Measurements of 1st, 2nd and 3rd azimuthal anisotropy in Cu+Au collisions

TAC seminar
May 26th 2016

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Outline

✓Introduction
- Quark Gluon Plasma(QGP)
- Azimuthal anisotropy
- CuAu collisions

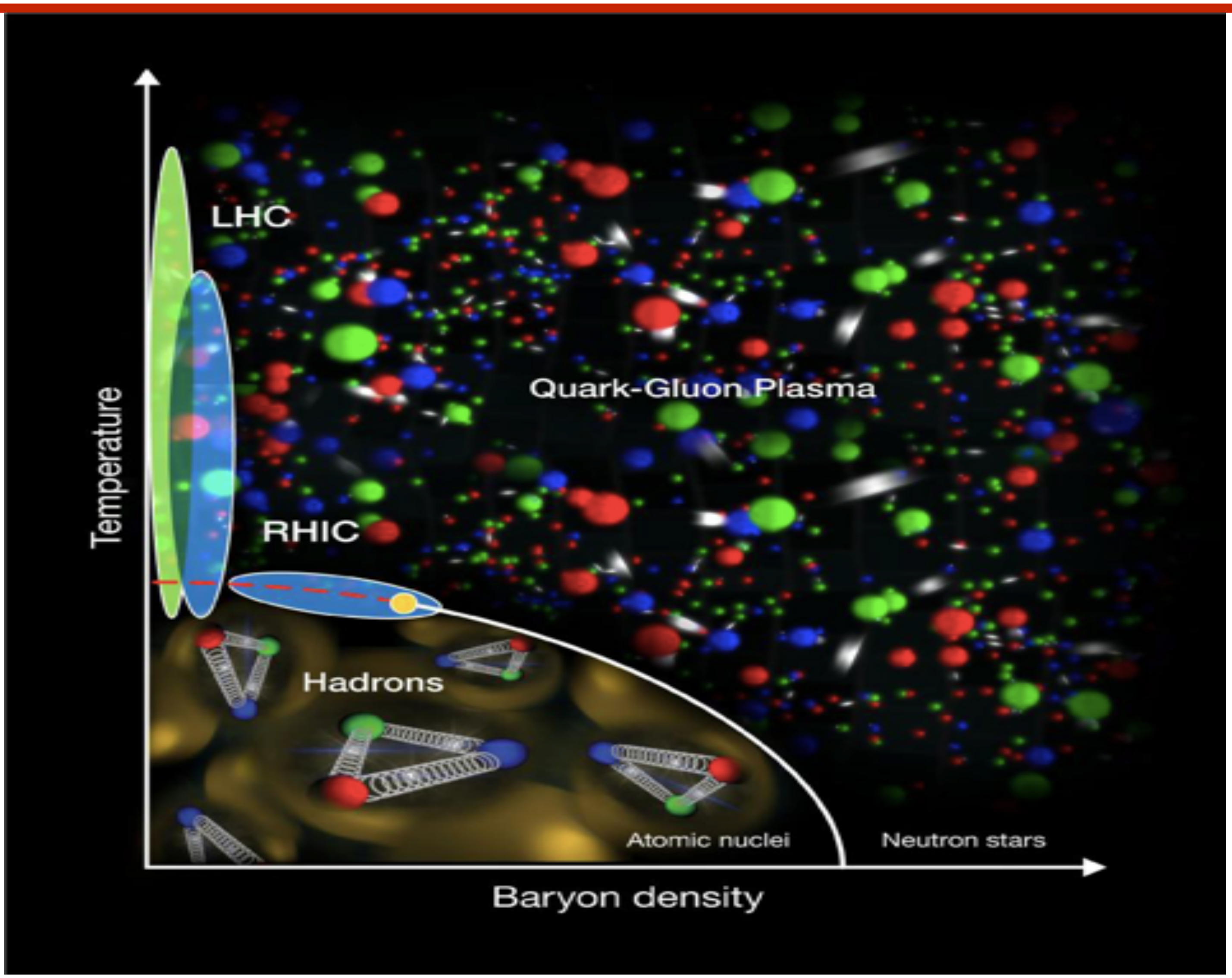
✓Experiment/Analysis
- PHENIX
- Centrality, event plane
- Simulation

✓Results/Discussions
- System size dependence
- Rapidity dependence
- Theory comparison

✓Summary

Introduction

Quark Gluon Plasma(QGP)



QGP is a state of nuclear matter

- **extremely high temperature, density**
- **consist of asymptotic free quarks and gluons**
- **Almost perfect liquid**

Predicted phase transition ε_c and T_c by Lattice QCD calculation

- **$T_c \sim 170$ MeV**
- **$\varepsilon_c \sim 1$ [GeV/fm^3]**

Relativistic Heavy Ion Collider(RHIC)

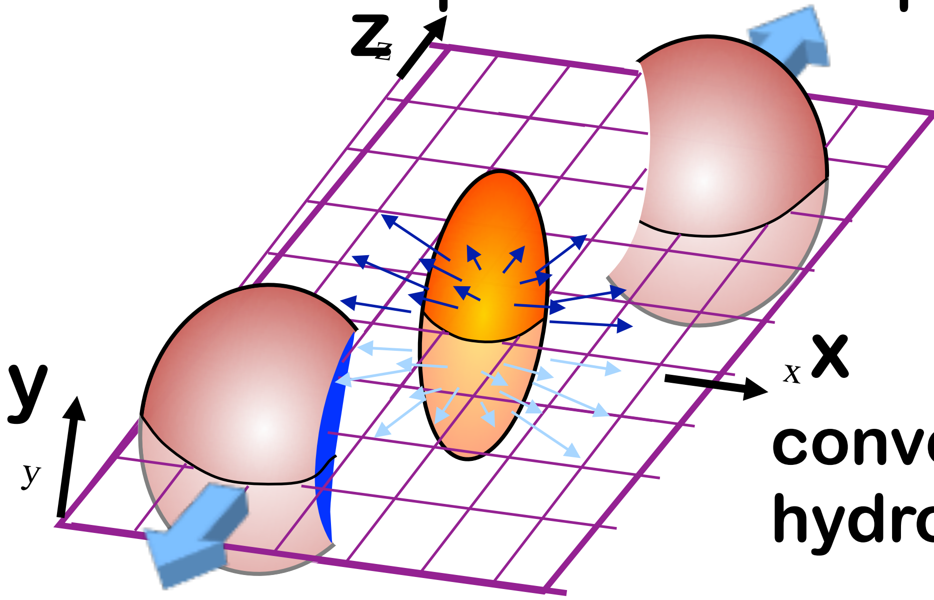
Species	Energies
Au+Au	200, 130, 62.4GeV 39, 27, 22.4GeV 19.6 14.6, 7.7GeV
Cu+Cu	200, 62.4, 22.4GeV
U+U	193GeV
Cu+Au	200GeV
3He+Au	200GeV
d+Au	200GeV
p+Au	200GeV
p+Al	200GeV
p+p	510, 500, 200GeV 62.4GeV

Wide range of species and energies

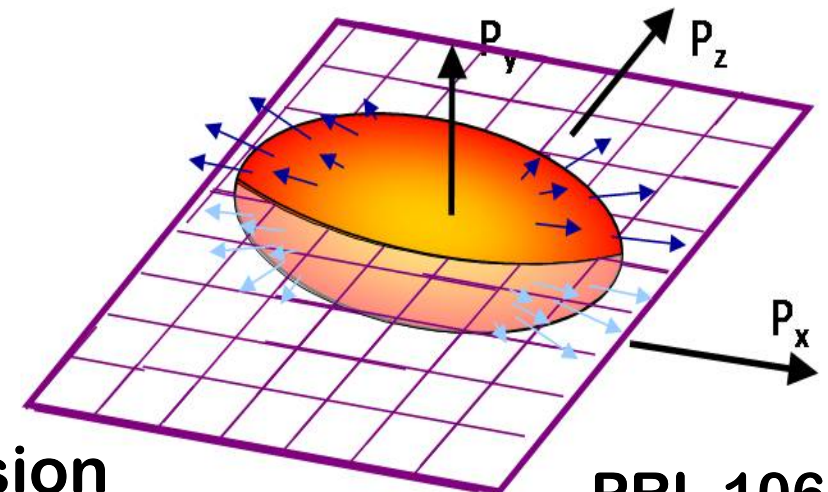
Relativistic heavy ion collision
is unique tool to form QGP
Au+Au 200GeV@RHIC
- ε_{Bj} ~ 5 [GeV/fm^3] > ε_c

Azimuthal anisotropy:Elliptic flow

Initial spatial anisotropy:ε_2

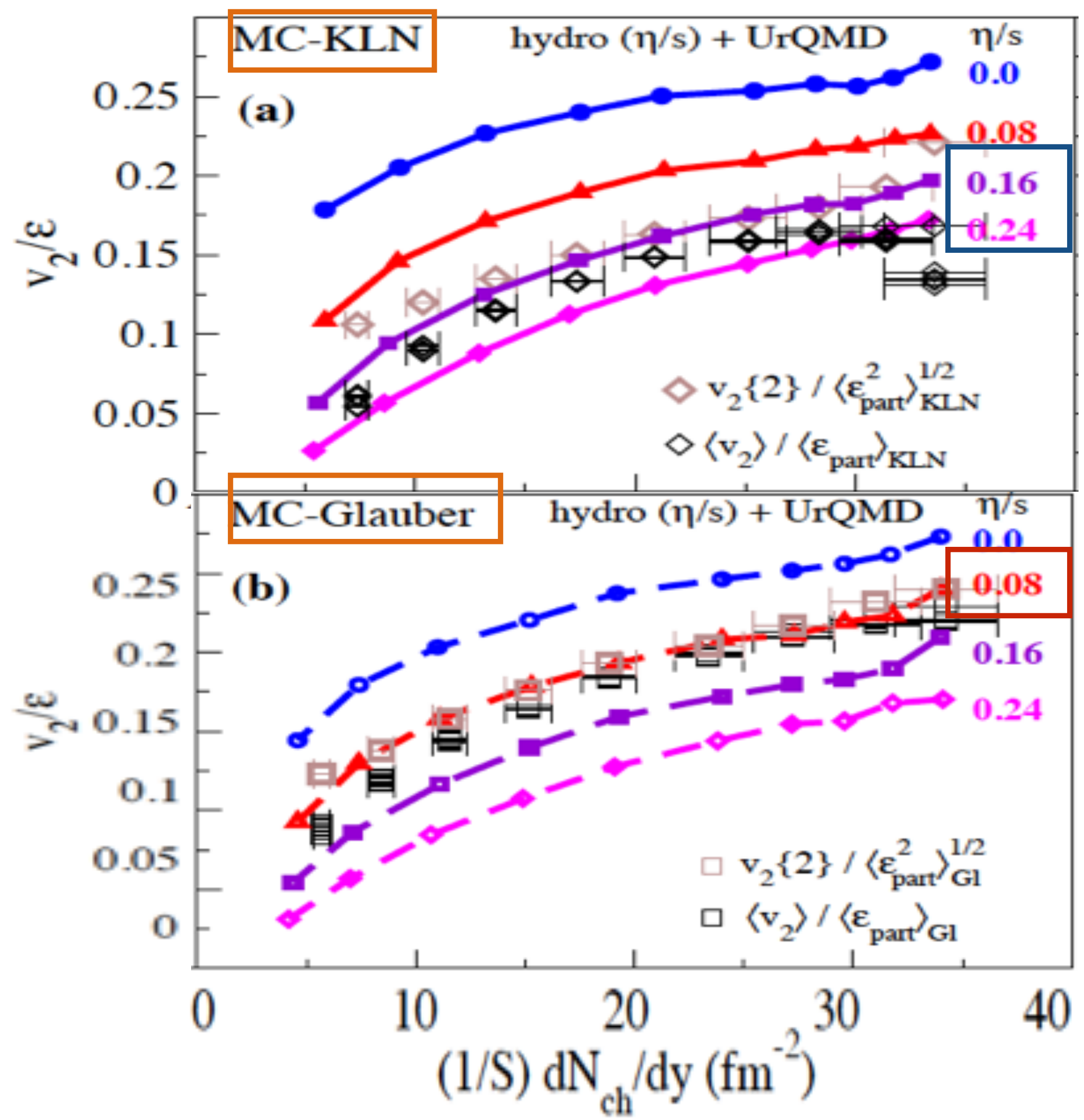


momentum anisotropy:v_2



converted though hydrodynamic expansion

PRL 106.192301

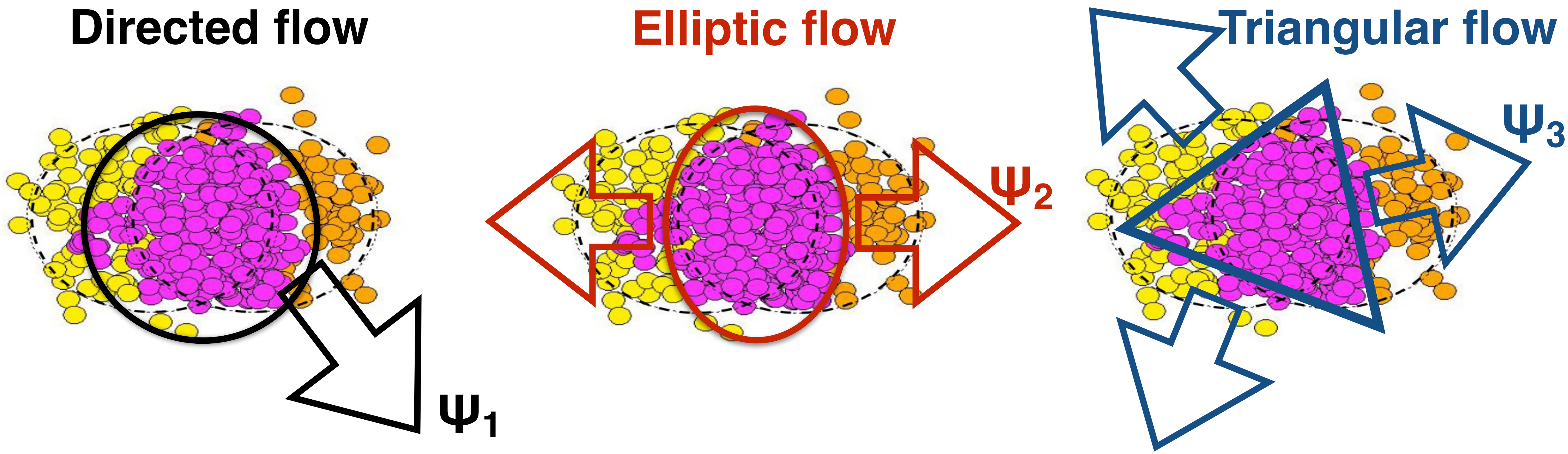


particle production will have
an elliptical azimuthal distribution.

- Non-isotropic pressure gradient

Sensitive to
-initial condition
(Glauber,KLN..etc.)
-viscosity
(η/s)

Azimuthal anisotropy:Directed,Triangular flow



Event by event, initial participant fluctuation can lead to

-Directed particle production anisotropy v_1

-Triangular particle production anisotropy v_3

-v_4, v_5, v_6

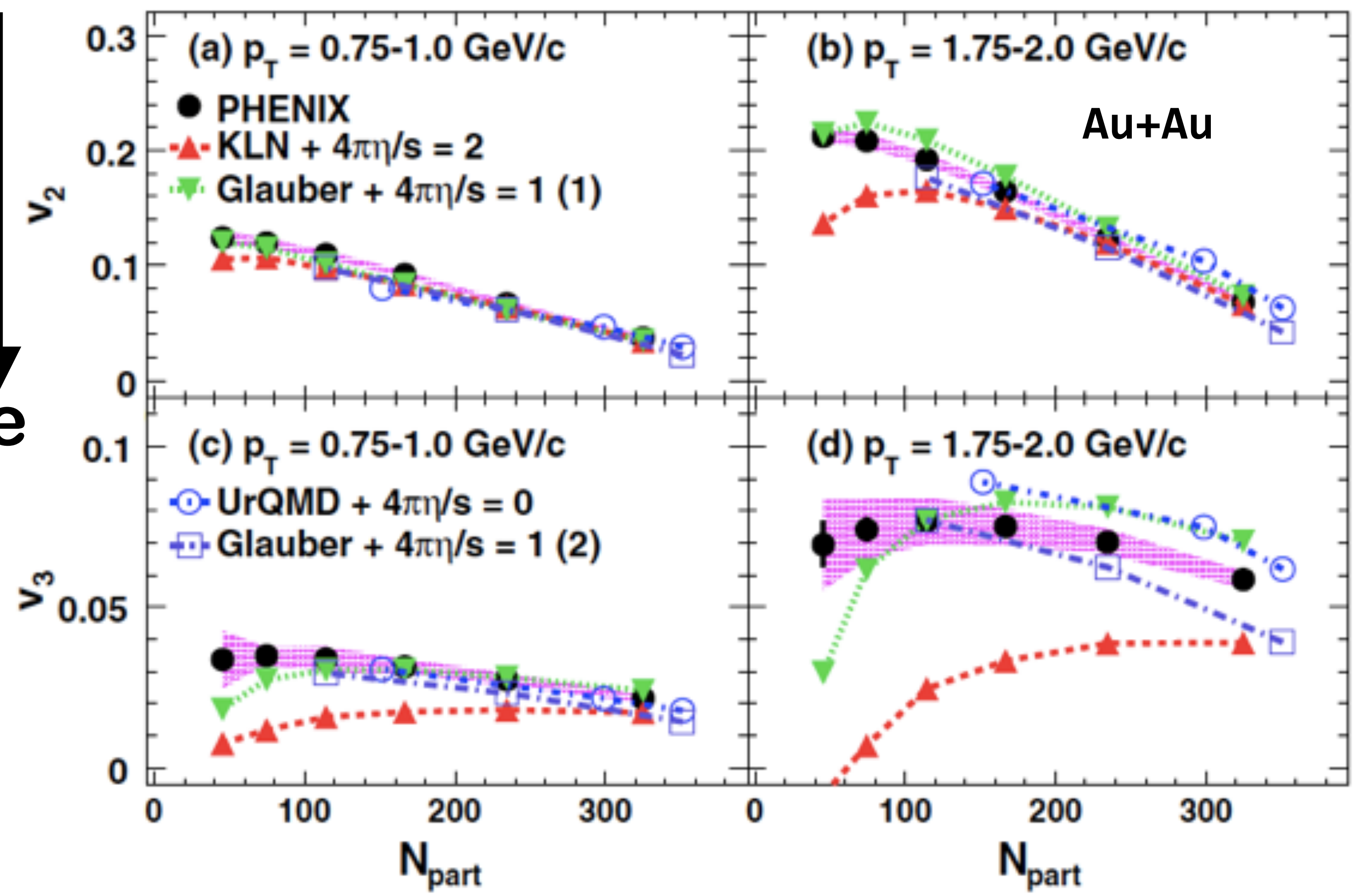
v_n constrain initial condition & viscosity

Theory model consists of
initial condition(collision)
+
hydro with viscosity(QGP)
+
hadron gas

time

initial condition		viscosity
KLN	+	$4\pi\eta/s=2$
Glauber	+	$4\pi\eta/s=1$
Glauber	+	$4\pi\eta/s=1$

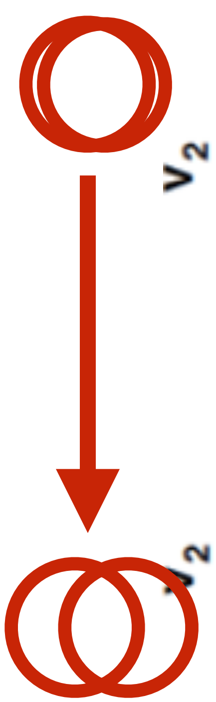
v2, v3 theory comparison

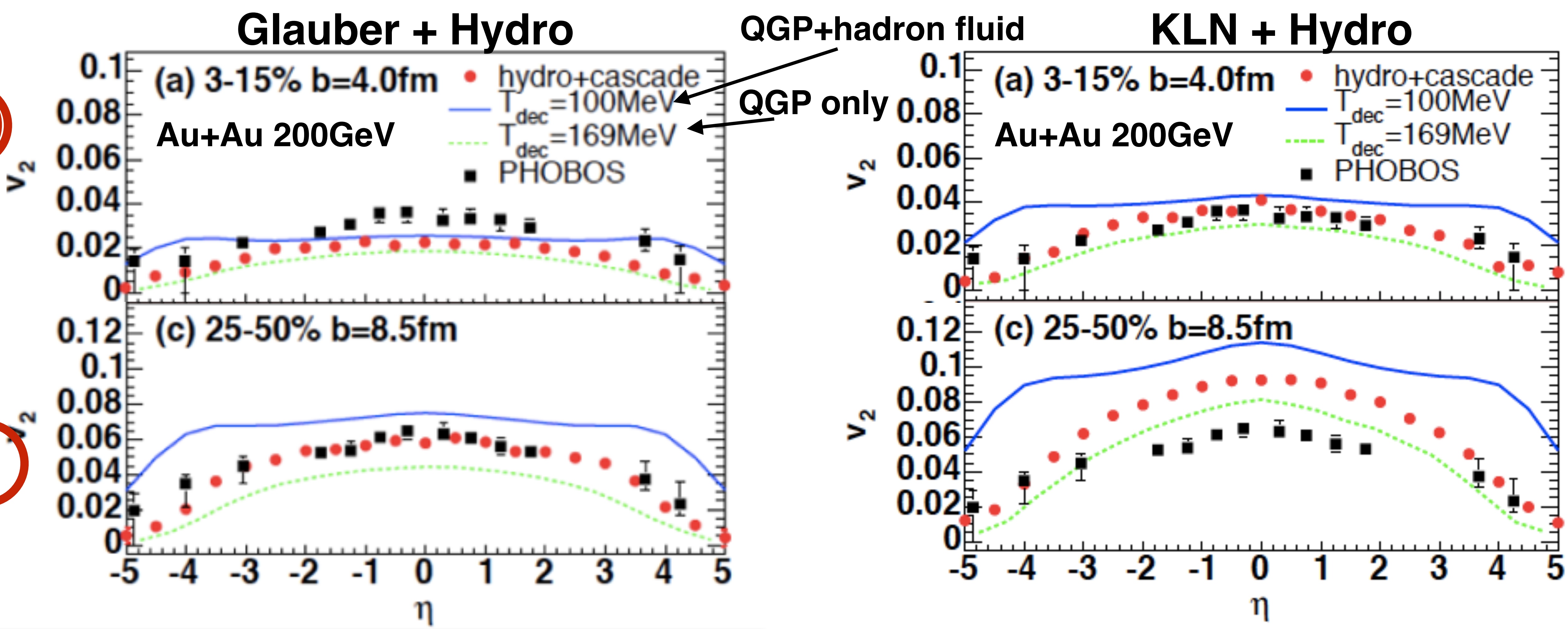


v2, v3 are sensitive to initial condition and viscosity of QGP
- Theoretically, initial condition and viscosity have uncertainty

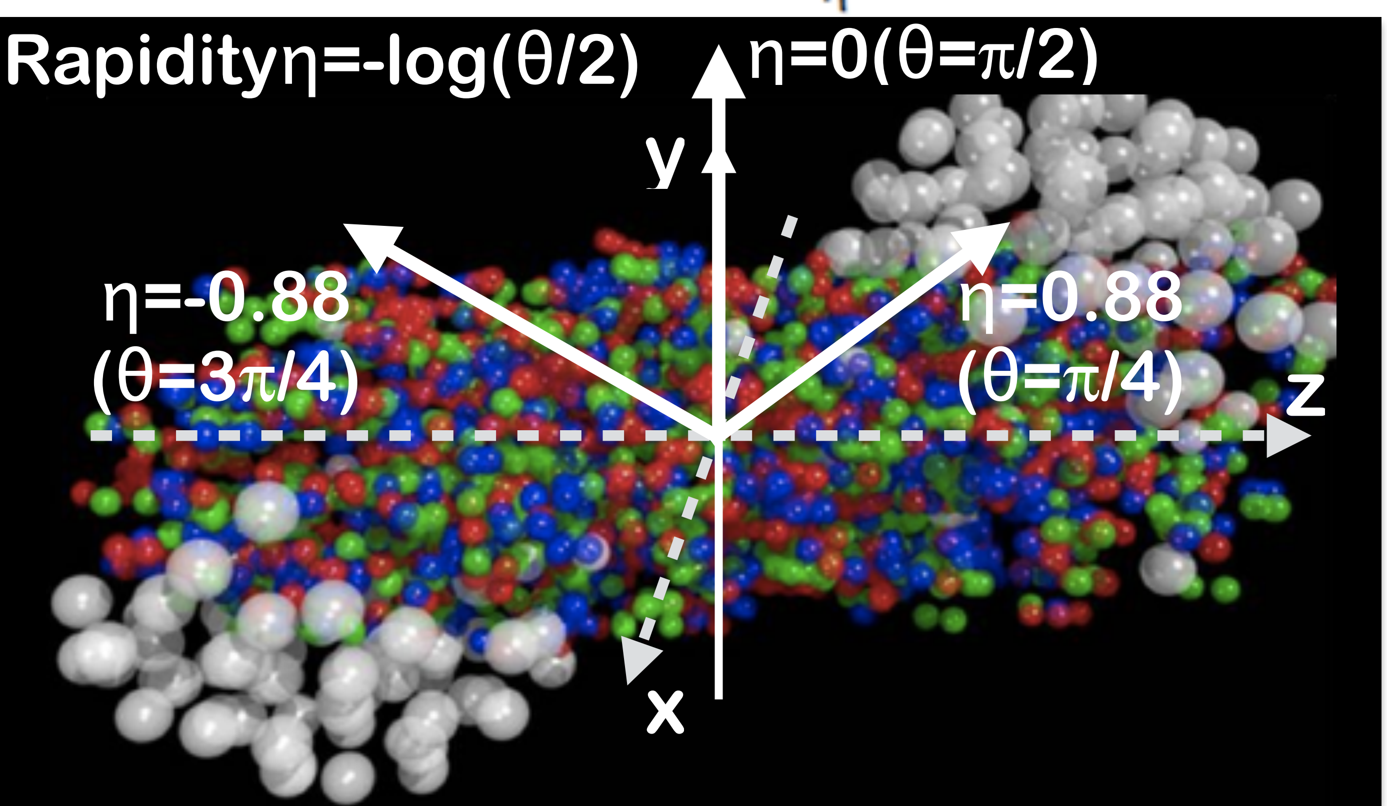
->v_n are good constraint of both of them

η dependence of v_2 with different initial conditions





Phys.Lett.B636:299-304



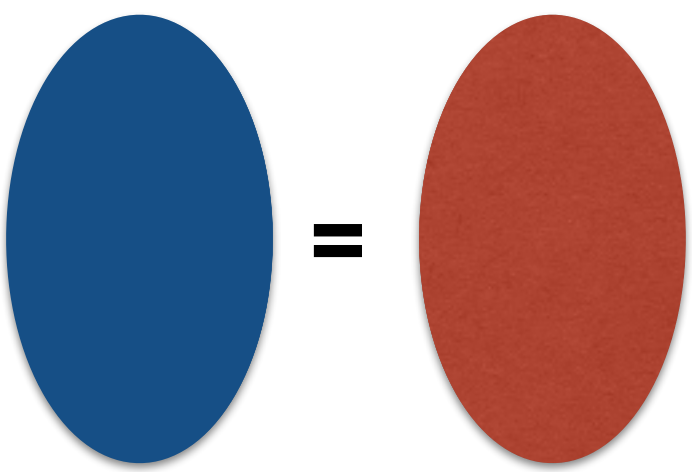
✓ $v_2(\eta)$ depends on initial condition

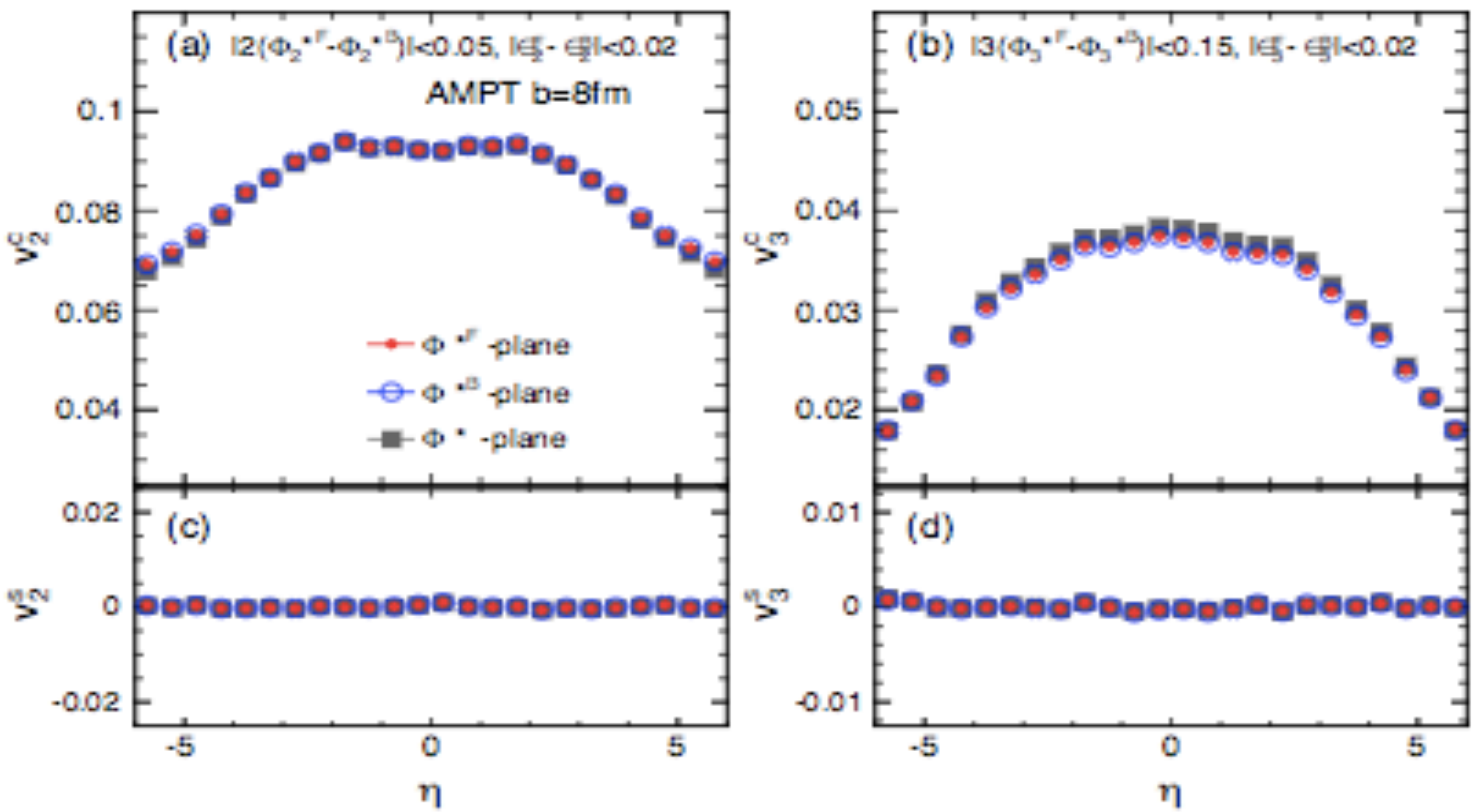
- v_2 with KLN > v_2 with Glauber
- v_2 with Glauber reproduce data in mid-central collision
- v_2 with KLN reproduce data in central collision

