Systematic Study of Elliptic Flow at RHIC-PHENIX

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PHENIX

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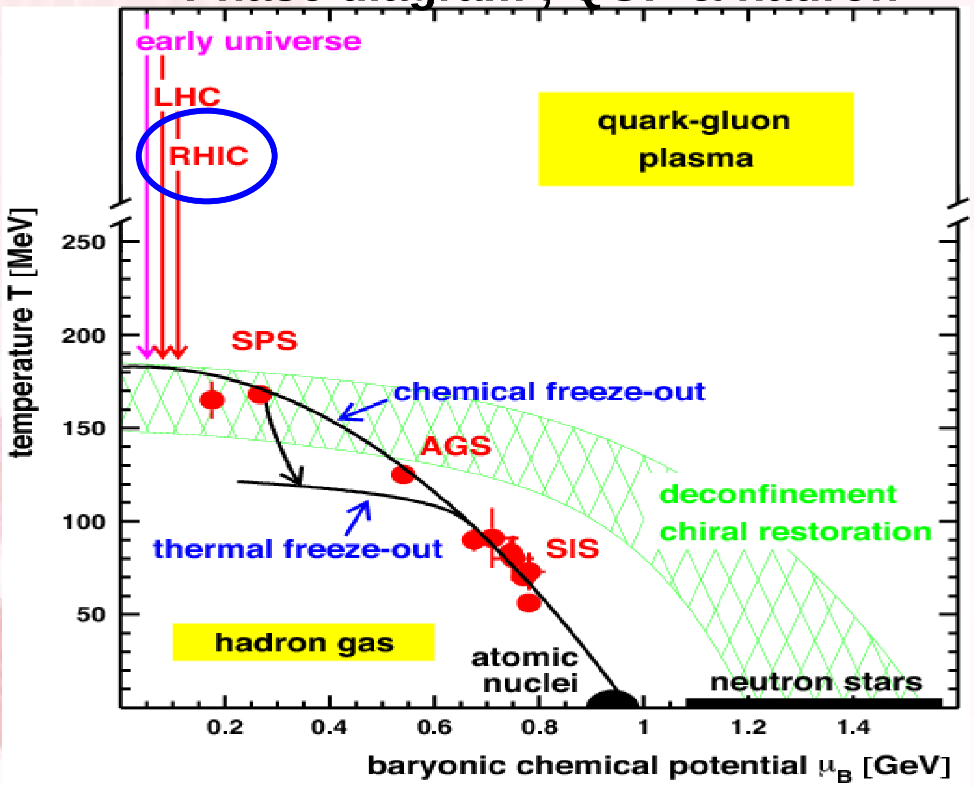
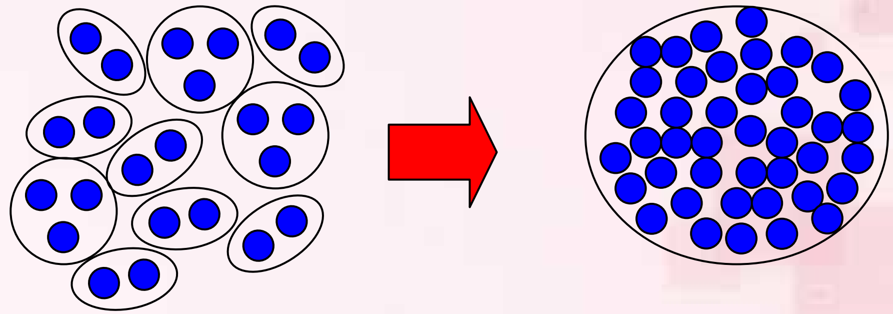
Quark Gluon Plasma (QGP)

