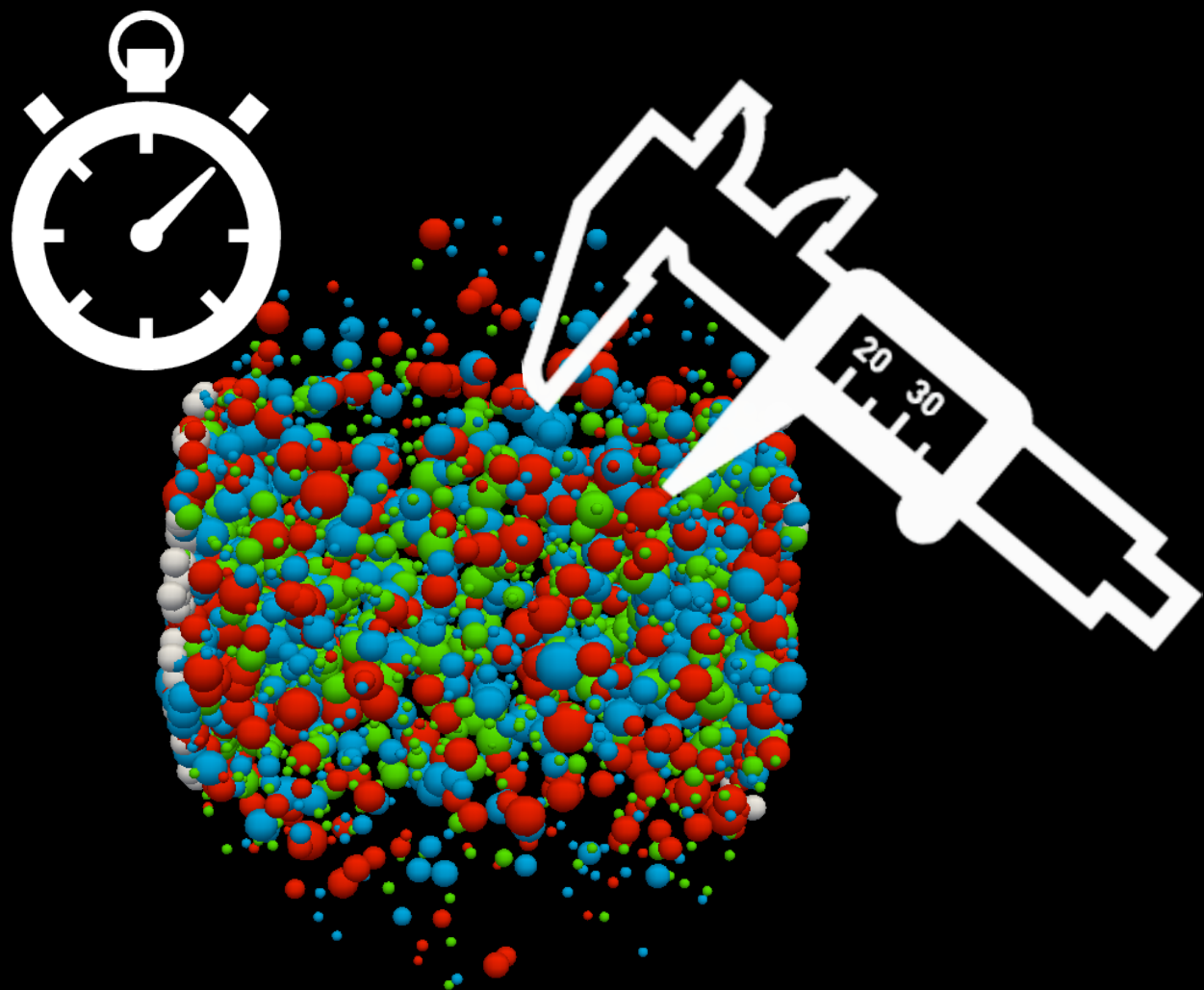


Measurements of Azimuthal Angle Dependence of HBT radii with respect to the event plane in $\sqrt{s_{NN}} = 2.76$ TeV Pb-Pb collisions at LHC-ALICE



田中 直斗

高エネルギー原子核実験グループ

2018. March. 5

Naoto Tanaka

High Energy Nuclear Physics
Group

Outline

✓ Introduction

- Quark Gluon Plasma
- HBT interferometry

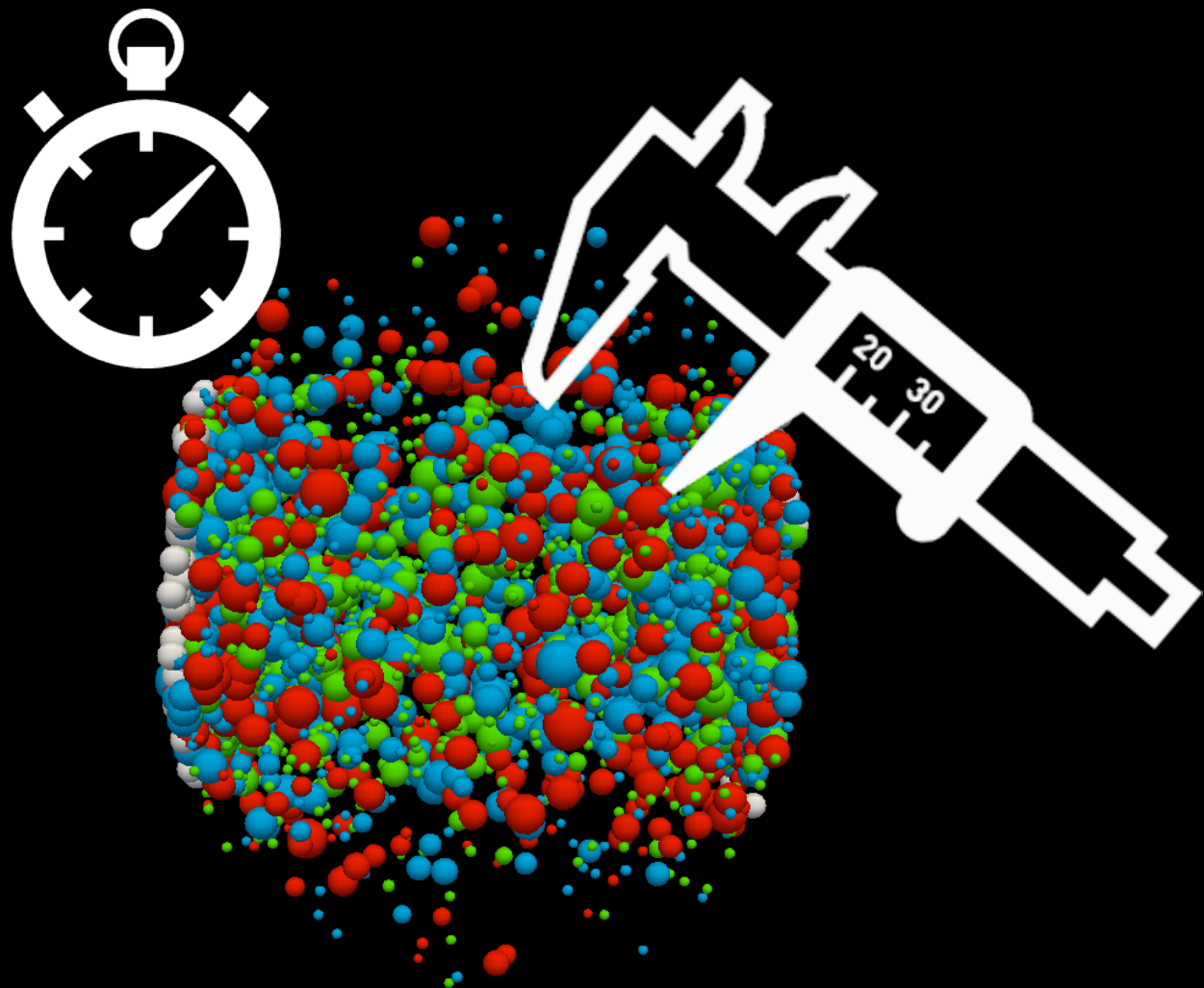
✓ Experiment & Analysis

- ALICE
- Analysis methods

✓ Result & Discussions

- Azimuthal angle dependence of HBT radii w.r.t. Ψ_2
- Azimuthal anisotropy (v_2 and v_3) with ESE cut
- Azimuthal angle dependence of HBT radii w.r.t. $\Psi_2 + q_2$ cut
- Interpretation with Blast-wave fit
- Azimuthal angle dependence of HBT radii w.r.t. Ψ_3
- Azimuthal angle dependence of HBT radii w.r.t. $\Psi_3 + q_3$ cut

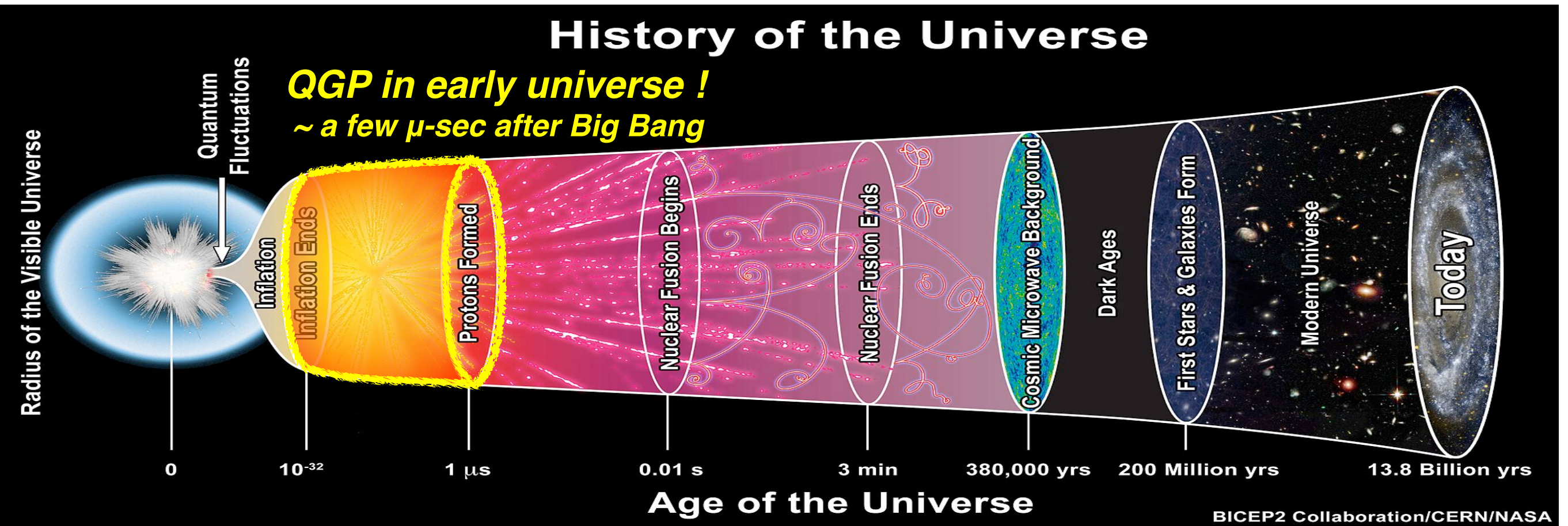
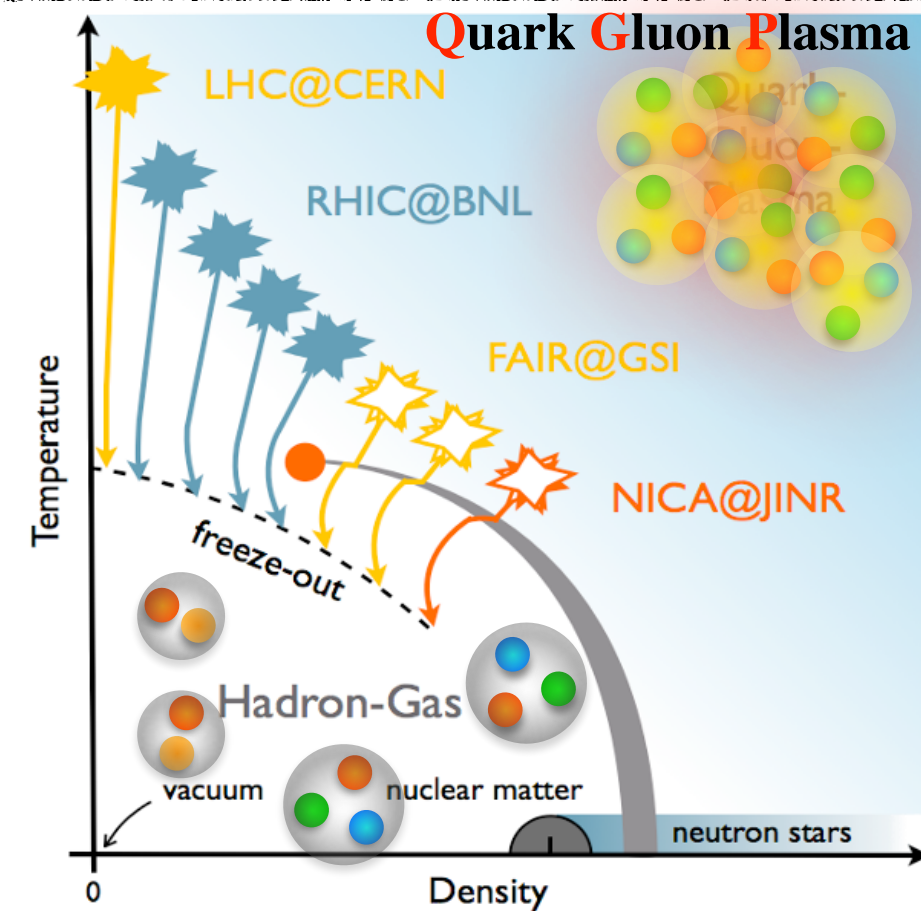
Introduction



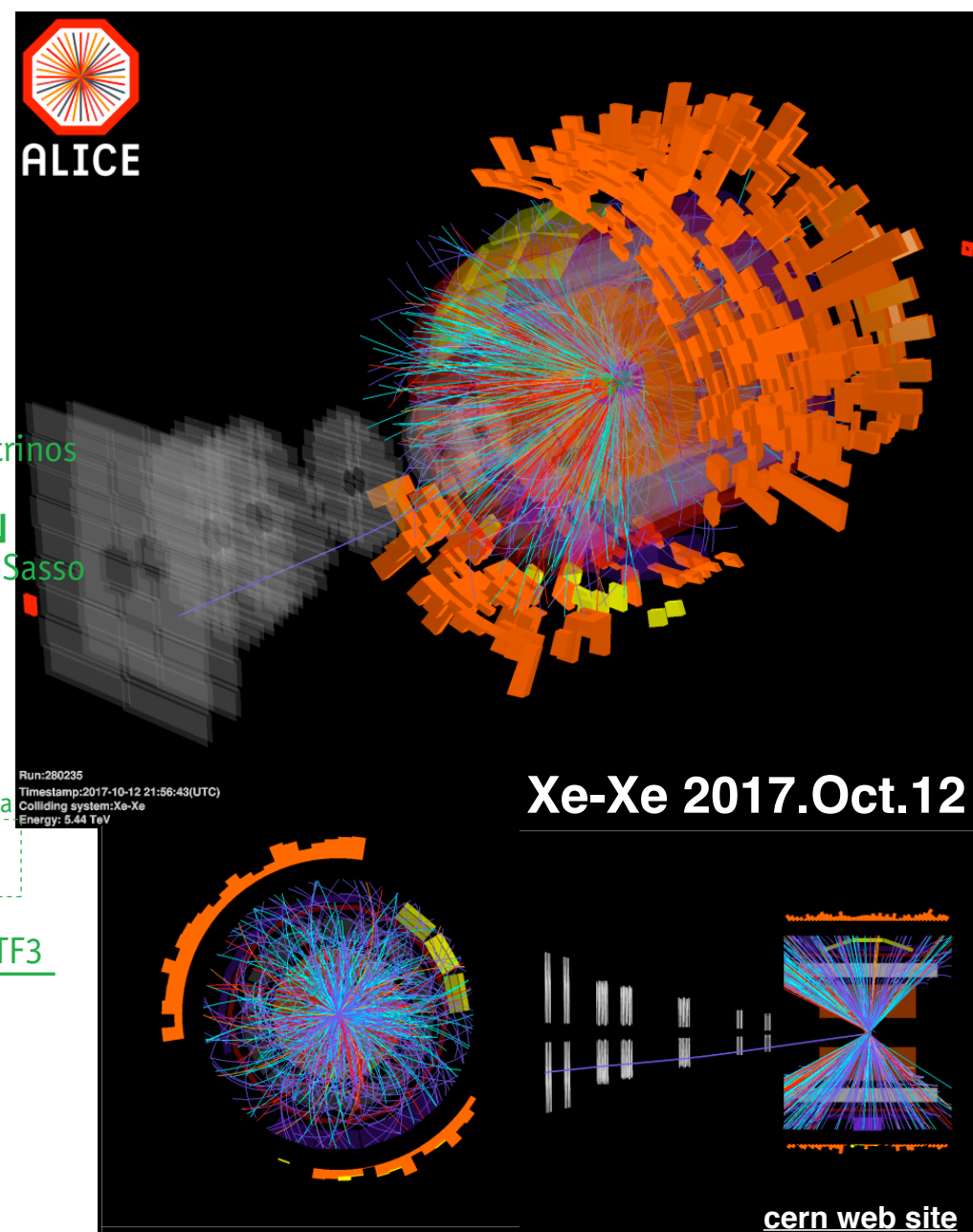
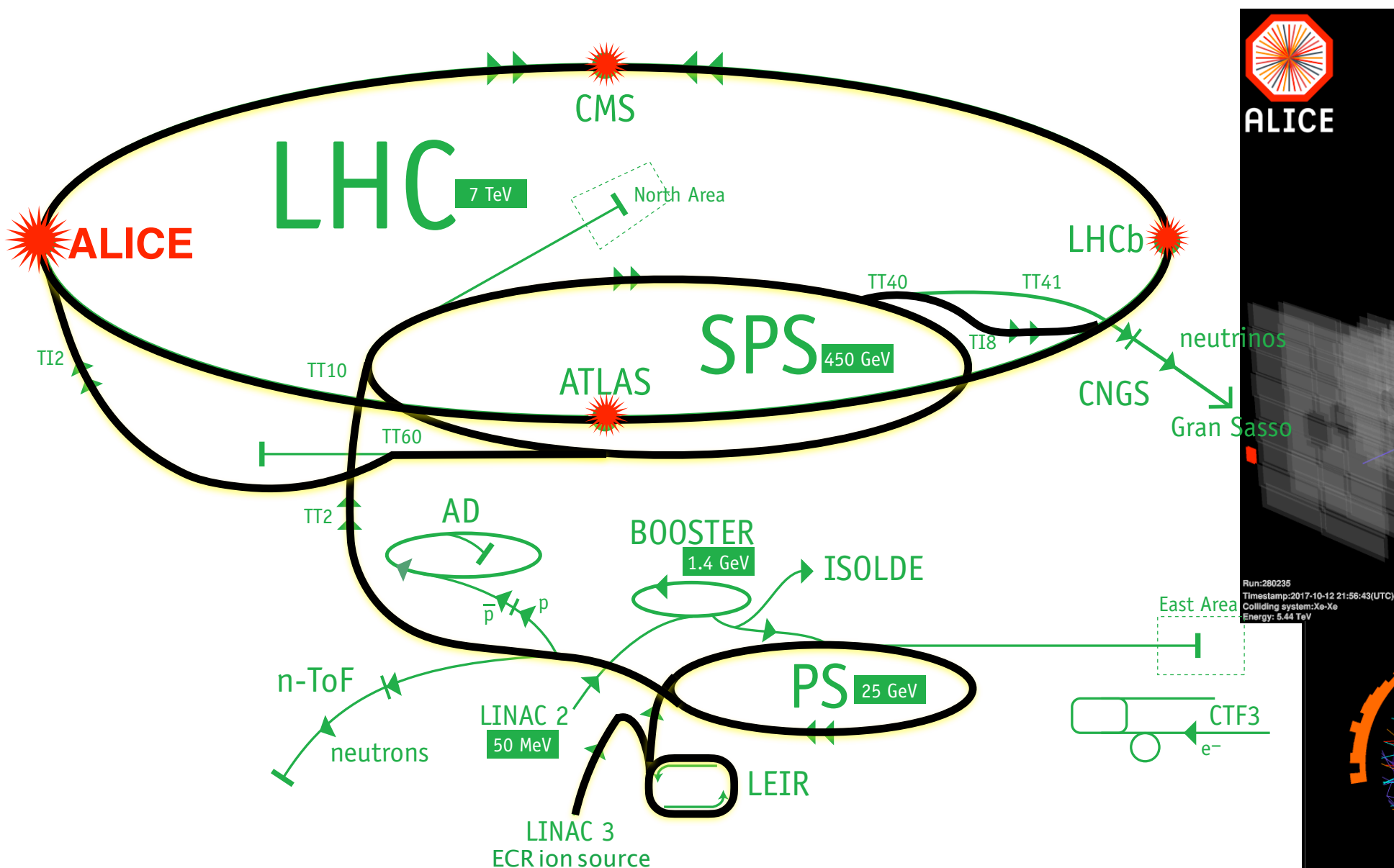
Quark Gluon Plasma (QGP)

- ✓ Extremely high temperature and density
 - ✓ quarks and gluons are deconfined from hadron
- ✓ Lattice QCD calculation predicts phase transition
 - ✓ $T_c \sim 170 \text{ MeV}$
 - ✓ $\epsilon_c \sim 1 \text{ GeV/fm}^3$
- ✓ QGP exists in early universe and neutron star

Important to understand History of the universe !



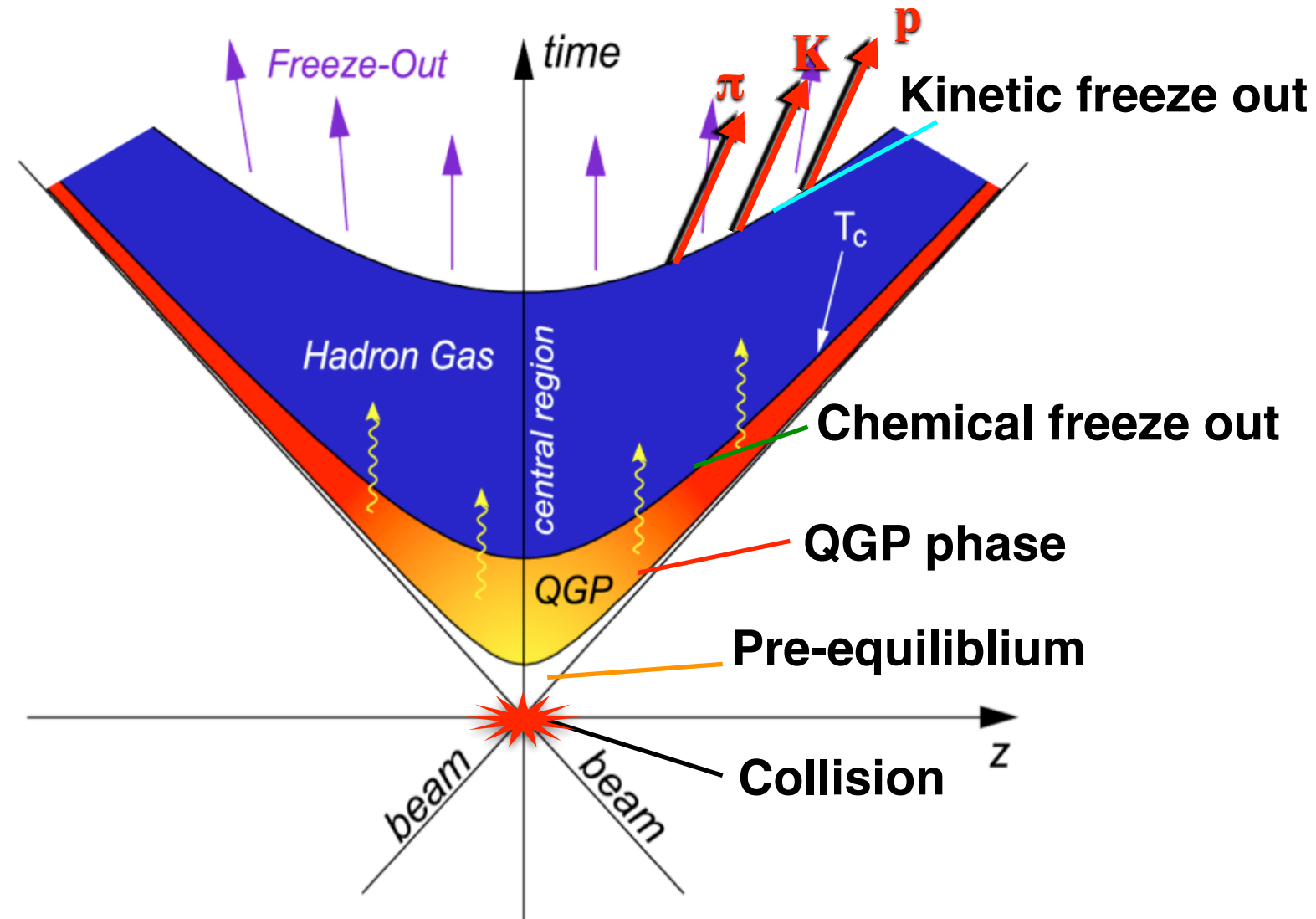
Heavy ion collision at LHC



Species	Collision energy
Pb-Pb	2.76, 5.02 TeV
p-Pb	5.02, 8.16 TeV
Xe-Xe	5.44 TeV
p-p	0.9, 2.76, 5.02, 7, 13 TeV

- ✓ Highest energy collision (energy dep.)
- ✓ E-loss in QGP with hard probes
- ✓ Detailed study of bulk property

Space time evolution



- ✓ To quantify the properties of QGP, a precise understanding of spatial and temporal evolution is required
- ✓ Freeze out time, emission duration, system size
 - ➔ HBT is a unique tool to measure the size and lifetime of the source

HBT interferometry

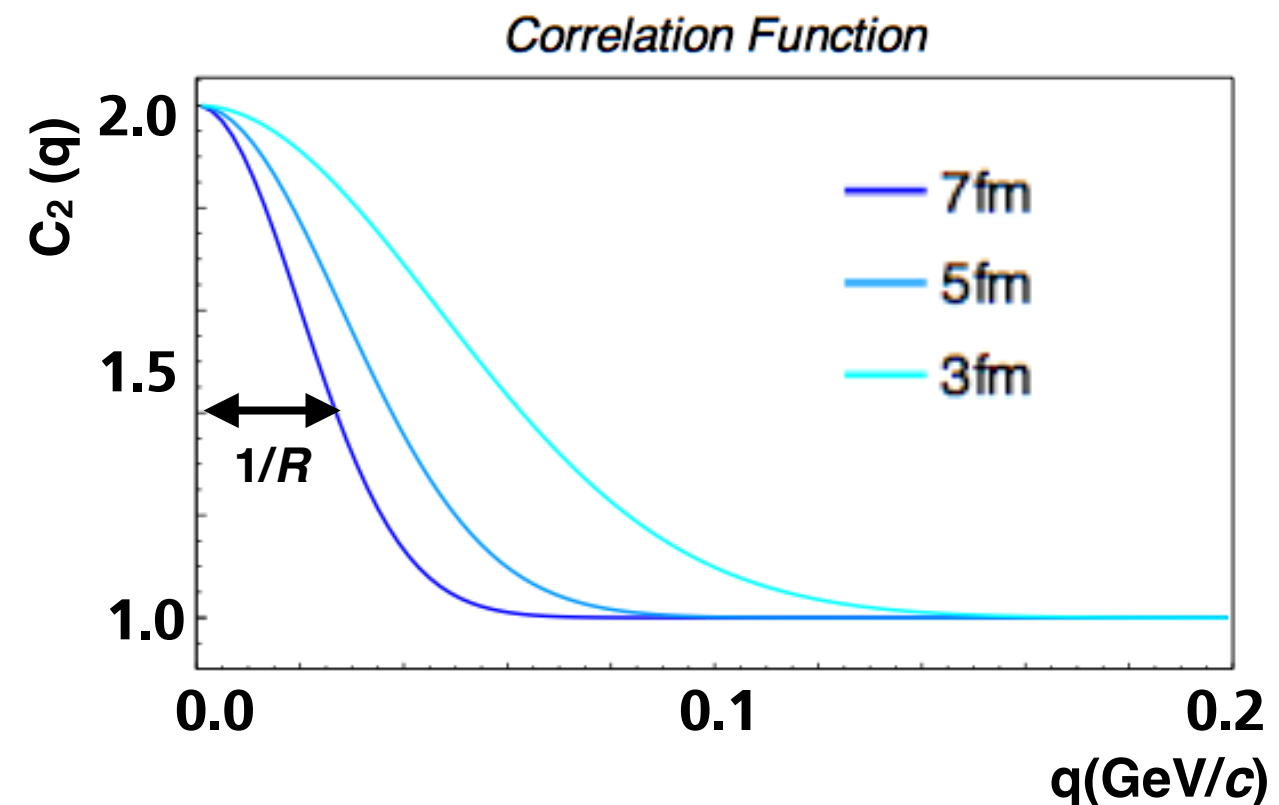
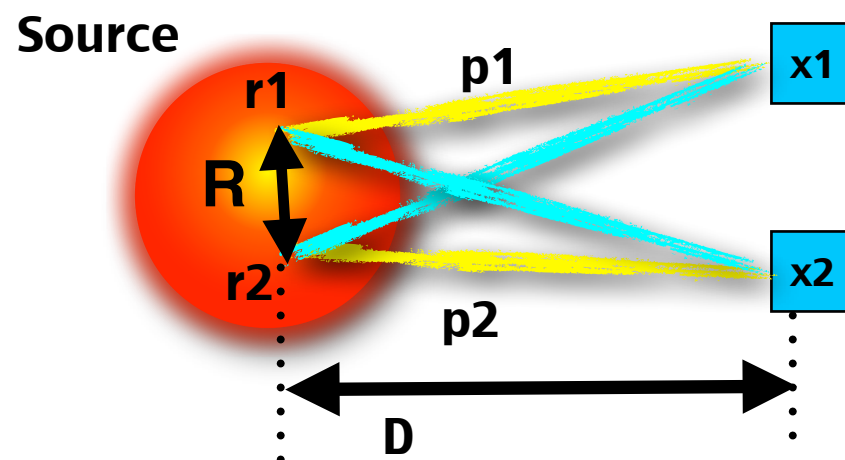
- ✓ **H**anbury **B**rown & **T**wiss (Femtoscscopy, Bose Einstein correlations)
- ✓ Measure the source size with correlation between two identical particles

$$\Psi_{12}(p_1, p_2) = \frac{1}{\sqrt{2}} \left(e^{ip_1(x_1-r_1)} e^{ip_2(x_2-r_2)} \pm e^{ip_1(x_1-r_2)} e^{ip_2(x_2-r_1)} \right) \quad \begin{array}{l} \checkmark \text{ Boson +} \\ \checkmark \text{ Fermion -} \end{array}$$

$$C_2 = \frac{P(p_1, p_2)}{P(p_1) P(p_2)} \approx 1 + |\tilde{\rho}(q)|^2 = 1 + \exp(-R^2 q^2) \quad \checkmark \mathbf{q = p_1 - p_2}$$

- ✓ Source distribution ρ is assumed to be gaussian

$$\rho(r) \equiv \exp\left(-\frac{r^2}{2R^2}\right)$$



3D HBT analysis

■ For **more detailed spatial information**, correlation function is expanded to 3-dimension

Bartsch-Pratt parametrization

$$C_2 = 1 + \lambda G$$

$$G = \exp(-R_x^2 q_x^2 - R_y^2 q_y^2 - R_z^2 q_z^2 - \Delta\tau q_0^2)$$

$$\approx \exp(-R_{side}^2 q_{side}^2 - \underline{\underline{(R_{out}^{\prime 2} + \beta_T \Delta\tau^2)} q_{out}^2} - R_{long}^2 q_{long}^2)$$

$$G = \exp(-R_{side}^2 q_{side}^2 - \underline{\underline{R_{out}^2}} q_{out}^2 - R_{long}^2 q_{long}^2 - 2R_{os}^2 q_{out} q_{side} - 2R_{sl}^2 q_{side} q_{long} - 2R_{ol}^2 q_{out} q_{long})$$

LCMS (**L**ongitudinal **C**o-**M**oving **S**ystem)

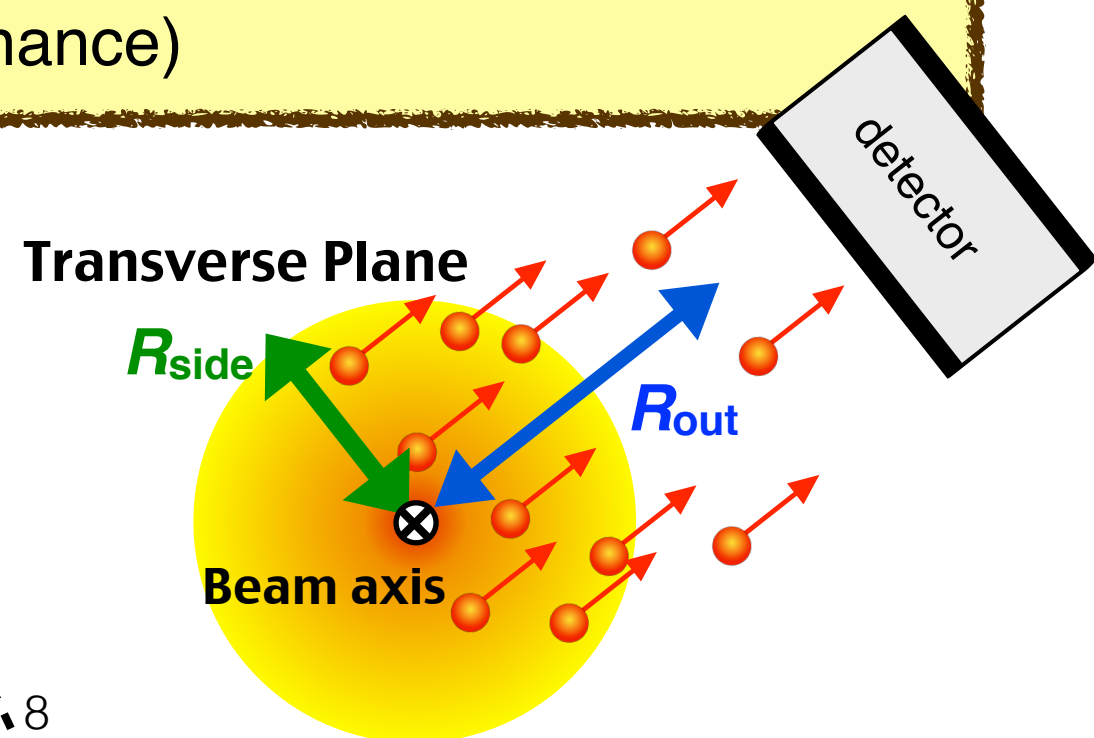
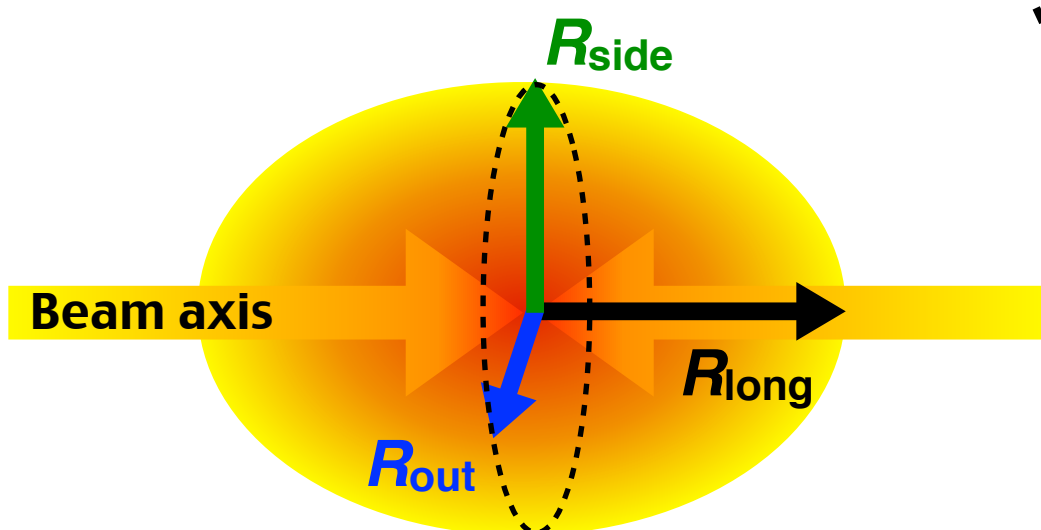
$$\checkmark \quad p_{z1} + p_{z2} = 0$$

R_{long} : source size along the longitudinal direction (beam direction)

R_{out} : source along the pair transverse momentum + **emission duration**

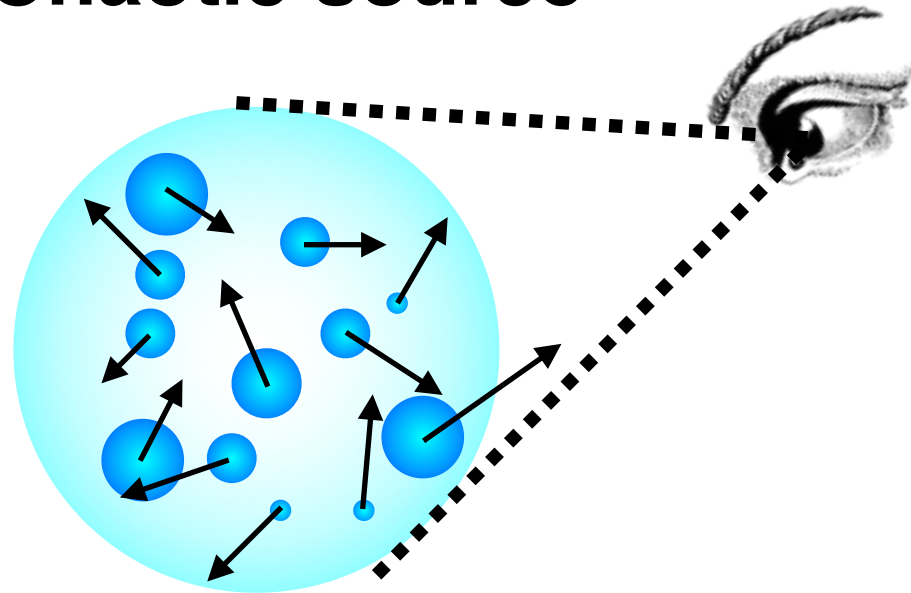
R_{side} : source size along the perpendicular to R_{out}

λ : chaoticity = (in coherence) – (resonance)



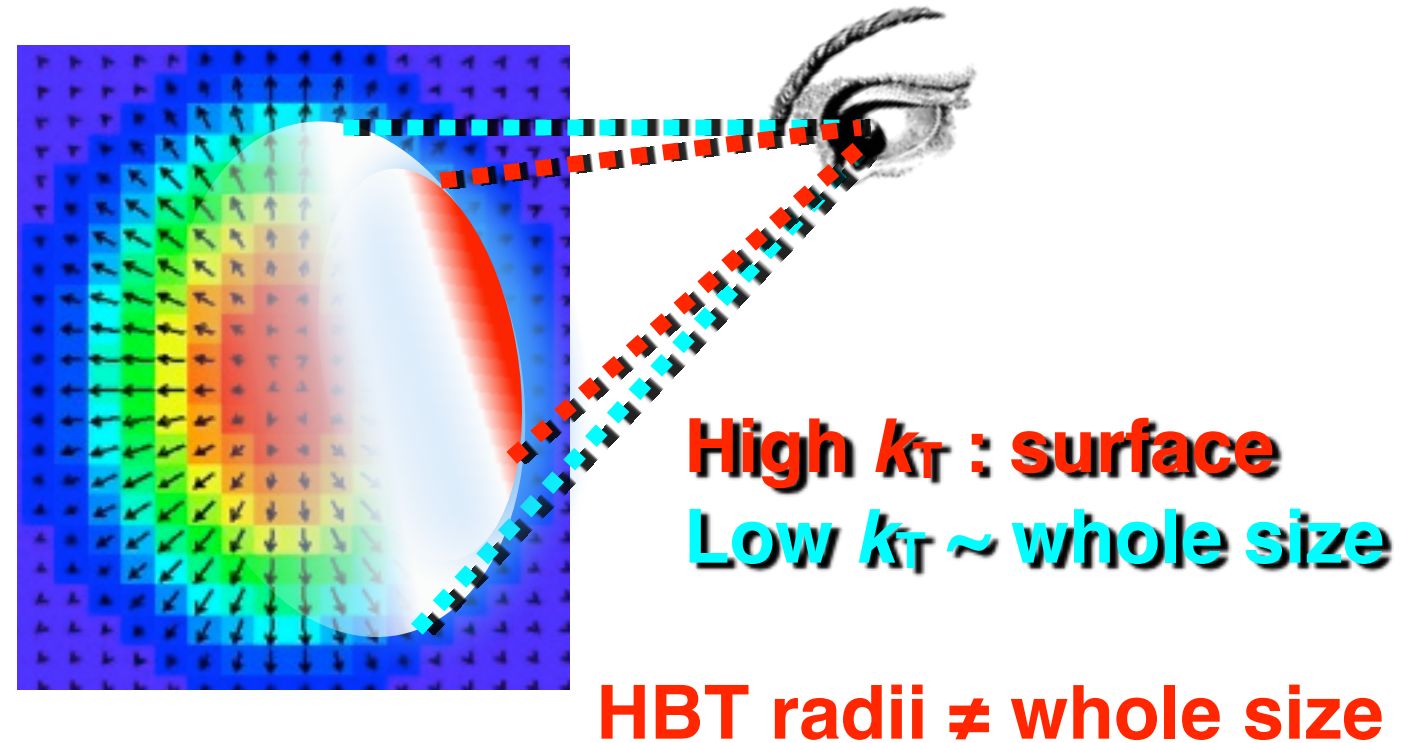
HBT radii = geometrical source size ??

Chaotic source



HBT radii = whole size

Dynamical source



- ◆ For static source, HBT radii = Geometrical source size
- ◆ For dynamical source, HBT radii = “Length of homogeneity region”
- ✓ HBT radii depends on Pair transverse momentum : k_T

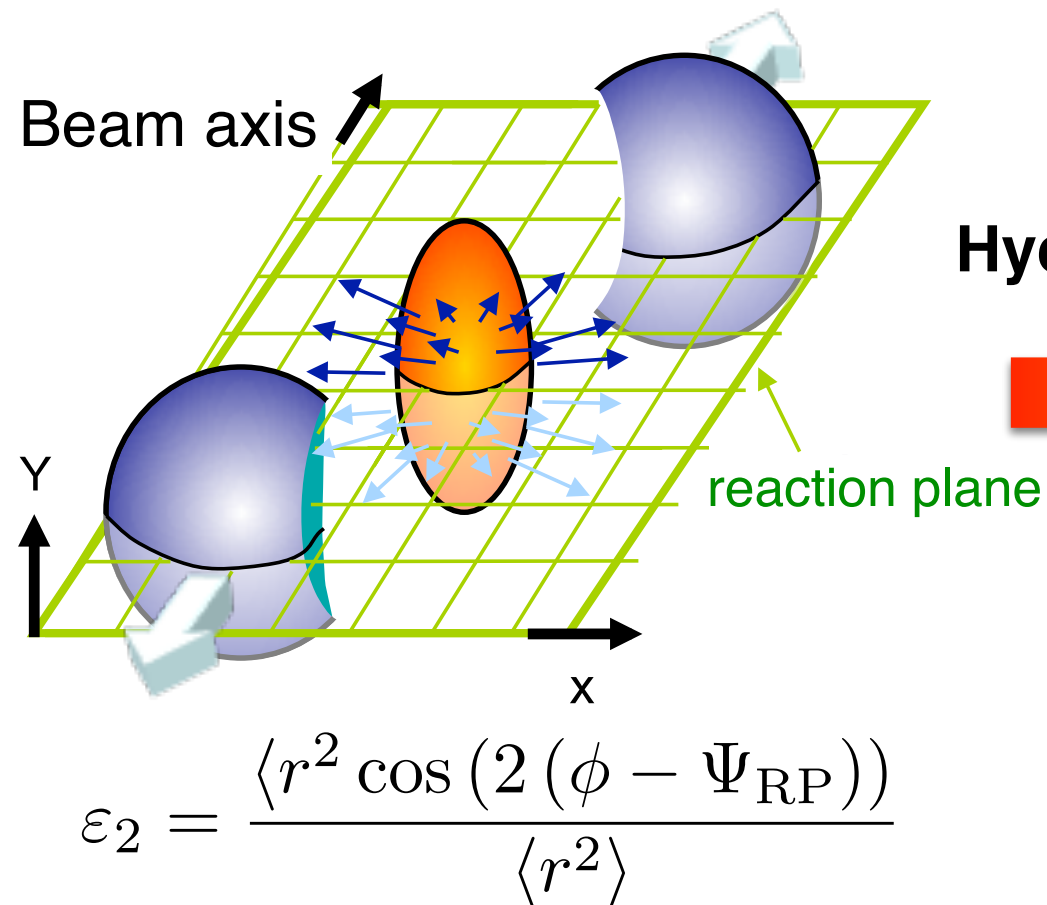
$$k_T = \frac{\vec{p}_{T1} + \vec{p}_{T2}}{2}$$

k_T dependence of HBT radii is important to understand system evolution

Azimuthal anisotropy

◆ Initial state

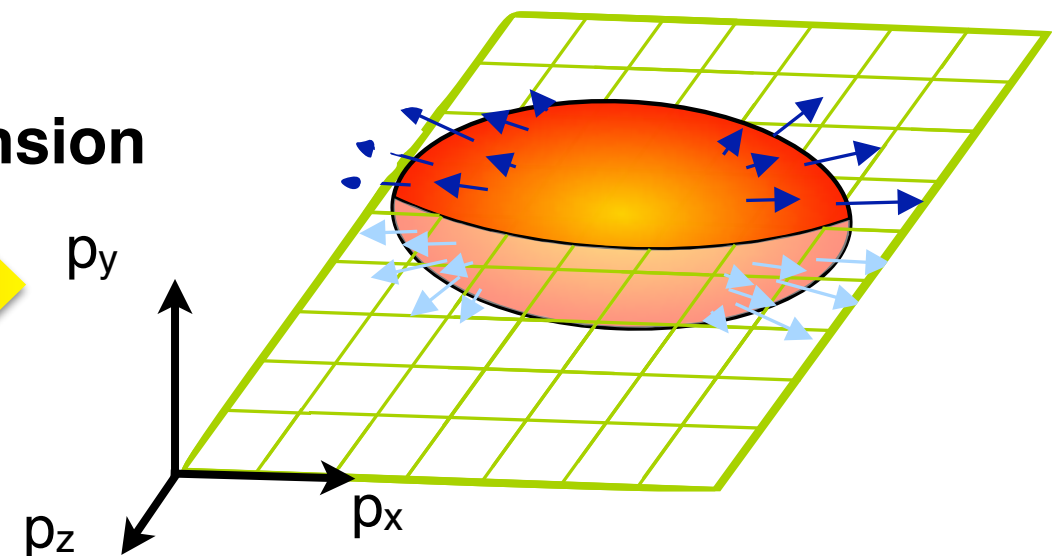
- Geometrical coordinate



◆ Freeze-out

- Momentum coordinate

Hydrodynamic expansion



$$v_2 = \langle \cos(2(\phi - \Psi_{RP})) \rangle$$

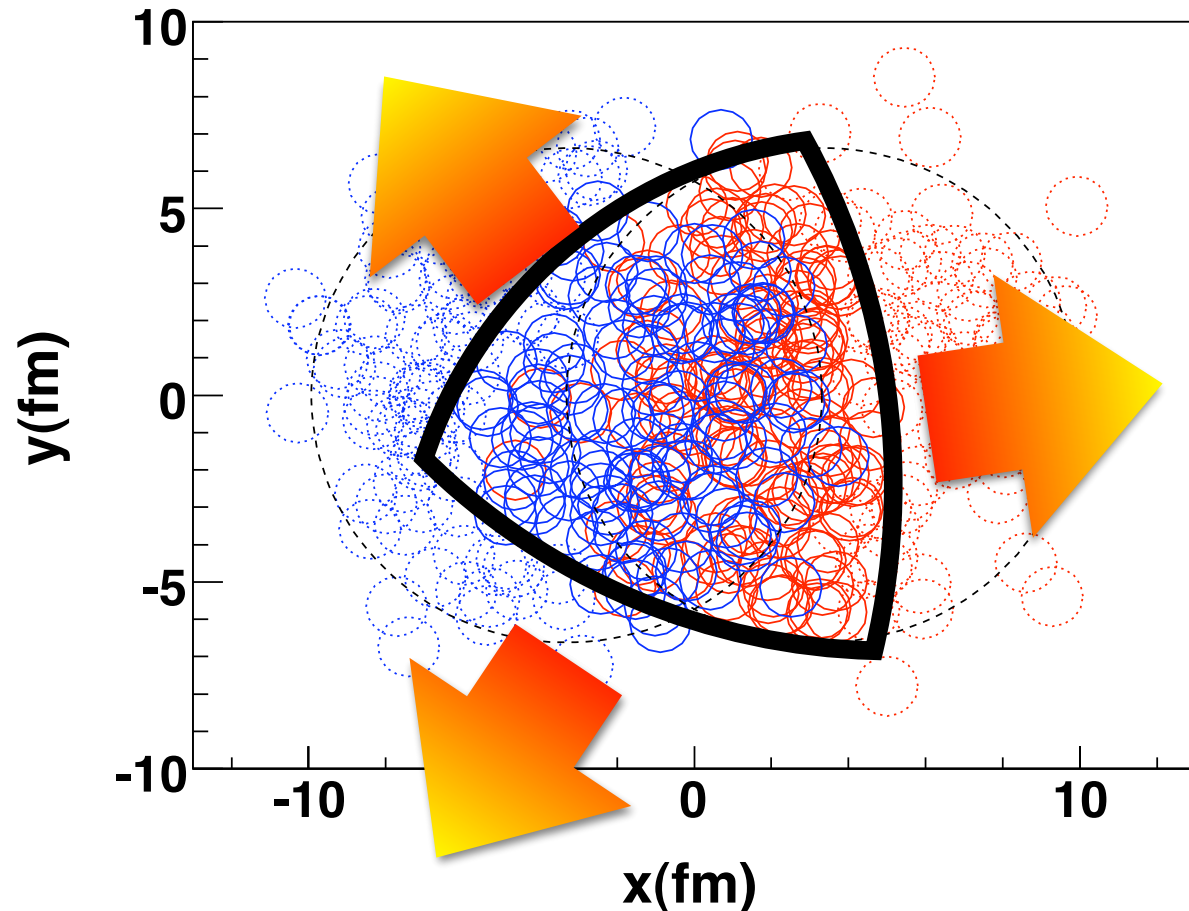
- ◆ Initial geometry ε_2 makes anisotropic pressure gradient
- ◆ Hydrodynamic expansion convert ε_2 to v_2

✓ **Sensitive to viscosity and initial geometry**

3rd-order azimuthal anisotropy

◆ Initial geometry

► MC Glauber simulation (arXiv:1408.2549)



$$\frac{dN}{d\phi} \propto 1 + 2v_2 \cos 2(\phi - \Psi_2) + 2v_3 \cos 3(\phi - \Psi_3)$$

◆ Initial participant shape \neq smooth elliptic shape

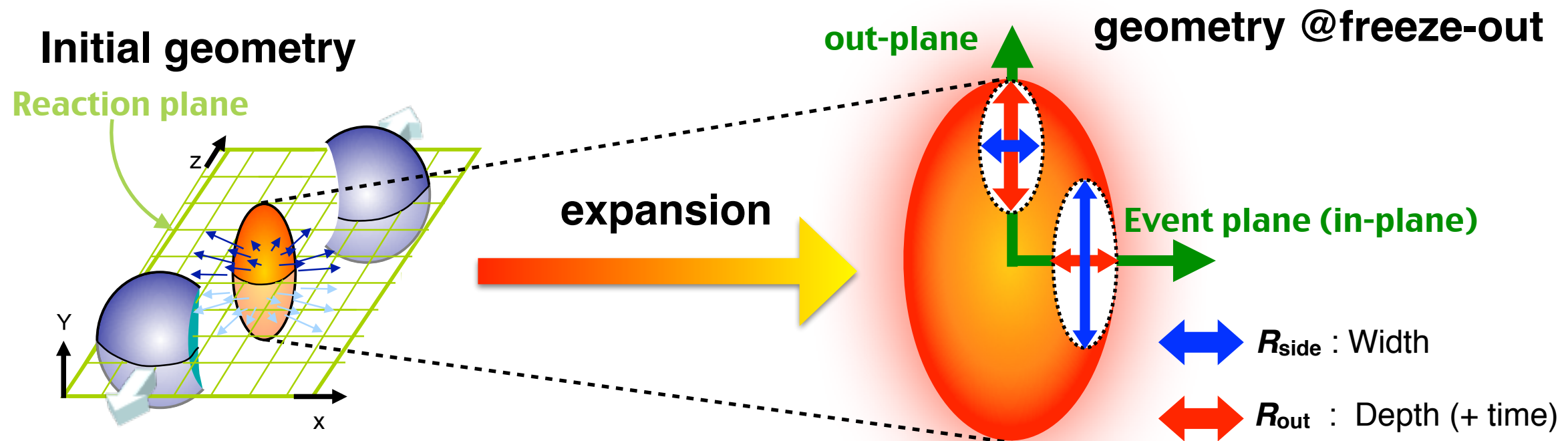
◆ Finite number of nucleons makes 3rd-order anisotropy

◆ Initial triangular shape is converted to v_3

◆ Initial geometry and viscosity are more sensitive to v_3 than v_2

Azimuthal angle dependence of HBT radii w.r.t. Ψ_2

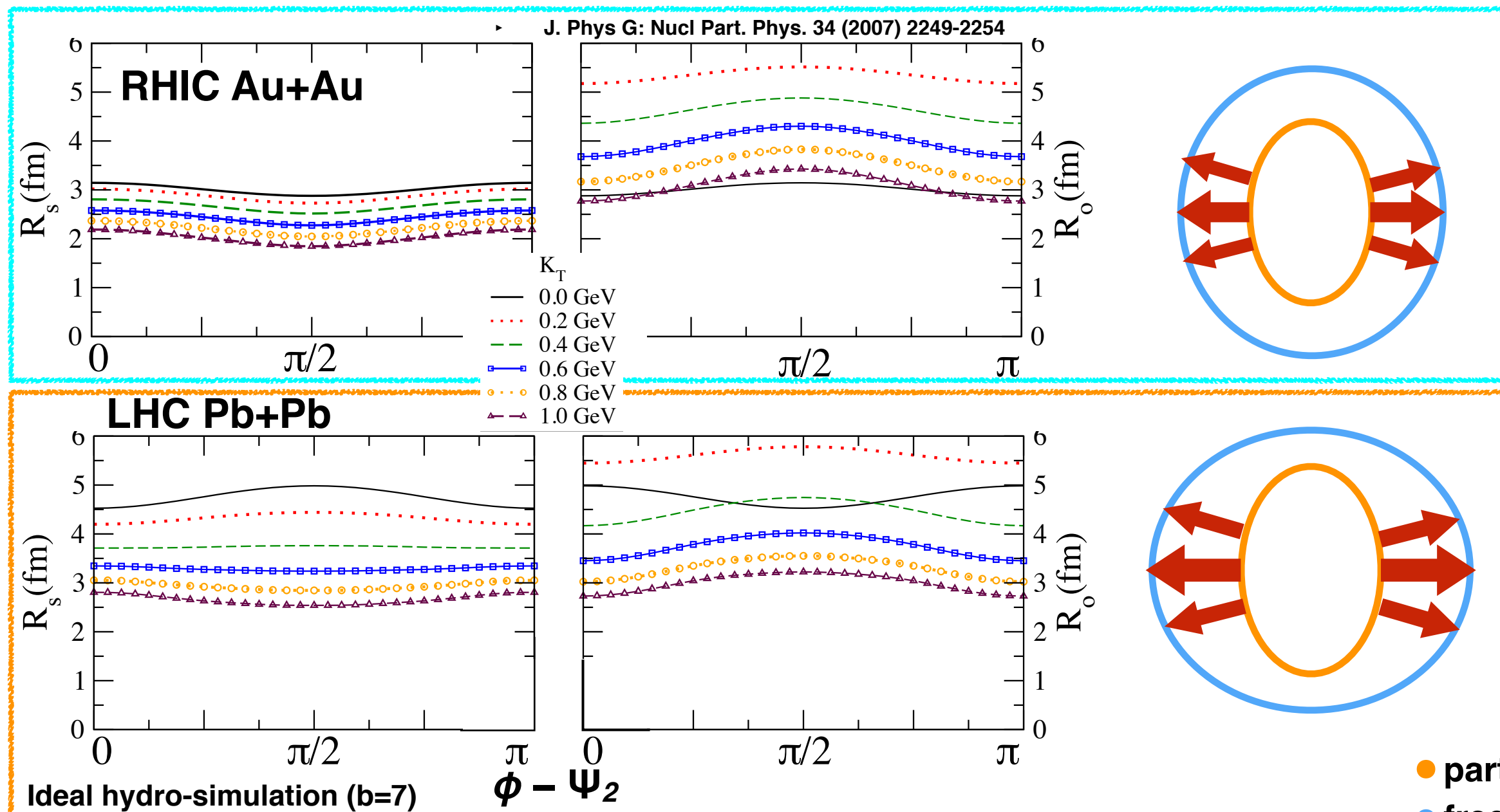
- ✓ Azimuthal angle dependence of 3D HBT radii gives us the **source shape** at freeze-out
- ✓ Event plane $\Psi_2 \sim$ reaction plane (determined with azimuthal anisotropy)
- ✓ Relation between initial and final source shape will constrain the freeze-out parameter
 - **flow velocity profile, system life time and viscosity**



- Spherical source
 - $R_{\text{side}} (\text{in-plane}) = R_{\text{side}} (\text{out-plane})$
- Out-plane elongated elliptic source
 - $R_{\text{side}} (\text{in-plane}) > R_{\text{side}} (\text{out-plane})$ & $R_{\text{out}} (\text{in-plane}) < R_{\text{out}} (\text{out-plane})$
- In-plane elongated elliptic source
 - $R_{\text{side}} (\text{in-plane}) < R_{\text{side}} (\text{out-plane})$ & $R_{\text{out}} (\text{in-plane}) > R_{\text{out}} (\text{out-plane})$

Final source eccentricity @ LHC energy

- Hydro model predicts R_{side} and R_{out} oscillate in phase at lower k_T
 - ✓ At RHIC energy, out-plane elongated elliptic shape still remains at freeze-out
 - ✓ Initial elliptic shape will be vanished or even reversed in LHC



✓ This effect can be observed in LHC ??

