QGP and Hadrons in Dense medium: a holographic view

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based on works with

X. Ge, Y. Matsuo, F. Shu, T. Tsukioka(APCTP), archiv:0806.4460

J. Hur, K.Kim (KIAS), archiv:0809.4541

S. Seo (HYU), JHEP 0804:010,2008

Introduction

- String theory began as a hadron theory
- After 40 years
  string theory is back to Str. Int. with
  the idea of gague/gravity duality.
- Can we do something for QCD?
- N=4 SYM has difference as much as
  similarity: SUSY, extra deg. of freedom
- appeal to universality-> hydro.

Universality and AdS/QCD

- High temperature phase
  -> Broken SUSY -> Share more with QCD
- a quantity in Hydrodynamics ->
  Low frequency and Long wave length
- As far as dual gravity is valid-> $\eta/s = 1/4\pi$
- perhaps, ads/qcd in medium is more relevant to qcd than in vacuum.

Plan

- Hydrodynamics I: Green function and Kubo formula [archiv:0806.4460]
- Hydrodynamics II: Metallic QGP [archiv:0809.4541]
- Baryons in finite chemical potential [JHEP 0804:010,2008]

Hydrodynamics I:
Transport coefficients

X. Ge, Y. Matsuo, F. Shu, T. Tsukioka(APCTP), archiv:0806.4460

- Finite temperature (and density)
  <-> (charged) black hole

- Linear Response theory:
  Transport coefficients <- Kubo formula
  <- zero frequency limit of G_R
  <- ads/cft boundary action.

AdS R-N black hole

$$S[g_{mn}, \mathcal{A}_m] = \frac{1}{2\kappa^2}\int d^5x\sqrt{-g}\Big(R - 2\Lambda\Big) - \frac{1}{4e^2}\int d^5x\sqrt{-g}\mathcal{F}_{mn}\mathcal{F}^{mn},$$

$$\frac{l^3}{\kappa^2} = \frac{N_c^2}{4\pi^2}. \qquad \frac{l}{e^2} = \frac{N_c N_f}{(2\pi)^2}, \qquad \frac{e^2}{\kappa^2} = \frac{N_c}{N_f} l^{-2}.$$

- Solution to the eq. of M:

$$ds^2 = \frac{r^2}{l^2}\Big(-f(r)(dt)^2 + \sum_{i=1}^{3}(dx^i)^2\Big) + \frac{l^2}{r^2 f(r)}(dr)^2,$$

$$\mathcal{A}_t = -\frac{Q}{r^2} + \mu,$$

$$f(r) = 1 - \frac{ml^2}{r^4} + \frac{q^2 l^2}{r^6}, \qquad \Lambda = -\frac{6}{l^2},$$

Thermodynamics

- temperature, entropy, energy

$$T = \frac{r_+^2 f'(r_+)}{4\pi l^2} = \frac{r_+}{\pi l^2}\Big(1 - \frac{1}{2}\frac{q^2 l^2}{r_+^6}\Big) \equiv \frac{1}{2\pi b}\Big(1 - \frac{a}{2}\Big),$$

$$s = \frac{2\pi r_+^3}{\kappa^2 l^3} = \frac{\pi l^3}{4\kappa^2 b^3},$$
$$\epsilon = \frac{3m}{2\kappa^2 l^3} = \frac{3l^3}{32\kappa^2 b^4}\Big(1 + a\Big),$$

$$p = \frac{\epsilon}{3},$$
$$\mu = \frac{Q}{r_+^2},$$
$$\rho = \frac{2Q}{e^2 l^3}.$$

- pressure, charge density and chemical pot.

Perturbations around RN

$$g_{mn} \equiv g^{(0)}_{mn} + h_{mn},$$
$$\mathcal{A}_m \equiv A^{(0)}_m + A_m,$$

- perturbed Eintein eq.

$$R^{(1)}_{mn} - \frac{1}{2} g^{(0)}_{mn} R^{(1)} - \frac{1}{2} h_{mn} R^{(0)} + h_{mn}\Lambda = \kappa^2 T^{(1)}_{mn}.$$

$$R^{(1)}_{mn} = \frac{1}{2}\Big(\nabla_k \nabla_m h_n{}^k + \nabla_k \nabla_n h_m{}^k - \nabla_k \nabla^k h_{mn} - \nabla_m \nabla_n h\Big)$$
$$R^{(1)} = g^{(0)kl} R^{(1)}_{kl} - h^{kl} R^{(0)}_{kl}$$

$$= \nabla_k \nabla_l h^{kl} - \nabla_k \nabla^k h - h^{kl} R^{(0)}_{kl},$$
$$T^{(1)}_{mn} = \frac{1}{e^2}\Big(- F^{(0)}_{mk} F^{(0)}_{nl} h^{kl} + \frac{1}{2} g^{(0)}_{mn} F^{(0)}_{kp} F^{(0)}{}_l{}^p h^{kl} - \frac{1}{4} h_{mn} F^{(0)}_{kl} F^{(0)kl}$$
$$+ F^{(0)}_{mk} F_n{}^k + F^{(0)}_{nk} F_m{}^k - \frac{1}{2} g^{(0)}_{mn} F^{(0)}_{kl} F^{kl}\Big),$$

- perturbed Maxwell eq.

$$
\begin{aligned}
0 &= \nabla_m \Big(F^{mn} - F^{(0)m}{}_k h^{nk} + F^{(0)n}{}_k h^{mk} + \frac{1}{2} F^{(0)mn} h \Big) \\
&= \frac{1}{\sqrt{-g^{(0)}}} \partial_m \Big\{ \sqrt{-g^{(0)}} \Big(g^{(0)mk} g^{(0)nl} (\partial_k A_l - \partial_l A_k) \\
&\qquad\qquad - F^{(0)m}{}_k h^{nk} + F^{(0)n}{}_k h^{mk} + \frac{1}{2} F^{(0)mn} h \Big) \Big\}.
\end{aligned}
$$

- Gauge choice and Fourier mode

$h_{rm}(x) = 0$ and $A_r(x) = 0$ gauges

$$
\begin{aligned}
h_{\mu\nu}(t,z,r) &= \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \, \mathrm{e}^{-i\omega t + ikz} h_{\mu\nu}(k,r), \\
A_\mu(t,z,r) &= \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \, \mathrm{e}^{-i\omega t + ikz} A_\mu(k,r),
\end{aligned}
$$

we choose the momenta which are along the z-direction.