Initial condition on target and projectile nuclei

PhysRevC.90.034915

$\varepsilon_n(+\eta) = \varepsilon_n(-\eta)$





✓Initial spatial anisotropy on target and projectile nuclei are same event

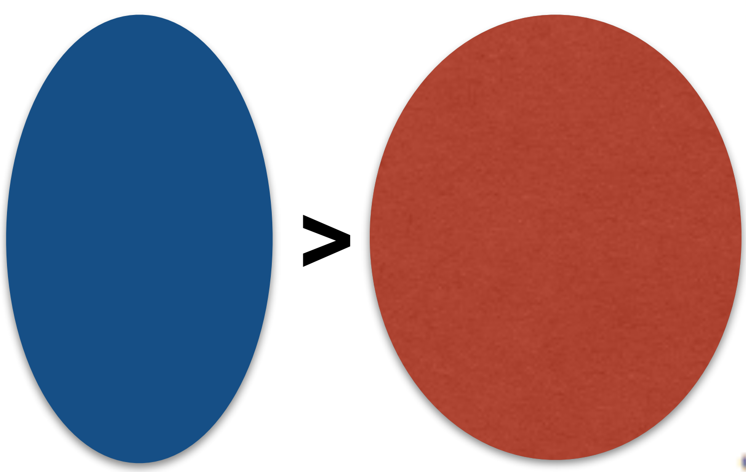
-Symmetric $v_n(\eta)$

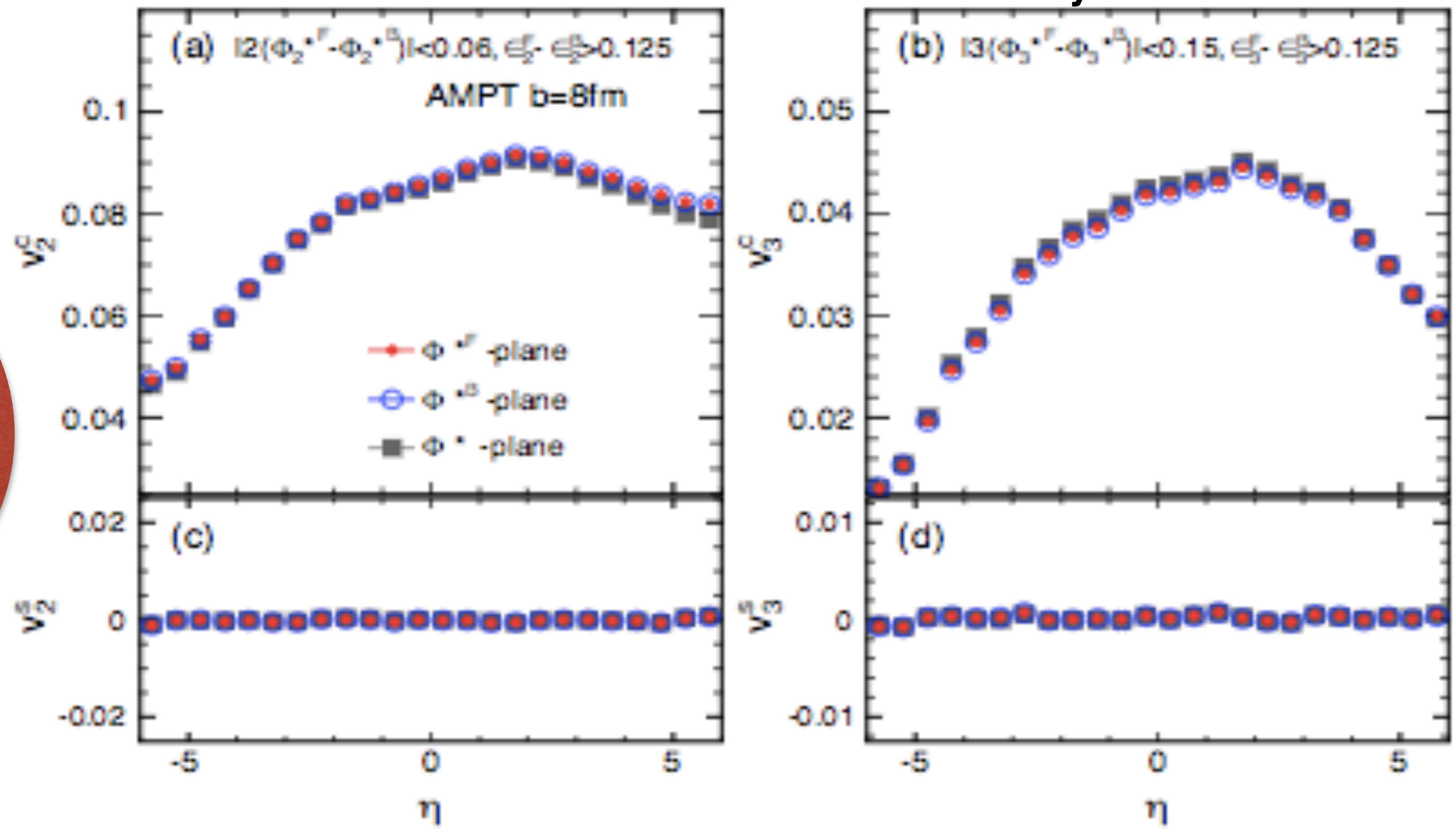
$\varepsilon_n(+\eta) = \varepsilon_n(-\eta)$ ➡ $v_n(+\eta) = v_n(-\eta)$

Initial condition on target and projectile nuclei

PhysRevC.90.034915

$\varepsilon_n(+\eta) > \varepsilon_n(-\eta)$



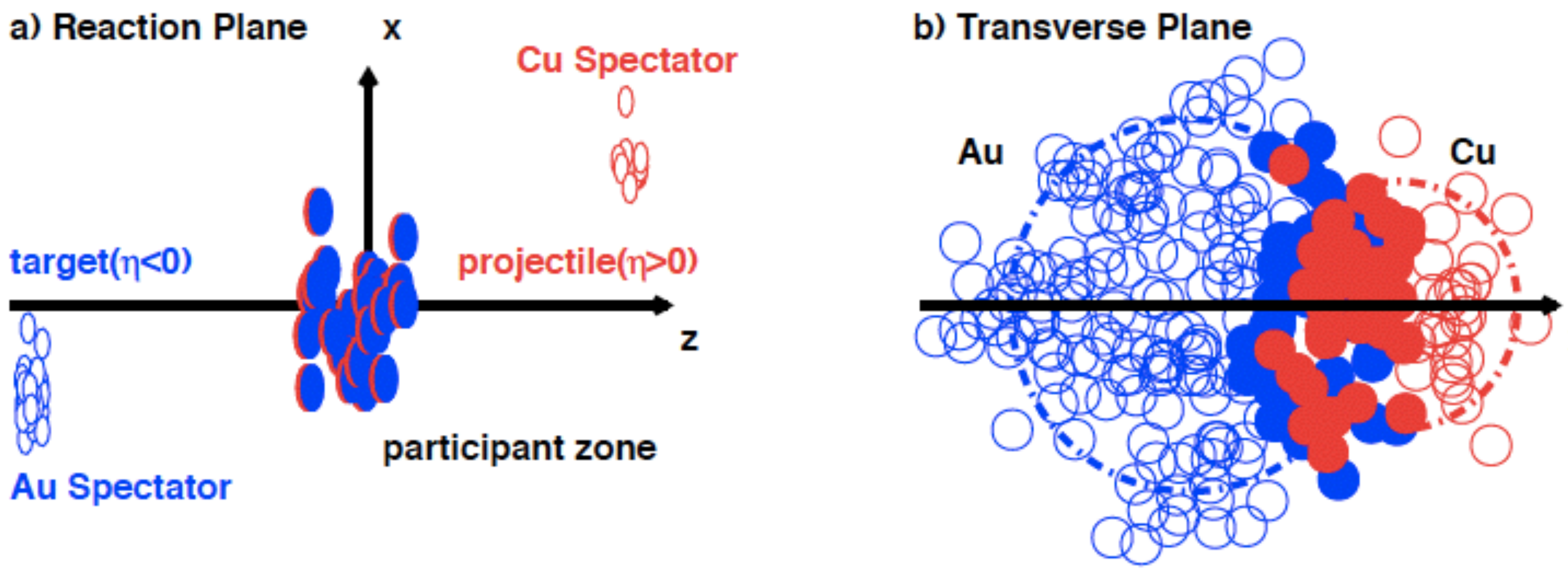


✓Initial conditions on target and projectile nuclei are not same event

$\varepsilon_n(+\eta) > \varepsilon_n(-\eta)$ ➡ $v_n(+\eta) > v_n(-\eta)$

✓Theory predicts the two different initial conditions survives collective expansion

Cu+Au collision



So far, vn have been studied in symmetric collision systems
First asymmetric Cu+Au collisions were operated in 2012

Asymmetric initial condition provides

- **Different left/right pressure gradient, particle production….**
- **Longitudinally, above characteristics could be different in Au-going/Cu-going**

-> Measurement of vn in asymmetric system could be good study of initial condition

My activity

M1~M2 (2011~2013)

Repair VTX @BNL
JPS Spring & Fall (Talk)
QM2012 (poster)
ATHIC 2012 (Talk)
AuAu flow analysis using VTX

D1(2013~2014)

Repair VTX @BNL
Shift taking & detector expert for Run 13, Run14

CuAu flow analysis

D2 (2014~2015)

QM2014(Talk)
JPS-DNP(Talk)
Shift taking & detector expert for Run 14, Run15

D3(2015~2016)

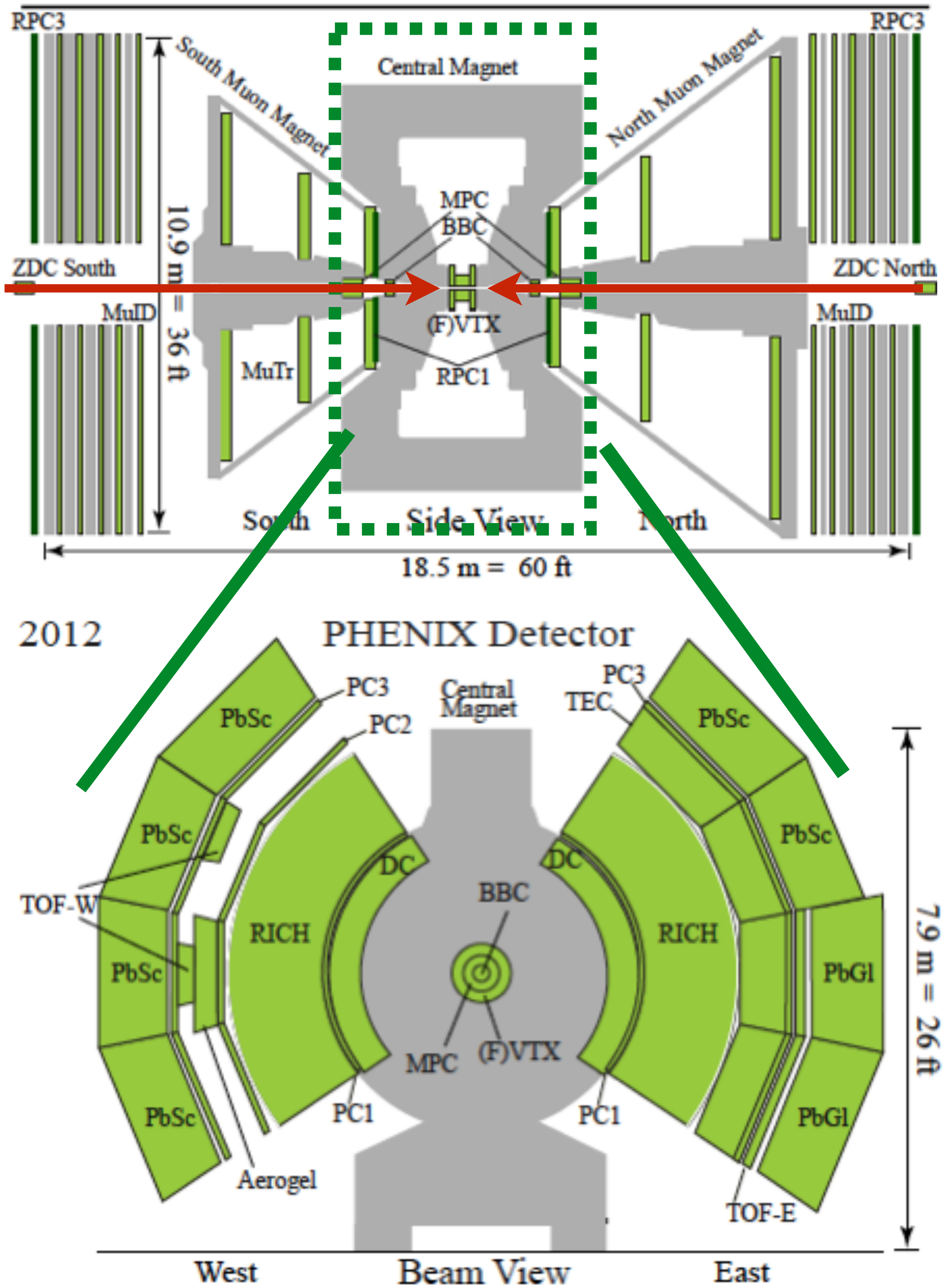
QM2015(Poster)
TGSW2015 (Talk)
WWND2016(Talk)
Submit CuAu flow paper to arXiv

D4 (2016~)

Domestic conference
Inter national conference
Hardware and shift

Experiment Analysis

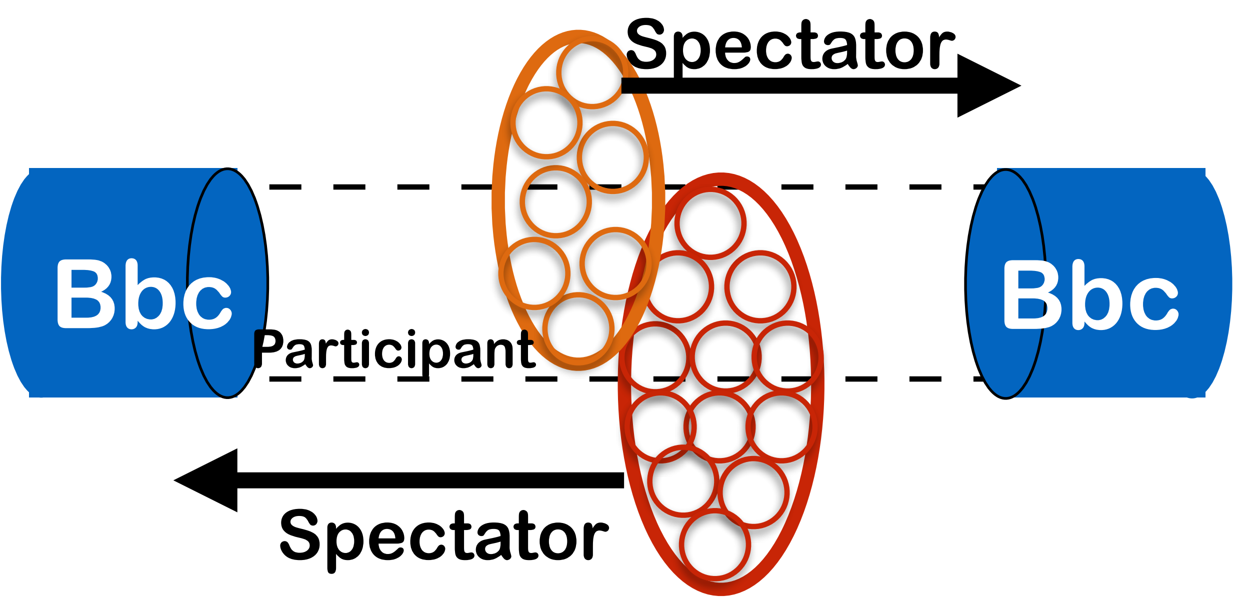
PHENIX detectors



Trigger, centrality, collision vertex
- **Beam Beam counter(BBC) (3<|η|<4)**
- **Zero degree calorimeter**
- **Shower max detector**

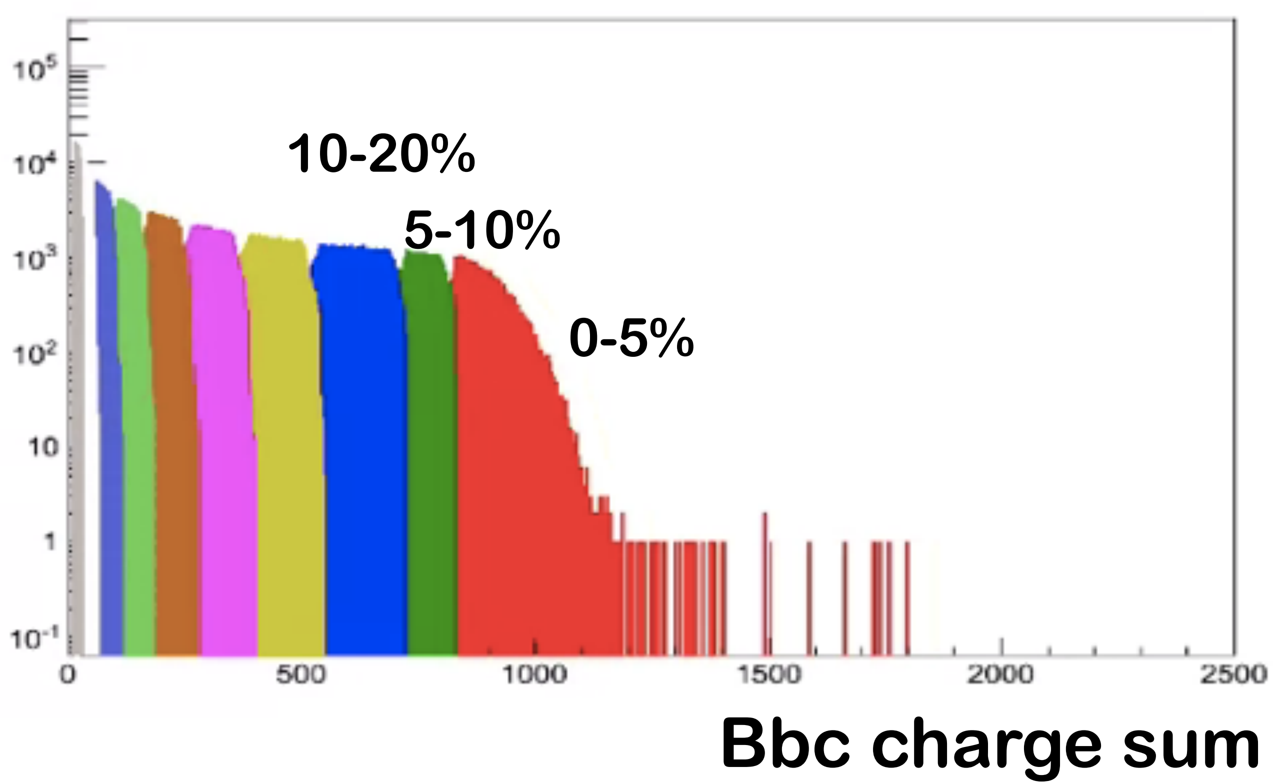
Charged particle Tracking
- **Drift Chamber(DC) (|η|<0.35)**
  - **Momentum**
- **Pad Chamber(PC) (|η|<0.35)**
  - **Hit position**
- **Electro magnetic calorimeter(EMC) (|η|<0.35)**
  - **Hit position**

Collision centrality



✓Overlap zone of two nuclei
- Impact parameter
- Number of participant nucleons
- Multiplicity

✓ Experimentally, overlap zone is classified by multiplicity in Bbc
- Multiplicity in Bbc
  - ∝ Number of participant nucleons
  - ∝ Overlap zone



✓ Each percentile contains same number of events
- Most central collision 0 %
- Most peripheral collisions 100%

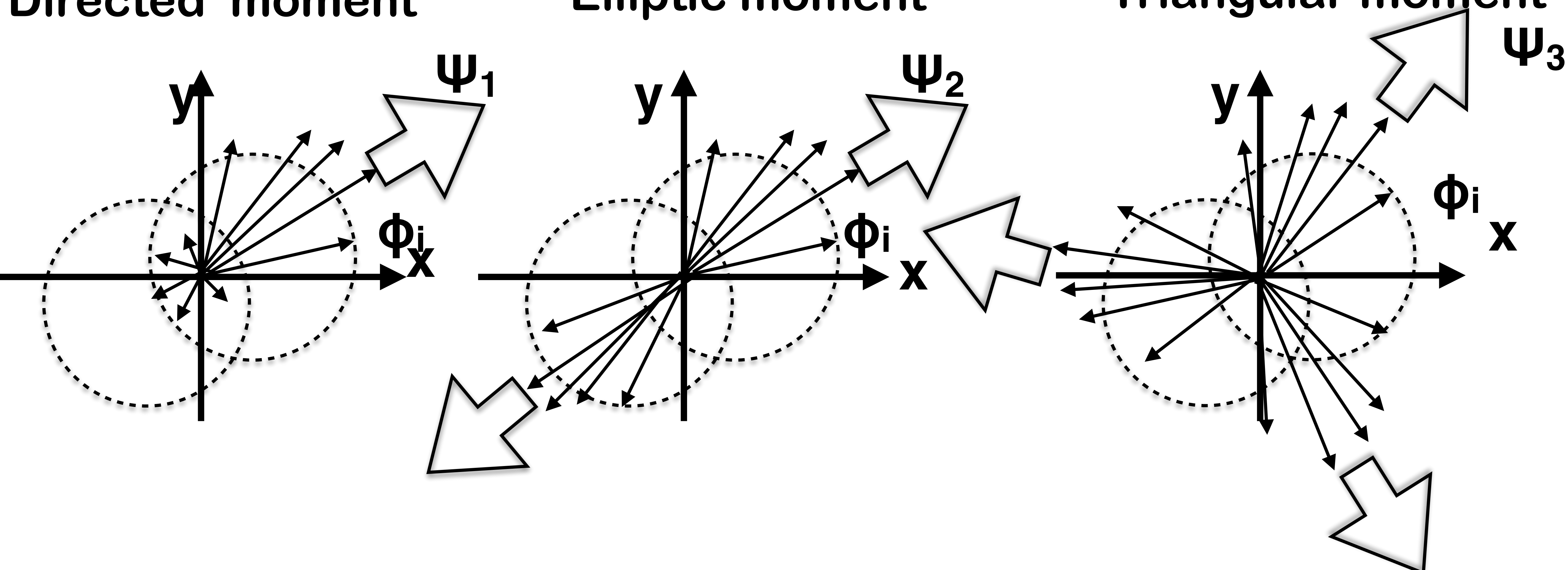
Anisotropy measurement via Event Plane method

Event plane(EP) method

- one of the flow measurement methods
- produced particles are measured with respect to EP
- EP is the azimuthal direction most particles are emitted to
- observed v_n is corrected by EP resolution

$$v_n = \frac{< cos(n[\phi - \Psi_n^{obs}]) >}{Res\{\Psi_n^{obs}\}}$$

Directed moment **Elliptic moment** **Triangular moment**



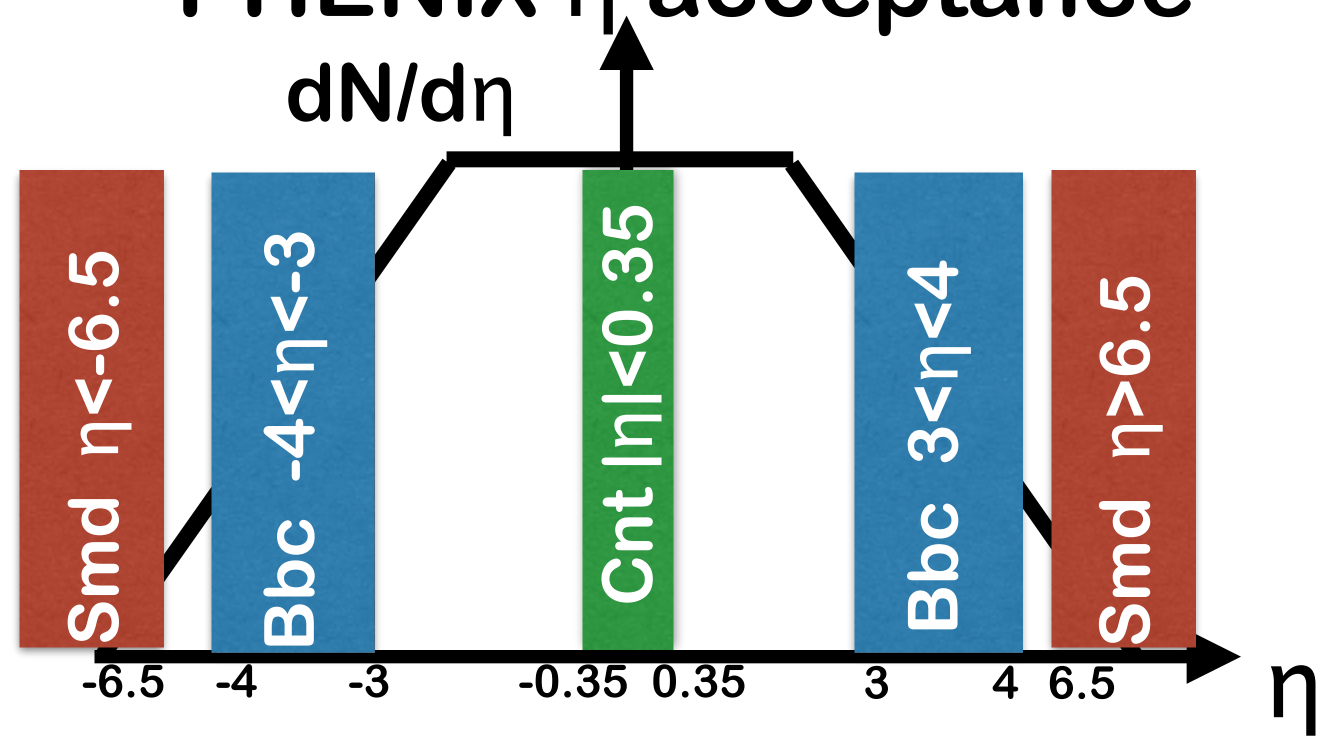
Event plane detectors and resolutions

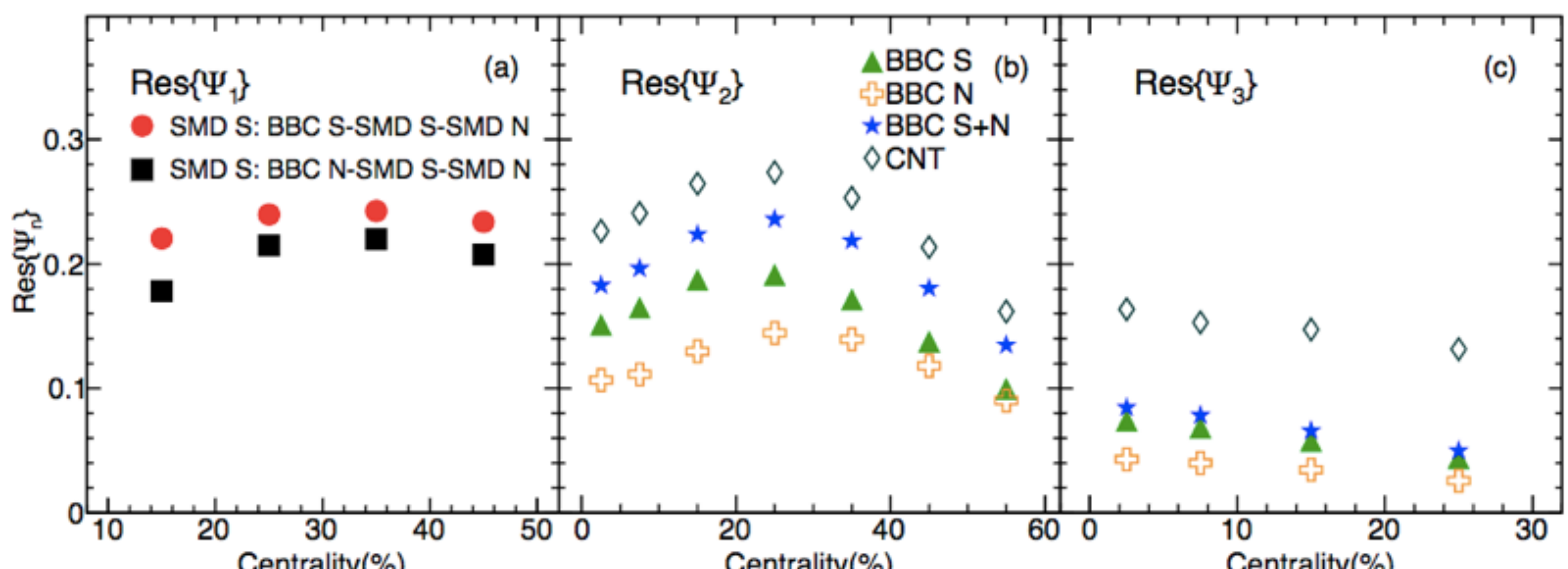
Event Plane detectors
- 2nd, 3rd Event plane
  - Bbc, Cnt
- 1st Event plane
  - Bbc, Smd

Event Plane resolution
-Estimated from EP correlations(3sub method)

$$Res\{\Psi^{obs}_{n,A}\} = \langle \cos(n[\Psi^{obs}_{n} - \Psi^{true}_{n}]) \rangle$$

PHENIX η acceptance



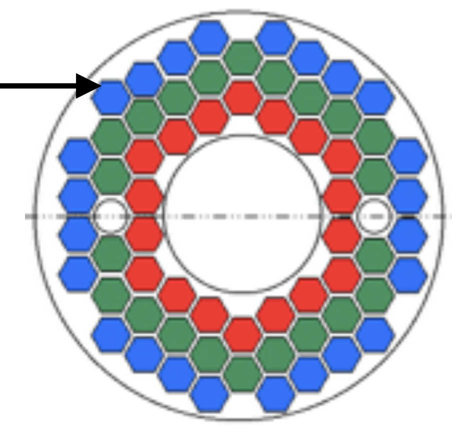


v_n measurement at Bbc(3<|η|<4)

✓ v_n is measured using 64 Bbc pmts

- Bbc can't reconstruct tracks
- pmt based v_n include back ground

pmt



✓ Run Giant simulation with PHENIX configuration

- pmt based v_n -> track based v_n

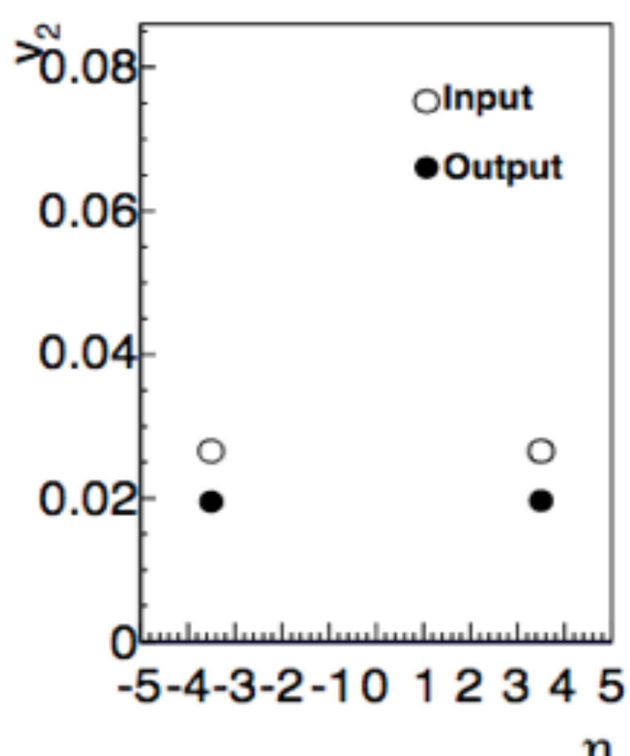
$$v_n^{track} = R_n * v_n^{pmt} \qquad R_n = \frac{v_{n,input}^{Sim}}{v_{n,output}^{Sim}}$$

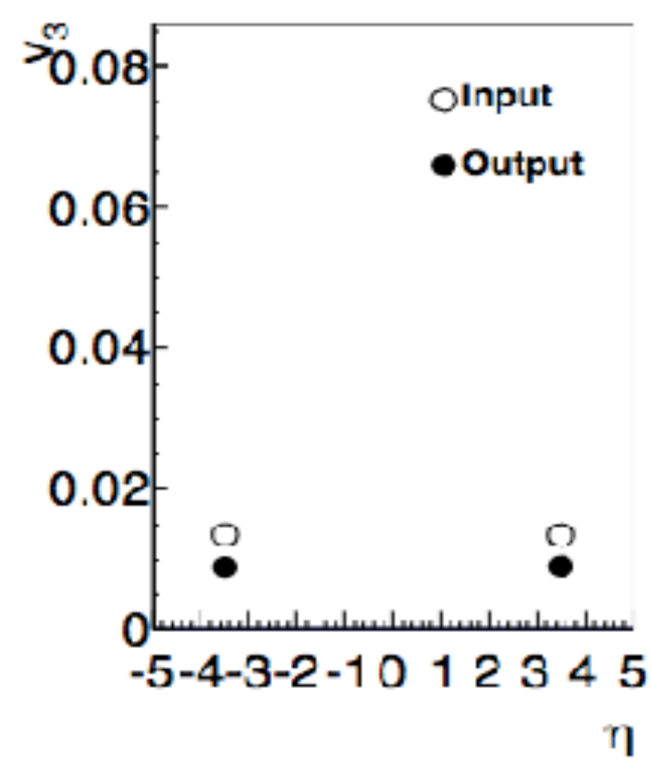
$v_{n,input}^{Sim}$
Input v_n from particle simulation

$v_{n,output}^{Sim}$
Output v_n from Giant simulation

✓ Correction factor R_n

- R_2 : 0.73
- R_3 : 0.65





✓ Systematic study

- dN/deta
- pT spectra
- vn(pt)
- vn(eta)