- participant
- freeze-out source
- flow

Final source **triangular shape** and HBT w.r.t. Ψ_3

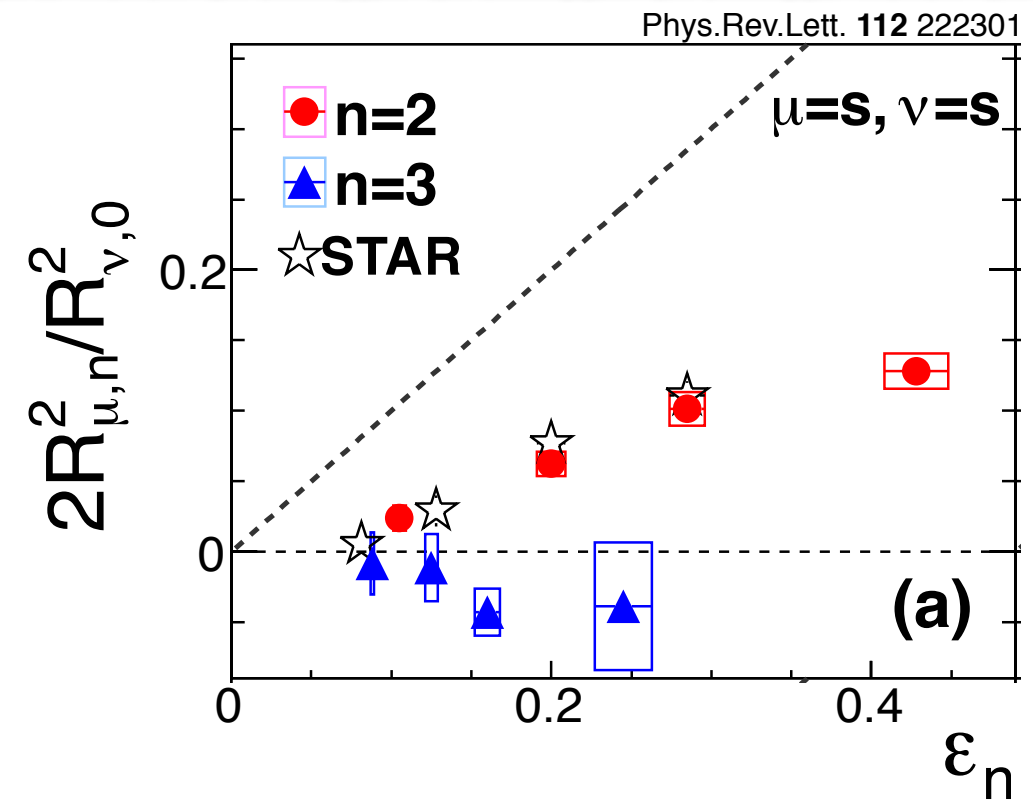
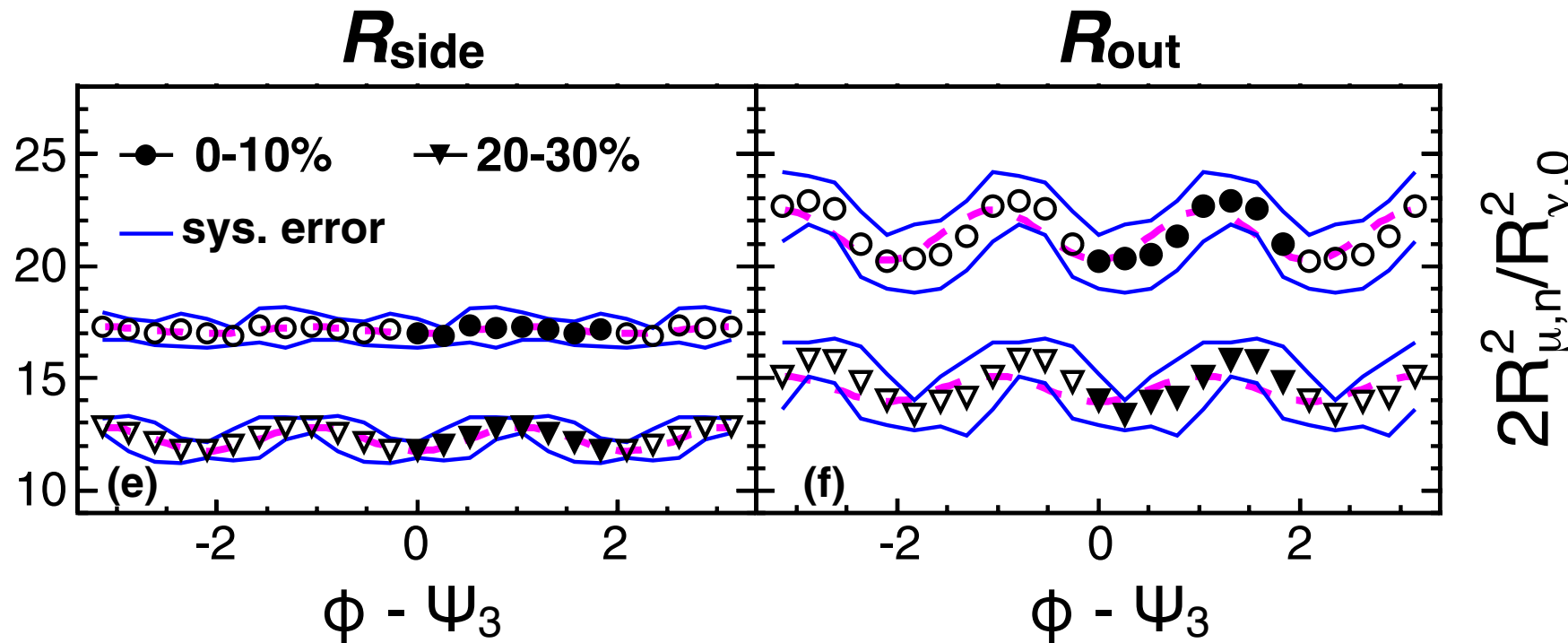
◆ Model suggests HBT w.r.t. Ψ_3 shows finite oscillation in expanding source

✓ S.Voloshin, J. Phys. G38, 124097

◆ First measurement of HBT w.r.t. Ψ_3 was measured @ PHENIX Au+Au 200GeV

✓ Negative or zero oscillation was observed in R_{side}

Au+Au 200GeV @ PHENIX



☑ In order to constrain the freeze-out parameters, to determine the oscillation sign and centrality dependence is indispensable !!

✓ Precise measurement @LHC is expected

Event shape engineering (ESE)

□ Event by event flow amplitude selection

J. Schukraft et al., Phys. Lett. B719, 394-398 (2013)

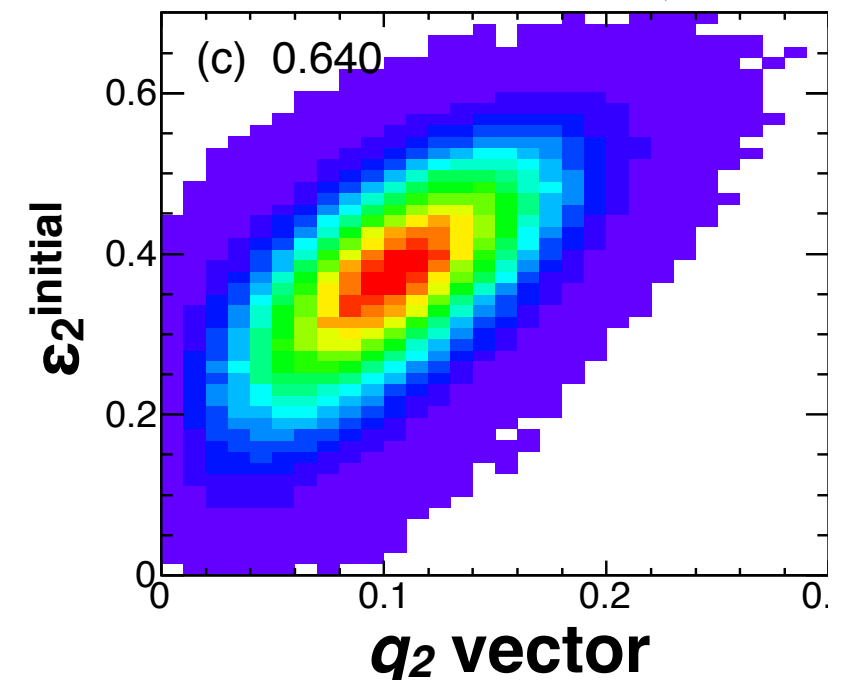
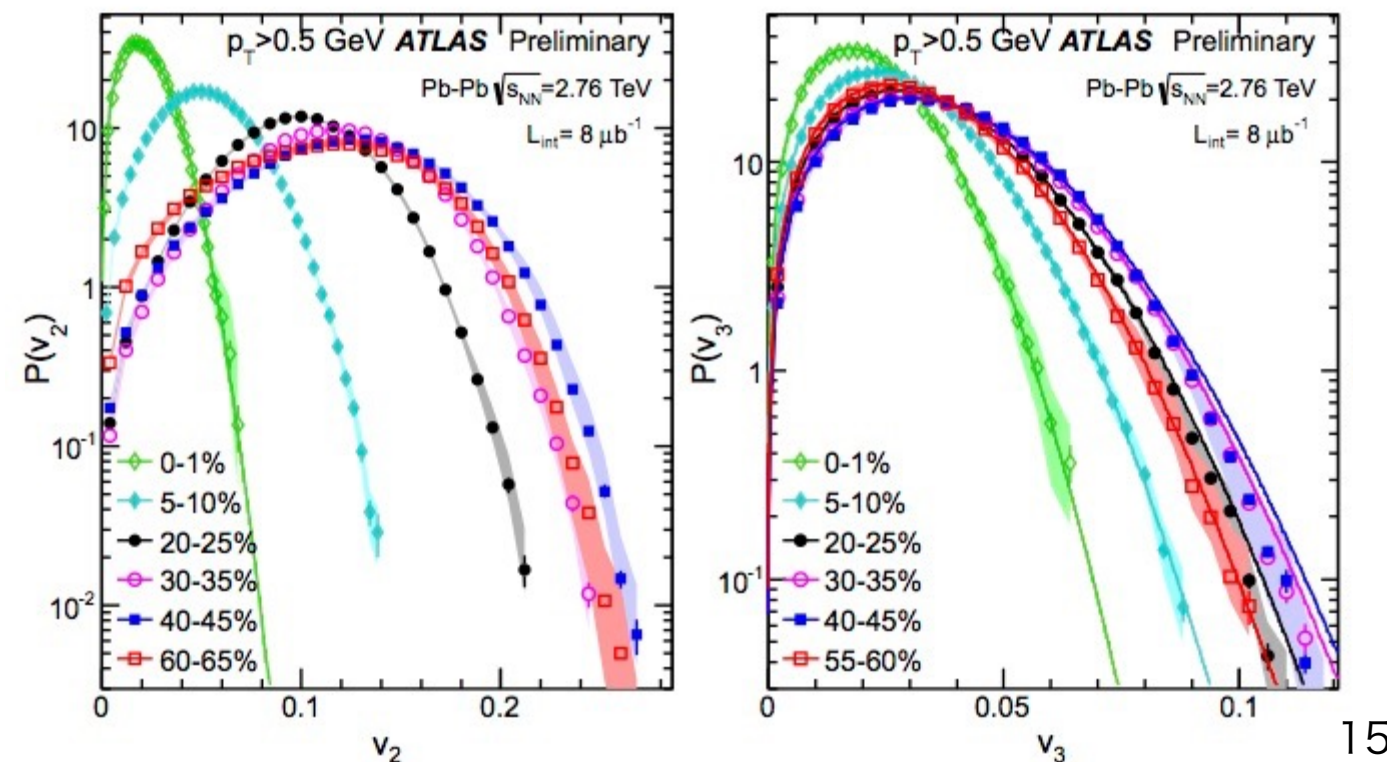
- v_2 and v_3 largely fluctuate within a fixed centrality
- **Event by event $v_2(v_3)$ fluctuation is selected with flow vector $q_2(q_3)$**
- ✓ Possibly control the initial eccentricity

★ q-vector is defined by $q_n = \sqrt{Q_{n,x}^2 + Q_{n,y}^2}$ $\left(\begin{array}{l} Q_{n,x} = \sum w_i \cos(n\phi) / \sum w_i \\ Q_{n,y} = \sum w_i \sin(n\phi) / \sum w_i \end{array} \right)$

- **Centrality is not best probe for initial geometry!! (system size also changes)**
- By applying ESE technique to other observables, **separation of volume effect and geometrical effect** can be allowed

□ Explicit correlation between q_2 and $\epsilon_2^{\text{initial}}$

- **AMPT simulation** ▸ J.Jia et al., arXiv:1430.6077



Motivation

□ Study the space time evolution of QGP with Azimuthally sensitive HBT and Event Shape Engineering in 2.76TeV Pb-Pb collisions

♦ Elliptic shape

- Measurements of azimuthally sensitive HBT w.r.t. Ψ_2 in LHC energy
- Relation between Initial and final source eccentricity with **ESE**
- Extract freeze out parameters with **Blast-wave fit**

♦ Triangular shape

- Measurements of azimuthally sensitive HBT w.r.t. Ψ_3 in LHC energy
- Relation between v_3 (initial triangular shape) and HBT oscillation with **ESE (q_3) selection**

My activity

Master

- DCAL construction
- EMCAL SRU work @cern
- DCAL commissioning
- Shift taking @ PHENIX
- Development of read out module with FPGA and Flash ADC
- KEK summer challenge M1->D4
- Development of radon detector ->D4
- Azimuthal angle dependence of HBT radii w.r.t. Ψ_2 and Ψ_3

Doctor

▸ Talk

JPS fall 2016

▸ Poster

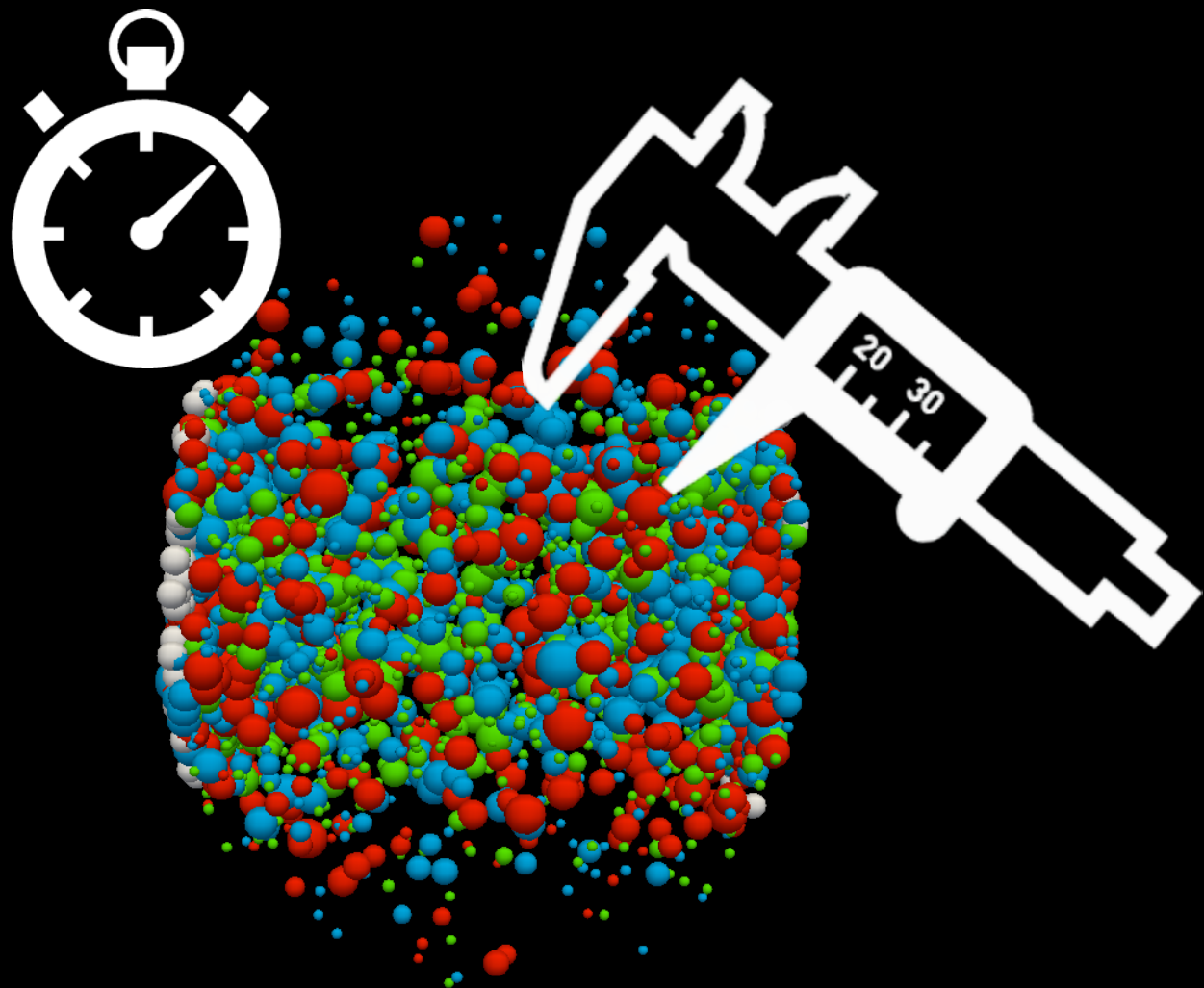
QM2016

▸ Talk

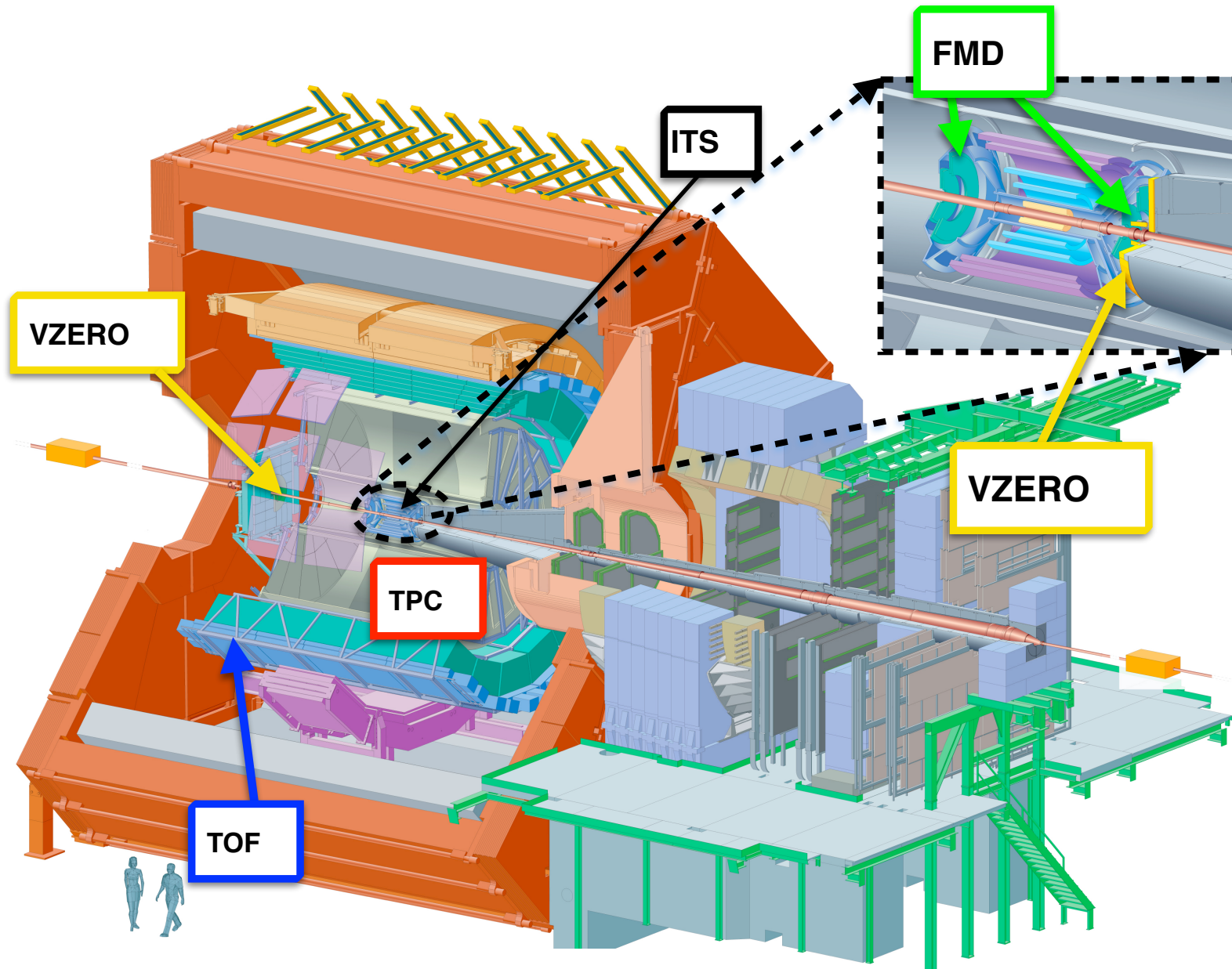
WPCF2017

- Shift taking @ ALICE
- HBT w.r.t. Jet axis
- HBT relative to Ψ_2 and Ψ_3 with ESE

Experiment & Analysis



ALICE Detector



In this analysis

VZERO

- ✓ Trigger & centrality
- ✓ $V0_A : 2.8 < \eta < 5.1$
- ✓ $V0_C : -3.7 < \eta < -1.7$

TPC & ITS

- ✓ Tracking & PID
- ✓ Vertex
- ✓ $|\eta_{\text{track}}| < 0.8$

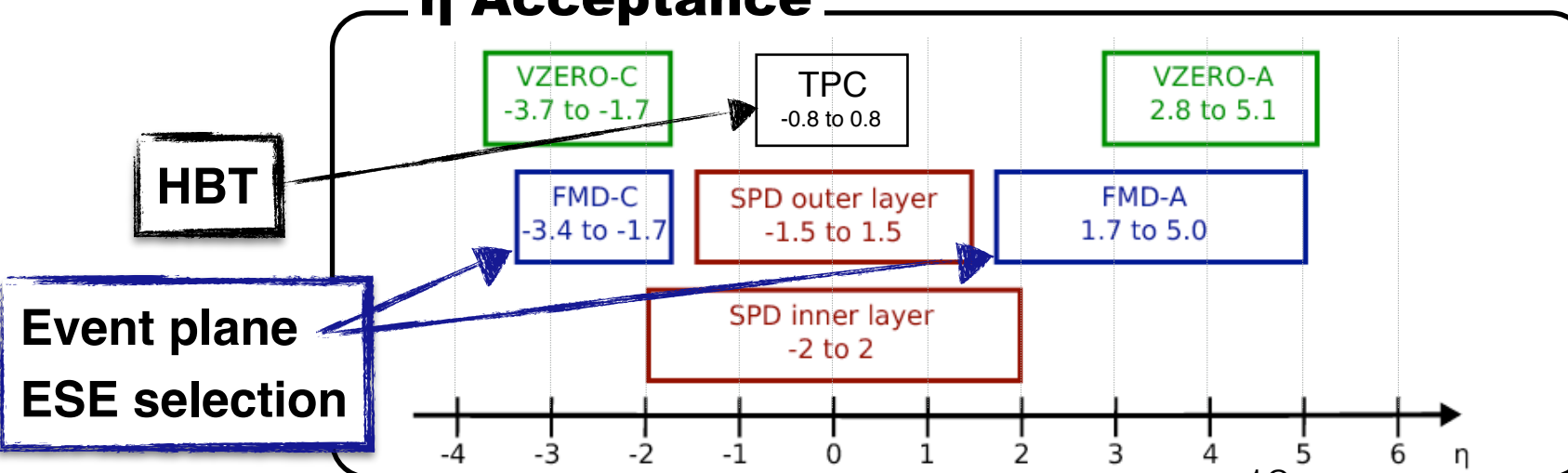
TOF

- ✓ PID
- ✓ $|\eta_{\text{track}}| < 0.8$

FMD

- ✓ Event plane
- ✓ $FMD_A : 1.7 < \eta < 5.0$
- ✓ $FMD_C : -3.4 < \eta < -1.7$

η Acceptance



Two track resolution

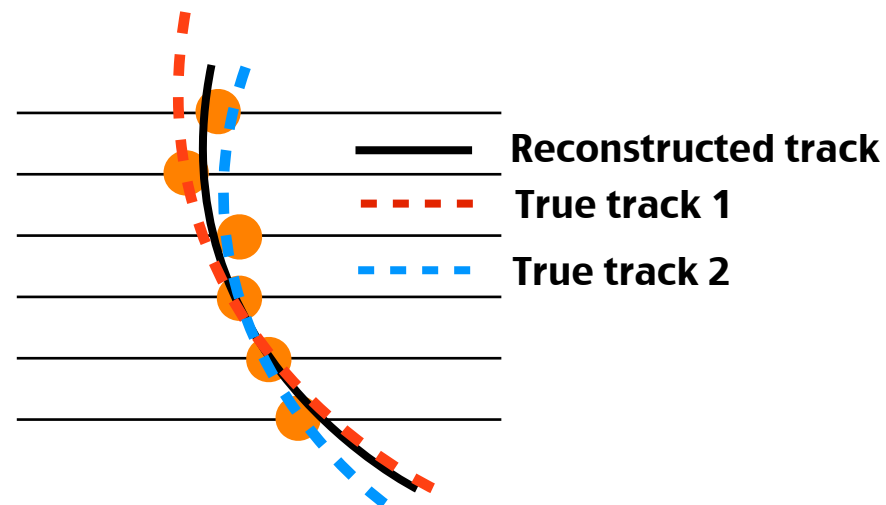
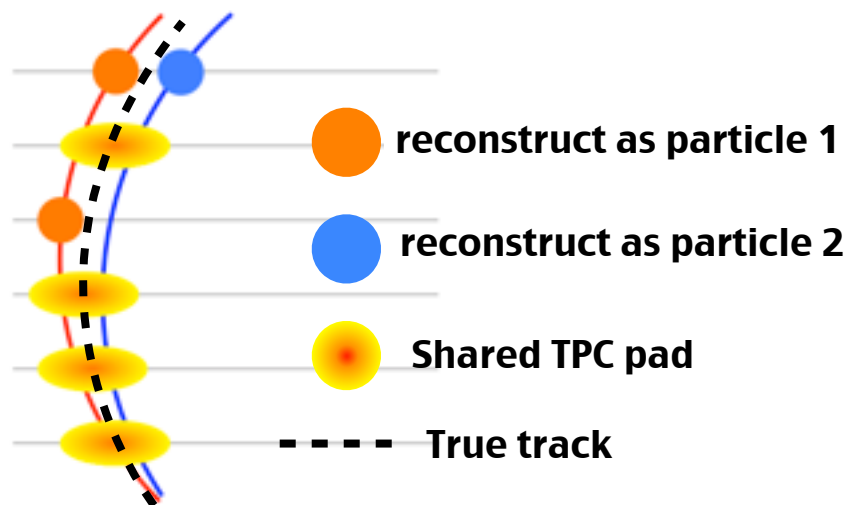
■ Due to the high multiplicity event

- **Track splitting**

- A track is falsely reconstructed as two tracks that are spatially close

- **Track merging**

- Two tracks that are spatially close are falsely reconstructed as one
- These effect modify measured correlation function



✓ Applied pair cut

- Fraction of shared TPC cluster $< 5\%$
- Angular distance in $\Delta\phi^*$, $\Delta\eta$

HBT for experimental approach

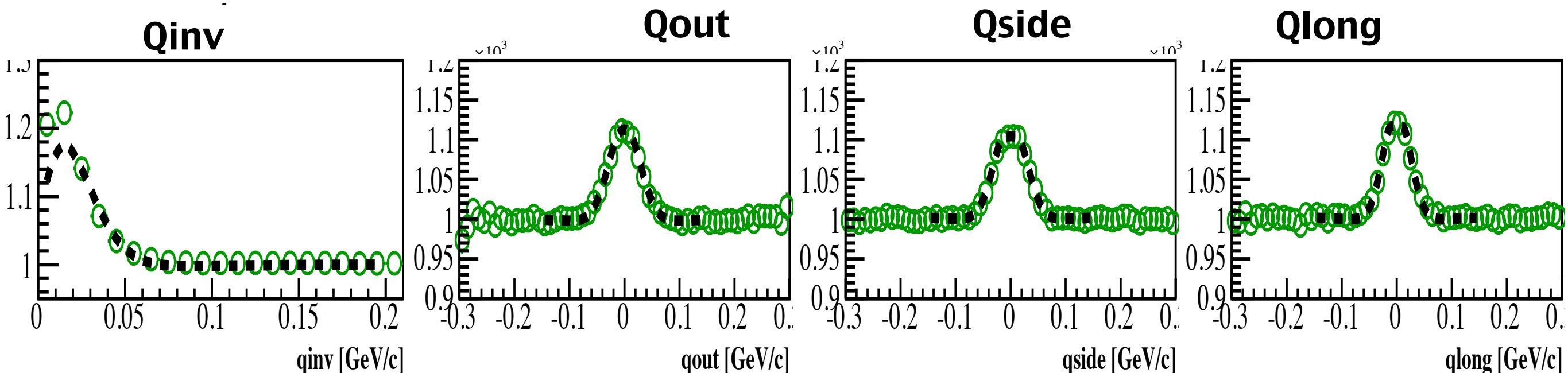
How to calculate correlation function C_2 in experiment

$$C_2 = \frac{P(p_1, p_2)}{P(p_1)P(p_2)} = \frac{Q_{Real}}{Q_{Mix}}$$

● Q_{Real} : pair in same event (HBT effect)
● Q_{Mix} : pair in different event (no HBT effect)

✓ Event Mixing

- Both real pairs and mixed pairs has detector efficiency
- HBT correlation is included only in real pairs
- C_2 has HBT correlation without **acceptance effect** and **detector efficiency**



Final state interaction and resonance

- Like-sign pairs that is spatially close are repulsive with Coulomb

- **Correlation function is suppressed for low q pairs**
- Coulomb weight is calculated with Coulomb wave function

$$\left[-\frac{\hbar^2 \nabla^2}{2\mu} + \frac{Z_1 Z_2 e^2}{r} \right] = E \Psi_c(r)$$

■ Resonance decay

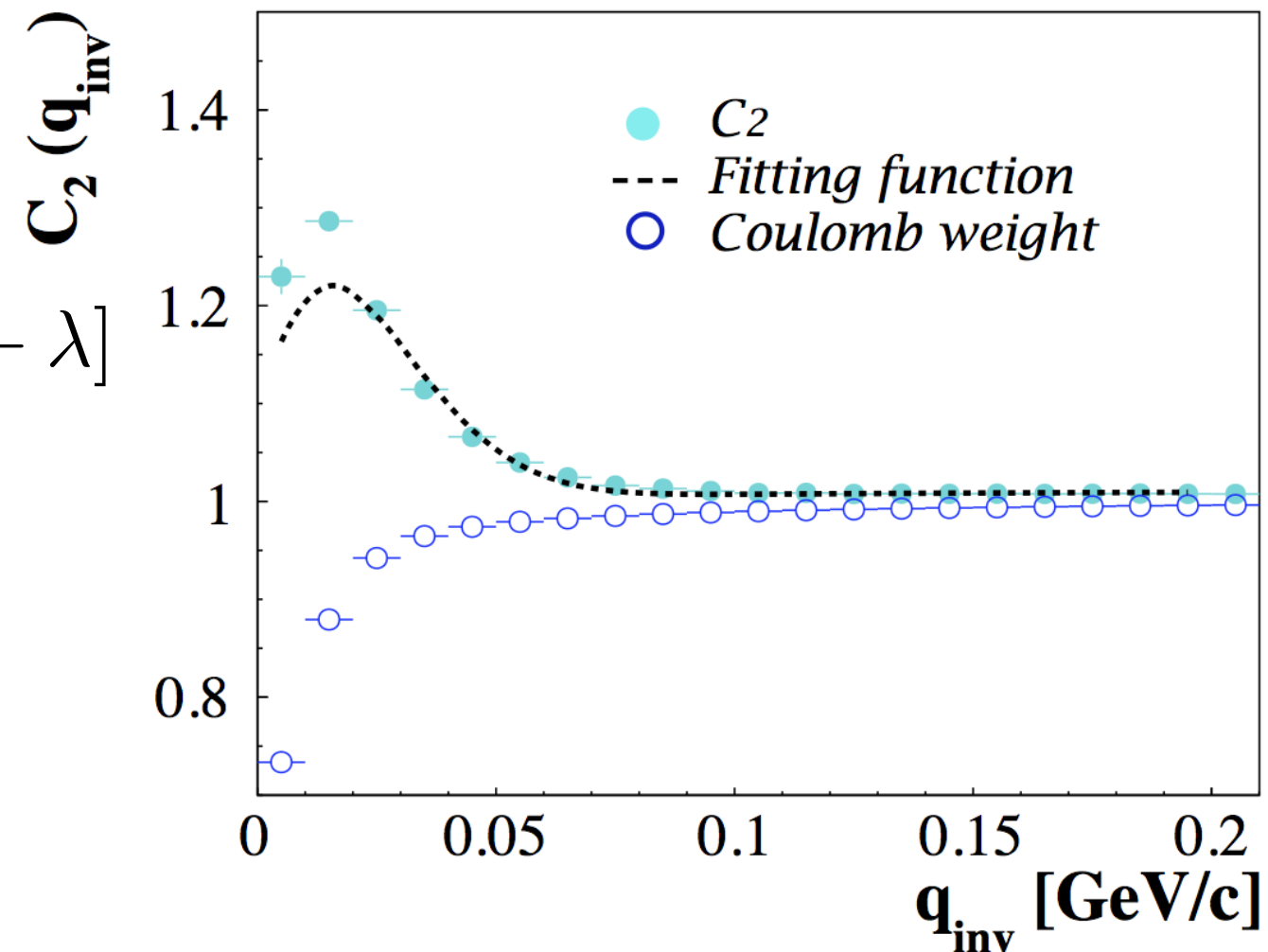
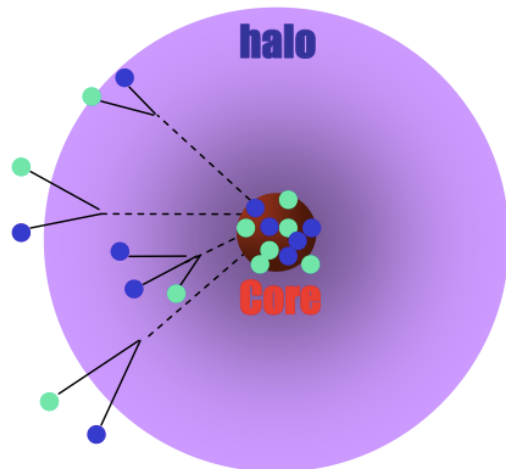
- λ in C_2 is sensitive to purity
- Core-halo model

$$C_2 = C_2^{core} + C_2^{halo}$$

$$= N [\lambda (1 + G) F_{coul}] + [1 + \lambda]$$

✓ G : HBT interferometry

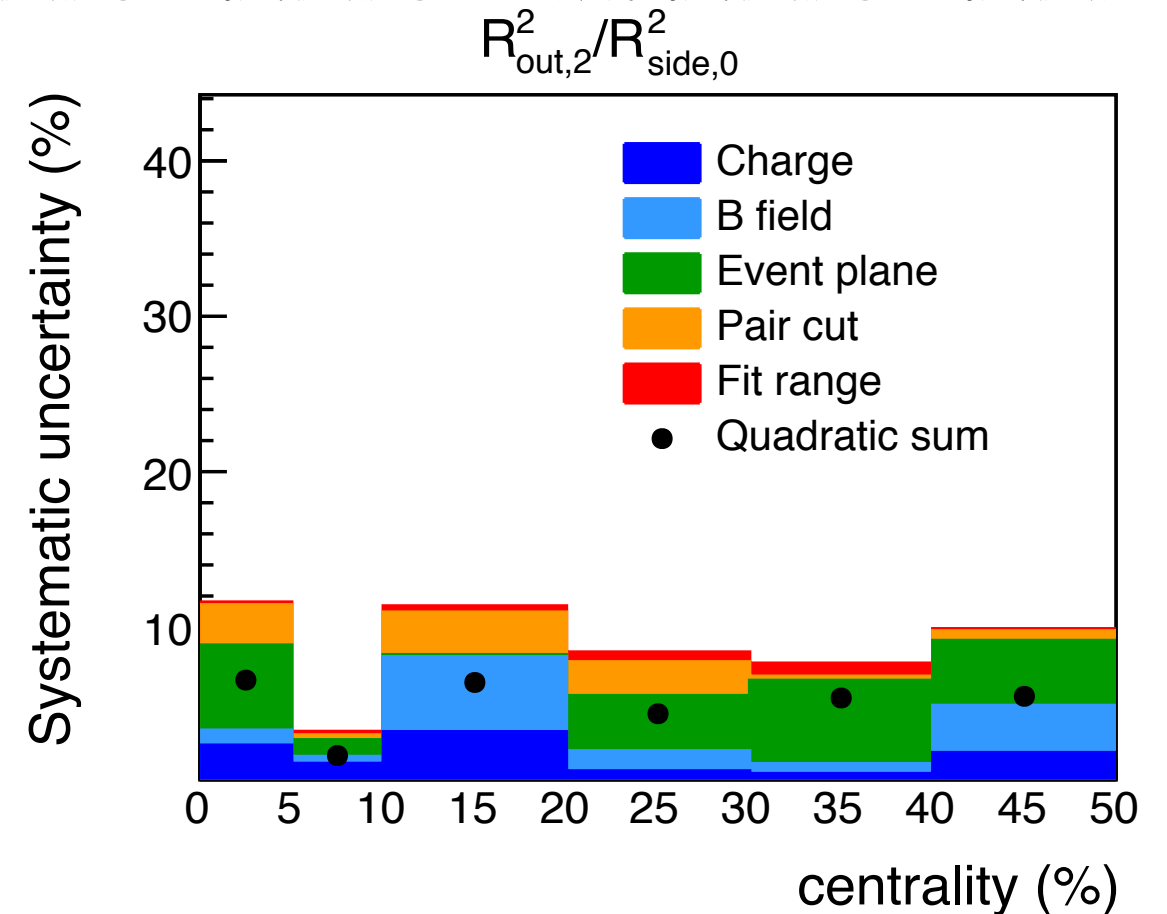
✓ F_{coul} : Coulomb interaction



Systematic uncertainties

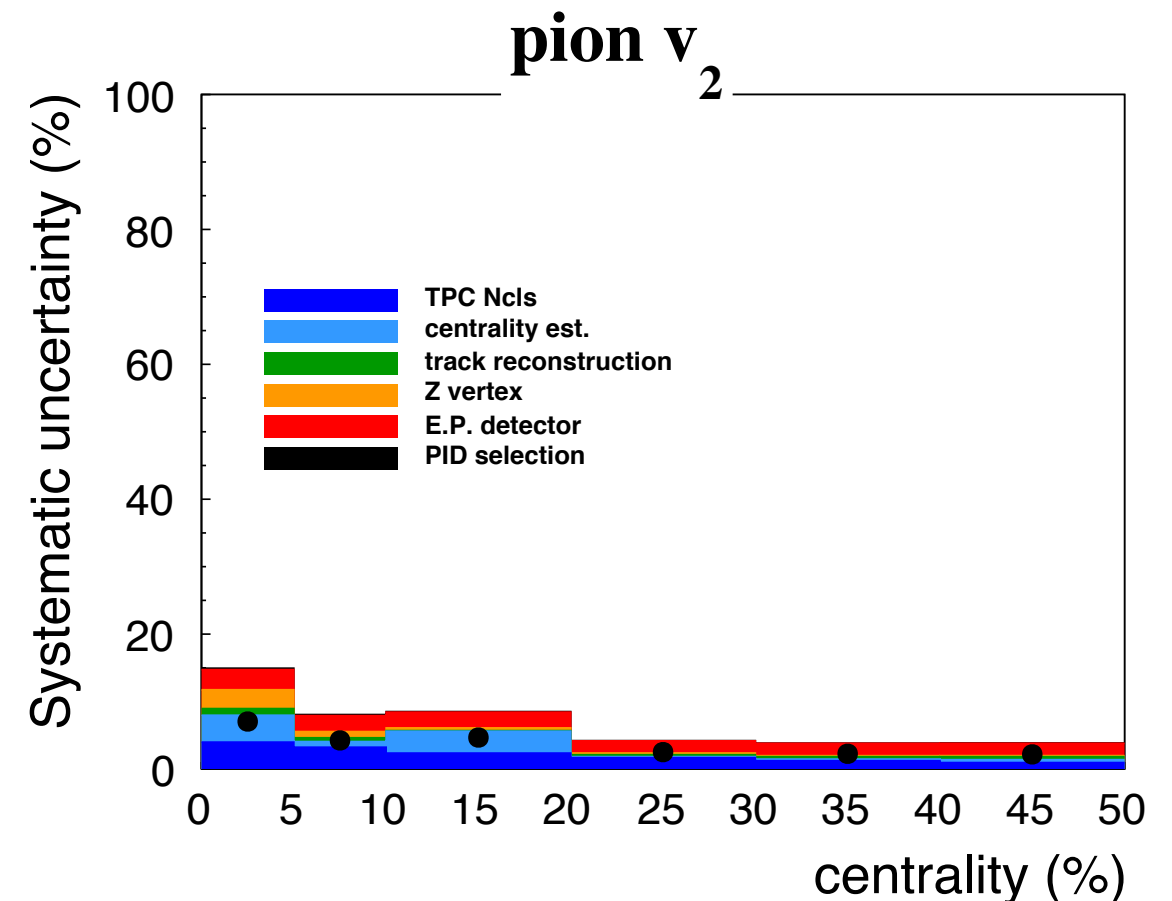
□ HBT analysis

- ✓ charge ($\pi^+\pi^+$ pair and $\pi^-\pi^-$ pair)
- ✓ B field (positive and negative)
- ✓ Event plane (VZERO detector)
- ✓ Pair cut (tight cut 3.5σ)
- ✓ Fit range (140, 130 MeV/c)

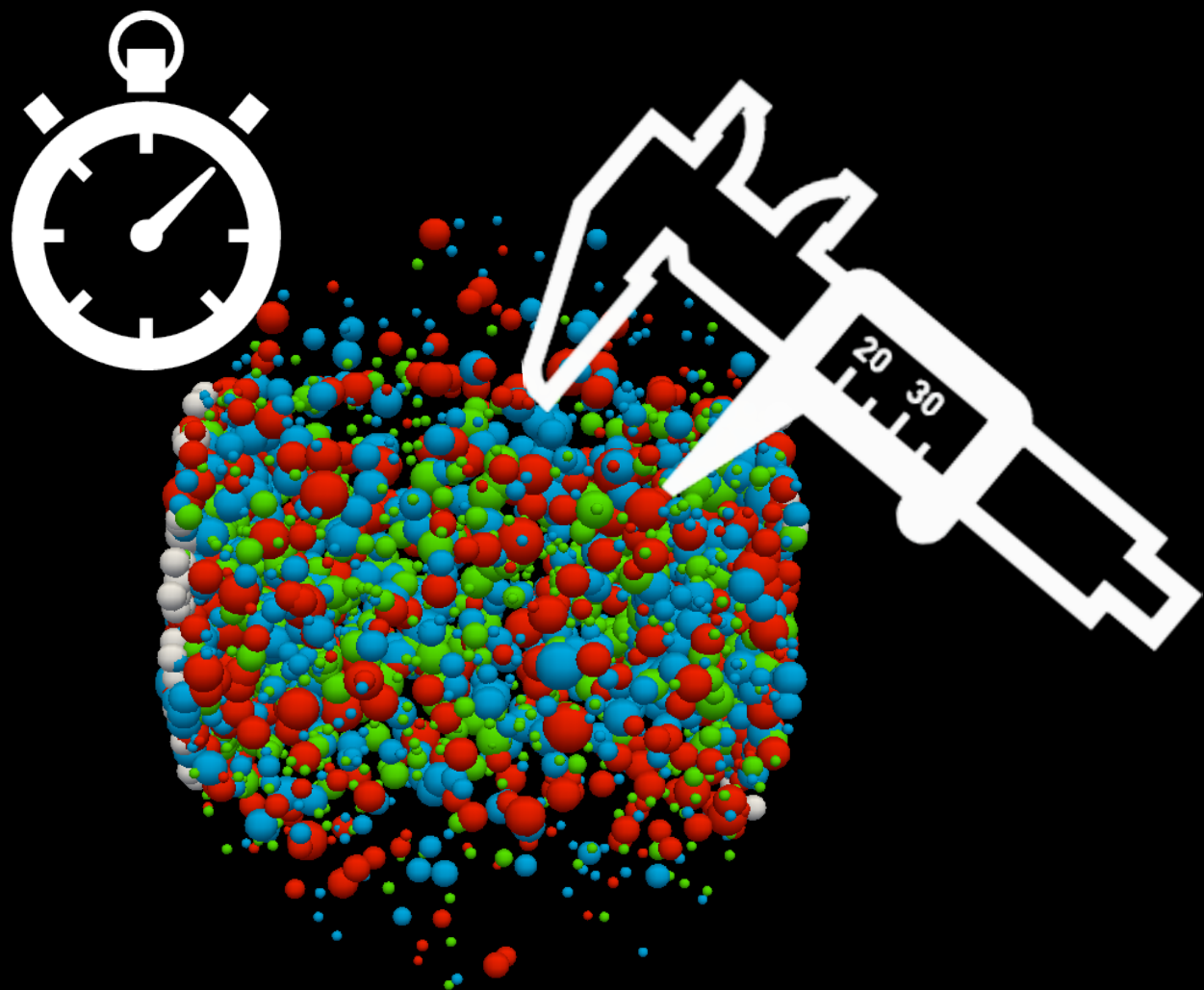


□ Flow analysis

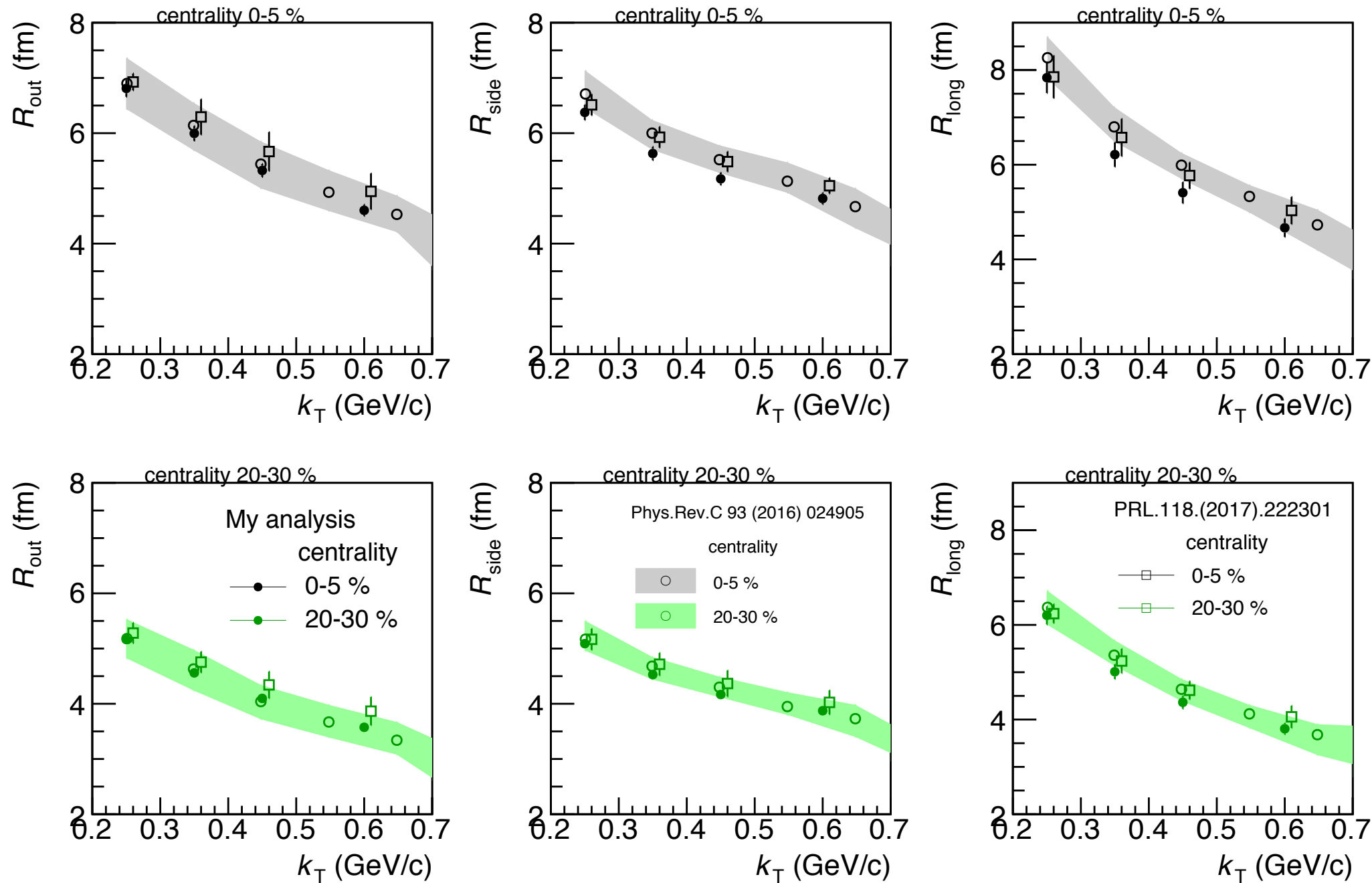
- ✓ TPC # of clusters (50, 90)
- ✓ Centrality estimator (ITS 2nd layer)
- ✓ Z-vertex (5, 10cm)
- ✓ Event Plane
 - ✓ FMDA, FMDC, V0A, V0C, V0AC,
 - TPC($-1 < \eta < -0.5$) and TPC($0.5 < \eta < 1.0$)
- ✓ PID probability (75, 90%)



Result & Discussion



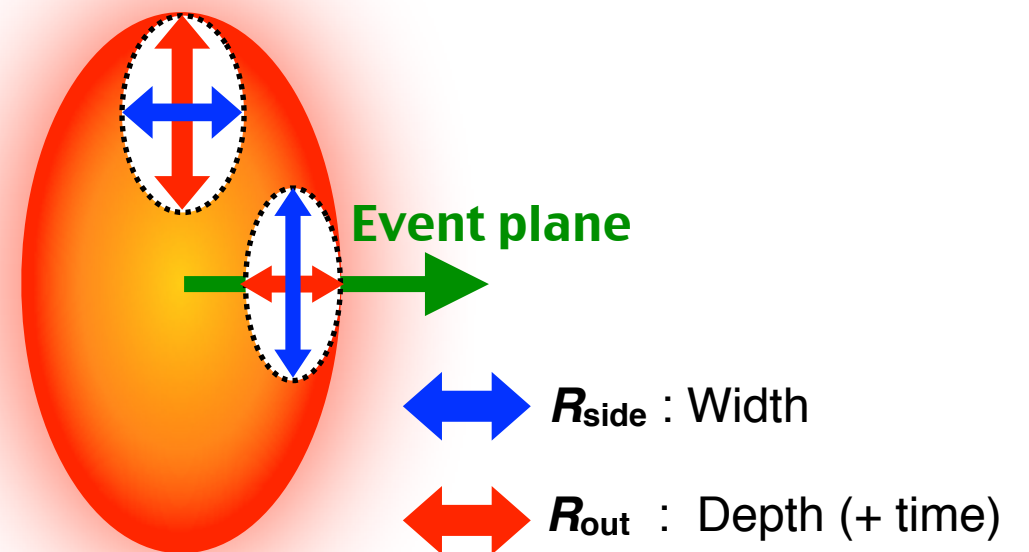
Consistency check with published results



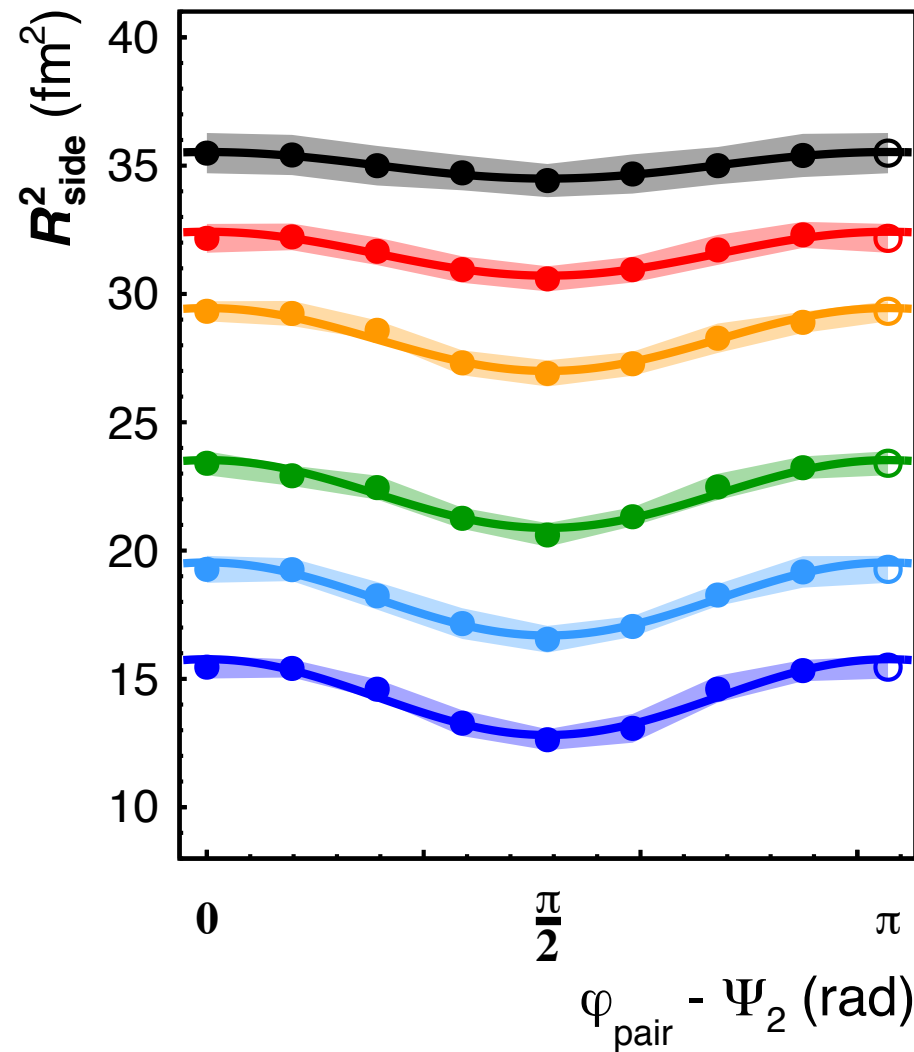
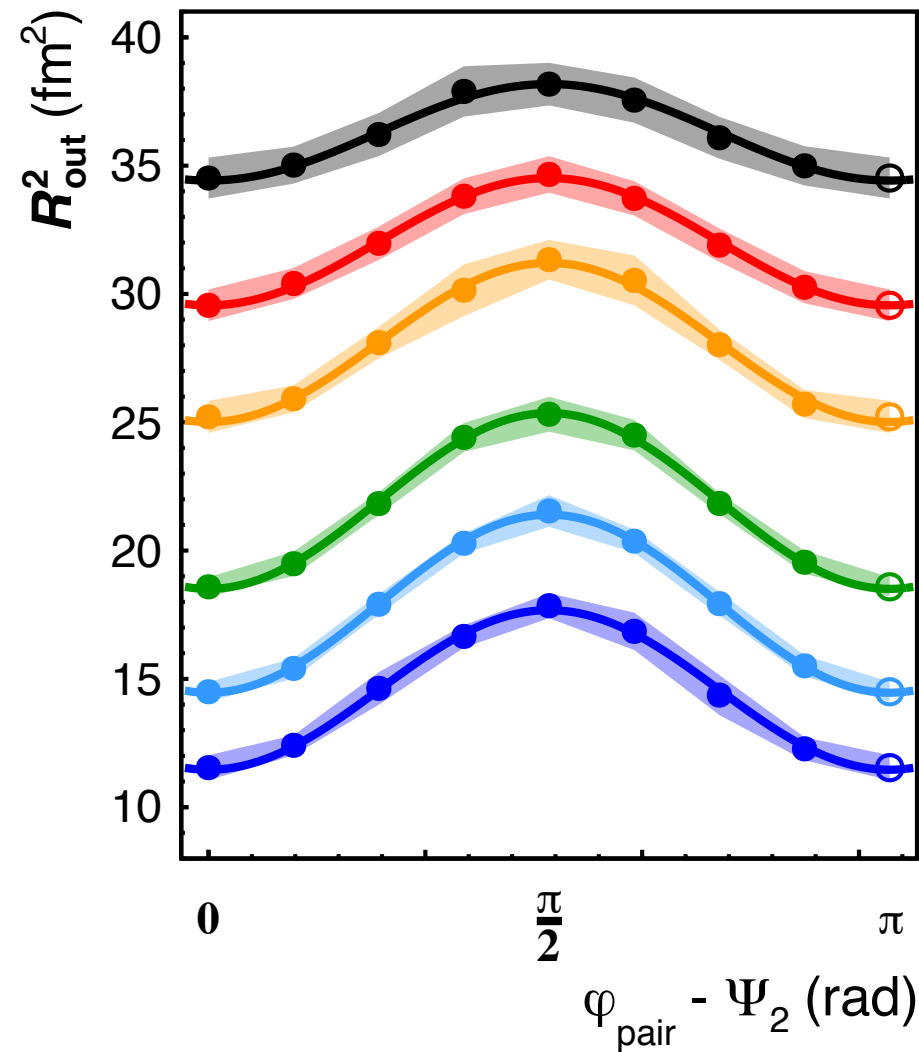
- R_{out} is fully consistent with published results
- R_{side} and R_{long} are systematically smaller than published result, but all results are consistent within systematic uncertainties

Second harmonics

geometry @freeze-out



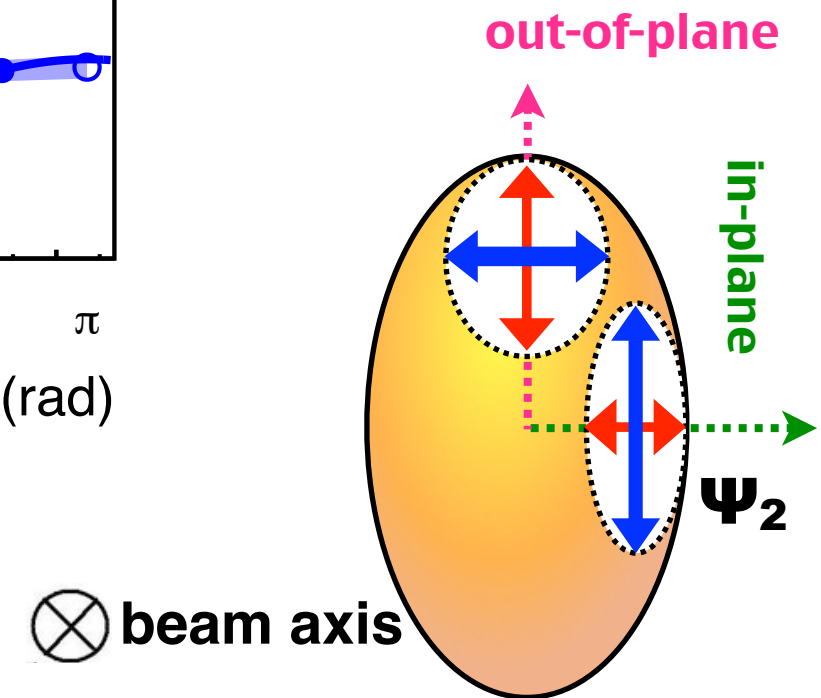
Azimuthal angle dependence of HBT w.r.t. Ψ_2



Pb-Pb $\sqrt{s_{NN}}=2.76$ TeV
 $\pi^+\pi^+$ and $\pi^-\pi^-$ combined
 $k_T : 0.2-1.5$ GeV/c
 centrality

- 0-5%
- 5-10%
- 10-20%
- 20-30%
- 30-40%
- 40-50%

- Fit function : $R^2_{\mu,0} + 2 R^2_{\mu,2} \cos(2(\phi_{\text{pair}} - \Psi_2))$
 - $R^2_{\mu,0}$: Average HBT radii
 - $R^2_{\mu,2}$: Oscillation amplitude

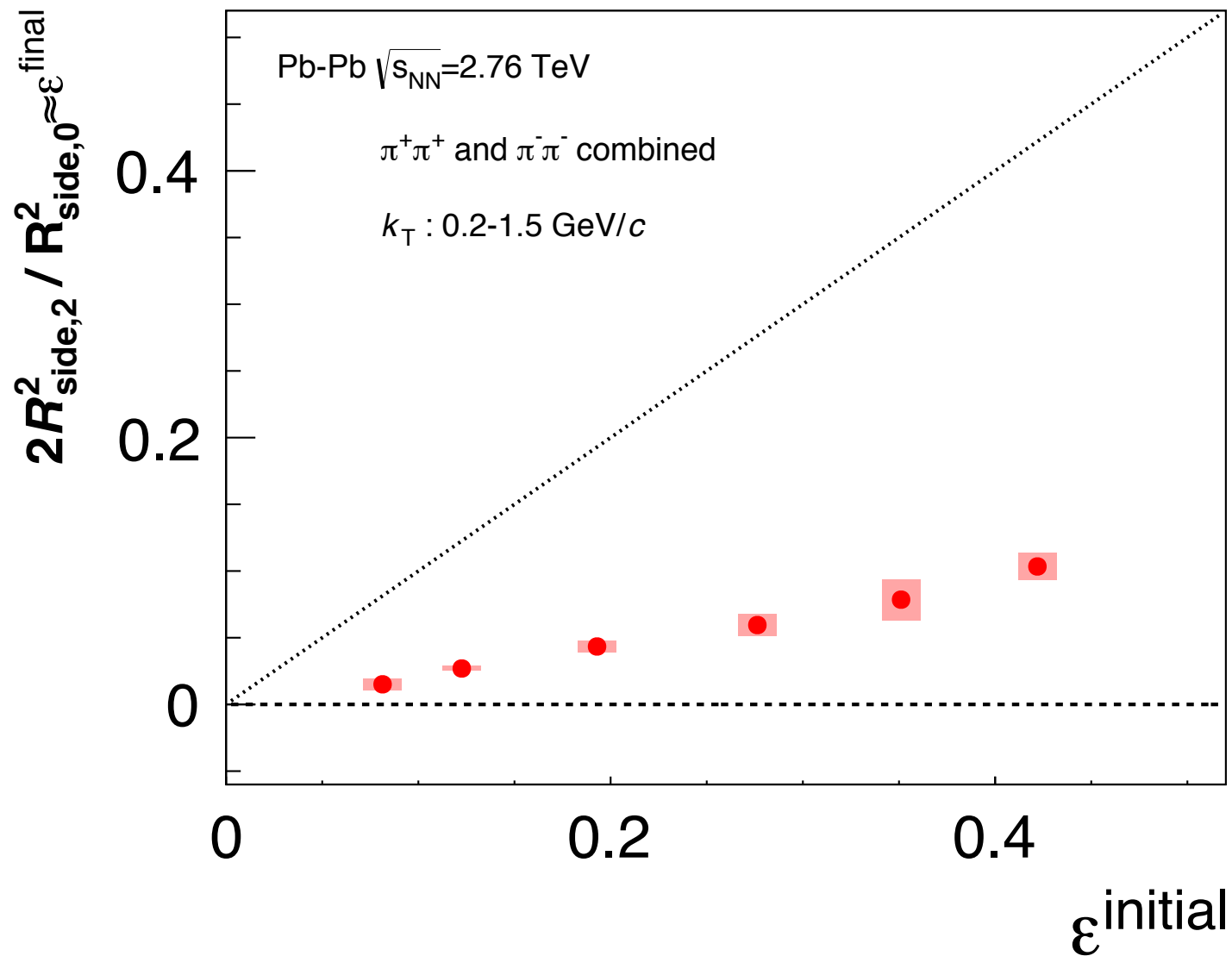


\longleftrightarrow R_{side} : width
 \longleftrightarrow R_{out} : depth + time

- Explicit oscillation can be seen in R_{out} , R_{side}
- Oscillation of R_{out} and R_{side} is out-phase
- R_{out} has larger oscillation than R_{side} . sensitivity to duration time !