Phase diagram ; QGP & hadron





- Prediction from Lattice QCD
  - T ~ 170 MeV
  - ε ~ 1.0 GeV/fm^3

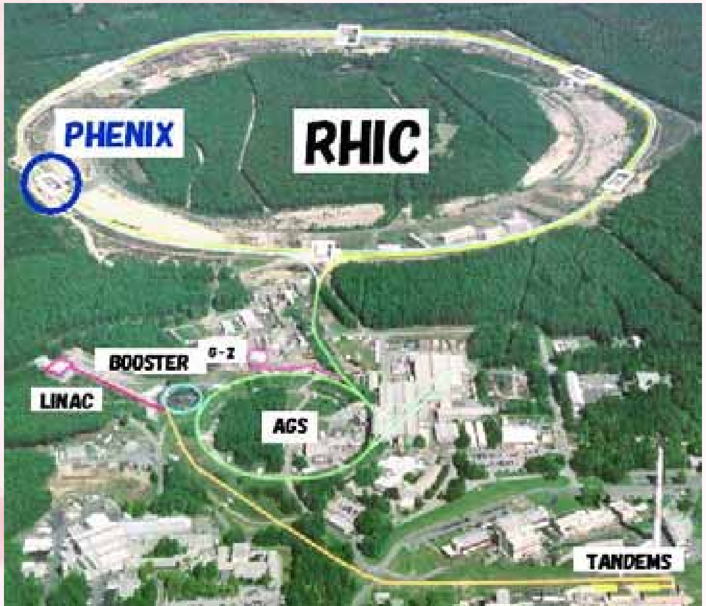
Quarks become de-comfined

Phase transition to QGP

* Normal Nucleus: ε ~ 0.2 GeV/fm^3

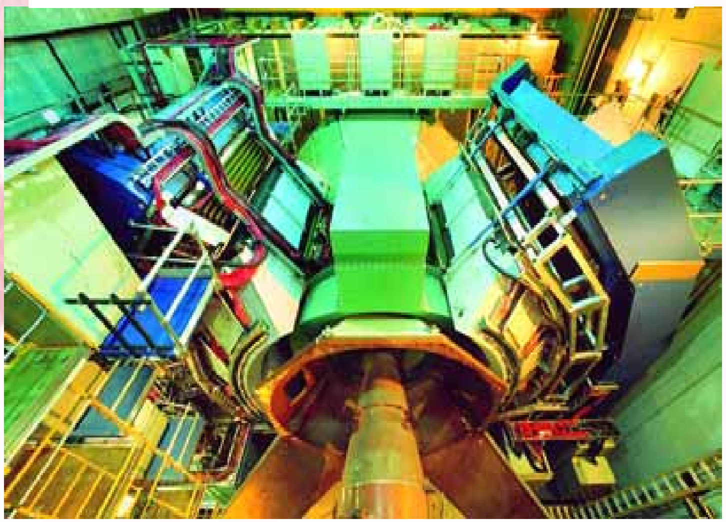
- High energy nuclear collision
  Au+Au $\sqrt{s}$=200GeV
  - RHIC : 5 ~ 15 GeV/fm^3

Relativistic Heavy Ion Collider (RHIC)

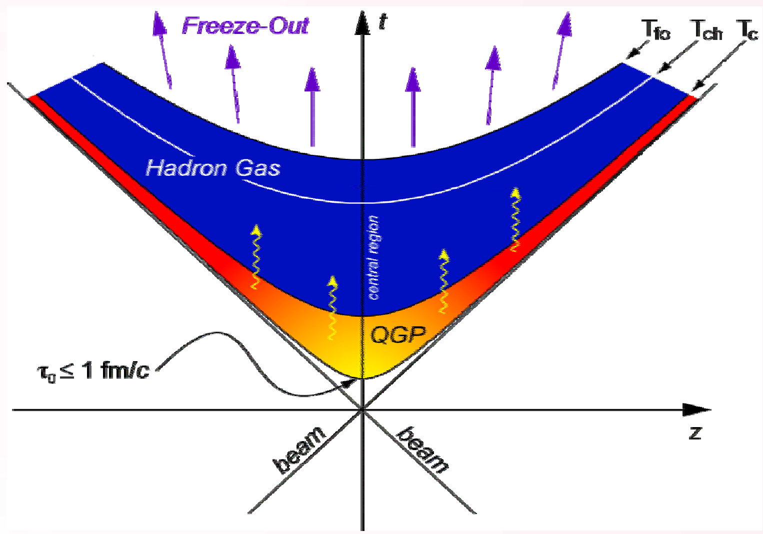


- Brookhaven National Laboratory
- First relativistic heavy ion collider in the world
- Circumference 3.83 km、2 rings
- Collision species (Au+Au, Cu+Cu, d+Au, p+p)
- Energy (A+A); up to 100 GeV/nucleon
- PHENIX is the one of the main experiment group