Initial spatial anisotropy

Glauber Monte Carlo simulation

-Wood Saxon density profile

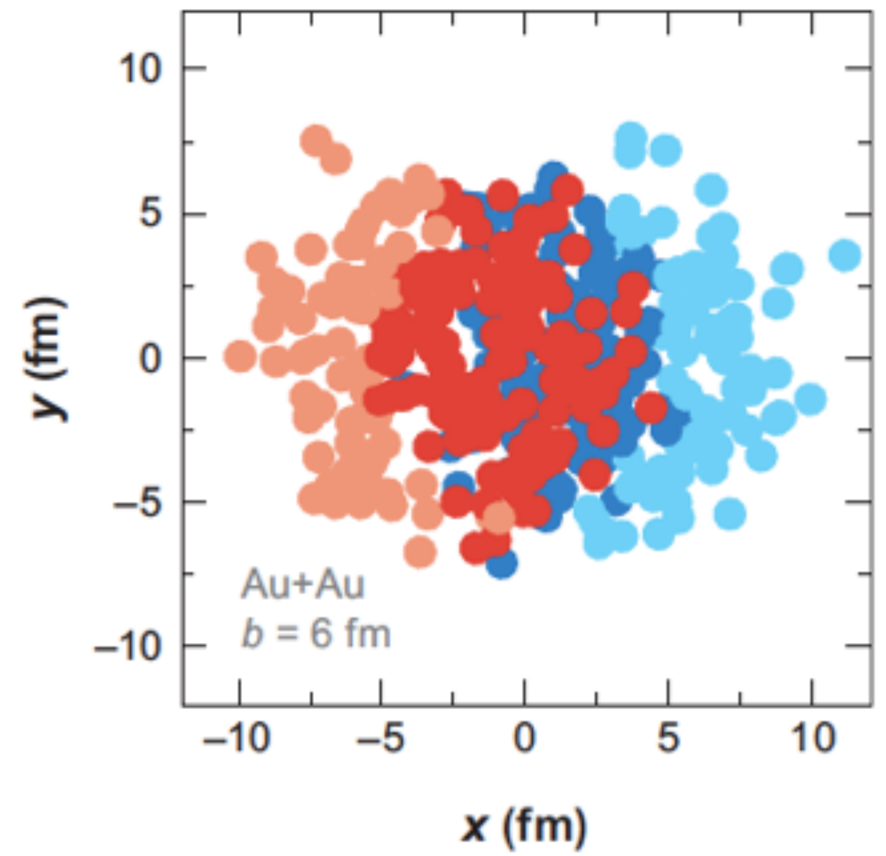
- collision is occurred, if $d < \sqrt{(\sigma_{nn}/\pi)}$

  d:distance between nucleons

  σ_{nn}:total cross section(pp collision)

$$\epsilon_n = \frac{\langle r^2 \cos[n(\phi - \Psi_{n,PP})] \rangle}{\langle r^2 \rangle}$$

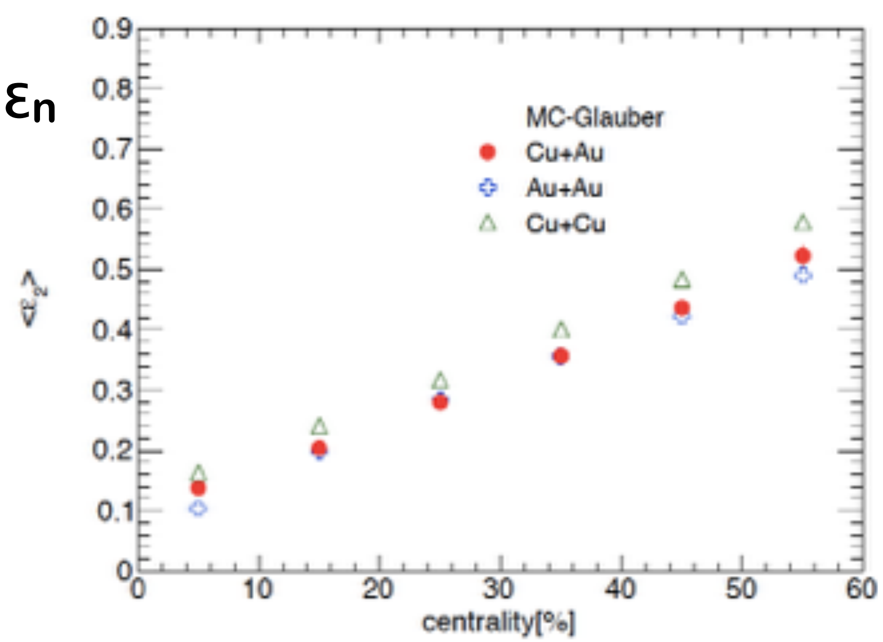
$$\Psi_{n,PP} = \frac{1}{n}[\tan^{-1}\frac{\langle r^2 \sin(n\phi)\rangle}{\langle r^2 \cos(n\phi)\rangle} + \pi]$$

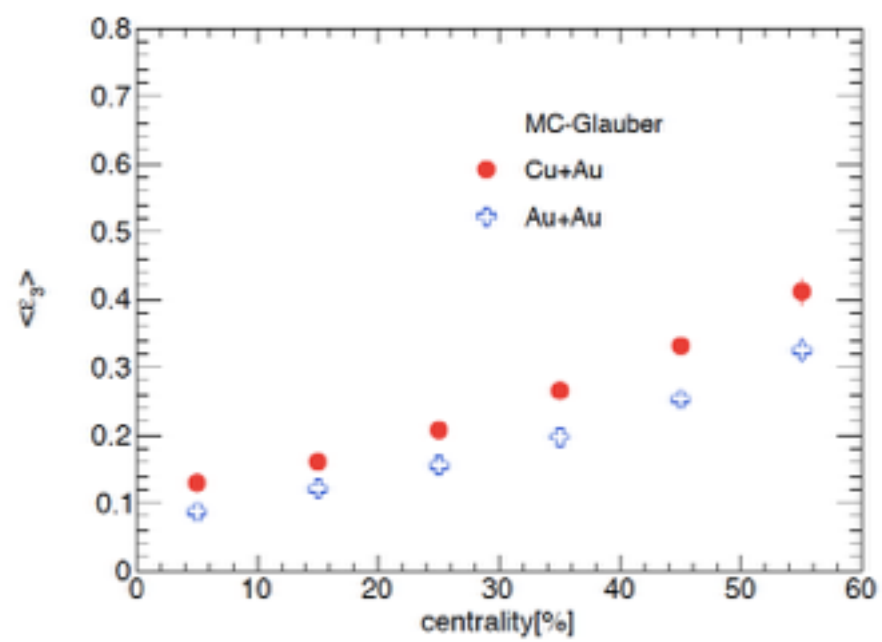


Centrality dependence of ϵ_n

ϵ_2:CuCu>CuAu~AuAu

ϵ_3:CuAu>AuAu

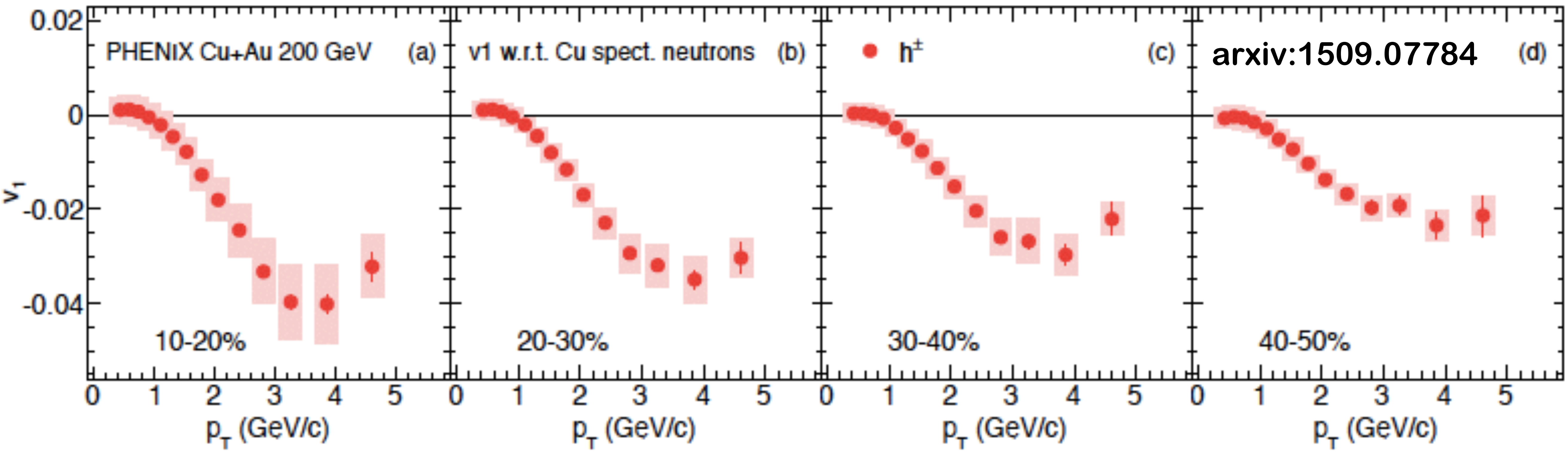




Results
Discussions

- v_1,v_2,v_3 at mid-η
- v_2,v_3 at large-η
- v_1,v_2,v_3 theory comparison

Charged hadron v_1

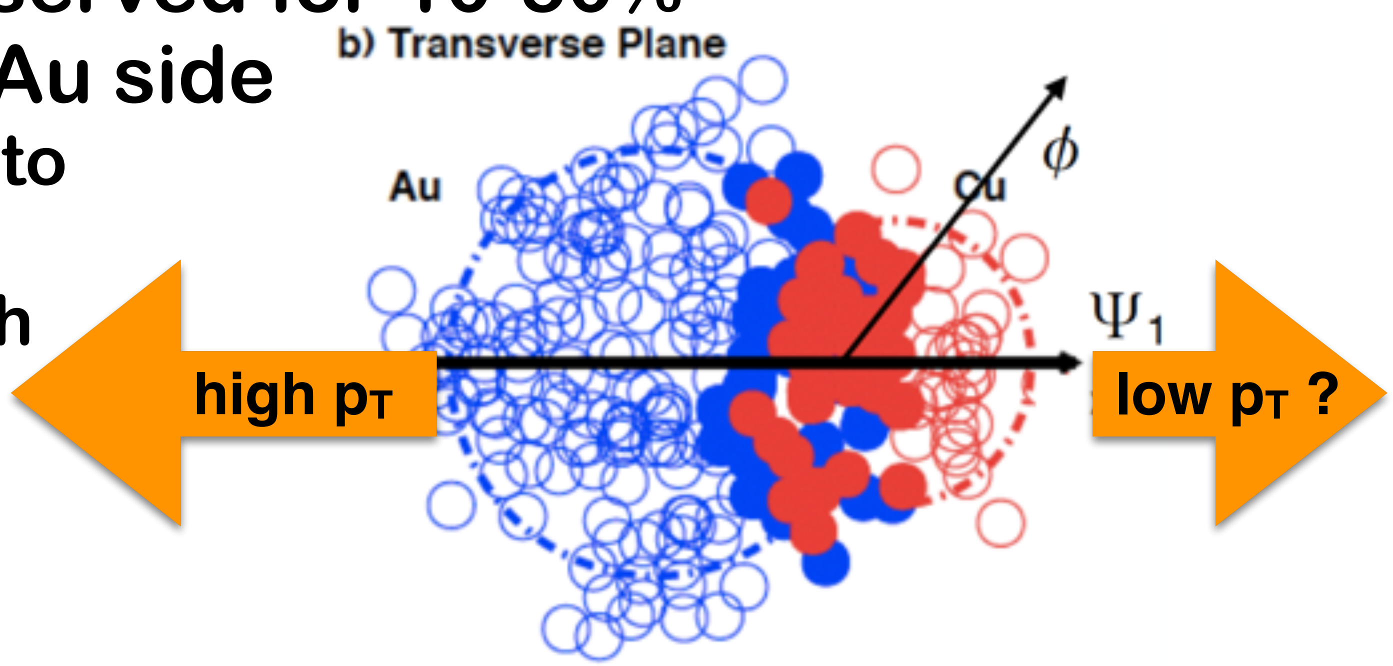


Sizable v1 at mid-rapidity is observed for 10-50%

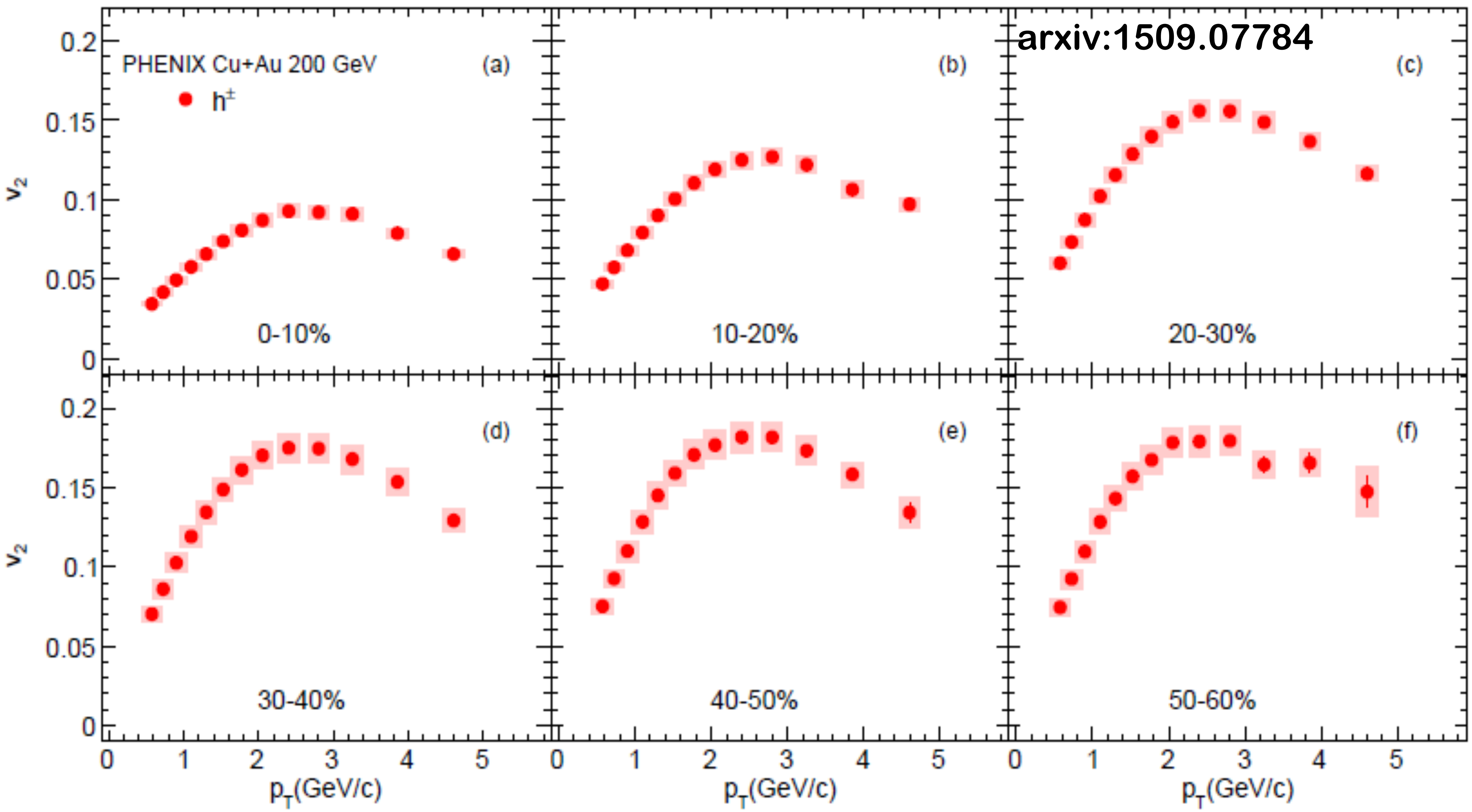
High pt particle are emitted to Au side

-Magnitude decreases from central to more peripheral events

-In peripheral events, Left/Right path length becomes similar



Charged hadron v_2

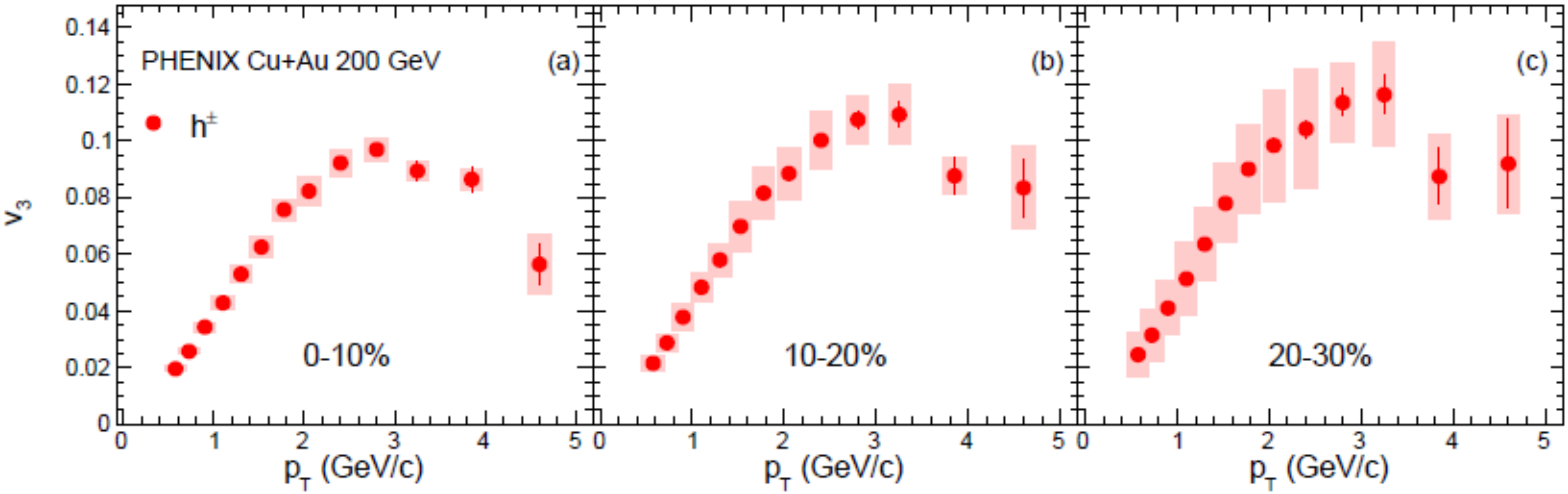


Similar p_T and centrality dependence of v_2 as seen in symmetric collisions

- Strong centrality dependence, magnitude increase from central to peripheral

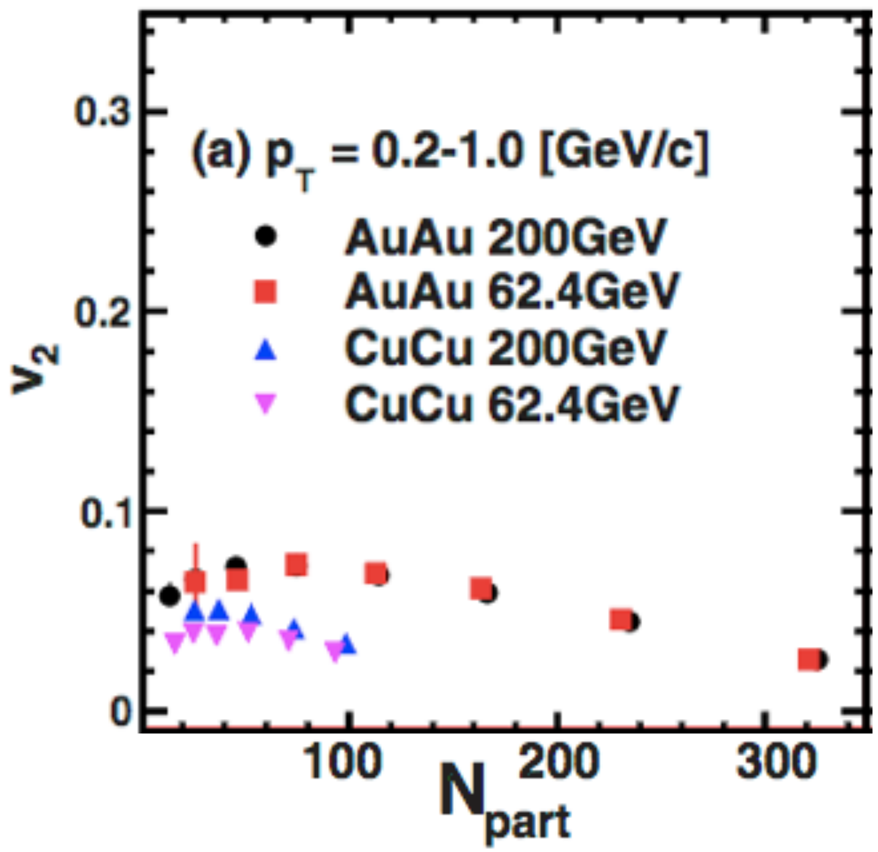
Charged hadron v_3

arxiv:1509.07784

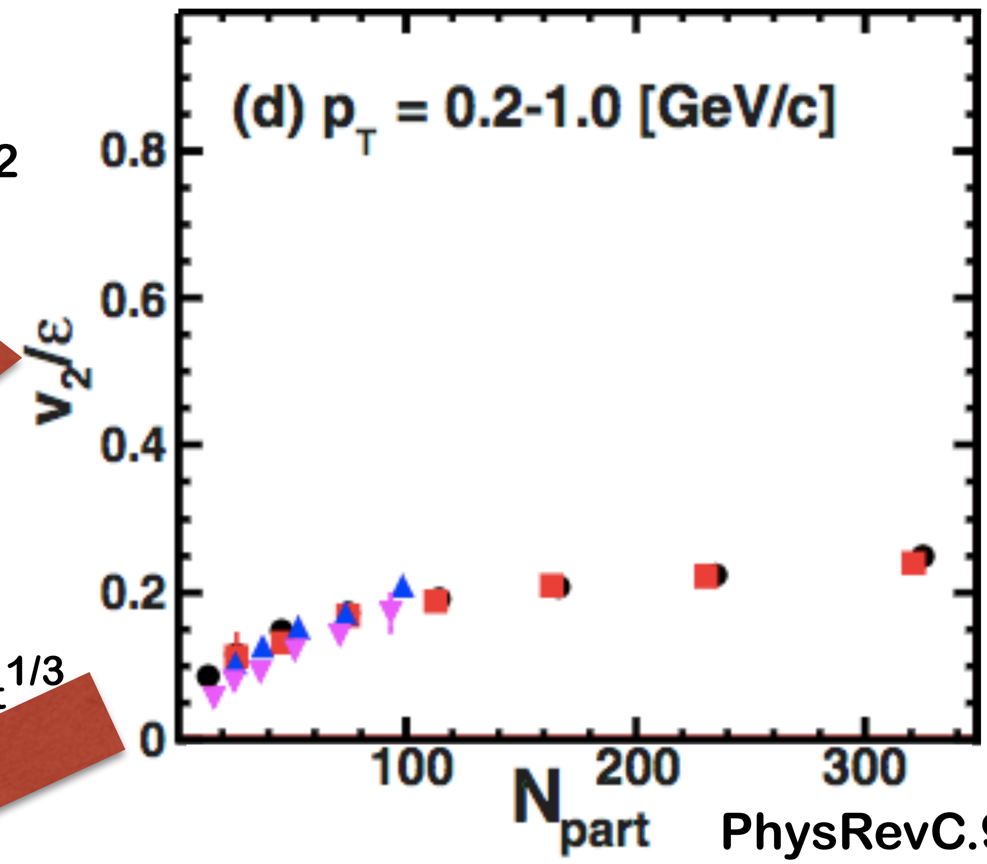


Similar pT and centrality dependence of v3 as seen in symmetric collisions
- Weak centrality dependence, magnitude slightly increase from central to peripheral

$\varepsilon_2 N_{part}^{1/3}$ scaling

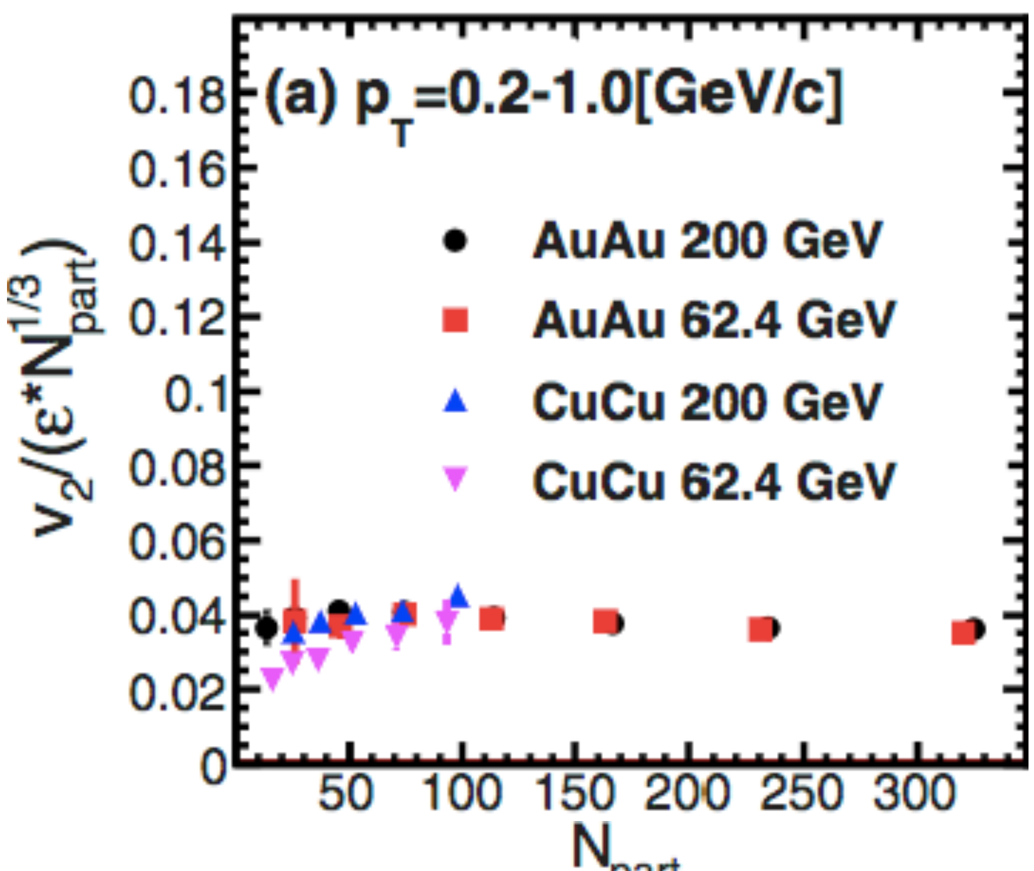


Sale with ε_2



PhysRevC.92.034913

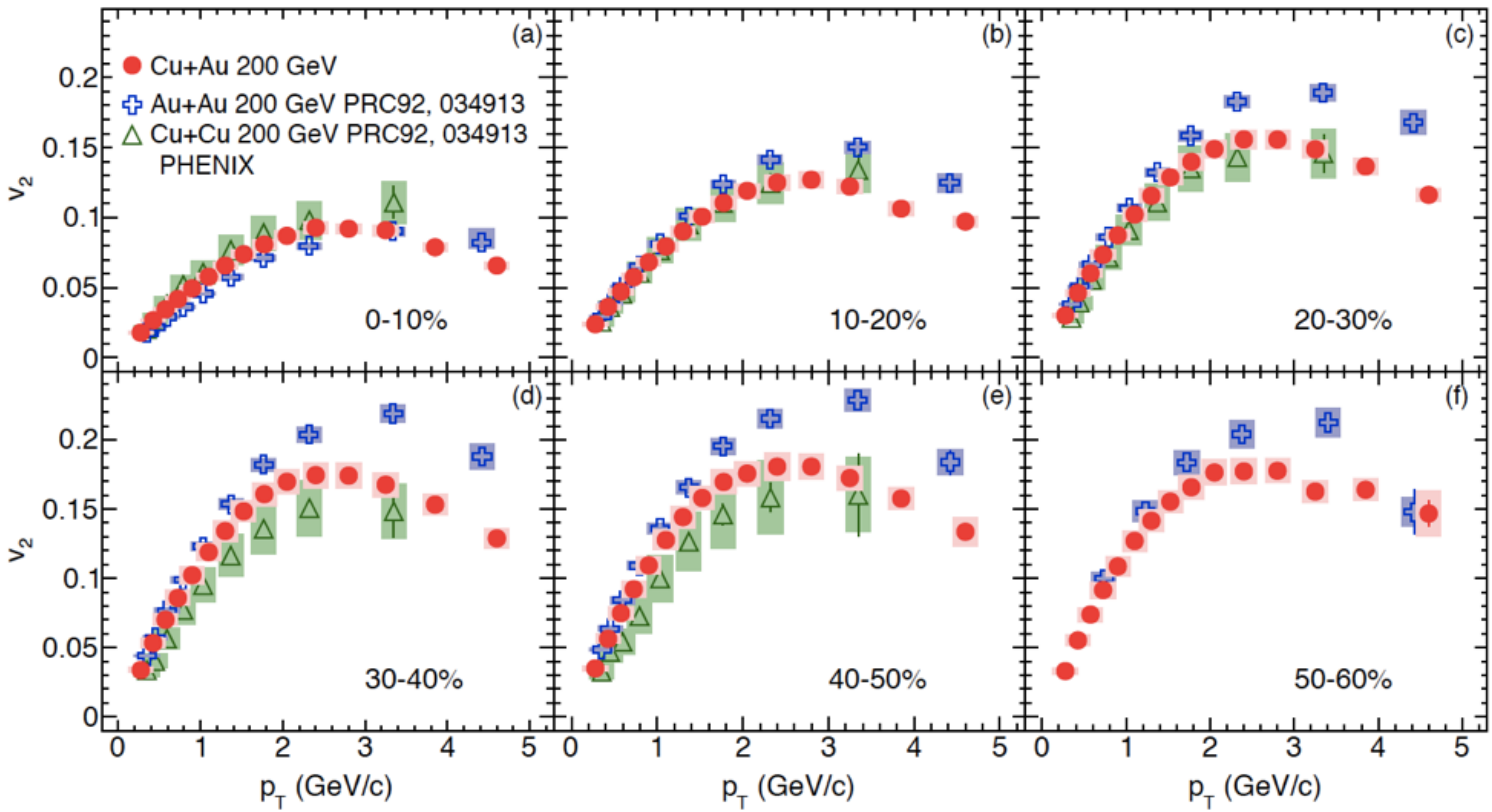
Sale with $N_{part}^{1/3}$



- √ v_2 is scaled with ε_2
  - Cancel initial geometry
    - ε_2 is initial geometry
  - Strong N_{part} dependence
- √ v_2 is scaled with ε_2 $N_{part}^{1/3}$
  - Cancel system size
    - $N_{part}^{1/3}$ is proportional to length scale or expansion time

