varepsilon^2}+\frac{\langle M_-\rangle}{\epsilon^2}\right)-\left(\frac{\langle M_+\rangle^2}{\varepsilon^2}+\frac{\langle M_-\rangle^2}{\epsilon^2}\right)+\left(\frac{\langle M_+\rangle}{\varepsilon}+\frac{\langle M_-\rangle}{\epsilon}\right)\\
&\quad-2\frac{\langle M_+M_-\rangle}{\varepsilon^2}+2\frac{\langle M_+\rangle\langle M_-\rangle}{\varepsilon^2}.
\end{aligned}
$$

$$
\begin{aligned}
K_{2,\mathrm{sep}}(\Delta\varepsilon) &\simeq K_{2,\mathrm{sep}}(0)+\left.\frac{\partial K_{2,\mathrm{sep}}}{\partial\Delta\varepsilon}\right|_{\Delta\varepsilon=0}\Delta\varepsilon+\mathcal{O}(\Delta\varepsilon^2).\\
&\simeq \frac{\langle M_+^2\rangle+\langle M_-^2\rangle}{\varepsilon^2}-2\left(\frac{\langle M_+^2\rangle-\langle M_-^2\rangle}{\epsilon^3}\right)\Delta\epsilon-\frac{\langle M_+\rangle+\langle M_-\rangle}{\varepsilon^2}+2\left(\frac{\langle M_+\rangle-\langle M_-\rangle}{\epsilon^3}\right)\Delta\epsilon\\
&\quad-\frac{\langle M_+\rangle^2+\langle M_-\rangle^2}{\varepsilon^2}+2\left(\frac{\langle M_+\rangle^2-\langle M_-\rangle^2}{\epsilon^3}\right)\Delta\epsilon+\frac{\langle M_+\rangle+\langle M_-\rangle}{\varepsilon}-2\left(\frac{\langle M_+\rangle-\langle M_-\rangle}{\epsilon^2}\right)\Delta\epsilon\\
&\quad-2\frac{\langle M_+M_-\rangle}{\varepsilon^2}+2\frac{\langle M_+\rangle\langle M_-\rangle}{\epsilon^2}+\mathcal{O}(\Delta\varepsilon^2).
\end{aligned}
$$

$$
\begin{aligned}
\Delta K_2 &= \left|K_{2,\mathrm{ave}}-K_{2,\mathrm{sep}}\right|\\
&\simeq \frac{2\Delta\varepsilon}{\epsilon^2}\left[\frac{\underline{\left(\langle M_+\rangle-\langle M_-\rangle\right)}-(\sigma_+^2-\sigma_-^2)}{\epsilon}-\left(\langle M_+\rangle-\langle M_-\rangle\right)\right]
\end{aligned}
$$

represented by “net-charge” terms

Difference ΔK2 is proportional to the difference of multiplicity.

Analytical calculation, C_3

- **Difference for C3 can be also calculated by similar approach**

$$
\begin{aligned}
K_{3,\mathrm{sep}}(\Delta\varepsilon) &= \frac{A_+}{(\varepsilon+\Delta\varepsilon)^3} - \frac{A_-}{(\varepsilon-\Delta\varepsilon)^3} - \frac{B_+}{(\varepsilon+\Delta\varepsilon)^2(\varepsilon-\Delta\varepsilon)} + \frac{B_-}{(\varepsilon+\Delta\varepsilon)(\varepsilon-\Delta\varepsilon)^2} \\
&\quad + \frac{C_+}{(\varepsilon+\Delta\varepsilon)^2} - \frac{C_-}{(\varepsilon-\Delta\varepsilon)^2} + \frac{D_+}{\varepsilon+\Delta\varepsilon} - \frac{D_-}{\varepsilon-\Delta\varepsilon},
\end{aligned}
$$