Classification of modes

- vector type: $h_{xt} \neq 0, \;\; h_{xz} \neq 0, \;\; (\text{others}) = 0$

$$\Big(\text{equivalently}, \; h_{yt} \neq 0, \;\; h_{yz} \neq 0, \;\; (\text{others}) = 0\Big)$$

- tensor type: $h_{xy} \neq 0, \;\; h_{xx} = -h_{yy} \neq 0, \;\; (\text{others}) = 0$
- scalar type: $h_{tz} \neq 0$, $h_{tt} \neq 0$, $h_{xx} = h_{yy} \neq 0$, and $h_{zz} \neq 0$, $(\text{others}) = 0$

- O(2) classification.
- We consider vector and scalar types

Vector type pert.

$$h_{xt}(x) \neq 0, \quad h_{xz}(x) \neq 0, \quad A_x(x) \neq 0, \quad (\text{others}) = 0.$$

- def. new variables:

$$u = r_+^2/r^2$$

$$B(u) \equiv \frac{A_x(u)}{\mu} = \frac{l^4}{4Qb^2} A_x(u)$$

$$h_t^x(r) = g^{(0)xx} h_{xt}(r) = (l^2/r^2) h_{xt}(r)$$

$$h_z^x(r) = g^{(0)xx} h_{xz}(r) = (l^2/r^2) h_{xz}(r)$$

$$h_t^x(r) = g^{(0)xx} h_{xt}(r) = (l^2/r^2) h_{xt}(r)$$

- eq. of Motion

$$0 = h_t^{x\prime\prime} - \frac{1}{u} h_t^{x\prime} - \frac{b^2}{uf}\left(\omega k h_z^x + k^2 h_t^x\right) - 3auB',$$

$$0 = kfh_z^{x\prime} + \omega h_t^{x\prime} - 3a\omega uB,$$

$$0 = h_z^{x\prime\prime} + \frac{(u^{-1}f)'}{u^{-1}f} h_z^{x\prime} + \frac{b^2}{uf^2}\left(\omega^2 h_z^x + \omega k h_t^x\right),$$

$$0 = B'' + \frac{f'}{f} B' + \frac{b^2}{uf^2}\left(\omega^2 - k^2 f\right) B - \frac{1}{f} h_t^{x\prime},$$

$$f(u) = (1-u)(1+u-au^2)$$

- Remark: q=0-> A-h decouples

Master equations

- $$0 = \Phi_\pm{}'' + \frac{(r^{-1}f)'}{r^{-1}f}\Phi_\pm{}' + \frac{l^4}{r^4 f^2}\Big(\omega^2 - k^2 f\Big)\Phi_\pm - \frac{l^8 C_\pm}{4b^4 r^6 f}\Phi_\pm$$

$$C_\pm = (1+a) \pm \sqrt{(1+a)^2 + 3ab^2k^2},$$

$$\Phi_\pm \equiv -\frac{8b^4}{l^8} r^5 h_t^{x\prime} + \Big(-\frac{3al^4}{4Qb^2} + \frac{C_\pm}{Q} r^2\Big) A_x$$

- Similarly tensor type has simple eq.

$$h_{xy}(x) \neq 0, \qquad h_{xx}(x) = -h_{yy}(x), \qquad (\text{others}) = 0.$$

$$0 = h_y^{x\prime\prime} + \frac{(r^5 f)'}{r^5 f} h_y^{x\prime} + \frac{l^4}{r^4 f^2}\Big(\omega^2 - k^2 f\Big) h_y^x.$$

Diffusion pole

$$0 = \Phi_\pm{}'' + \frac{(u^2 f)'}{u^2 f}\Phi_\pm{}' + \frac{b^2}{uf^2}\Big(\omega^2 - k^2 f\Big)\Phi_\pm - \frac{C_\pm}{f}\Phi_\pm$$

- Infalling BC at horizon

$$\Phi_-(u) = (1-u)^\nu F_-(u) \qquad \nu = -i\frac{\omega}{4\pi T},$$

- perturbative solution near boundary

$$F_-(u) = F_0(u) + \omega F_1(u) + k^2 G_1(u) + \mathcal{O}(\omega^2,\ \omega k^2),$$

$$F_0(u) = C, \quad (\text{const.}),$$

$$F_1(u) = iCb\Bigg\{\frac{1+2a-2a^2}{2\sqrt{1+4a}(2-a)}\Bigg(\log\left(\frac{1-\frac{1-2au}{\sqrt{1+4a}}}{1-\frac{1-2a}{\sqrt{1+4a}}}\right) - \log\left(\frac{1+\frac{1-2au}{\sqrt{1+4a}}}{1+\frac{1-2a}{\sqrt{1+4a}}}\right)\Bigg)$$
$$+1-\frac{1}{u}+\frac{1}{2(2-a)}\log\left(\frac{1+u-au^2}{2-a}\right)\Bigg\},$$

$$G_1(u) = \frac{Cb^2}{2(1+a)}\Big(-1+\frac{1}{u}\Big).$$

On-shell action

$$S[h_t^x, h_z^x, B] = \frac{l^3}{32\kappa^2 b^4}\int \frac{\mathrm{d}^4 k}{(2\pi)^4}\Big\{\frac{1}{u}h_t^x(-k,u)h_t^{x\prime}(k,u) - \frac{1}{u^2}h_t^x(-k,u)h_t^x(k,u)$$
$$-\frac{f(u)}{u}h_z^x(-k,u)h_z^{x\prime}(k,u) + \frac{f(u)}{u^2}h_z^x(-k,u)h_z^x(k,u)$$
$$-3af(u)B(-k,u)\Big(B'(k,u) - \frac{1}{f(u)}h_t^x(k,u)\Big)\Big\}\Big|_{u=0}^{u=1}$$

$$h_t^{x\prime}(\varepsilon) = -b^2\Big(\omega k (h_z^x)^0 + k^2 (h_t^x)^0\Big)$$
$$+\frac{\varepsilon}{i\omega - \frac{b}{2(1+a)}k^2}\Big\{b\Big(\omega k (h_z^x)^0 + k^2 (h_t^x)^0\Big) + 3ia\omega (B)^0 + \mathcal{O}(\omega^2 k, \omega k^2)\Big\}$$
$$+\mathcal{O}(\varepsilon^2),$$

- Others are similar.

Correlation functions

$$G_{xt\ xt}(\omega,k) = \frac{l^3}{16\kappa^2 b^3}\left(\frac{k^2}{i\omega - Dk^2}\right),$$

$$G_{xt\ xz}(\omega,k) = G_{xz\ xt}(\omega,k) = -\frac{l^3}{16\kappa^2 b^3}\left(\frac{\omega k}{i\omega - Dk^2}\right),$$

$$G_{xz\ xz}(\omega,k) = \frac{l^3}{16\kappa^2 b^3}\left(\frac{\omega^2}{i\omega - Dk^2}\right),$$

$$G_{xt\ x}(\omega,k) = G_{x\ xt}(\omega,k) = -\frac{2Q}{e^2 l^3}\left(\frac{i\omega}{i\omega - Dk^2}\right),$$

$$G_{xz\ x}(\omega,k) = G_{x\ xz}(\omega,k) = \frac{Qb}{(1+a)e^2 l^3}\left(\frac{\omega k}{i\omega - Dk^2}\right),$$

$$G_{x\ x}(\omega,k) = \frac{3al}{4(1+a)b^2 e^2}\left(\frac{i\omega}{i\omega - Dk^2}\right) - \frac{(2-a)^2 l}{8(1+a)^2 b e^2} i\omega,$$

- Diffusion constant: $D = \frac{b}{2(1+a)}$

diffusion const.