System size dependence of v_2

arxiv:1509.07784



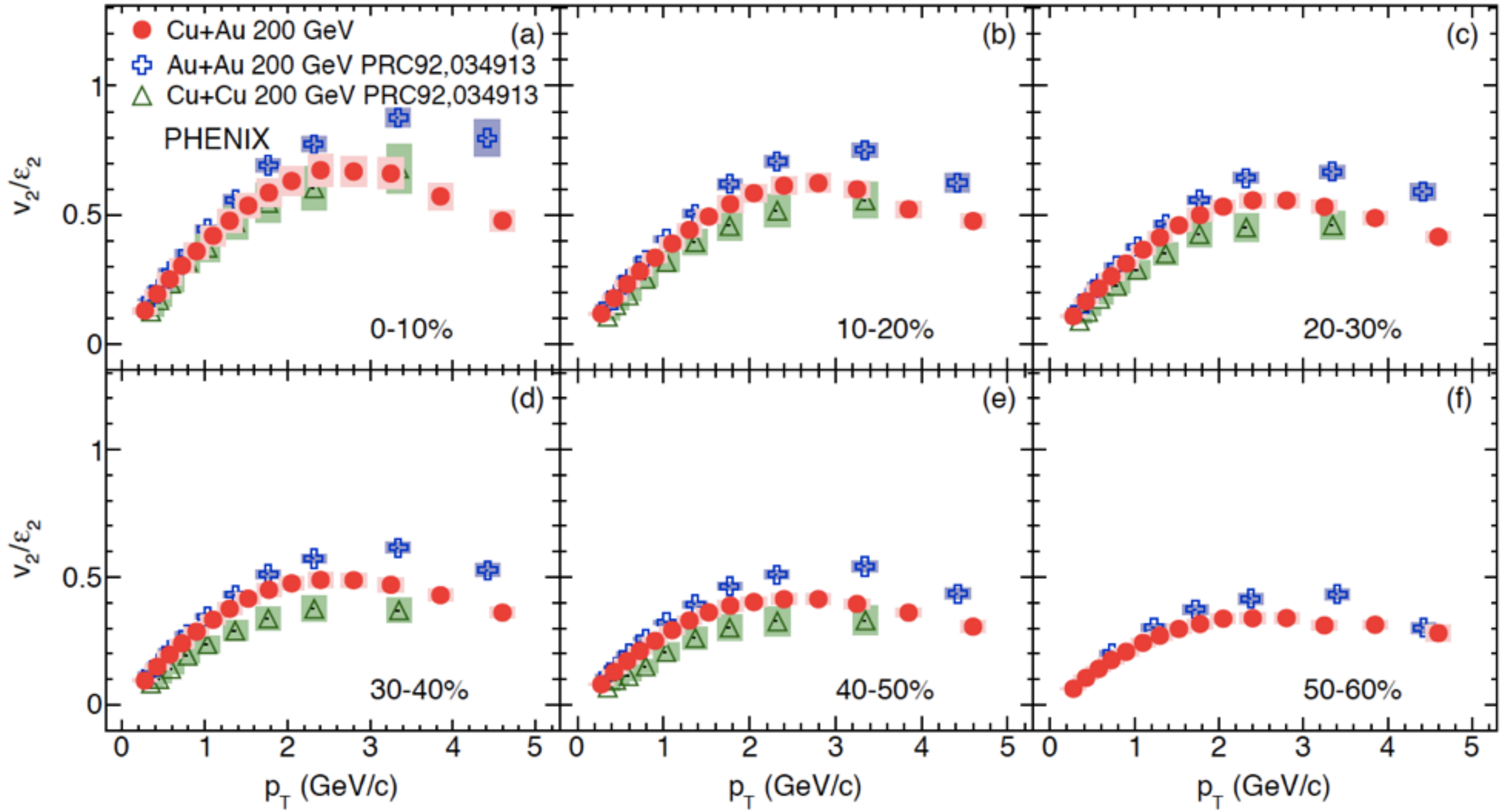
v_2 for different systems has similar centrality and pT dependence

v_2 in CuAu is always between those in AuAu and CuCu

Except in 0-10%, v_2 are not ordered according to ε_2

System size dependence of v_2

arxiv:1509.07784



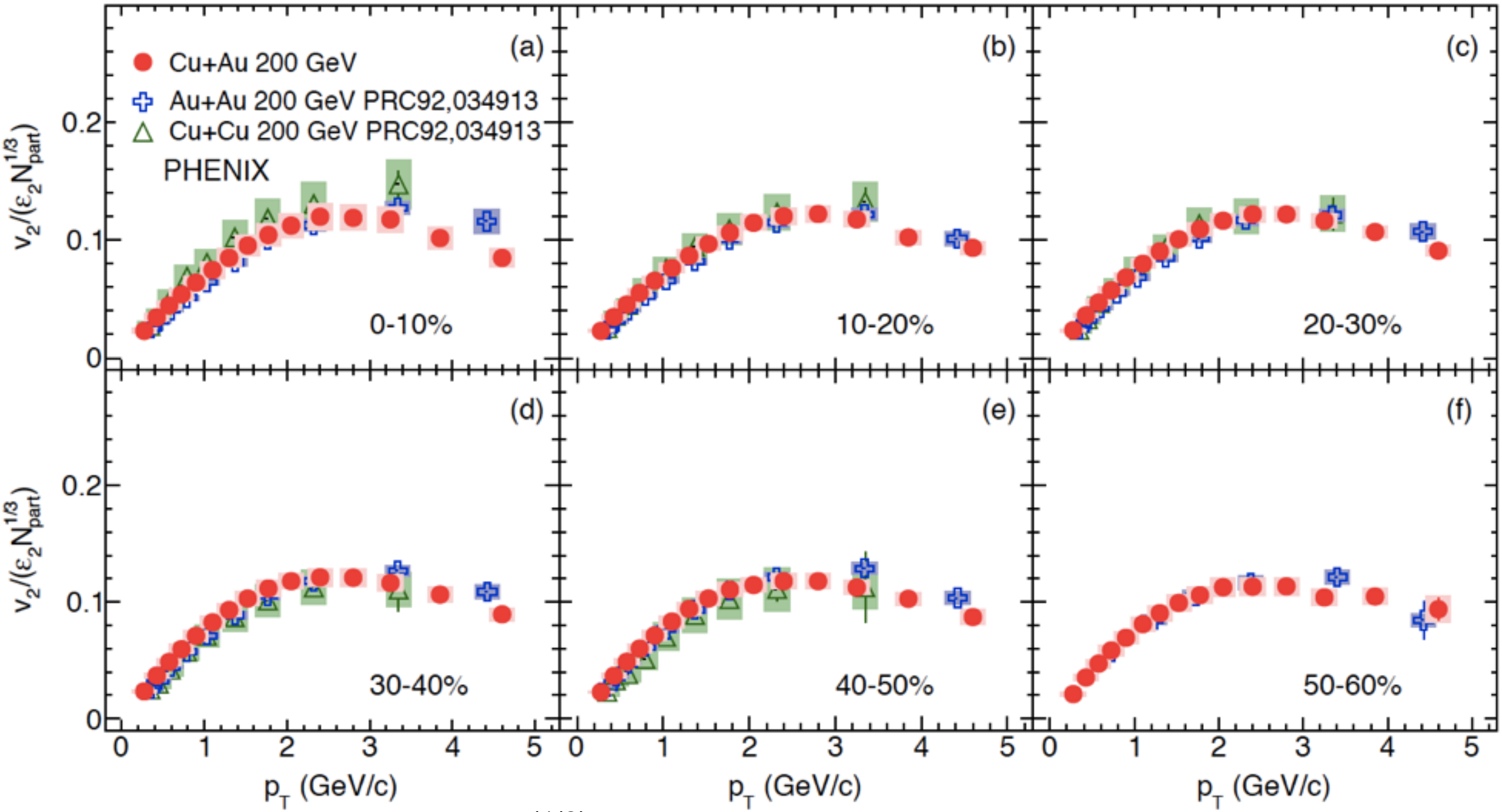
v_2 scaled with ε_2 (initial ellipticity)

For all centrality, order of magnitude of v2 is AuAu>CuAu>CuCu

->System size contribute magnitude of v_2

Scaling v_2 with $\varepsilon_2 * N_{part}^{(1/3)}$

arxiv:1509.07784



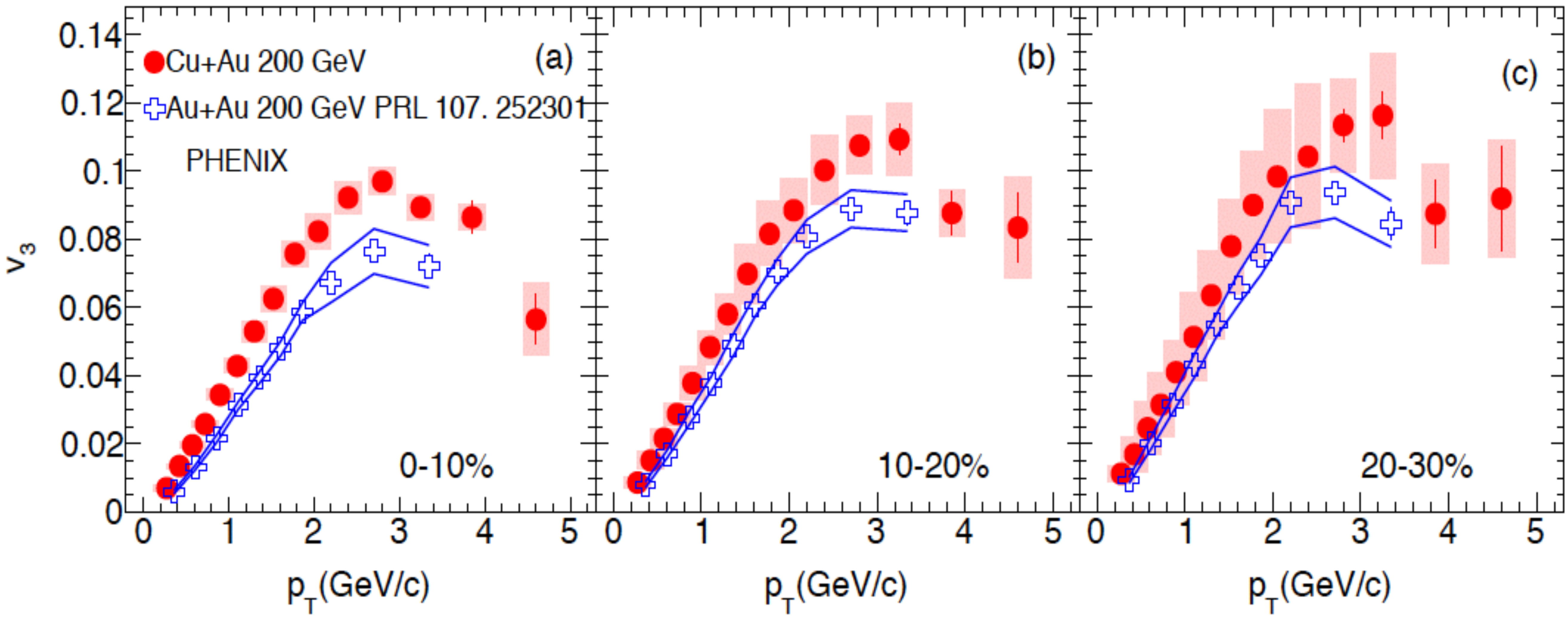
v_2 is scaled with $\varepsilon_2 N_{part}^{(1/3)}$

-$N_{part}^{(1/3)}$ is proportional to length scale or expansion time

$\varepsilon_2 N_{part}^{(1/3)}$ scaling works well in CuAu!

System size dependence of v_3

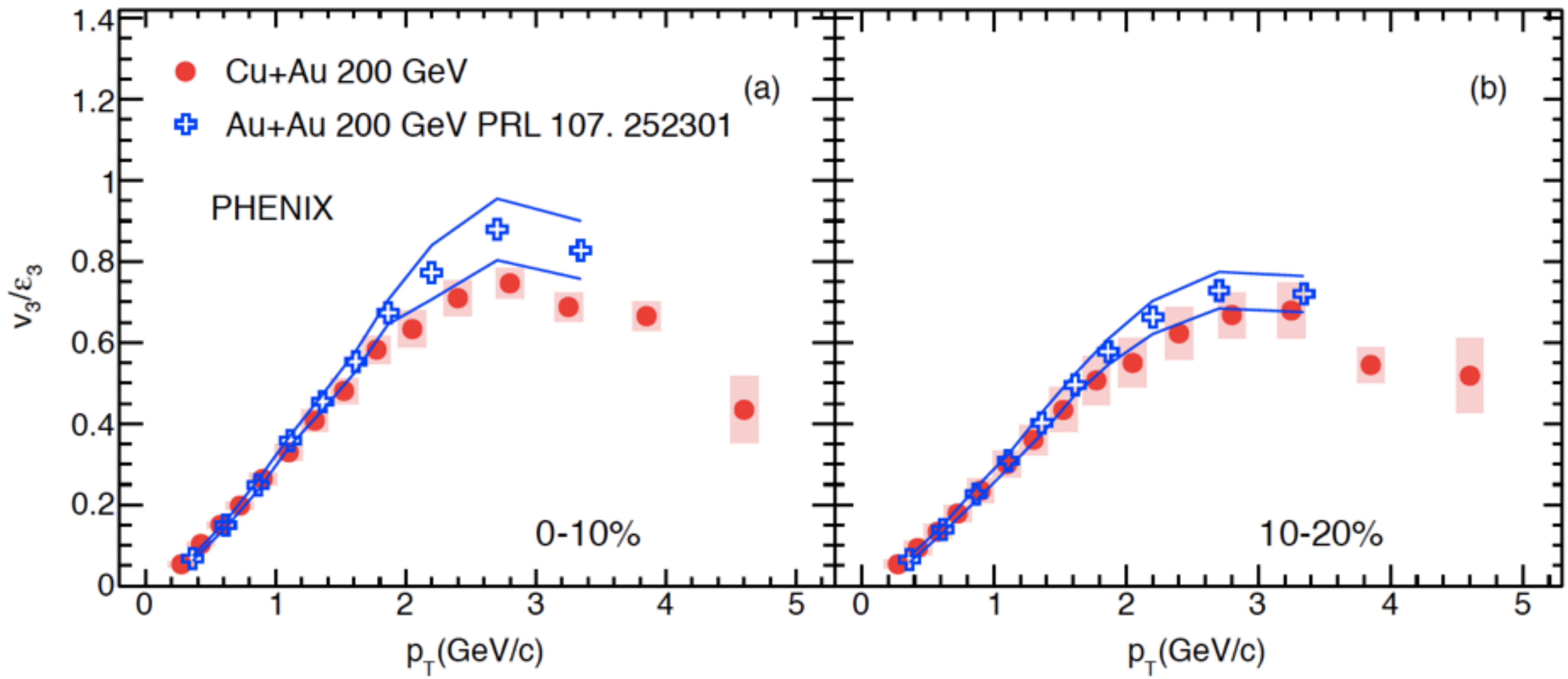
arxiv:1509.07784



v_3 for different systems has weak centrality dependence
v_3 in CuAu is always bigger than those in AuAu
Unlike v_2, v_3 are ordered according to ε_3
->v_3 doesn't depend on system size

System size dependence of v_3

arxiv:1509.07784



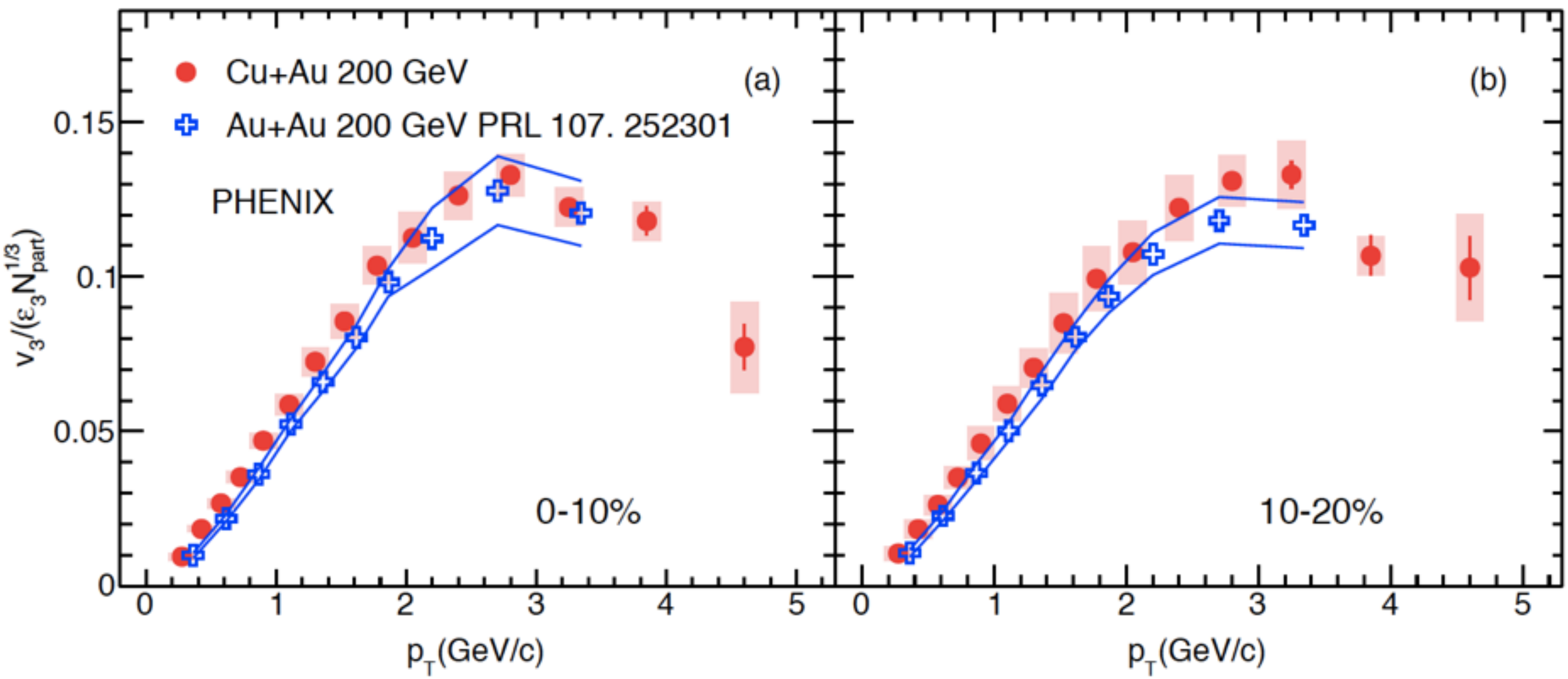
v_3 scaled with ε_3 for

Unlike v_2, scaled v_3 are good agreement between AuAu and CuAu

-For 0-10%, the deviation is seen pT>(2.5GeV/c)

Scaling v3 with ε3*Npart$^{(1/3)}$

arxiv:1509.07784

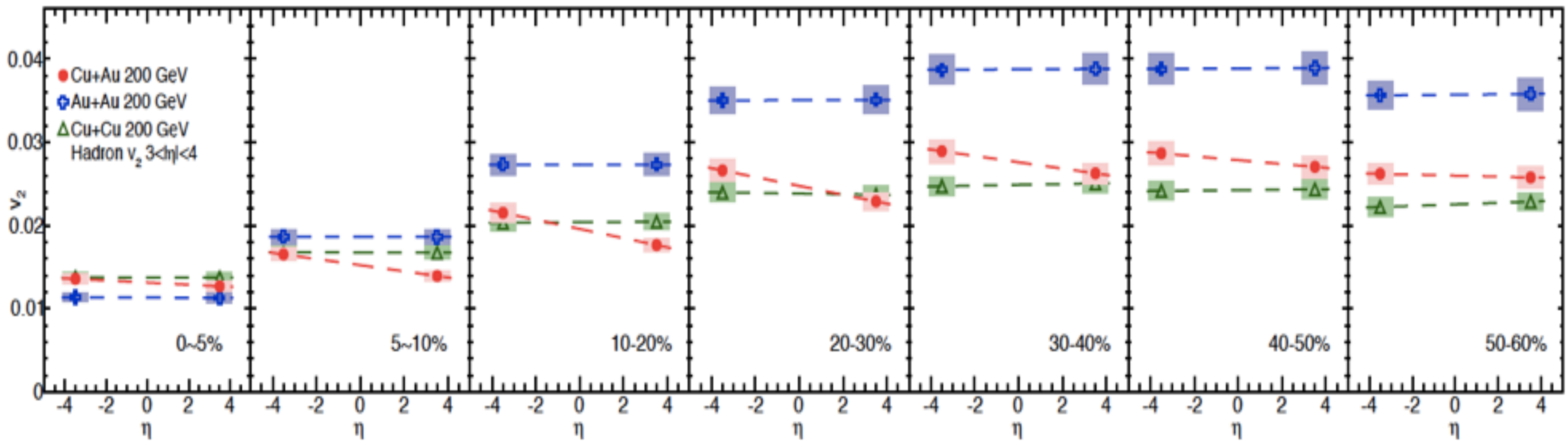


v_3 is scaled with $\varepsilon_3 N_{part}^{(1/3)}$
-$N_{part}^{(1/3)}$ is proportional to length scale or expansion time
$\varepsilon_3 N_{part}^{(1/3)}$ scaling works well in v_3!

Results
Discussions

- v_1,v_2,v_3 at mid-η
- v_2,v_3 at large-η
- v_1,v_2,v_3 theory comparison

System size dependence of v_2 at F/B



Centrality dependence of v_2 is seen for all collision systems

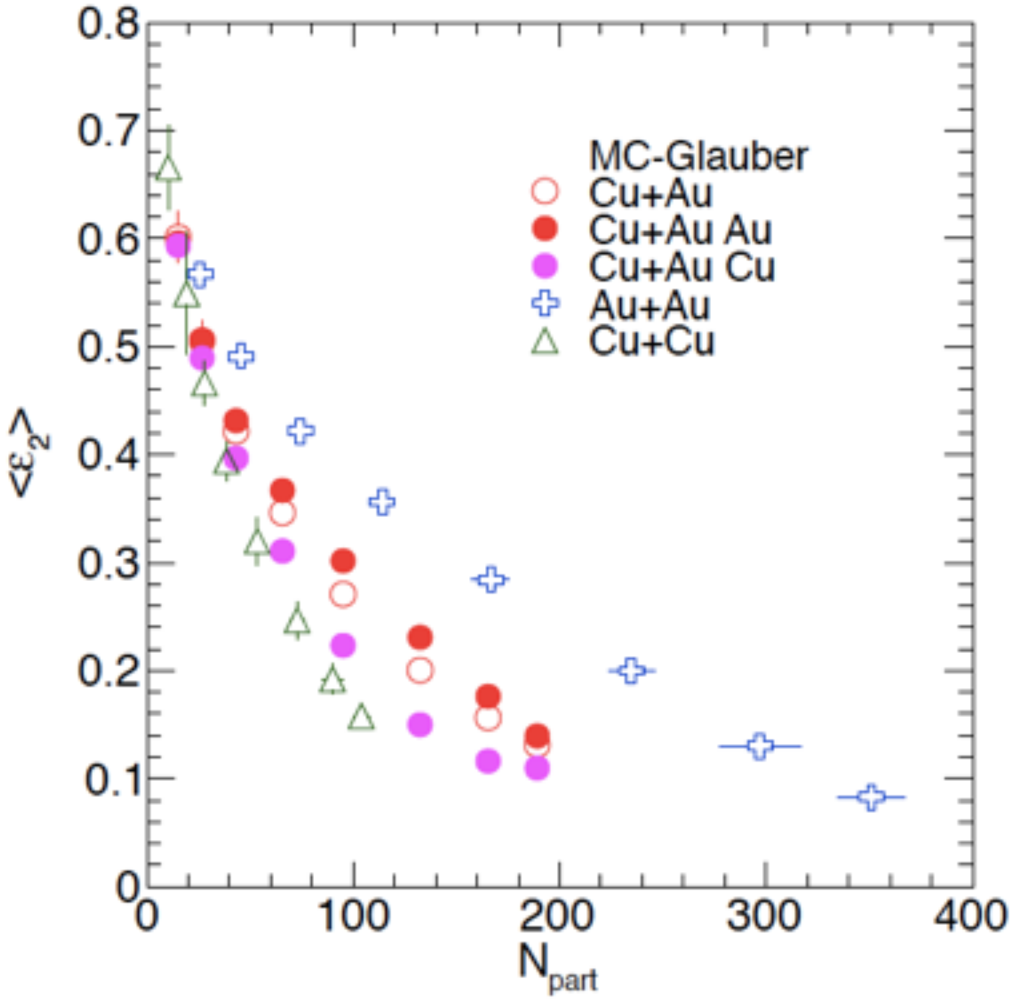
In CuAu collisions,

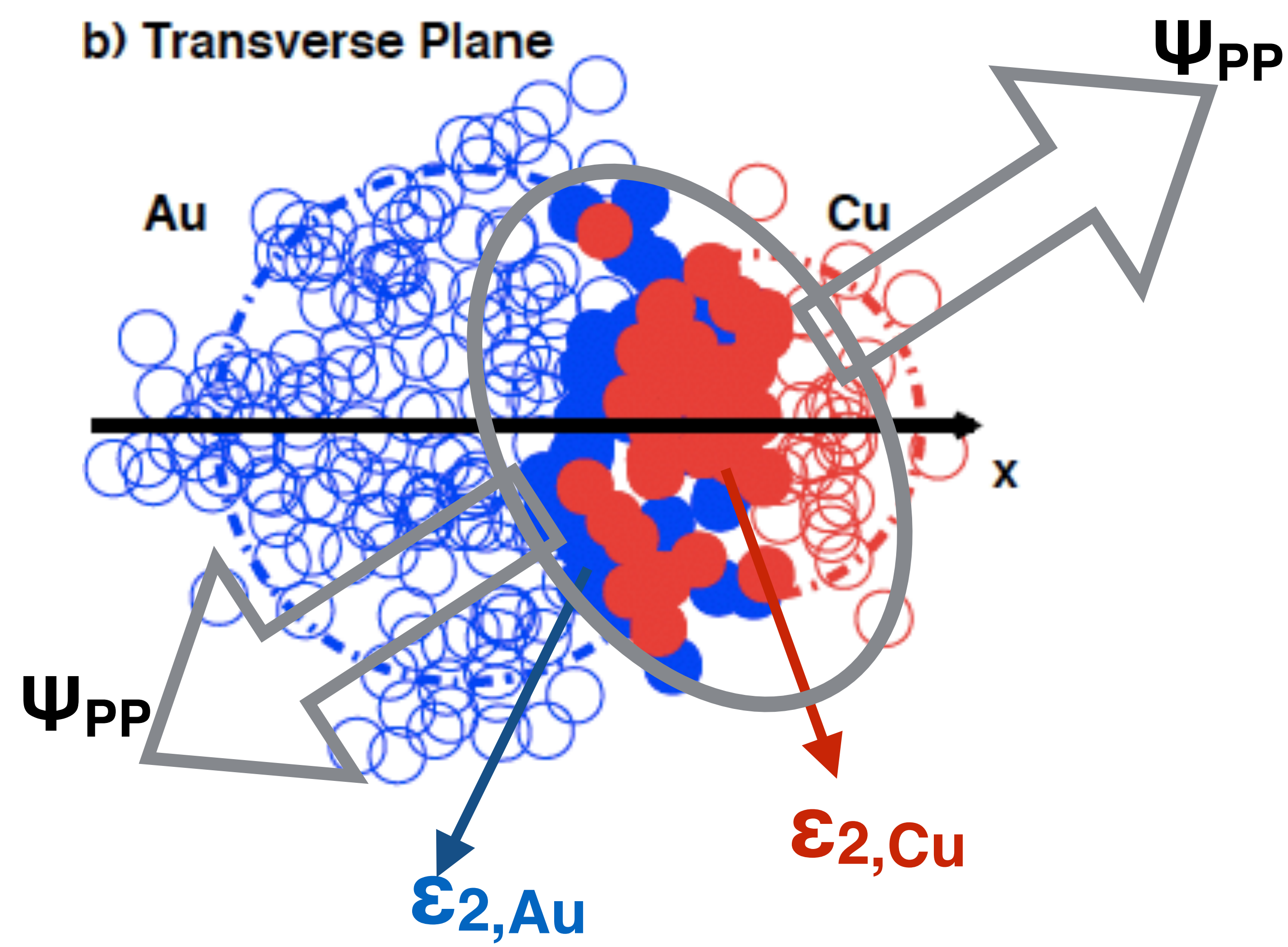
-central & peripheral collisions: v2(Au-going) ~ v2(Cu-going)

-mid-central collisions : v2(Au-going) > v2(Cu-going)

->caused by different initial geometries in Au and Cu ?

Calculation of $\varepsilon_{2,Au}$ and $\varepsilon_{2,Cu}$





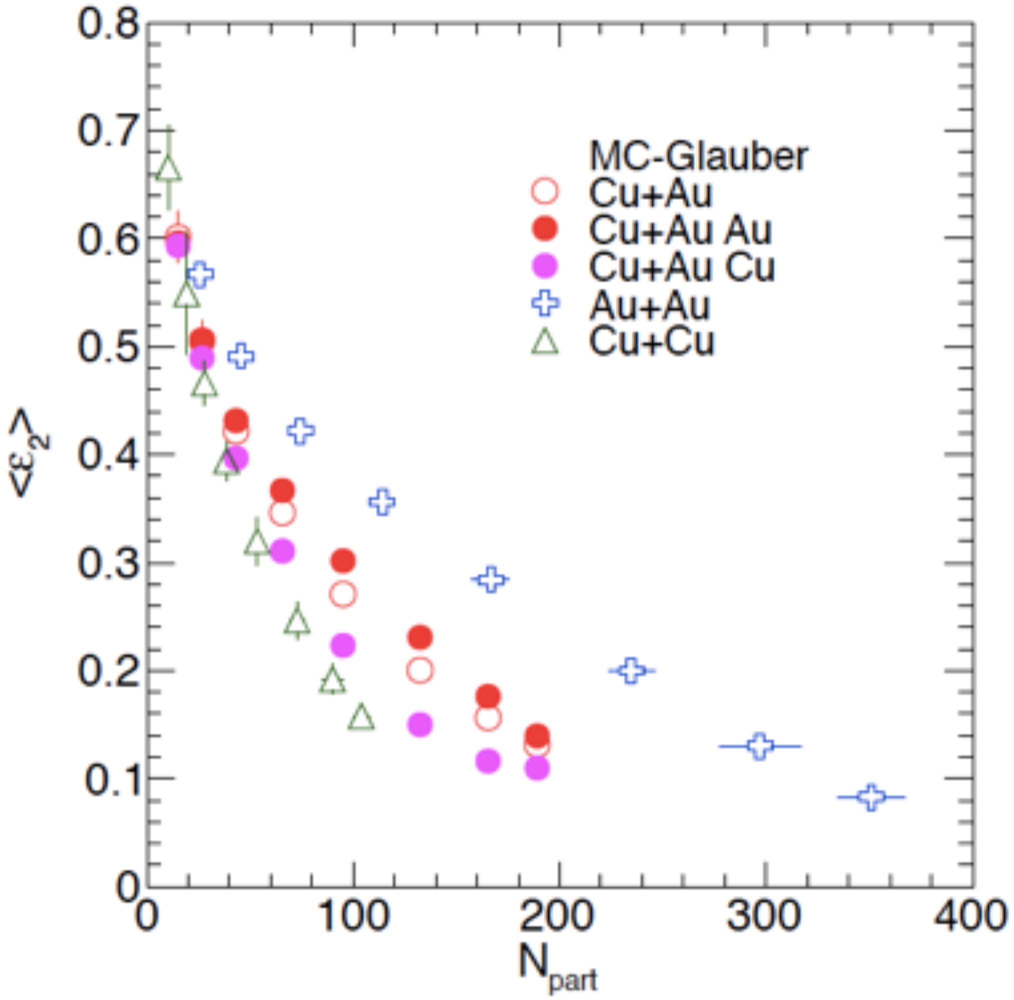
In CuAu, $\varepsilon_{2,Cu}$ and $\varepsilon_{2,Au}$ calculated by Cu and Au nucleons separately.