PHENIX Experiment



Time-evolution after collision



Thermal freezeout

Chemical freezeout

hadronization

QGP

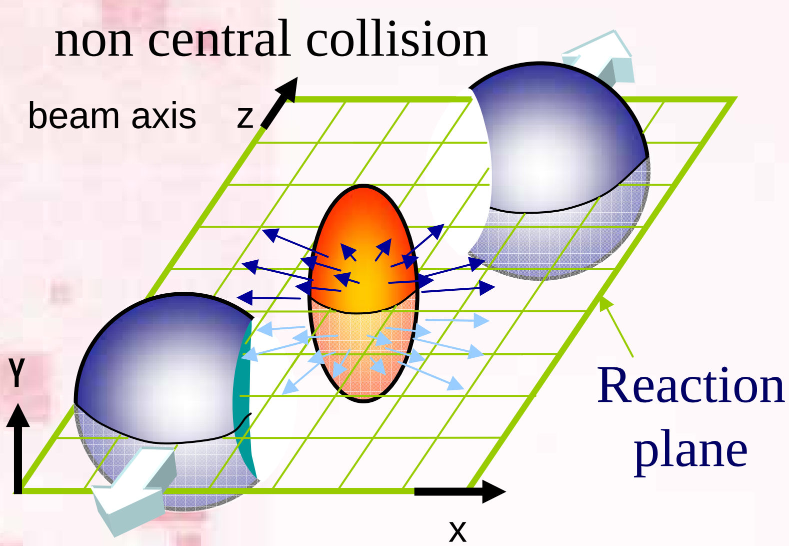
thermal equilibrium

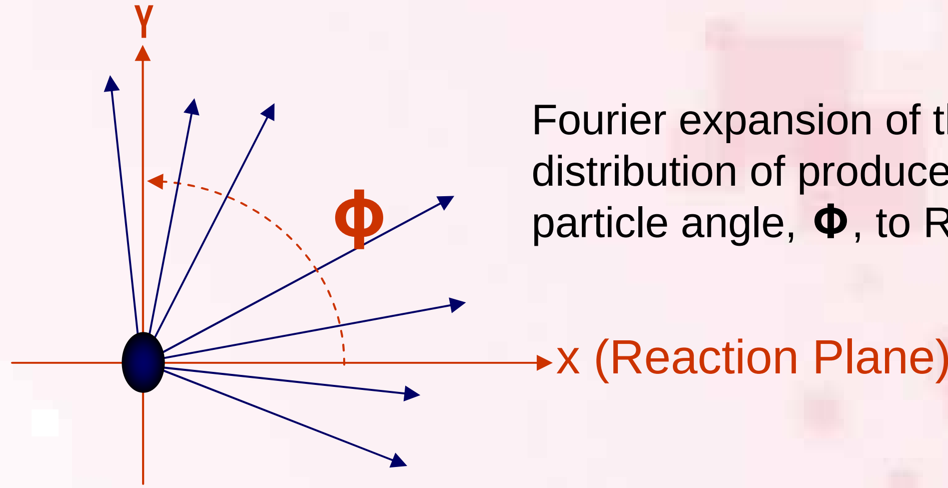
collision

Elliptic Flow (v_2)

v_2 is the strength of the elliptic anisotropy of produced particles.

A sensitive probe for studying properties of the hot dense matter made by heavy ion collisions.





Fourier expansion of the distribution of produced particle angle, **Φ**, to RP

$$\frac{dN}{d\varphi} = n\{1 + 2\underline{v_2}\cos[2(\varphi - \Phi_{RP})]\}$$

v_2 is the coefficient of the second term
→ indicates ellipticity

If yield is (x direction)>(y direction), v_2 >0.

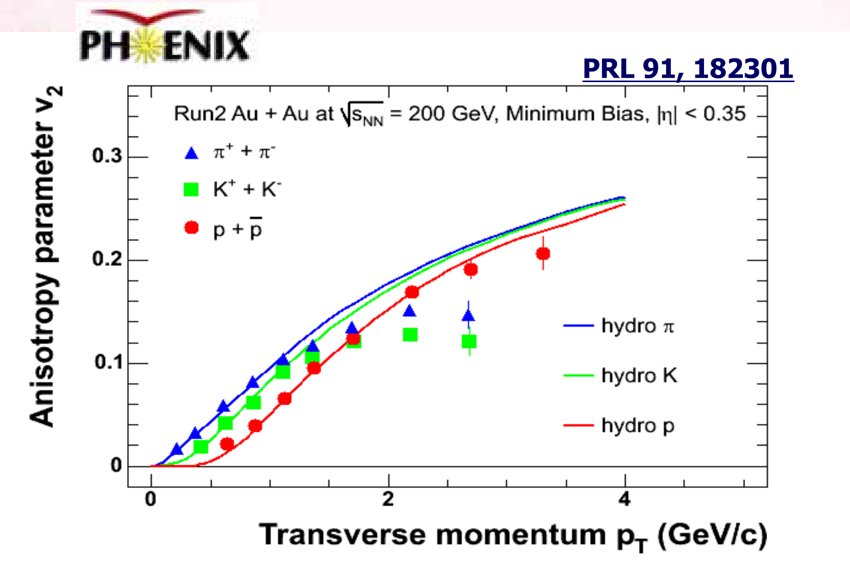
The initial geometrical anisotropy is transferred by the pressure gradients into a momentum space anisotropy → the measured v_2 reflects the dense matter produced in the collisions.