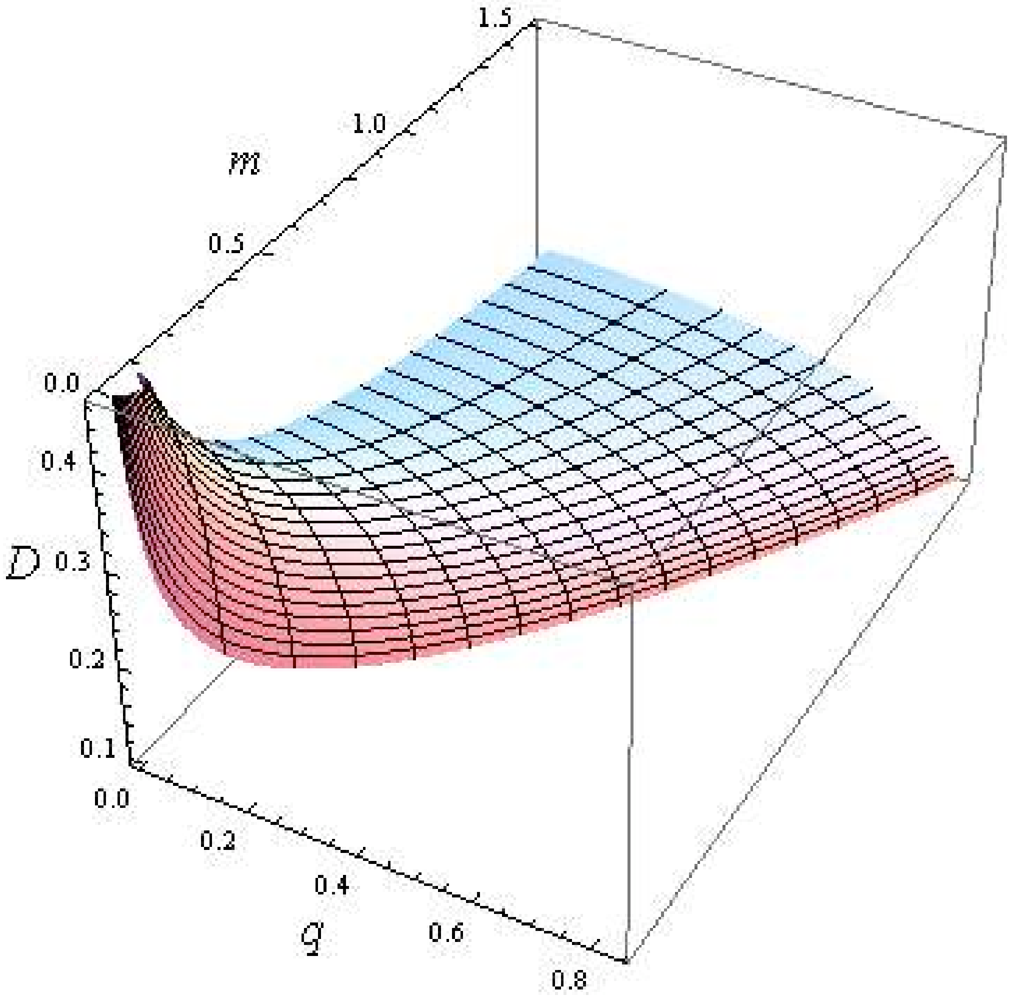


Figure 1: D vs. q and m ($l = 1$)

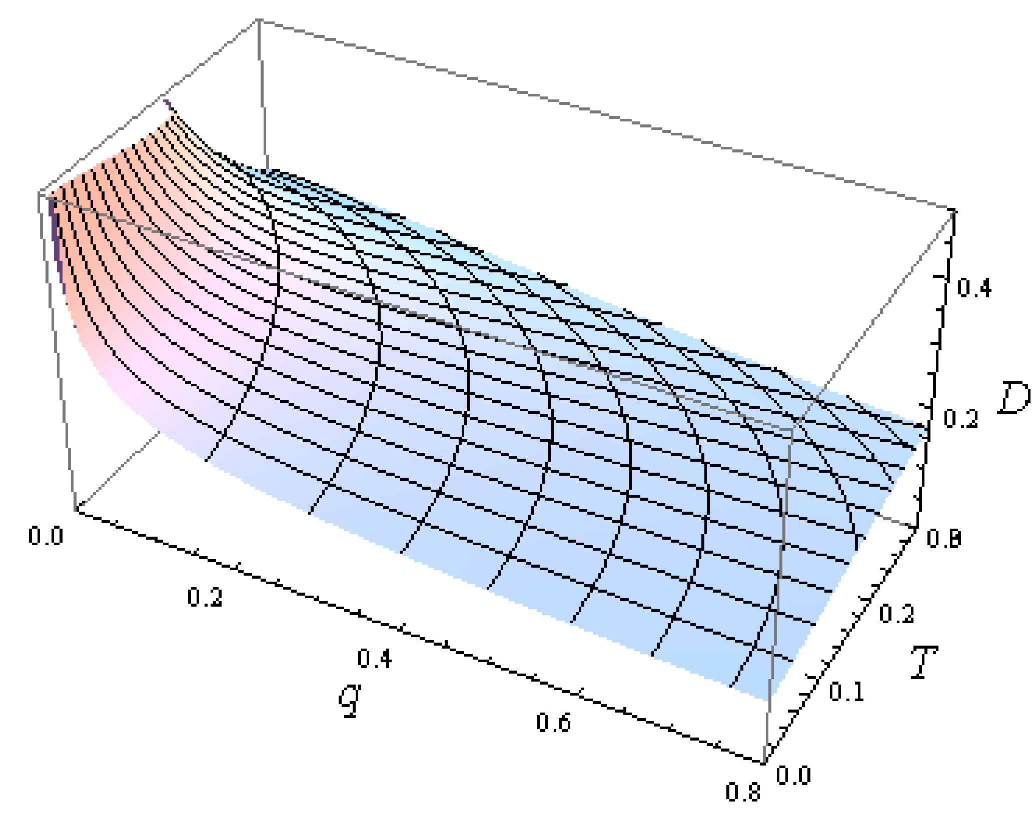


Figure 2: D vs. q and T ($l = 1$)

shear Viscosity

- Kubo formula

$$\eta = -\lim_{\omega\to 0}\frac{\mathrm{Im}(G(\omega,0))}{\omega}$$

- Green functions for tensor pert.

$$G_{xy\ xy}(\omega,k) = G_{xx\ xx}(\omega,k) = G_{yy\ yy}(\omega,k) = -\frac{l^3}{16\kappa^2 b^3}\Big(i\omega + bk^2\Big),$$

- Shear viscosity

$$\eta = \frac{N_c^2}{8\pi^2}r_+^3 = \frac{\pi N_c^2 T^3}{8}\left(\frac{1}{2}+\sqrt{\frac{1}{4}+\frac{1}{6}\left(\frac{\mu}{g\pi T}\right)^2}\right)^3$$

Transport coefficients.