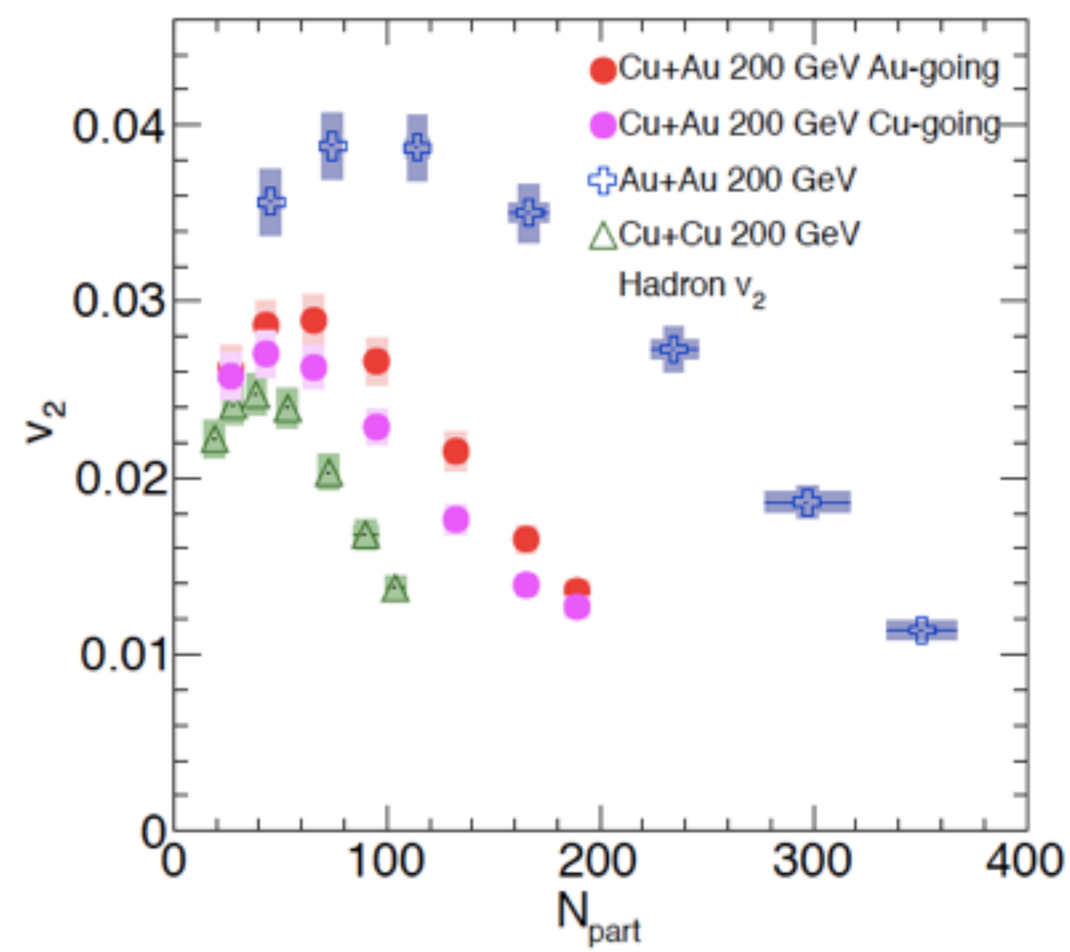
- $\varepsilon_{2,Cu}$ and $\varepsilon_{2,Au}$ are determined with respect to Ψ_{PP}

- Ψ_{PP} is determined by all participant nucleons

- $\varepsilon_{2,Au} > \varepsilon_{2,CuAu} > \varepsilon_{2,Cu}$

Comparison of ε_2 and v_2



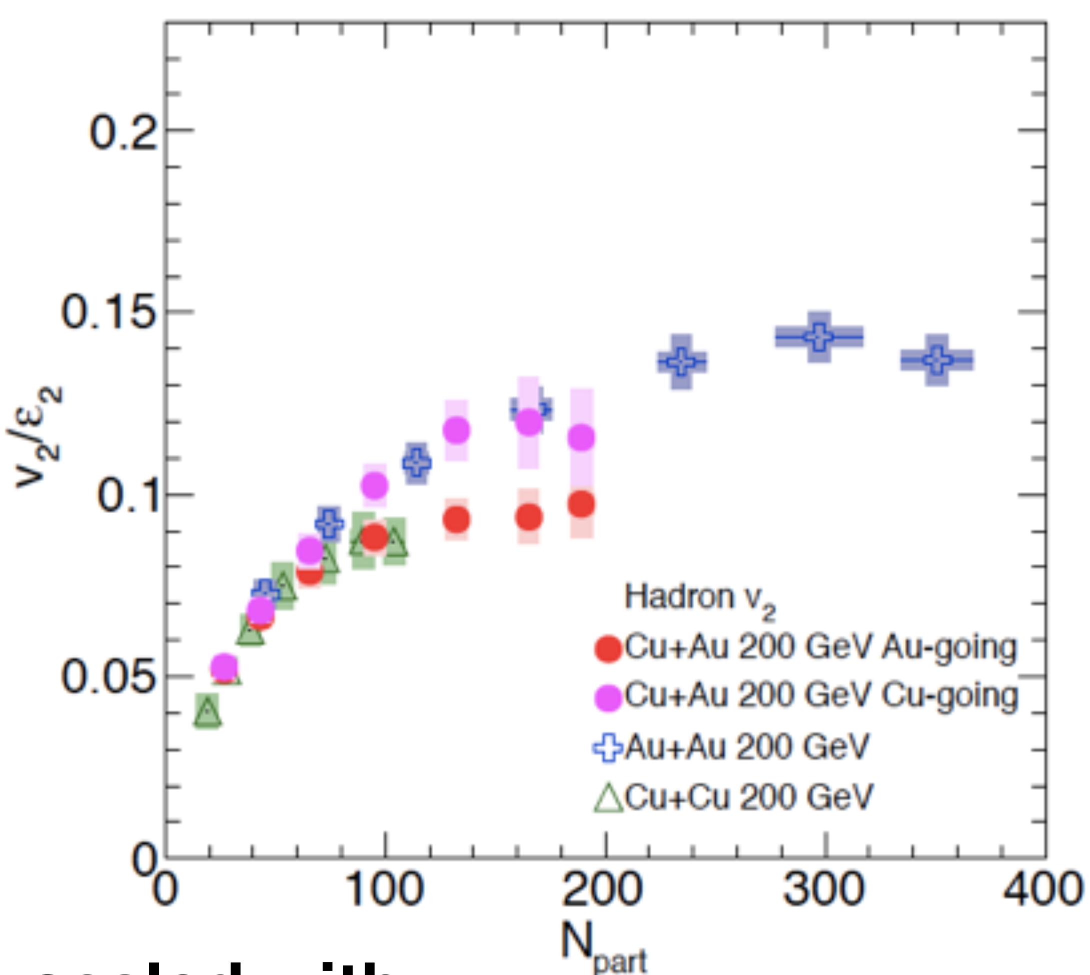


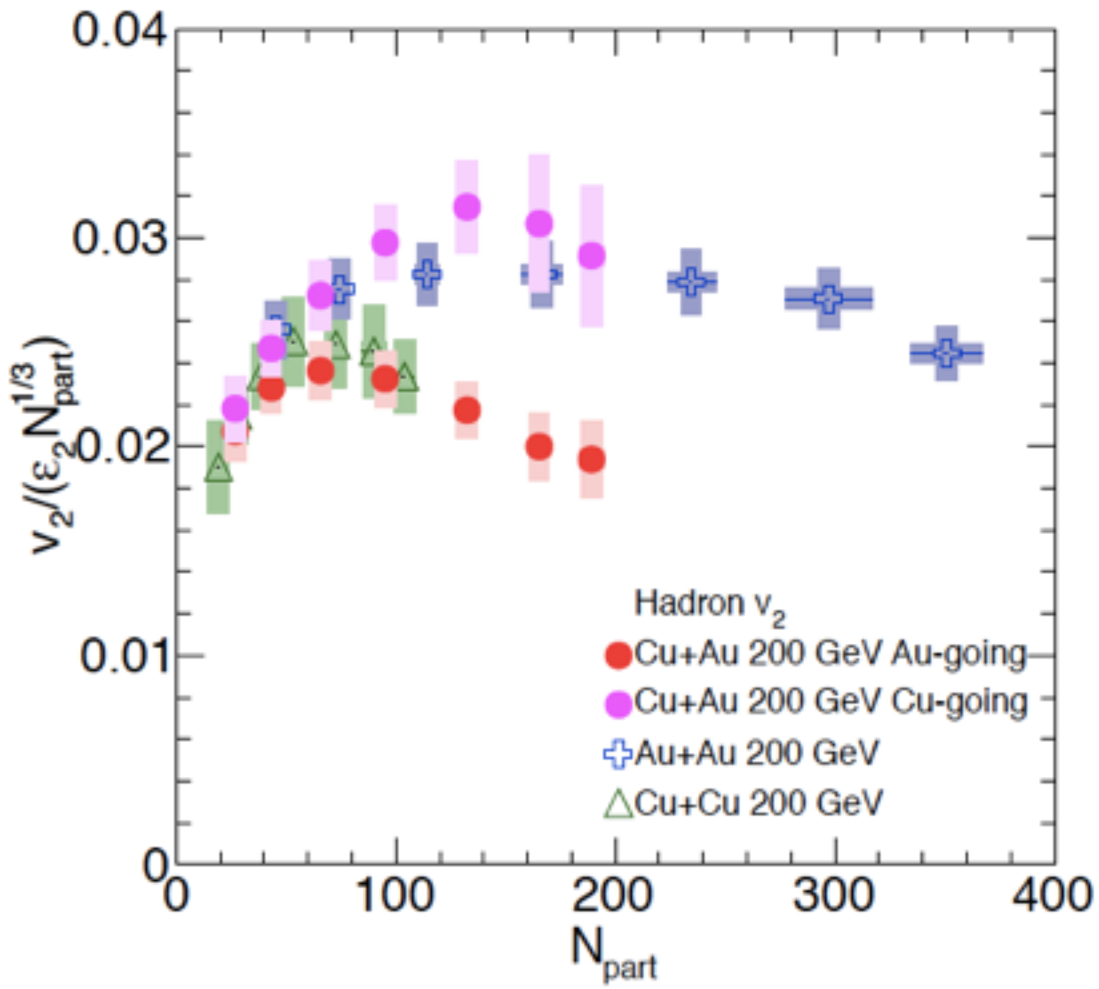
In CuAu, $\varepsilon_{2,Cu}$ and $\varepsilon_{2,Au}$ calculated by Cu and Au nucleons separately.

- **$\varepsilon_{2,Cu}$ and $\varepsilon_{2,Au}$ are determined with respect to Ψ_{PP}**
- **Ψ_{PP} is determined by all participant nucleons**
- **$\varepsilon_{2,Au} > \varepsilon_{2,CuAu} > \varepsilon_{2,Cu}$**

-> $v_{2,Au} > v_{2,Cu}$
-> Target nucleons geometry and projectile nucleons geometry survive hydrodynamic expansion

Scaling with $\varepsilon_{2,Cu}$, $N_{part,Cu}$ and $\varepsilon_{2,Au}$, $N_{part,Au}$





v_2 scaled with ε_2

-In CuAu, v_2 is scaled with $\varepsilon_{2,Au}$ and $\varepsilon_{2,Cu}$ separately

- v_2(Cu-going) in CuAu : consistent with those in AuAu and CuCu

- v_2(Au-going) in CuAu : consistent with those in AuAu and CuCu for lower N_{part}
inconsistent with those in AuAu for higher N_{part}

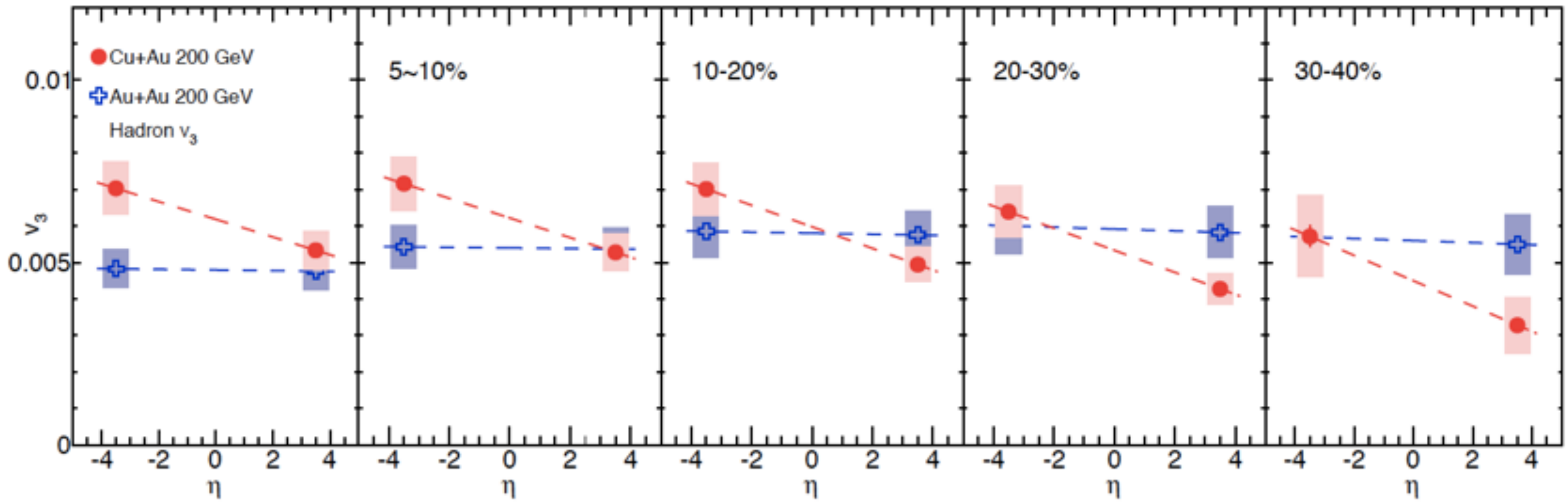
v_2 scaled with $\varepsilon_2 N_{part}^{(1/3)}$

- In CuAu, $N_{part,Au}^{(1/3)}$ and $N_{part,Cu}^{(1/3)}$ are calculated separately

- In AuAu,CuCu, $N_{part}/2$ is used.

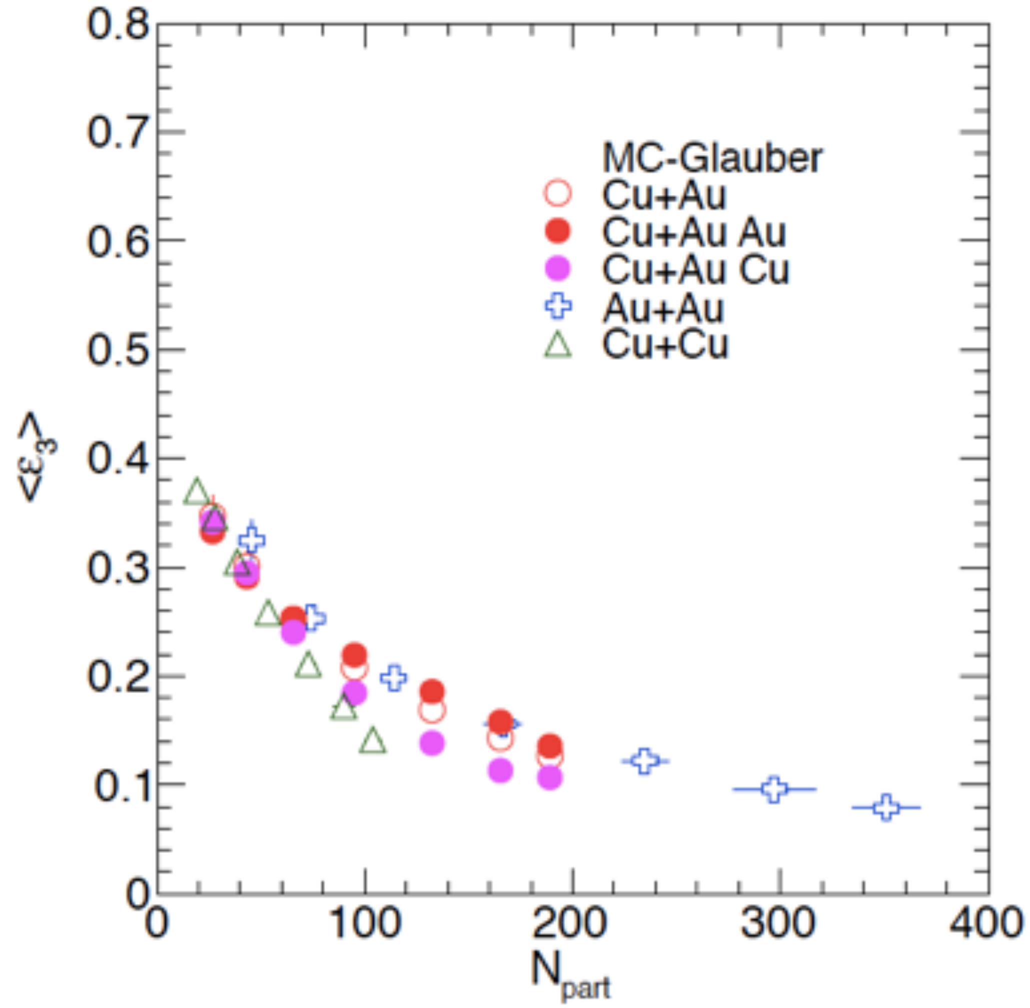
-v_2(Au-going) are not consistent for higher N_{part}

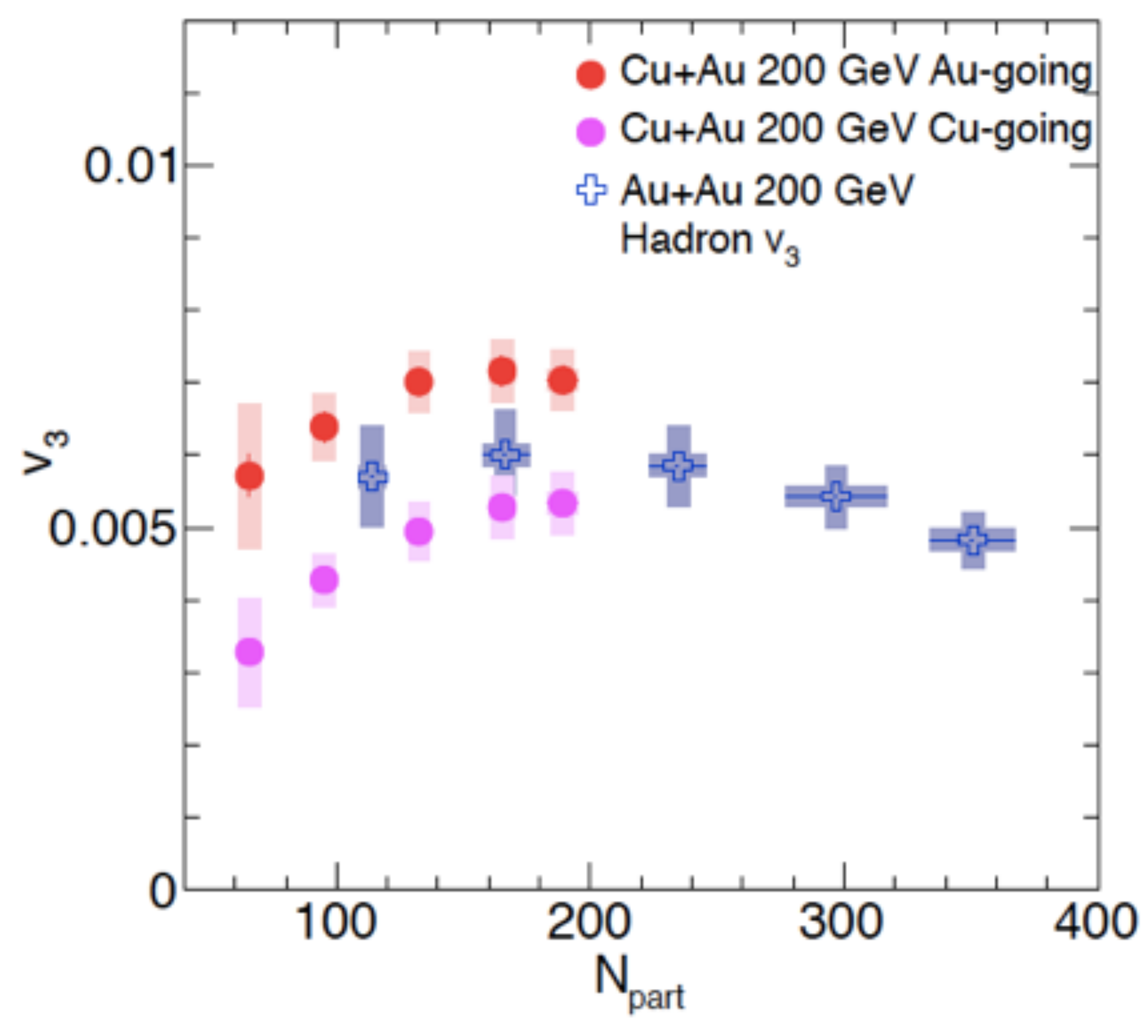
System size dependence of v_3 at F/B



Weak centrality dependence of v_3 is seen for all collision systems
AuAu:same centrality dependence as seen mid η
CuAu: v_3 decrease as centrality decrease
In CuAu collisions, v_3(Au-going) > v_3(Cu-going) for all centrality bins
-> Like v_2, the different initial geometry cause the different v_3 ?

System size dependence of v_3 at F/B





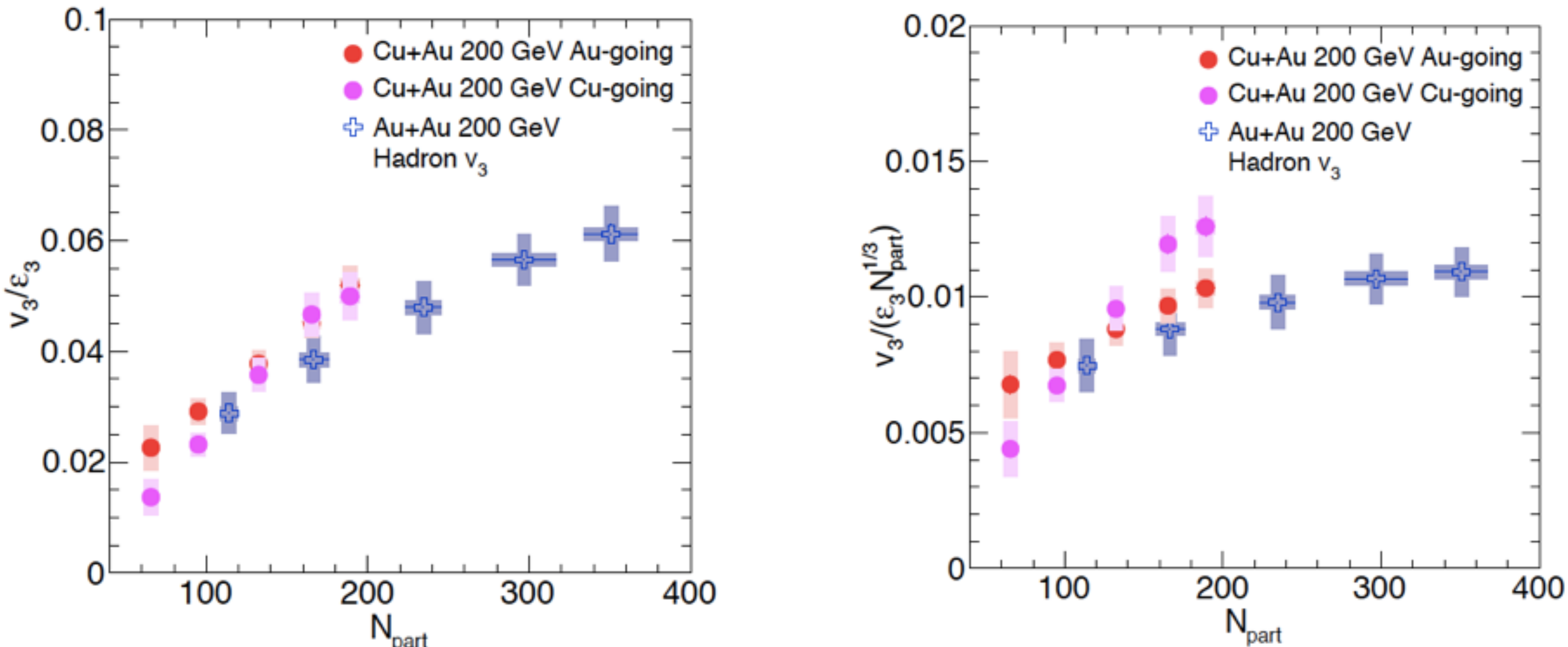
In CuAu, $\varepsilon_{3,Cu}$ and $\varepsilon_{3,Au}$ calculated by Cu and Au nucleons separately.

- **$\varepsilon_{3,Cu}$ and $\varepsilon_{3,Au}$ are determined with respect to Ψ_{PP}**
- **Ψ_{PP} is determined by all participant nucleons**
- **$\varepsilon_{3,Au} > \varepsilon_{3,CuAu} > \varepsilon_{3,Cu}$**

  -> $v_{3,Au} > v_{3,Cu}$

  -> Target nucleons geometry and projectile nucleons geometry survive hydrodynamic expansion

Scaling with $\varepsilon_{3,Cu}$, $N_{part,Cu}$ and $\varepsilon_{3,Au}$, $N_{part,Au}$



v_3 scaled with ε_3

- **v_3(Au-going) and v_3(Cu-going) are consistent with each other but in consistent with those in AuAu for higher N_{part}**
- **v_3(Au-going) and v_3(Cu-going) are inconsistent for lower N_{part}**

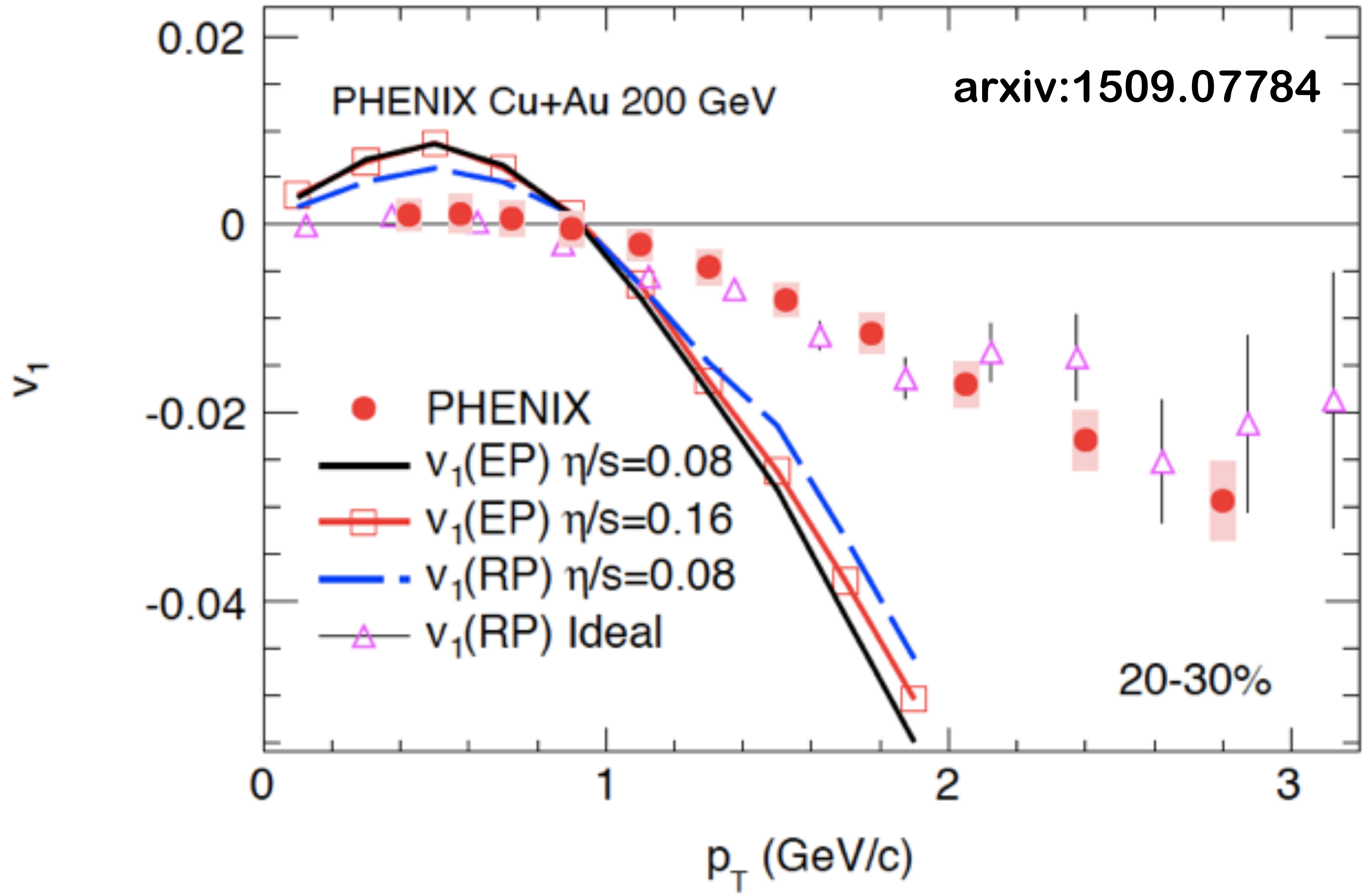
v_3 scaled with $\varepsilon_3 N_{part}^{(1/3)}$

- **In CuAu, $N_{part,Au}^{(1/3)}$ and $N_{part,Cu}^{(1/3)}$ are calculated separately**
- **In AuAu, $N_{part}/2$ is used**
- **v_3(Cu-going) is inconsistent with those in AuAu and v_3(Au-going)**

Results
Discussions

- v_1,v_2,v_3 at mid-η
- v_2,v_3 at large-η
- v_1,v_2,v_3 theory comparison

MC-Glauber E-by-E hydro v1(pt) at mid-η



In hydro calculation,

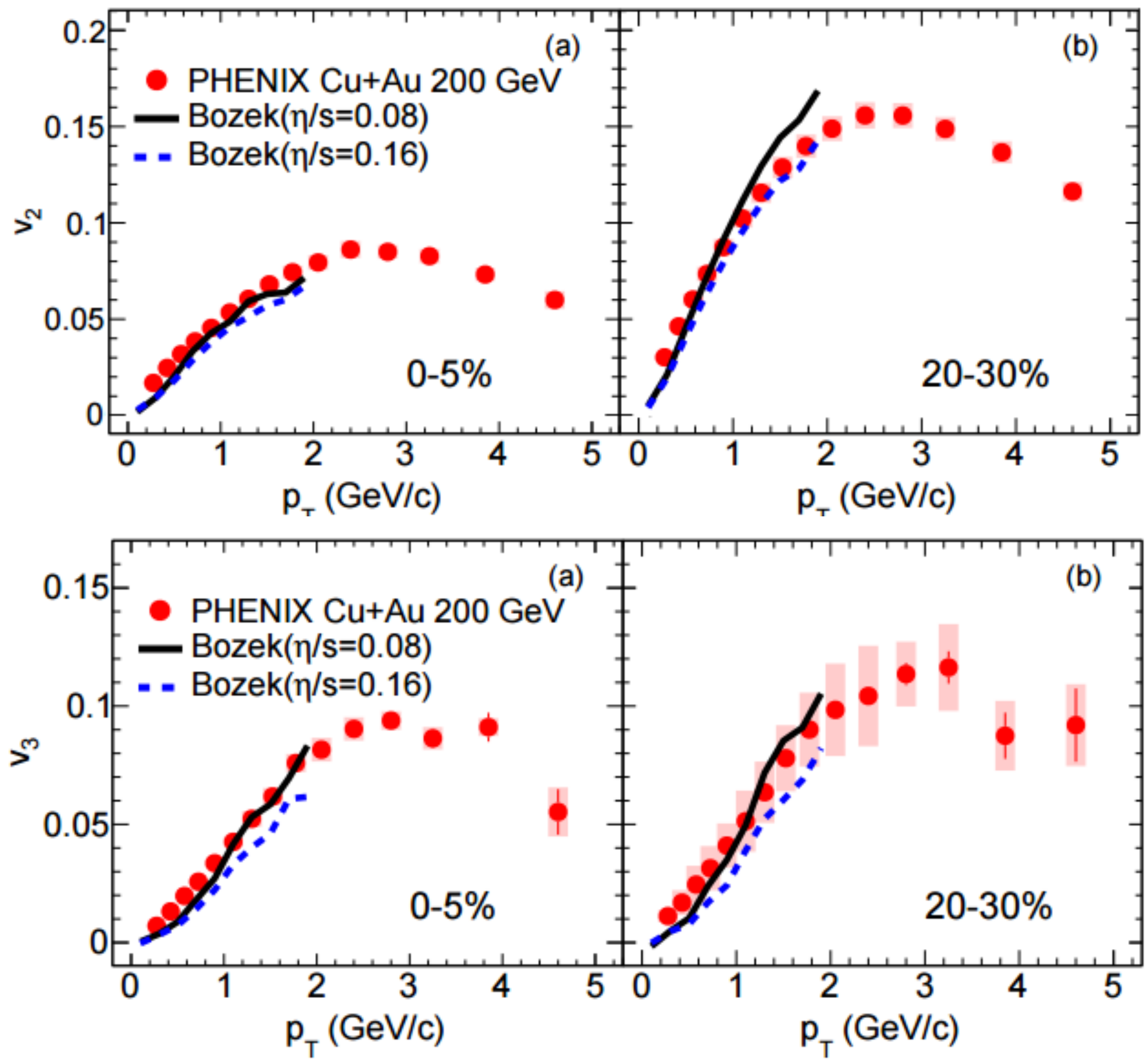
-More low pT , particles are emitted to Cu side