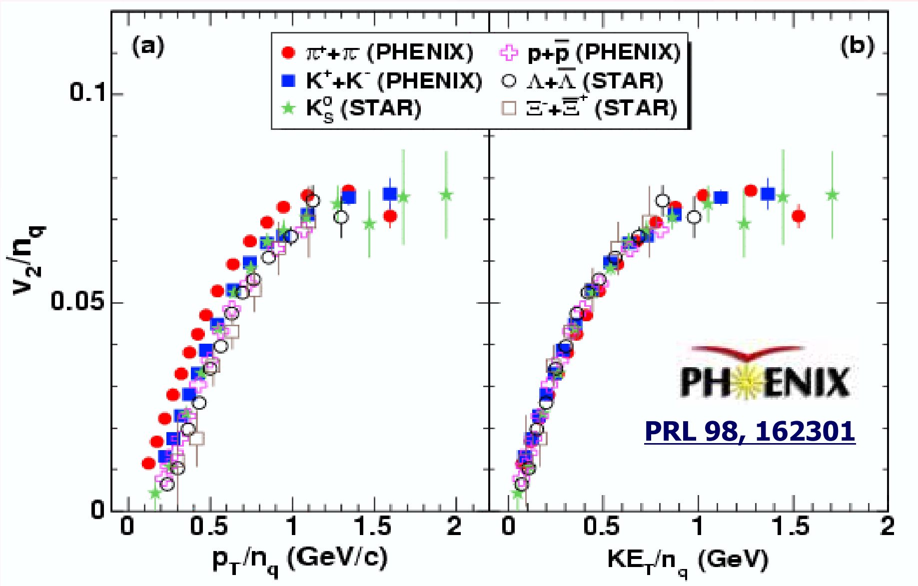
Motivation

From the results at 200GeV

v_2 at low p_T (< ~2 GeV/c) → can be explained by a hydro-dynamical model
v_2 at mid p_T (<4~6 GeV/c) → is consistent with recombination model

The results are consistent with Quark number +KE_T scaling.





How about other systems and energies !?

KE_T = mT-m0

Results

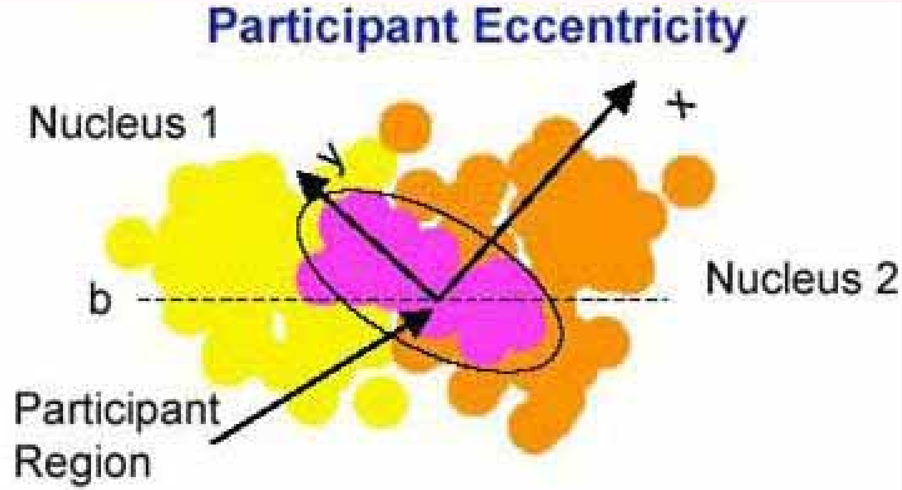
- Energy dependence
- System size dependence
  - Eccentricity scaling
- Universal v_2
  - Quark number + K_{ET} scaling
  - Universal scaling