- Thermal conductivity

$$\kappa_T = -\frac{(\epsilon+p)^2}{\rho^2 T}\lim_{\omega\to 0}\frac{\mathrm{Im}(G(\omega,0))}{\omega},$$
$$= 2\pi^2\frac{N_c}{N_f}\frac{\eta T}{\mu^2}$$

- Consistency:

$$D = \frac{\eta}{\epsilon+p},$$

$$\frac{\eta}{s} = \frac{1}{4\pi}.$$

viscosity

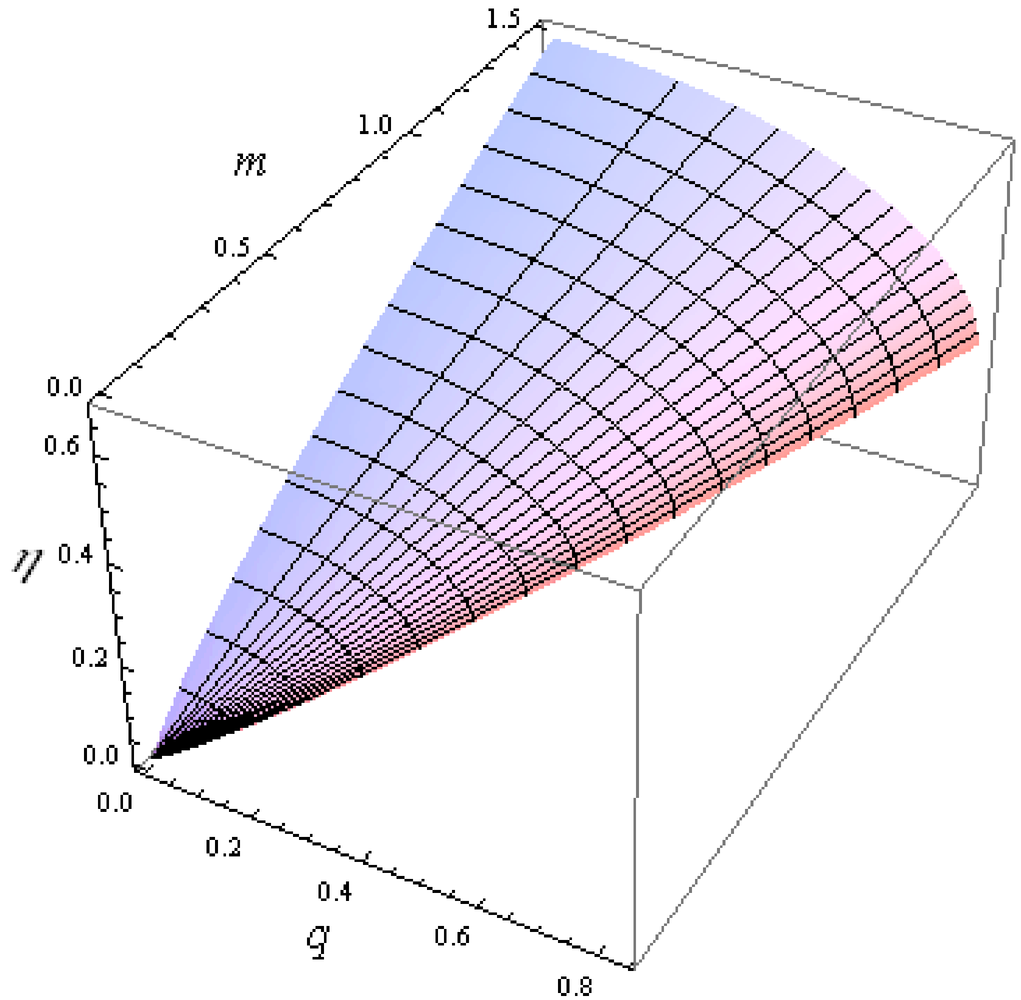


Figure 3: η vs. q and m ($\kappa = l = 1$)

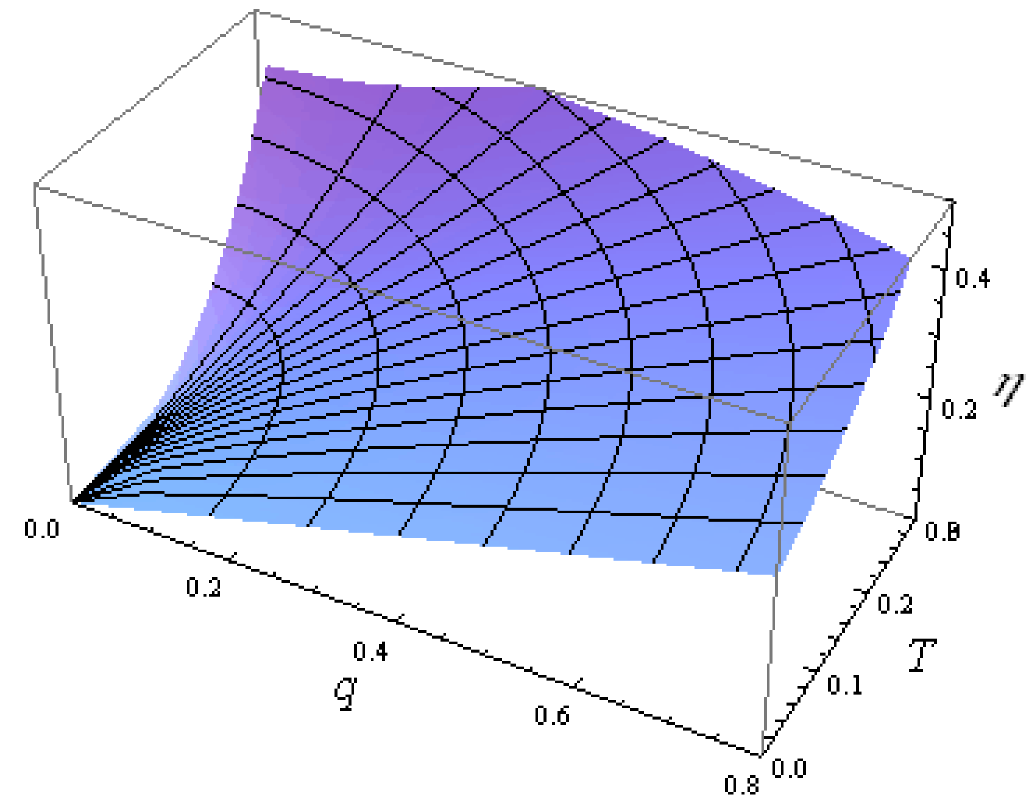


Figure 4: η vs. q and T ($\kappa = l = 1$)