✓ R_{out} is sensitive to β_T ($R^2_{\text{out}} = R^2_{\text{out}}^* + \beta_T \Delta\tau^2$)

Initial ϵ_2 v.s. final ϵ_2

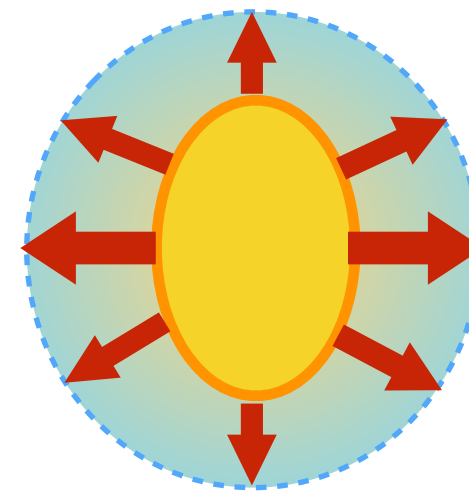


Final source Eccentricity

$$\epsilon_{final} = 2 \frac{R_{side,2}}{R_{side,0}} = -2 \frac{R_{out,2}}{R_{side,0}} = 2 \frac{R_{os,2}}{R_{side,0}}$$

- ϵ_{final} is extracted via HBT oscillation ($k_T \rightarrow 0$)

F. Retiere and M.A.Lisa, PRC70.044907

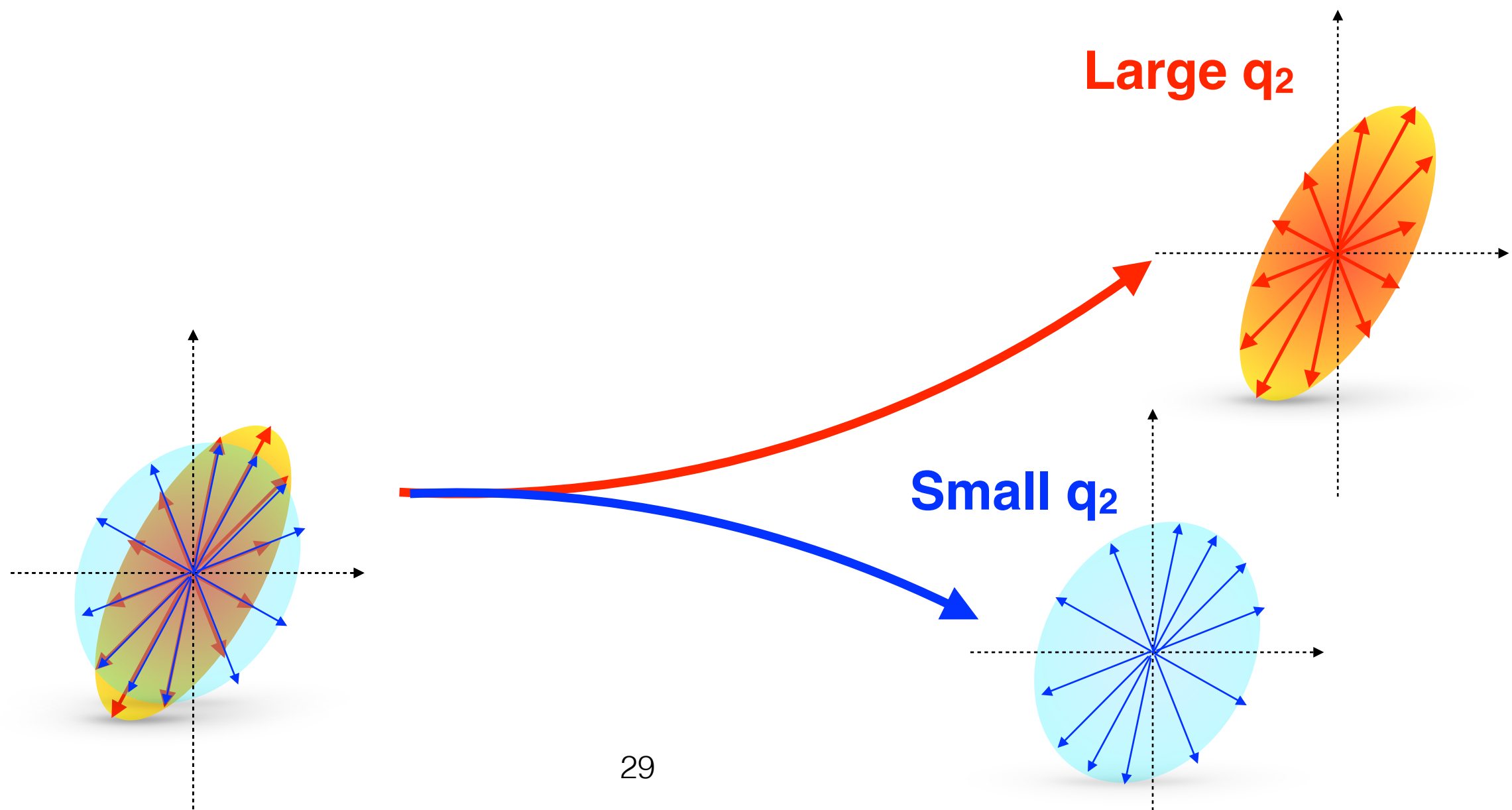


- participant
- freeze-out source
- flow

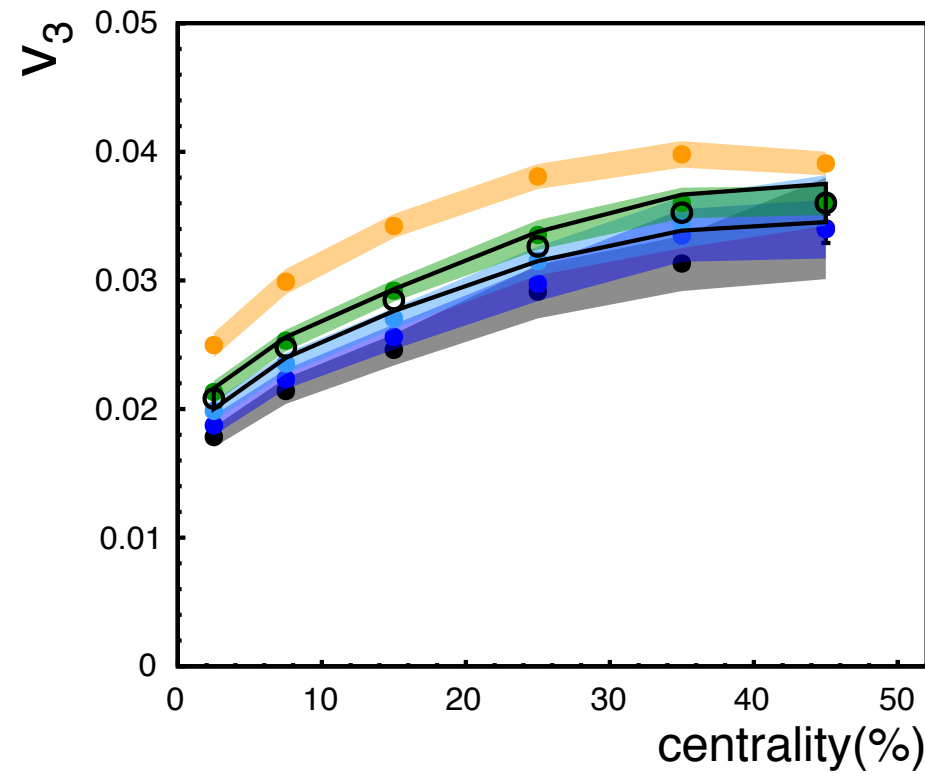
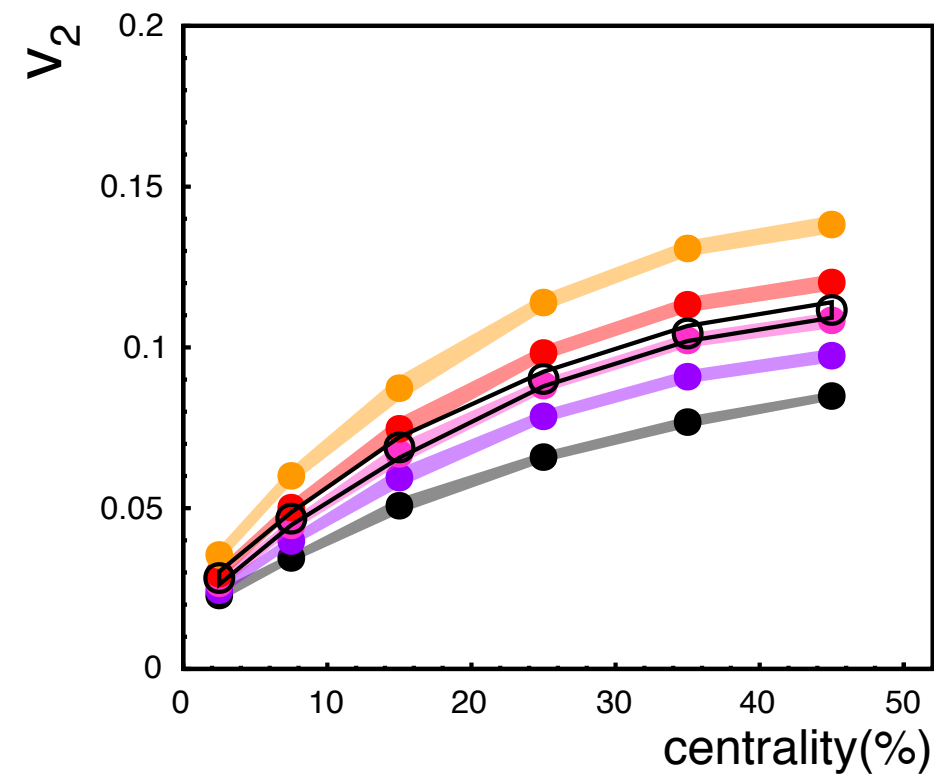
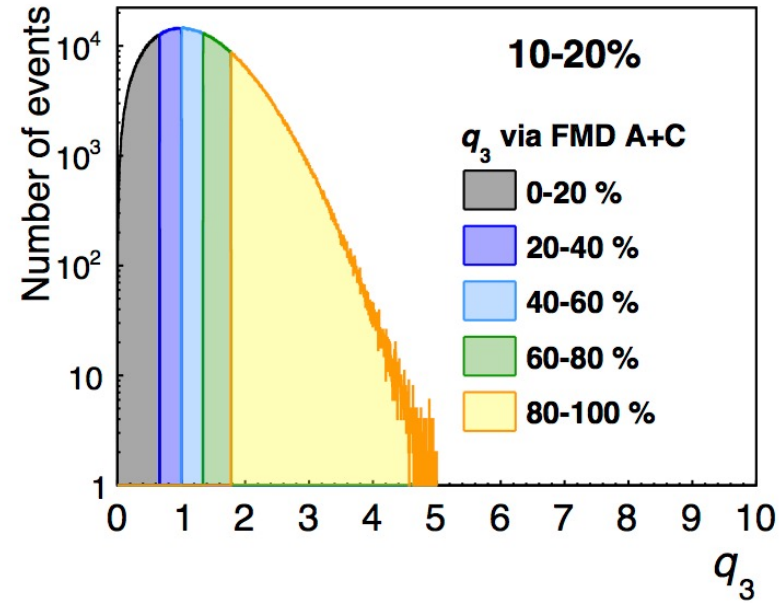
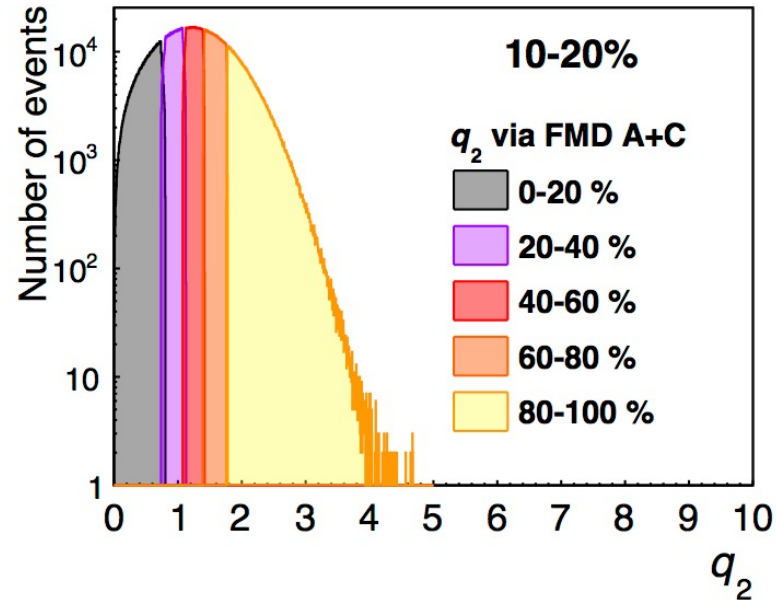
- ▶ Initial elliptic shape has strongly diluted with radial flow and elliptic flow
- ▶ Final source eccentricity linearly increases with increasing $\epsilon_{initial}^2$
 - ➔ Dilution effect linearly increases to $\epsilon_{initial}^2$?
- ▶ Even in most central collision , $2R_{side,2}^2 / R_{side,0}^2 > 0$
 - ➔ Initial out-plane elongated elliptic shape still remains at freeze-out time

Event Shape Engineering

v_n cut (initial ε_n selection)



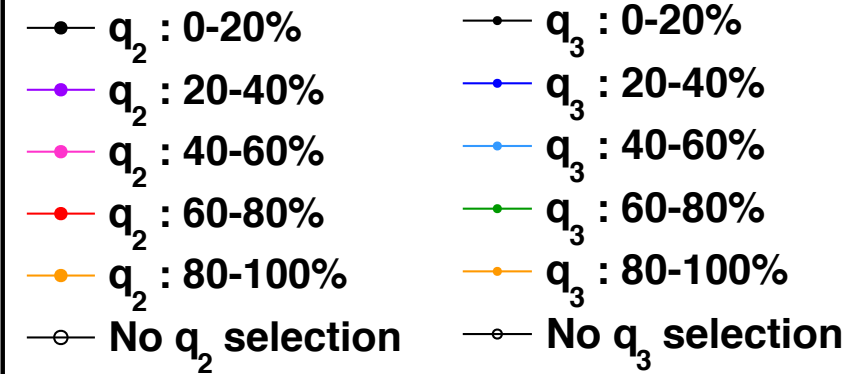
v_n for each 20% Event shape q_2, q_3 selection



Pb-Pb $\sqrt{s_{NN}}=2.76$ TeV

Ψ_n determined via FMD A+C

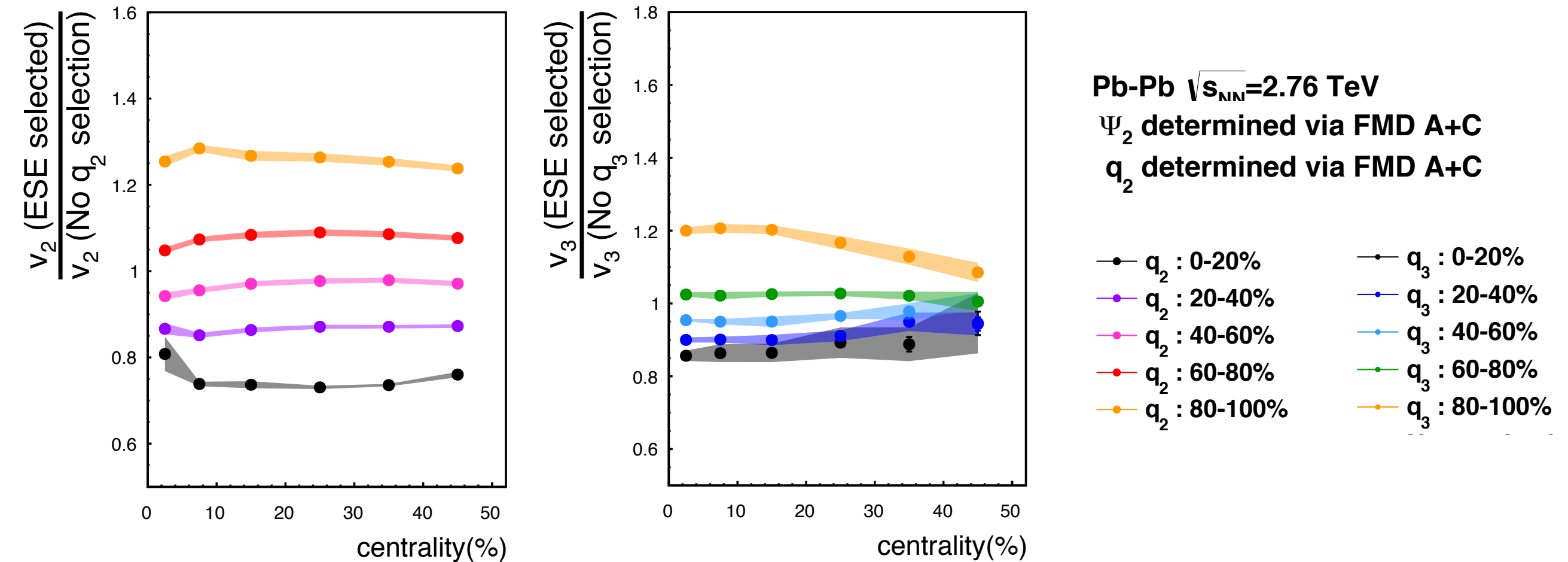
q_n determined via FMD A+C



□ 0-20% (80-100%) q_n selection denotes top (bottom) 20% cut

□ Event by event v_2 and v_3 fluctuations can be selected with ESE cut

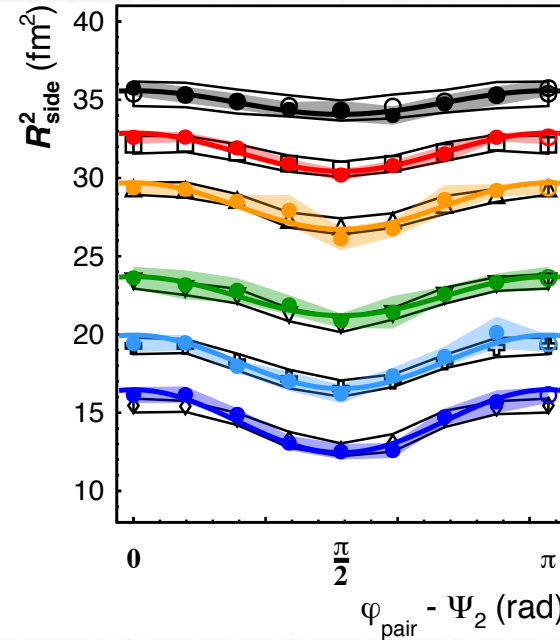
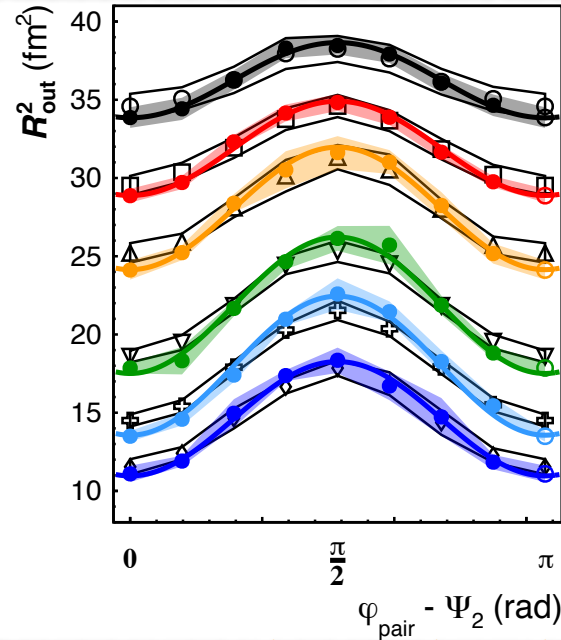
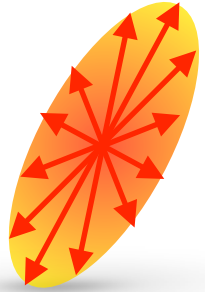
Ratio of v_2 and v_3 (ESE selected / No ESE cut)



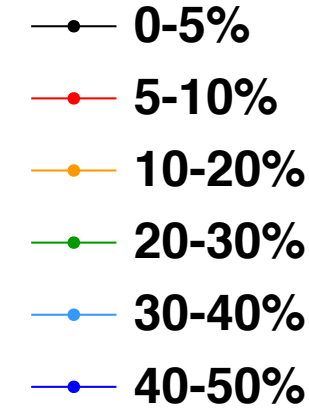
- Both q_2 and q_3 , enhancement (suppression) with ESE cut can be found
- Ratio of v_2 slightly depends on centrality
- Enhancement(suppression) effect becomes smaller from central to peripheral collisions in ratio of v_3
- “Selectivity “ of q_2 cut is larger than that of q_3
 - ✓ Centrality dependence and difference of selectivity can be interpreted with Event plane resolution

q_2 selection + azimuthal angle dependence of HBT w.r.t. Ψ_2

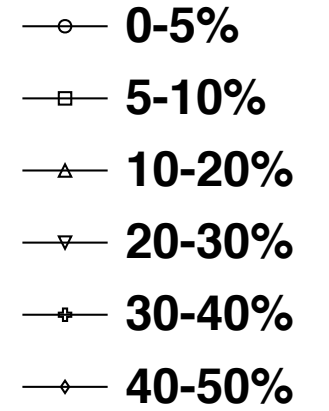
Top 20% q_2



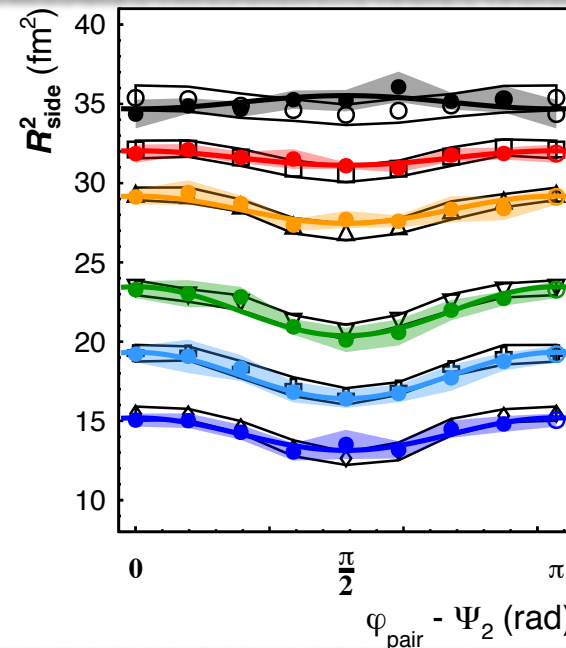
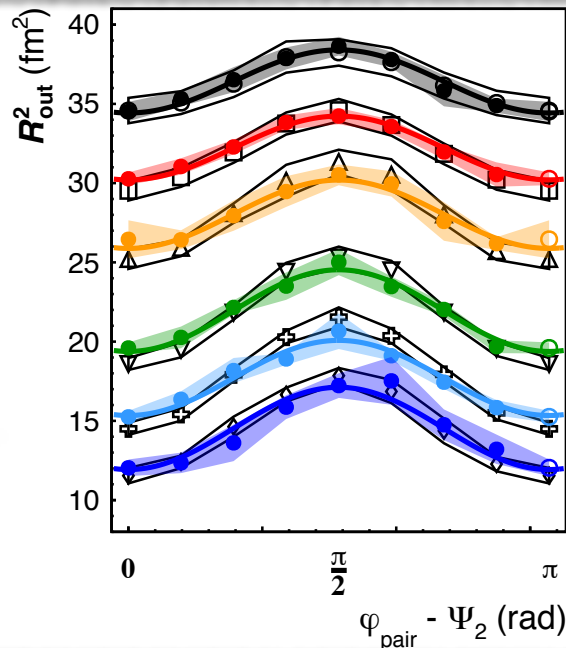
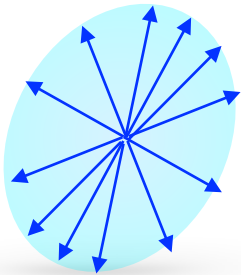
q_2 cut
centrality



No q_2 selection
centrality

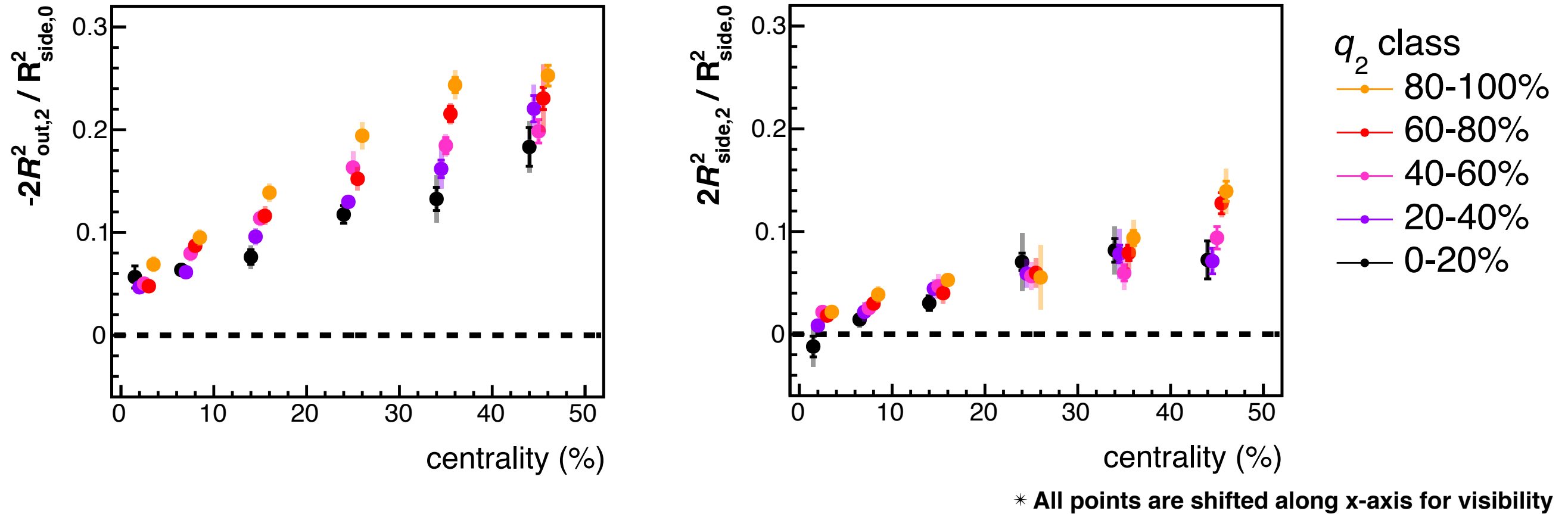


Bottom 20% q_2



- 20% q_2 selection enhanced(suppressed) oscillation of R_{out} and R_{side}
 - Correlation between v_2 and ε_2 final can be found
- For smallest q_2 selection(0-20%)
 - R_{side} could have positive sign oscillation

Relative amplitude of HBT radii (2nd harmonics)



- ◆ $-2R_{out,2}^2 / R_{side,0}^2$ increase from small q_2 to large q_2 in all centrality
- ◆ $2R_{side,2}^2 / R_{side,0}^2$ ($\sim \epsilon_2^{\text{final}}$) increases with increasing q_2 in 0-20% and 40-50% collisions
- ◆ In most centrality 0-5% and smallest q_2 , $2R_{side,2}^2 / R_{side,0}^2$ ($\sim \epsilon_2^{\text{final}}$) shows negative (or zero)
 - ➔ Elliptic shape at freeze out might be vanished or even reversed

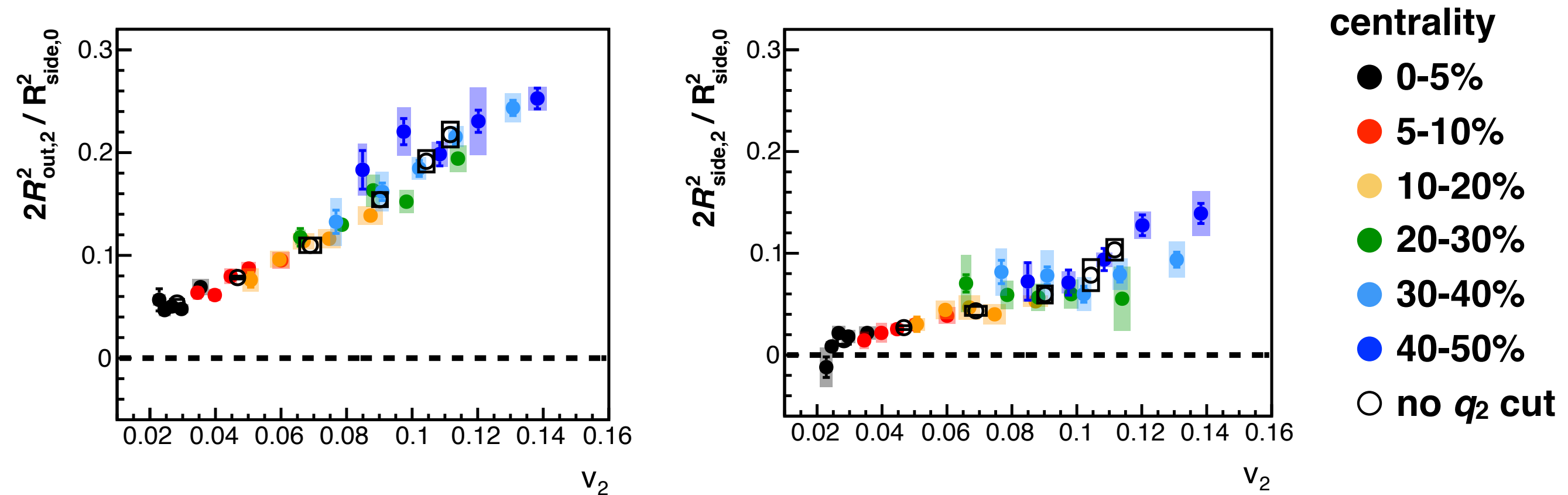
v_2 scaling to Relative amplitude of HBT radii

✓ Difference of event by event initial geometry fluctuation can not be reflected to “centrality”

✓ More sensitive probe is necessary !

◆ Empirical correlation between v_2 , $\varepsilon_2^{\text{initial}}$ and energy density

◆ $v_2 \propto \varepsilon_2^{\text{initial}} \cdot f(dN/d\eta)$



✓ Both relative amplitudes of R_{out} and R_{side} are scaled with v_2

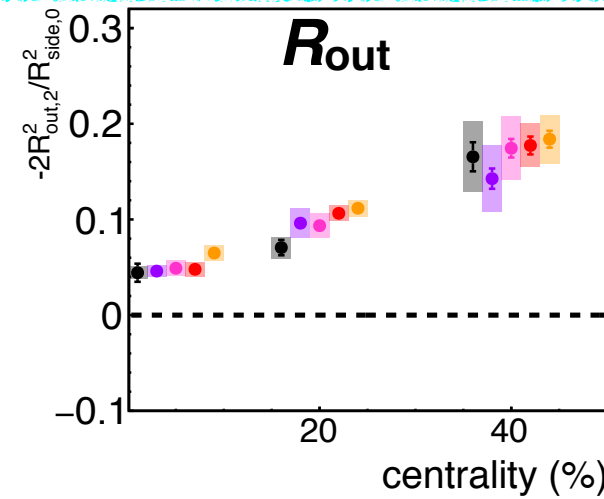
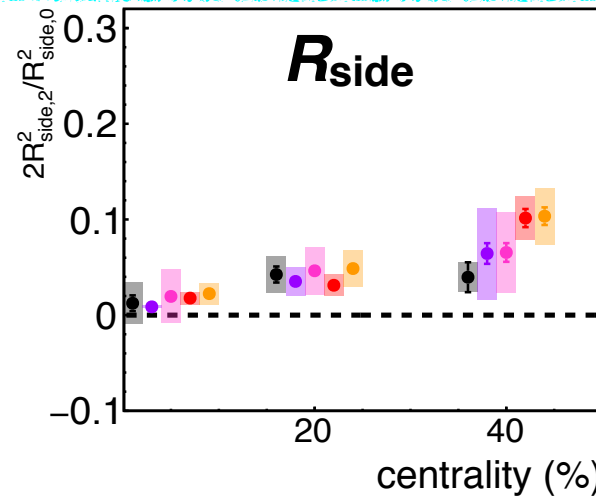
✓ In a same $dN/d\eta$ (centrality), correlation between v_2 and relative amplitudes of HBT radii \sim correlation between $\varepsilon_2^{\text{initial}}$ and $\varepsilon_2^{\text{final}}$

✓ In centrality 20-40%, $2R_{\text{side},2}^2 / R_{\text{side},0}^2$ ($\sim \varepsilon_2^{\text{final}}$) does not depends on $\varepsilon_2^{\text{initial}}$

k_T dependence of relative amplitude of HBT radii (2nd harmonics)

* All points are shifted along x-axis for visibility

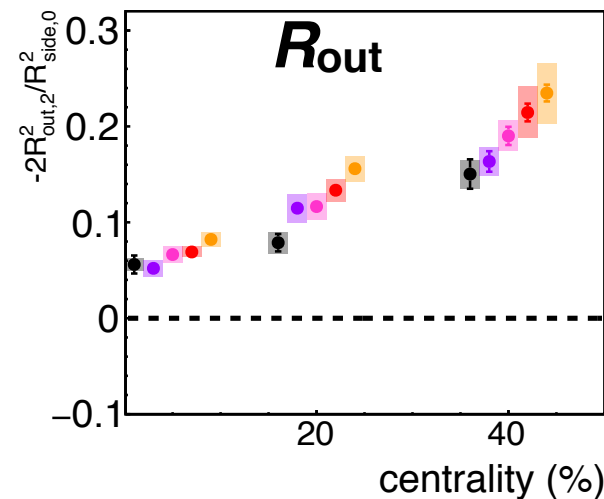
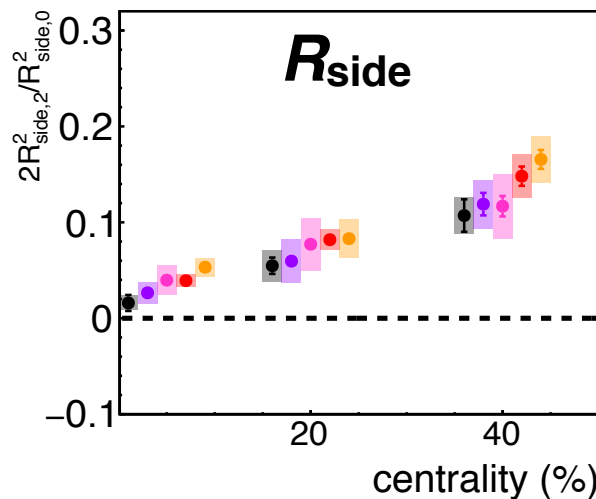
low k_T
 $0.2 < k_T \text{ (GeV/c)} < 0.3$



q_2 class

- 80-100%
- 60-80%
- 40-60%
- 20-40%
- 0-20%

high k_T
 $0.4 < k_T \text{ (GeV/c)} < 1.5$



- ◆ $-2R_{\text{out},2}^2 / R_{\text{side},0}^2$ increase from small q_2 to large q_2 in all k_T
- ◆ Insensitivity of $2R_{\text{side},2}^2 / R_{\text{side},0}^2$ ($\sim \epsilon_2^{\text{final}}$) to q_2 can be found in low k_T and mid central collisions
- ◆ In mid central collisions, elliptic shape at freeze-out might not be changed by initial elliptic shape
- ➔ correlation between initial geometry and collective flow could generate this effect

Blast-wave fit for spectra, v_2 and HBT radii

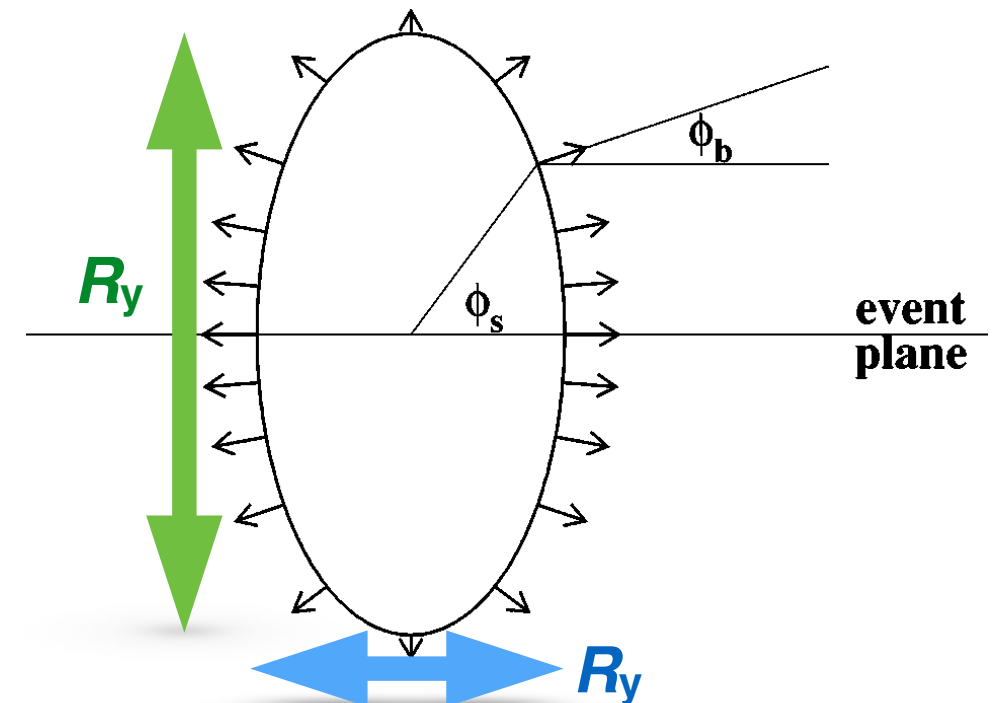
- What makes the difference of relative amplitude of HBT radii with q_2 selection ?
 - ◆ Blast-wave model allows us to extract spatio-temporal parameters, analytically
- Blast-wave model is based on the hydrodynamical model
- Extended to Azimuthally sensitive HBT interferometry (Phys. Rev. C 70 044907)

★ Blast-wave parameters

- Freeze out temperature : T_f
- Flow velocity : $\rho(r, \phi_s) = r (\rho_0 + \rho_2 \cos(2\phi_b))$
- Transverse extents R_x, R_y
- System lifetime : τ_0
- Emission duration : $\Delta\tau$

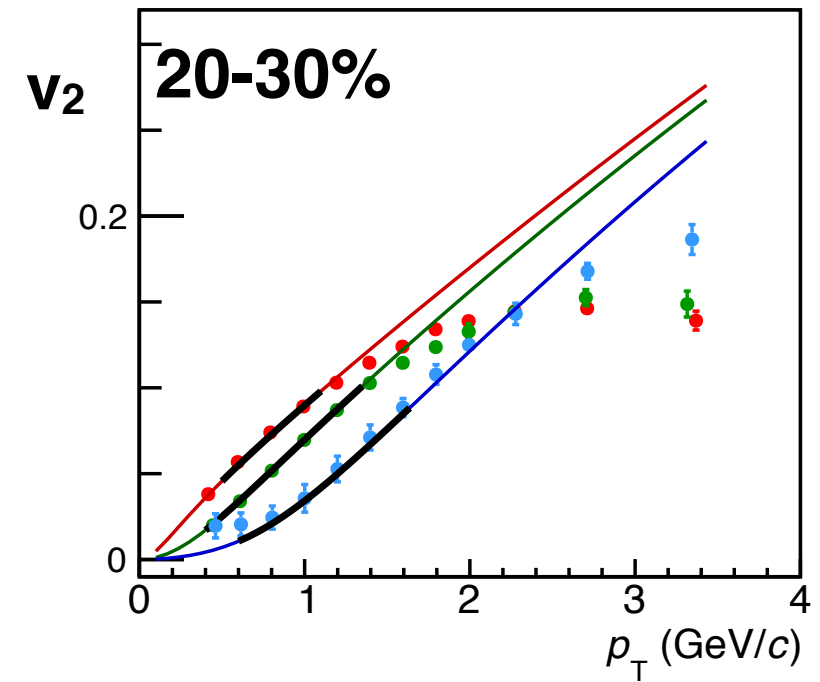
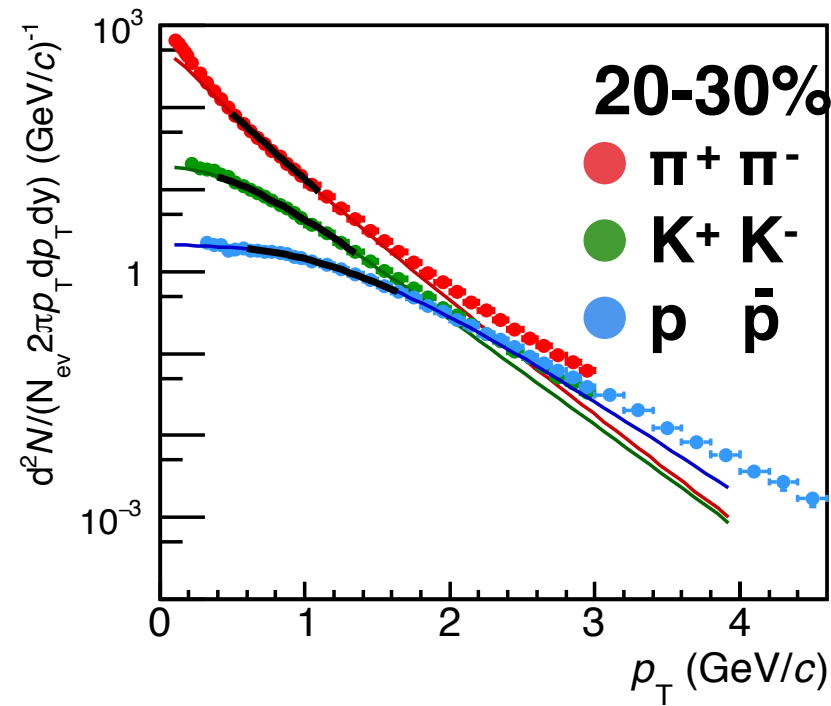
$$\Omega(r, \phi_s) = \frac{1}{1 + e^{(\tilde{r}-1)/a_s}}$$

$$\tilde{r}(r, \phi_s) \equiv \sqrt{\frac{(r \cos(\phi_s))^2}{R_x^2} + \frac{(r \sin(\phi_s))^2}{R_y^2}}$$

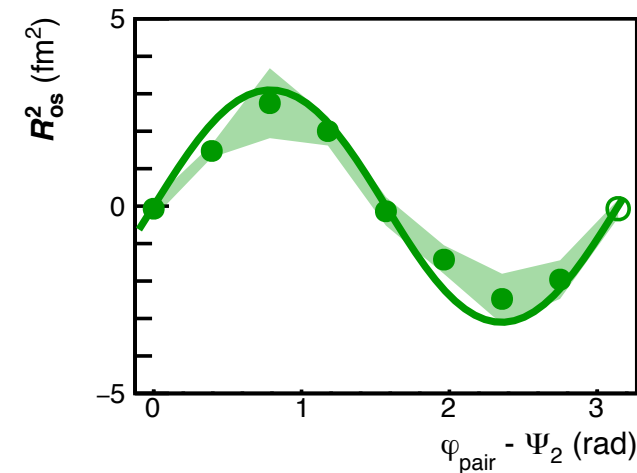
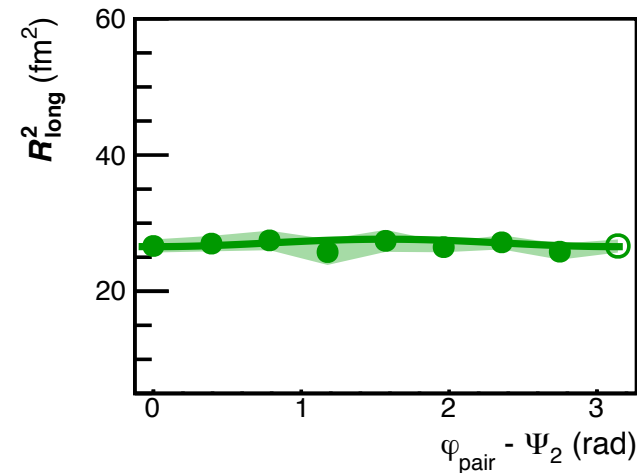
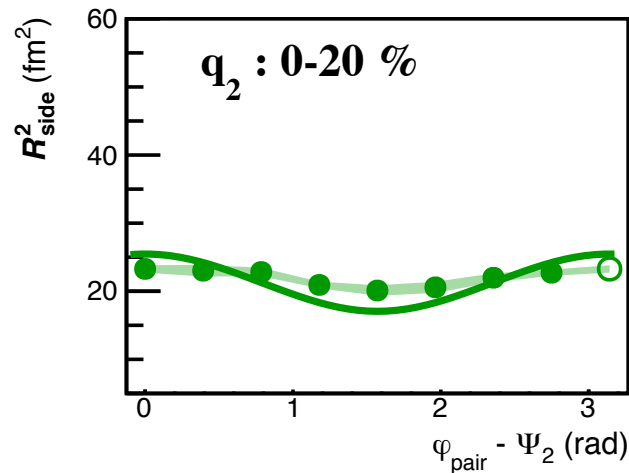
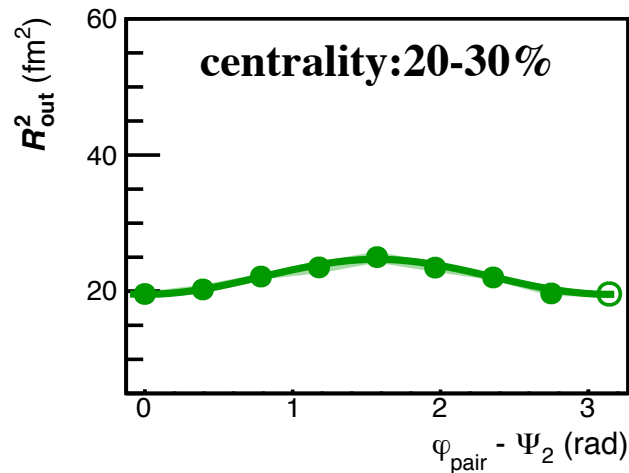


Blast wave fit for Spectra, v_2 and HBT radii

- ▶ T_f , ρ_0 is determined with π , K, p spectra (independent of v_2 and HBT)
 - ✓ assuming that T_f and ρ_0 don't change within systematic uncertainties
- ▶ ρ_2 , R_x , R_y/R_x , τ_0 , $\Delta\tau$ are determined with simultaneous fitting of v_2 and HBT fit

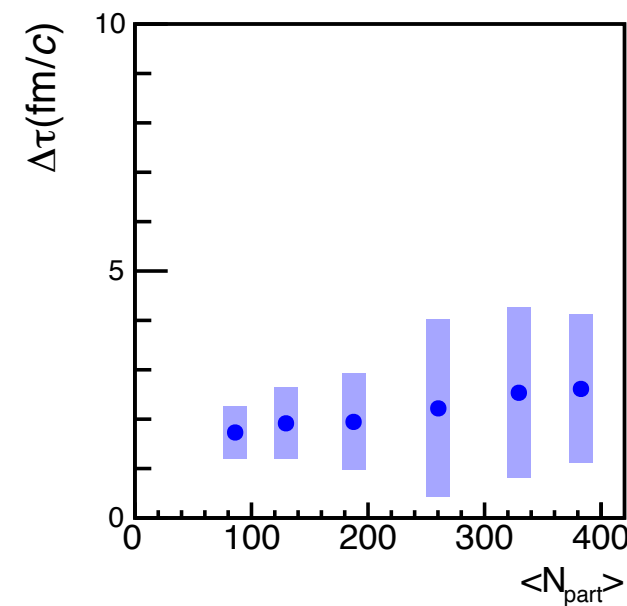
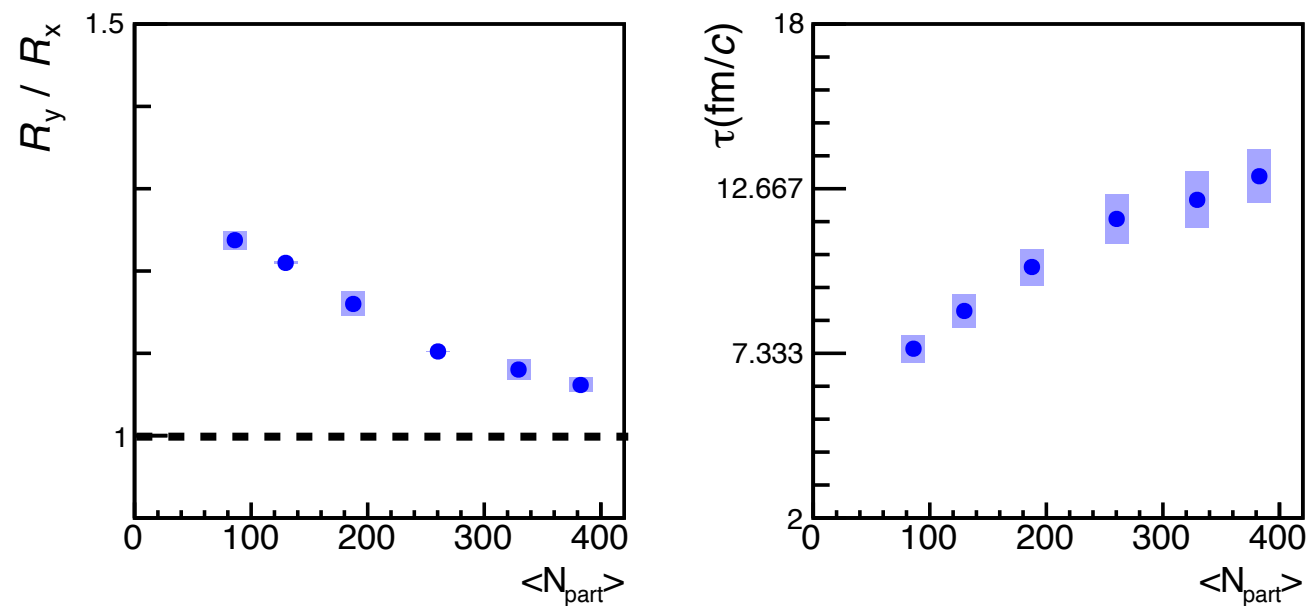
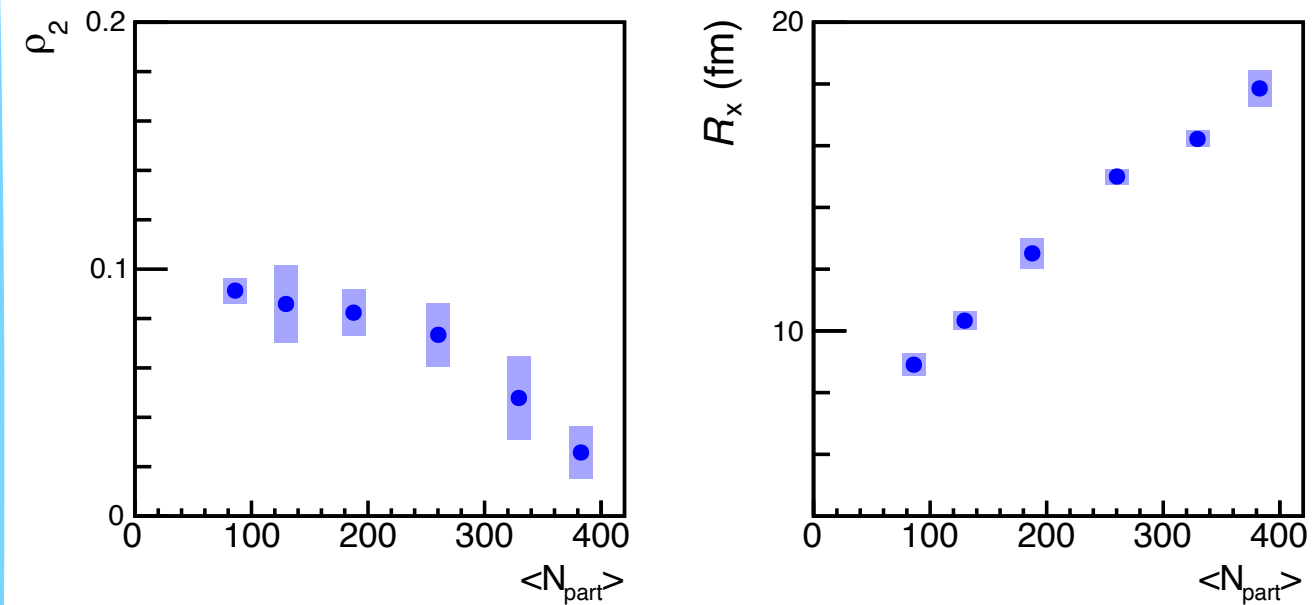
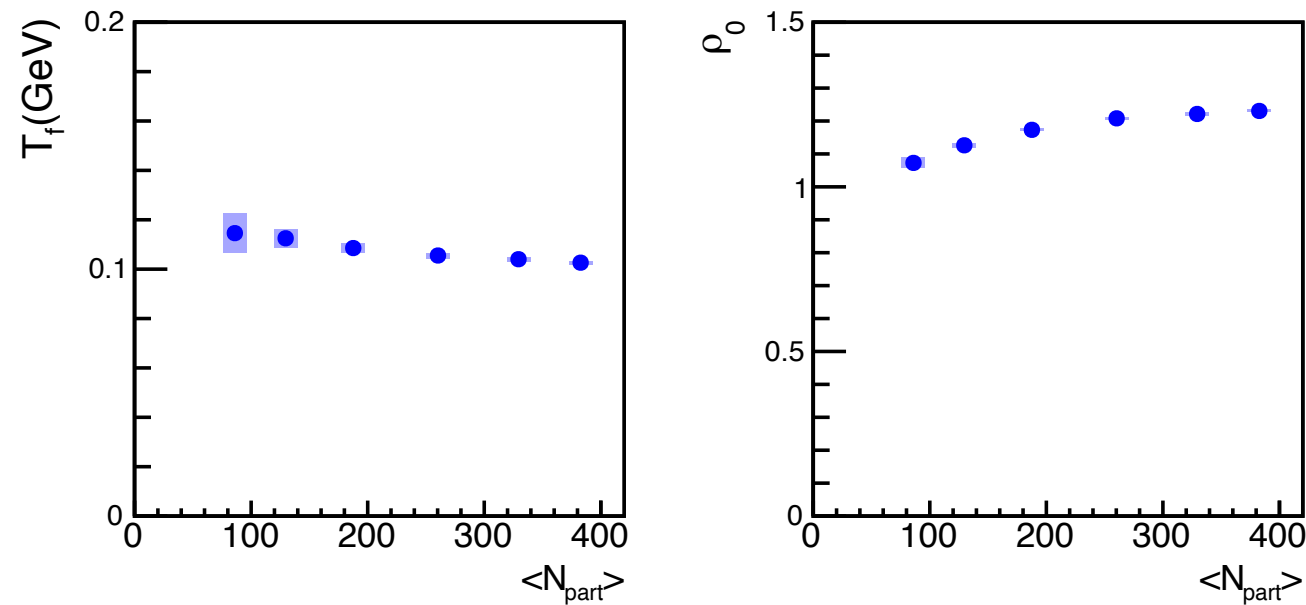


❖ spectra (Phys.Rev.C 88, 044910 [2013])



Extracted Blast Wave parameters

✓ Fully consistent with published result



Pb-Pb $\sqrt{s_{\text{NN}}} = 2.76$ TeV

$\pi^+\pi^+$ and $\pi^-\pi^-$

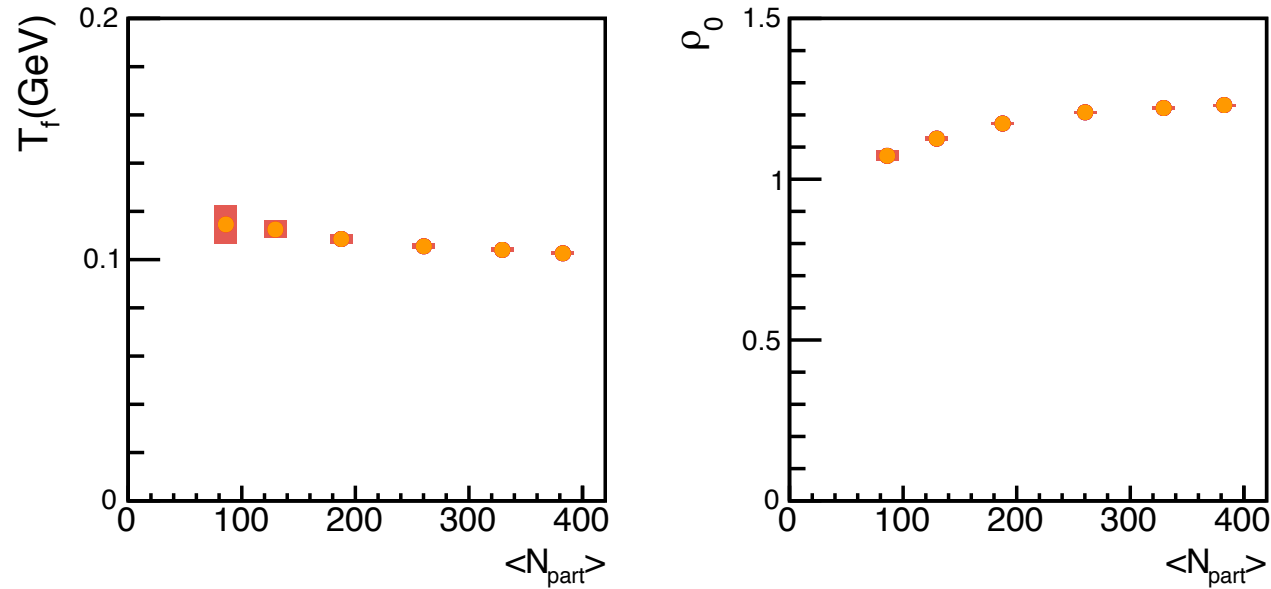
$k_T : 0.2-1.5$ GeV/c

- ◆ Source size(R_x) and freeze out time (τ_0) increases as a function of $\langle N_{\text{part}} \rangle$
- ◆ Emission duration slightly increase with increasing $\langle N_{\text{part}} \rangle$
- ◆ R_y/R_x , T_f and ρ_2 decrease from peripheral to central