<words>

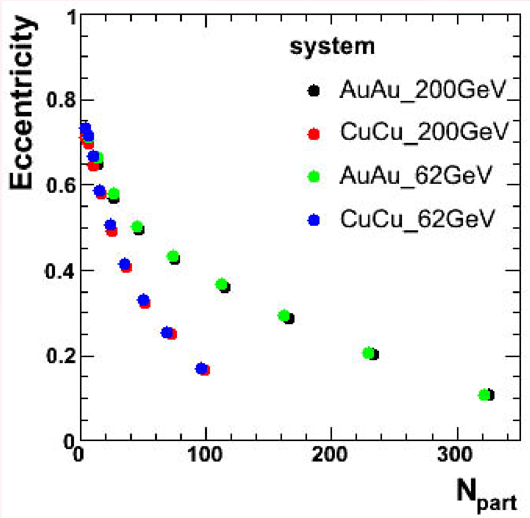
N_{part} --- Number of nucleons participating the collision
N_{coll} --- Number of binary collisions
eccentricity(ε) --- geometirical eccentricity of participant nucleons

$$\varepsilon = \frac{\langle y^2 \rangle - \langle x^2 \rangle}{\langle y^2 \rangle + \langle x^2 \rangle}$$

-Nucleus formed by wood-Saxon shape
-Monte-Carlo simulation with Glauber model
- Participant eccentricity which is calculated with long and short axis determined by distribution of participants at each collision.



ε vs. N_{part}



Comparison Table

	Energy	Particle species	System Size (CuCu, AuAu)	Centrality
scaling		n_q+K_{ET}		
AuAu 200		●		
AuAu 62				
CuCu 200				
CuCu 62				

● **Already known**

● **Is going to check next**

Comparison Table

Size

	Energy	Particle species	System (CuCu, AuAu)	Centrality
scaling		$n_q + K_{ET}$		
AuAu 200	●	●		
AuAu 62				
CuCu 200	●			
CuCu 62				