Thermal conductivity

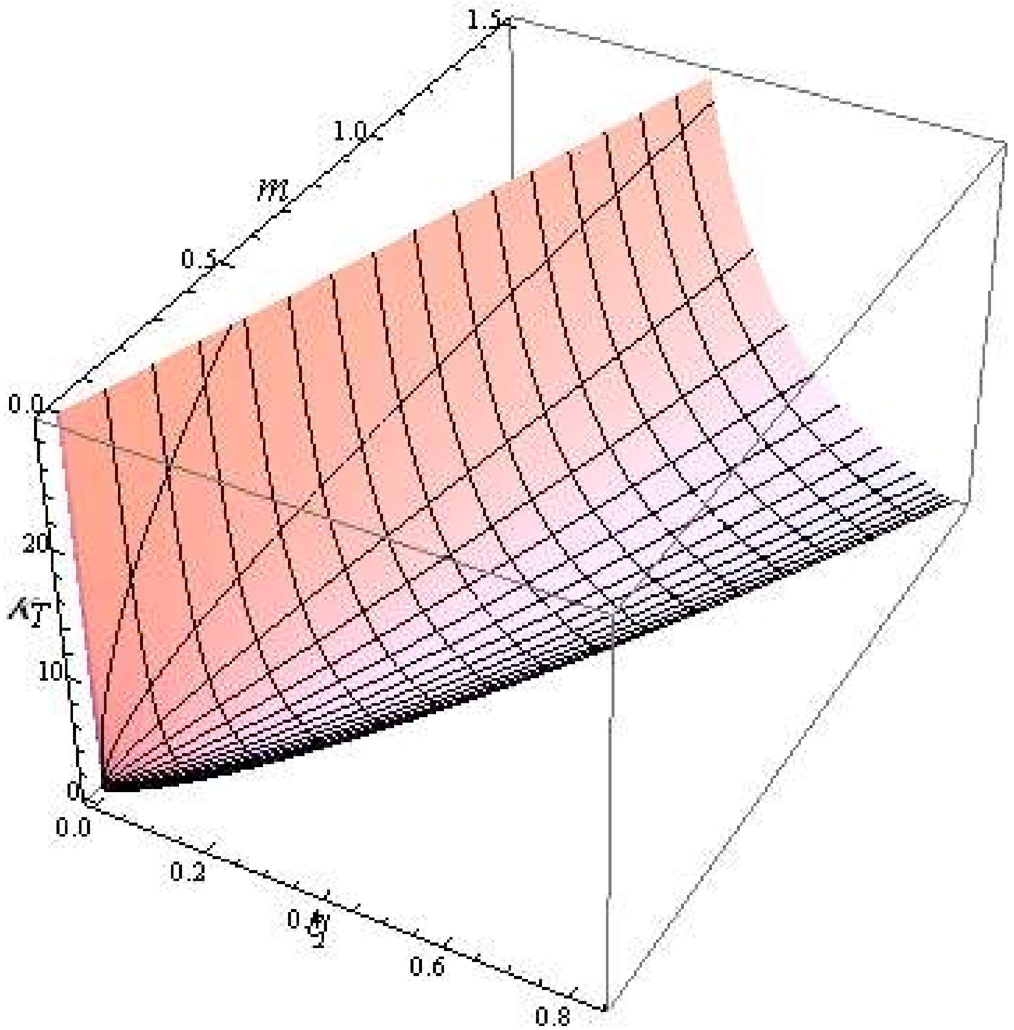


Figure 5: κ_T vs. q and m ($\kappa = l = 1$)