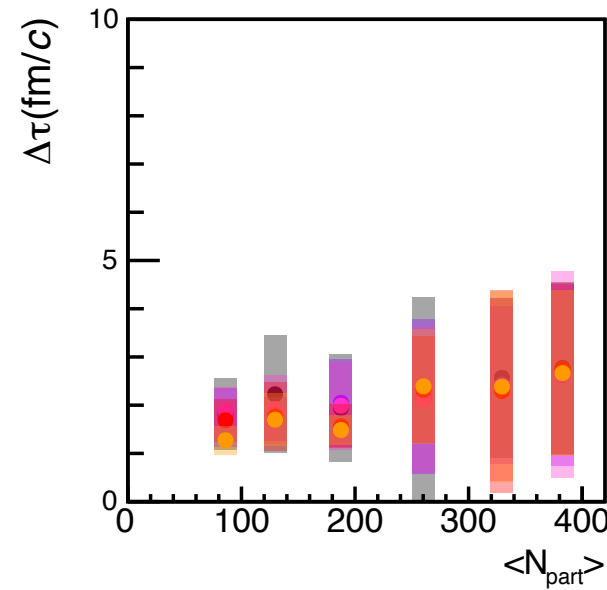
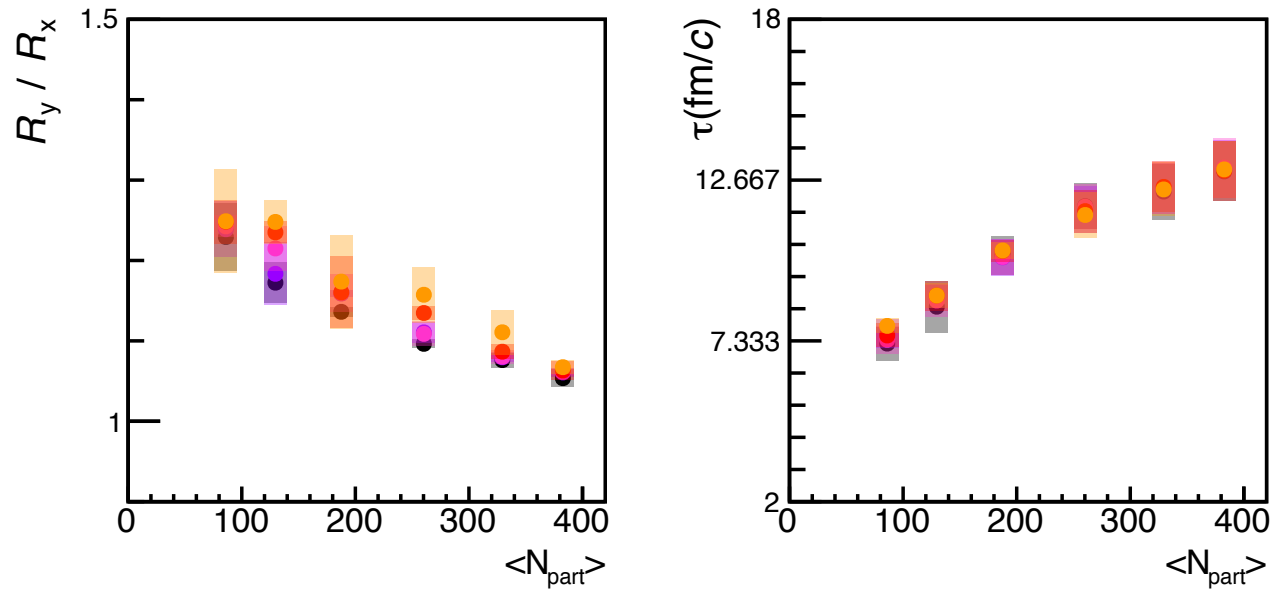
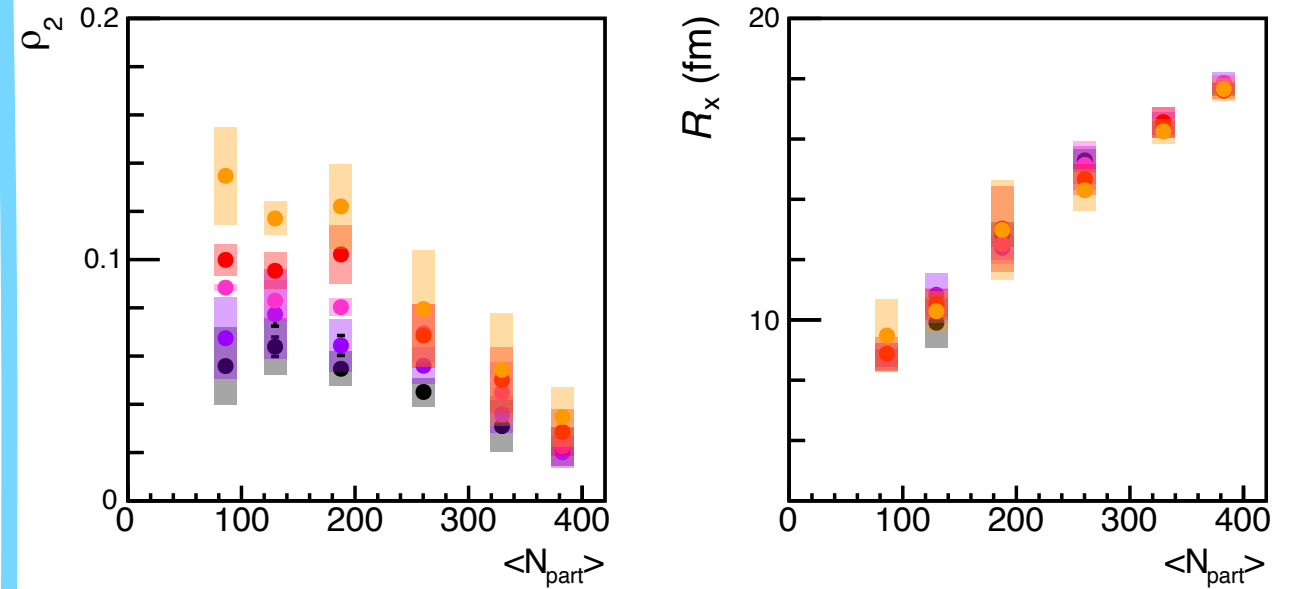
✓ Out-plane elongated elliptic shape can be found even in most central events

Extracted Blast Wave parameters with ESE q_2 cut

◆ Spectra fit



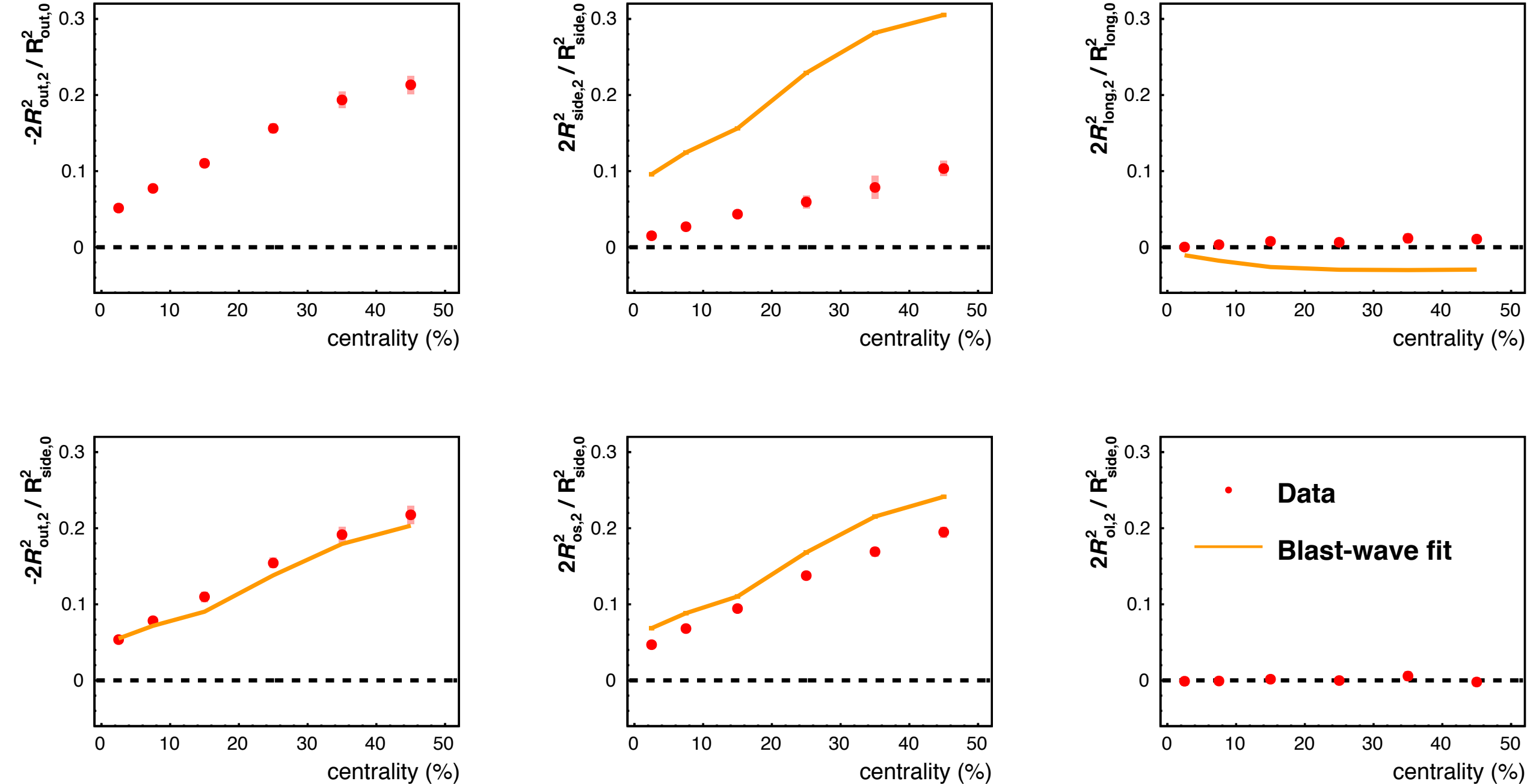
◆ v_2 and HBT fit



Pb-Pb $\sqrt{s_{\text{NN}}} = 2.76$ TeV
 $\pi^+\pi^+$ and $\pi^-\pi^-$
 $k_T : 0.2-1.5$ GeV/c
 q_2
 • 0-20%
 • 20-40%
 • 40-60%
 • 60-80%
 • 80-100%

- ◆ ρ_2 significantly changes with q_2 selection
- ◆ R_y/R_x , τ slightly changes with q_2 cut, but consistent within systematic uncertainties
- ◆ No significant difference can be found in R_x , $\Delta\tau$

Comparison with Blast-wave model (relative amplitude)



- ◆ $-2R_{\text{out},2}^2 / R_{\text{side},0}^2$ is consistent within a systematic uncertainty
- ◆ $2R_{\text{side},2}^2 / R_{\text{side},0}^2$ of Blast-wave is much larger than that of Data
- ◆ $2R_{\text{os},2}^2 / R_{\text{side},0}^2$ of Blast-wave is slightly larger than that of Data
- ★ More realistic model is necessary

Summary-1

- ✦ **Azimuthal angle dependence of HBT relative to 2nd order event plane**
 - Explicit oscillation can be found in R_{out} , R_{side}
 - Initial elliptic shape is strongly diluted by collective flow, but out-plane elongated elliptic shape still remains at freeze-out

- ✦ **ESE applies to measurements of azimuthal anisotropy (v_2 and v_3)**
 - Both v_2 and v_3 are enhanced (suppressed) with large (small) q_2 and q_3 cut
 - “Selectivity” of q_2 to v_2 is larger than that of q_3 to v_3

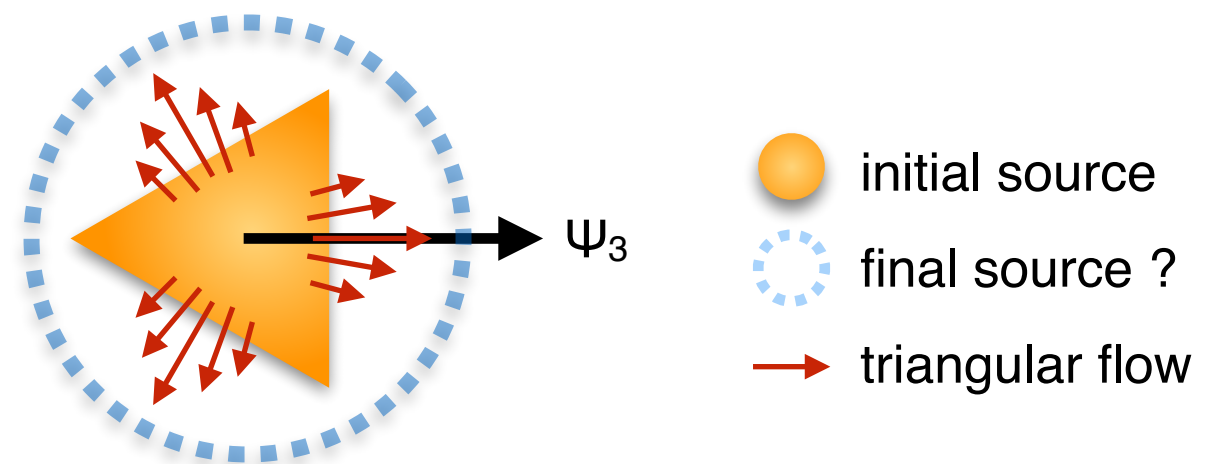
- ✦ **ESE applies to HBT measurements w.r.t. 2nd order event plane**
 - First measurement of q_2 selection + HBT
 - Relative amplitudes of R_{out} , R_{side} and R_{os} vary with q_2 selection
 - In most central collisions 0-5% and smallest q_2 , relative amplitude of R_{side} is negative or zero
 - Modification with q_2 was scaled with v_2 , but q_2 dependence of R_{side} could change in centrality 20-40%

Summary-2

♦ Interpretation with Blast-wave model

- ρ_2 explicitly changes with q_2 selection
- R_y/R_x , system life time slightly changes with q_2 selection, but no significant change was found within systematic uncertainties
- Oscillation of R_{side} can not be reproduced with Blast-wave model

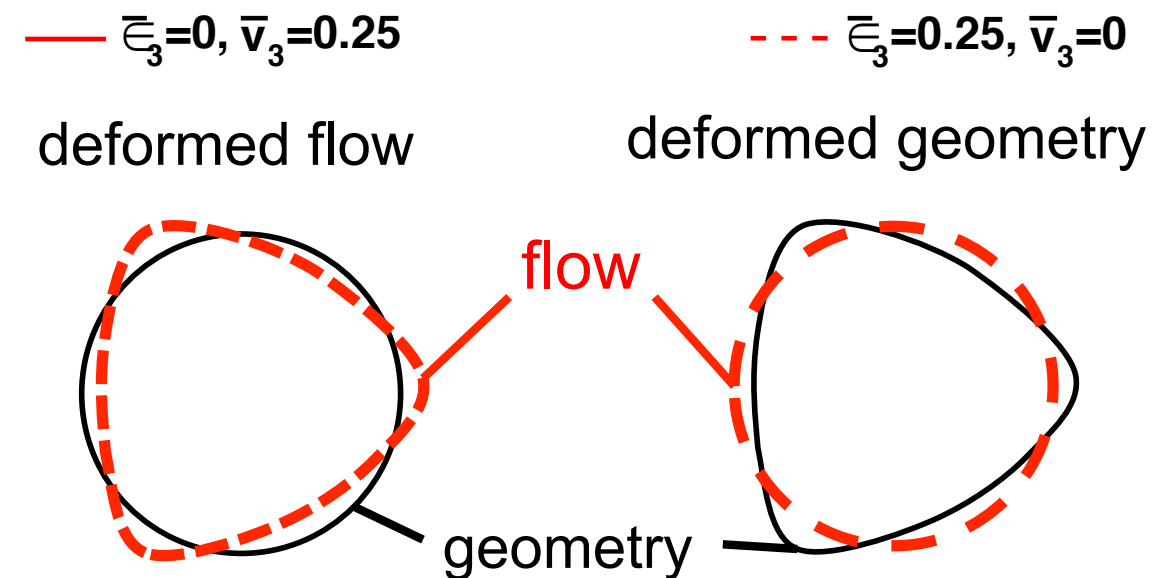
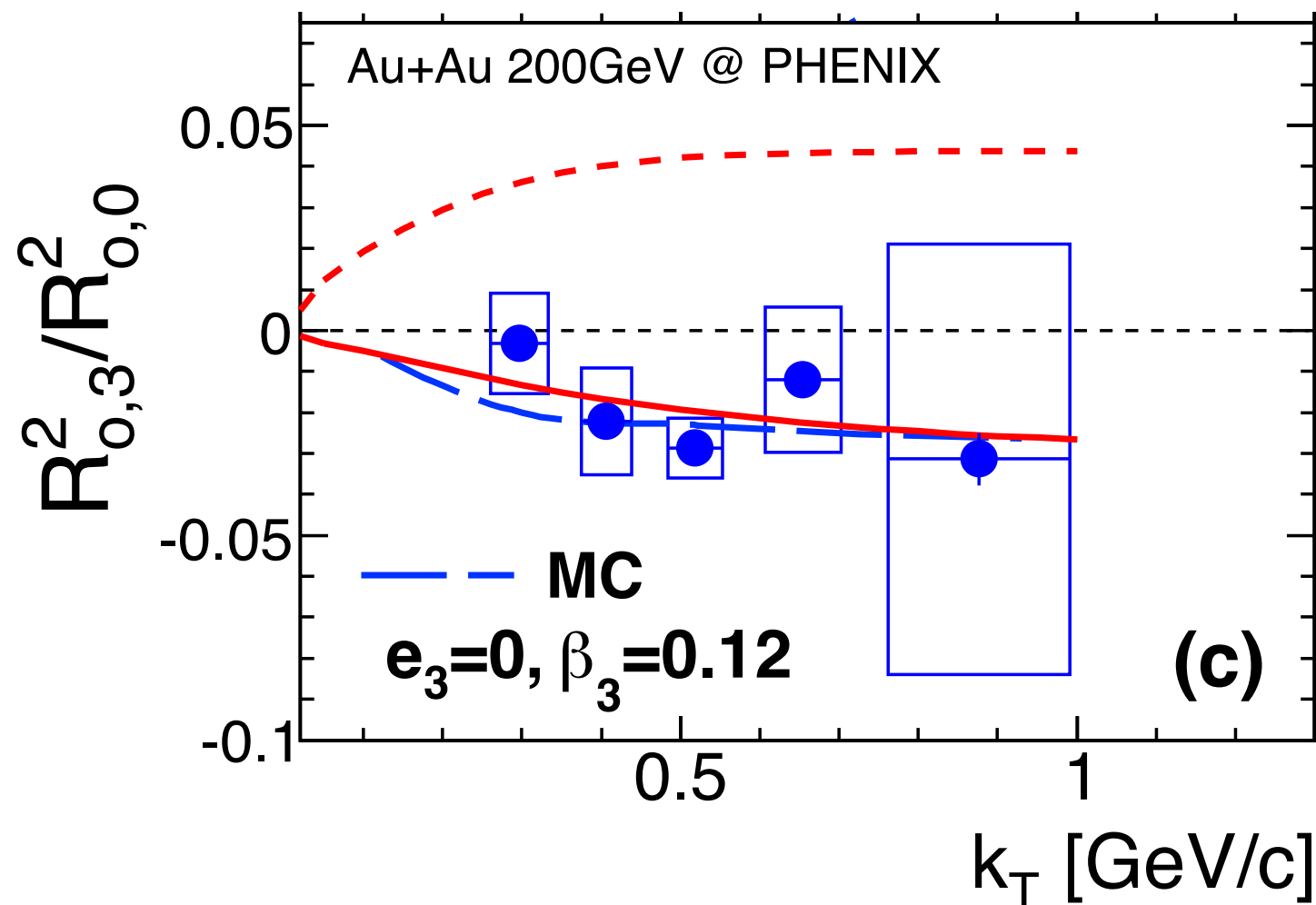
Triangular shape @ freeze-out



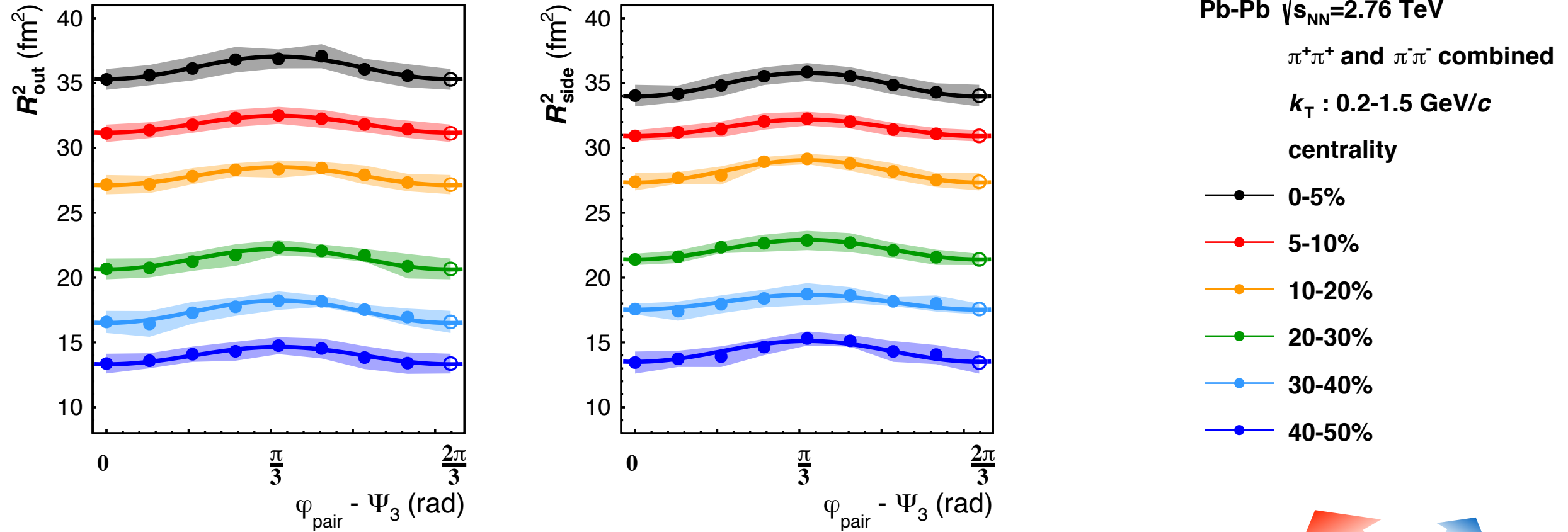
Oscillation of HBT w.r.t. Ψ_3 is flow dominant

- k_T dependence of 3rd-order oscillation at PHENIX is described by flow dominant case of Gaussian toy model

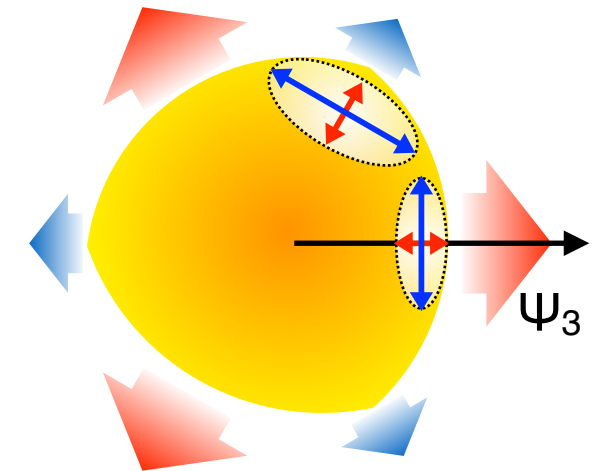
➔ q_3 + Azimuthal angle dependence of HBT radii is direct measurement of relation between v_3 and oscillation of HBT radii



Azimuthal angle dependence of HBT radii w.r.t. Ψ_3

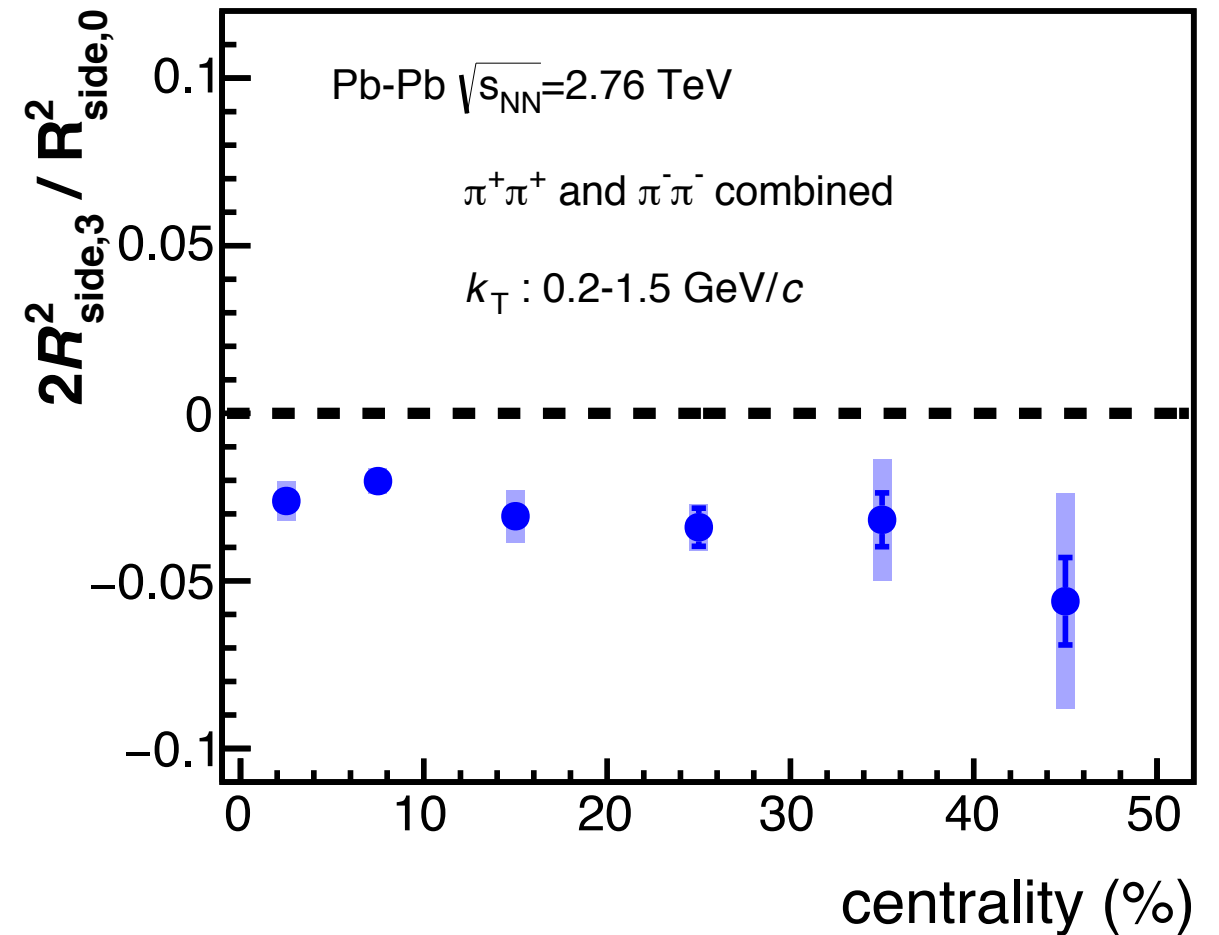
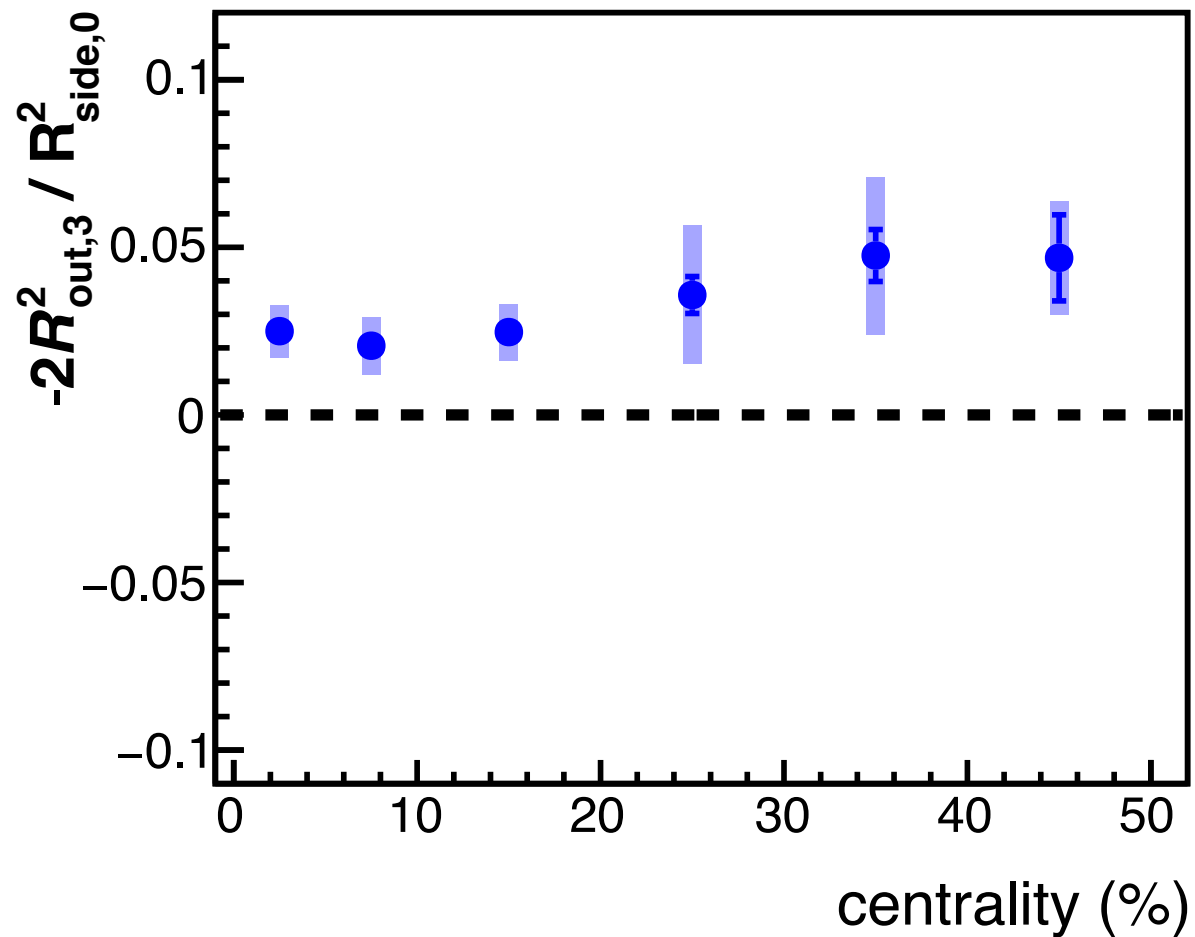


- Fit function
- $R_{\mu,0}^2 + 2 R_{\mu,3}^2 \cos(3(\varphi_{pair} - \Psi_3))$
- $R_{\mu,0}^2$: Average HBT radii, $R_{\mu,3}^2$: Oscillation amplitude



- Oscillations w.r.t. Ψ_3 are observed in R_{out} and R_{side}
- **R_{out} and R_{side} oscillations have same sign**
 - Consistent to PHENIX result in Au+Au 200GeV collisions (PRL112.222301)
 - **Similar behaviour to HBT w.r.t. Ψ_2 in most central smallest q_2**
 - When the initial eccentricity is small, oscillation of R_{out} and R_{side} could have same sign.

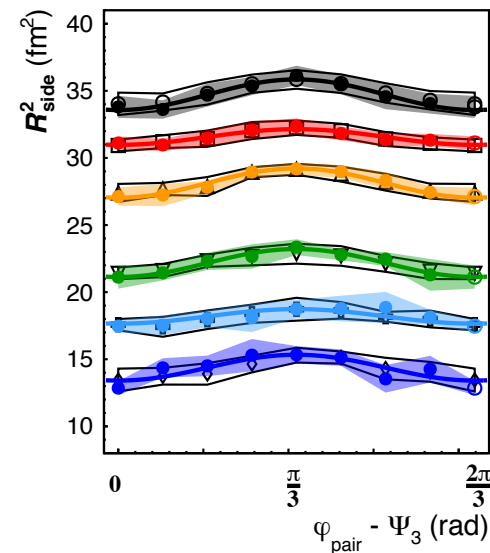
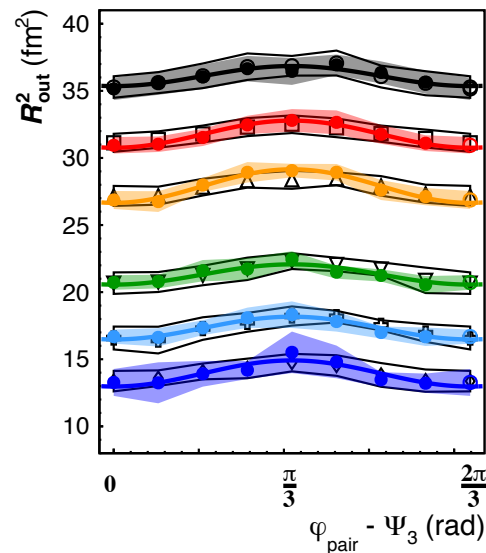
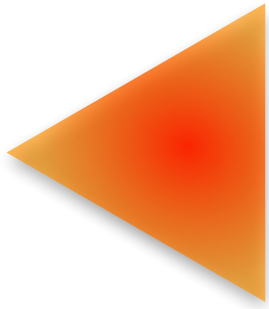
3rd harmonic oscillation of HBT radii



- ✓ $-2R^2_{out,3} / R^2_{side,0}$ has positive value in all centrality
- ✓ $2R^2_{side,3} / R^2_{side,0}$ has negative value in all centrality
- ✓ $R^2_{out,3} / R^2_{side,0}$ slightly increase with increasing centrality
- ✓ $R^2_{side,3} / R^2_{side,0}$ slightly becomes smaller from central to peripheral

Azimuthal HBT w.r.t. Ψ_3 with ESE

Top 20% q_3



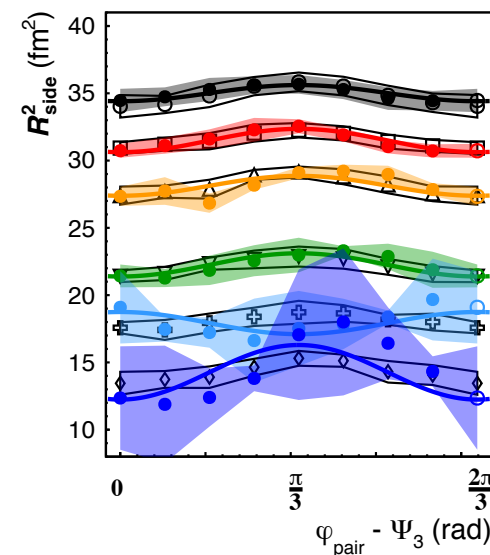
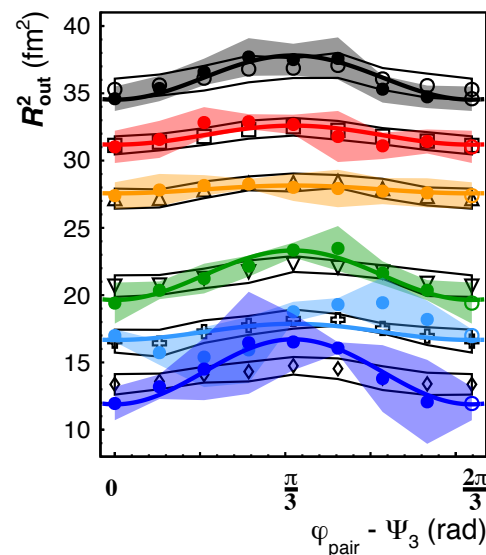
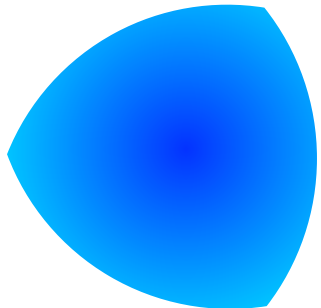
q_3 cut
centrality

- 0-5%
- 5-10%
- ▲— 10-20%
- ▼— 20-30%
- +— 30-40%
- ◆— 40-50%

No q_3 selection
centrality

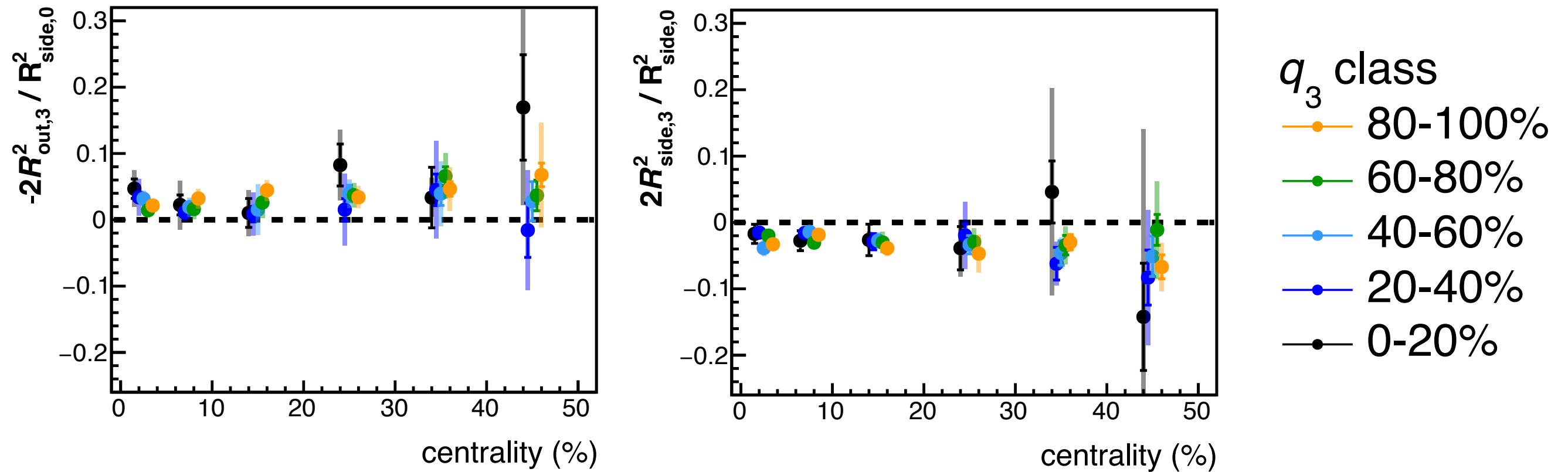
- 0-5%
- 5-10%
- △— 10-20%
- ▽— 20-30%
- +— 30-40%
- ◆— 40-50%

Bottom 20% q_3



- Top(Bottom) 20% q_3 vector selection is applied to HBT w.r.t. Ψ_3
- No significant effect on the R_{out} oscillation can be observed by large q_3 selection, though v_3 changes with q_3 selection

Relative amplitude of HBT radii (3rd harmonics)



- ▶ **No significant change has been observed in relative amplitudes ($R^2_{\text{out},3} / R^2_{\text{side},0}$ and $R^2_{\text{side},3} / R^2_{\text{side},0}$), though v_3 is enhanced $\sim 15\%$**
 - ➔ **Different behaviour to HBT w.r.t. $\Psi_2 + q_2$ selection**
 - ▶ **Triangular shape at freeze-out seems to be saturated by radial flow and triangular flow**

Summary-3

- ✦ **Azimuthal angle dependence of HBT relative to 3rd order event plane**
 - Small but finite oscillation can be found in R_{out} , R_{side}
 - Relative amplitude of R_{out} has positive and that of R_{side} has negative value
 - Both $R_{\text{out},3}^2 / R_{\text{side},0}^2$ $R_{\text{side},3}^2 / R_{\text{side},0}^2$ has small centrality dependence
- ✦ **ESE applies to HBT measurements w.r.t. 3rd order event plane**
 - First measurement of q_3 selection + HBT
 - No significant change can be found in relative amplitudes of R_{out} and R_{side} with q_3 selection

Conclusion

- ✦ **Azimuthal angle dependence of HBT relative to 2nd-order event plane**
 - ✦ Initial elliptic shape strongly diluted, but out-plane elongated elliptic shape still remains at freeze-out
 - ✦ Final eccentricity enhanced(suppressed) with q_2 ($\epsilon_2^{\text{initial}}$) selection
 - ✦ q_2 dependence could be different in centrality 20-40%
- ✦ **Azimuthal angle dependence of HBT relative to 3rd order event plane**
 - ✦ $-2R_{\text{out},3}^2 / R_{\text{side},0}^2 > 0$ and $2R_{\text{side},3}^2 / R_{\text{side},0}^2 < 0$ in all centrality. This will constrain the freeze-out parameters
 - ✦ Relative amplitudes of HBT radii w.r.t. Ψ_3 did not change with q_3 selection. it might indicate freeze-out triangular shape saturates

Back up

Event plane resolution

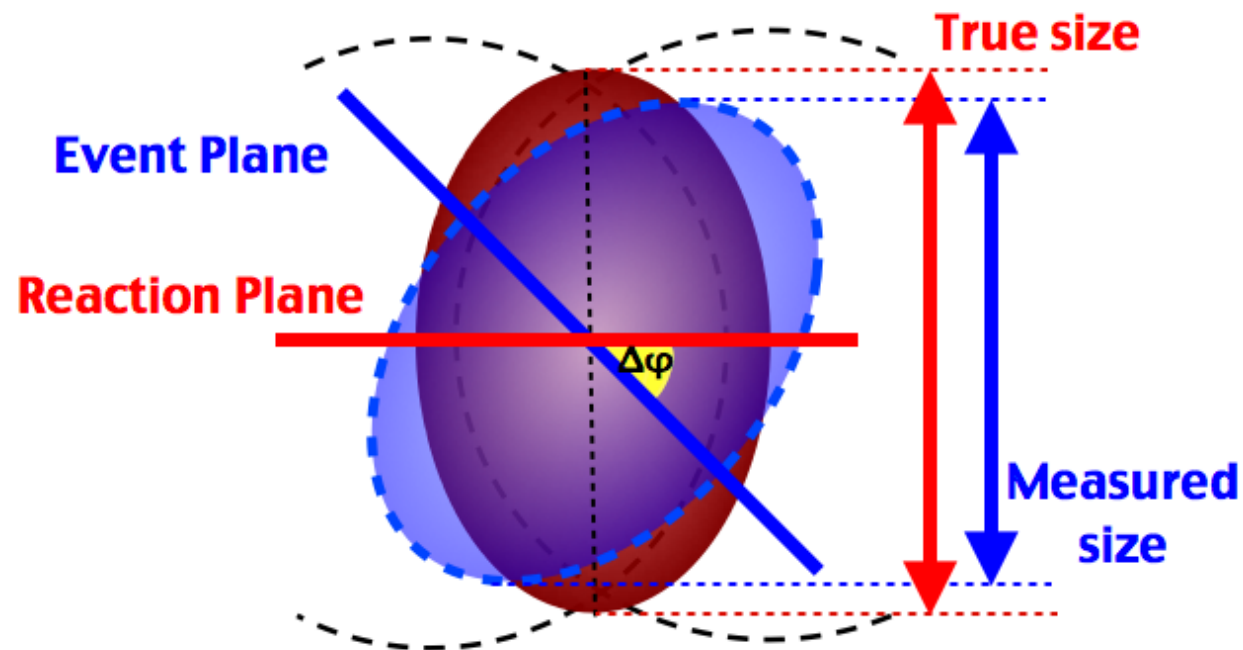
■ Event Plane Resolution Correction (Phys. Rev. C66, 044903 (2002))

$$N(q, \phi_j) = N_{exp}(q, \phi_j) + 2 \sum_{n=1}^{n_{bins}} \xi_{n,m}(\Delta) [N_{c,n}^{exp}(q) \cos(n\phi_j) + N_{s,n}^{exp}(q) \sin(n\phi_j)]$$

$$N_{c,n}^{exp}(q) \cos(n\phi_j) = \langle N_{exp}(q, \phi_j) \cos(n\phi) \rangle = \frac{1}{n_{bins}} \sum_{n=1}^{n_{bins}} N_{exp}(q, \phi_j) \cos(n\phi_j)$$

$$N_{s,n}^{exp}(q) \sin(n\phi_j) = \langle N_{exp}(q, \phi_j) \sin(n\phi) \rangle = \frac{1}{n_{bins}} \sum_{n=1}^{n_{bins}} N_{exp}(q, \phi_j) \sin(n\phi_j)$$

$$\xi_{n,m}(\Delta) = \frac{n\Delta/2}{\sin(n\Delta/2) \langle \cos(n(\Psi_n^m - \Psi_n^{true})) \rangle} \rightarrow \text{event plane resolution}$$



- correction for q-distribution with EP resolution

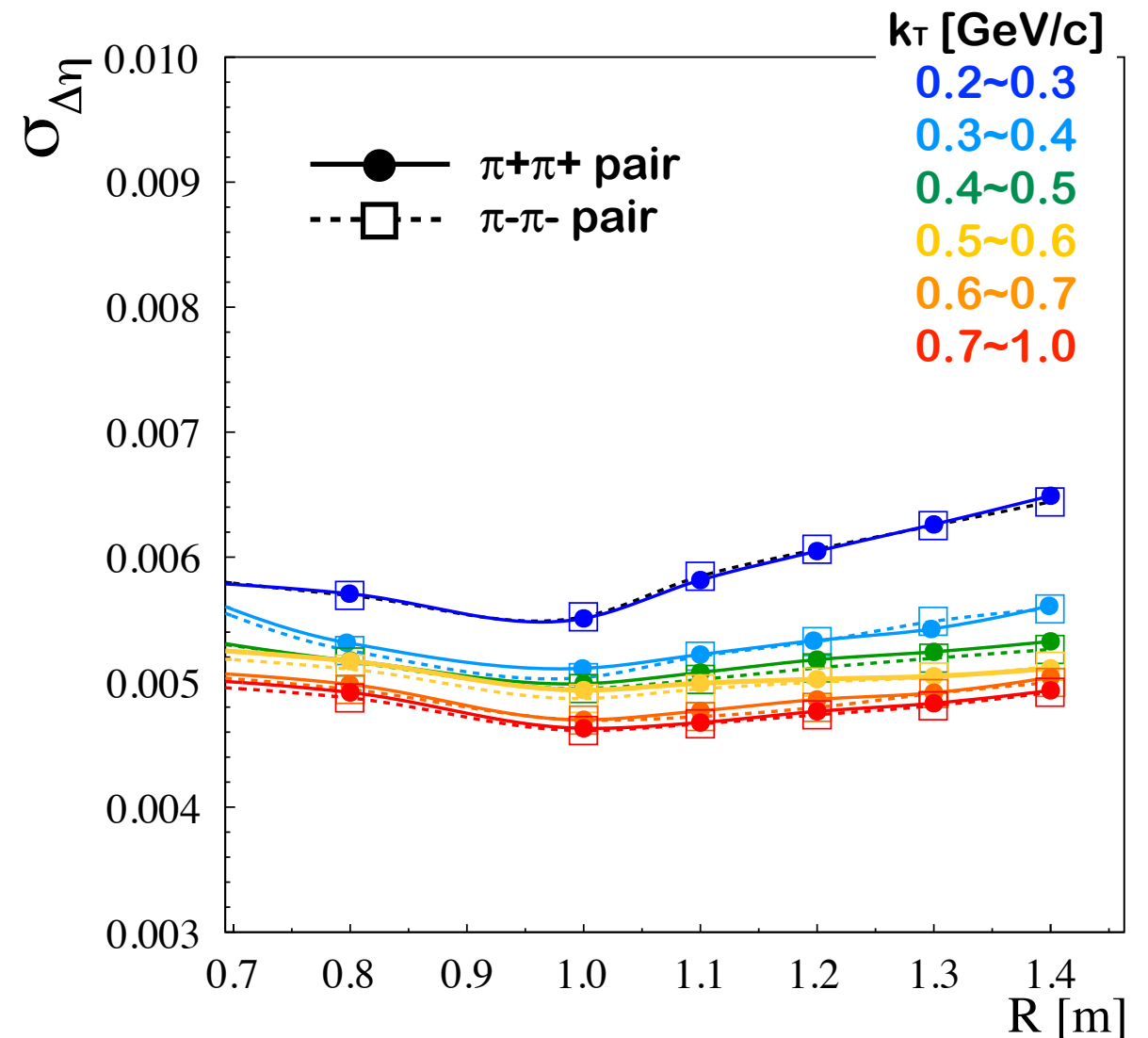
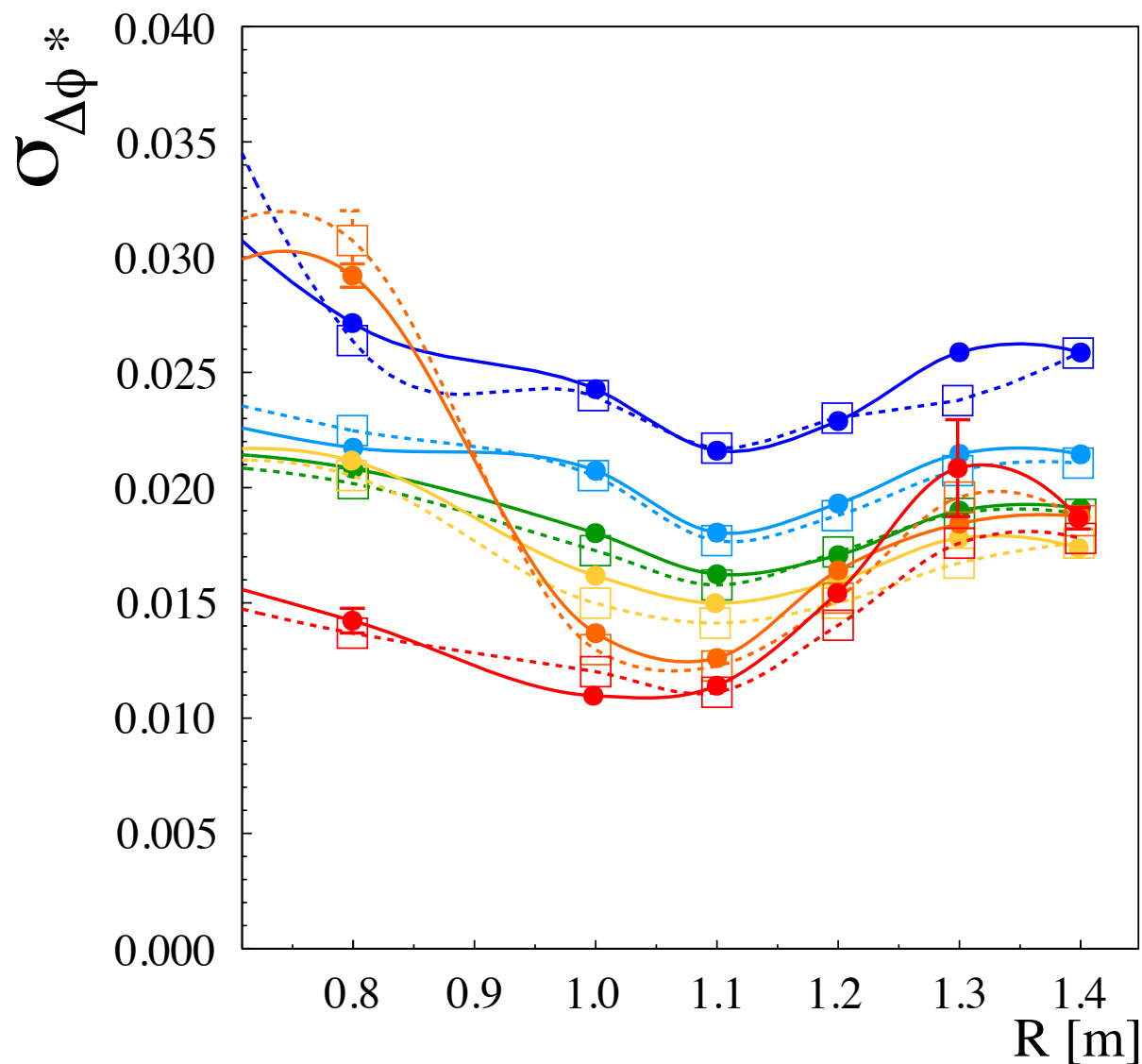
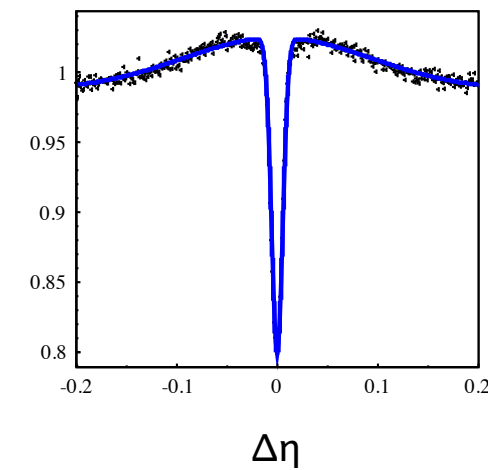
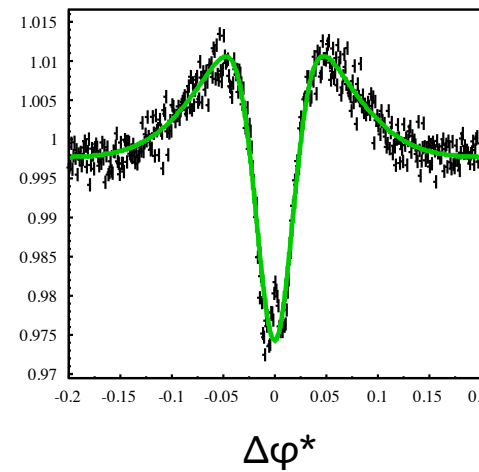
Angular distance in $\Delta\varphi^* \Delta\eta$

• Optimized Pair cut

■ $\Delta\varphi^*, \Delta\eta$ cut @ $R = 1.1$ [m]

- 3σ of gaussian cut

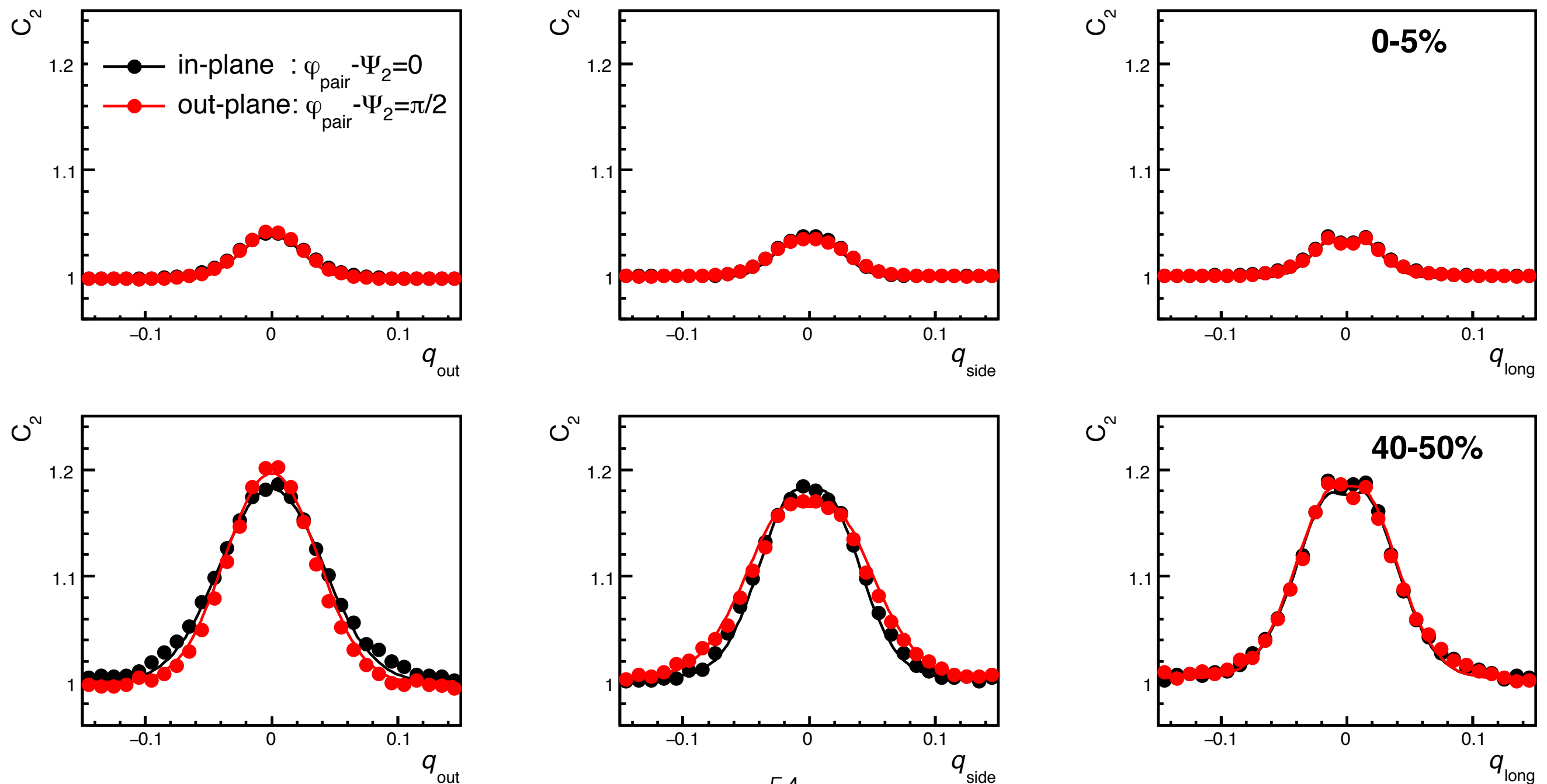
- $|\Delta\varphi^*| < 0.066$ && $|\Delta\eta| < 0.018$



Correlation function (HBT w.r.t. Ψ_2)

◆ 1D projection of 3D correlation functions

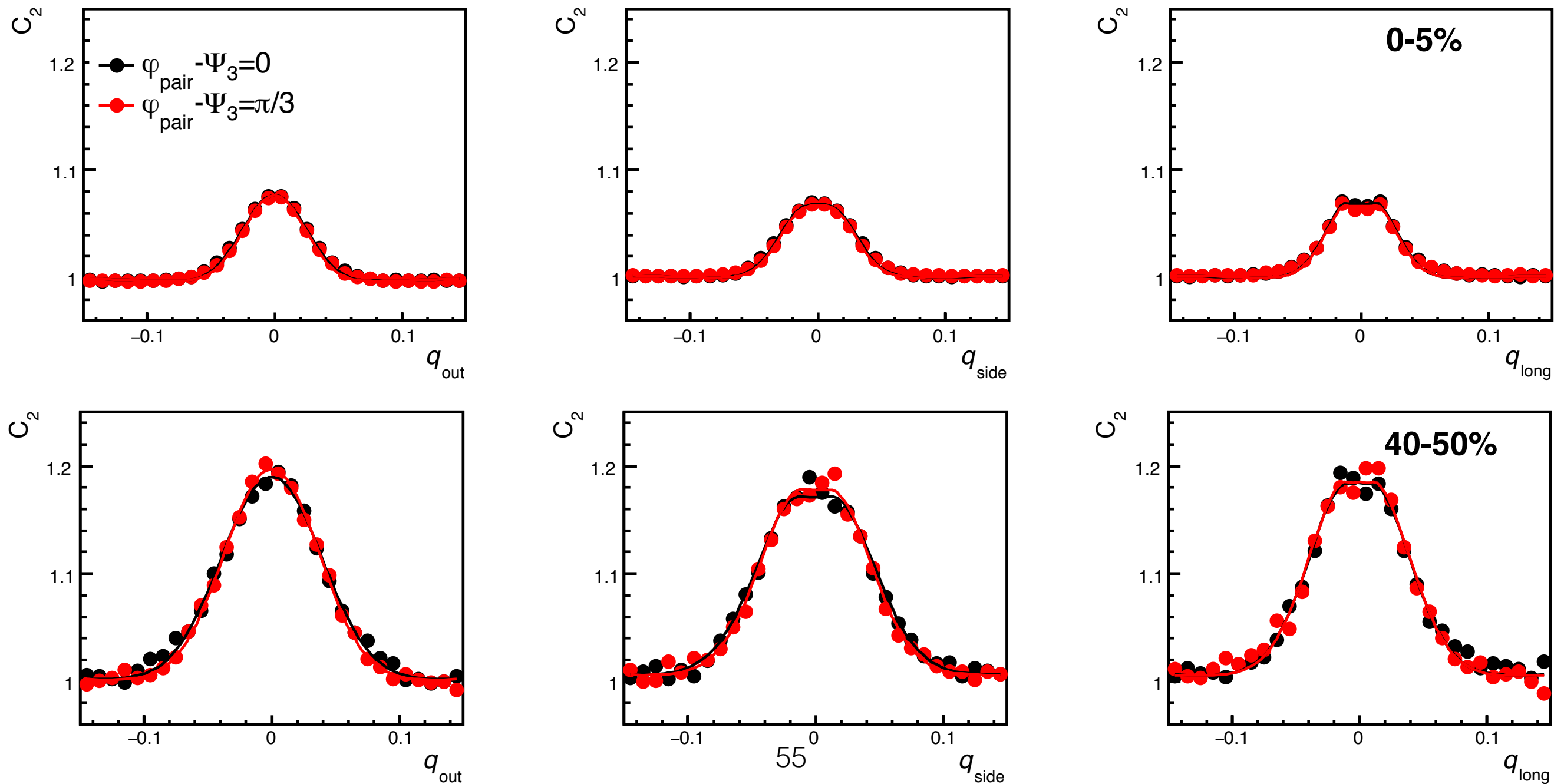
- ▶ Projections of the other component within 50 MeV/c
- ▶ Different HBT radii can be found in peripheral collisions (oscillation of radii)