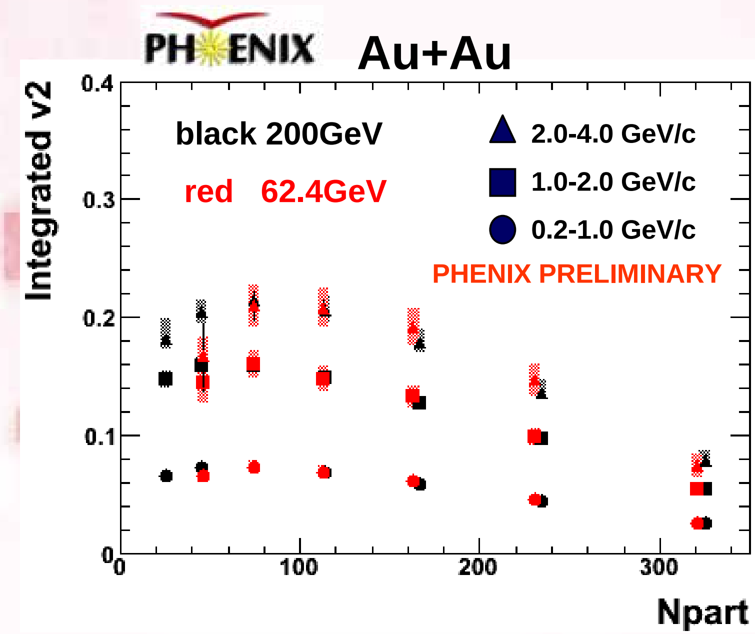
● **Already known**

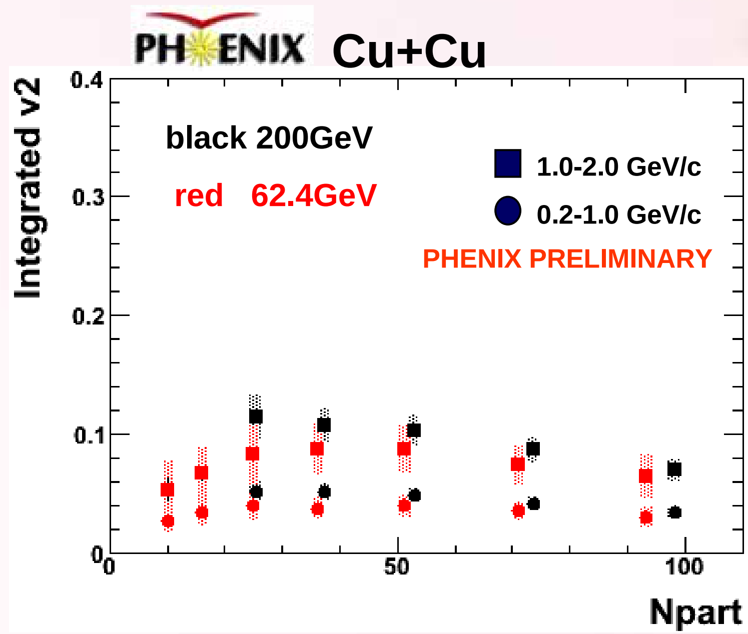
● **Is going to check next**

Energy dependence

Comparison of
√s = 62.4 and 200 GeV

- **dependence of centrality (Npart)**
- **compare the results in Cu + Cu which is smaller collision size than Au+Au**
- **comparison of PID hadrons. pi/K/p →next page**