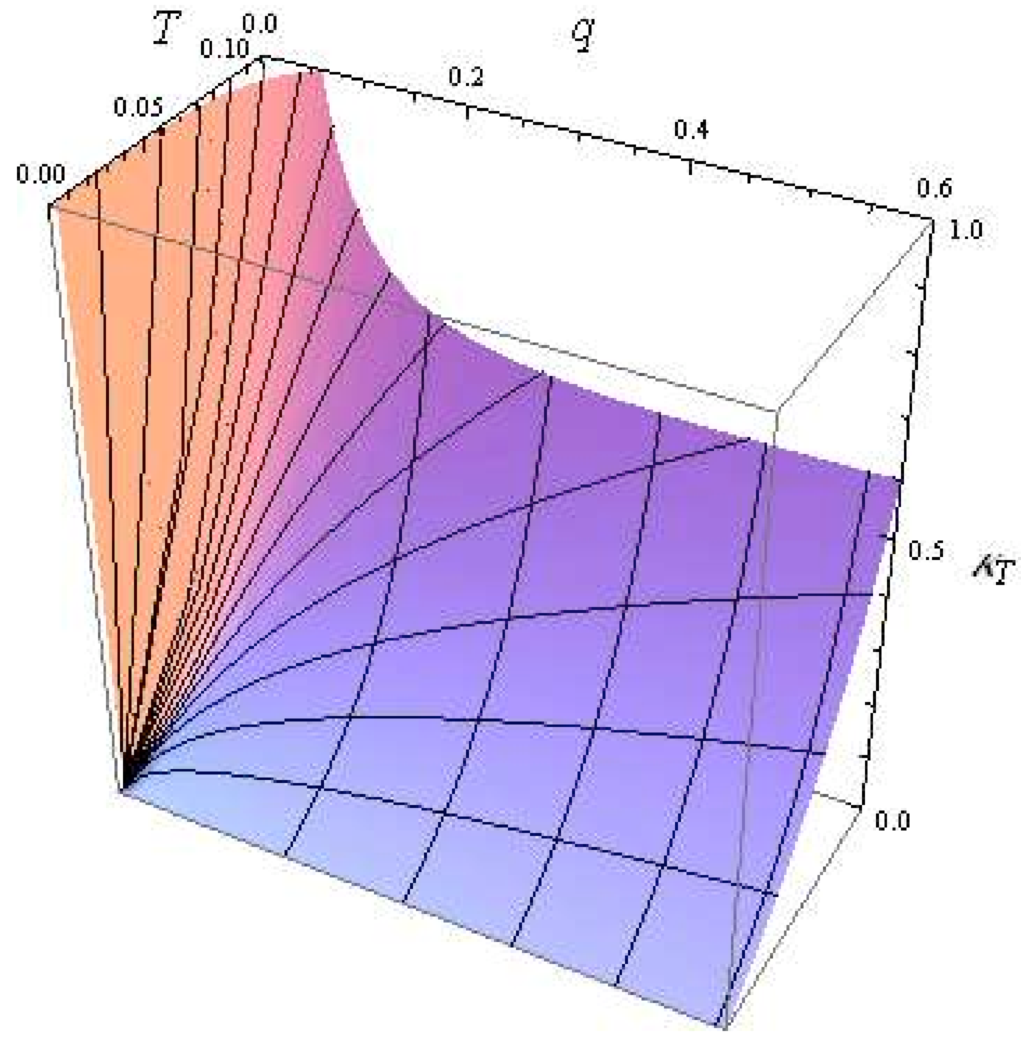


Figure 6: κ_T vs. q and T ($\kappa = l = 1$)

Charged black hole & baryon density:

archiv:0707.2919, SJS

- Consider a D-brane that fills the whole space. then the response of the metric to the charge is the same whether it is R-charge or baryon charge.
- Unique minimal coupling between Maxwell & Einstein fields.
- Therefore we can interpret the the charge of the RN black hole as the baryon charge density.

Hydrodynamics II:

archiv:0809.4521, Hur+Kim+SJS

- Einstein eq. in the bulk
  <-> Fluid dynamics in the boundary.

  This method yields:

thermal conductivity κ and the electrical conductivity

$$\kappa = \frac{\pi^2 T^3 r_+^7}{4g^2 M^2}, \qquad \sigma = \frac{\pi^2 e^2 T^2 r_+^7}{4g^2 M^2}.$$

Consistency bet. I and II

- From the Hydro I (Kubo formula)

$$\kappa = \frac{r_+}{4g^2}\frac{(2-a)^2}{(1+a)^2}, \quad \text{with } a = Q^2/r_+^6$$

- We can show that this is the same as

$$\kappa = \frac{\pi^2 T^3 r_+^7}{4g^2 M^2}$$

Wiedermann-Franz law

$$\frac{\kappa}{\sigma} = T/e^2$$

$$\kappa_T = \left(\frac{\epsilon + P}{\rho T}\right)^2 \kappa = 4\pi^2 \cdot \frac{g^2}{16\pi G} \cdot \frac{\eta T}{\mu^2}.$$

- This indicates the Metallic property of sQGP
- Can we measure the conductivity of sQGP?

Medium dependence of conductivities

$$\frac{\kappa}{T^2} = \frac{\sigma}{Te^2} = \frac{2\pi^2}{g^2}\frac{1+\sqrt{1+\frac{2}{3}(\frac{\mu}{g\pi T})^2}}{\left(1-3\sqrt{1+\frac{2}{3}(\frac{\mu}{g\pi T})^2}\right)^2}$$