where constant terms are defined as

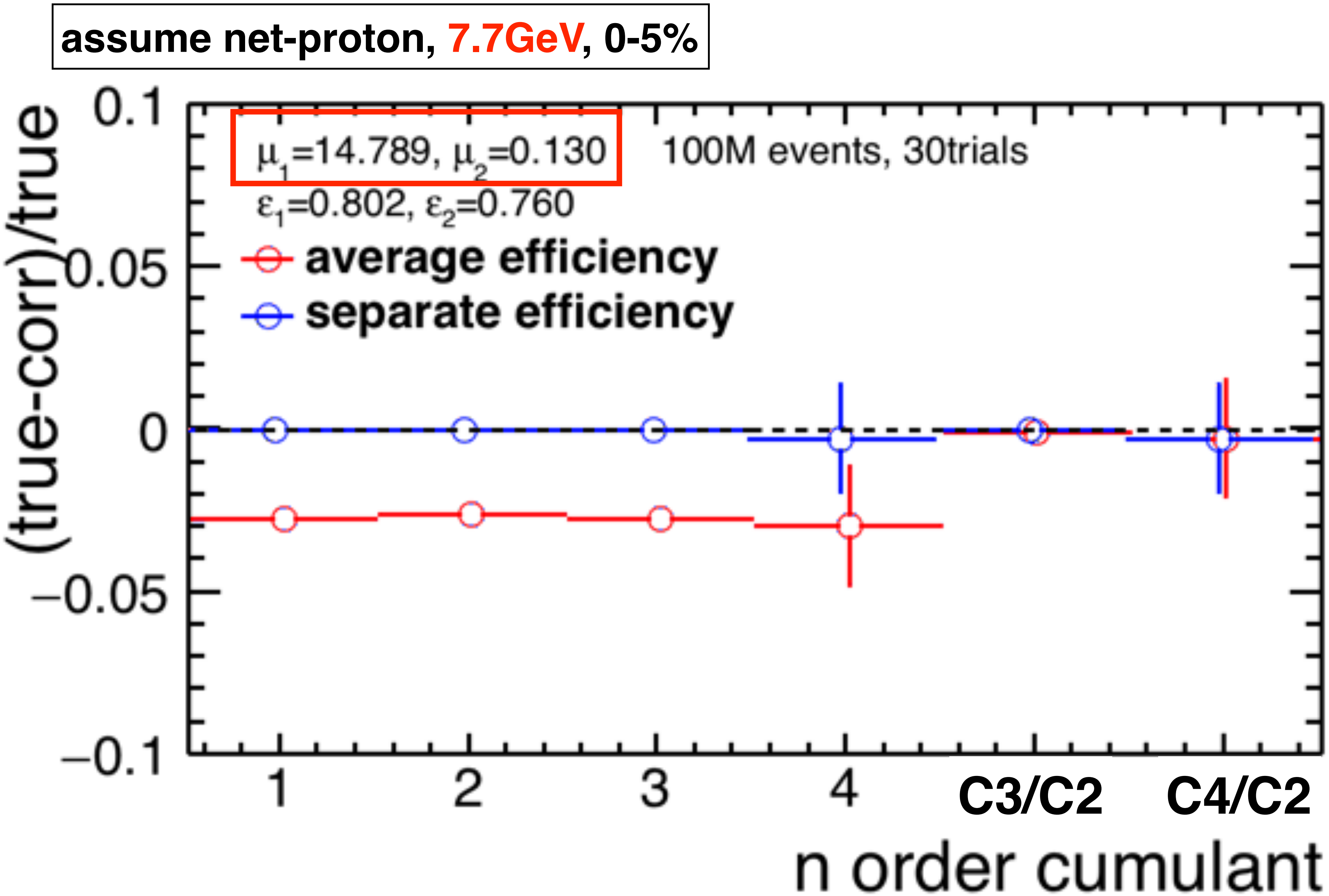
$$
\begin{aligned}
A_\pm &= \langle M_\pm^3\rangle + 2\langle M_\pm\rangle - 3\langle M_\pm^2\rangle - 3\langle M_\pm\rangle\left(\langle M_\pm^2\rangle - \langle M_\pm\rangle\right) + 2\langle M_\pm\rangle^3, \\
B_\pm &= 3\langle M_\pm^2 M_\mp\rangle - 3\langle M_\pm M_\mp\rangle - 3\langle M_\mp\rangle\left(\langle M_\pm^2\rangle - \langle M_\pm\rangle\right) - 6\langle M_\pm\rangle\langle M_\pm M_\mp\rangle + 6\langle M_\pm\rangle^2\langle M_\mp\rangle, \\
C_\pm &= 3\left(\langle M_\pm^2\rangle - \langle M_\pm\rangle^2 - \langle M_\pm\rangle\right), \\
D_\pm &= \langle M_\pm\rangle.
\end{aligned}
$$