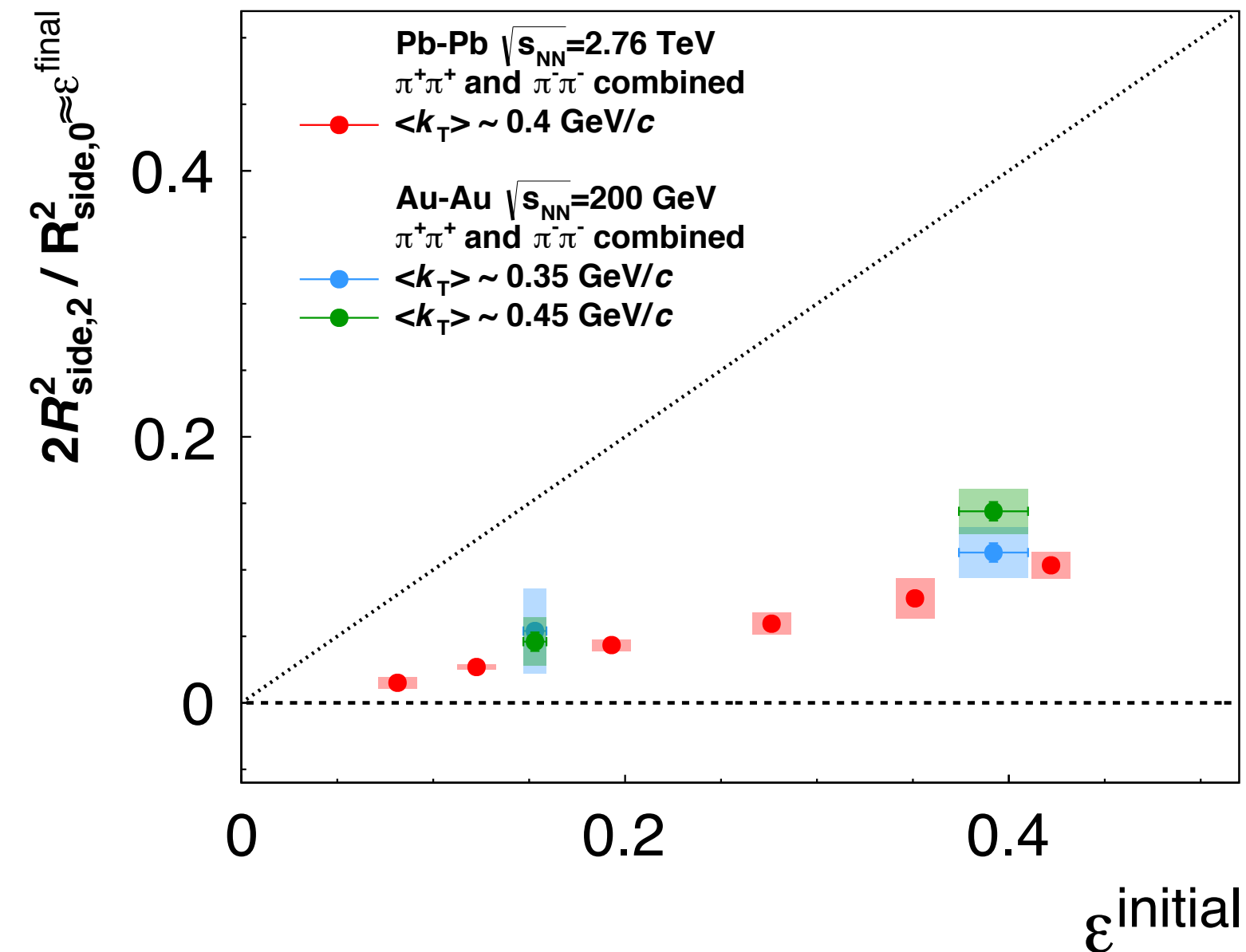
Correlation function (HBT w.r.t. Ψ_3)

◆ 1D projection of 3D correlation functions

- ▶ Projections of the other component within 50 MeV/c
- ▶ Different HBT radii can be found in peripheral collisions (oscillation of radii)

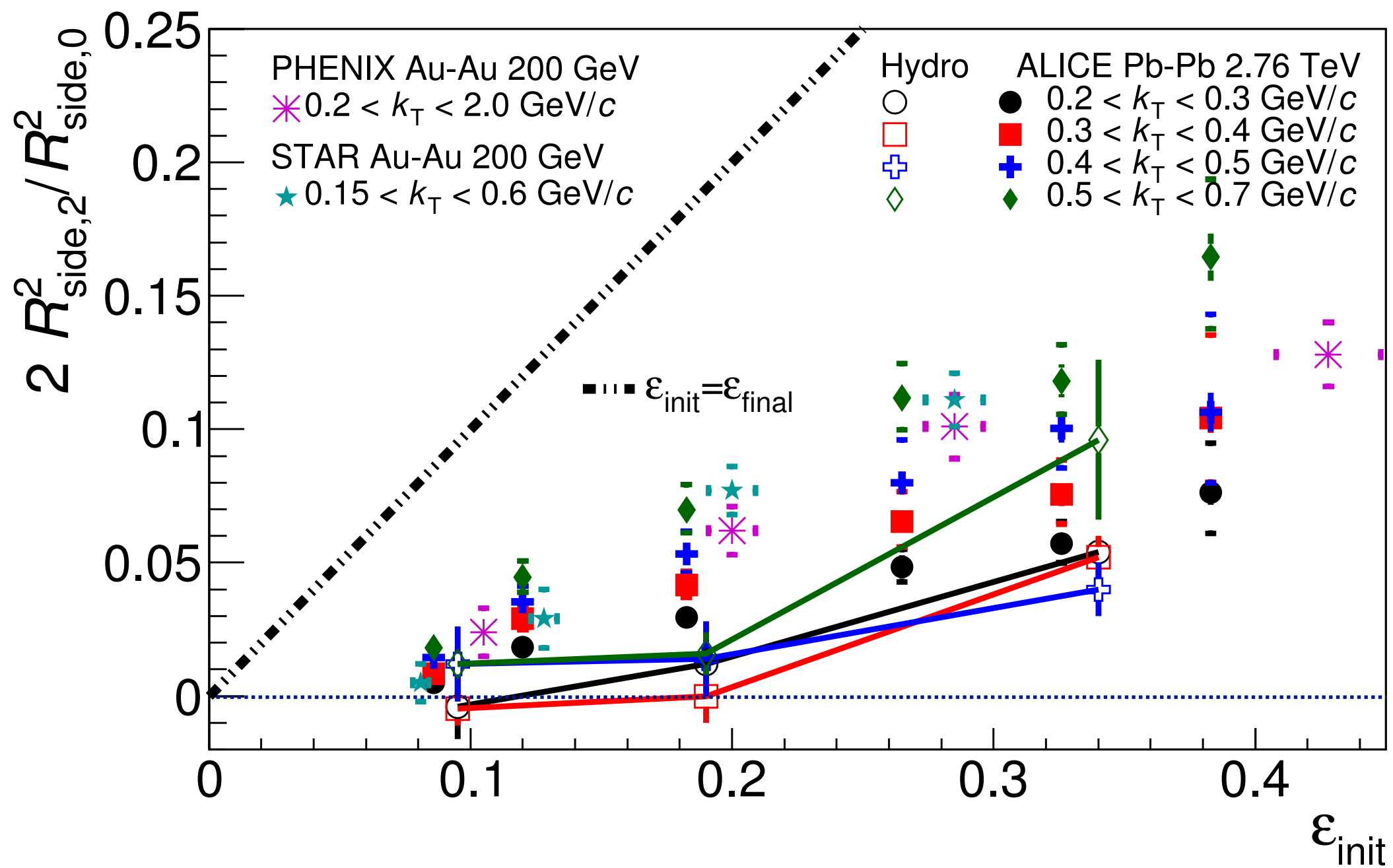


Comparison with result in Au-Au 200GeV

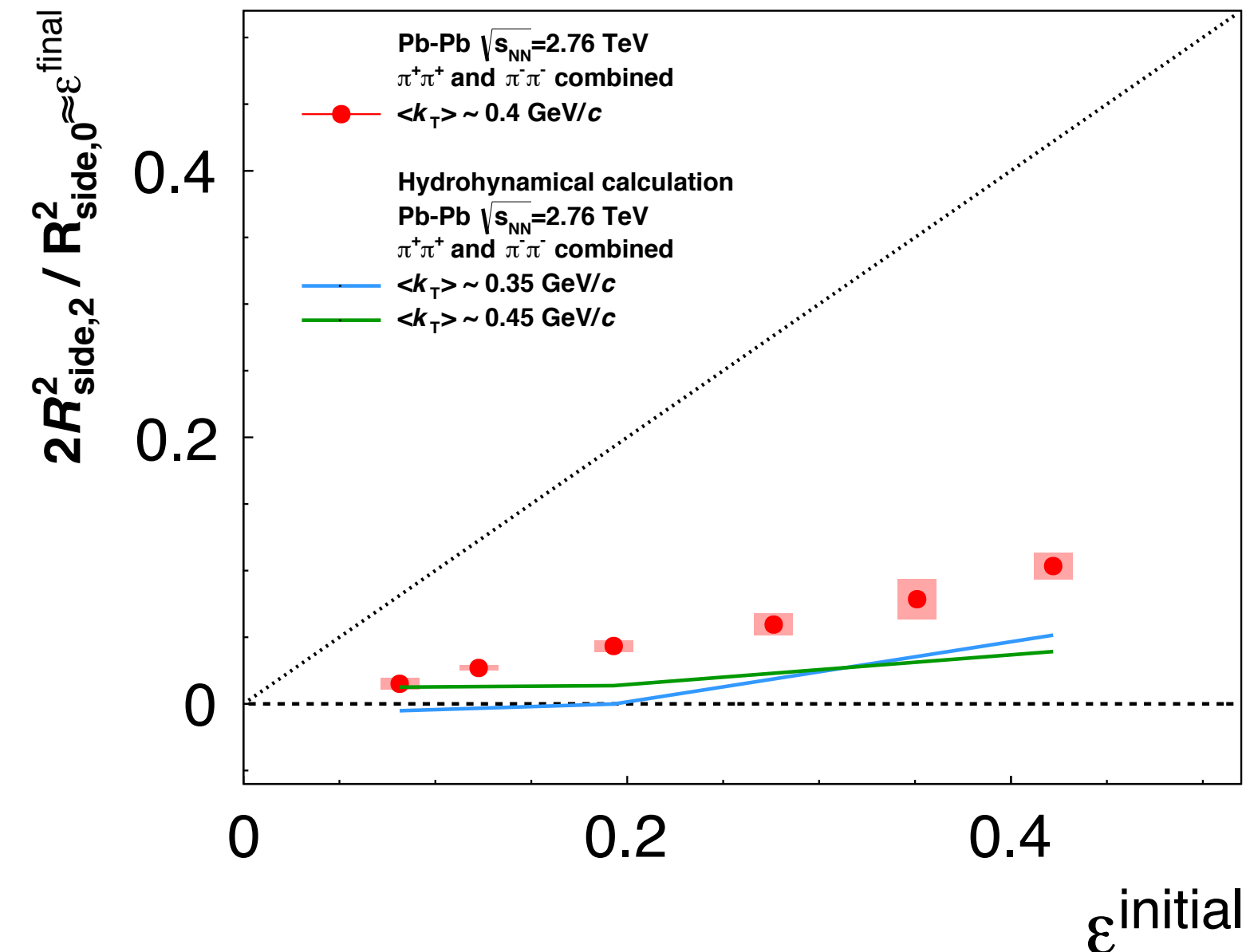


- Dilution effect at LHC is stronger than that at RHIC
 - Due to larger collective flow in LHC energy
- Difference between RHIC and LHC is remarkable at peripheral collisions

k_T dependence of $\varepsilon_2^{\text{final}}$ in Pb-Pb 2.76 TeV



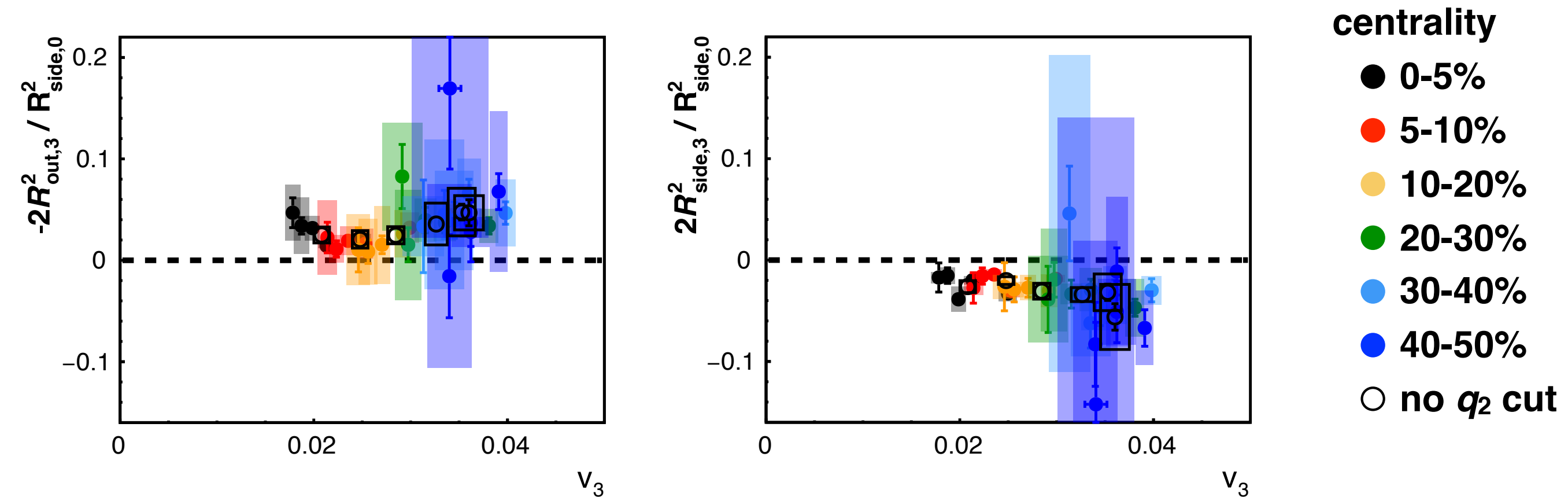
Comparison with hydrodynamical calculation



- Hydrodynamical calculation is qualitatively consistent to data
- Final eccentricity at freeze-out of hydrodynamical prediction is slightly smaller than experimental result
- Difference between model and data is larger in peripheral than central collisions

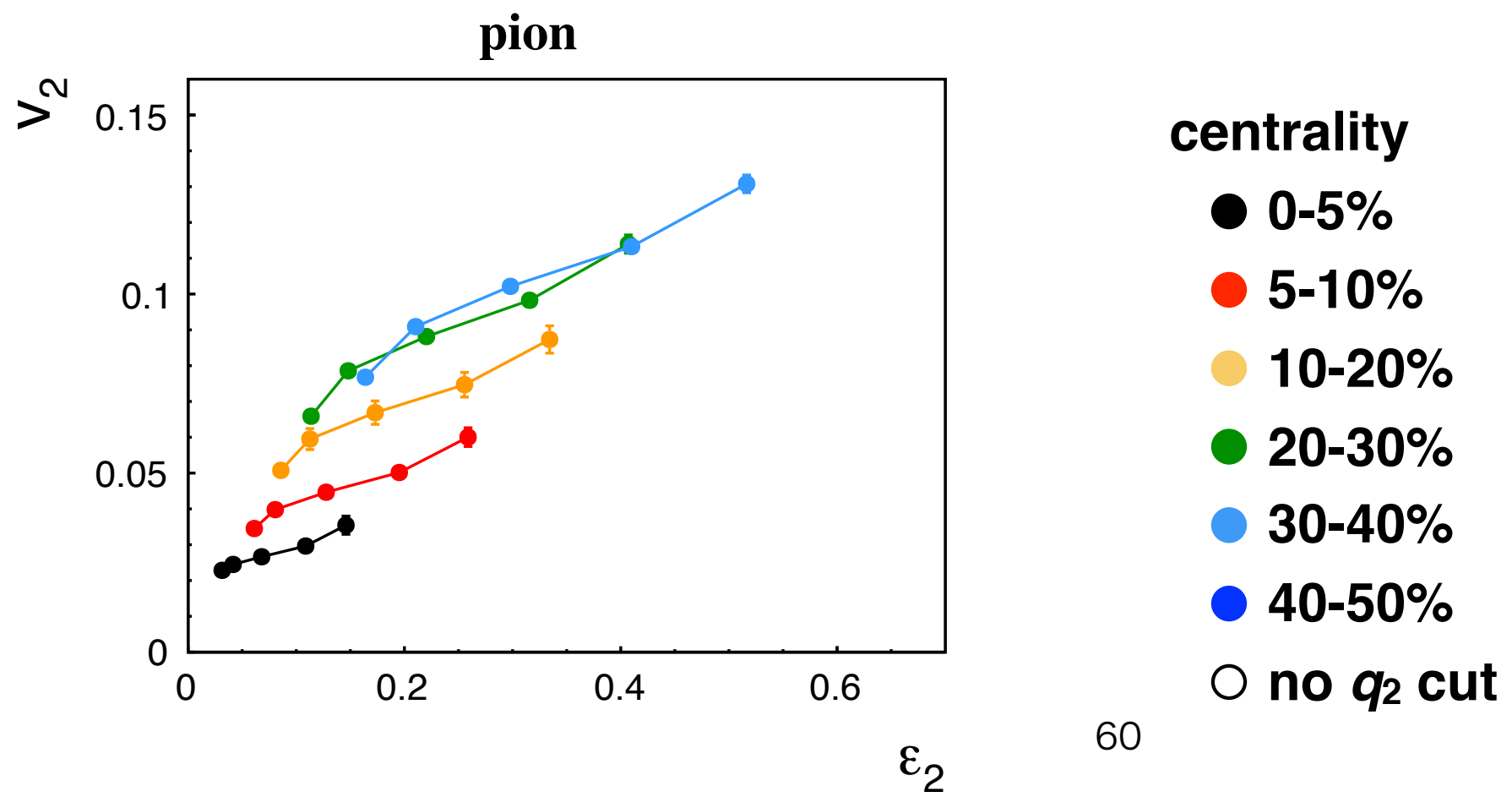
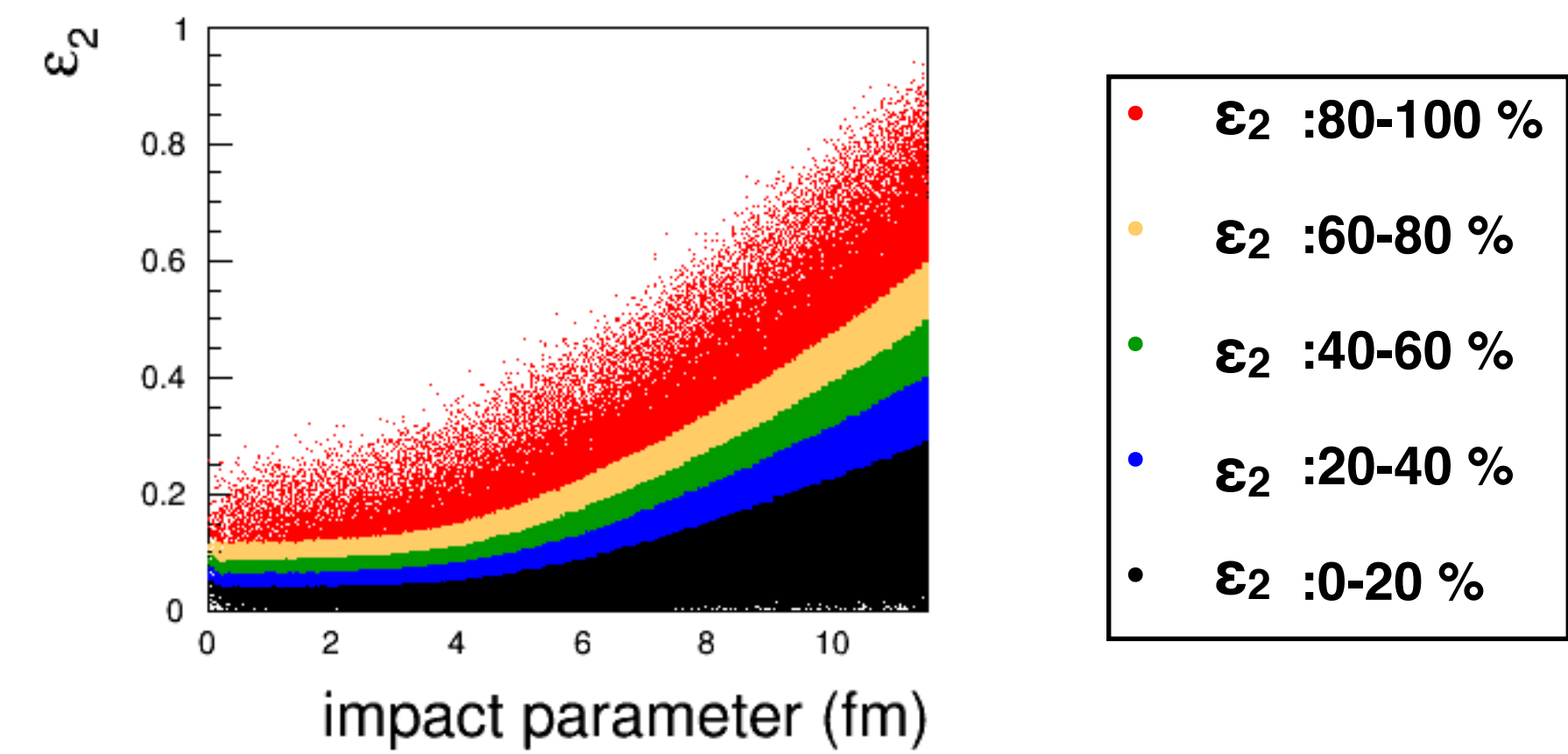
v_3 scaling to Relative amplitude of HBT radii

- ✓ Difference of event by event initial geometry fluctuation can not be reflected to “centrality”
- ✓ More sensitive probe is necessary !

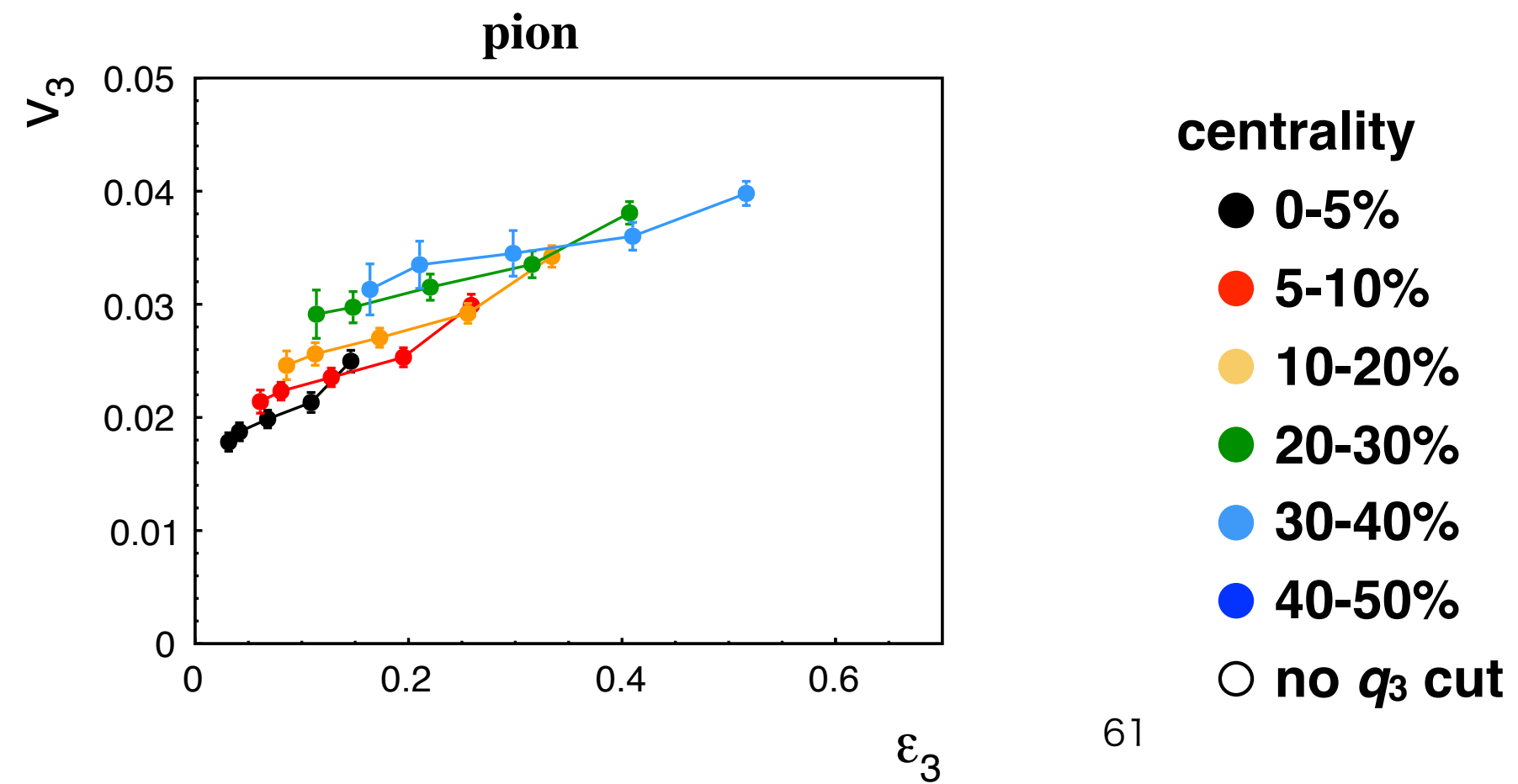
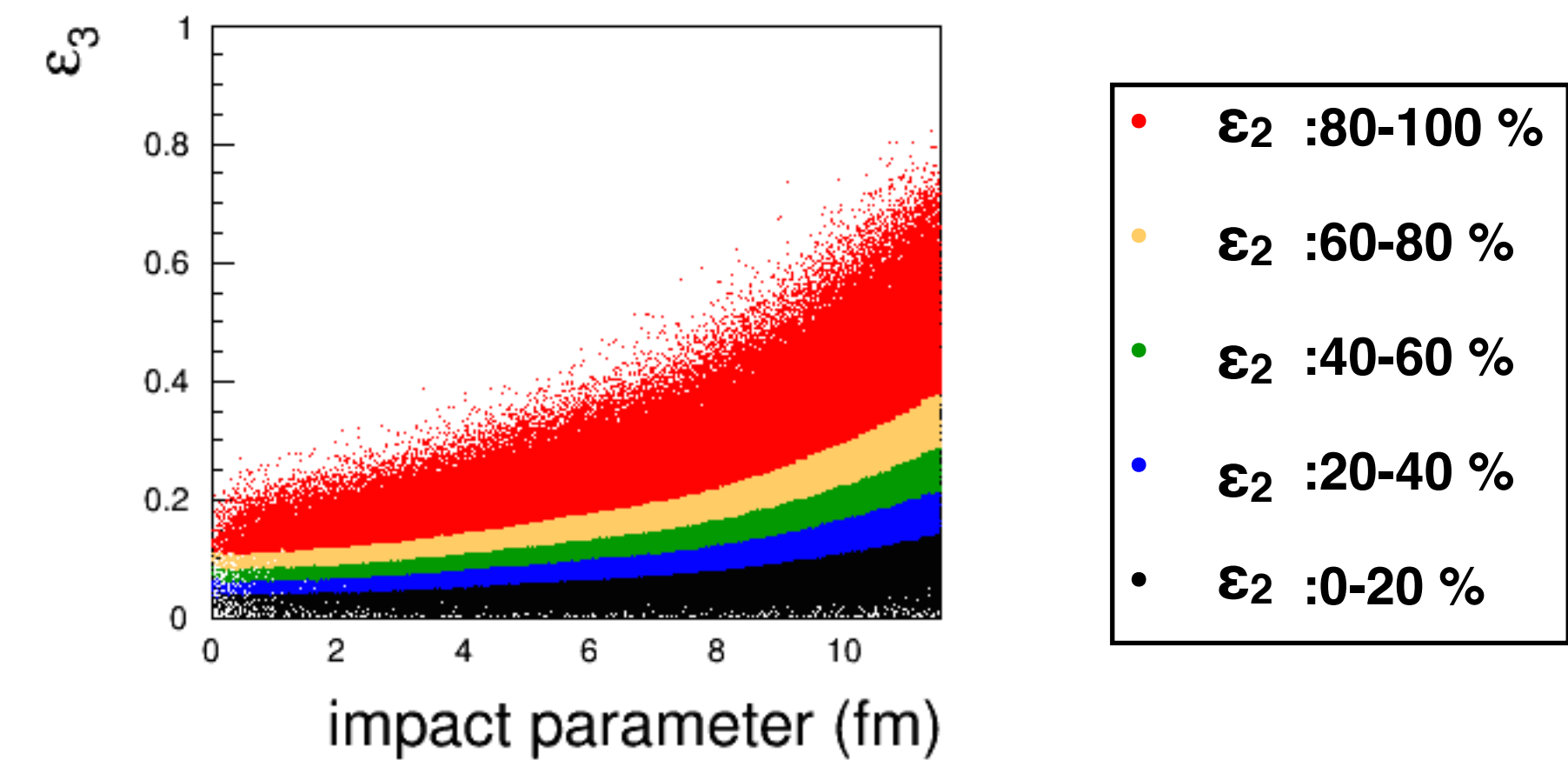


- ✓ Both relative amplitudes of R_{out} and R_{side} are scaled with v_3
- ✓ Non linear correlation between $-2R_{out,3}^2 / R_{side,0}^2$ and v_3
- ✓ No significant correlation can be found between $-2R_{side,2}^2 / R_{side,0}^2$ and v_3

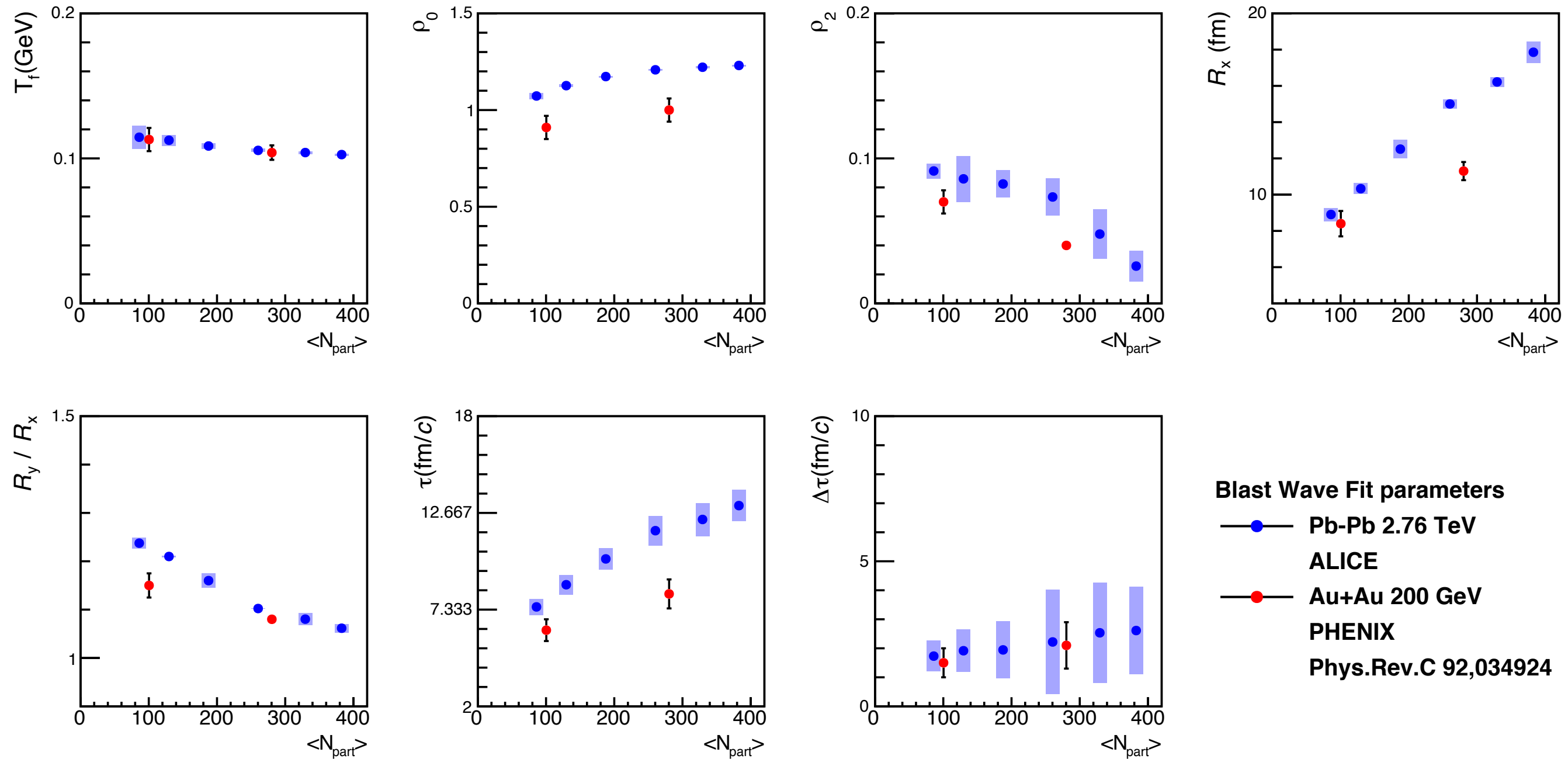
Correlation between v_2 and $\epsilon_2^{\text{initial}}$ (Glauber)



Correlation between v_3 and $\epsilon_3^{\text{initial}}$ (Glauber)

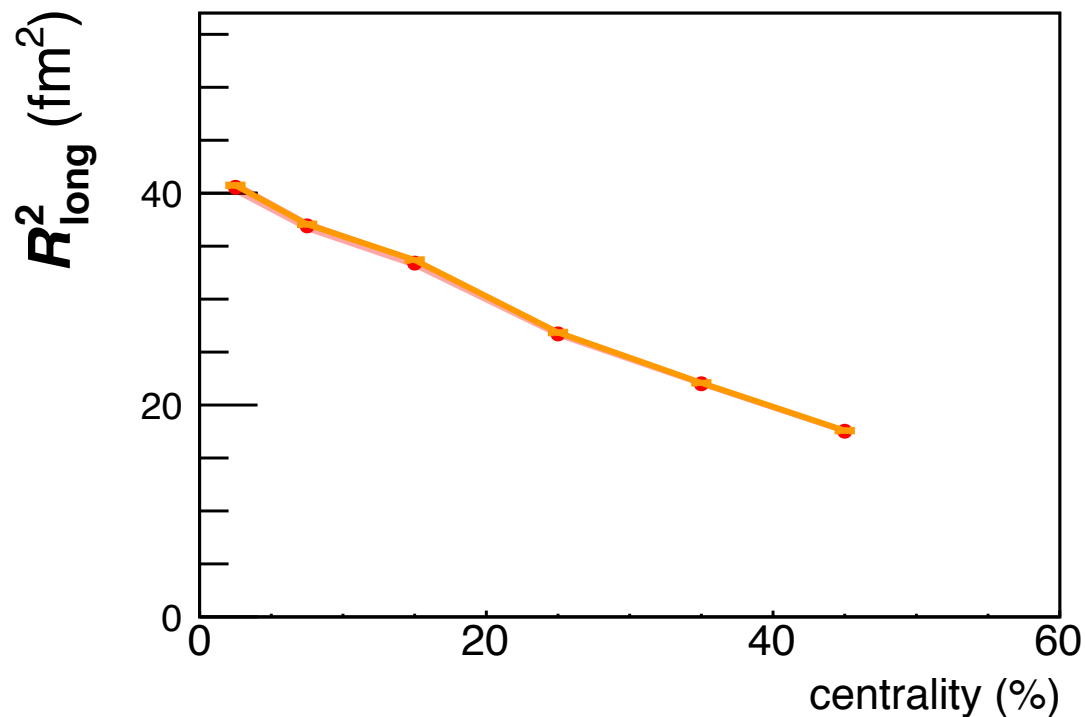
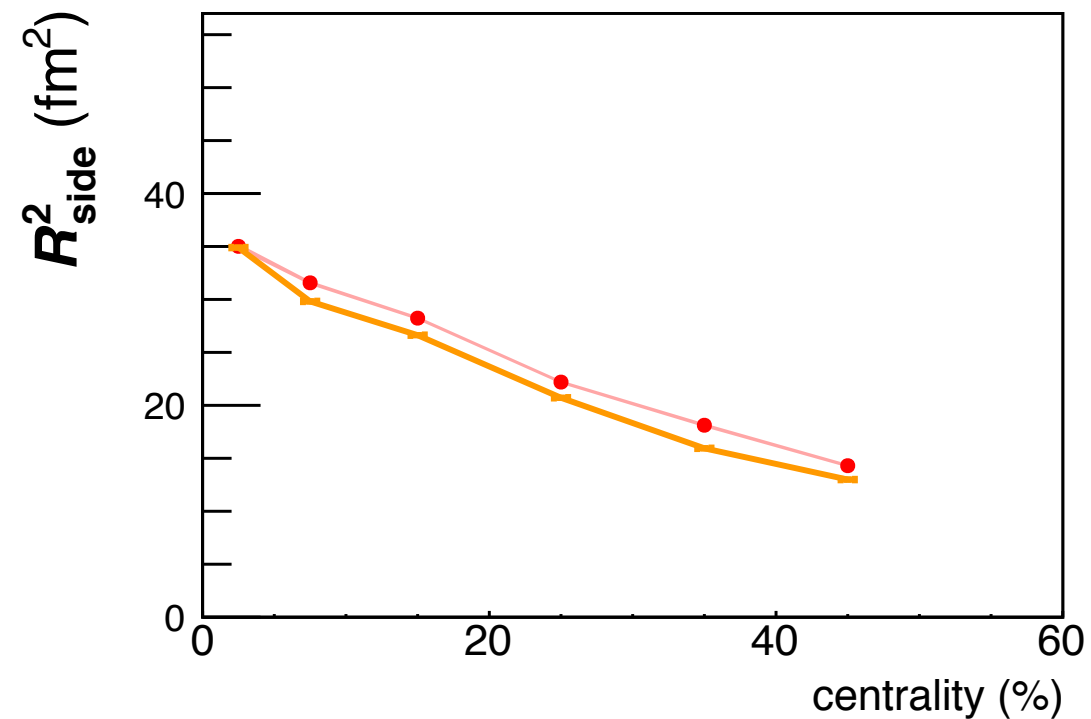
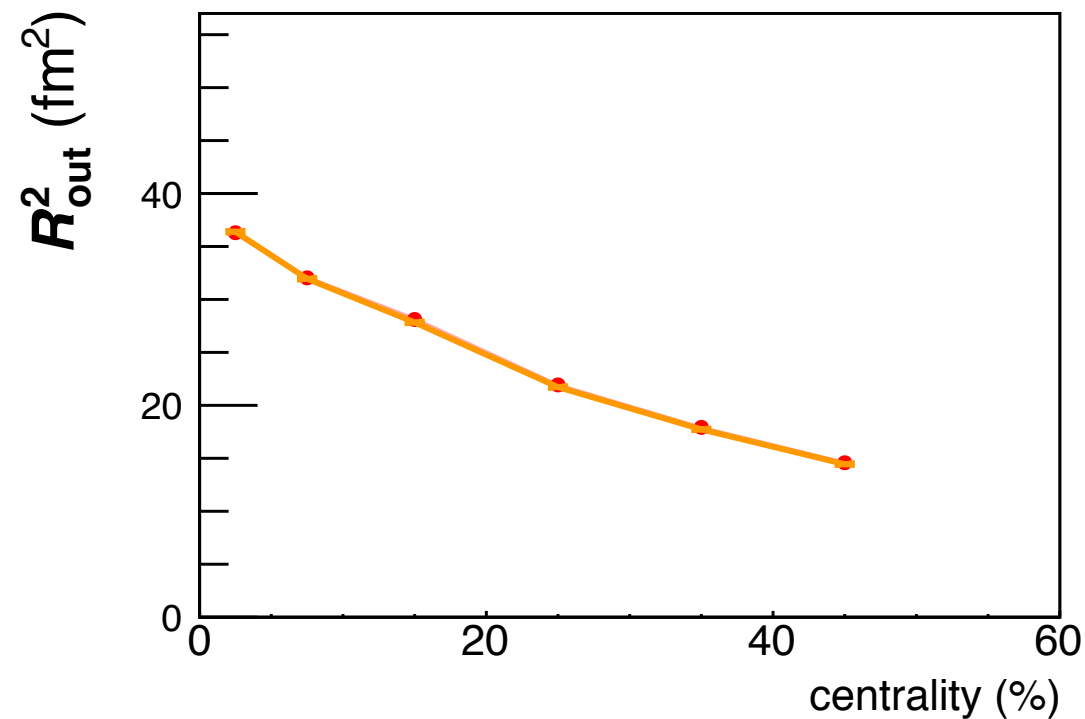


Blast-wave parameters (comparison with PHENIX)



- ◆ Freeze out temperature(T_f), eccentricity(R_y/R_x), duration($\Delta\tau$) : ALICE ~ PHENIX
- ◆ Flow velocity (ρ_0 and ρ_2) and system life time : ALICE > PHENIX

Comparison with Blast-wave model (average HBT radii)



Pb-Pb $\sqrt{s_{\text{NN}}}$ =2.76 TeV

$\pi^+\pi^+$ and $\pi^-\pi^-$ combined

k_T : 0.2-1.5 GeV/ c

• Data

— Blast-wave fit

◆ R_{out} and R_{long} of Blast-wave fit is consistent to that of Data

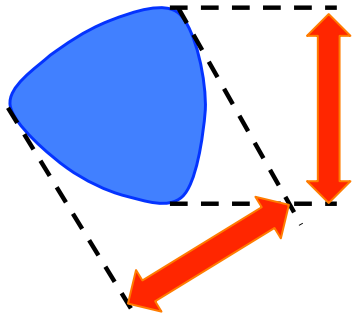
◆ R_{side} of Blast-wave fit is slightly smaller than that of Data

Final source **triangular shape** and HBT w.r.t. Ψ_3

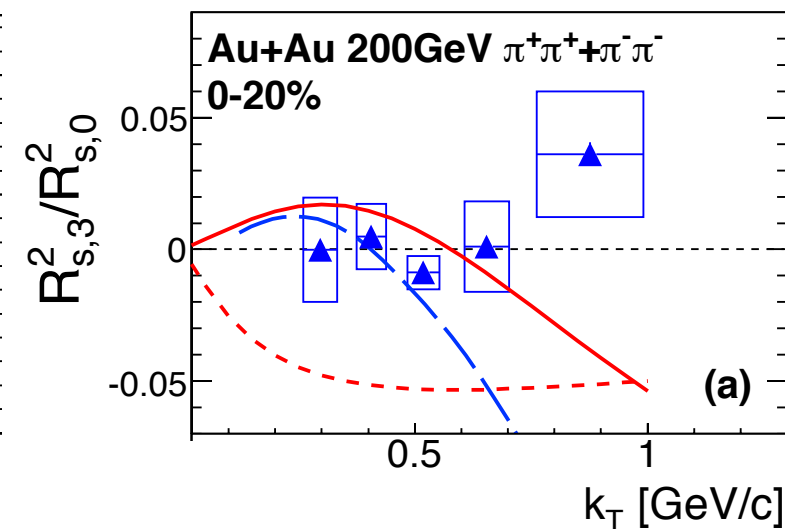
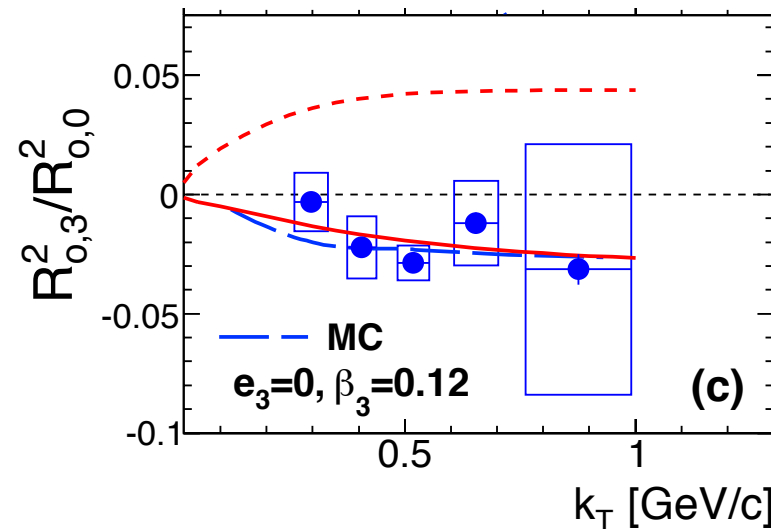
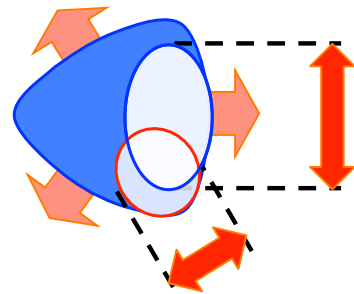
► **Triangularity cannot be directly obtained from HBT w.r.t. Ψ_3**

→ Both triangular flow and geometrical triangularity make 3rd order oscillation of HBT radii

• Static source



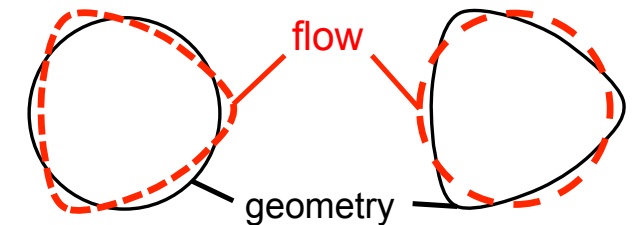
• Dynamic source



► **HBT w.r.t. Ψ_3 in Au+Au 200GeV collisions**

- A. Adare et al., PRL112.222301
- MC simulation of two extreme case
- HBT oscillation could be explained by “deformed flow” at RHIC

— $\bar{\epsilon}_3=0, \bar{v}_3=0.25$ deformed flow
 - - - $\bar{\epsilon}_3=0.25, \bar{v}_3=0$ deformed geometry



PRC88,044914

→ Any hint of sign change of ϵ_3 under larger collective flow at LHC ??

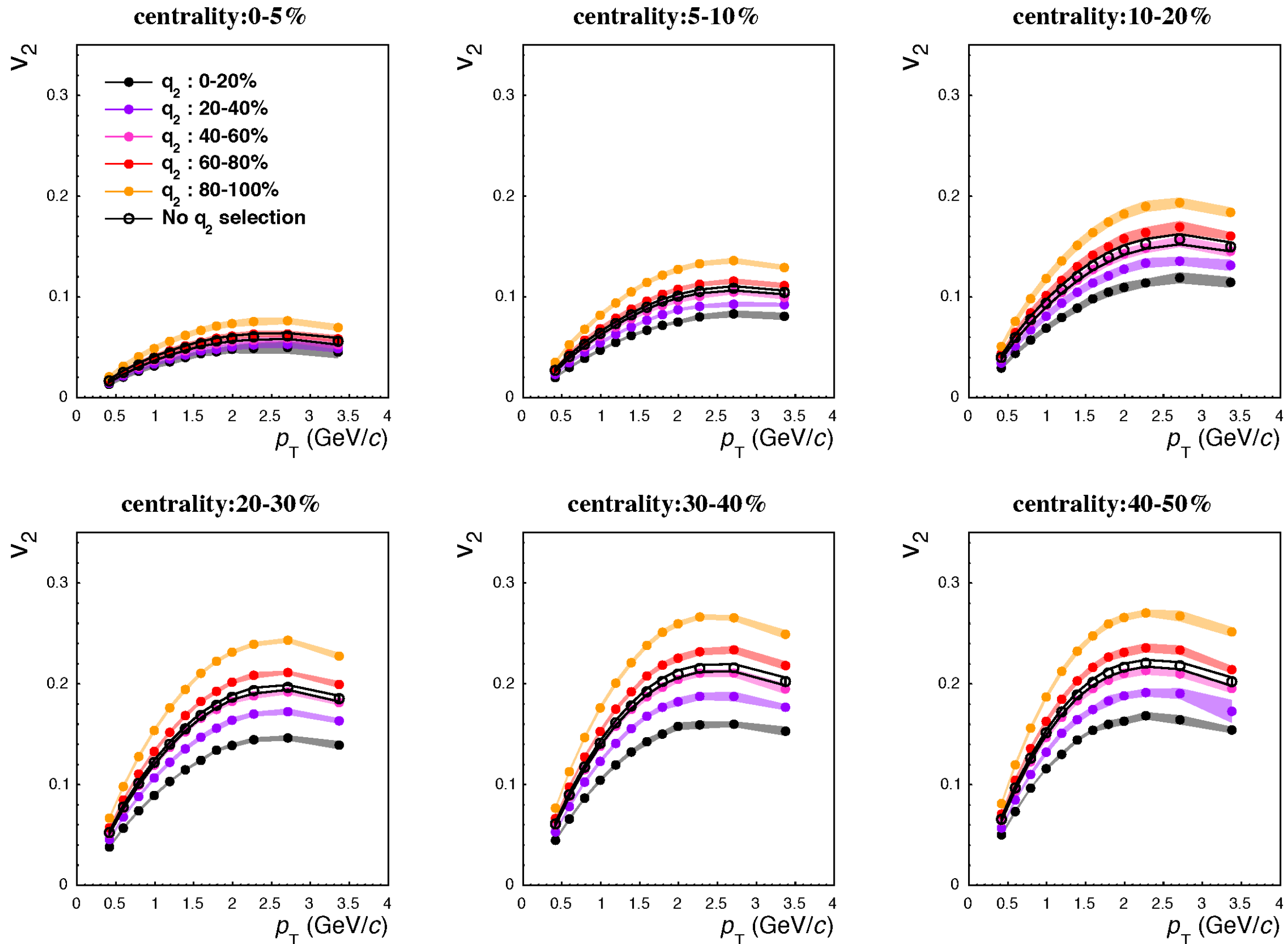
☑ Detailed analysis is necessary for understanding final source triangularity

◆ k_T dependence of Azimuthally differential femtoscopy w.r.t. Ψ_3

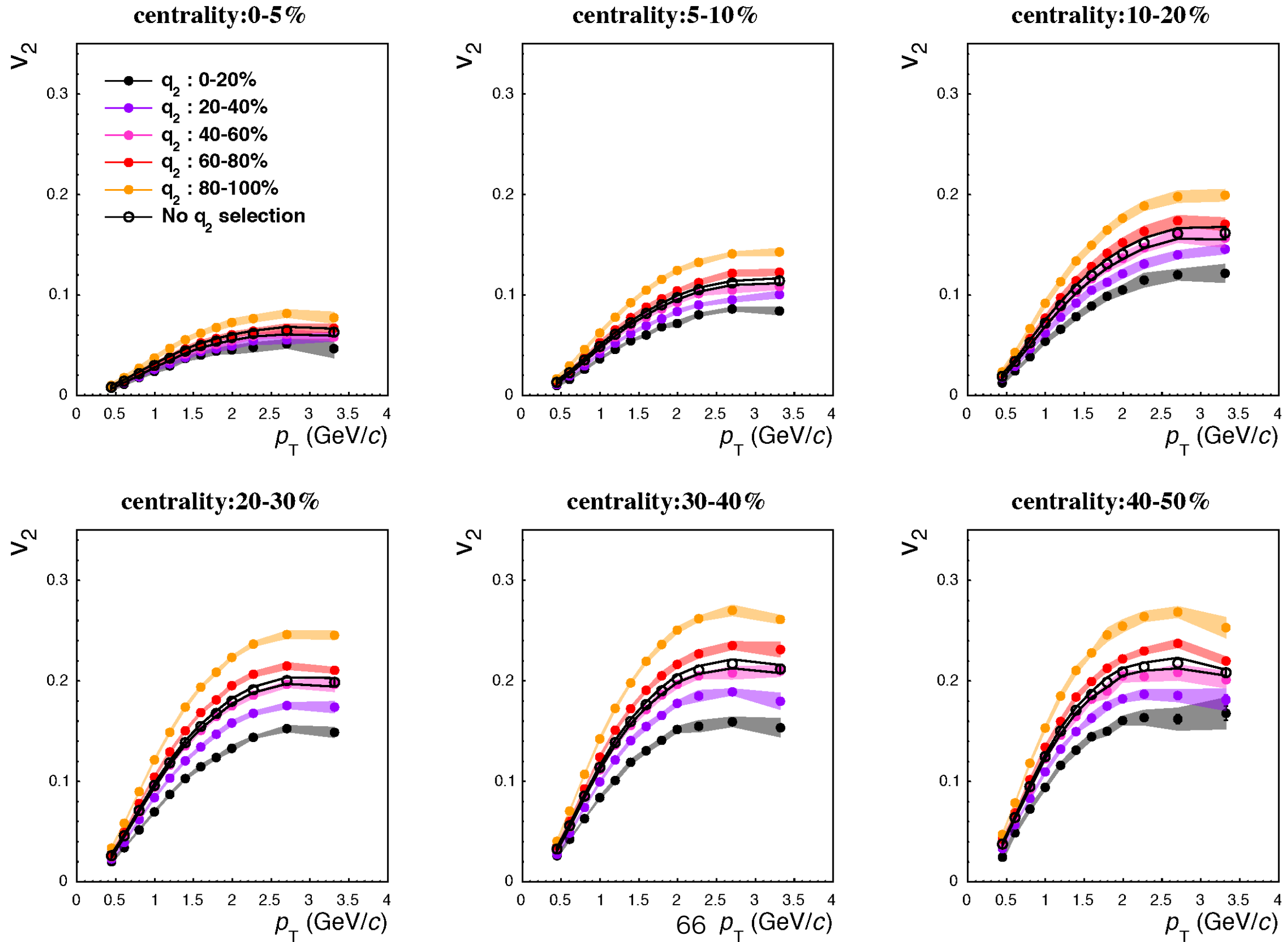
→ **High multiplicity and good E.P. resolution in ALICE Pb-Pb collisions !**

◆ **Direct measurement of correlation between geometrical and flow information**

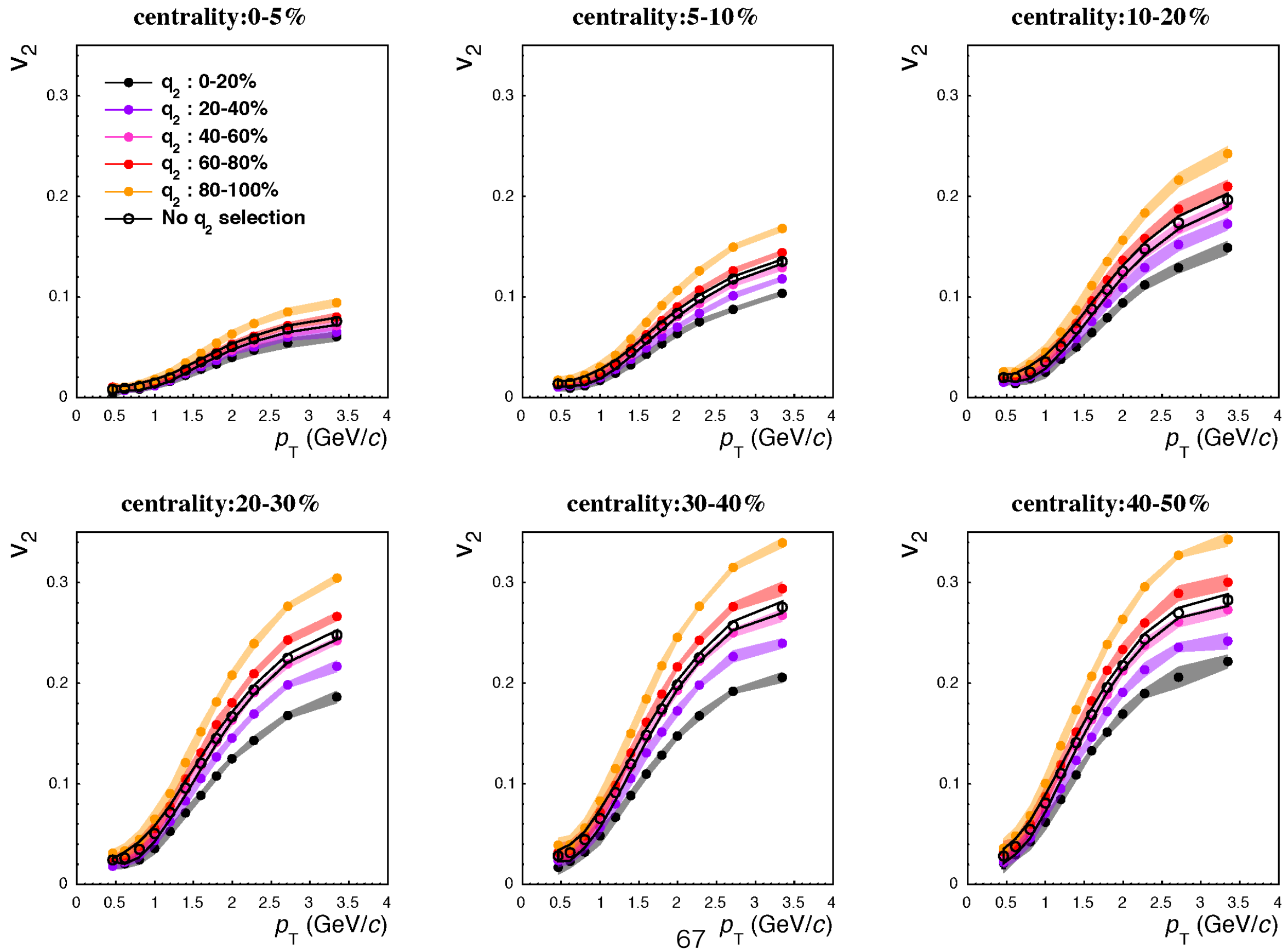
v_2 (p_T dependence) for each 20% q_2 selection (pion)