-More high pT, particles are emitted to Au side

-Ideal hydro reproduce experimental data well

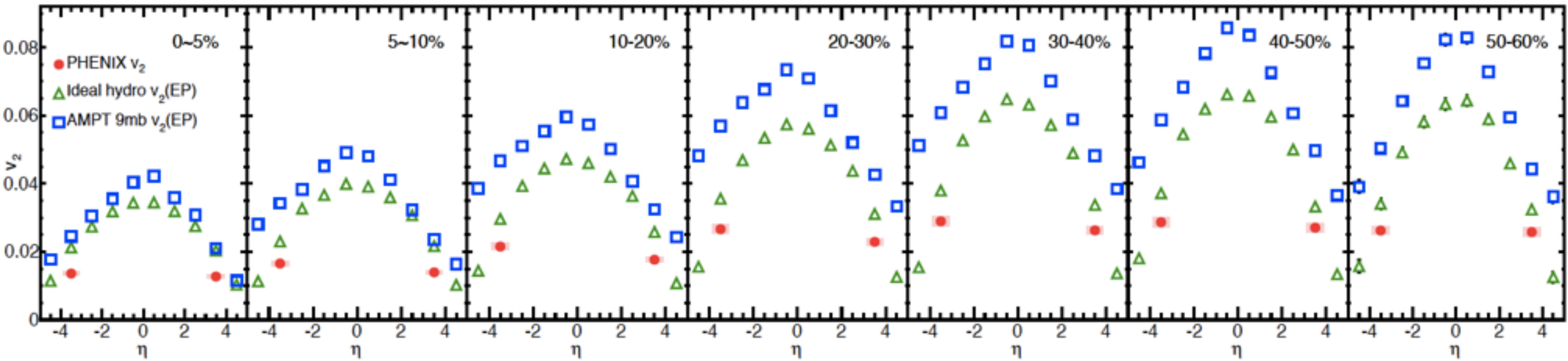
MC-Glauber E-by-E hydro v_2, v_3 at mid-η

arxiv:1509.07784



For both centrality, both value of η/s agree with data

Parton cascade and hydro $v_2(\eta)$



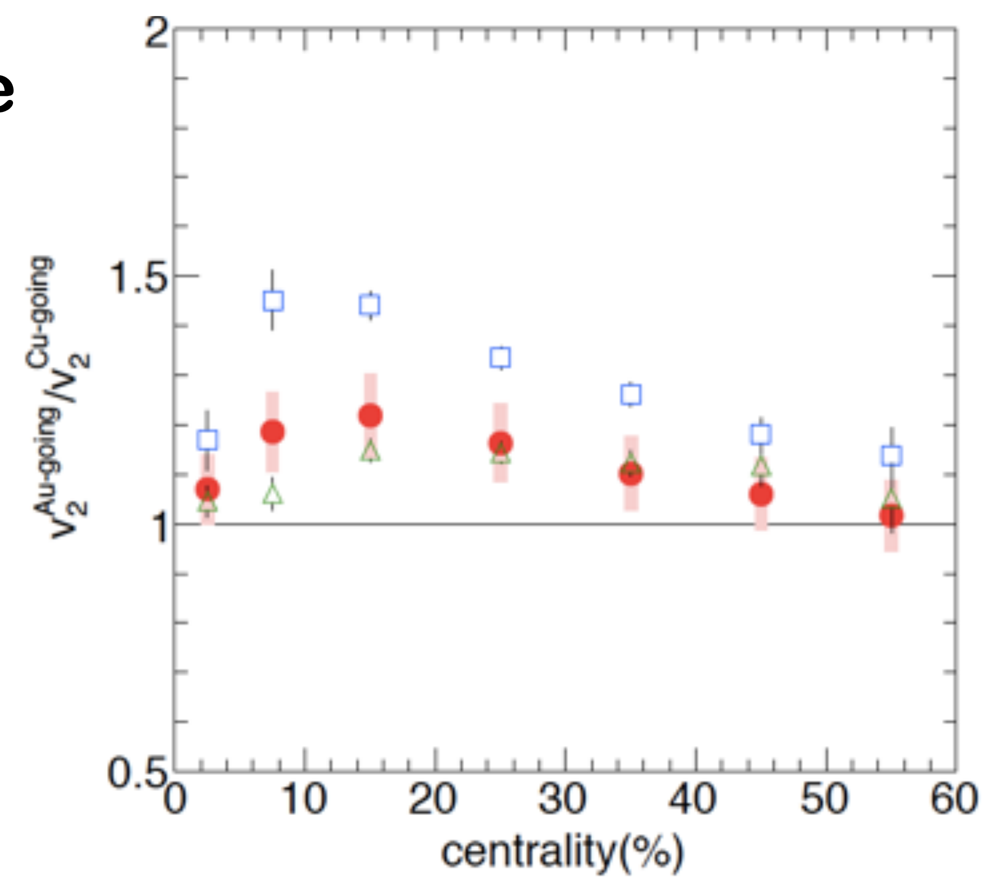
AMPT(parton cascade and hadron cascade) and Ideal hydrodynamic predict different v_2 value in Au going and Cu-going side

->Theory model predict different initial geometry survive collective expansion

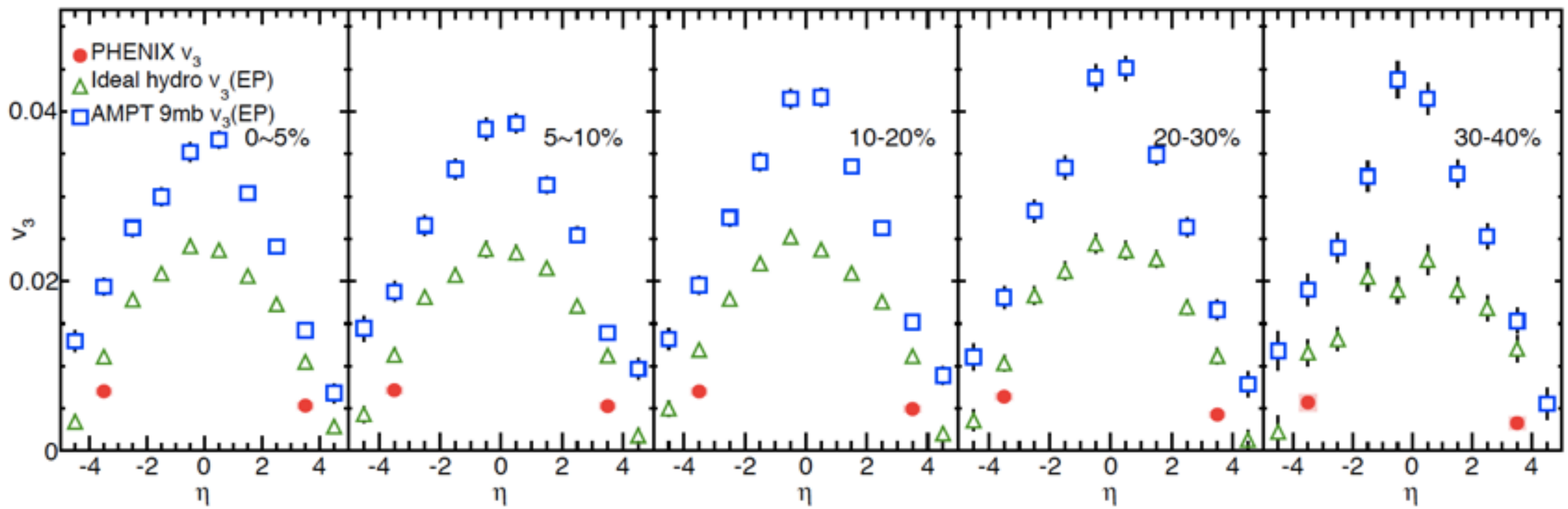
In peripheral and central collisions,

- data and hydro calculation show v_2 (Au-going) ~ v2(Cu-going)
- AMPT show v_2 (Au-going) > v_2(Cu-going)

->Hydro calculation reproduce the ratio of v_2 (Au-going) and v_2(Cu-going)

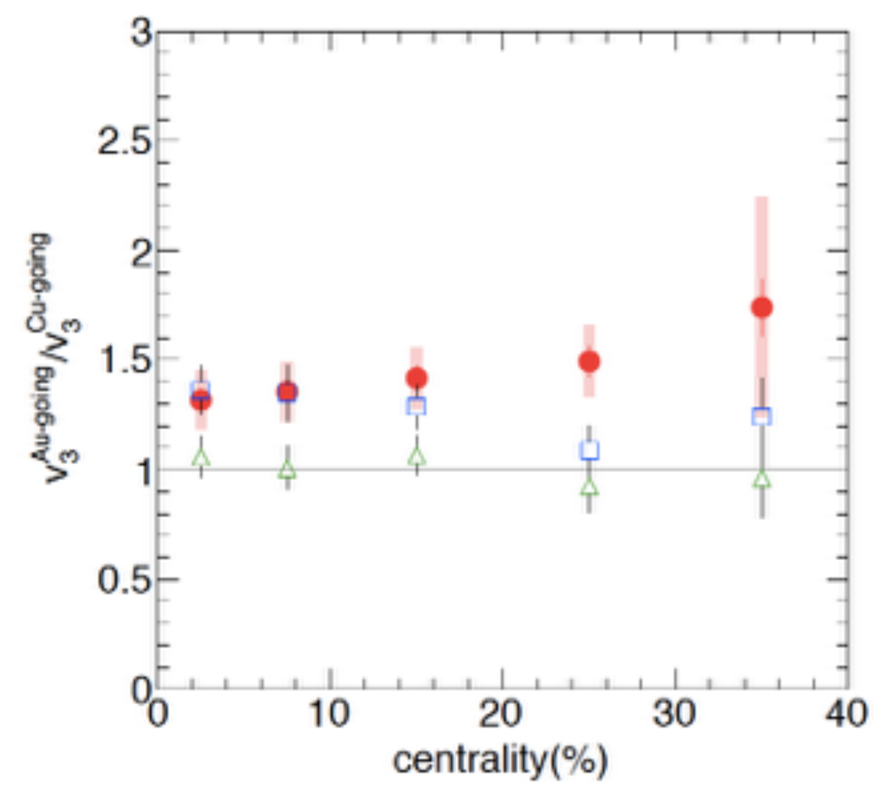


Parton cascade and hydro $v_3(\eta)$



Unlike v_2
-data and AMPT show
  v_3 (Au-going) > v_3(Cu-going)
-Hydro calculation show
  v_3(Au-going) ~ v_3(Cu-going)

->Hydro calculation doesn't reproduce the ratio of v_3 (Au-going) and v_3(Cu-going)



Summary

✓ v_2, v_3

-mid-η

v_2 and v_3 are similar centrality and p_T dependence as seen in symmetric collisions

Empirical scaling of $\varepsilon_2 N_{part}^{1/3}$ works v_2 in CuAu, CuCu, AuAu

Empirical scaling of $\varepsilon_3 N_{part}^{1/3}$ works v_3 in AuAu, CuAu

- Large-η

v_2, v_3 show different magnitude at forward/backward η originating from different initial participant anisotropy in Cu and Au nuclei

Empirical scaling of $\varepsilon_n N_{part}^{1/3}$ doesn't work well

- Theory comparison (mid-η)

Hydro calculation needs η/s(0.08-016) to reproduce v_2, v_3

- Theory comparison(large-η)

Ideal hydro and AMPT doesn't agree with magnitude of v_2, v_3

Ideal hydro reproduce the ratio of v_2(Au) and v_2(Cu)

But Ideal hydro doesn't show the difference of v_3(Au) and v_3(Cu)

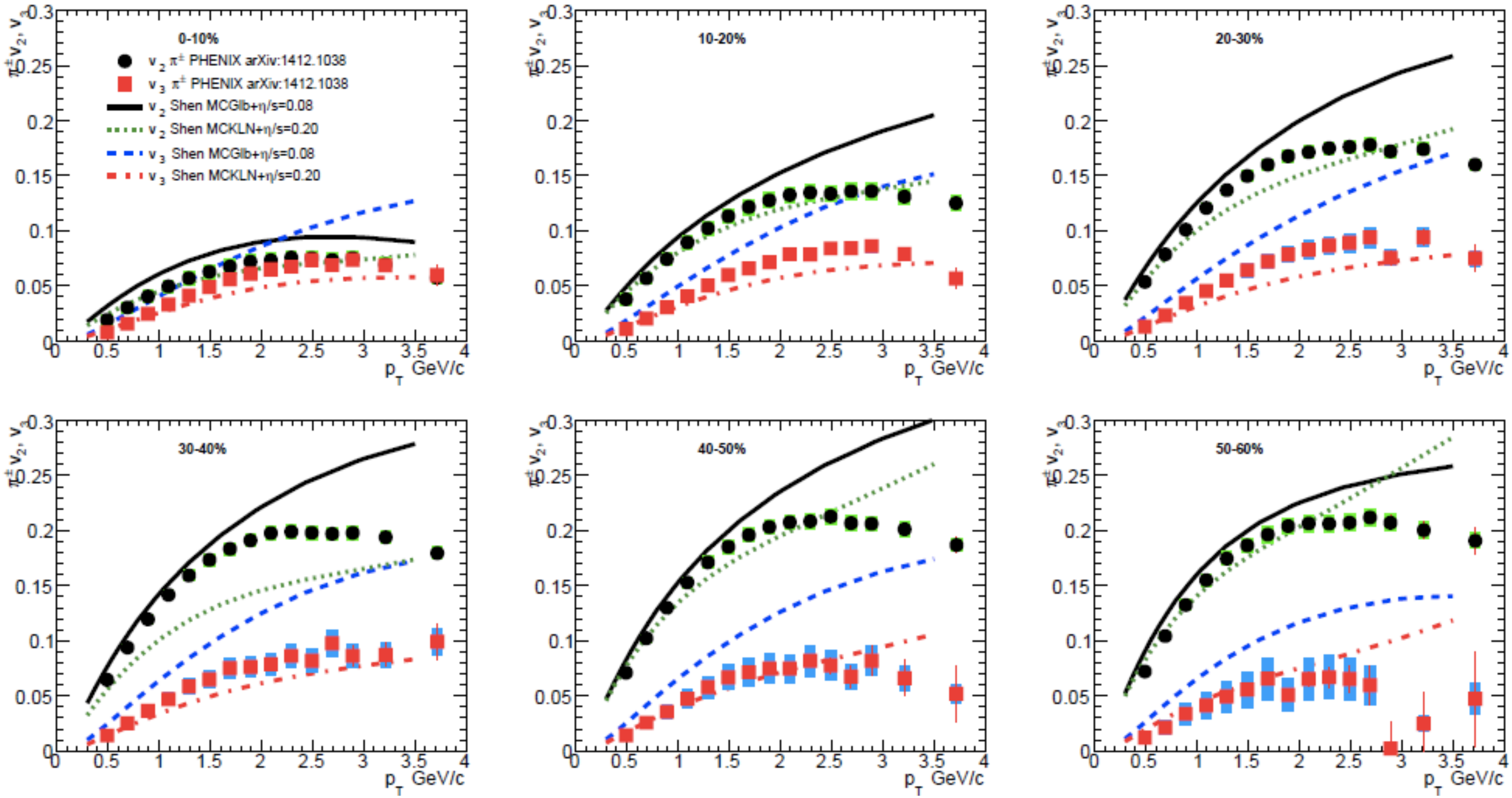
✓ -v_1

High p_T particles are emitted to Au side

Ideal hydro calculation agree with v_1

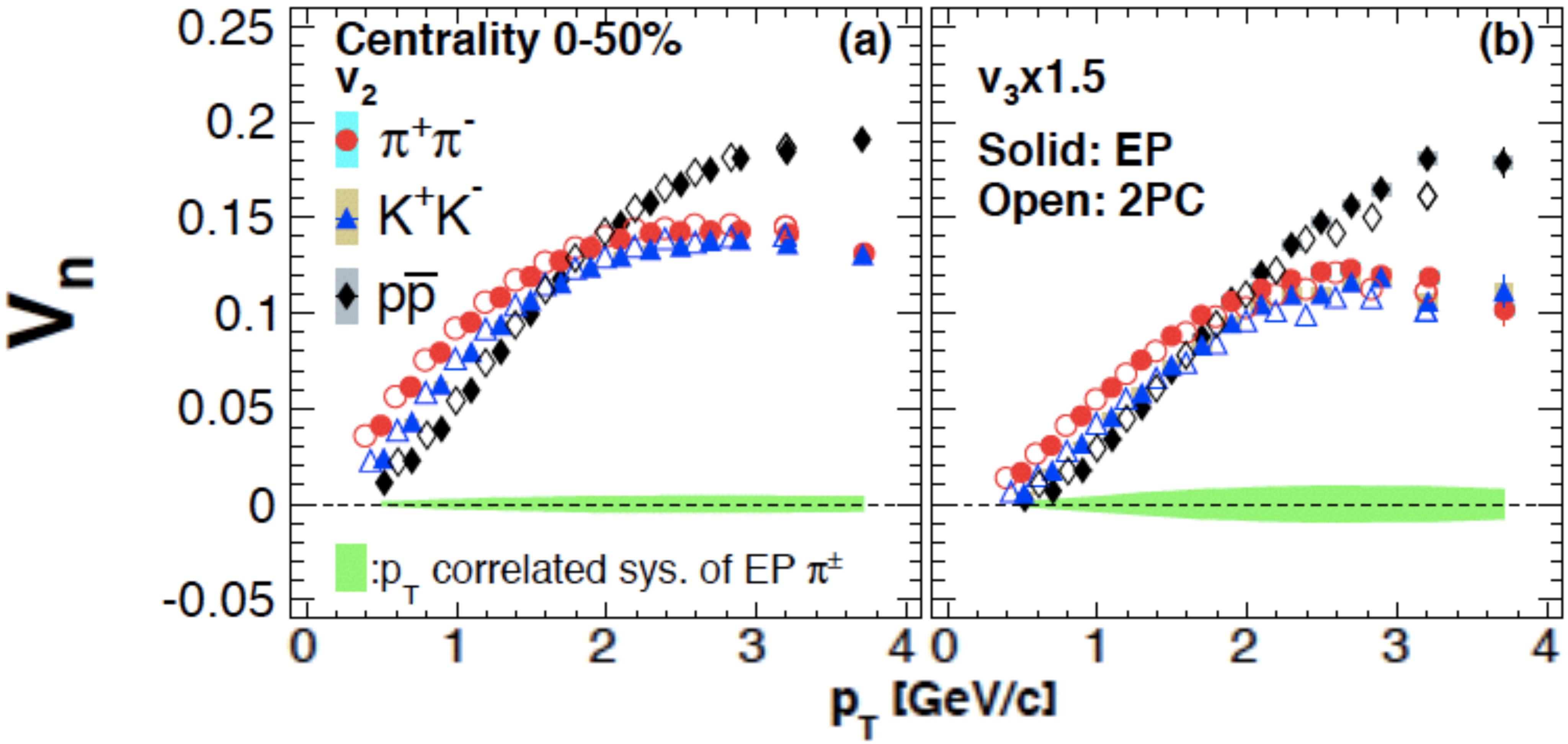
Back Up

Charged pion v2, v3 in AuAu



pi, K, p flow

arXiv:1412.1038

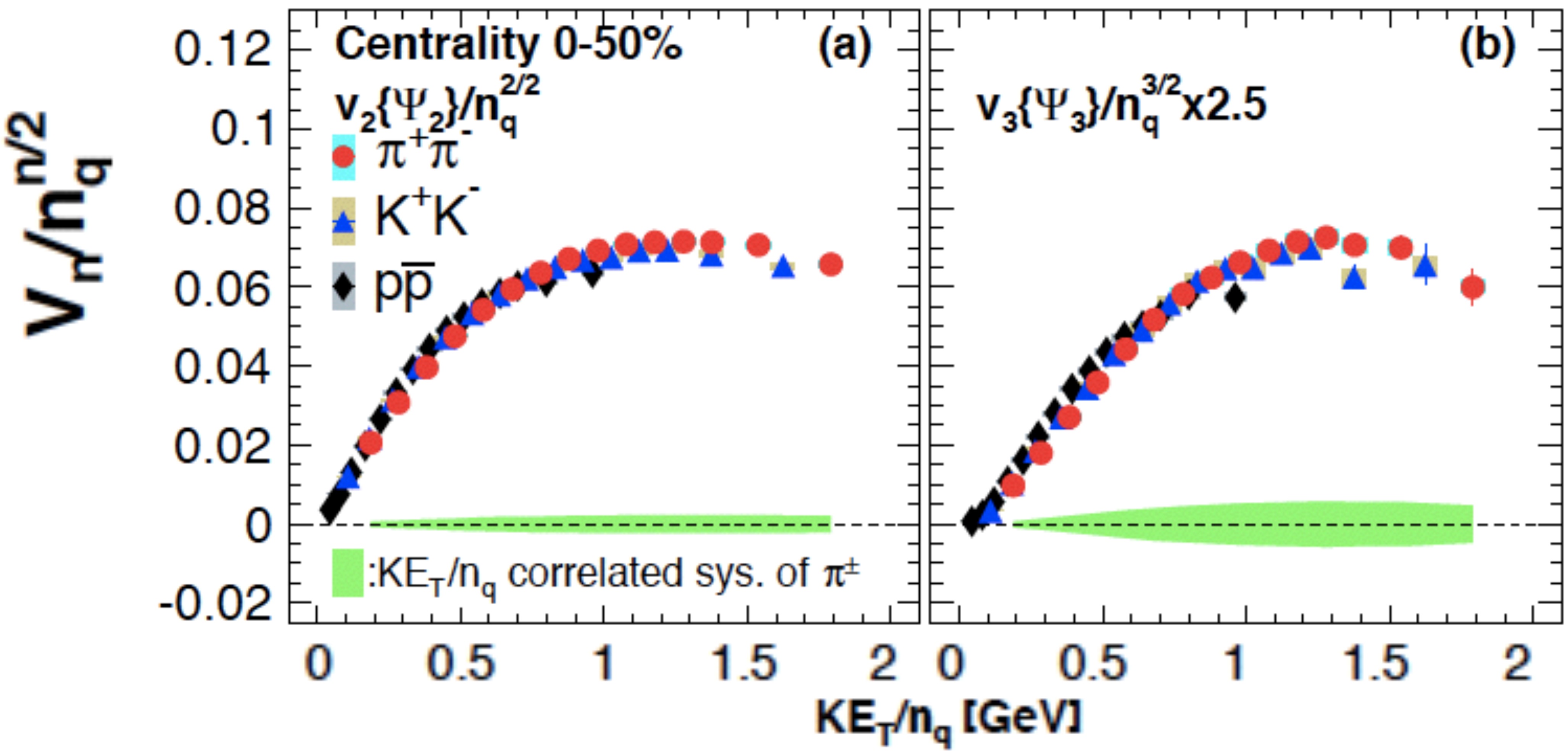


v_2,v_3 have similar particle dependence

v_3 scaled with $n_q^{3/2}$

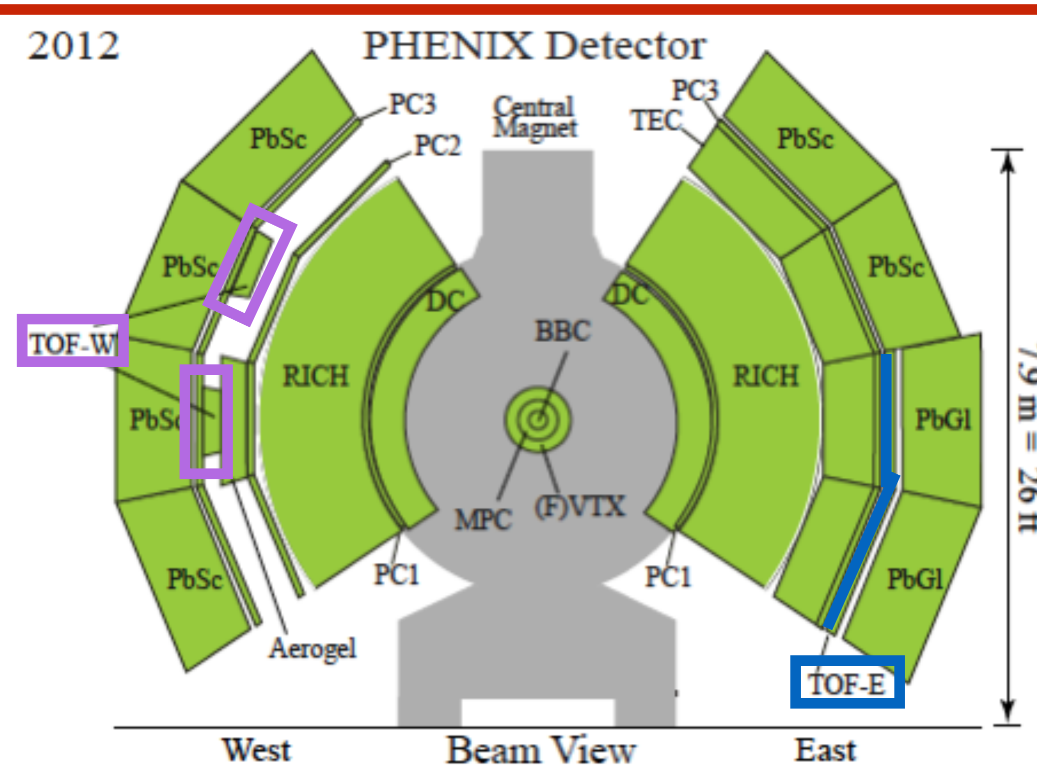
Scaling property : quark number scaling

arXiv:1412.1038



v_2,v_3 have similar particle dependence
v_3 scaled with $n_q^{3/2}$

Track identification at CNT($|\eta|<0.35$)