$$
\boxed{
\begin{aligned}
\Delta K_3 &= K_{3,\mathrm{ave}} - K_{3,\mathrm{sep}} \\
&\simeq K_{3,\mathrm{ave}} - \left[K_{3,\mathrm{sep}}(0) + \left.\frac{\partial K_{3,\mathrm{sep}}}{\partial\Delta\varepsilon}\right|_{\Delta\varepsilon=0}\Delta\varepsilon + \mathcal{O}(\Delta\varepsilon^2)\right]. \\
&= -\left.\frac{\partial K_{3,\mathrm{sep}}}{\partial\Delta\varepsilon}\right|_{\Delta\varepsilon=0}\Delta\varepsilon + \mathcal{O}(\Delta\varepsilon^2) \\
&= \left[\frac{1}{\varepsilon^4}\left[3\underline{(A_+ + A_-)} - \underline{(B_+ + B_-)}\right] + \frac{2}{\varepsilon^3}\underline{(C_+ + C_-)} + \frac{2}{\varepsilon^2}\underline{(D_+ + D_-)}\right]\Delta\varepsilon.
\end{aligned}
}
$$

represented by “multiplicity” terms

Difference ΔK3 is proportional to the sum of multiplicity.

Order dependence (7.7GeV)

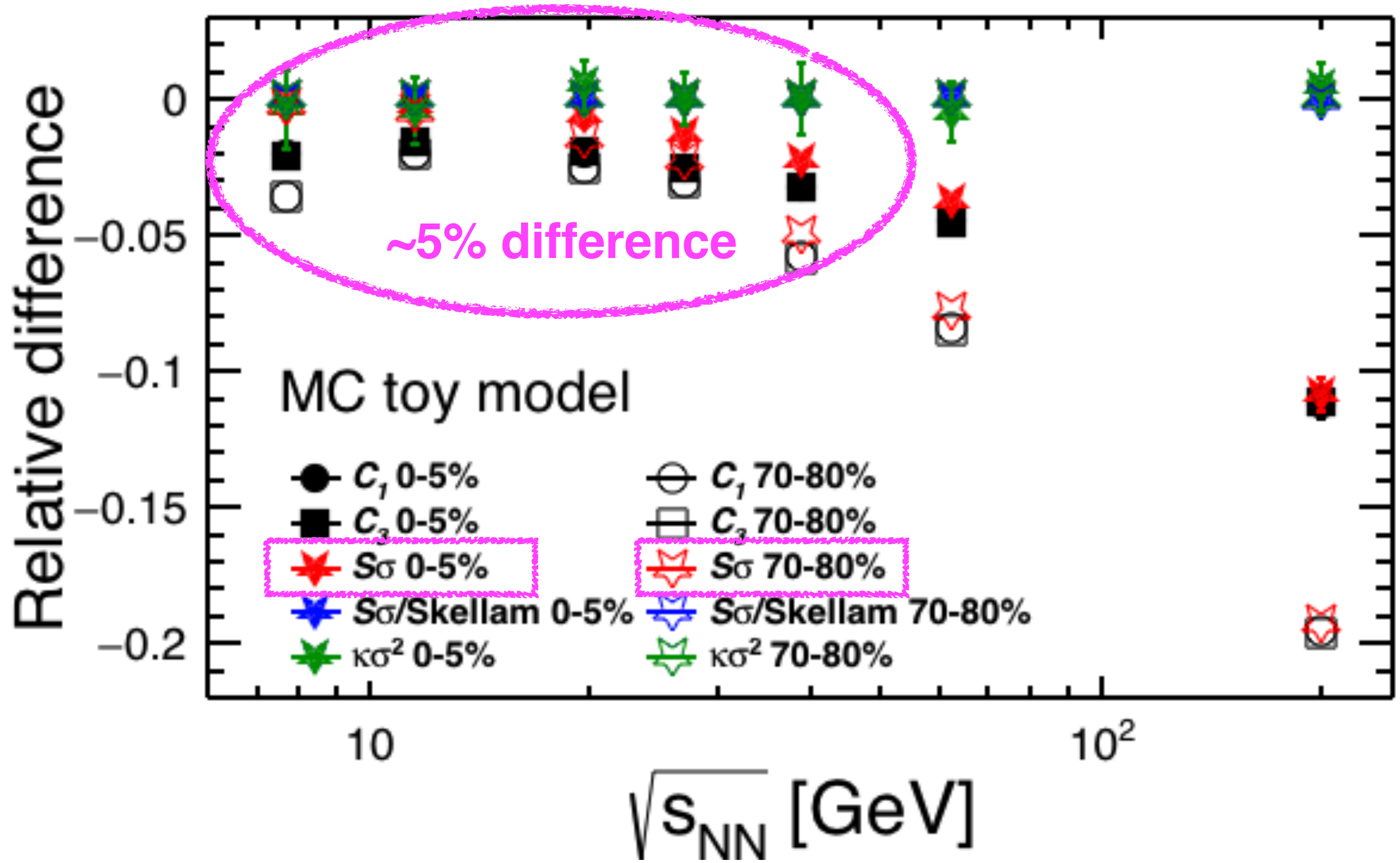


- <M+>-<M-> becomes large at low energy

  → **difference of even order is as large as odd order**

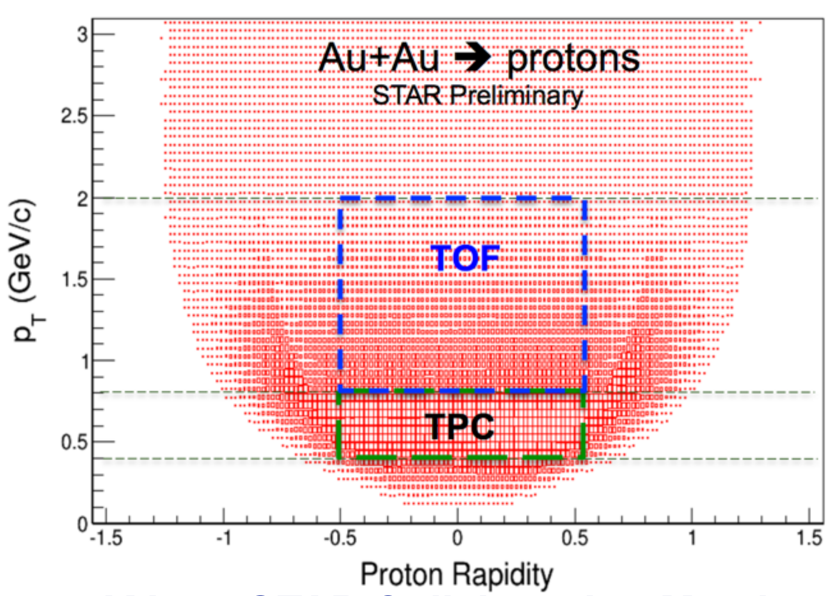
Beam energy dependence

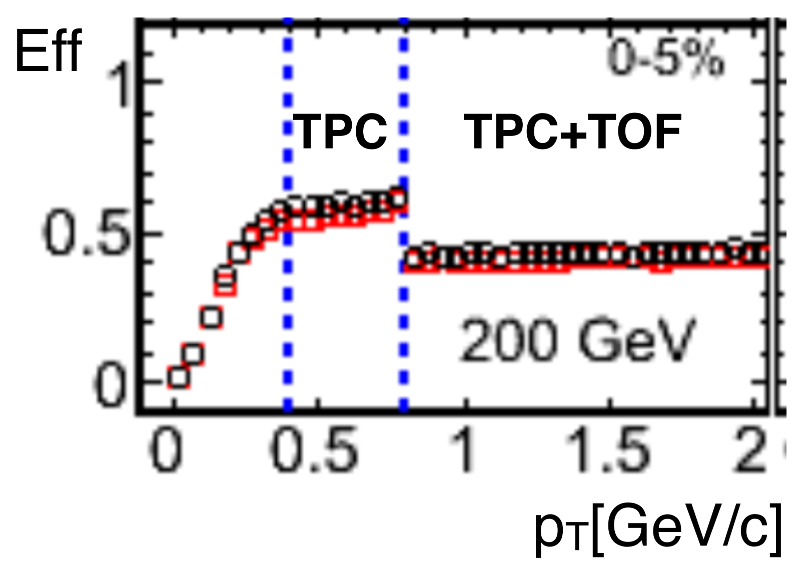
- Relative difference is calculated assuming parameters for each BES energy.
- There is less than 5% difference for *S*σ at experimental interest region(~39GeV).
- **Conclusions of published paper won't be changed.**