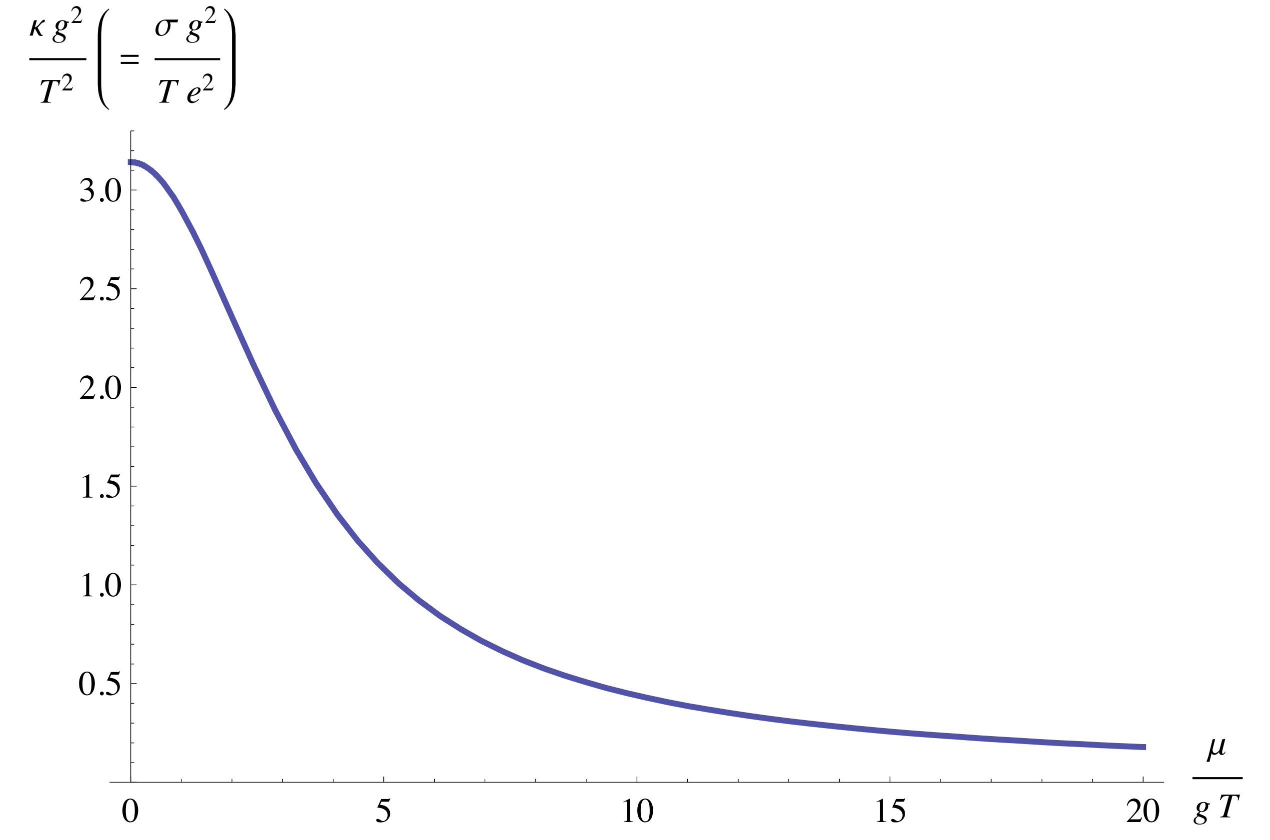
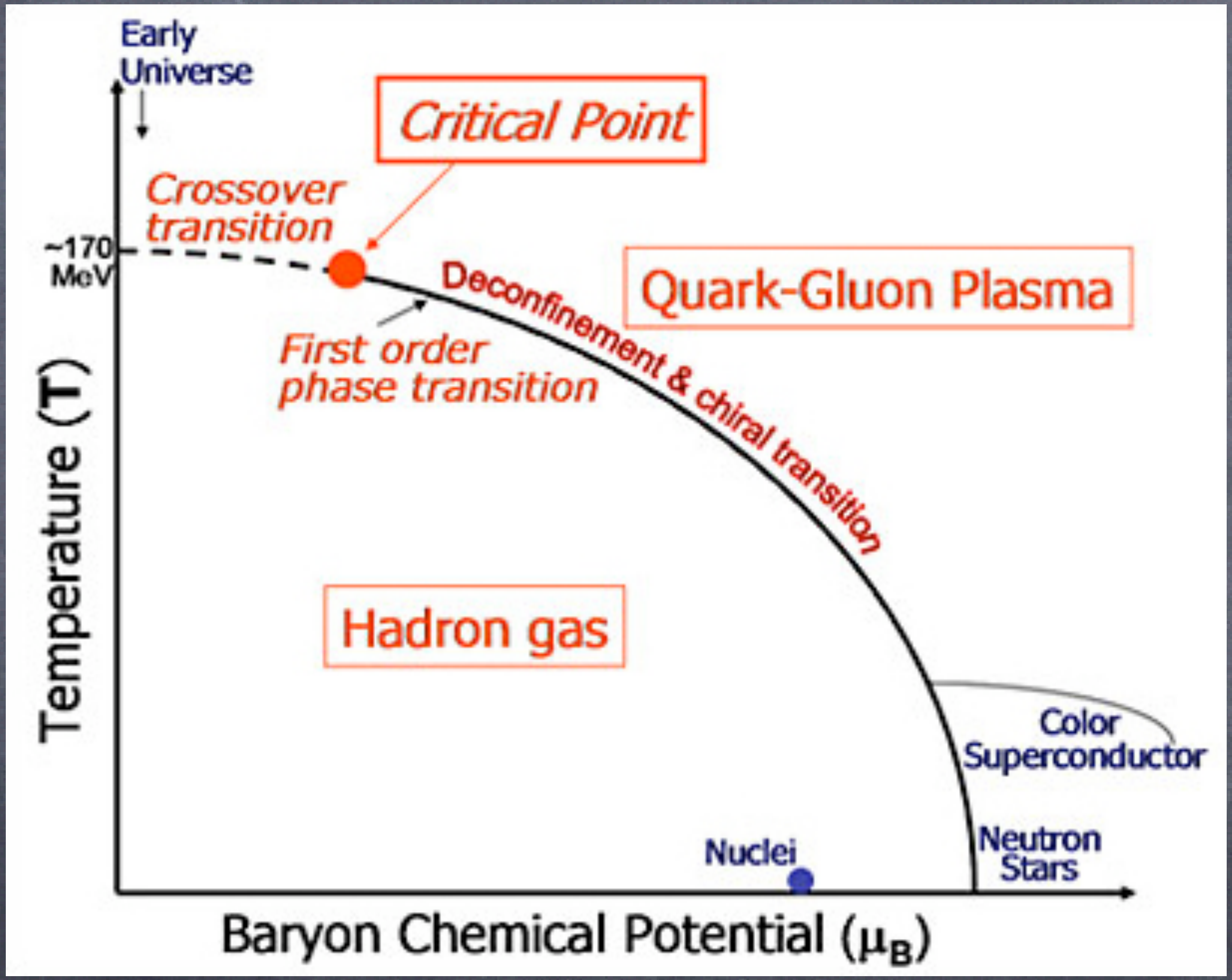


Figure 1: The coefficient of thermal conductivity or The electrical conductivity

So far, QGP regime
what about hadrons?



Baryon mass in finite density:

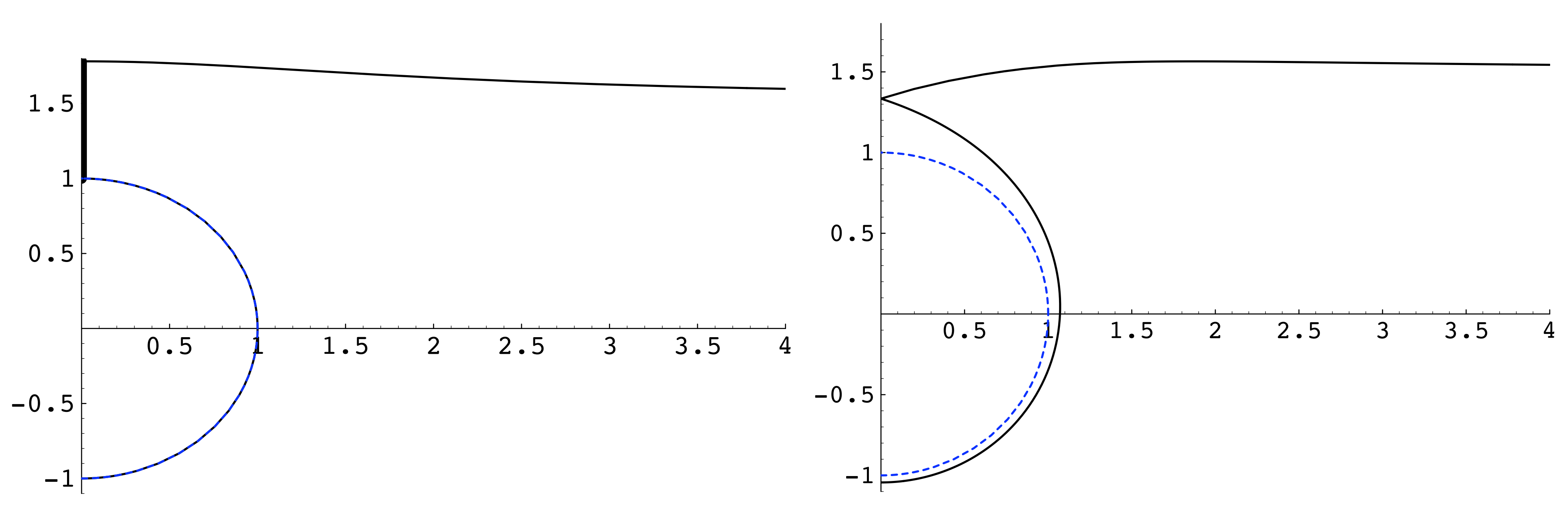
JHEP0804:010,2008, Seo +SJS

- In general, interested in fininte density and temperature.
- For QGP, black hole background and T is easy. string provide baryon charge.
- In confining regime, temperature dependence is hard to encode due to large N nature of ads/cft.
- set T=0 with non-zero density.