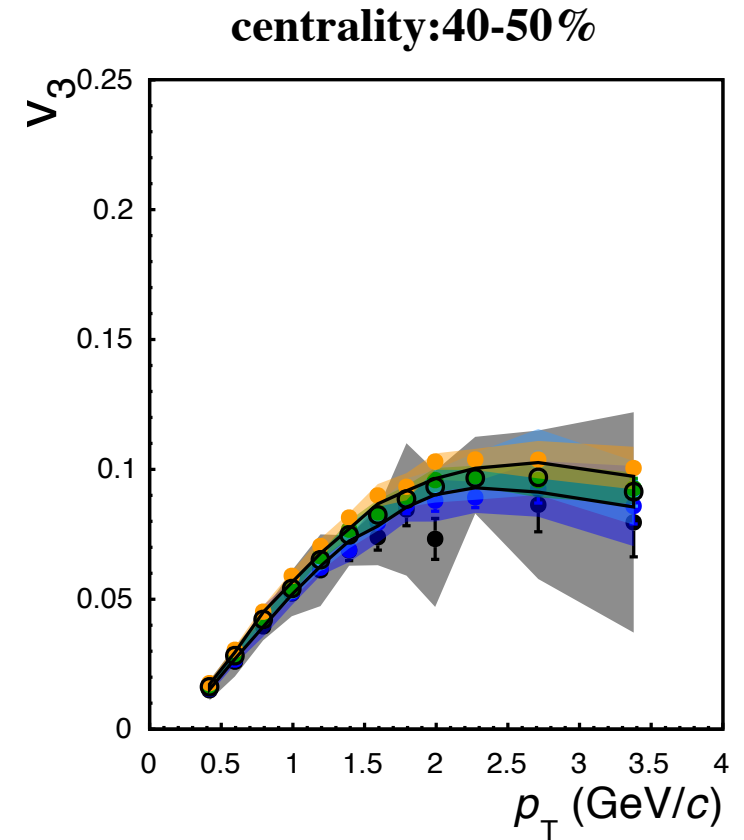
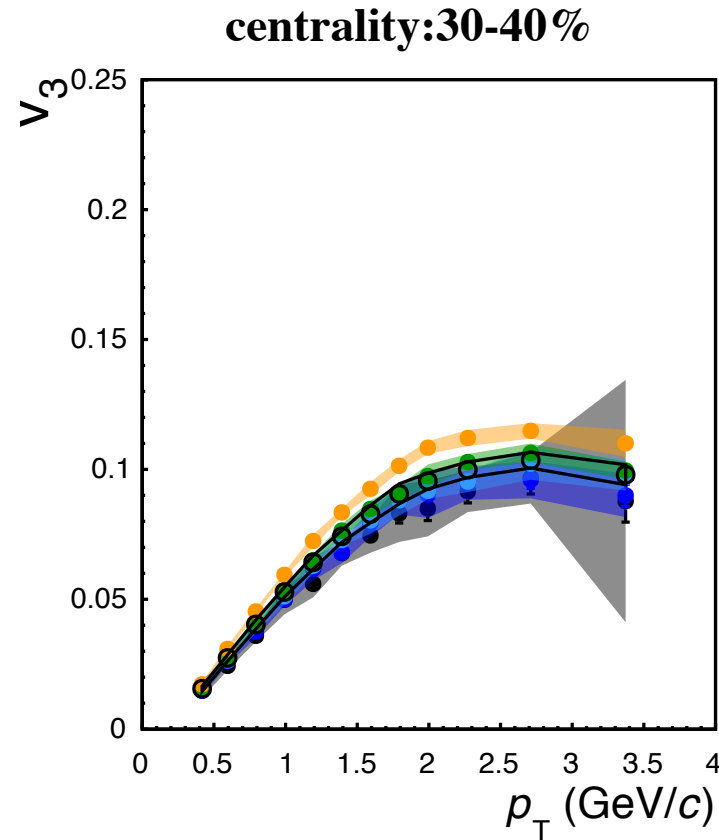
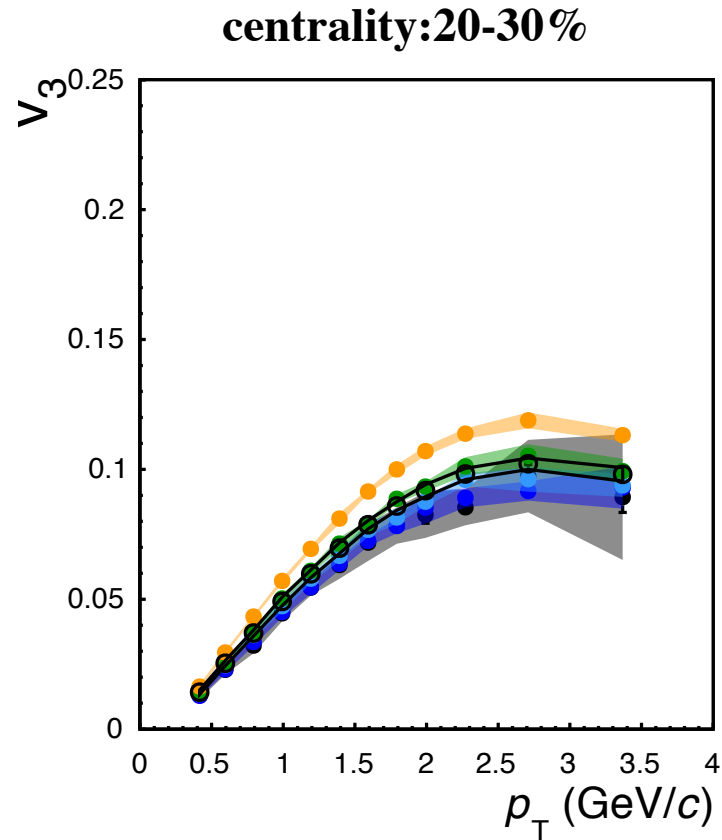
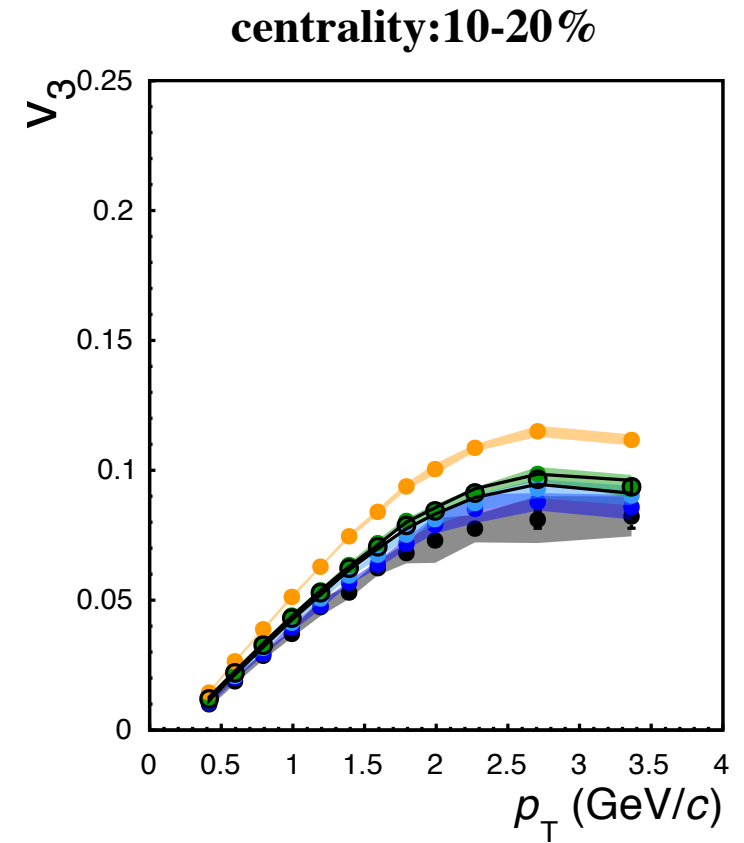
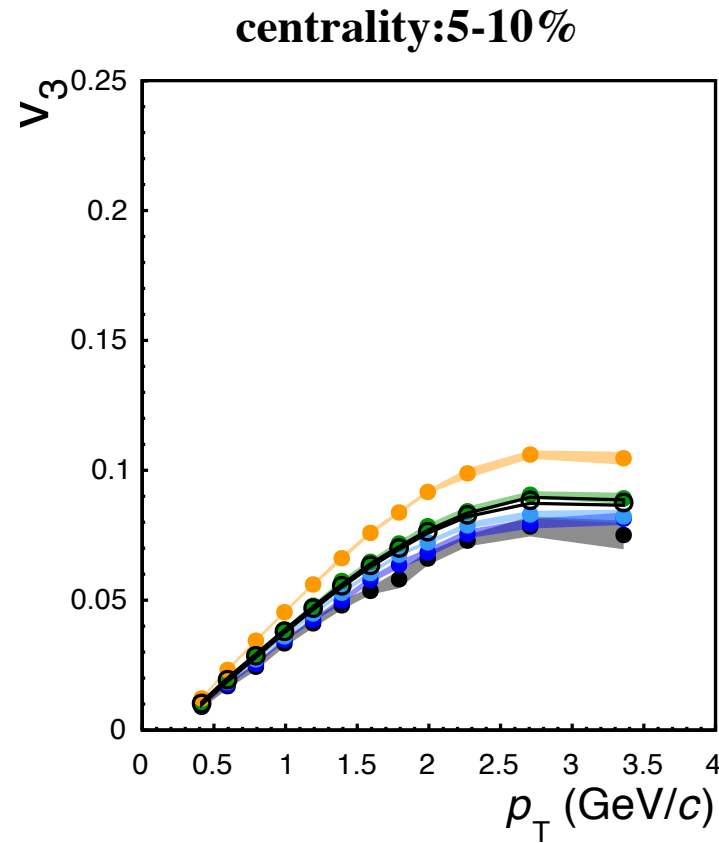
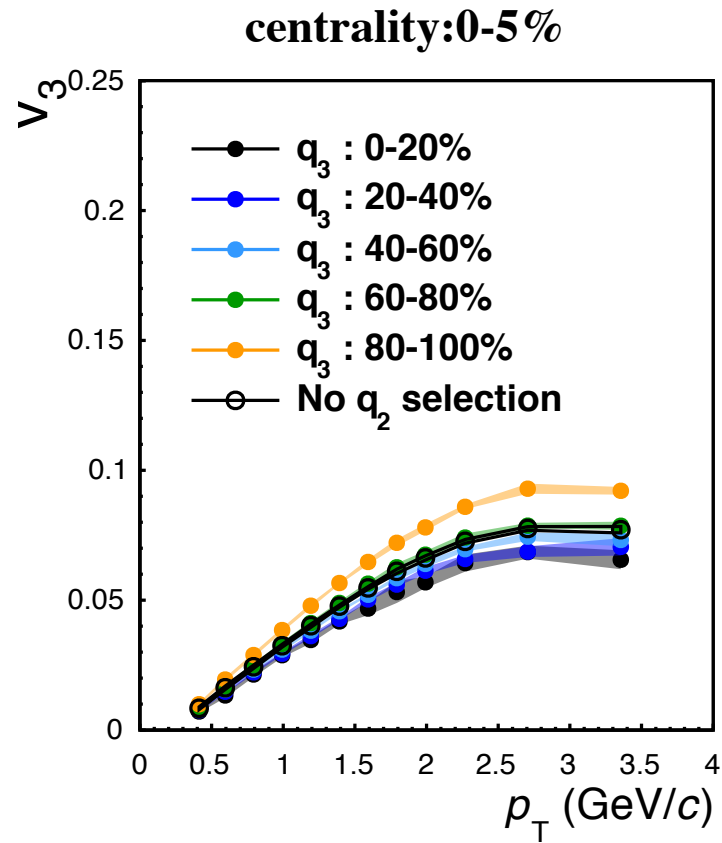
v_2 (p_T dependence) for each 20% q_2 selection (kaon)



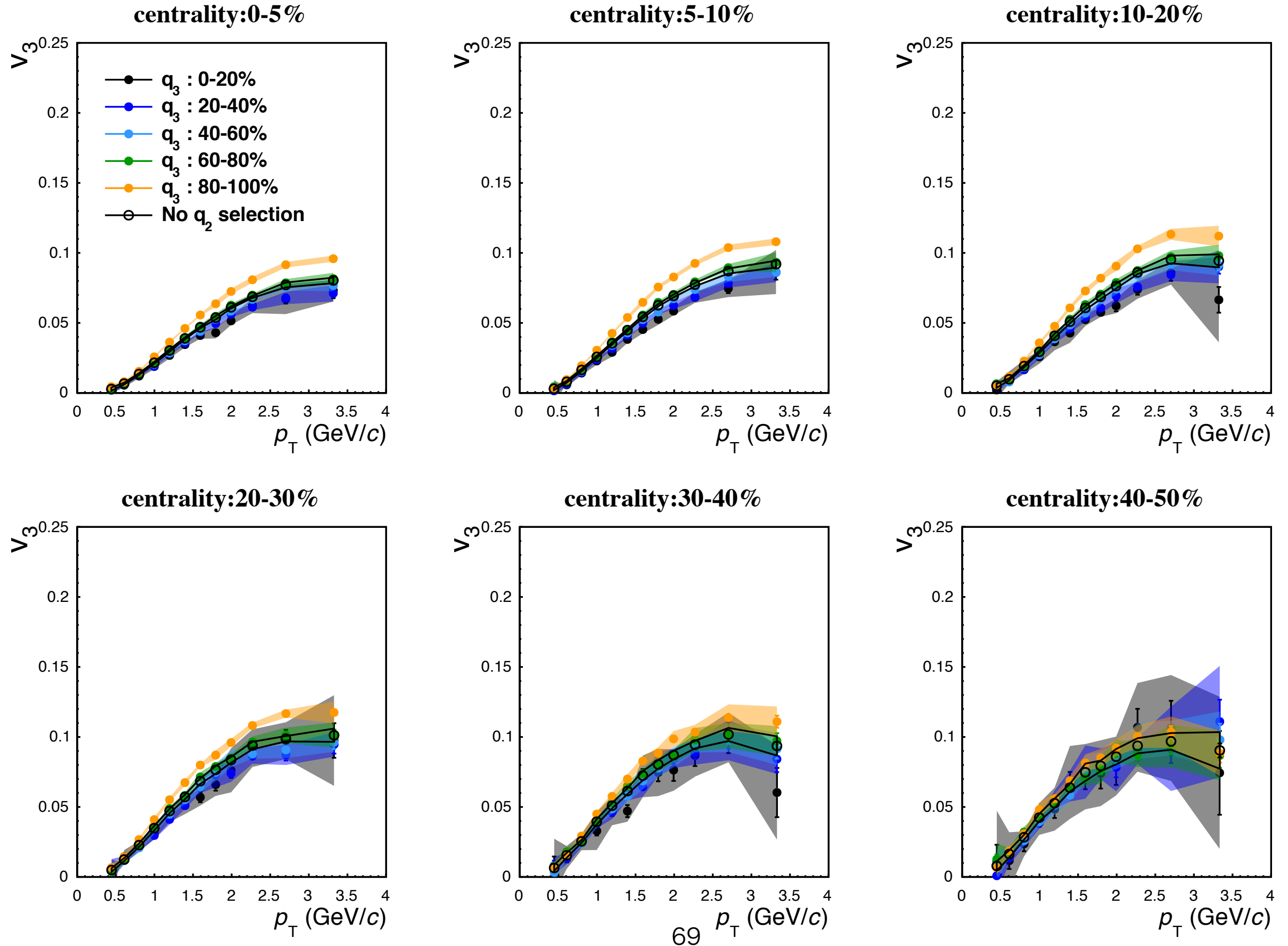
v_2 (p_T dependence) for each 20% q_2 selection (proton)



v_2 (p_T dependence) for each 20% q_2 selection (pion)

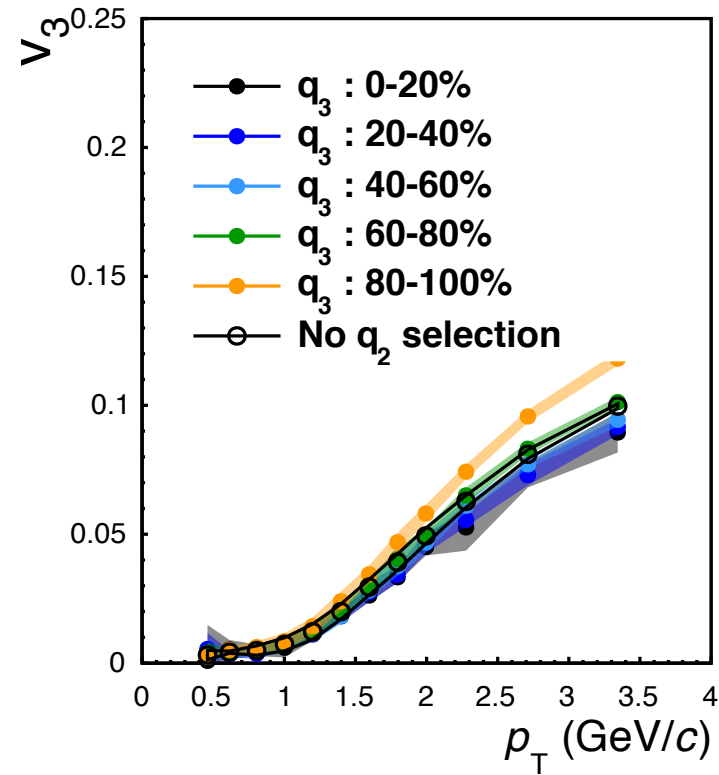


v_2 (p_T dependence) for each 20% q_2 selection (kaon)

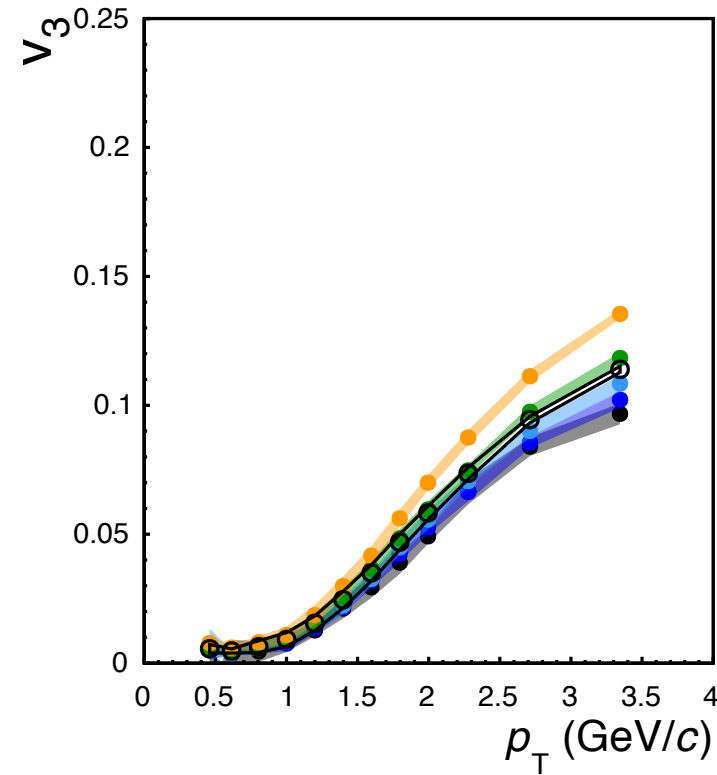


v_2 (p_T dependence) for each 20% q_2 selection (proton)

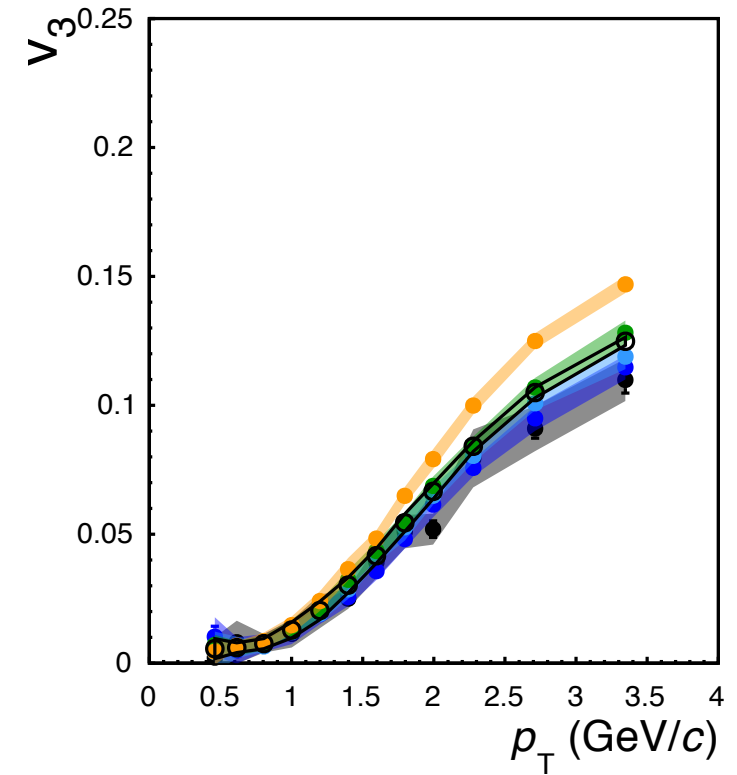
centrality:0-5%



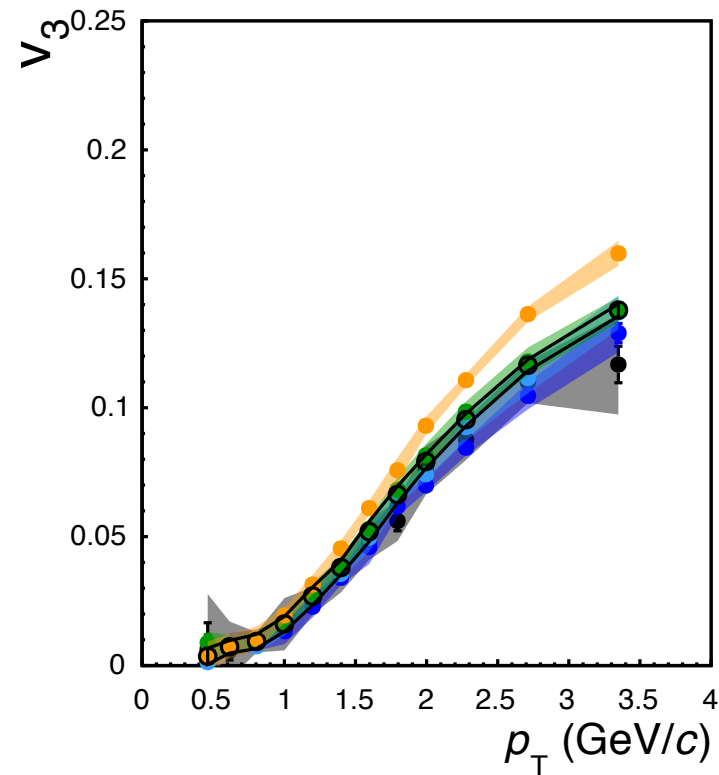
centrality:5-10%



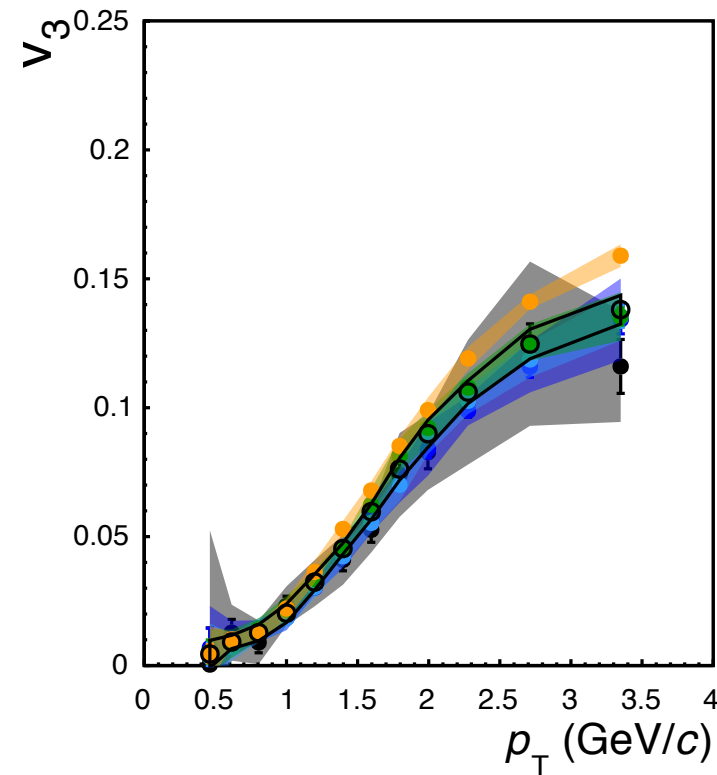
centrality:10-20%



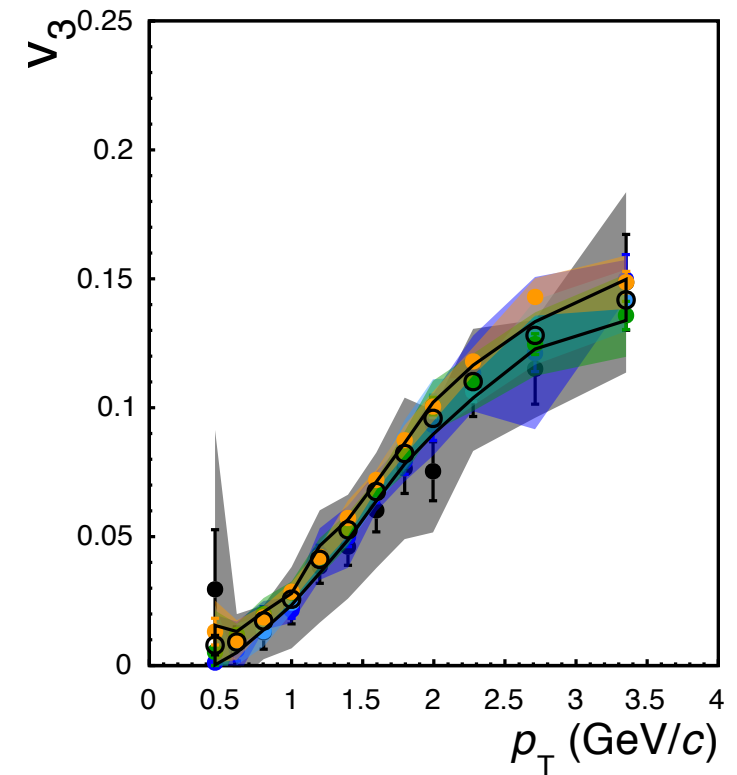
centrality:20-30%



centrality:30-40%



centrality:40-50%



Blast wave fit for π , K, p Spectra

★ Fitting for pT spectra

- positive and negative particle
- π , K, p
- 6 particles pT spectra (simultaneous)

- pion
- Kaon
- proton

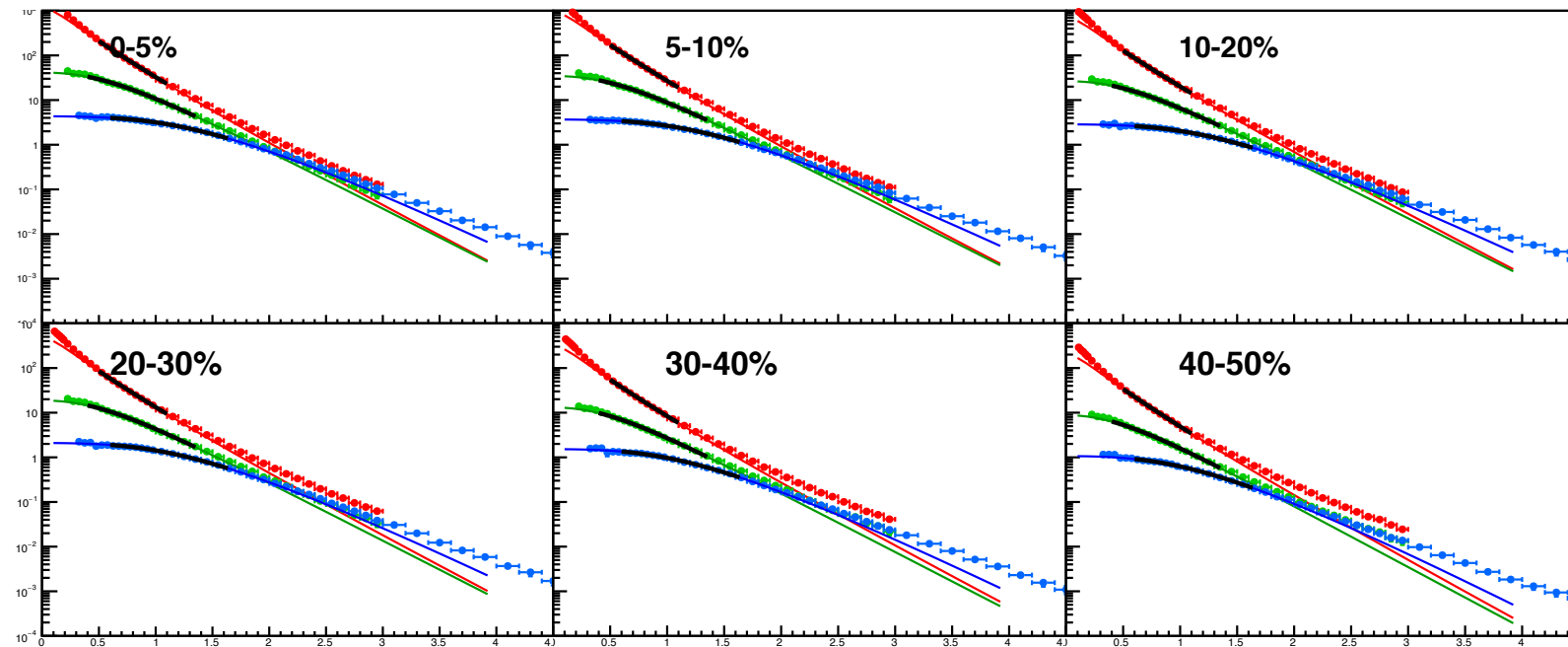
★ 2 Parameters

- T_f : Kinetic freeze out temperature
- p_0 : Transverse rapidity
- p_2 : 2nd order modulation
- τ_0 : Freeze out time
- $\Delta\tau$: Emission duration

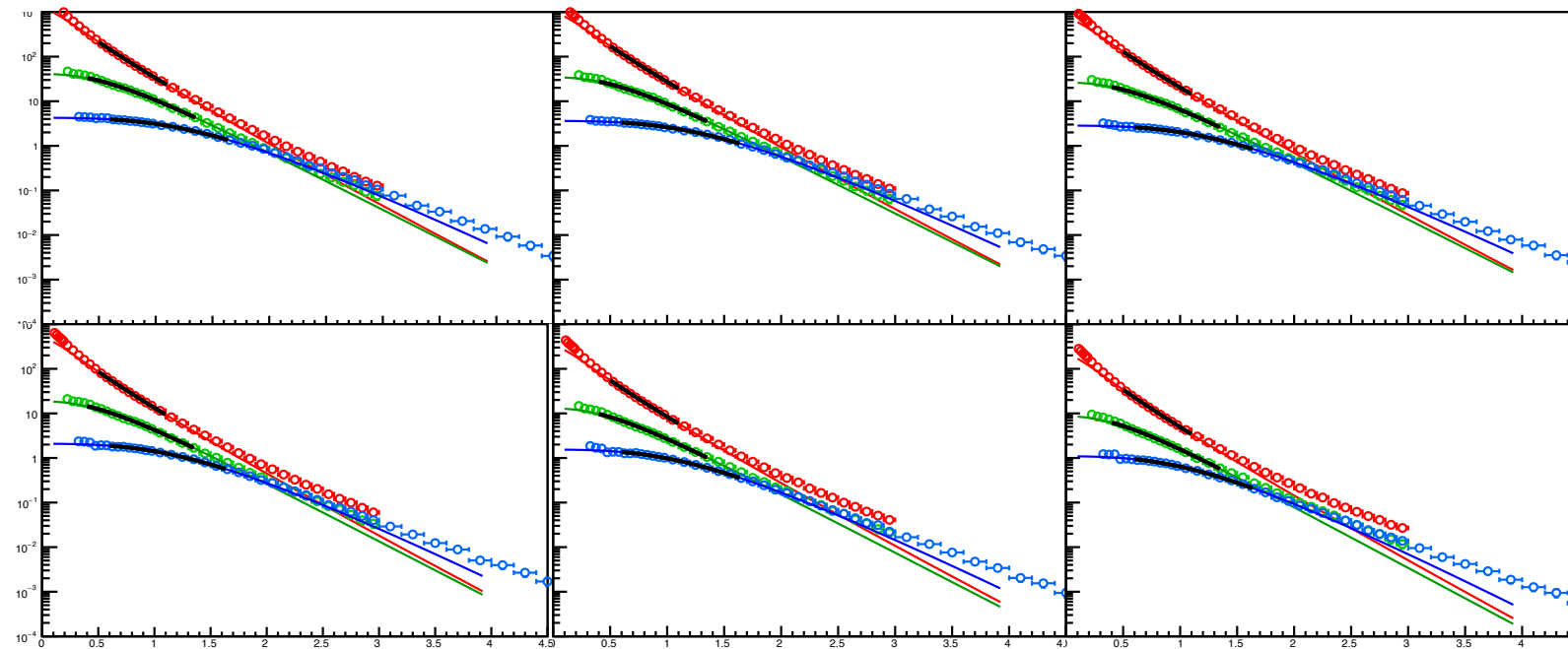
★ Fit function for spectra

$$\frac{dN}{p_T dp_T} = 2(2\pi)^{3/2} \tau_0 \Delta\tau m_T \int_0^{2\pi} d\phi_s \int_0^\infty r dr \Omega(r, \phi_s) I_0(\alpha) K_1(\beta)$$

★ Positive



★ Negative



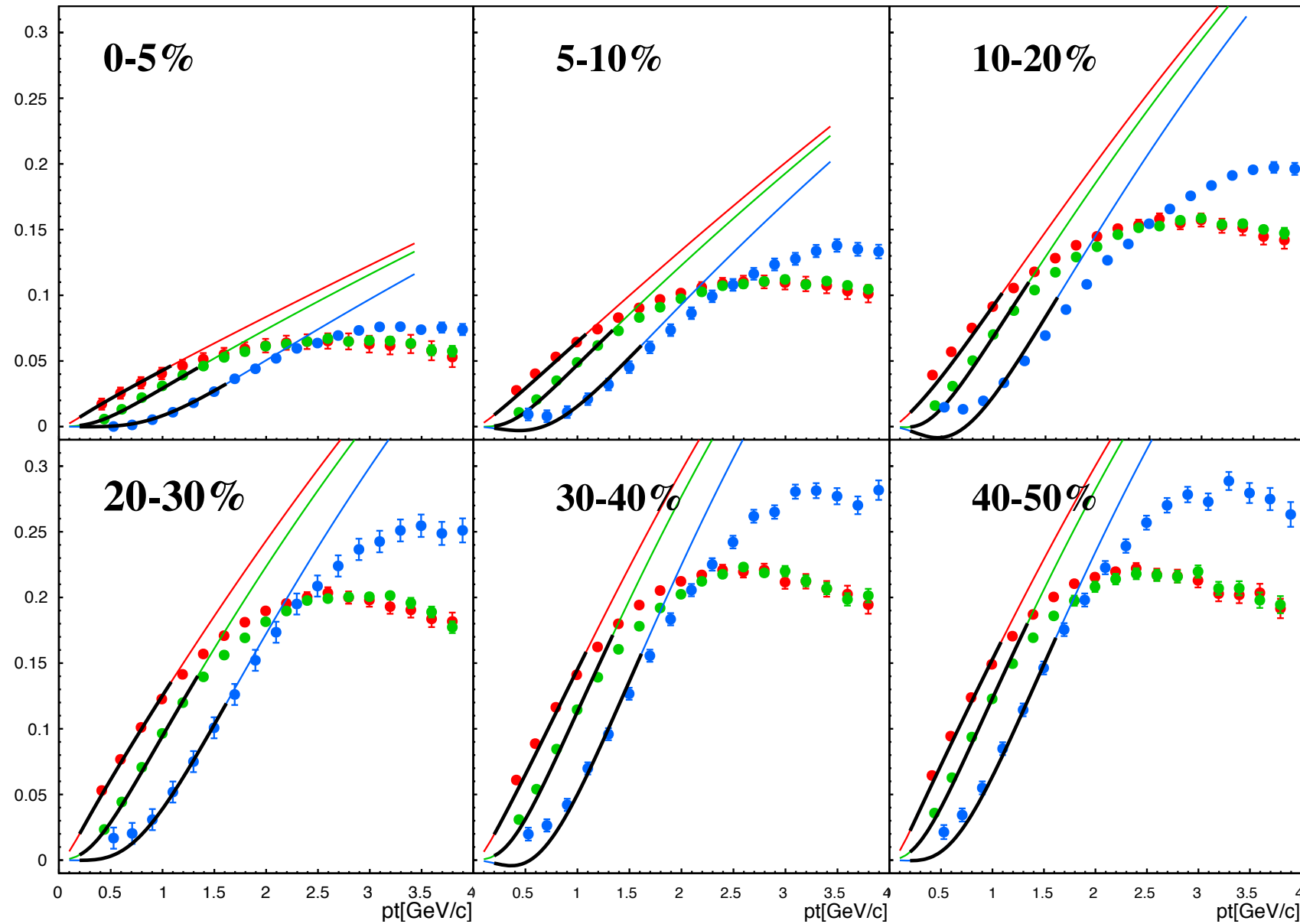
Blast wave fit for PID v_2

★ Fitting for pT dependence of π , K, p v_2

- pion
- Kaon
- proton

★ 4 Parameters

- T_f : Kinetic freeze out temperature
- p_0 : Transverse rapidity
- p_2 : 2nd order modulation
- R_x, R_y : Transverse size



★ Fit function for v_2

$$v_2(p_T, m) = \frac{\int_0^{2\pi} d\phi_p \int_0^\infty r dr \Omega(r, \phi_s) K_1(\beta) \cos(2\phi_b) I_2(\alpha)}{\int_0^{2\pi} d\phi_s \int_0^\infty r dr \Omega(r, \phi_s) I_{0/2}(\alpha) K_1(\beta)}.$$

Blast wave fit for HBT radii

✓ HBT radii relative to Ψ_2

★ 7 Parameters

- **Tf** : Kinetic freeze out temperature
- **p0** : Transverse rapidity
- **p2** : 2nd order modulation in transverse flow
- **Rx, Ry** : Transverse size of the source
- **τ0** : Freeze out time
- **Δτ** : Emission duration

$$\langle f(x) \rangle = \frac{\int d^4x f(x) S(x, K)}{\int d^4x S(x, K)},$$
$$\tilde{x}^\mu = x^\mu - \langle x^\mu \rangle,$$

★ Fit function for HBT

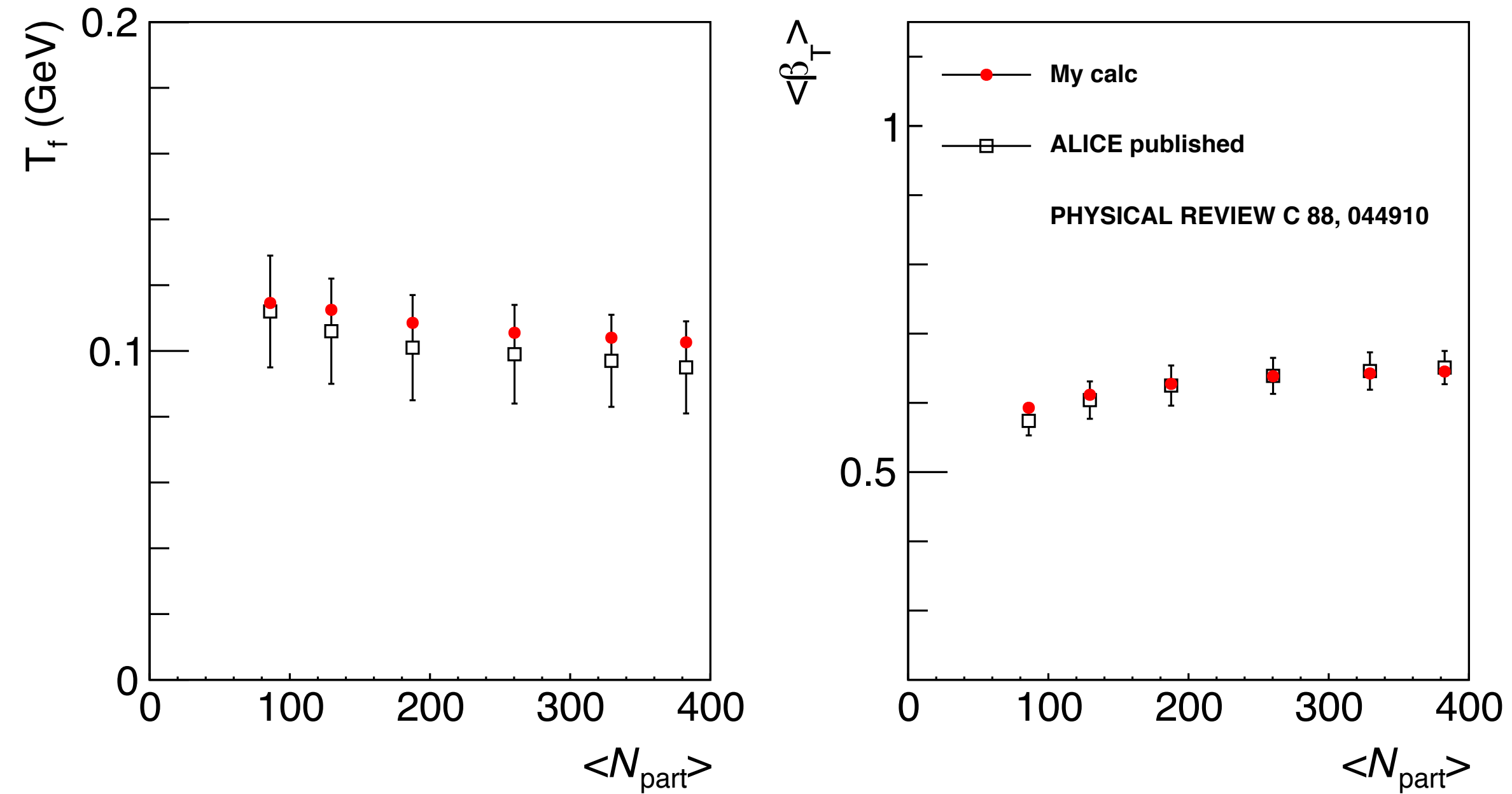
$$R_s^2 = \frac{1}{2}(\langle \tilde{x}^2 \rangle + \langle \tilde{y}^2 \rangle) - \frac{1}{2}(\langle \tilde{x}^2 \rangle - \langle \tilde{y}^2 \rangle) \cos(2\phi_p) - \langle \tilde{x}\tilde{y} \rangle \sin(2\phi_p),$$

$$R_o^2 = \frac{1}{2}(\langle \tilde{x}^2 \rangle + \langle \tilde{y}^2 \rangle) + \frac{1}{2}(\langle \tilde{x}^2 \rangle - \langle \tilde{y}^2 \rangle) \cos(2\phi_p) + \langle \tilde{x}\tilde{y} \rangle \sin(2\phi_p),$$
$$-2\beta_T(\langle \tilde{t}\tilde{x} \rangle \cos \phi_p + \langle \tilde{t}\tilde{y} \rangle \sin \phi_p) + \beta_T^2 \langle \tilde{t}^2 \rangle,$$

$$R_{os}^2 = \langle \tilde{x}\tilde{y} \rangle \cos(2\phi_p) - \frac{1}{2}(\langle \tilde{x}^2 \rangle - \langle \tilde{y}^2 \rangle) \sin(2\phi_p) + \beta_T(\langle \tilde{t}\tilde{x} \rangle \sin \phi_p - \langle \tilde{t}\tilde{y} \rangle \cos \phi_p),$$

$$R_l^2 = \langle \tilde{z}^2 \rangle - 2\beta_l \langle \tilde{t}\tilde{z} \rangle + \beta_l^2 \langle \tilde{t}^2 \rangle,$$
$$= \langle \tilde{z}^2 \rangle,$$

Blast Wave parameters (comparison with ALICE published)

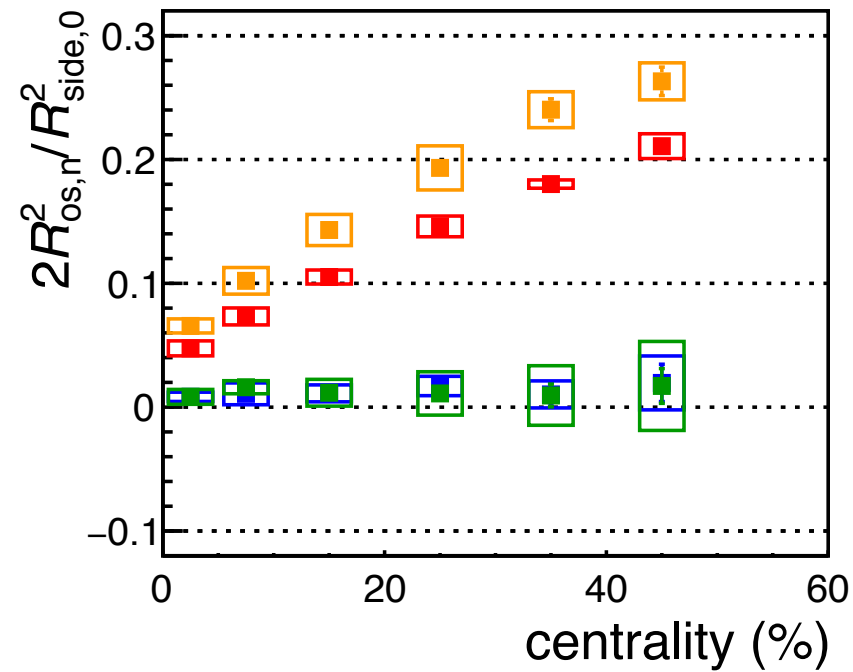
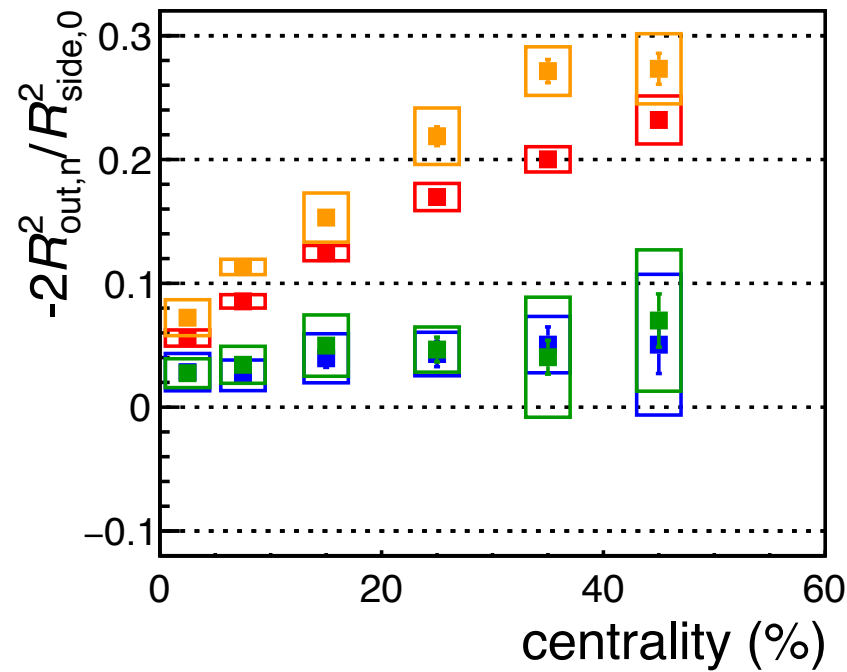
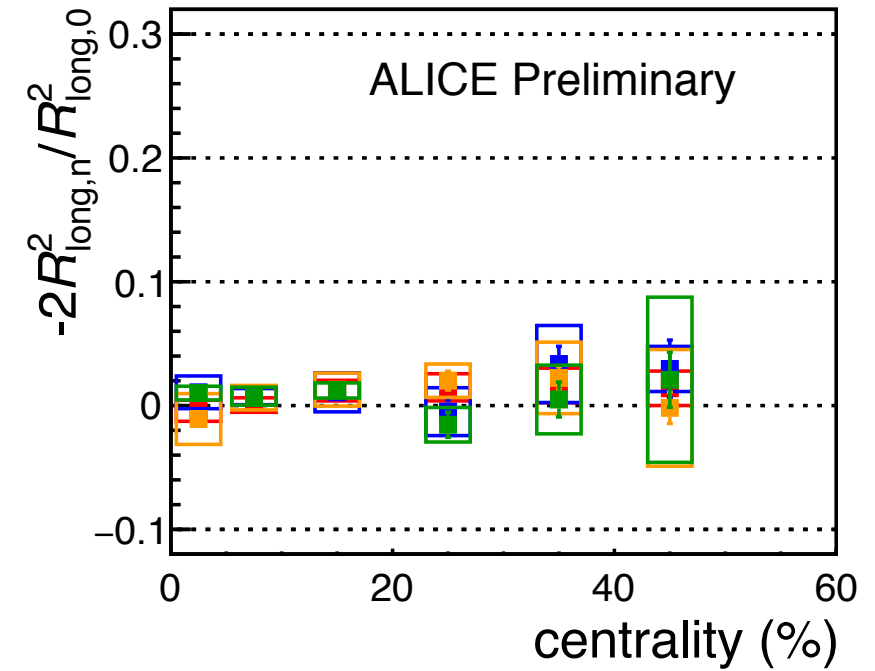
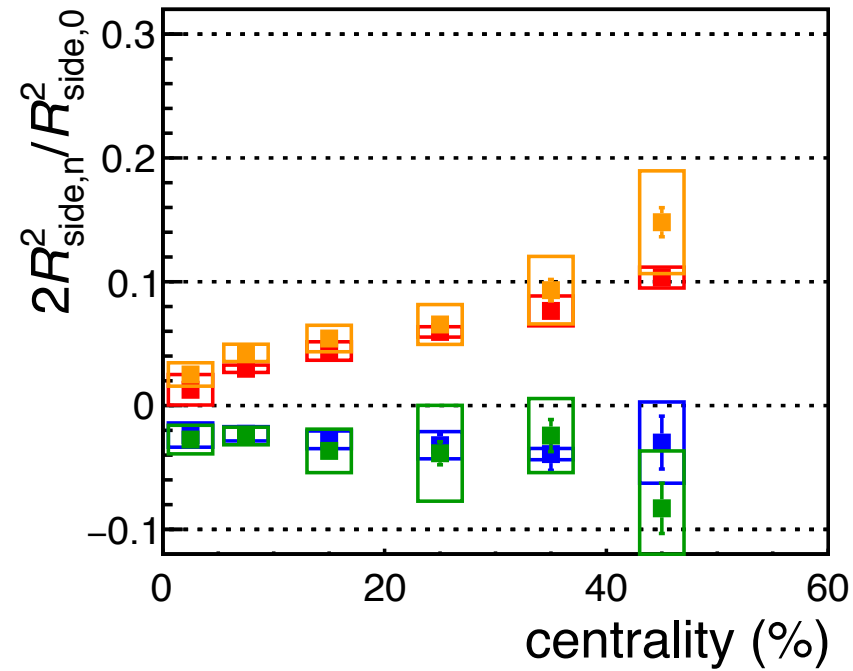
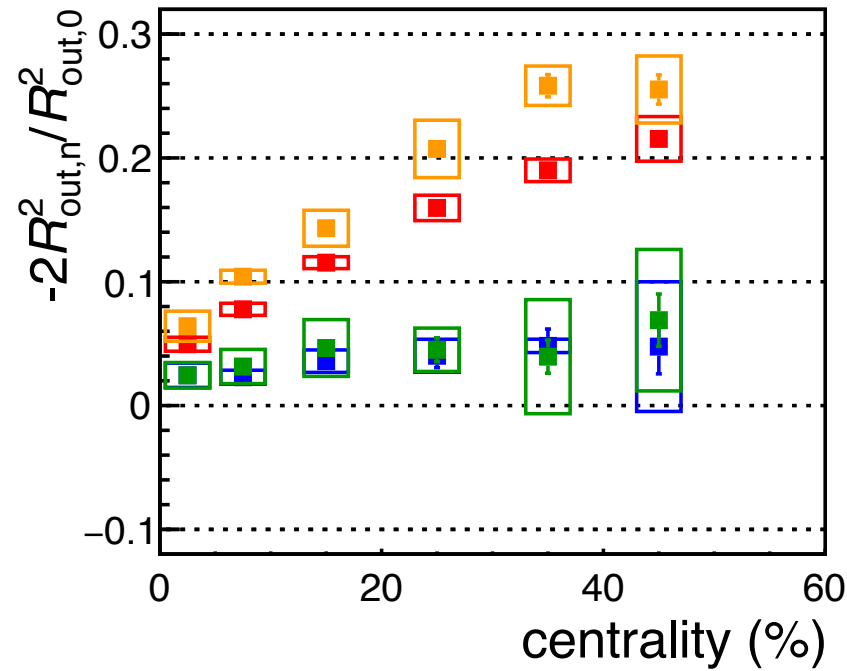


◆ Fully consistent within the systematic uncertainties

$$\langle \beta_T \rangle = \int_0^{2\pi} d\phi \int_0^1 dr \tanh((\rho_0 + \rho_2 \cos(2\phi)) r^n) r (1 + 2s_2 \cos(2\phi))$$

$$s_2 = \frac{1}{2} \frac{(R_y/R_x)^2 - 1}{(R_y/R_x)^2 + 1}$$

HBT relative to Ψ_n with ESE(q_n cut)



Pb-Pb $\sqrt{s_{NN}}=2.76$ TeV

$\pi^+\pi^+$ and $\pi^-\pi^-$ pair combined

$k_T:0.2-1.5(\text{GeV}/c)$

q vector cut via FMD A+C side

—■— 20% large q_2 cut, $n=2$

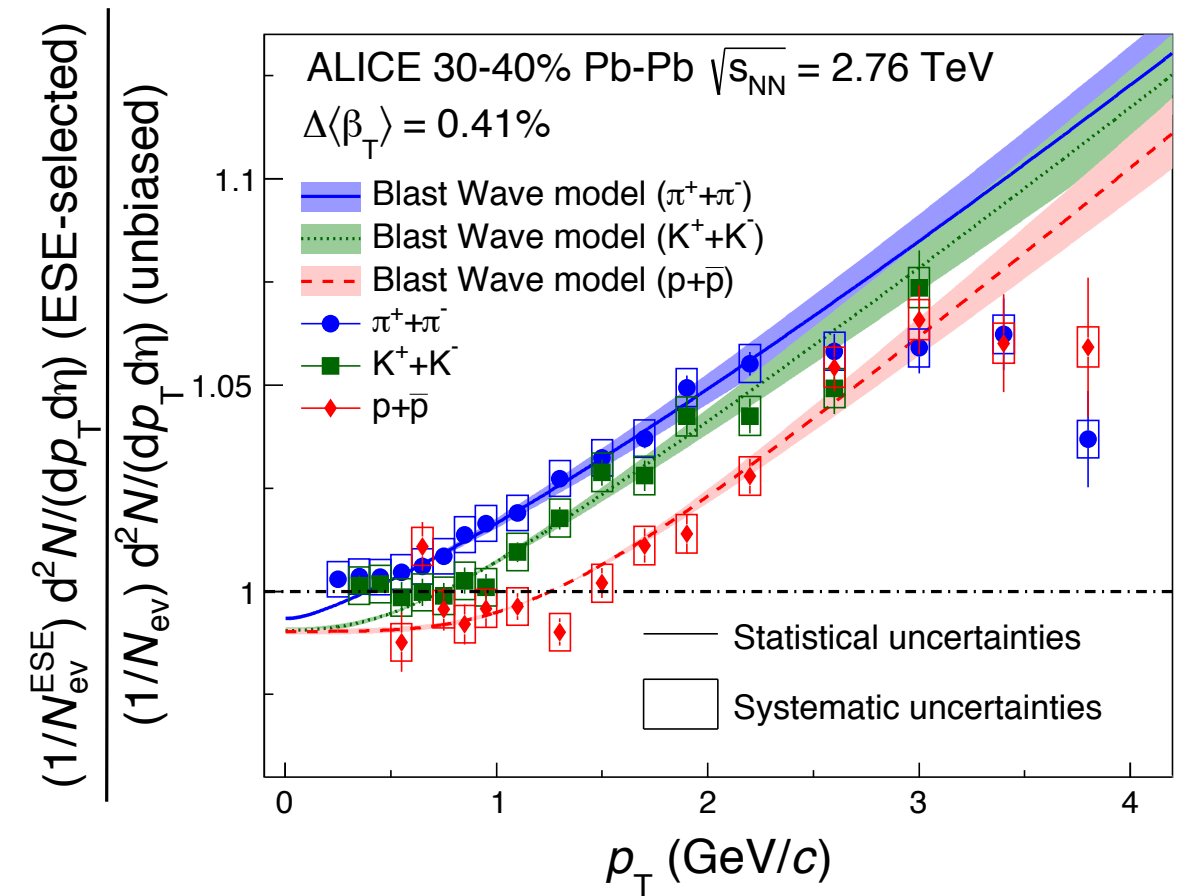
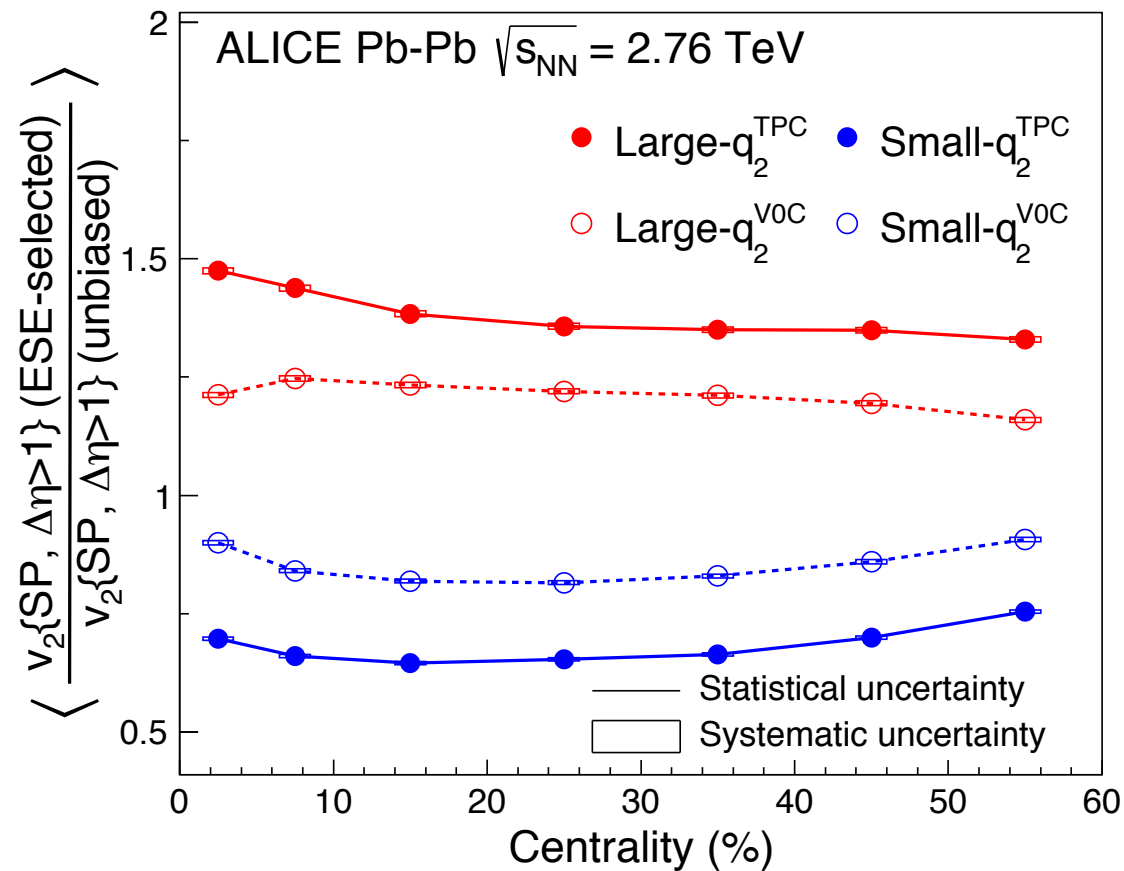
—■— No q_2 cut, $n=2$