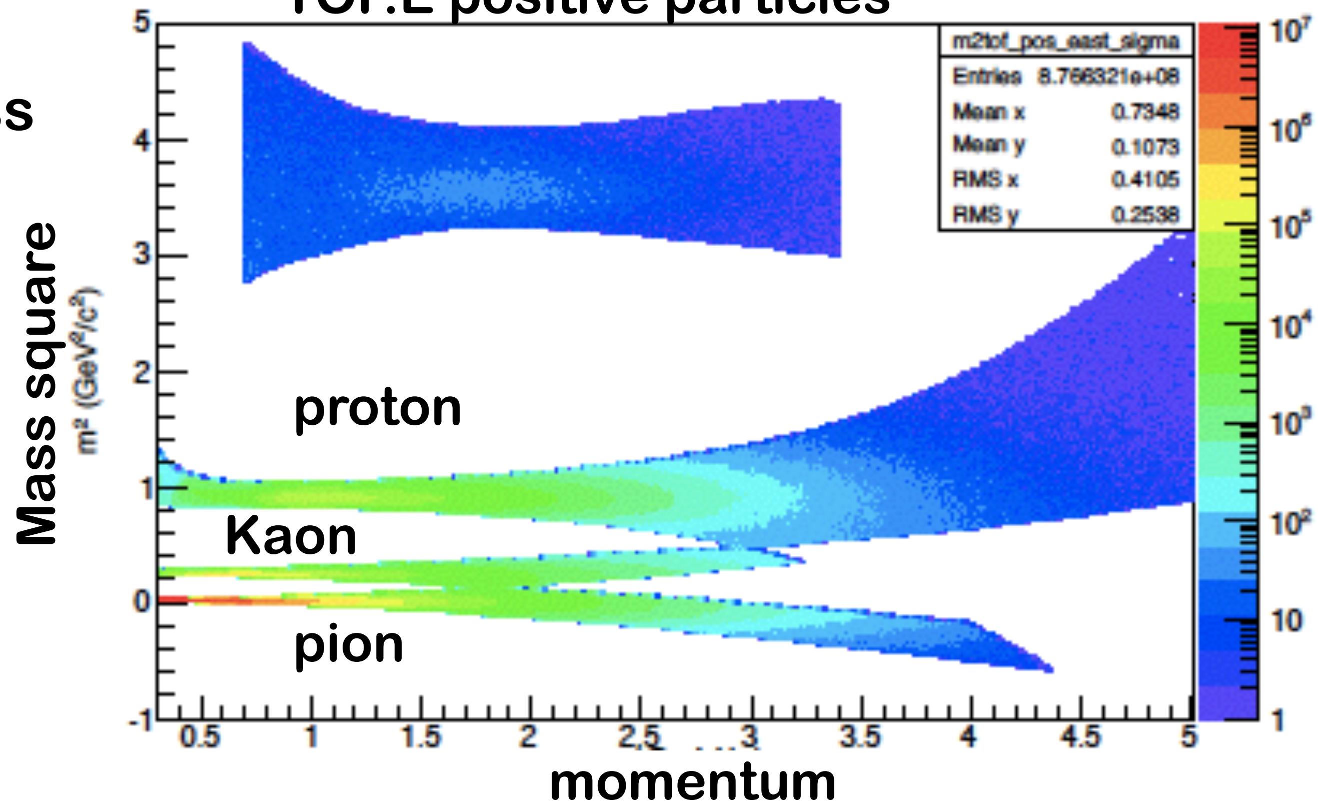
TOF.E and TOF.W are used

-TOF.E : Scintillation counter 130ps
-TOF.W : MRPC 95ps

Time of flight method

$$m^2 = p^2\left(\left(\frac{ct}{L}\right)^2 - 1\right)$$

m:particle mass, p:momentum, L:flight pass
c:light velocity, t:time of flight



Charged pi,K,p

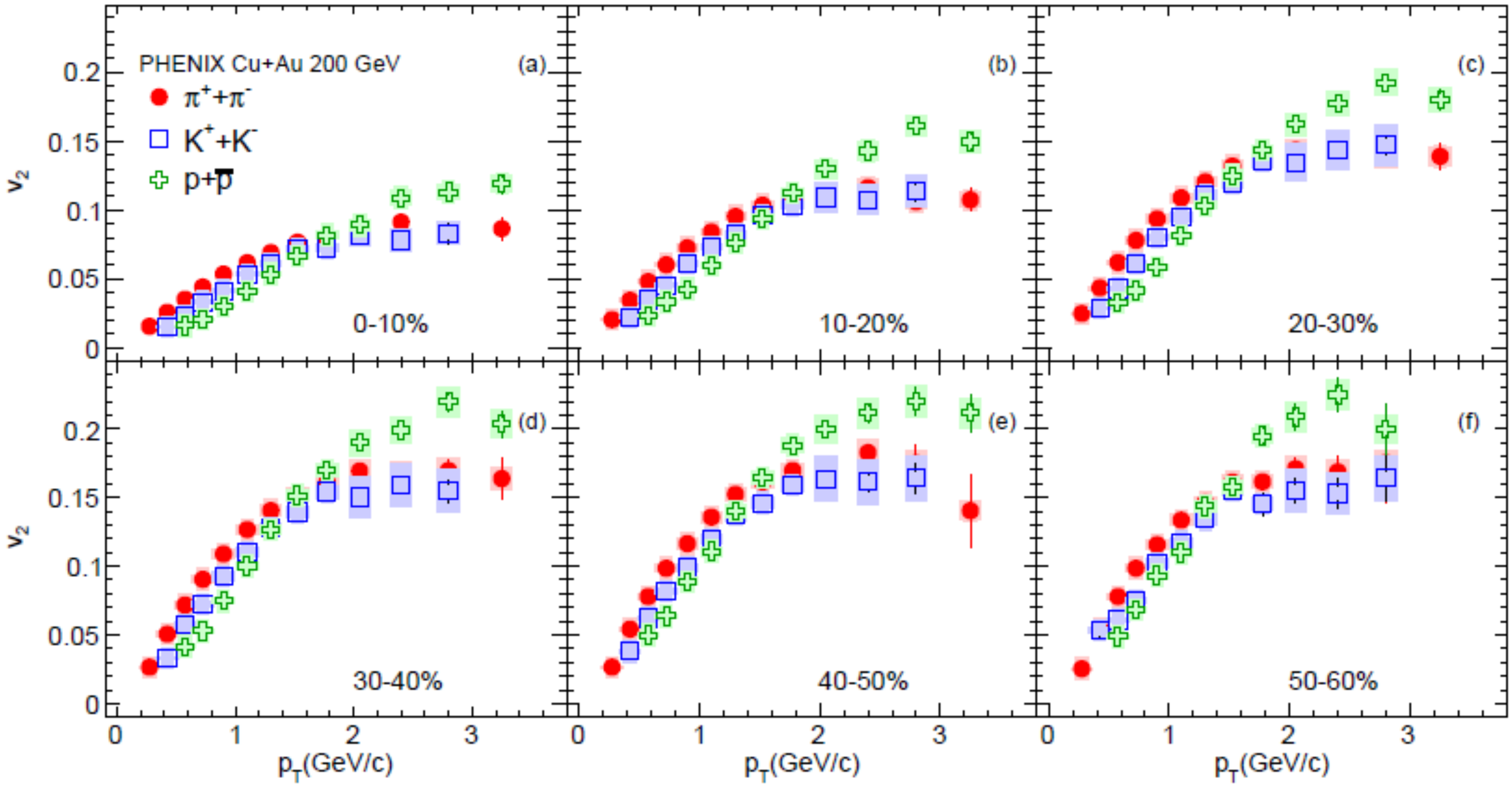
-pi/K up to 3GeV
-K/p up to 4GeV

Results
&
Discussions

- System size dependence
- PID v_n
- Rapidity dependence
- Theory comparison

Identified particle v_2 in Cu+Au

arxiv:1509.07784

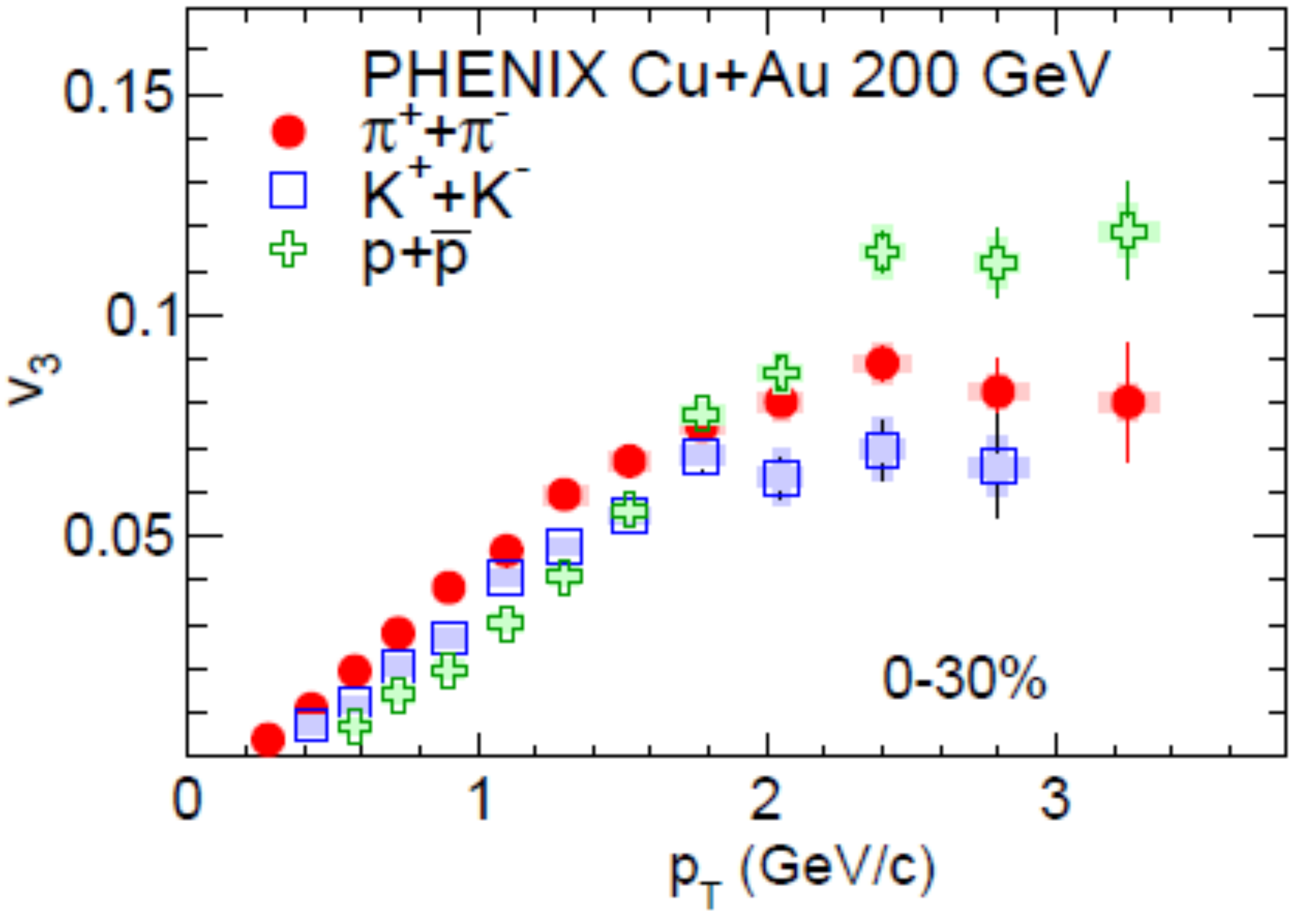


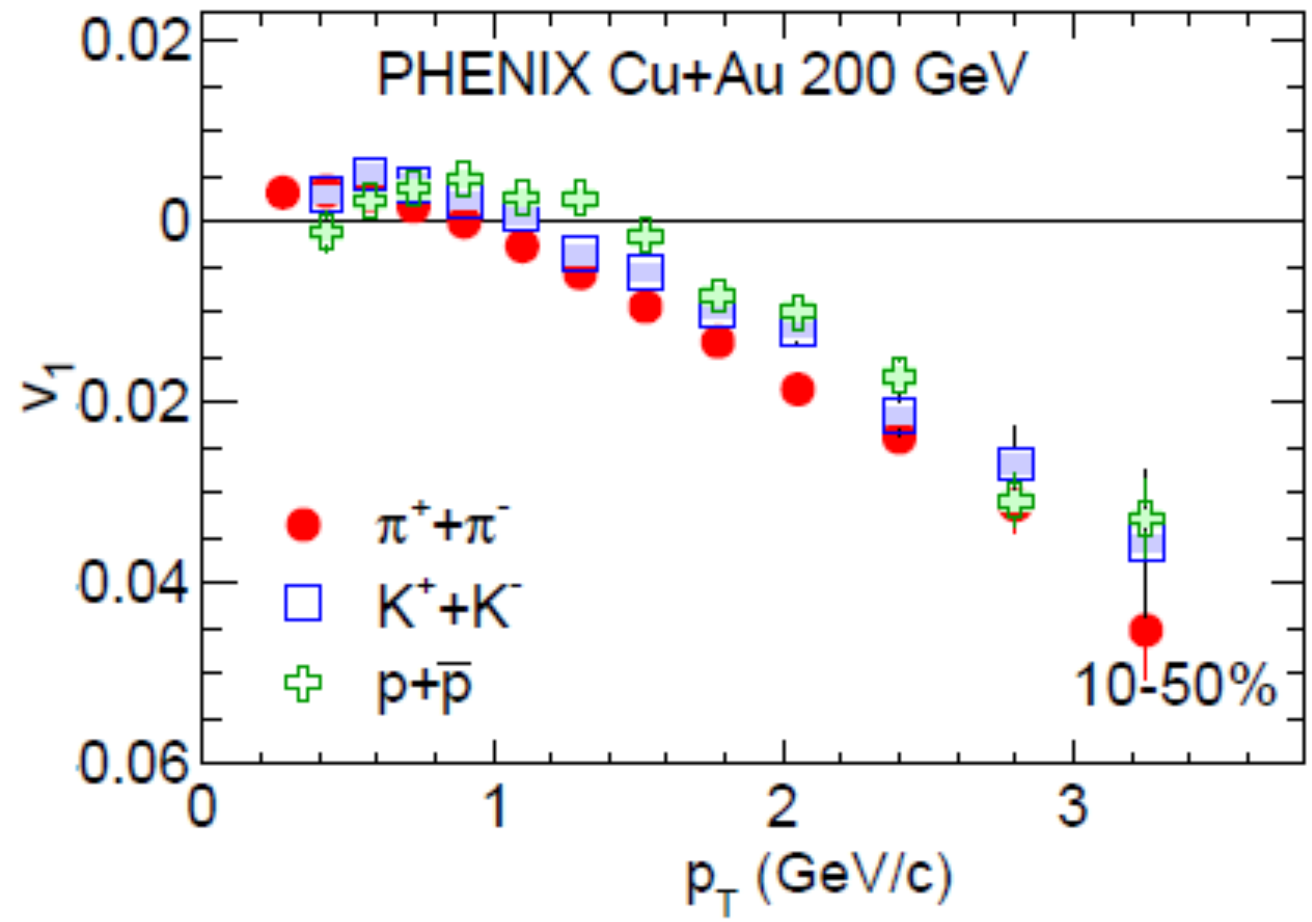
Mass ordering at low p_T for v_2 for all centralities

Baryon and meson splitting at mid-p_T is seen

Identified particle v_1, v_3 in Cu+Au

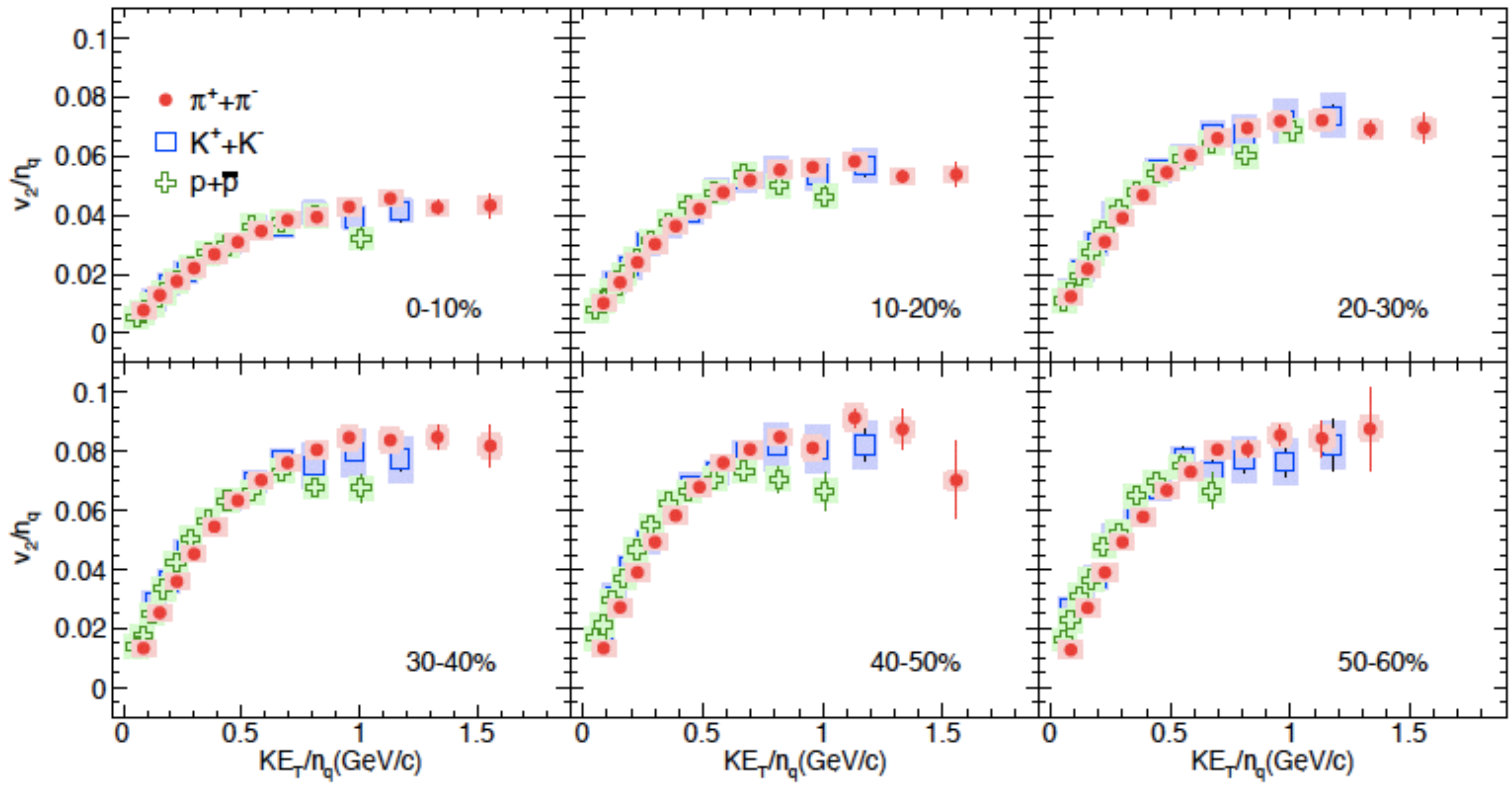
arxiv:1509.07784





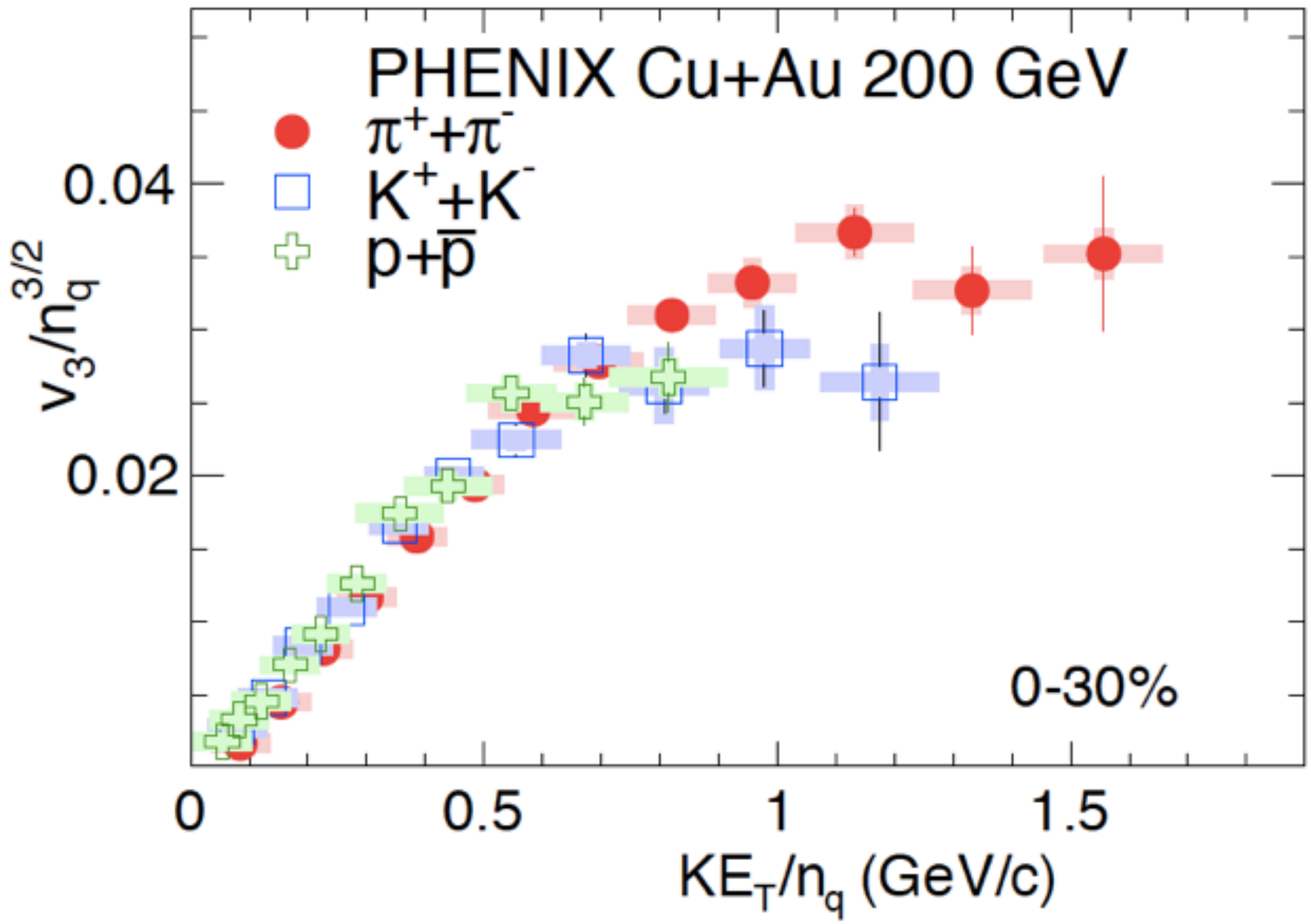
Same particle dependence of v_3 is seen as seen in v_2
Mass ordering is also seen for v_1
- At 1<pT<2.5GeV, Mass ordering is seen
- At low and high pt region, baryon v_1 ~ meson v_1
-Not same trend as seen in v_2, v_3

Quark Number Scaling of v2



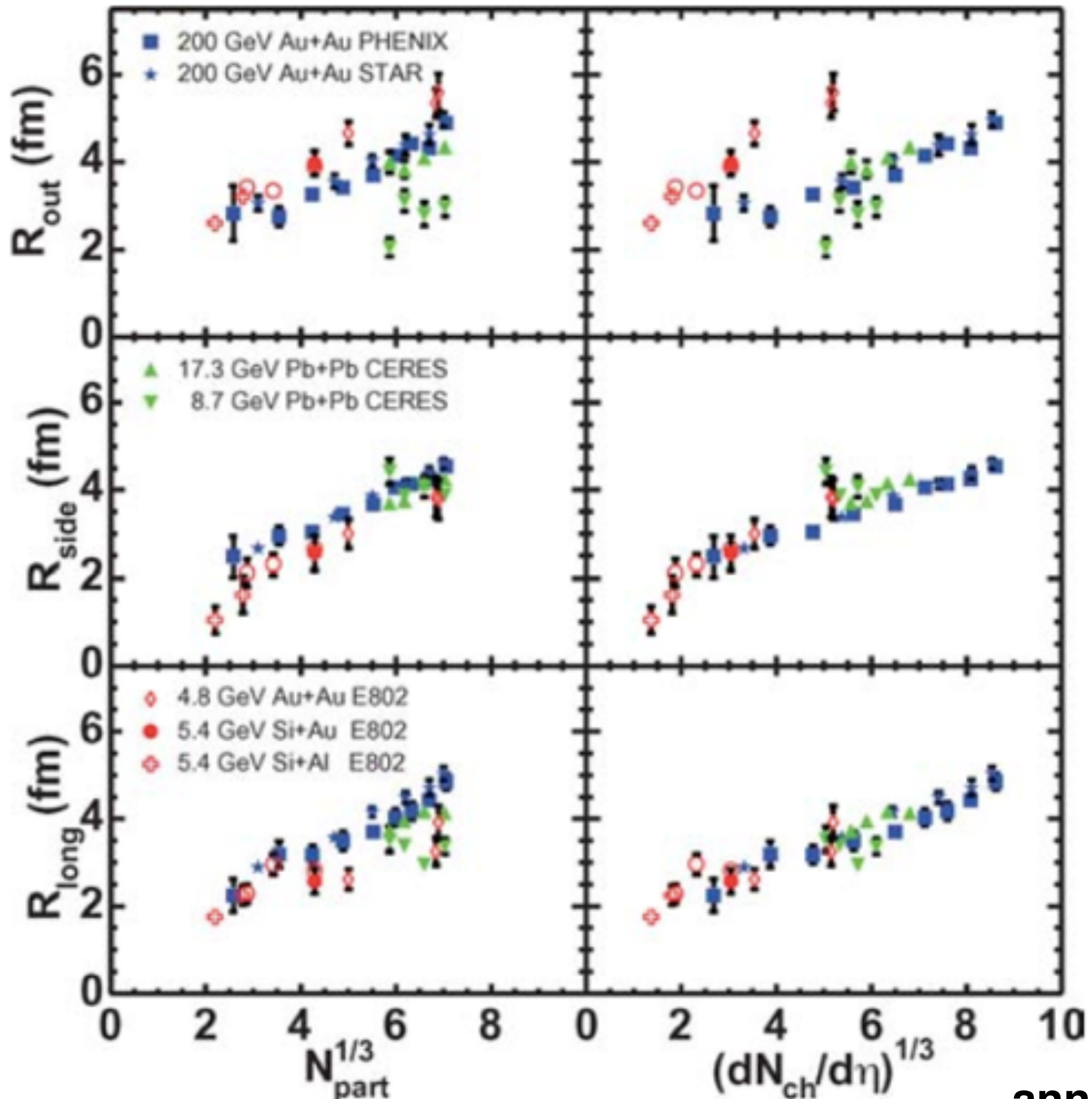
Quark Number Scaling works v2 in CuAu

Quark Number Scaling for v3

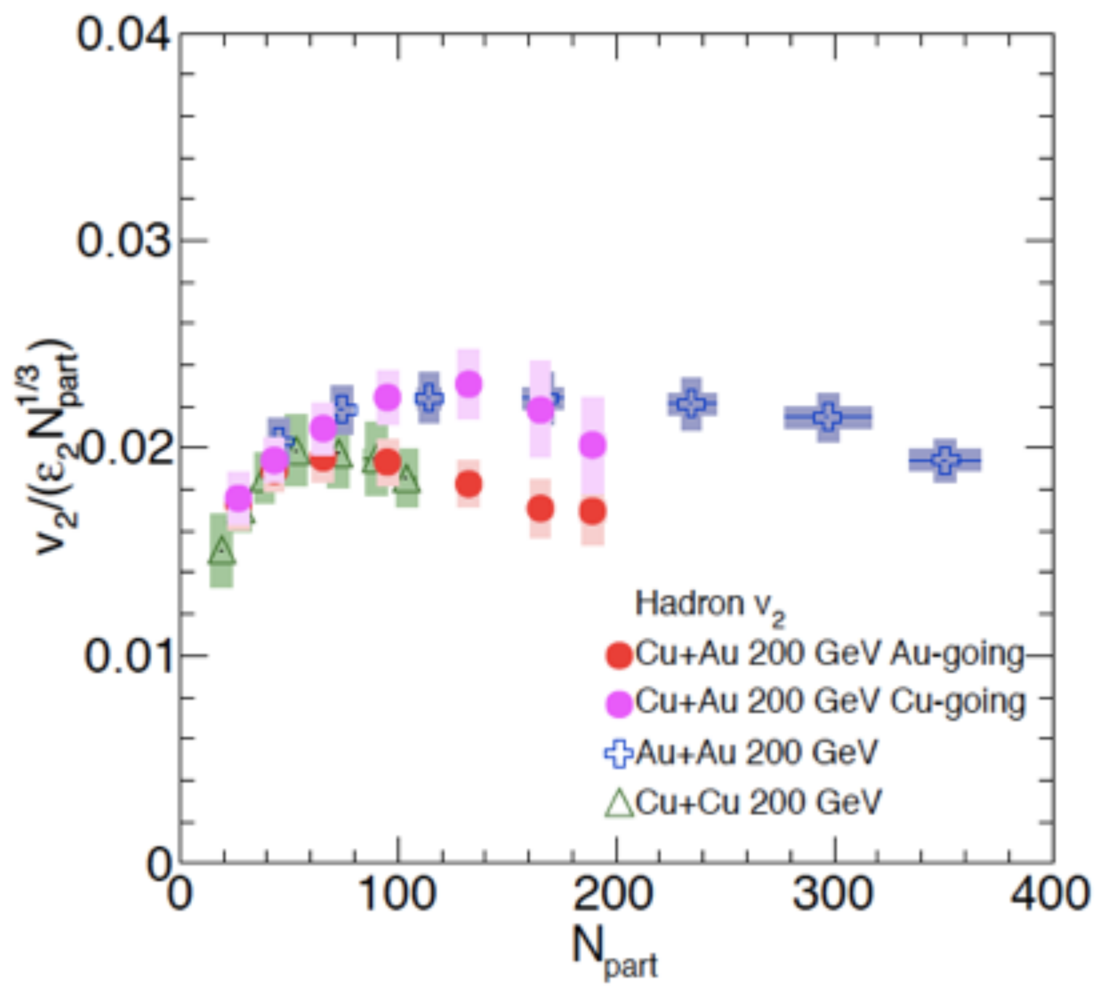


Quark Number Scaling work v3 in CuAu

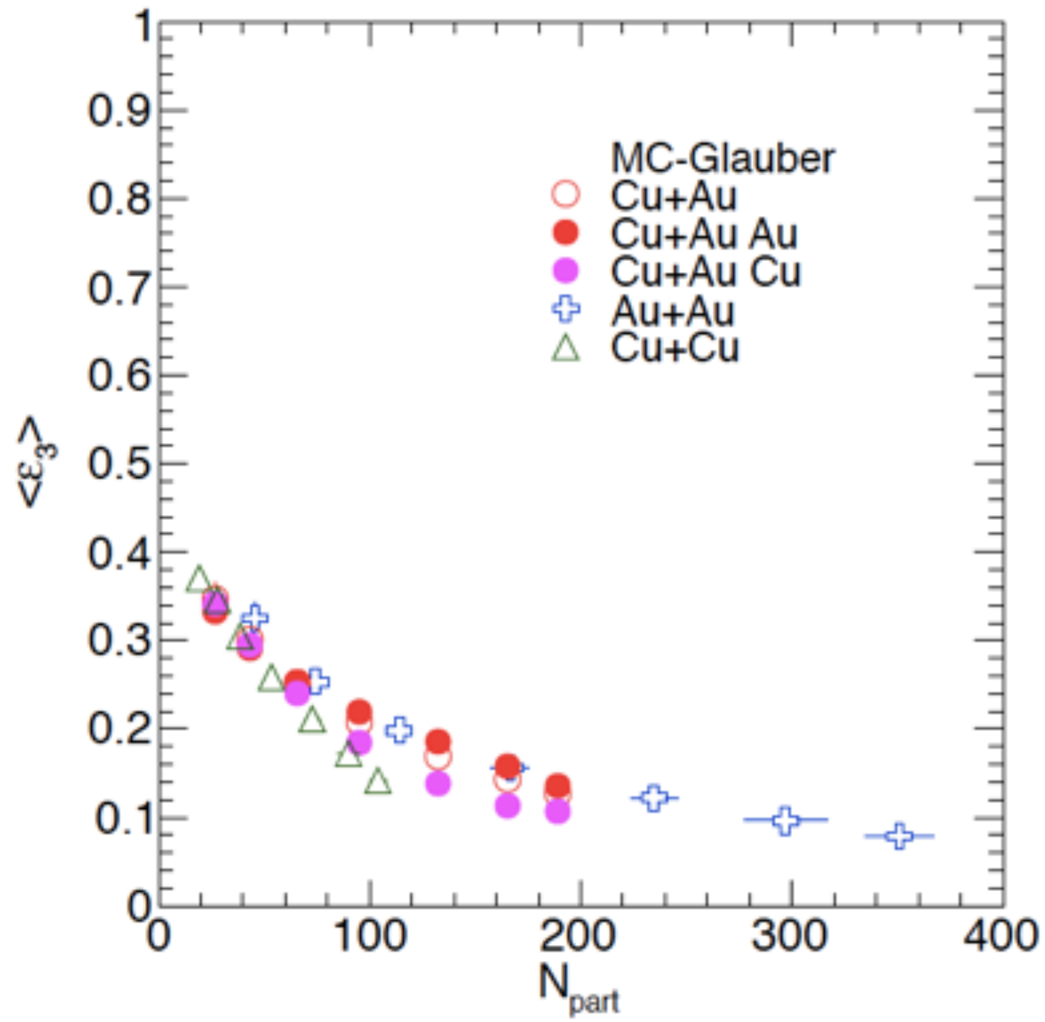
$N_{part}^{1/3}$ is proportional system size