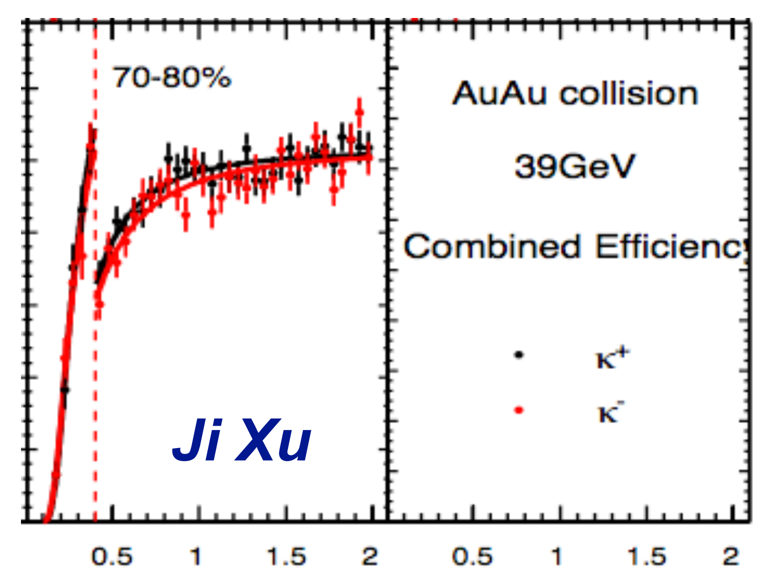


Efficiency correction in case of arbitrary number of phase space

- Efficiency correction method has been developed by V.Koch(PRC 86(2012) 044904) and M.Kitazawa(PRC (2012)86 024904).
- In recent net-proton analysis, phase space is divided into two phase spaces of protons, one is the **low p_T region where only TPC is used for PID**, the other is the **high p_T region where TPC+TOF is used for PID**.
- Efficiency correction method **in case of 4(2+2) phase spaces** has been established by X.Luo.







X.Luo, STAR Collaboration Meeting at Stony Brook University , June 1-6, 2015

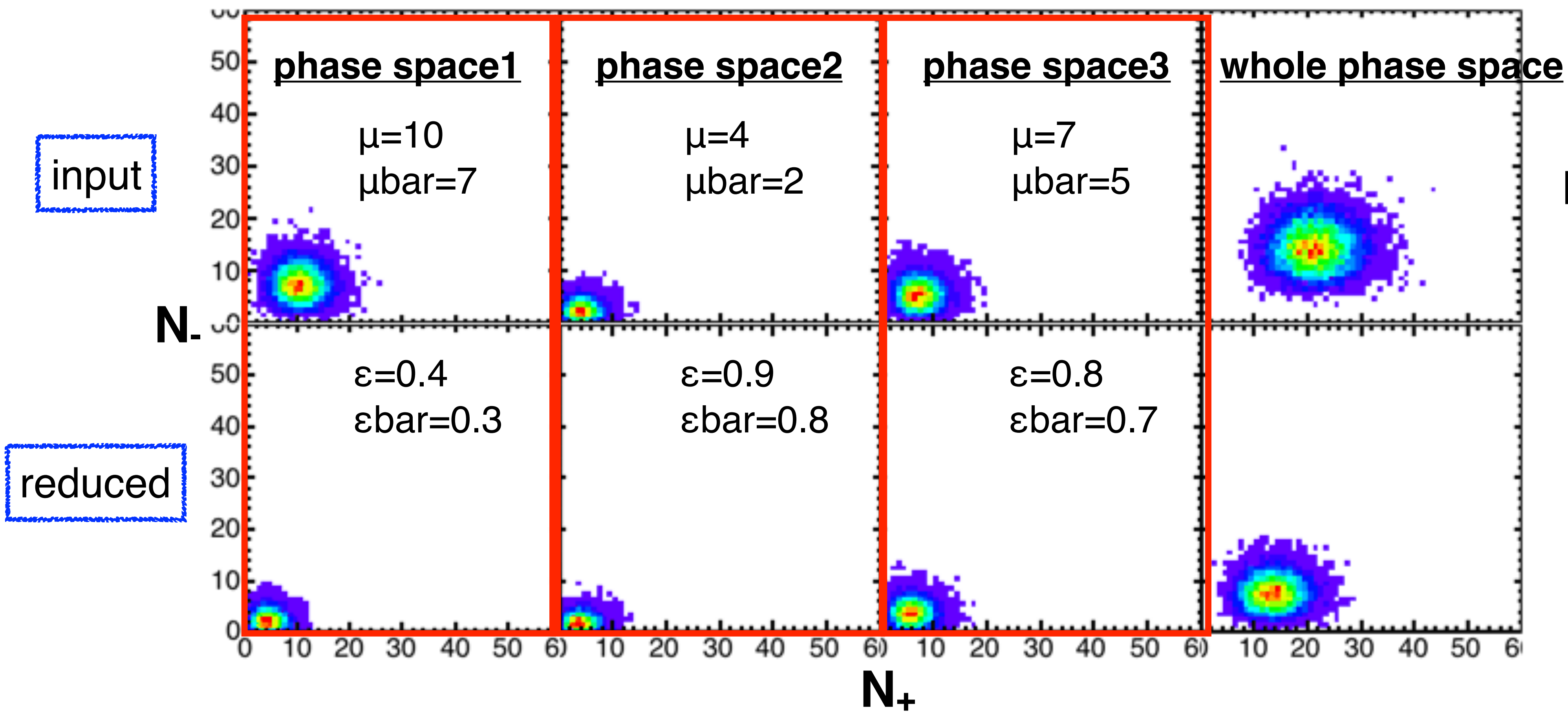
- Efficiency correction **in case of arbitrary number of phase spaces** has been established by H.Masui, which is based on factorial moments, so that one can calculate cumulants easily by C++.