—■— 20% large q_3 cut, $n=3$

—■— No q_3 cut, $n=3$

Spectra + Event shape engineering

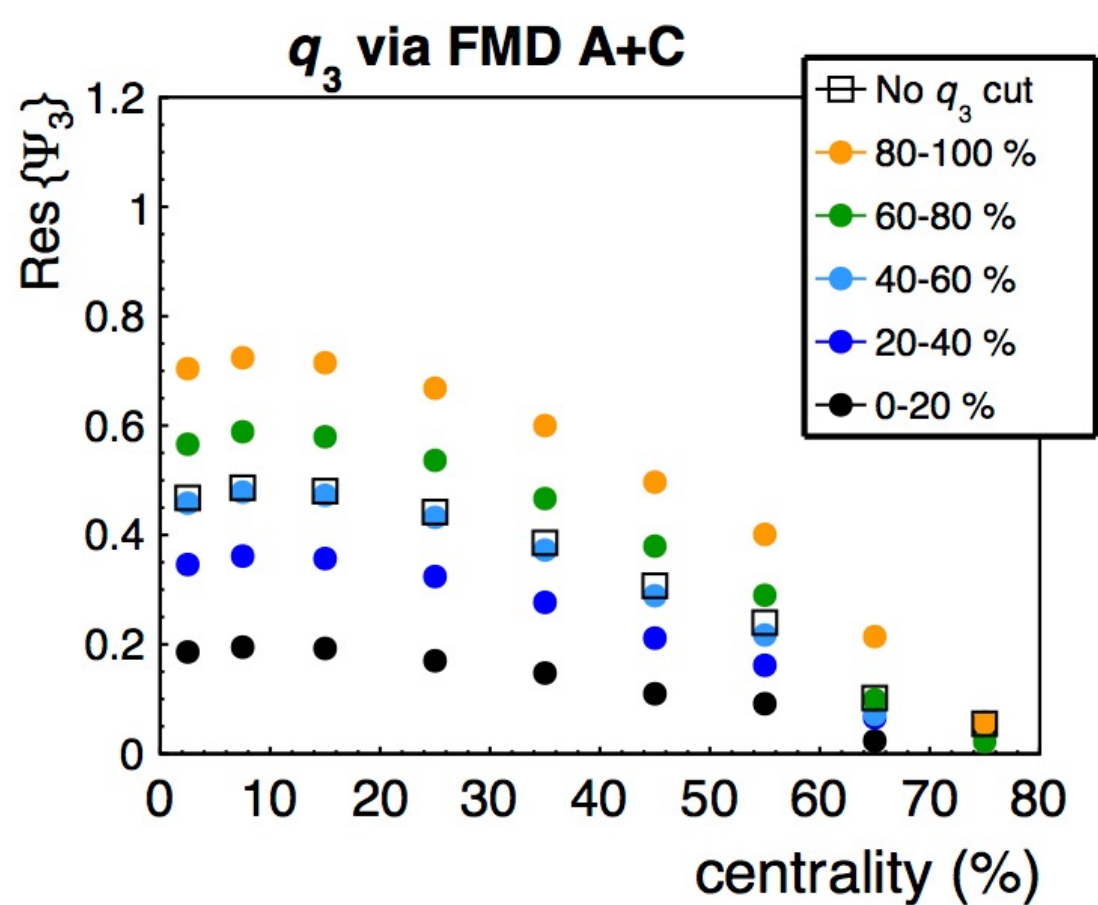
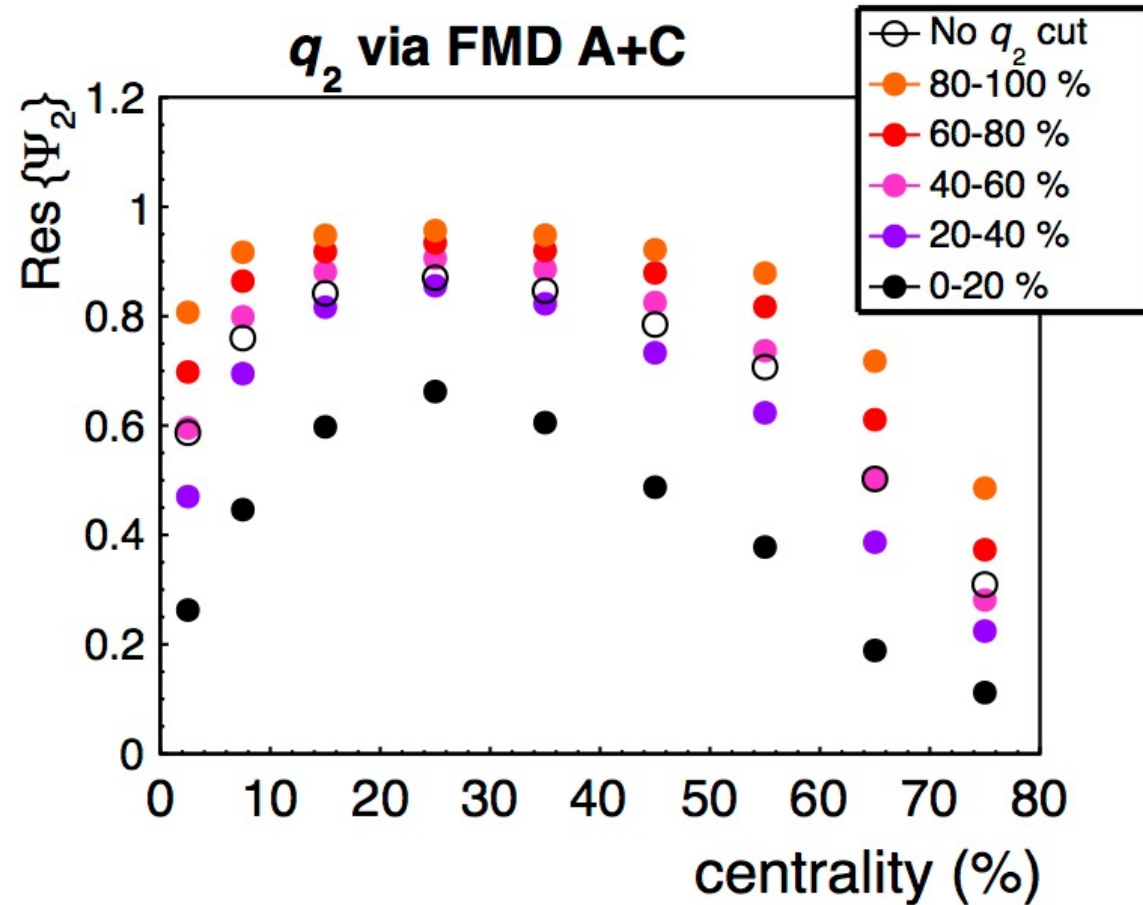
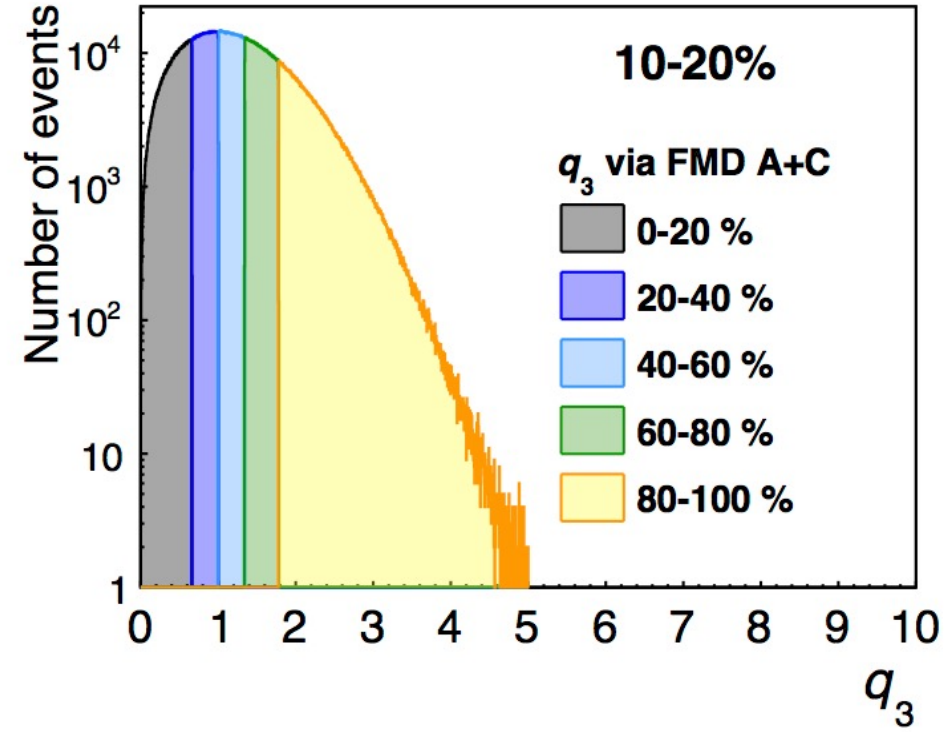
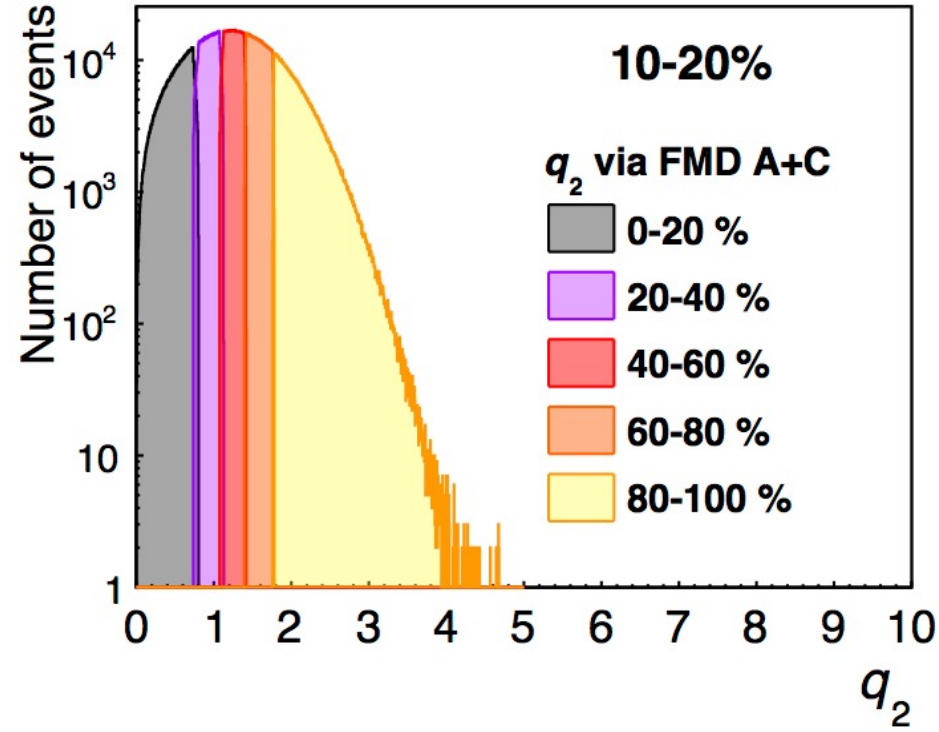
◆ Positive correlation between $\langle v_2 \rangle$ and $\langle p_T \rangle$



- ◆ v_2 ratio with q_2 large(small) cut
 - ✓ large q_2^{TPC} top10% (bottom10%)
 - ✓ large q_2^{VZERO} top10%(bottom10%)

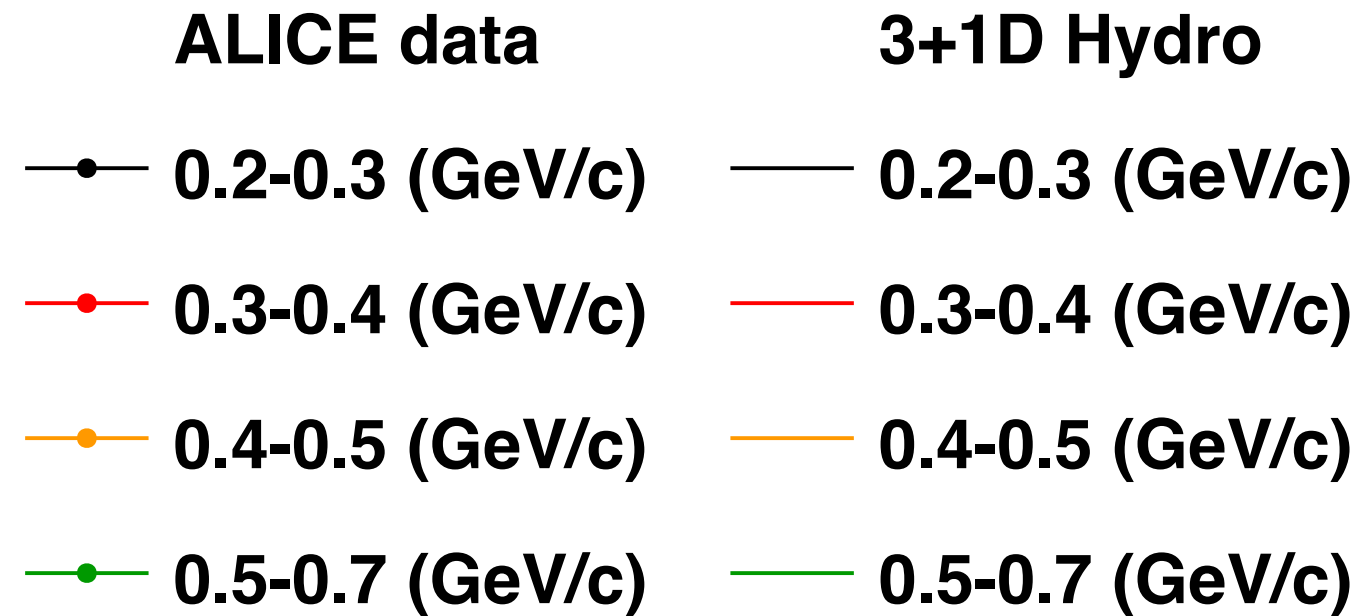
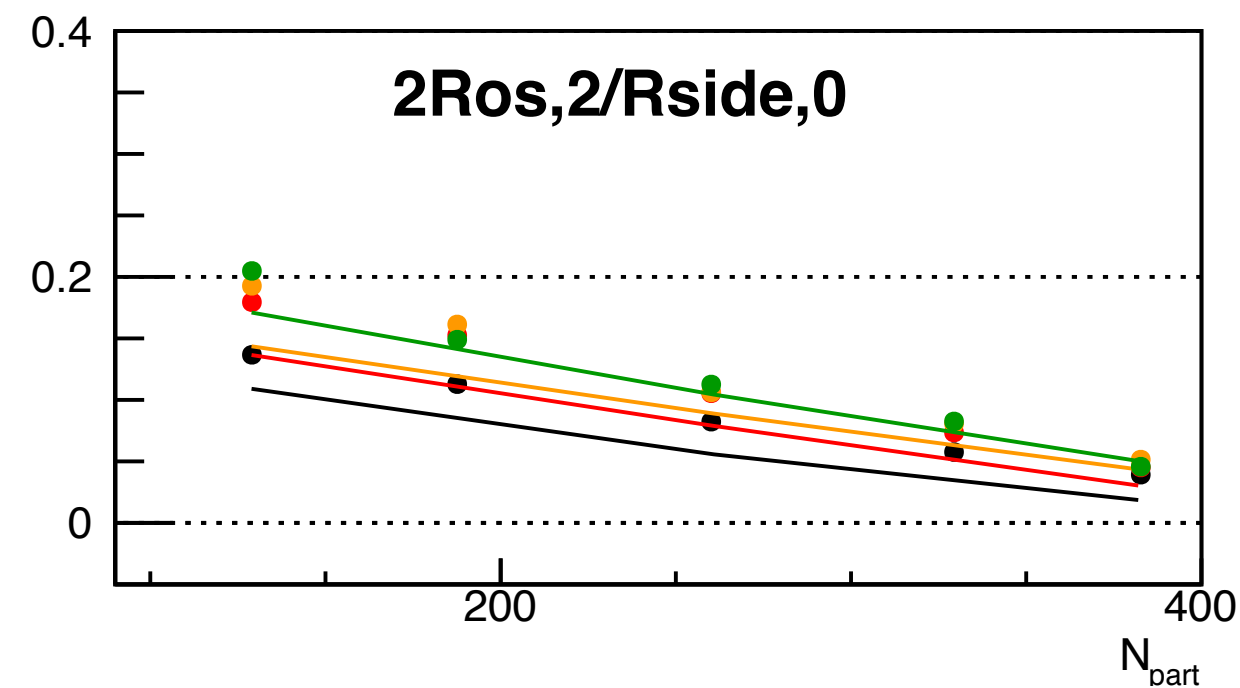
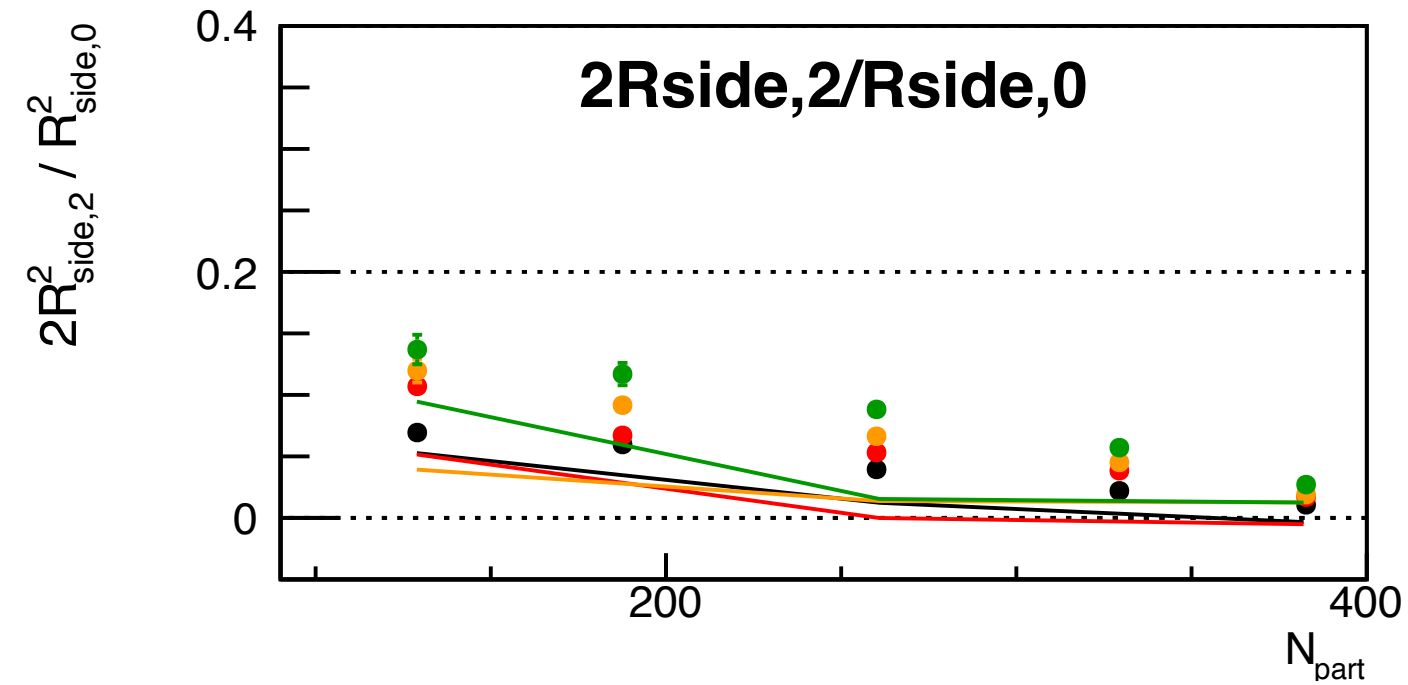
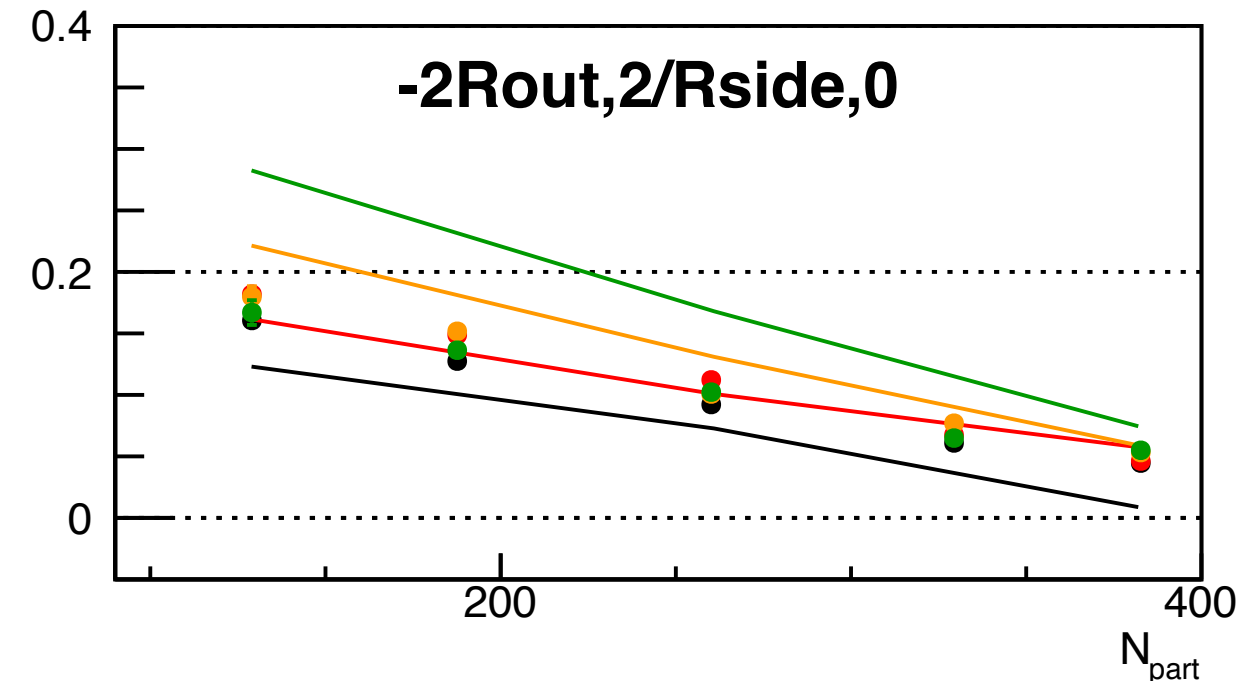
- ◆ Ratio of p_T distribution of π , K , p
- ◆ q_2^{TPC} top 10% cut ($|\ln| < 0.4$)
- ◆ Blast wave model comparison
 - ✓ $\Delta\langle\beta_T\rangle = +0.41\%$

E.P. resolution with q_n cut



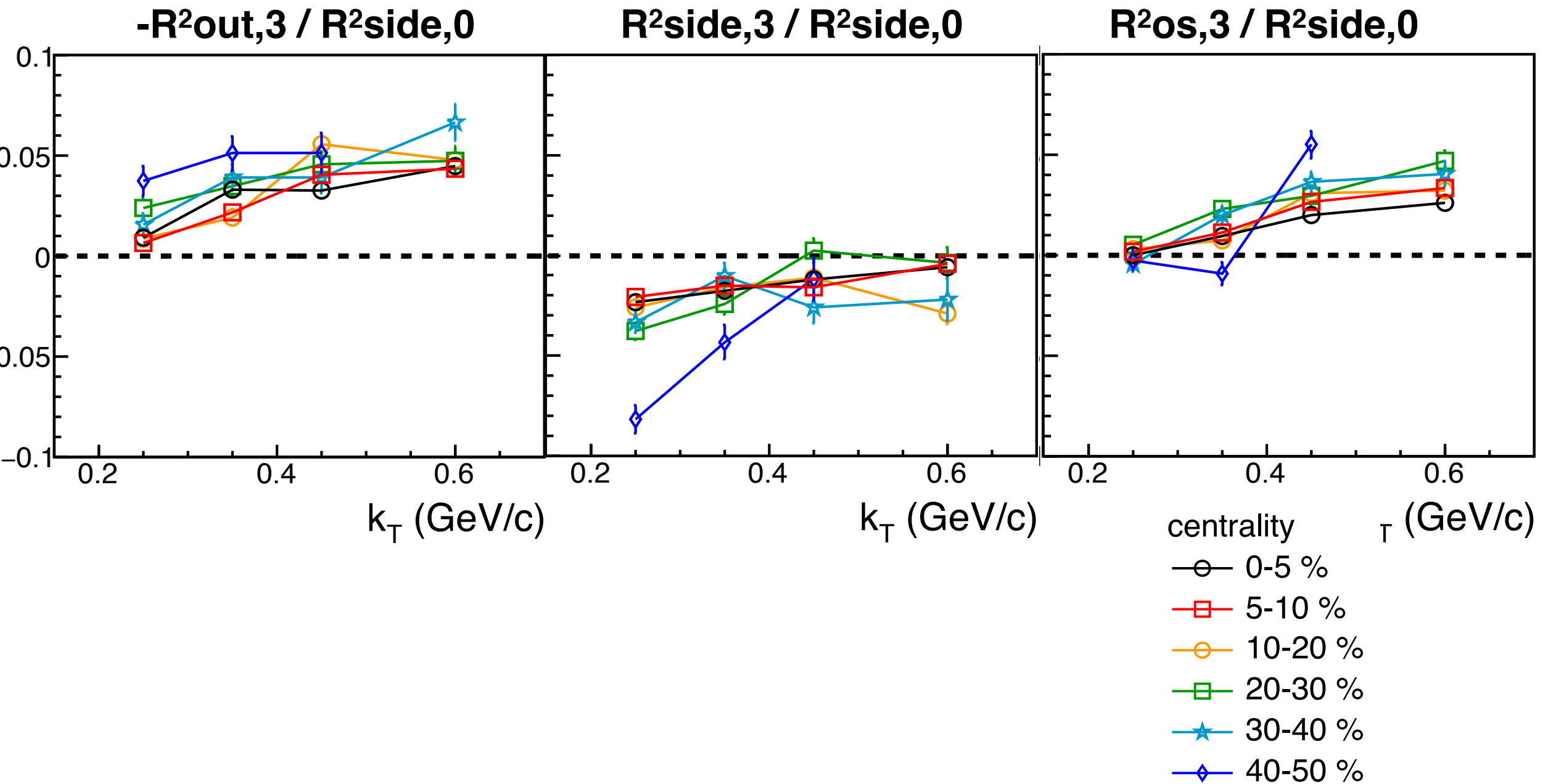
2nd harmonic oscillation amplitude of HBT radii

(P. Bozek, J. Phys. G38, 124097)



- ◆ Hydro calculation cannot reproduce $R_{out,2}^2 / R_{side,0}^2$ small k_T dependence
 - ◆ N_{part} dependence of $R_{out,2}^2 / R_{side,0}^2$ is very similar though
- ◆ $R_{out,2}^2 / R_{side,0}^2$ in lowest k_T is consistent but not in high k_T (under estimate)
- ◆ $R_{os,2}^2 / R_{side,0}^2$ is well reproduced

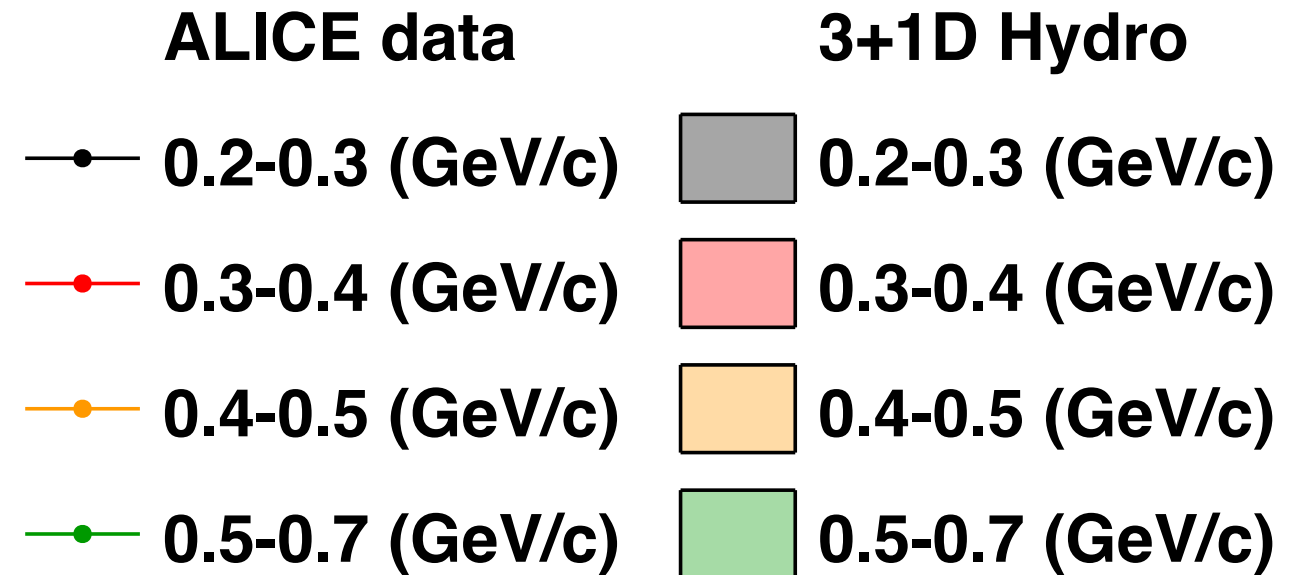
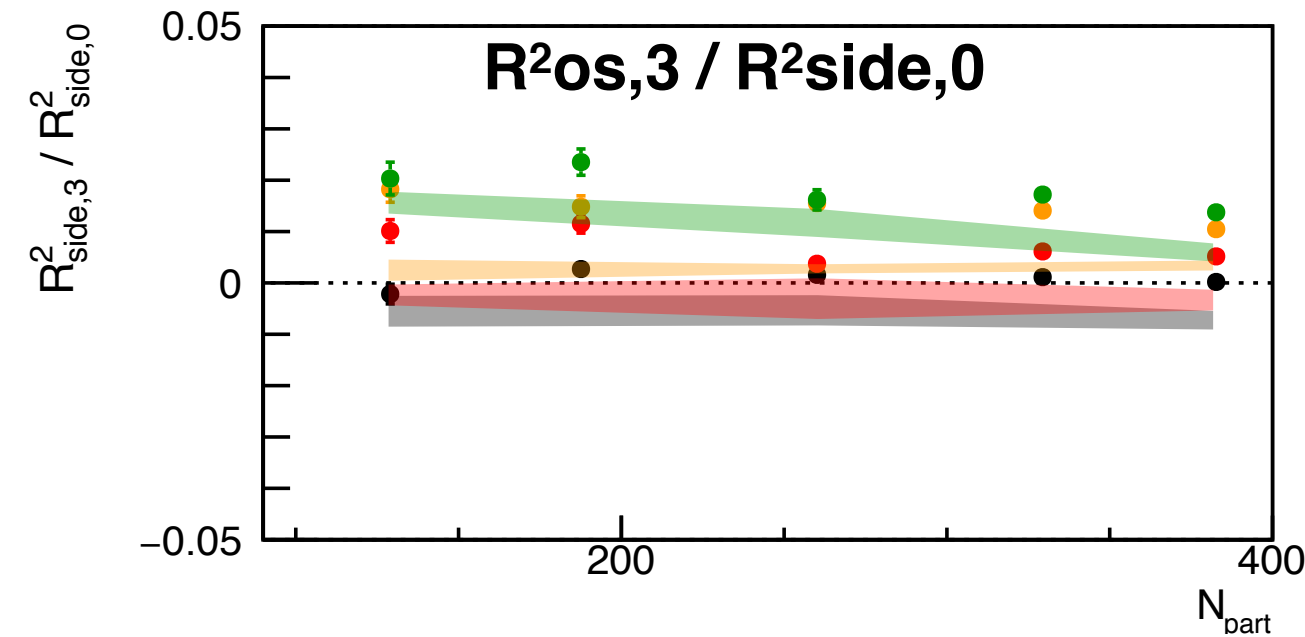
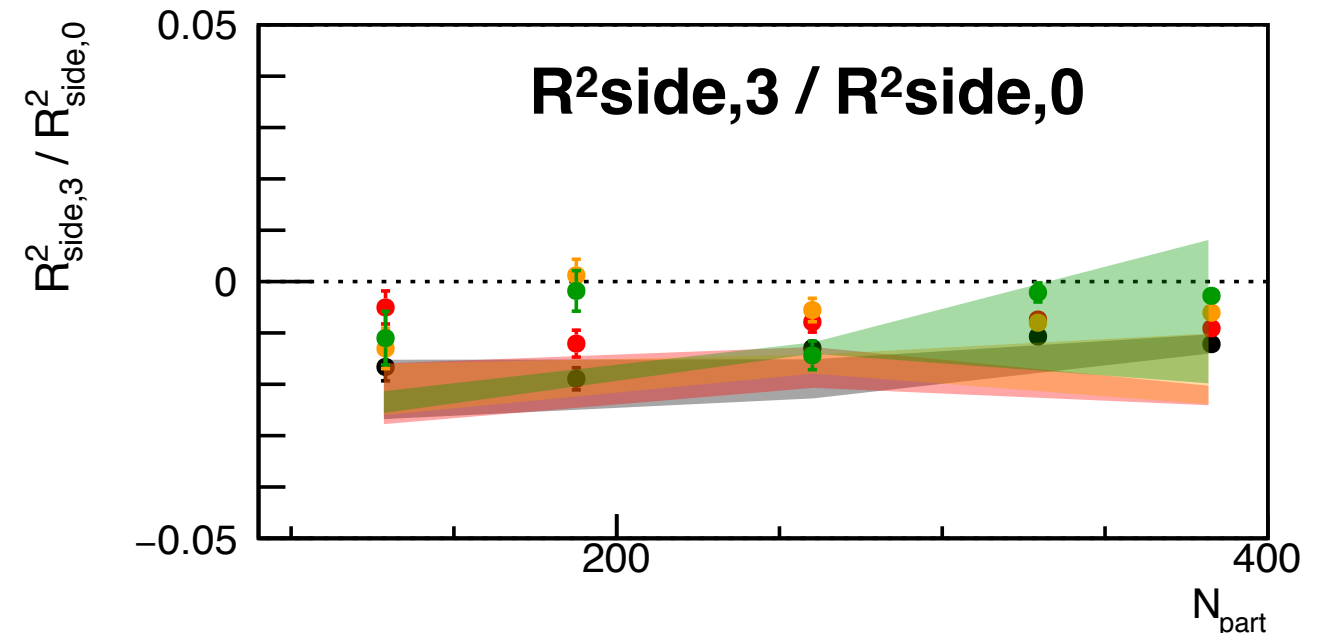
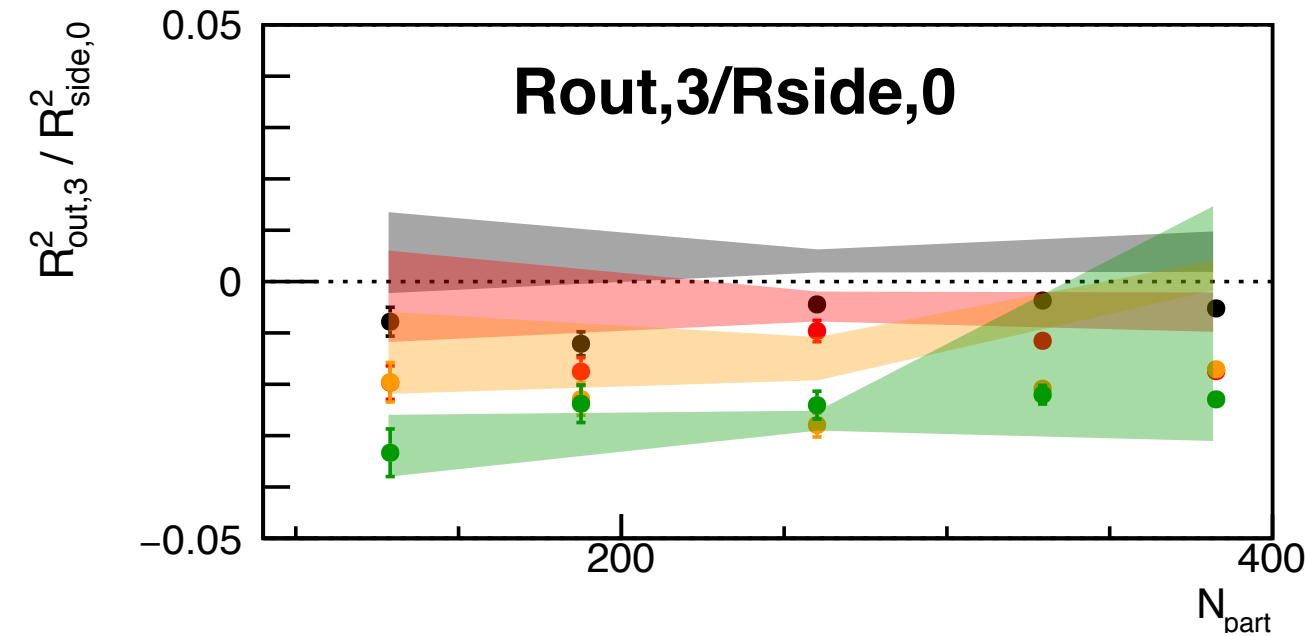
Relative amplitude of HBT radii w.r.t. Ψ_3 k_T dependence



- ✓ Relative amplitude of R_{out} becomes larger with increasing k_T
- ✓ R_{side} oscillation decreases from low k_T to high k_T
- ✓ R_{os} shows explicit k_T dependence and R_{os} oscillation is 0 at $k_T=0$

3rd harmonic oscillation amplitude of HBT radii

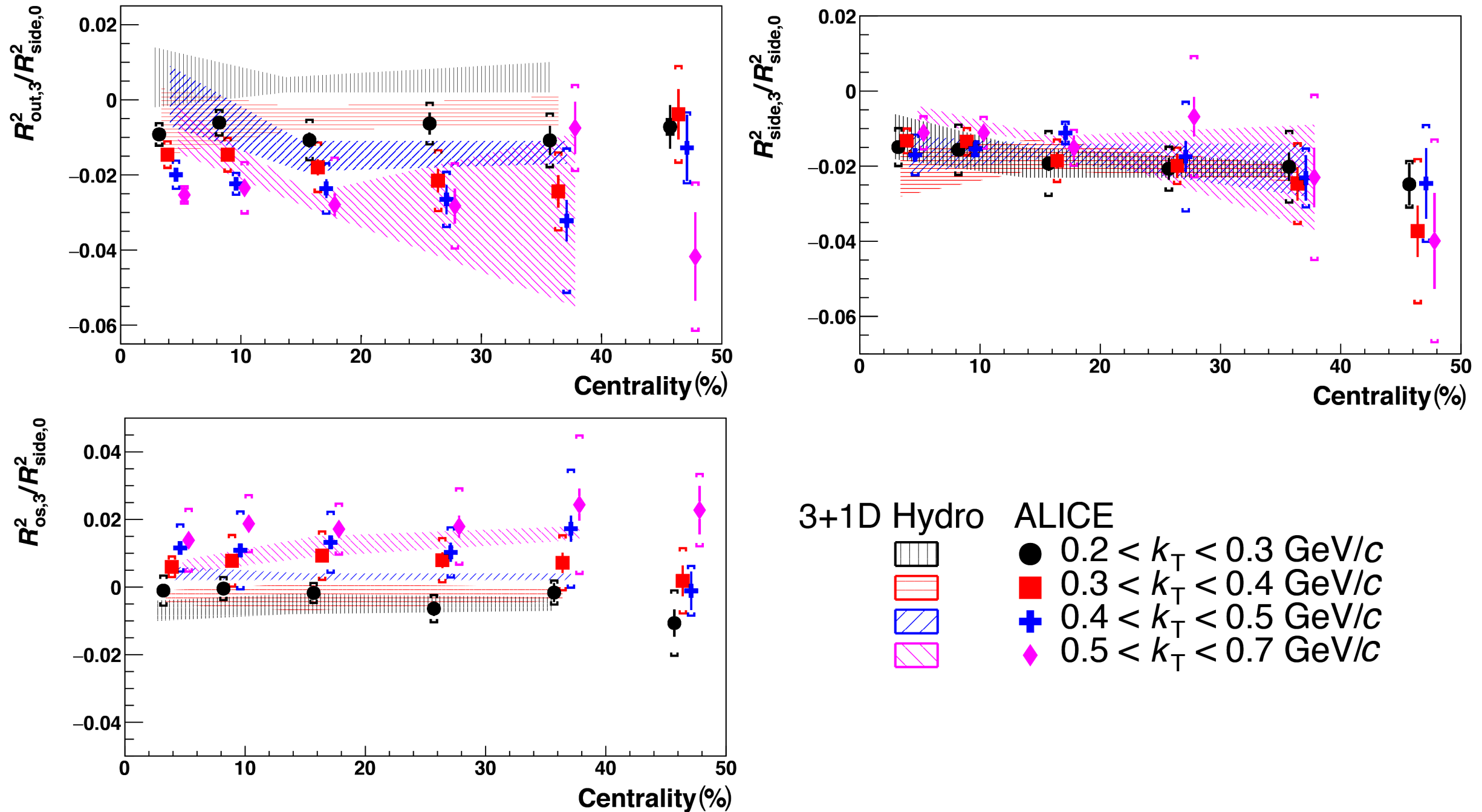
(P. Bozek, J. Phys. G38, 124097)



- ◆ N_{part} dependence in Hydro calc. and Data are qualitative consistent
- ◆ R^2_{out} oscillation is consistent in high k_T (Hydro calc. at low p_T is opposite sign)
- ◆ R^2_{side} oscillation is consistent in low k_T (Hydro calc. can't reproduce k_T dependence)
- ◆ Low k_T of R_{os} oscillation with Hydro calc is underestimate

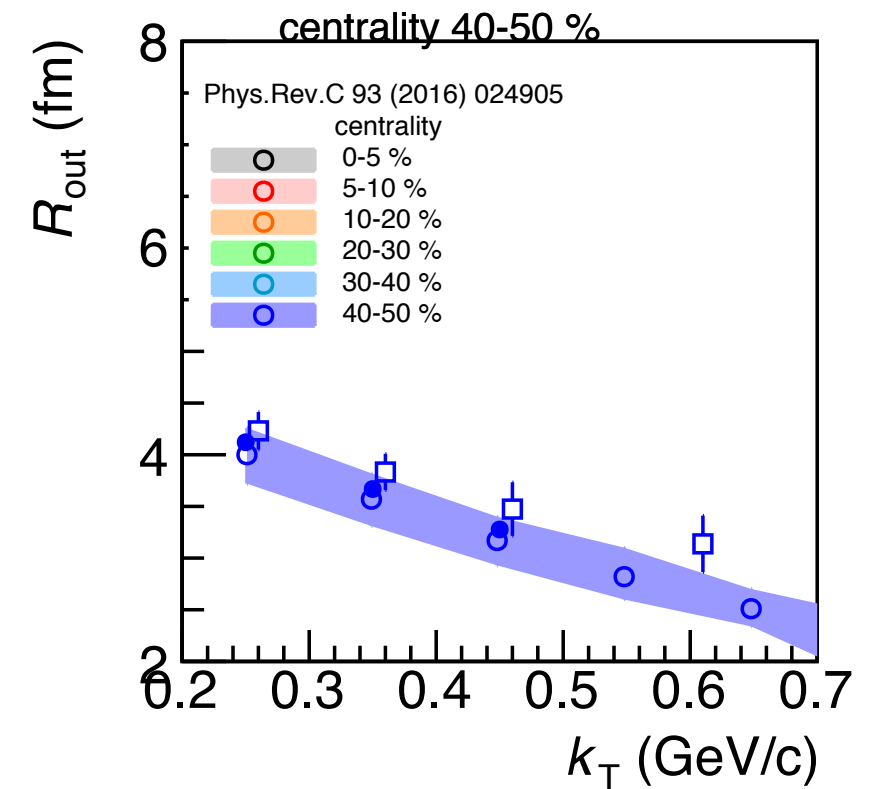
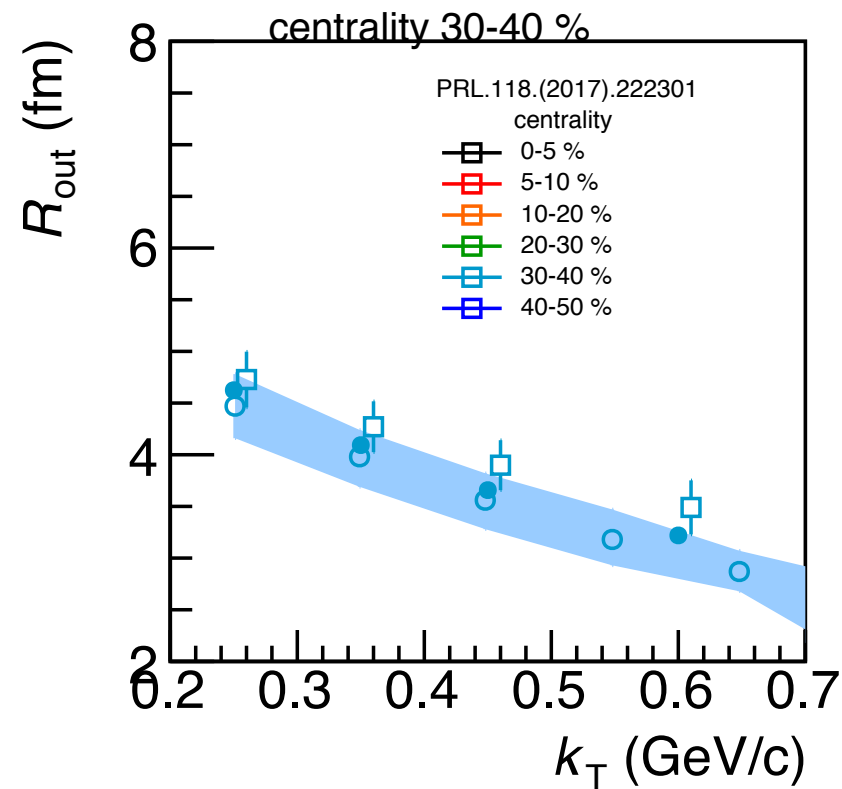
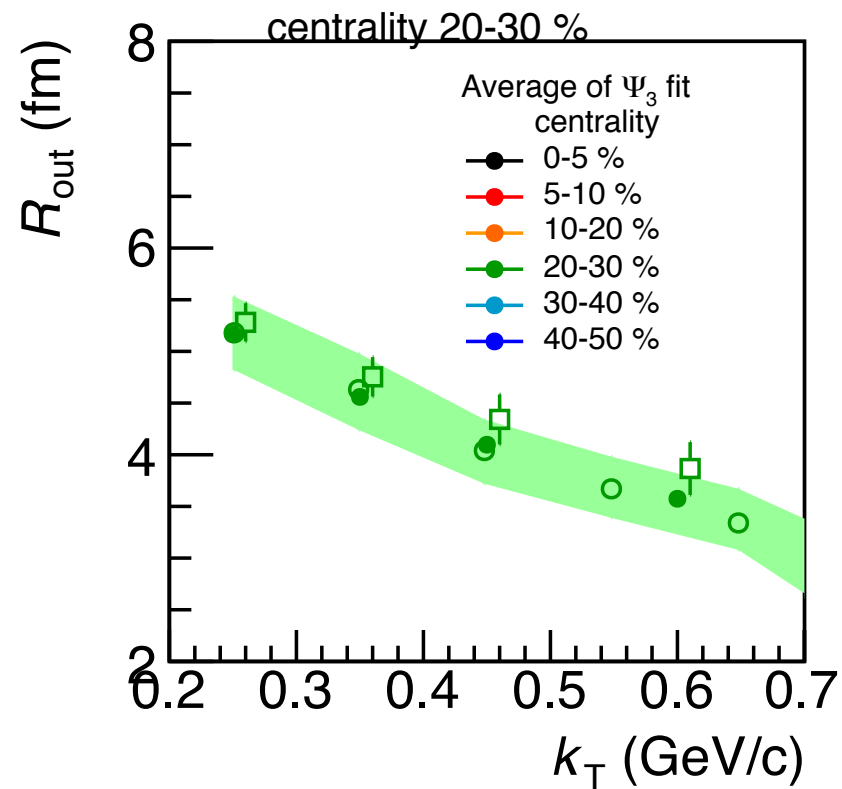
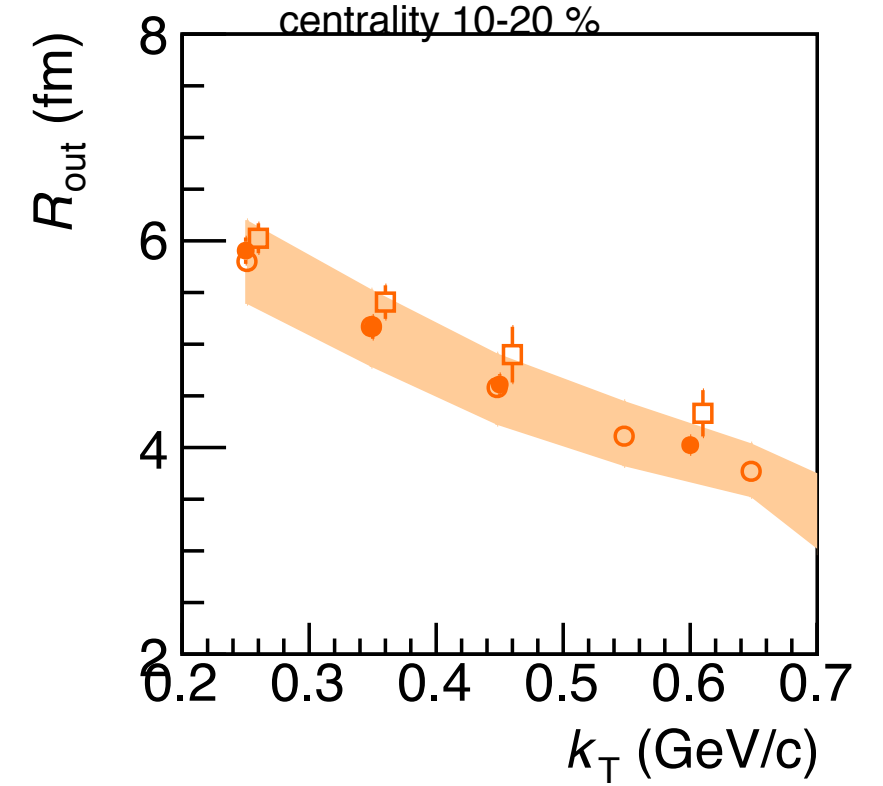
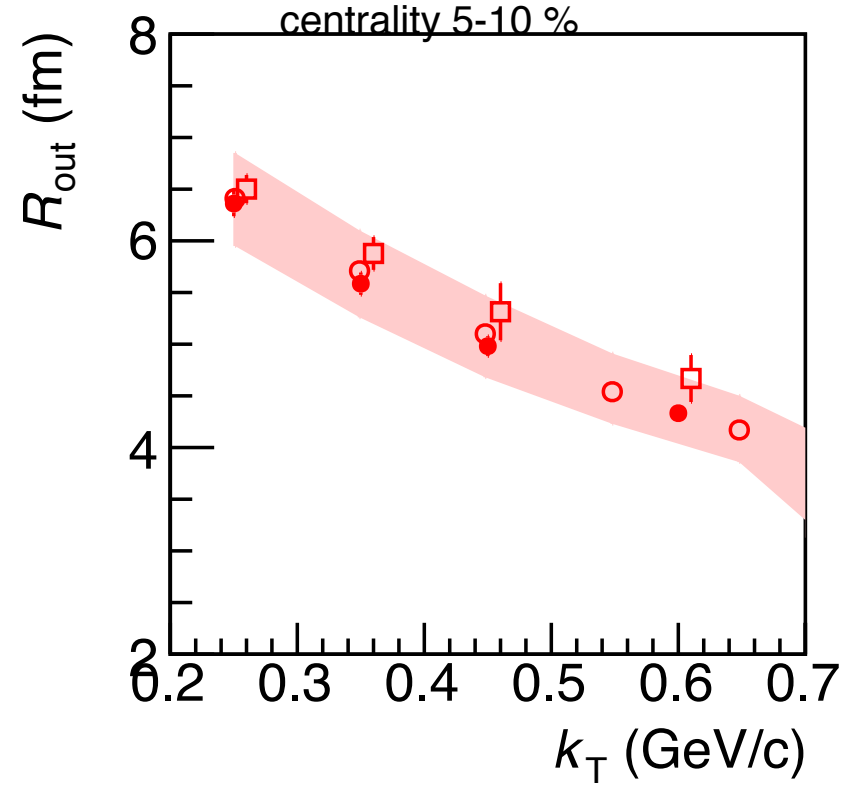
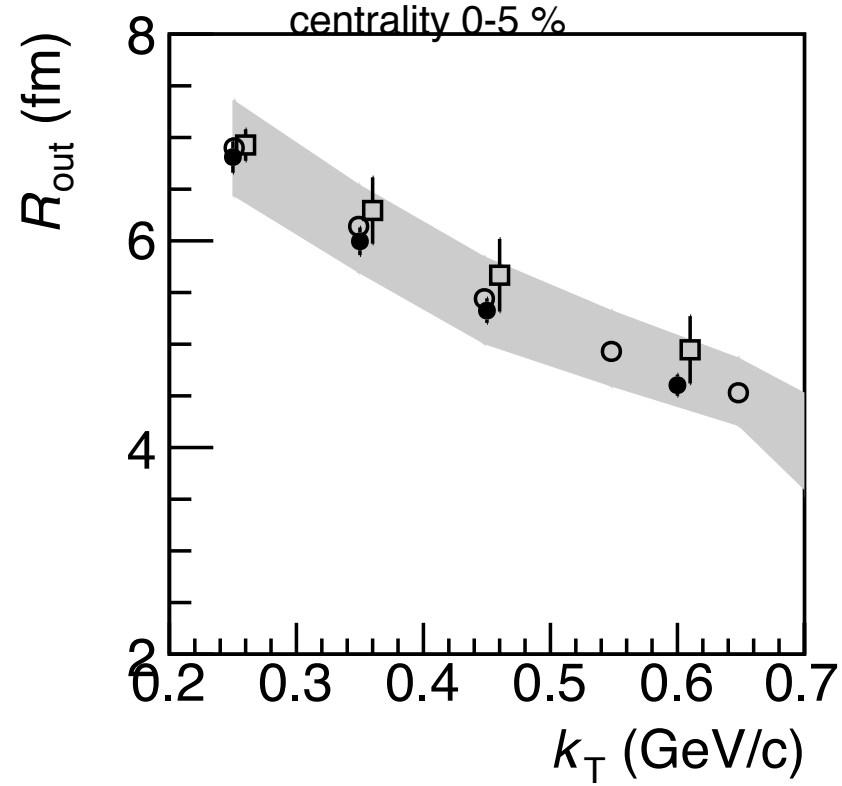
3rd harmonic oscillation amplitude of HBT radii

(P. Bozek, J. Phys. G38, 124097)

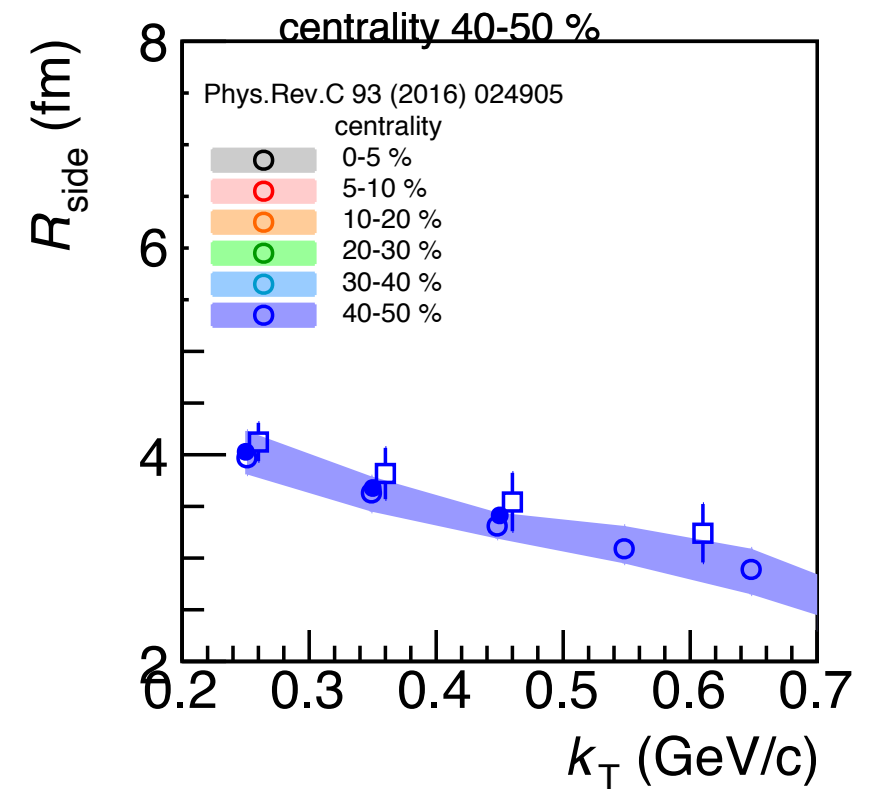
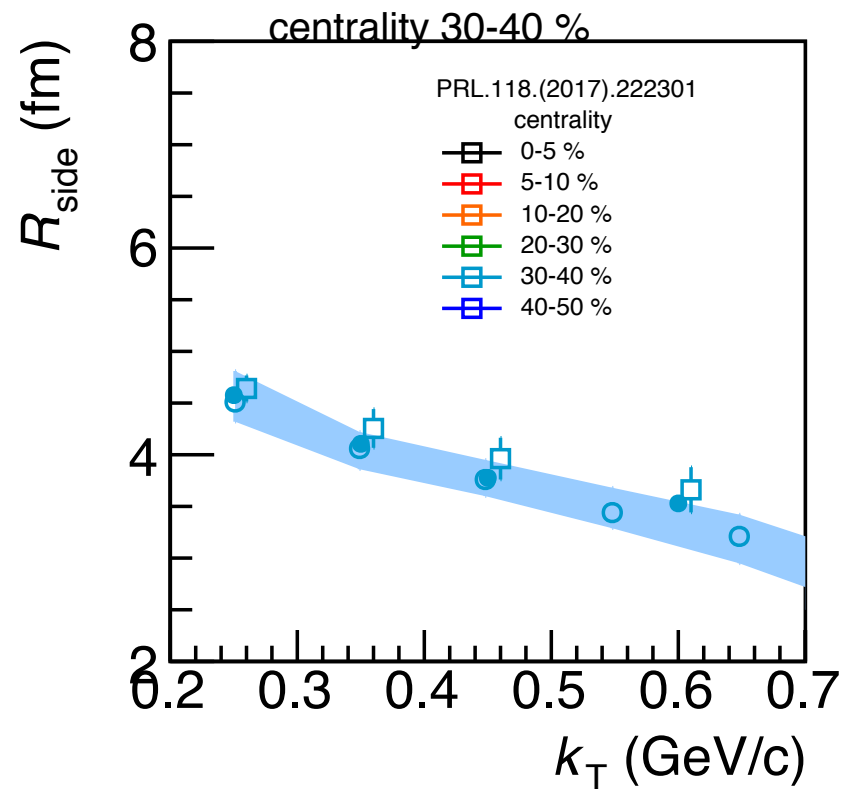
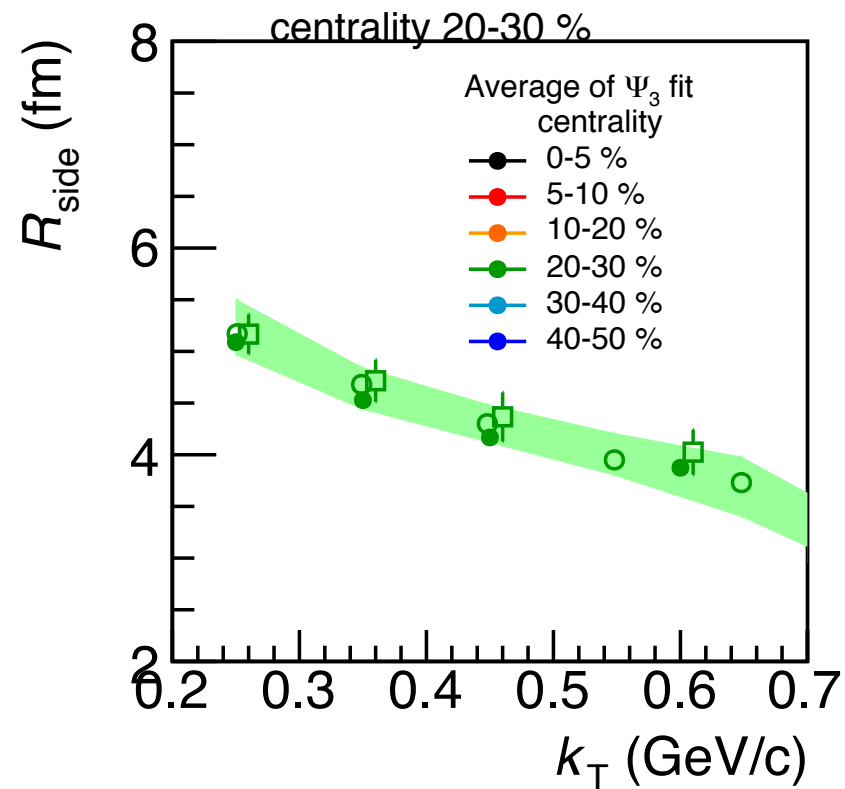
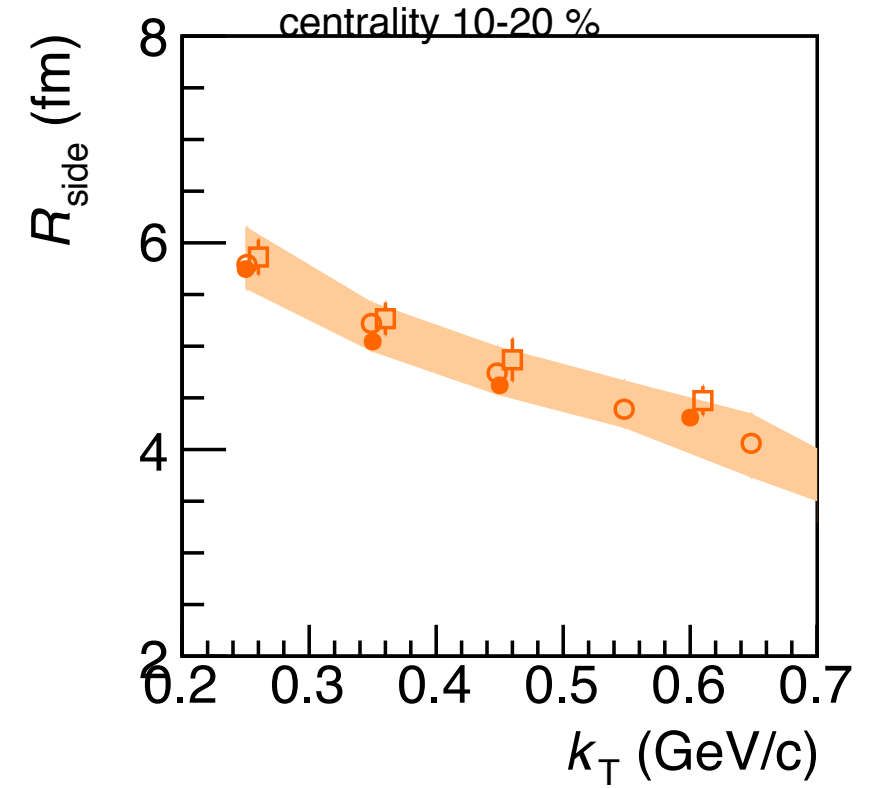
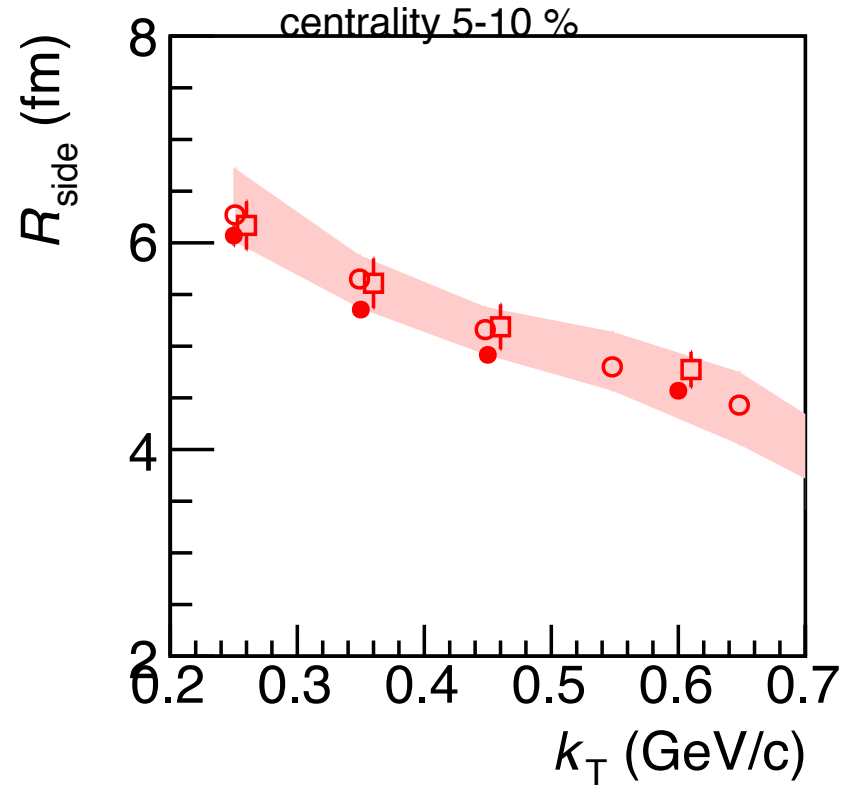
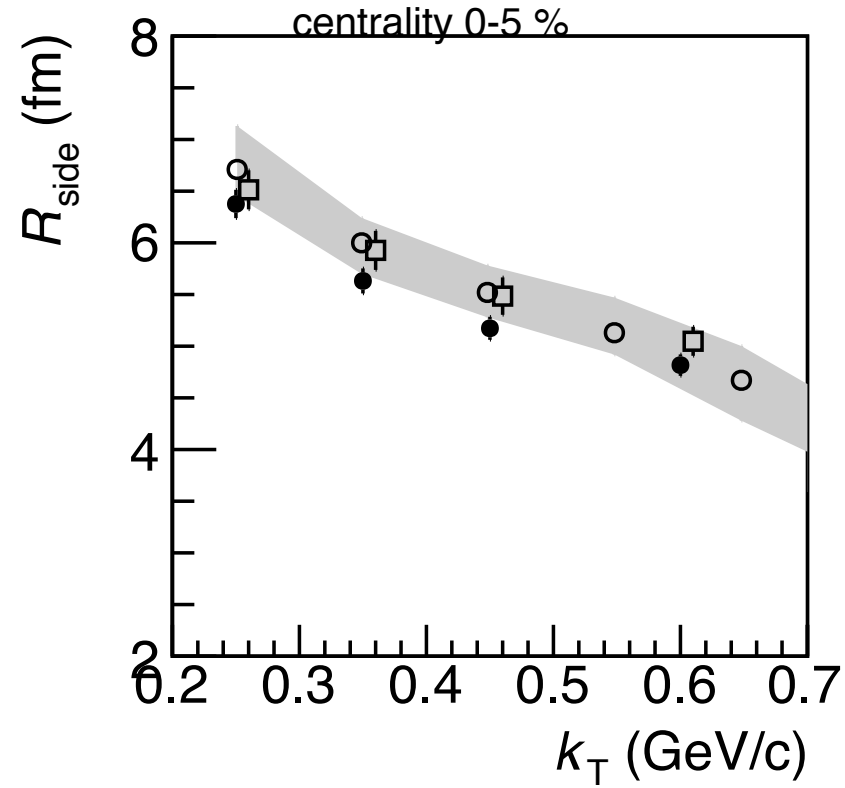