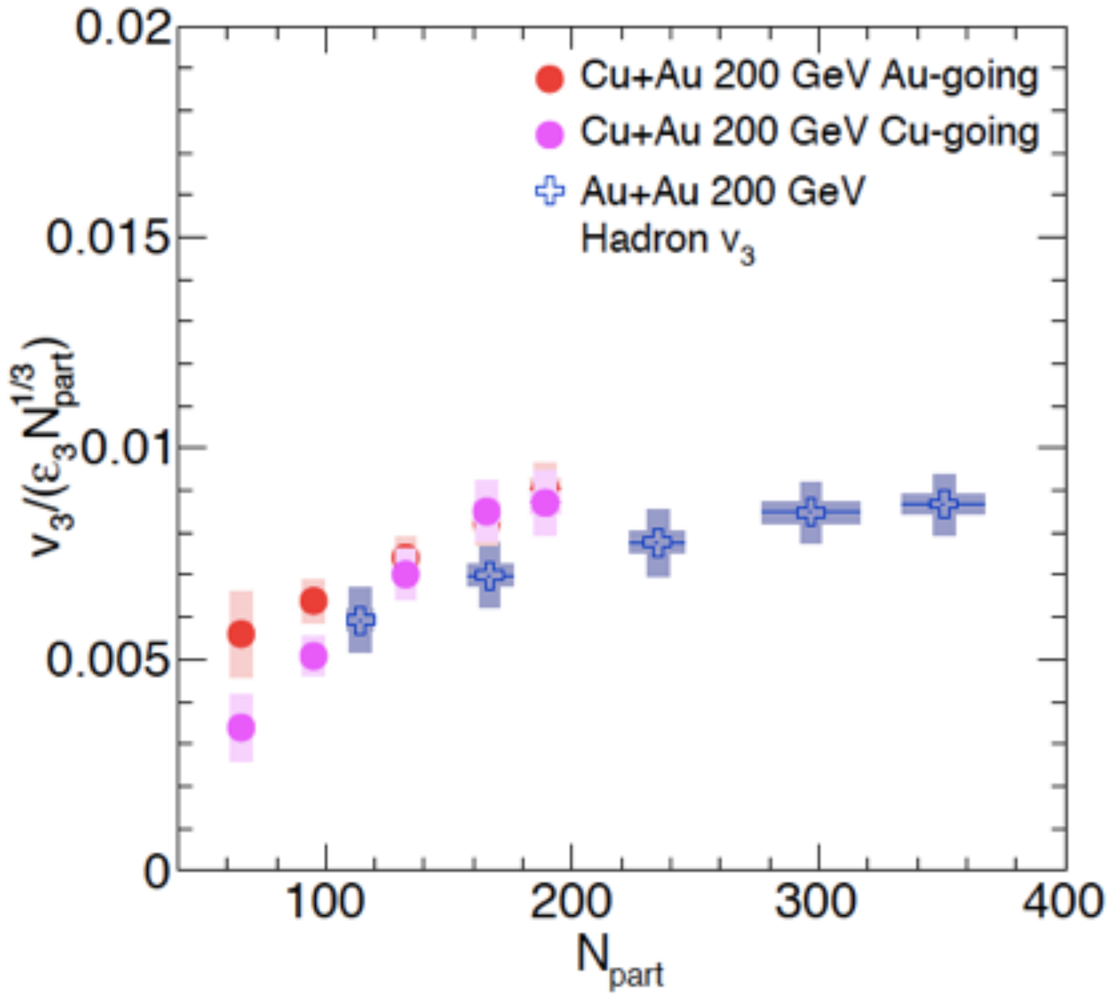


annurev.nucl.55.090704.151533



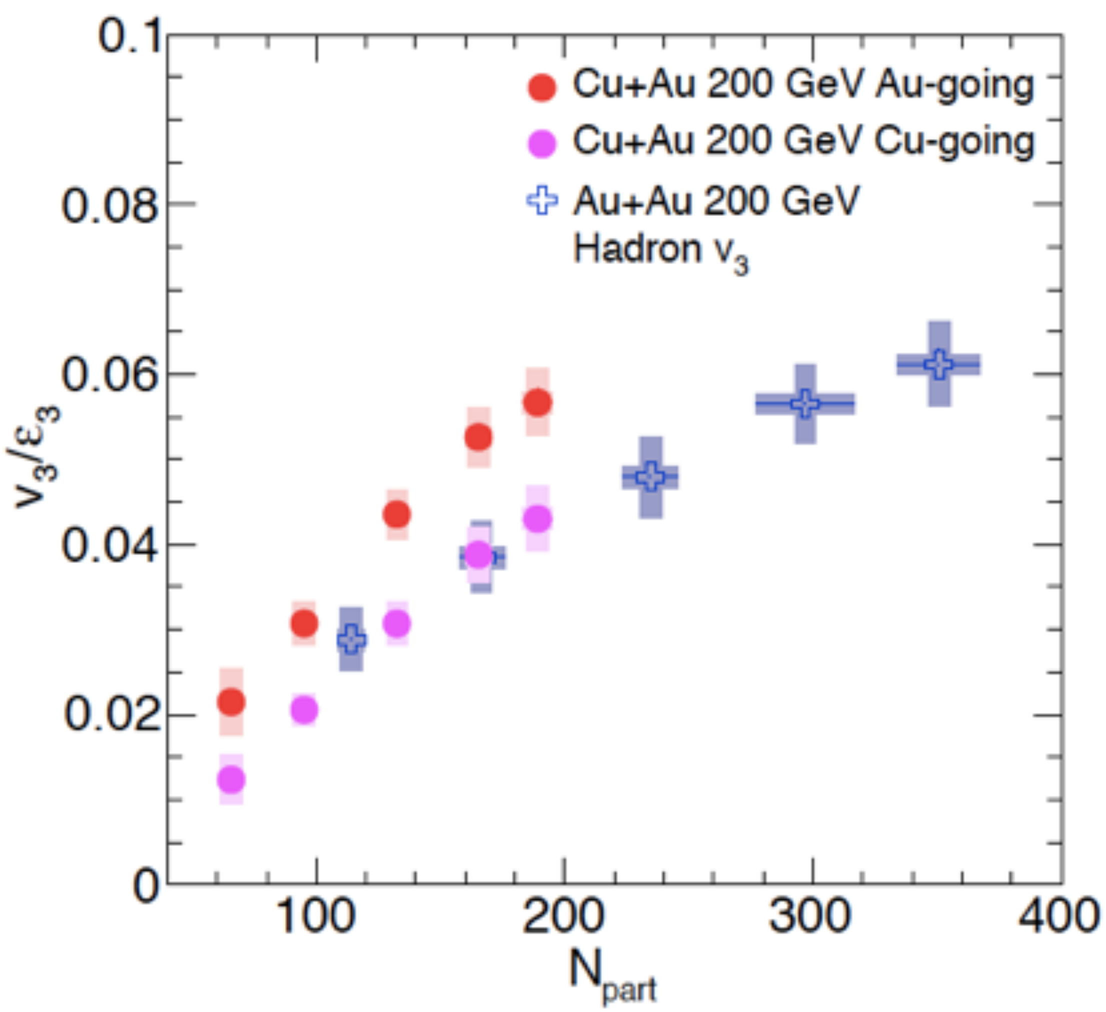
dif eCu, eAu
same NpartCu and NpartAu

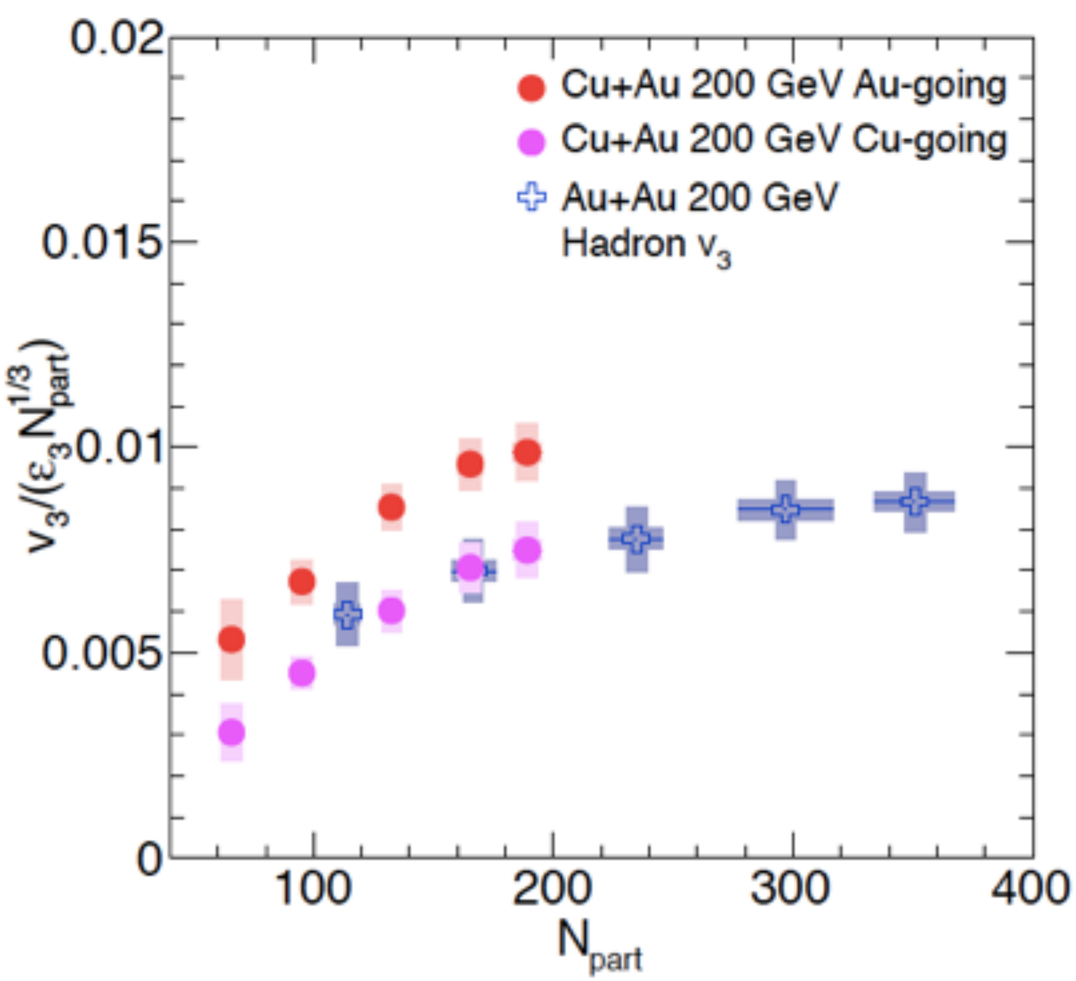




dif eCu, eAu
same NpartCu and NpartAu

Scaling with $\varepsilon_{3,CuAu}$ and $N_{part, CuAu}$





v_3 scaled with ε_3

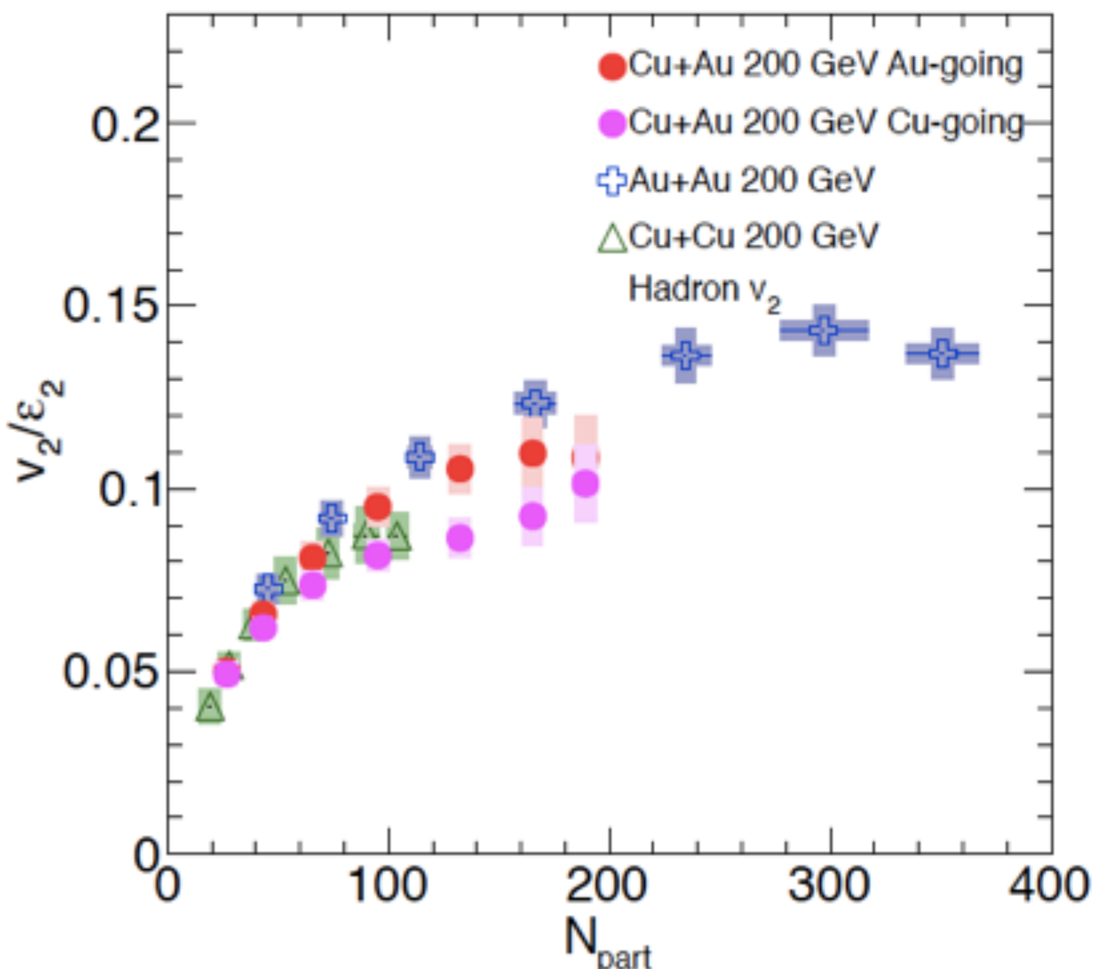
-v_3(Au-going) is not consistent with AuAu v_3

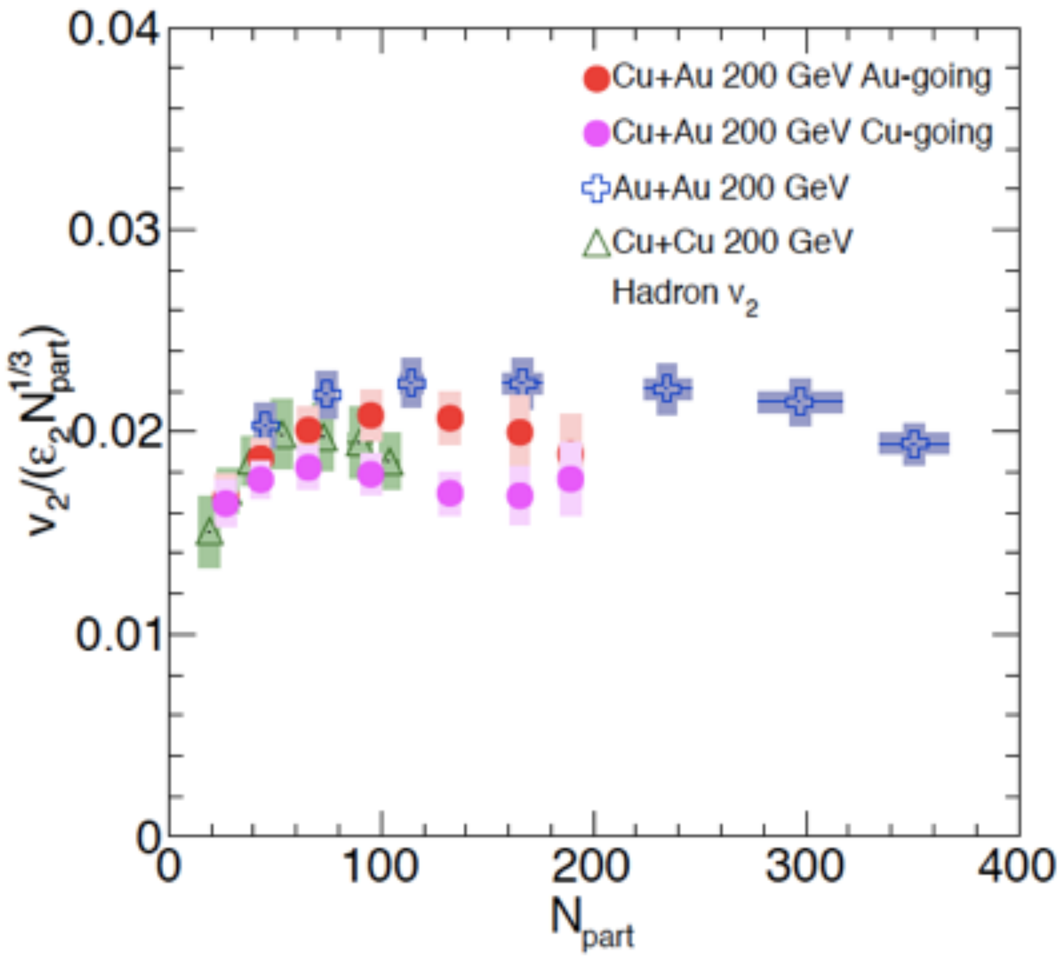
-v_3(Cu-going) is consistent with AuAu v_3

v_3 scaled with $\varepsilon_3 N_{part}^{(1/3)}$

-v_3(Au-going) is not consistent

Scaling with $\varepsilon_{2,CuAu}$ and $N_{part, CuAu}$





v_2 scaled with $\varepsilon_{2,AB}$

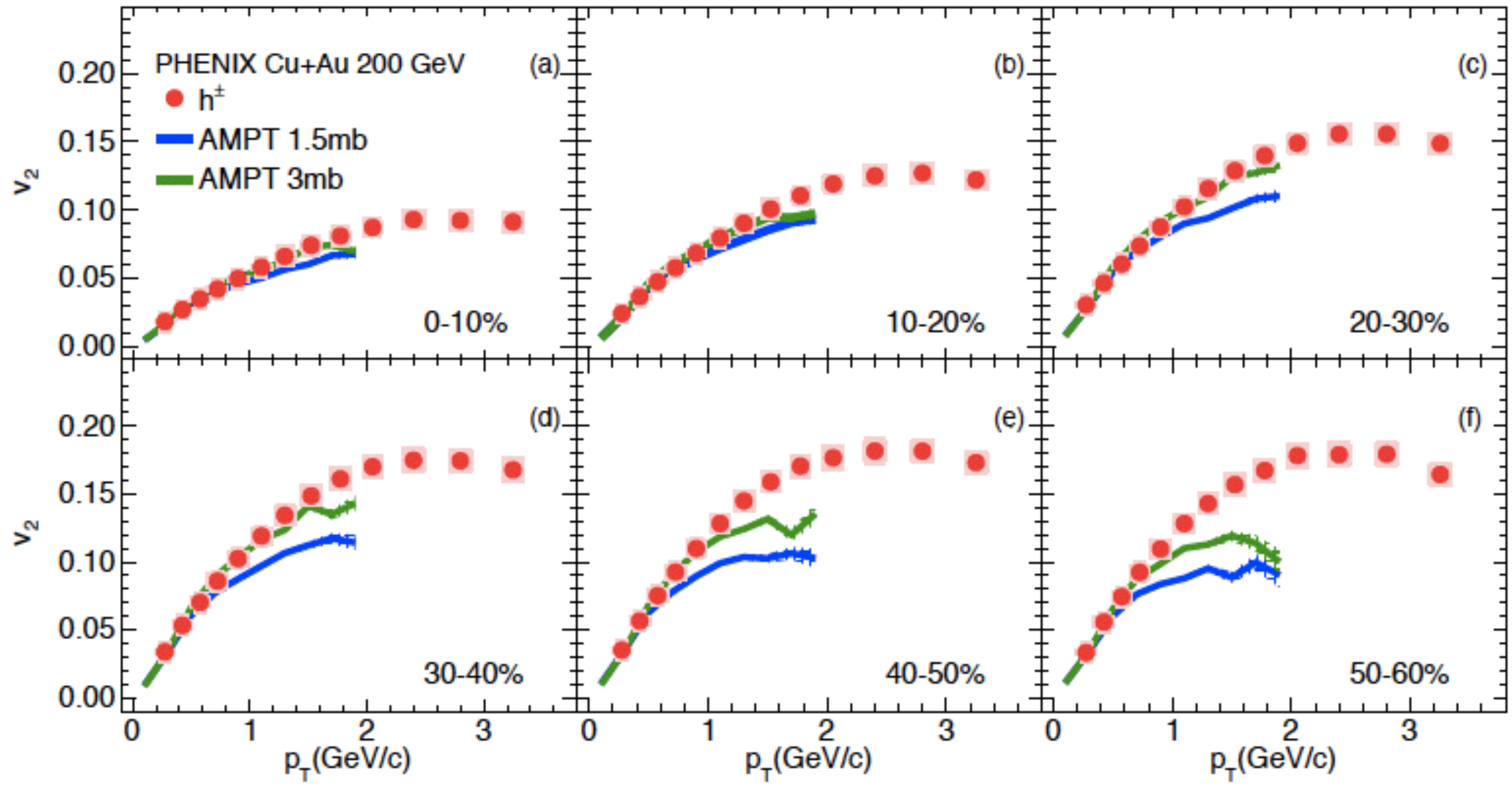
- **Scaled CuCu and AuAu v_2 show universal curve**
- **v_2(Au-going) is consistent with CuCu, AuAu v_2**
- **v_2(Cu-going) is not consistent with CuCu, AuAu v_2 except for peripheral**

v_2 scaled with $\varepsilon_2 N_{part}^{(1/3)}$

- **v_2(Cu-going) are not consistent for mid-central collisions**

Comparison to AMPT v2

arxiv:1509.07784



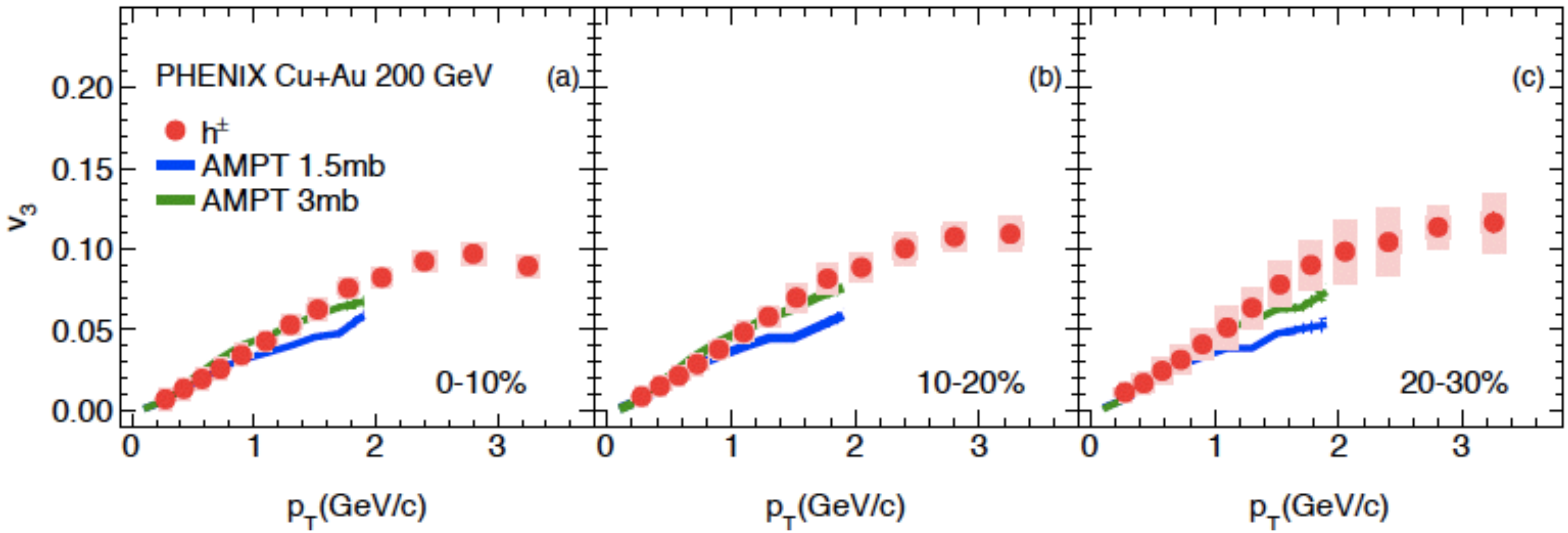
AMPT with 3mb reproduce v2

-In 0-30%, up to 2GeV

-In 30-60%, up to 1GeV

Comparison to AMPT v3

arxiv:1509.07784



AMPT with 3mb reproduce v3
-In 0-30%, up to 2GeV

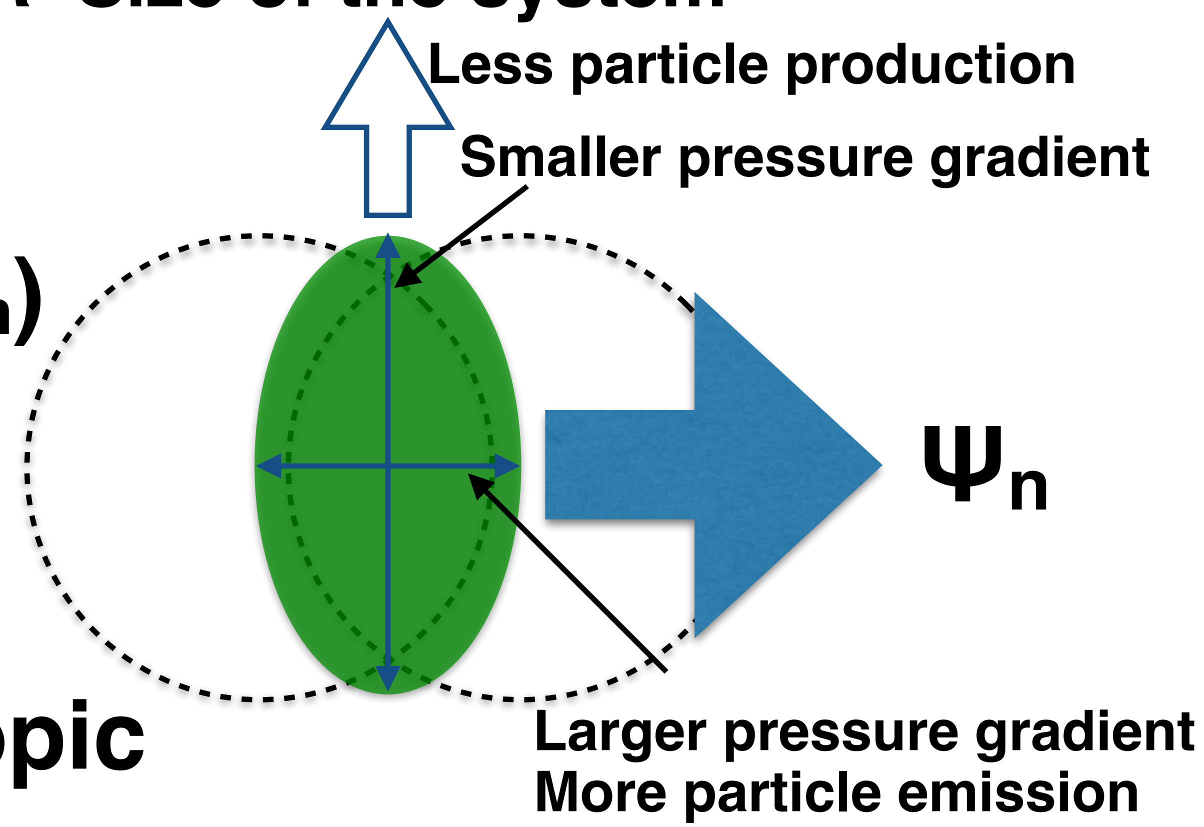
Azimuthal anisotropic flow

In relativistic heavy ion collisions, the azimuthal distribution of produced particles is anisotropic

Anisotropic initial overlap region(ε_n)

If $\lambda \ll R$, pressure gradient ΔP becomes anisotropic.

λ=Mean free path R=Size of the system



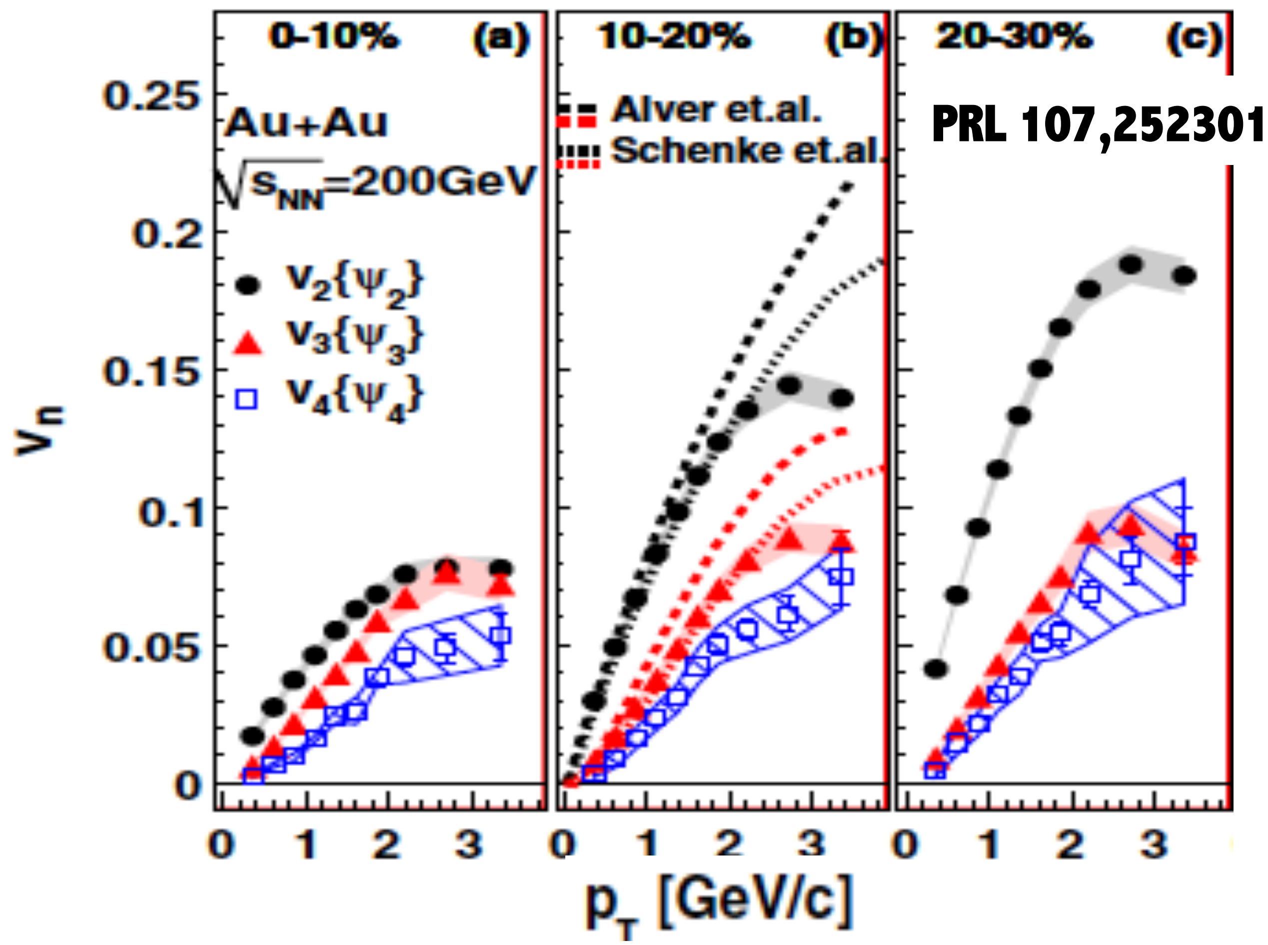
Anisotropic particle emission(v_n)

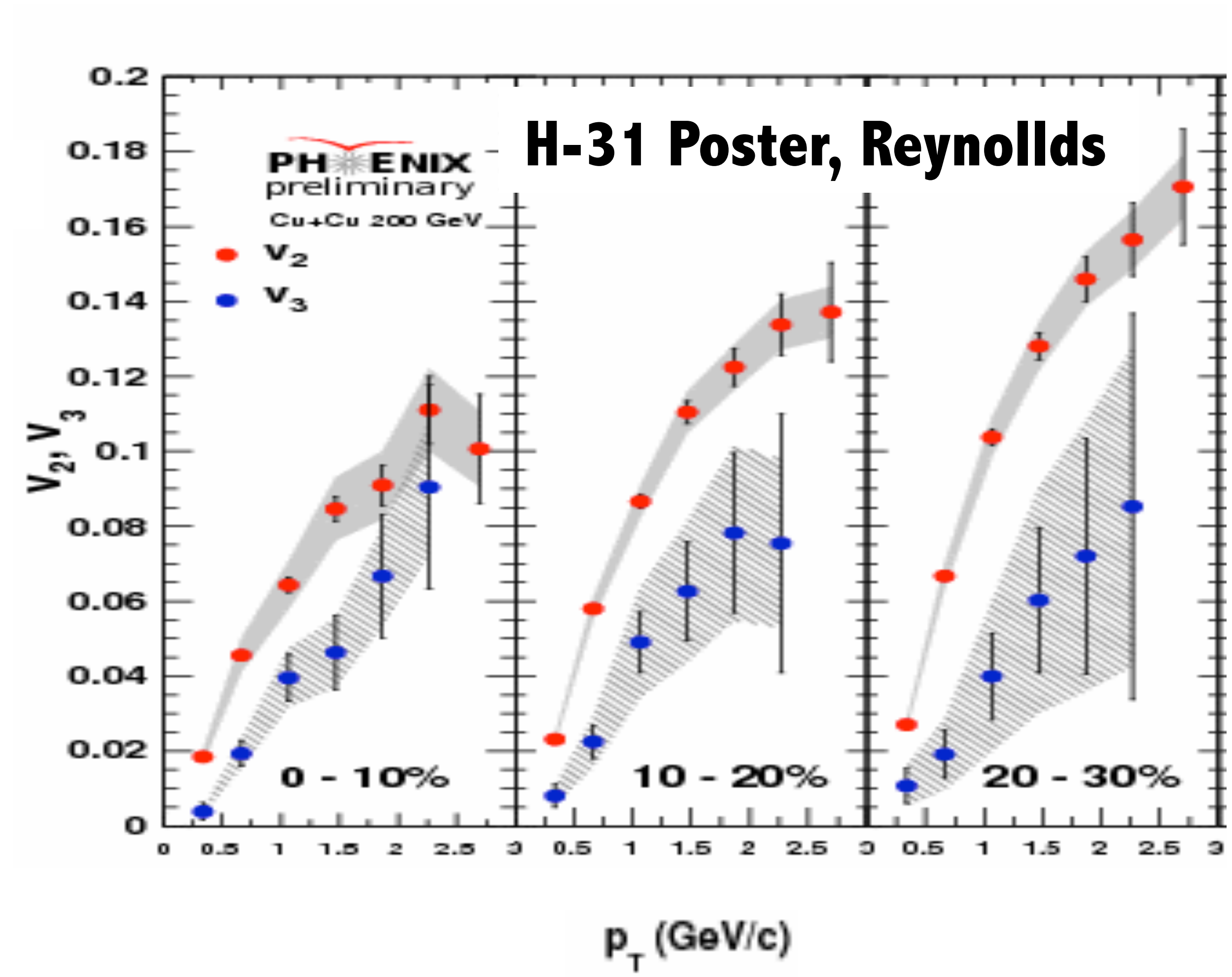
Expansion to short axis direction
by anisotropic pressure gradient

Magnitude of azimuthal anisotropic particle emission is evaluated as

$v_n = \langle \cos(n[\phi - \Psi_n]) \rangle$ ellipticity with respect to Ψ_n

Flow in symmetric collisions system





3sub method

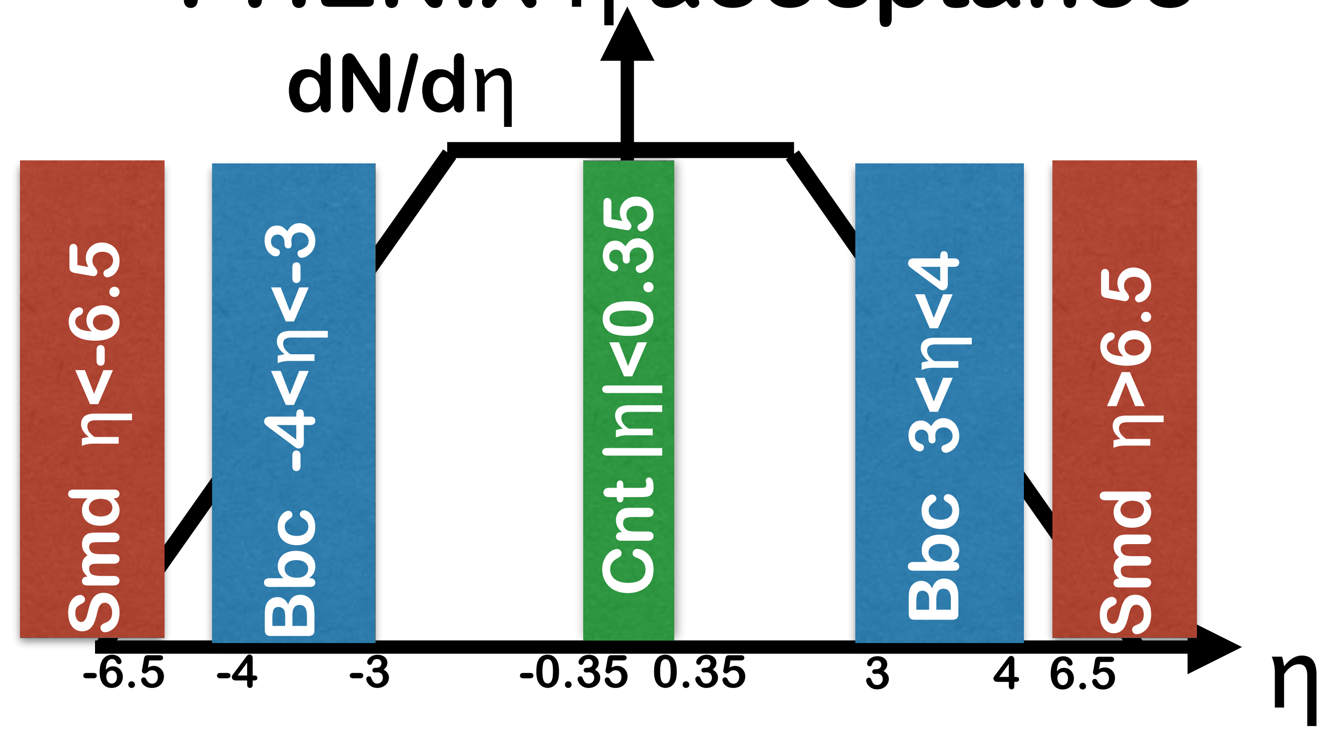
Event Plane detectors

- 2nd, 3rd Event plane
  - Bbc, Cnt
- 1st Event plane
  - Bbc, Smd

Event Plane resolution

-Estimated from EP correlations(3sub method)

PHENIX η acceptance



$$Res\{\Psi_{n,A}^{obs}\} = \sqrt{\frac{< cos(n[\Psi_{n,A}^{obs} - \Psi_{n,B}^{obs}]) >< cos(n[\Psi_{n,A}^{obs} - \Psi_{n,C}^{obs}]) >}{< cos(n[\Psi_{n,B}^{obs} - \Psi_{n,C}^{obs}]) >}}$$

$$\begin{aligned} < \cos(n[\Psi_{n,A}^{obs} - \Psi_{n,B}^{obs}]) > &= < \cos(n[\Psi_{n,A}^{obs} - \Psi_{n}^{true} + \Psi_{n}^{true} - \Psi_{n,B}^{obs}]) > \\ &= < \cos(n[\Psi_{n,A}^{obs} - \Psi_{n}^{true}]) >< \cos(n[\Psi_{n}^{true} - \Psi_{n,B}^{obs}]) > \\ &= Res\{\Psi_{n,A}\}Res\{\Psi_{n,B}\} \end{aligned}$$