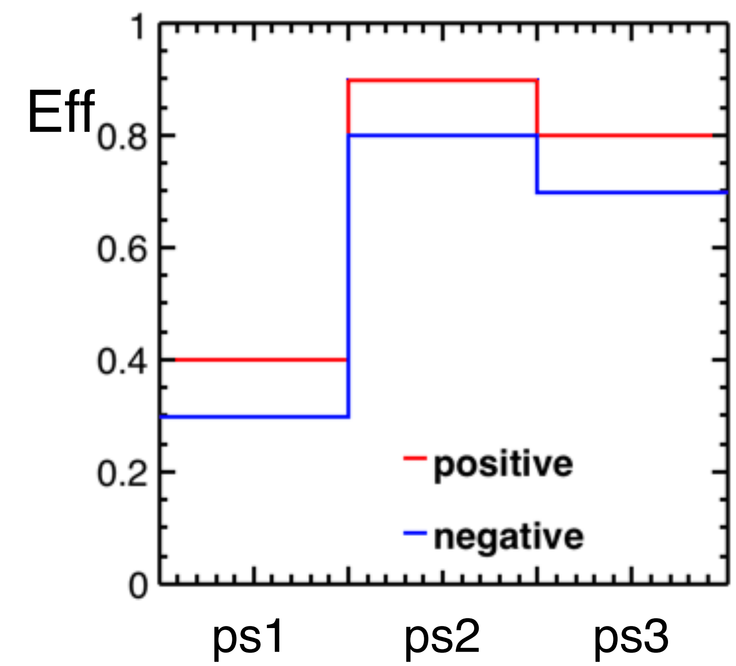
See also : A.Bzdak and V.Koch PRC 91(2015)2,027901

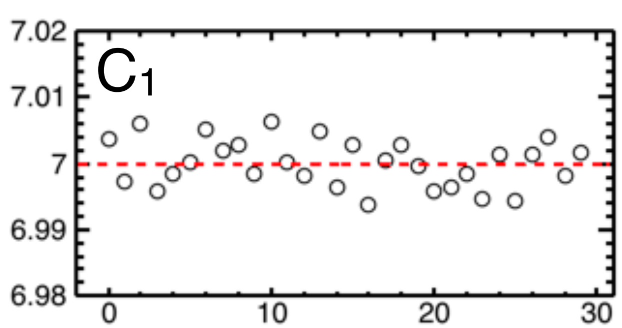
- This might be useful to extend rapidity window more than [-0.5,+0.5] for net-proton, or to correct more accurately at the part of **low p_T region for net-kaon and net-charge**.

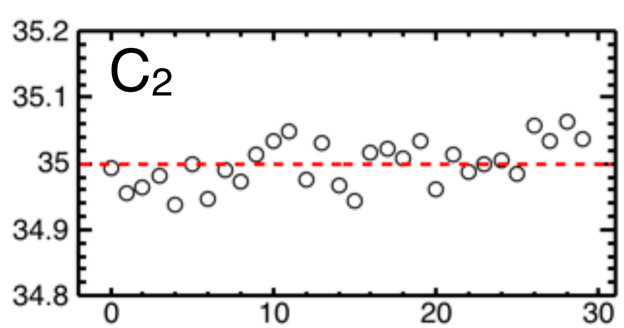
Simple MC toy model (3+3)