- ◆ N_{part} dependence in Hydro calc. and Data are qualitatively consistent
- ◆ R^2_{out} oscillation is consistent in high k_T (Hydro calc. at low p_T is opposite sign)
- ◆ R^2_{side} oscillation is consistent in low k_T

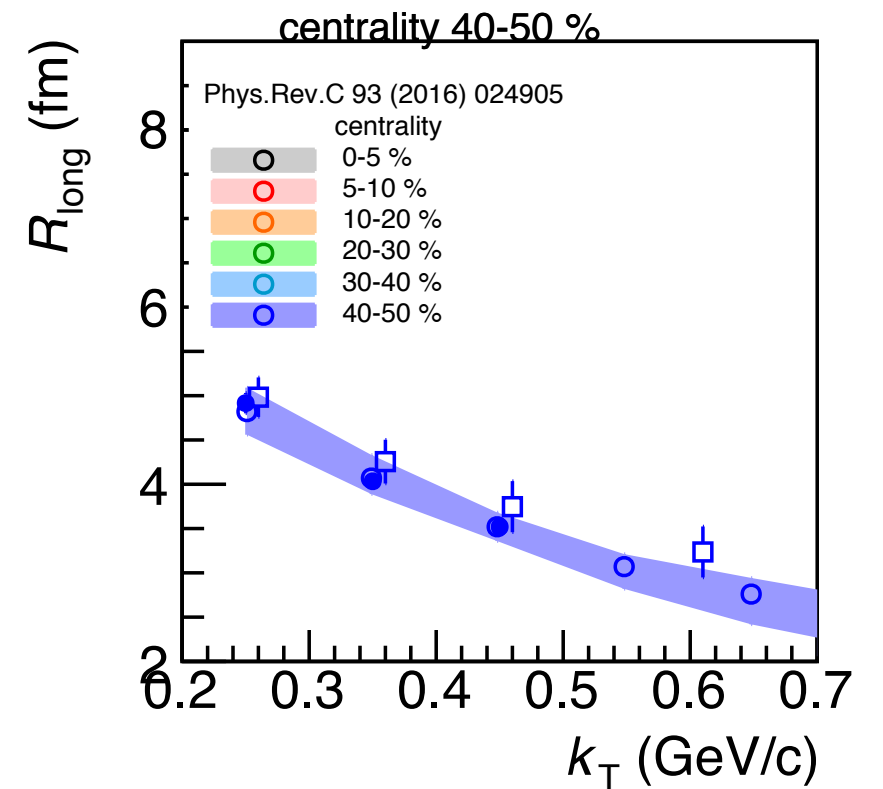
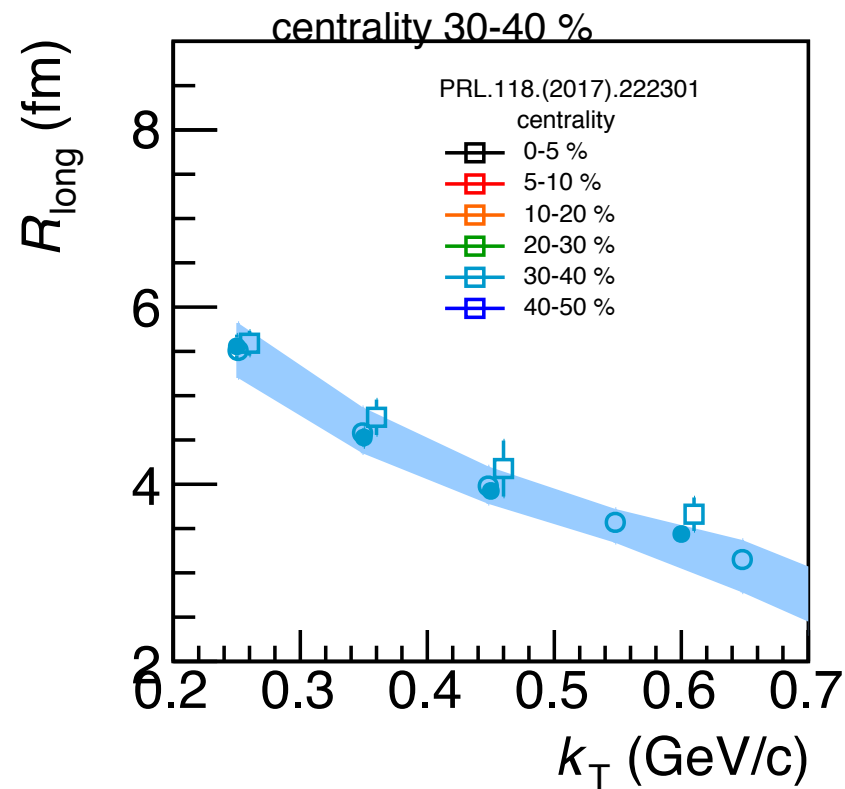
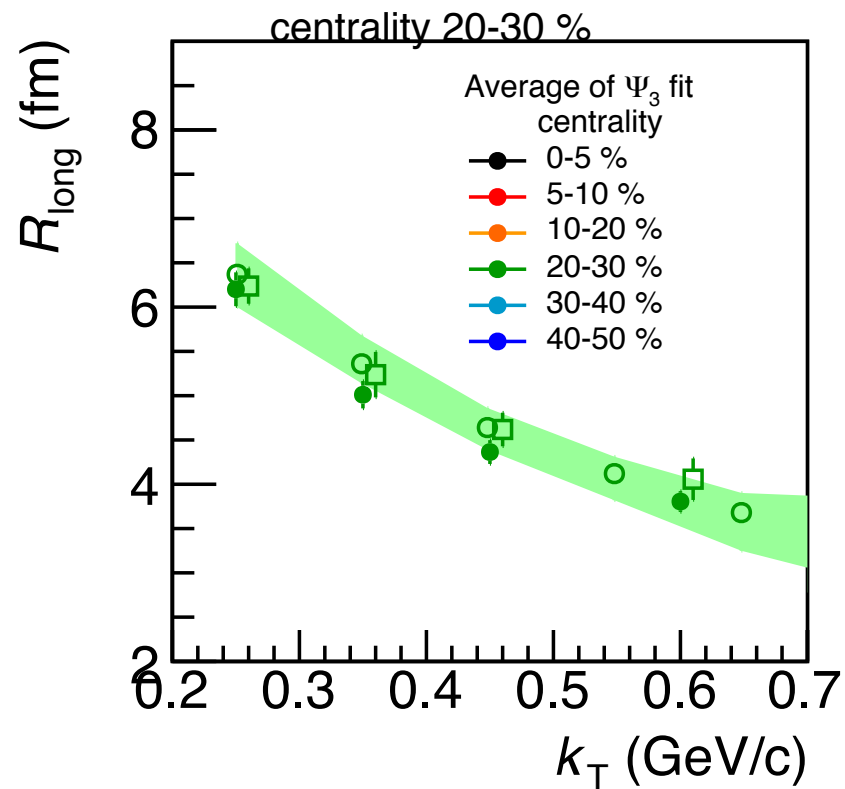
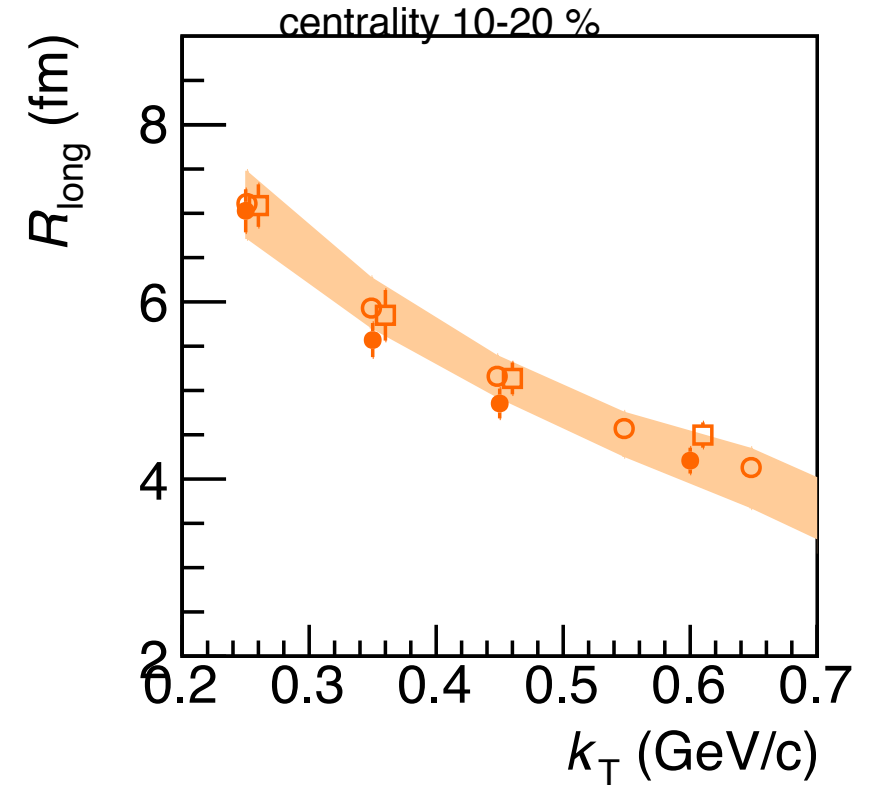
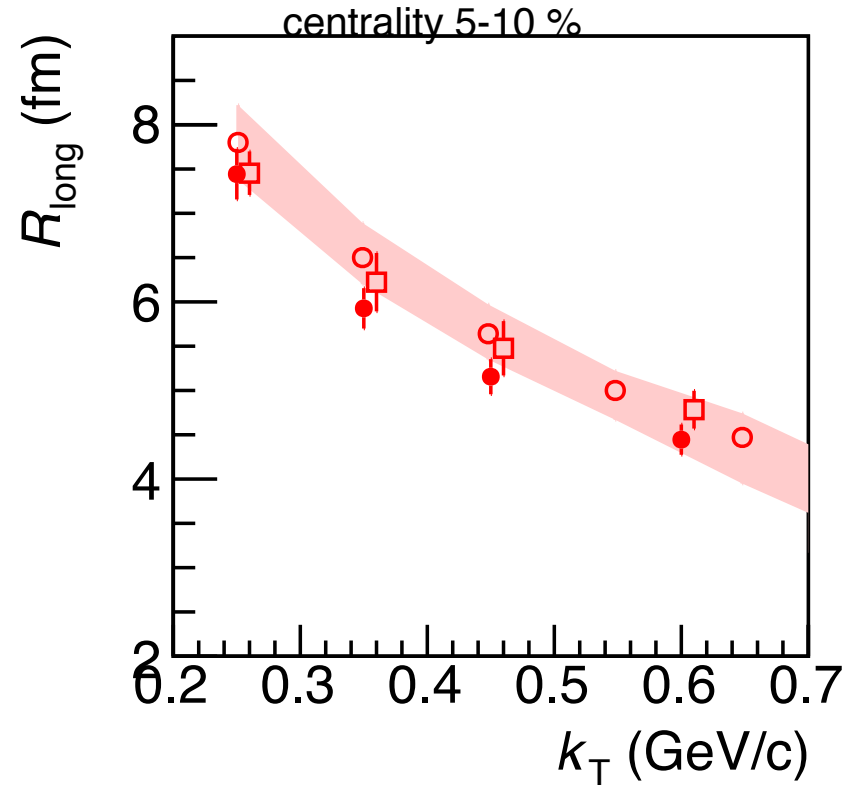
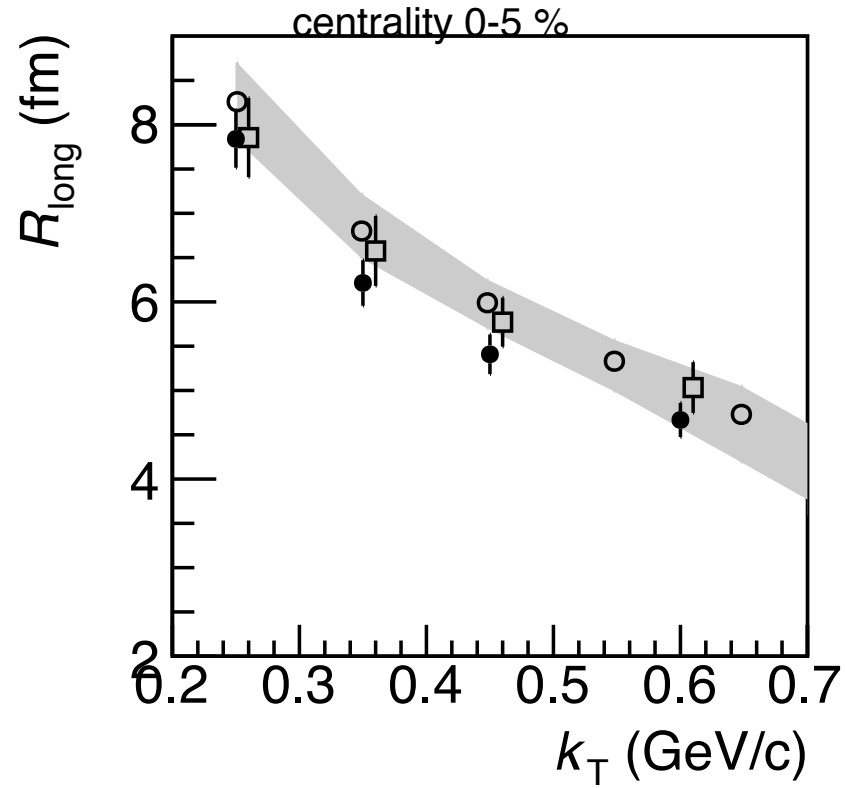
Consistency check of R_{out}



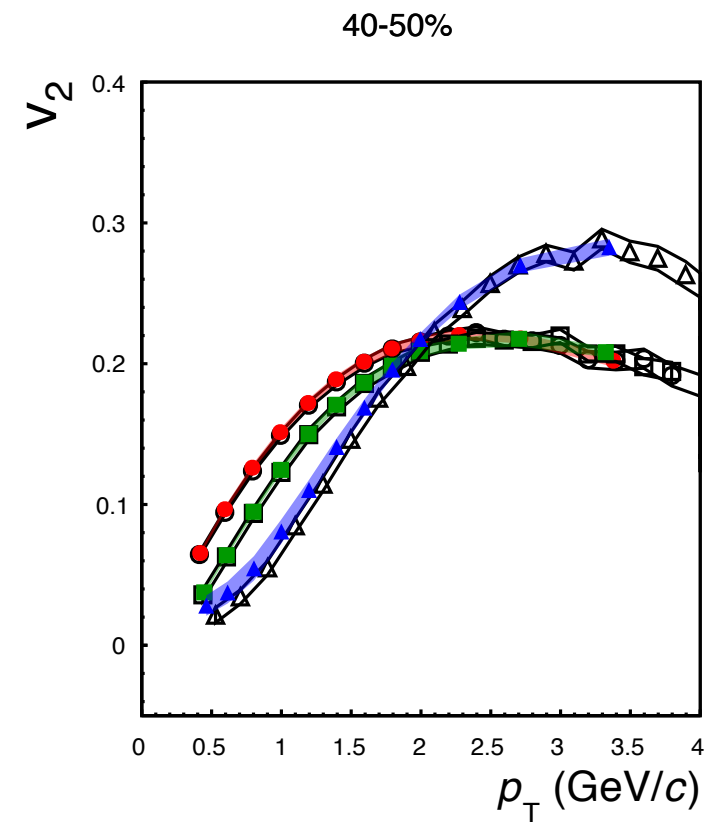
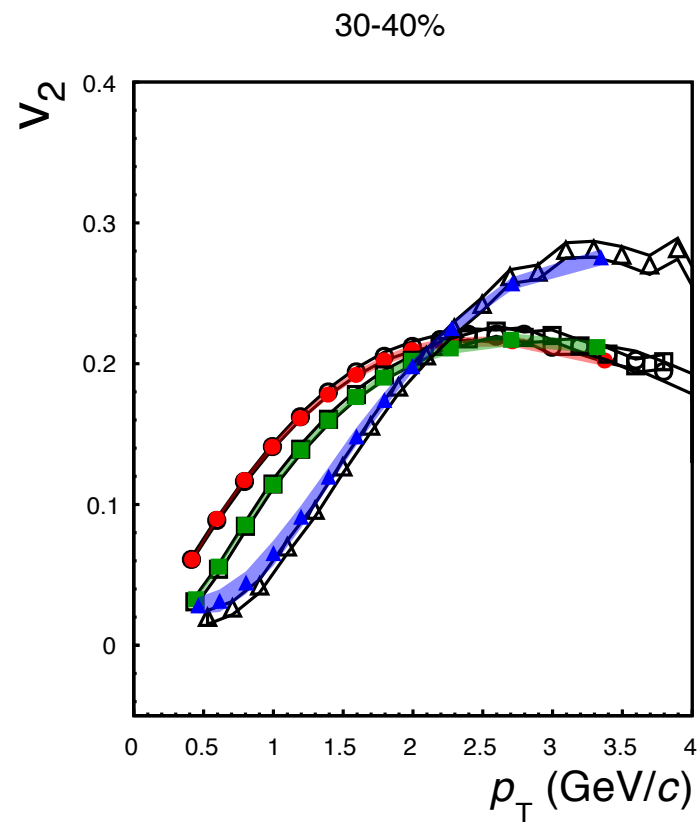
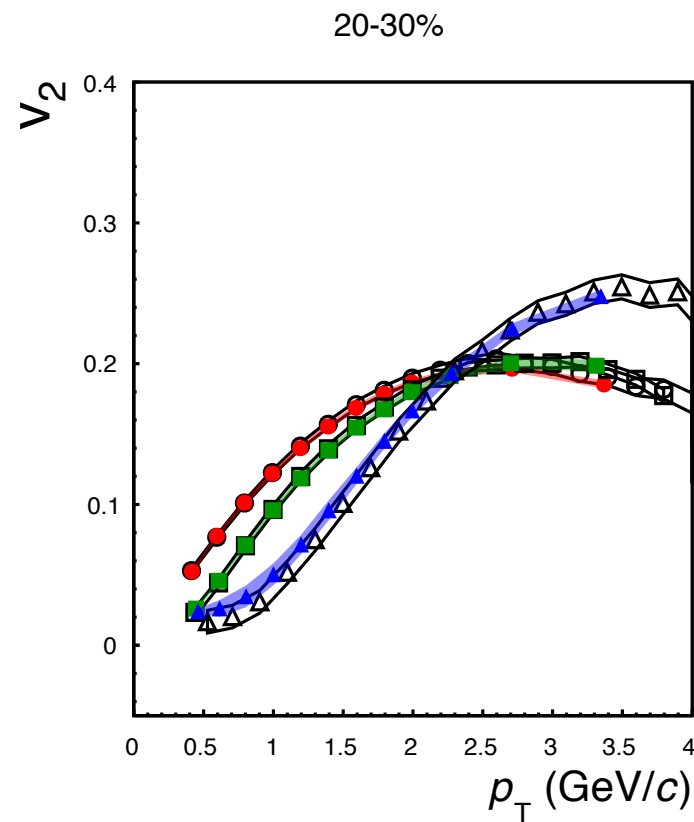
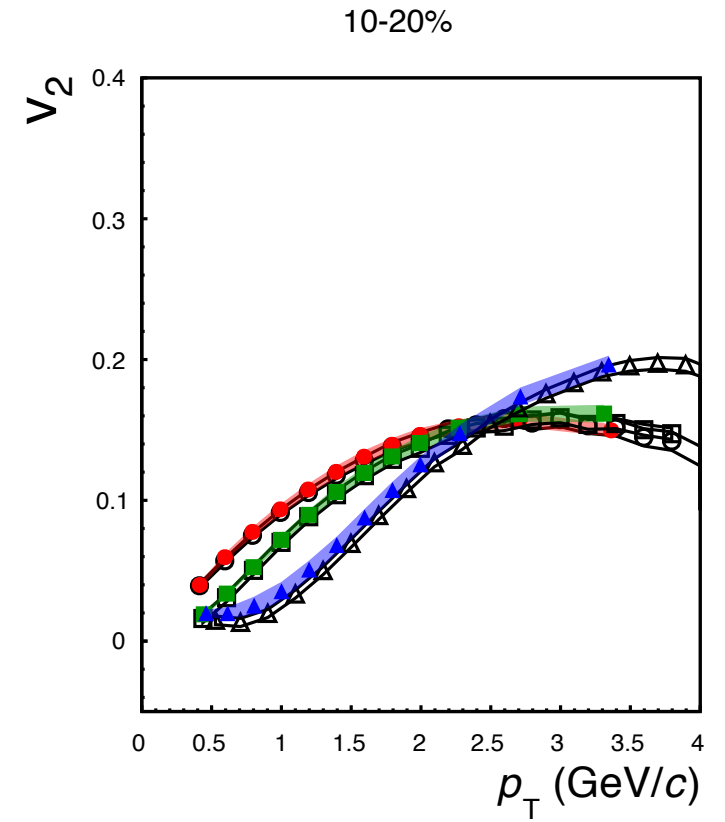
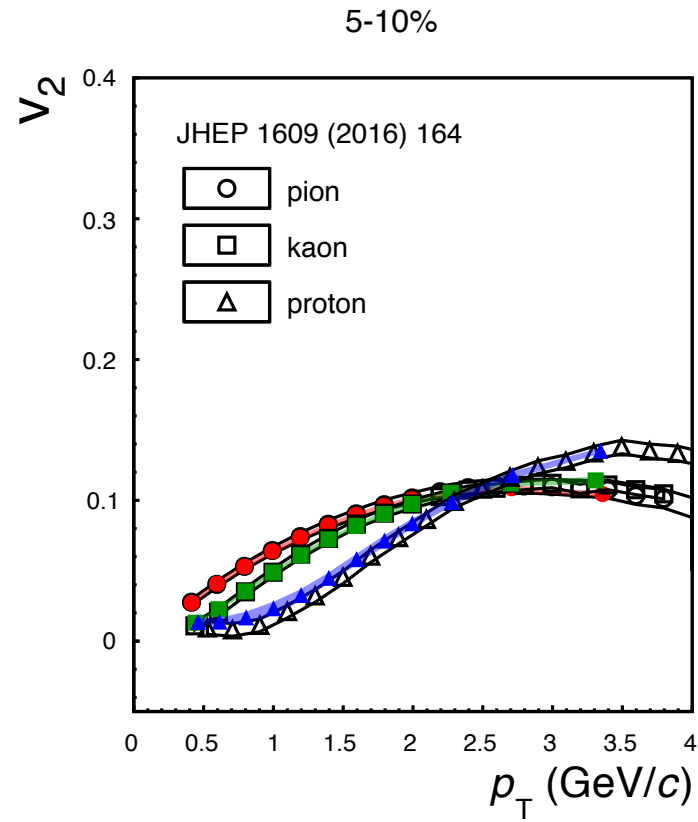
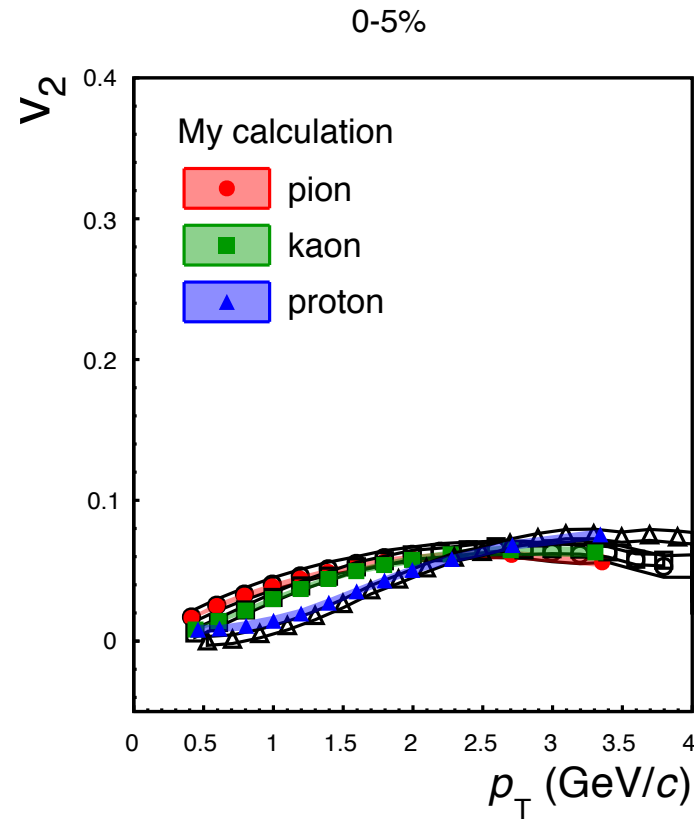
Consistency check of R_{side}



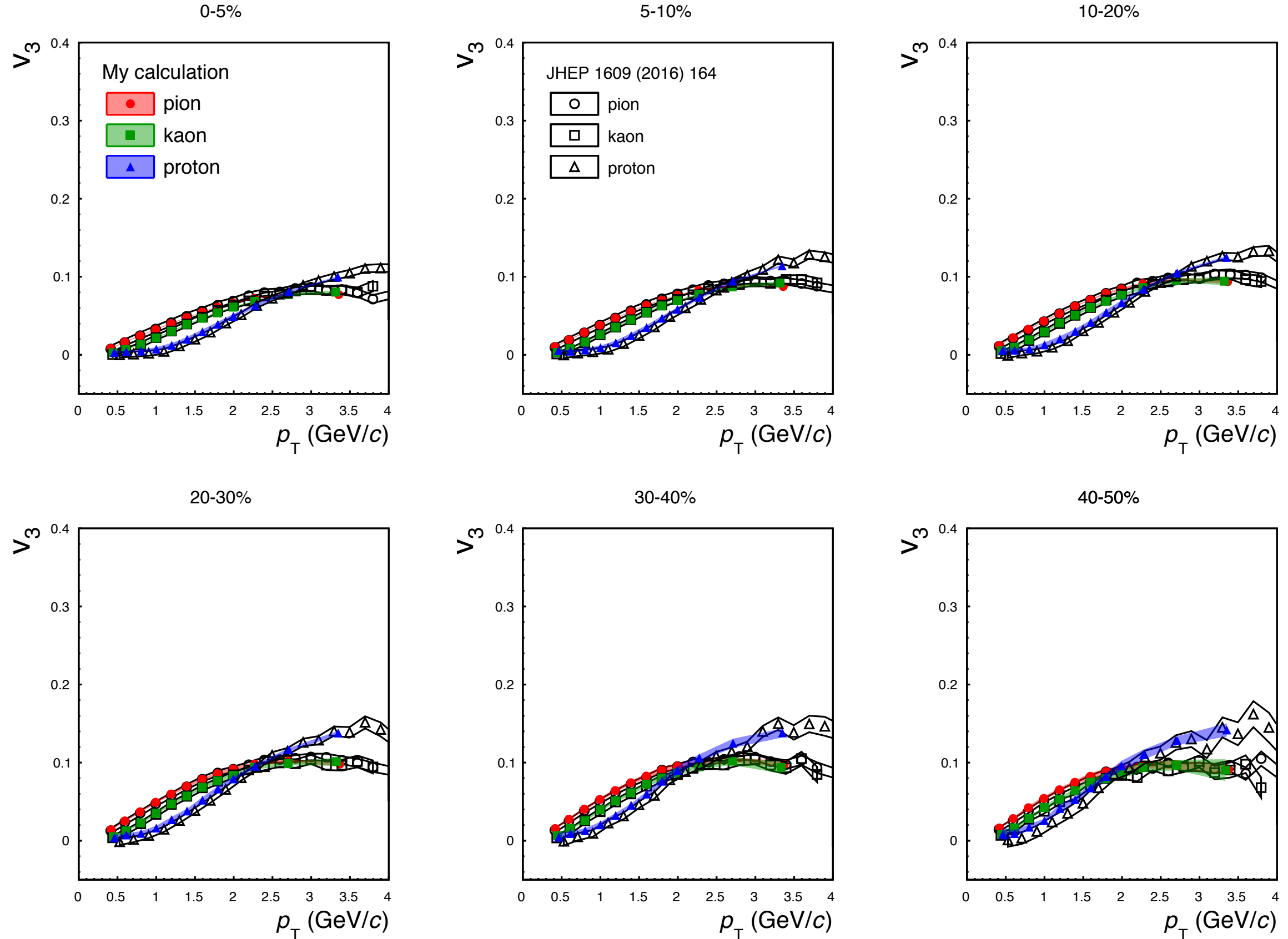
Consistency check of R_{long}



Consistency check of v_2

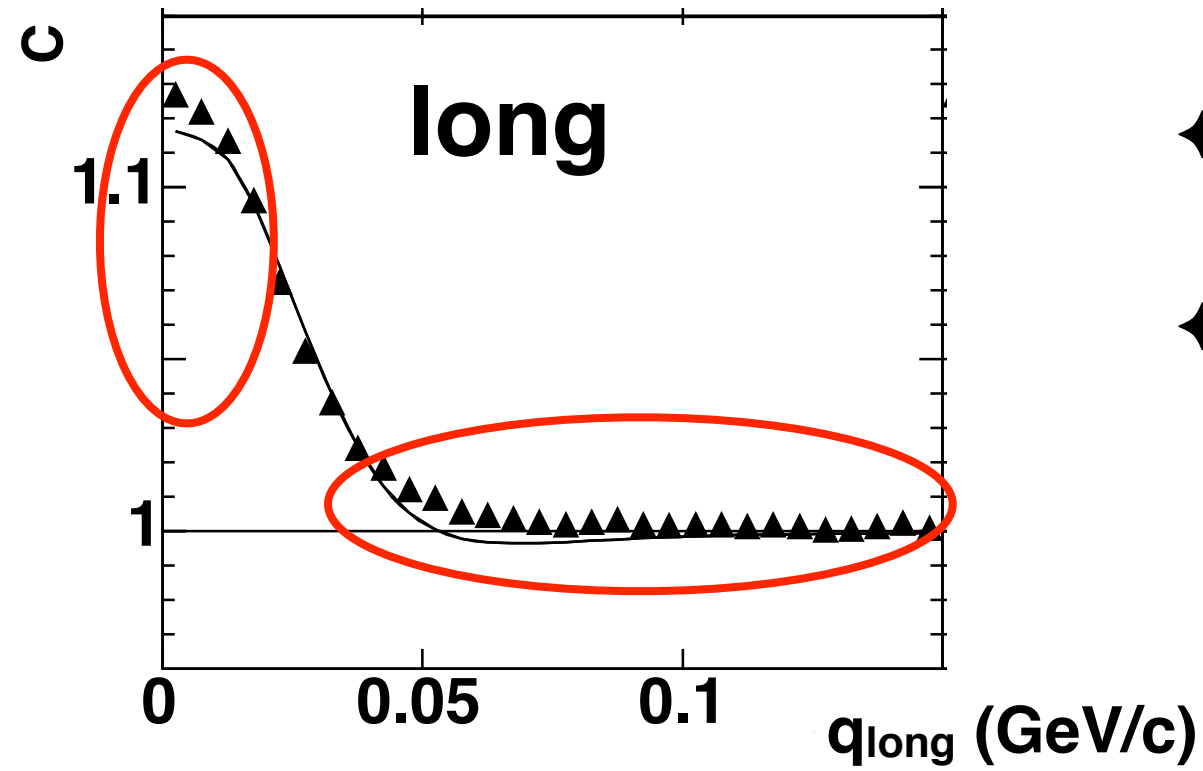
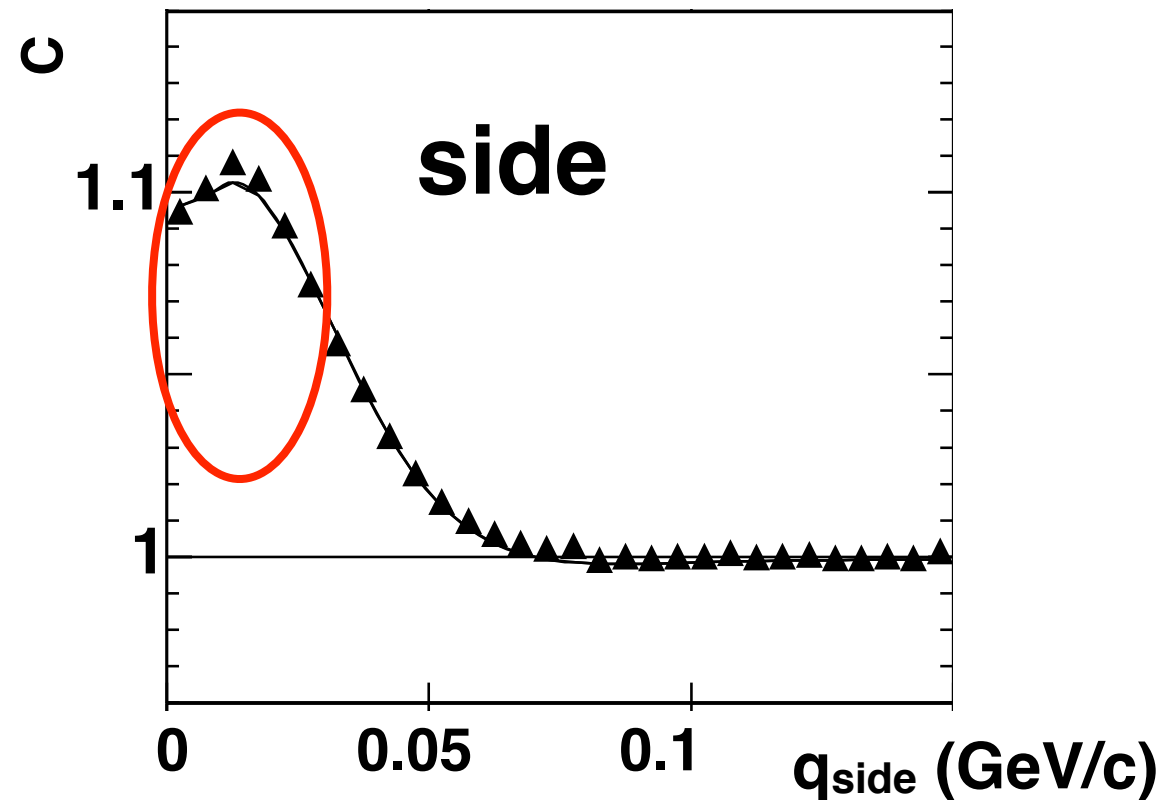
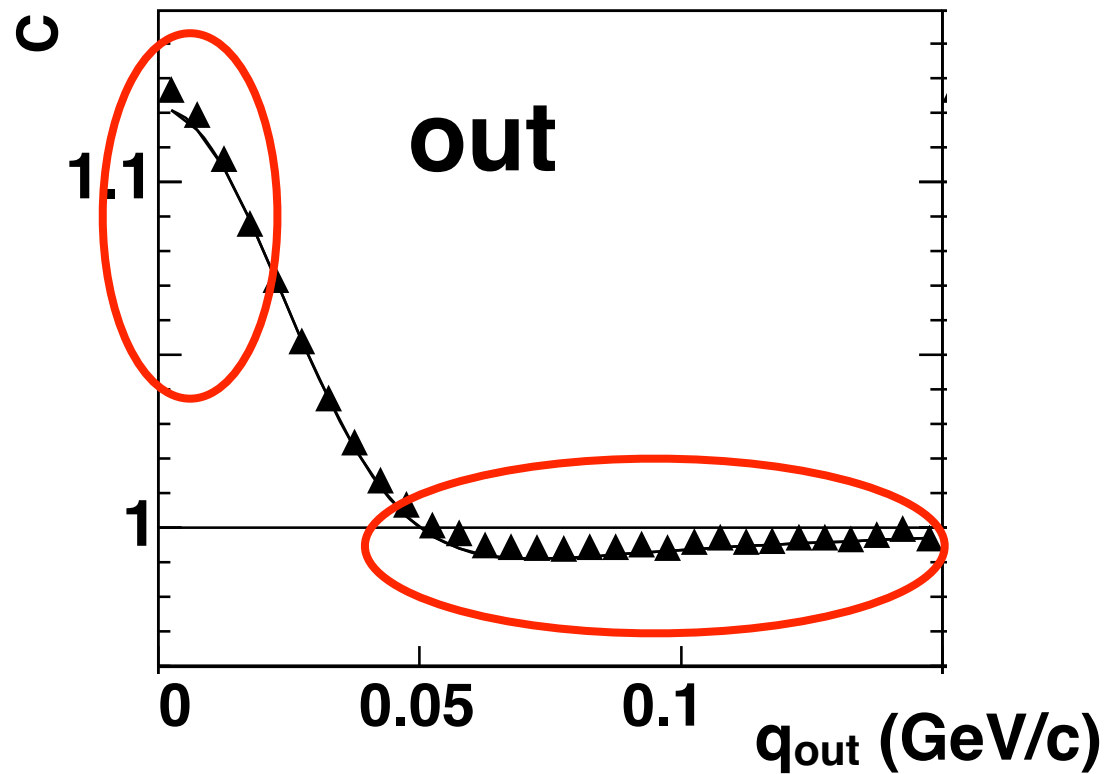


Consistency check of v_3



Measured correlation function is really gaussian ??

- ◆ Source distribution is assumed as “Gaussian distribution”
 - But measured correlation function is not “completely Gaussian”



◆ Au-Au collisions at $\sqrt{s_{\text{NN}}} = 200\text{GeV}$

- ◆ Non gaussian features can be found in
 - small q ($q < 0.02$)
 - $q : 0.04 - 0.15$ GeV/c

◆ J.Adams et al. PRC 71, 044906

Edgeworth expansion

- ◆ Correlation function is extended to non-gauss distribution
 - model independent method

$$C_2(q) = \boxed{1 + \exp(-q^2 R^2)} \times \boxed{\left(1 + \sum_n \frac{\kappa_n}{n! (\sqrt{2})^n} H_n(qR) \right)}$$

• Gauss distribution

• Non gaussian term

▸ n^{th} -order Edgeworth expansion

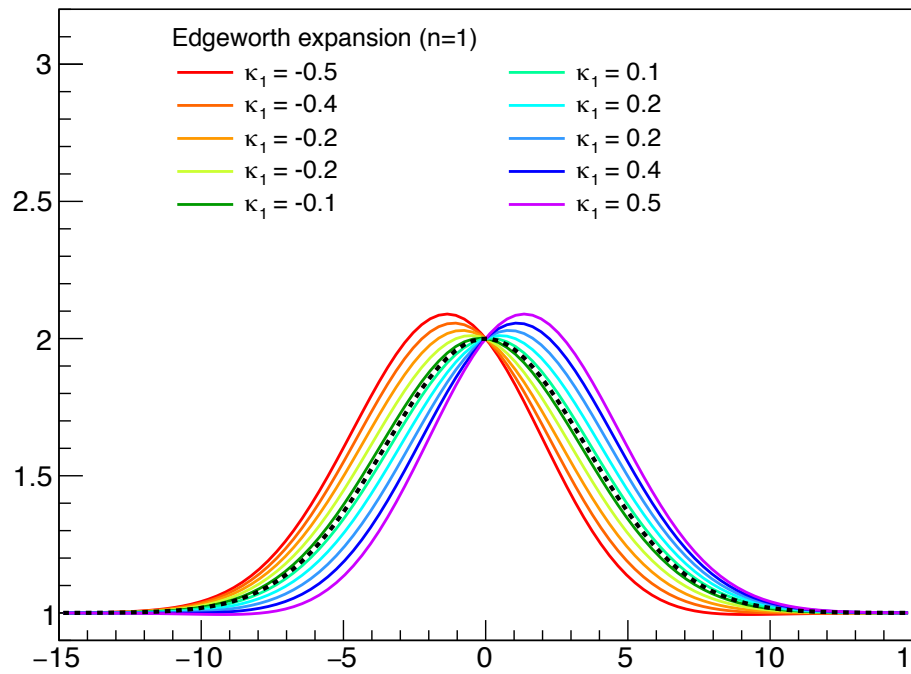
$$H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n} e^{-x^2}$$

Edgeworth expansion

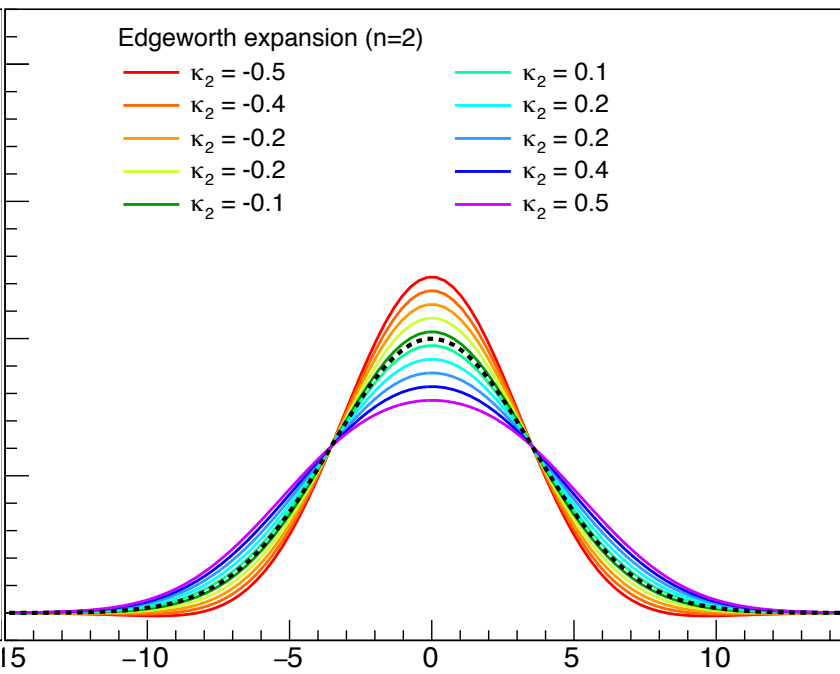
◆ How Edgeworth expansion modified gaussian shape (1st - 6th) ??

- Default gaus function is $1 + \exp(-x^2 / 25)$ **mean:0, sigma:5, height:1**

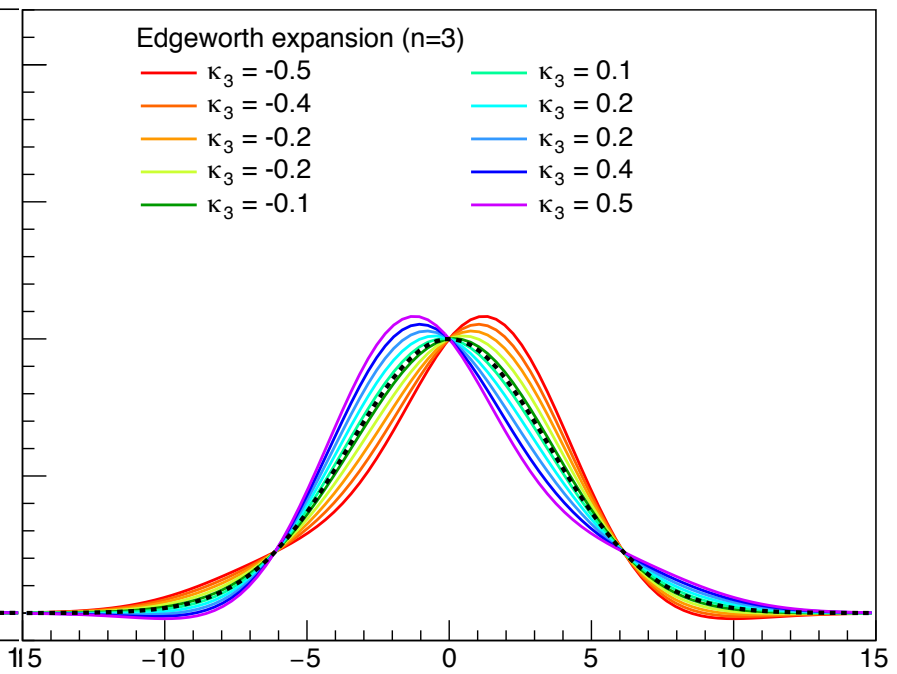
1st : mean



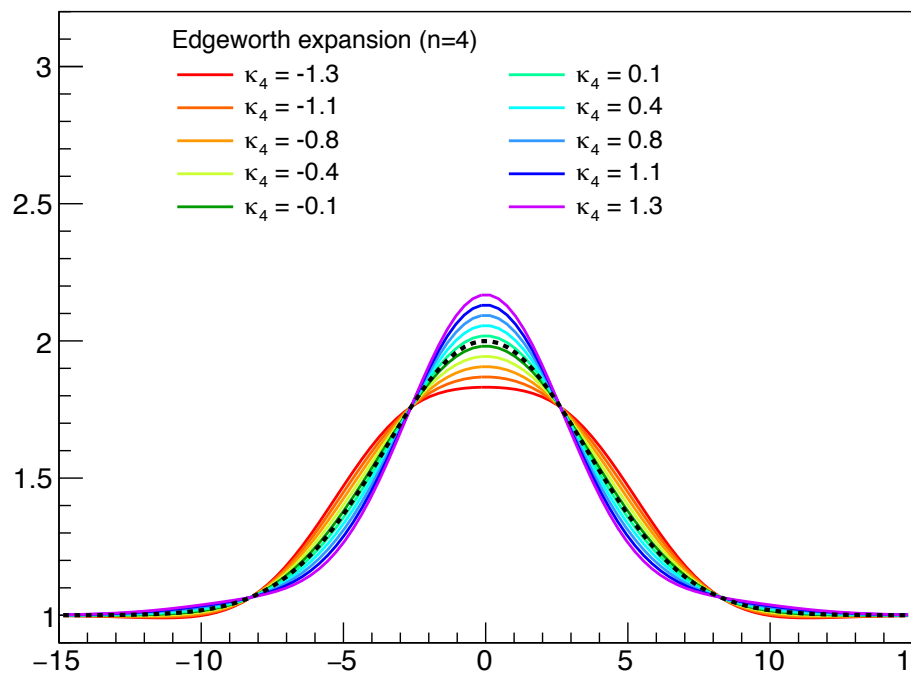
2nd : sigma



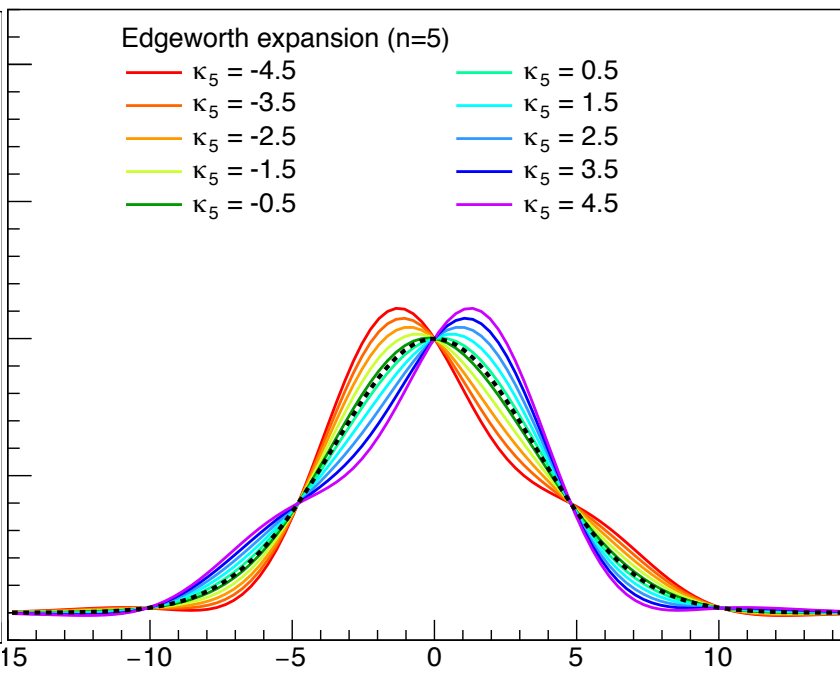
3rd : asymmetry



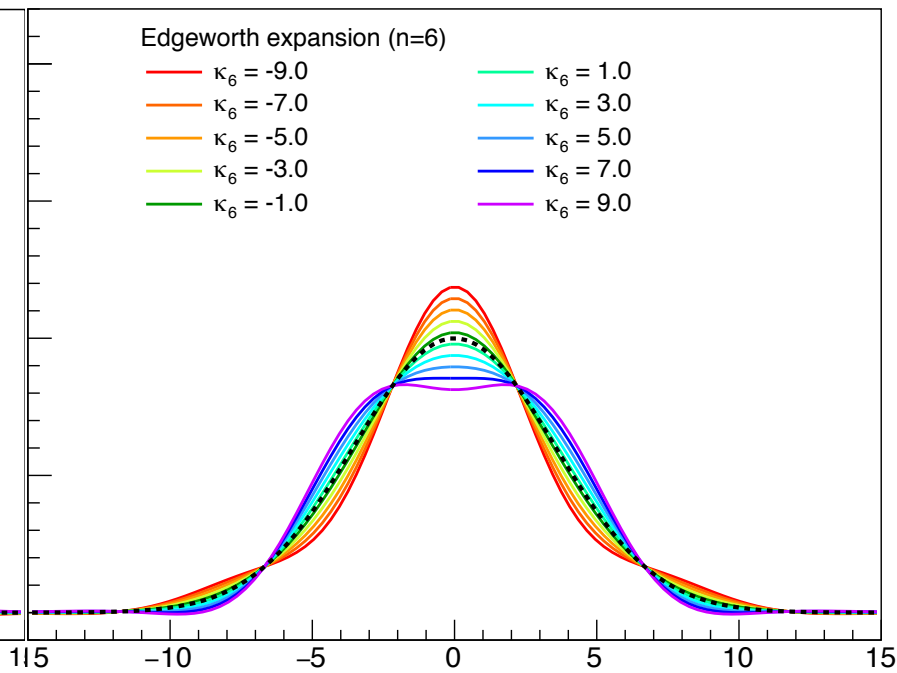
4th : sharpness



5th : asymmetry



6th : flatness



Correlation function with Edgeworth expansion

◆ Correlation function with 4th and 6th Edgeworth expansion

► Identical particles must be invariant under $(q_{\text{out}}, q_{\text{side}}, q_{\text{long}}) \rightarrow (-q_{\text{out}}, q_{\text{side}}, q_{\text{long}})$

$$C_2(q_{\text{out}}, q_{\text{side}}, q_{\text{long}}) = (1 - \lambda) + \lambda K_{\text{coul}} (1 + G \times \text{NGF})$$

$$G = \exp \left(-q_{\text{out}}^2 R_{\text{out}}^2 - q_{\text{side}}^2 R_{\text{side}}^2 - q_{\text{long}}^2 R_{\text{long}}^2 - q_{\text{out}} q_{\text{side}} R_{\text{os}}^2 - q_{\text{out}} q_{\text{long}} R_{\text{ol}}^2 - q_{\text{side}} q_{\text{long}} R_{\text{sl}}^2 \right)$$

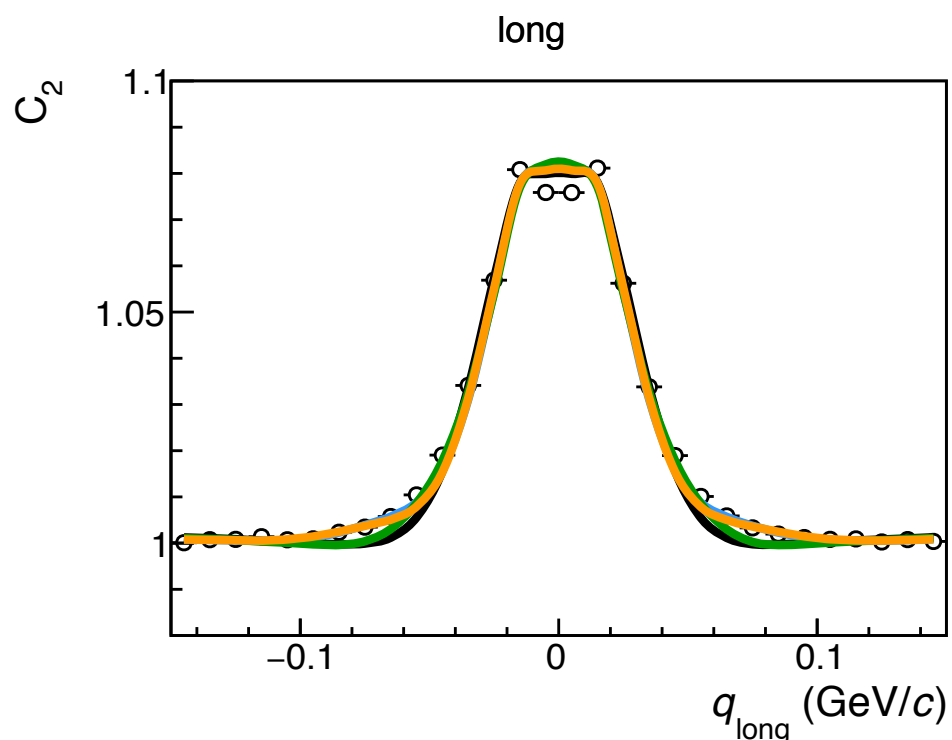
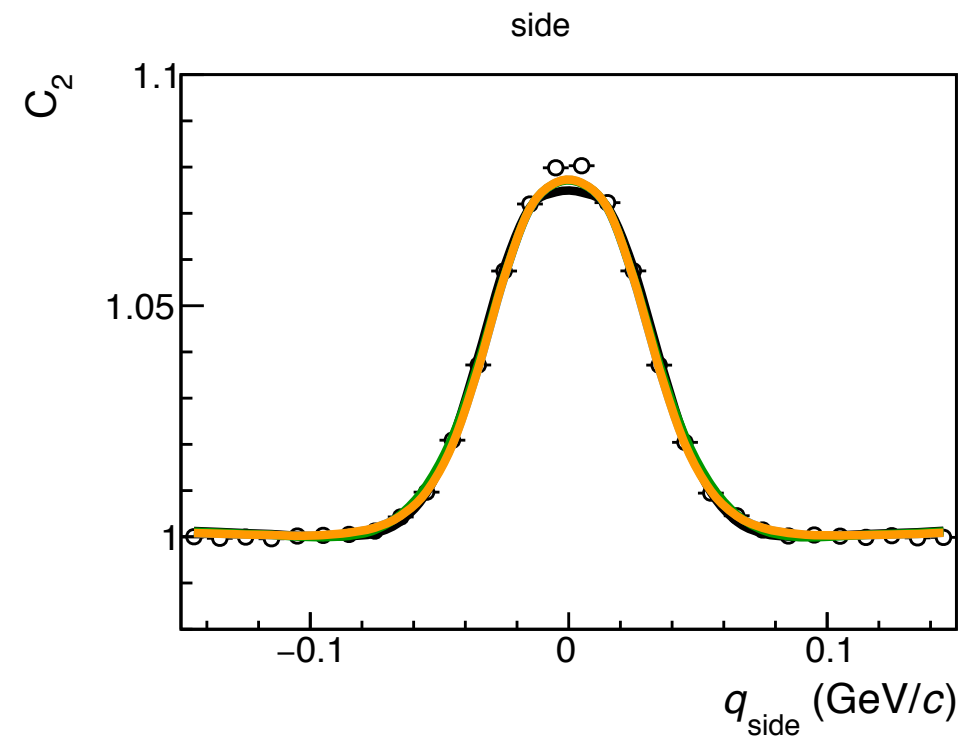
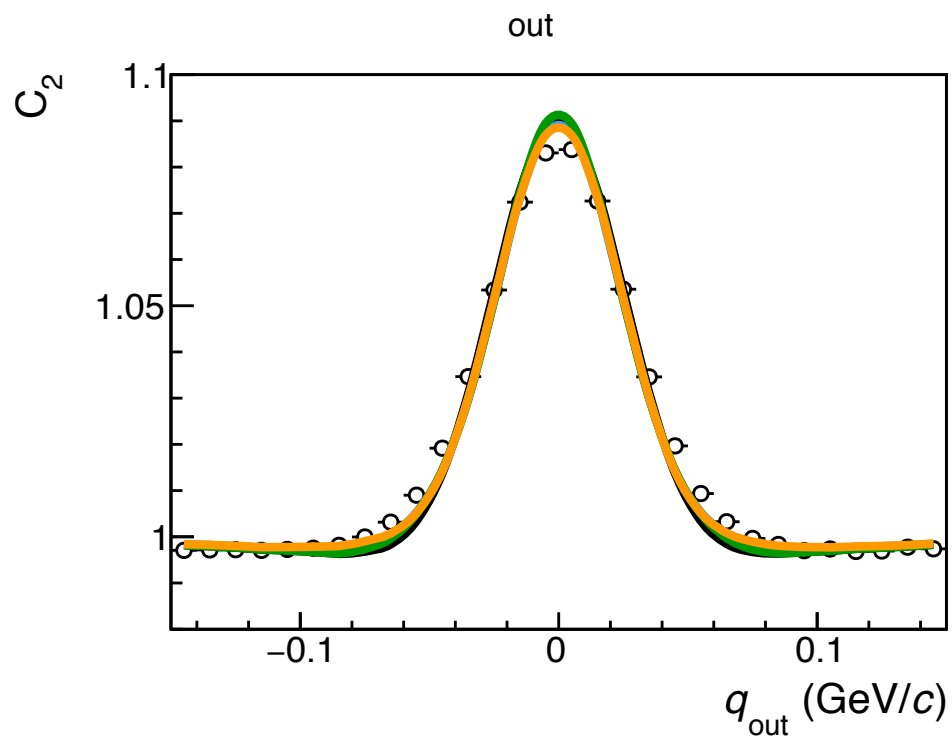
$$\begin{aligned} \text{NGF} = & \left(1 + \sum_{n=4,6}^{\infty} \frac{\kappa_n}{n! (\sqrt{2})^n} H_n(q_{\text{out}} R_{\text{out}}) \right) \\ & \times \left(1 + \sum_{n=4,6}^{\infty} \frac{\kappa_n}{n! (\sqrt{2})^n} H_n(q_{\text{side}} R_{\text{side}}) \right) \\ & \times \left(1 + \sum_{n=4,6}^{\infty} \frac{\kappa_n}{n! (\sqrt{2})^n} H_n(q_{\text{long}} R_{\text{long}}) \right) \end{aligned}$$

$$H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n} e^{-x^2}$$

Correlation function with Edgeworth expansion

◆ Correlation function with 4th and 6th Edgeworth expansion

- Gauss (+ coulomb) distribution could not describe tail shape of out, long
- 4th-order Edgeworth expansion is effective to non-gaussian shape of C_2



— No Expansion

— κ_4

— κ_6

— κ_4 and κ_6

Azimuthal angle dependence of HBT radii

- ◆ Extracted HBT radii as a function of $\phi_{\text{pair}} - \Psi_2$
 - Edgeworth expansion makes smaller extracted HBT radii
 - Not only average HBT radii but also oscillation has changed in R_{out}