Holographic view of baryon density

- In 4d field theory: chemical potential is a constant gauge potential.
- In holographic view: 5d electric potential, phi, is a created by the charge, which is the string end points.
- choose the value at the horizon to be 0.
- phi at boundary is the chemical pt.

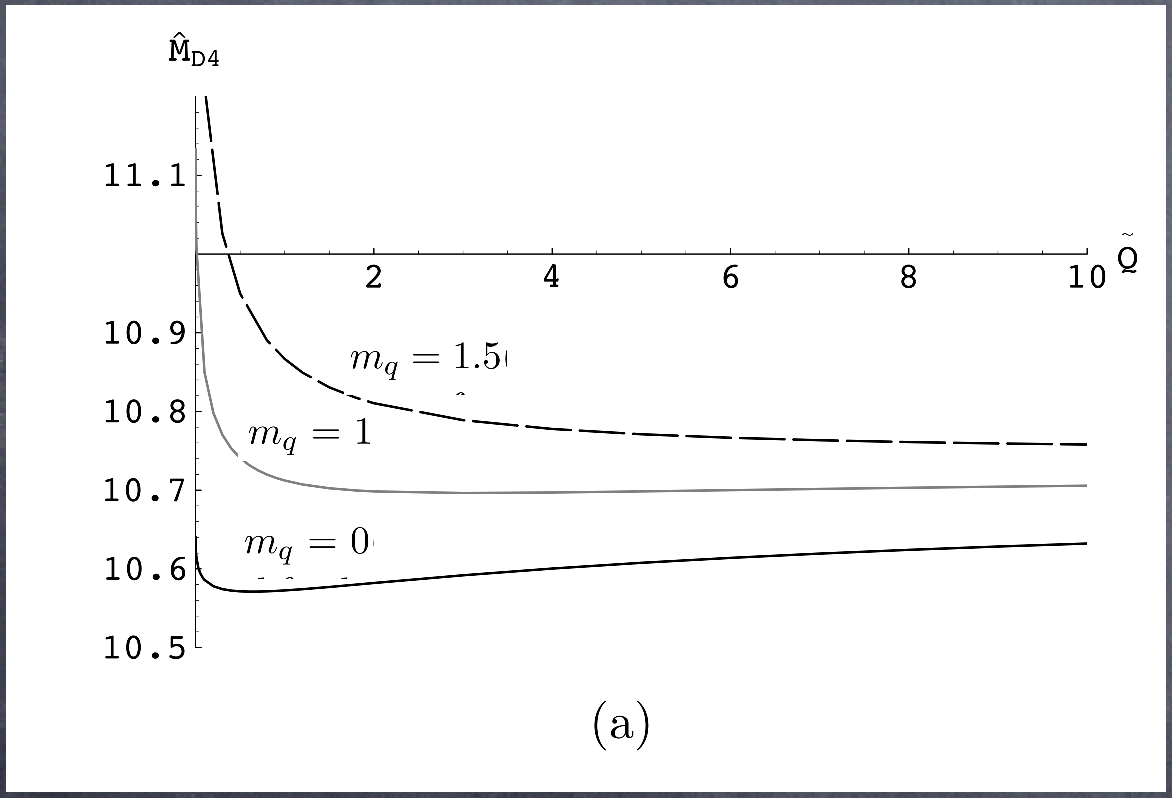
- a Baryon is a compact brane connected to flavor branes by N_c strings : fig. a
- strings are energy costly -> D branes deform to reduce the string length 0: fig. b
- Force balancing condition gives connection condi



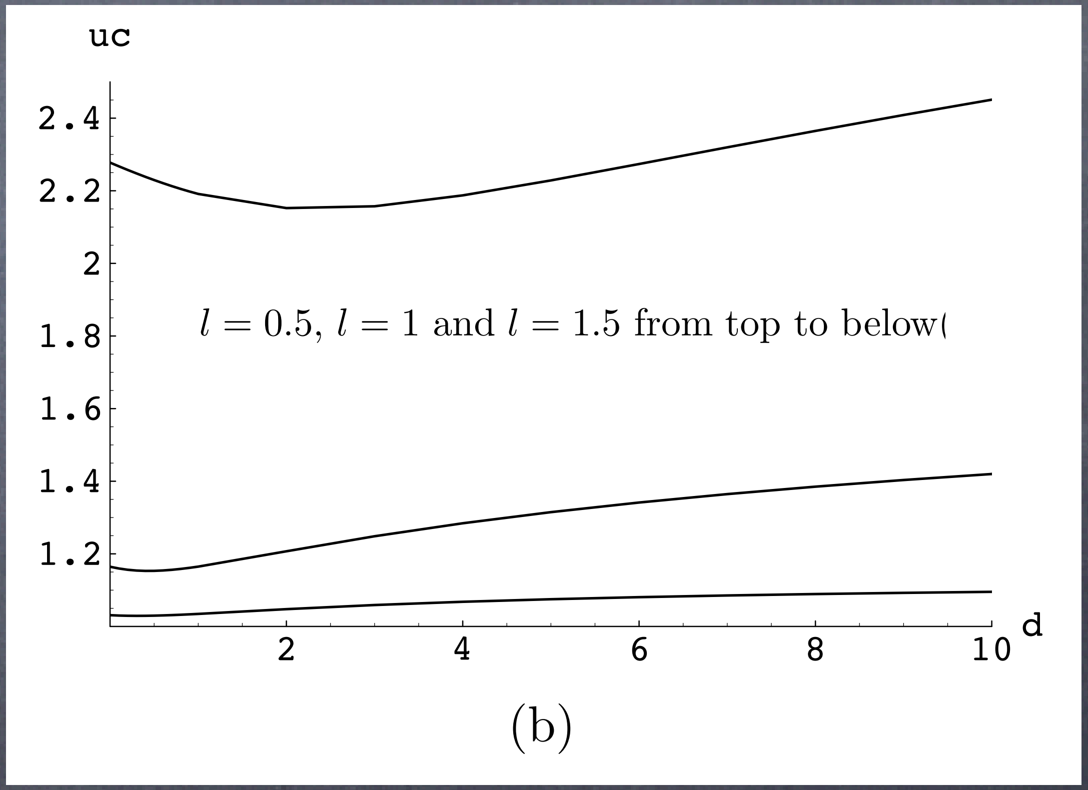
(a) (b)

Baryon mass in medium:

It falls but not in small current quark mass



(a)



(b)

D4/D6

D4/D8/$\bar{D}$8 $m_q = 0$

Conclusion

- Green functions and Transport coefficients calculated.
- QGP may have a another surprising property: Metallic conductivity!
- baryon mass falls in medium, perhaps.
- Gravity dual of gauge theory may become a powerful tool for the heavy ion collision.