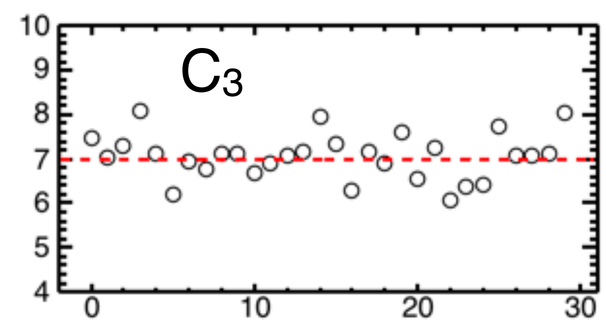
Poisson, 10M events, 30 trials

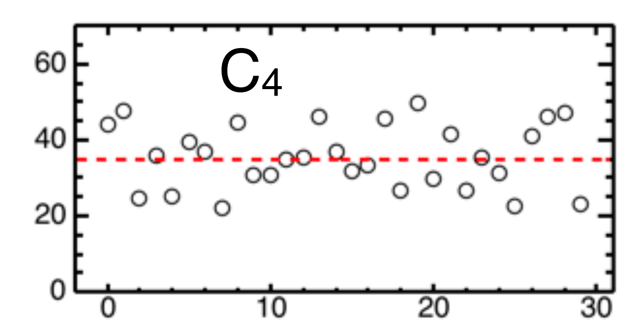








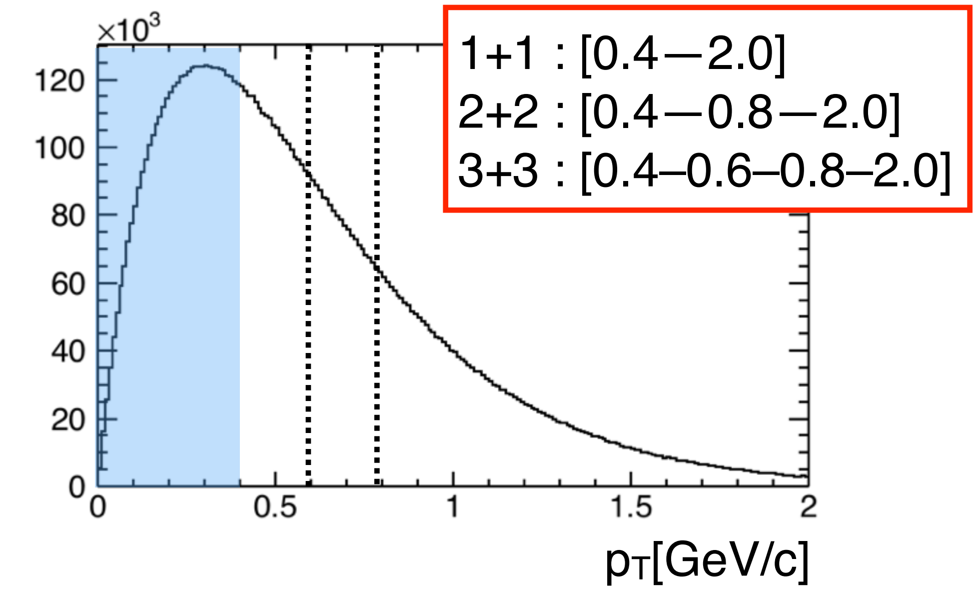


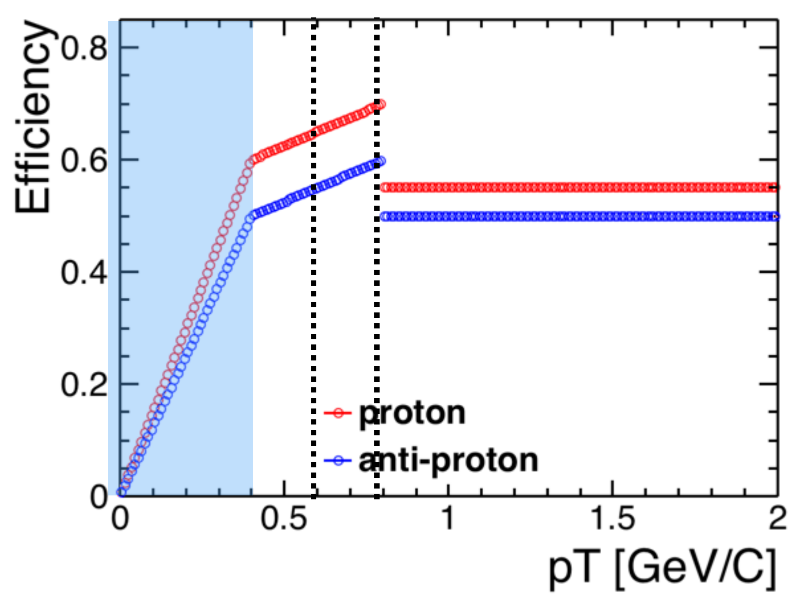


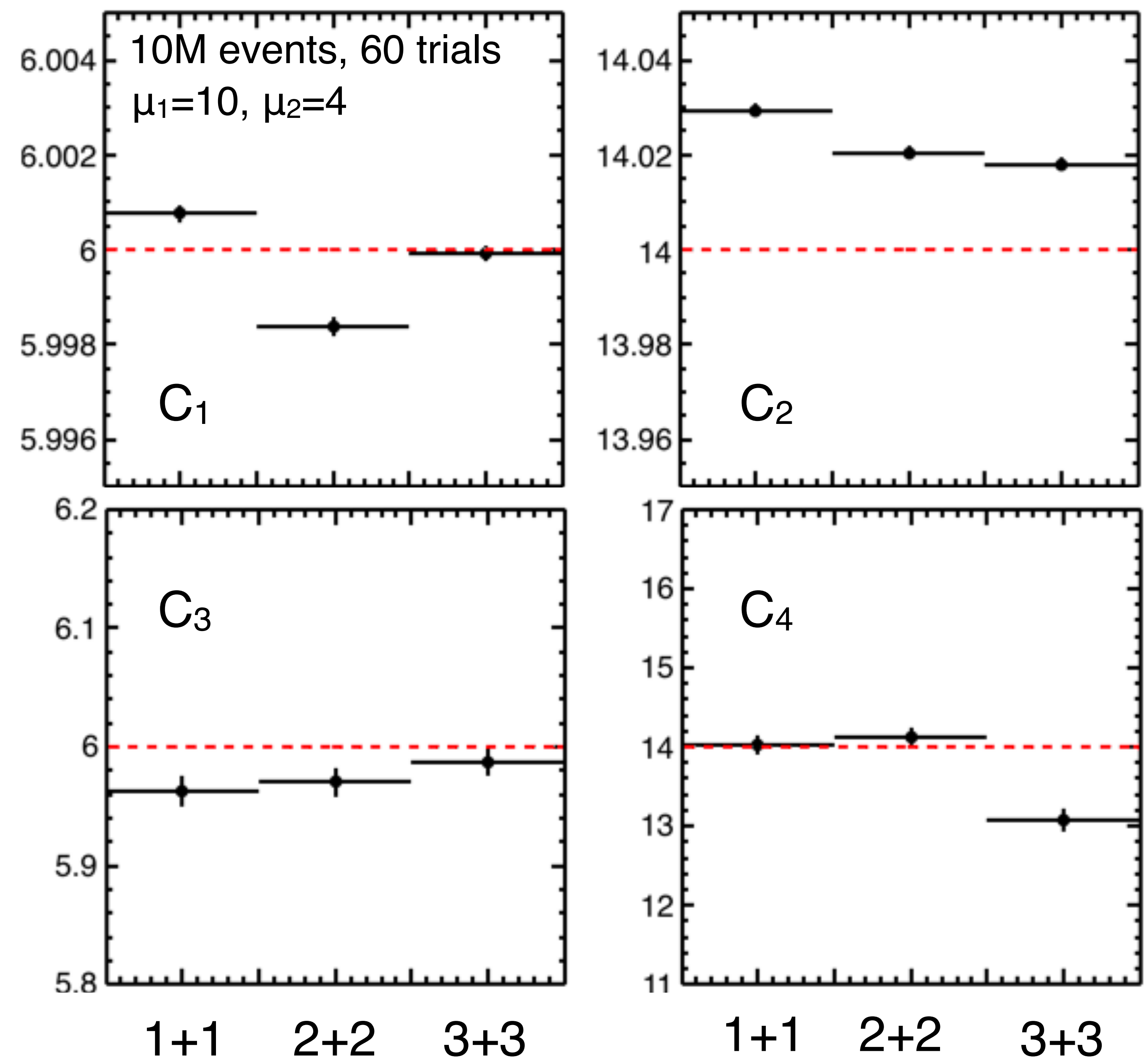
- Efficiency correction seems to work well.

Impose p_T spectra

- MC toy model including p_T dependence of efficiency, roughly assuming net-proton analysis.
- 3+3 is the best for C1, C2 and C3.
- Further study assuming net-charge or net-kaon analysis will be done.







Summary

- Importance of separated efficiencies was shown by MC toy models and analytical calculations.
  → Summary of published net-proton paper don't seem to be change
- The number of phase spaces used for efficiency correction were increased to 3+3.
  → Further study including low p_T region will be done.

Event and track selection

Run10, $\sqrt{s_{NN}}$=200GeV

Event selection

- $|V_z|<30$cm
- $V_r<2$cm
- Request at least one TOF Matched tracks with $\beta>0.1$ within Refmult

statistical errors are estimated by bootstrap method(100 times)

Centrality determination

- $|\eta|<1.0$
- gDCA<3.0
- nHitsFit>10
- if TOF matched
  - $n\sigma_p<-3$ && $m^2<0.4$
- else
  - $n\sigma_p<-3$

Track selection

- $|n\sigma_p|<2$
- nHitsFit/nFitPoss > 0.52
- $|y|<0.5$
- $0.4<p_T<0.8$
- gDCA<1.0cm
- nHitsFit>20
- nHitsDedx>5