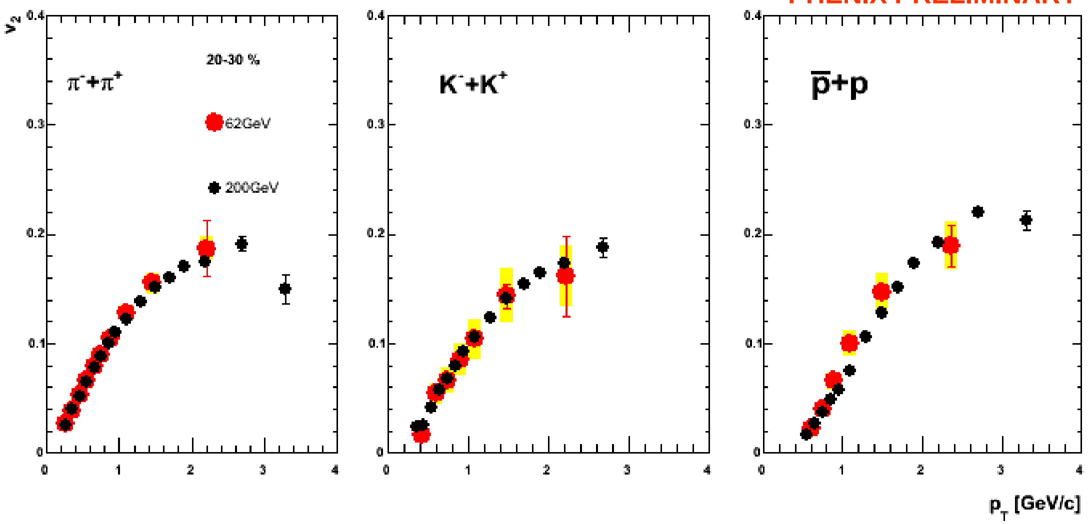


v_2 of 200GeV and 62GeV are consistent

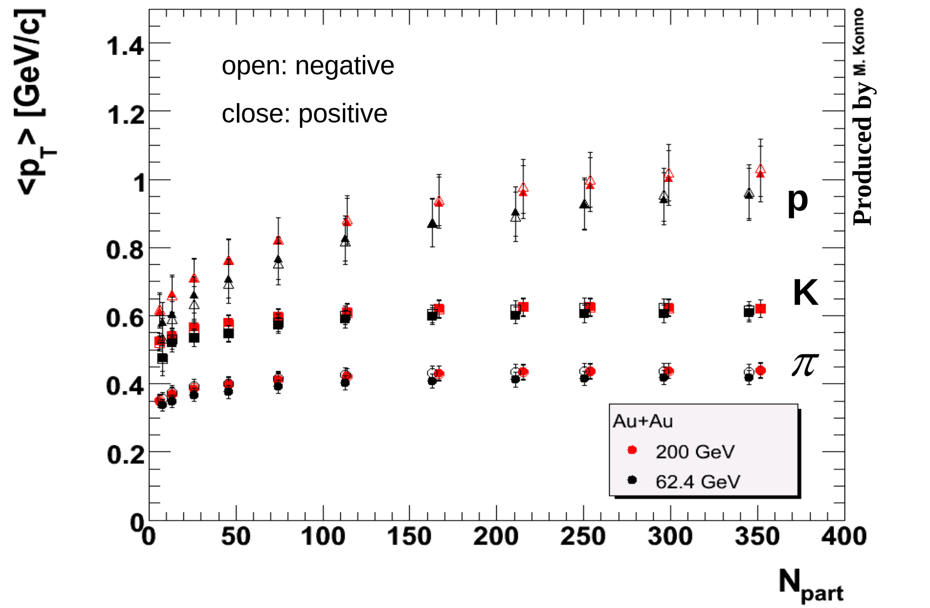
Energy dependence

- **identified hadrons (π/K/p)**
- **p_T dependence**

Au+Au v_2 vs. p_T



Mean p_T



<p_T> of 62.4 GeV and 200 GeV are consistent within errors on pi/K/p̄.
Therefore v_2 agree at any p_T region in figures.

v_2 of $\sqrt{s}$ = 17GeV (SPS) decreases to about 50% of RHIC energies.
Higher collision energy has larger v2 up to RHIC energy.
Above 62.4 GeV, v_2 is saturated.
→ indicate the matter reached thermal equilibrium state at RHIC

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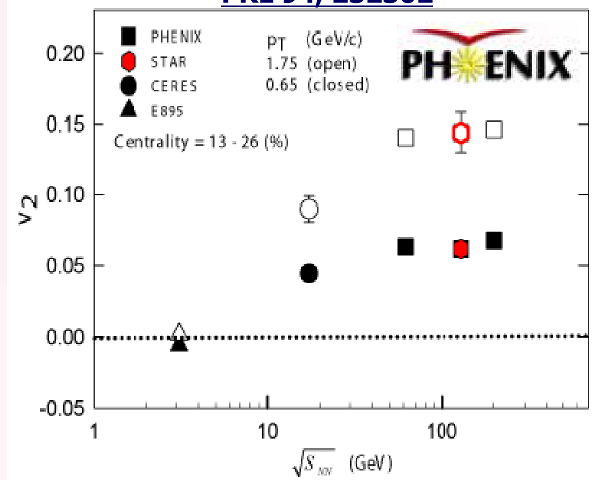


System Size Dependence

➢Eccentricity Scaling

What can change the size of collision system.

- Species of collision nucleus (Au+Au ,Cu+Cu)
- Centrality

Comparison Table

	Energy	Particle species	Size: System (CuCu, AuAu)	Size: Centrality
scaling	no change	n_q+KE_T		
AuAu 200	◉	●	◌	◌
AuAu 62			◌	◌
CuCu 200	◉		◌	◌
CuCu 62			◌	◌

● **Already known**

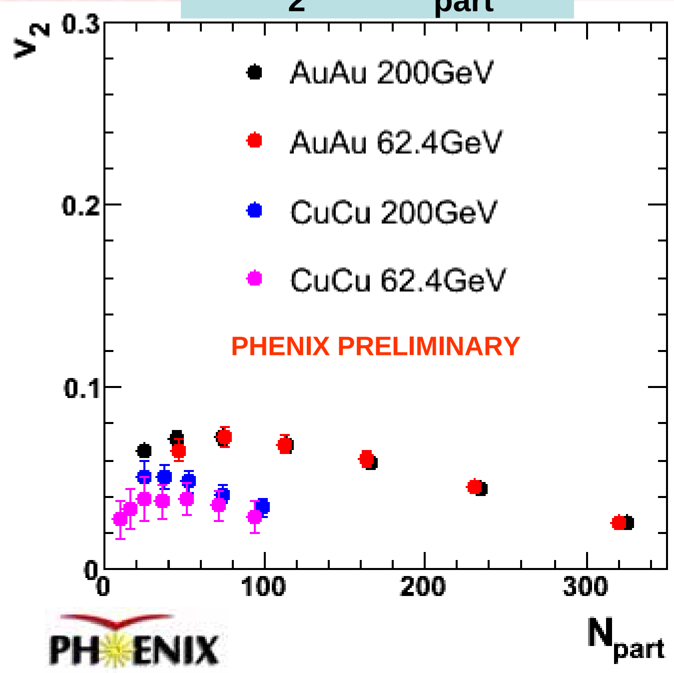
◌ **Is going to check next**

◉ **checked**

System size dependence

Compare v_2 normalized by eccentricity (ε) in the collisions of different size.

v_2 vs. N_{part}

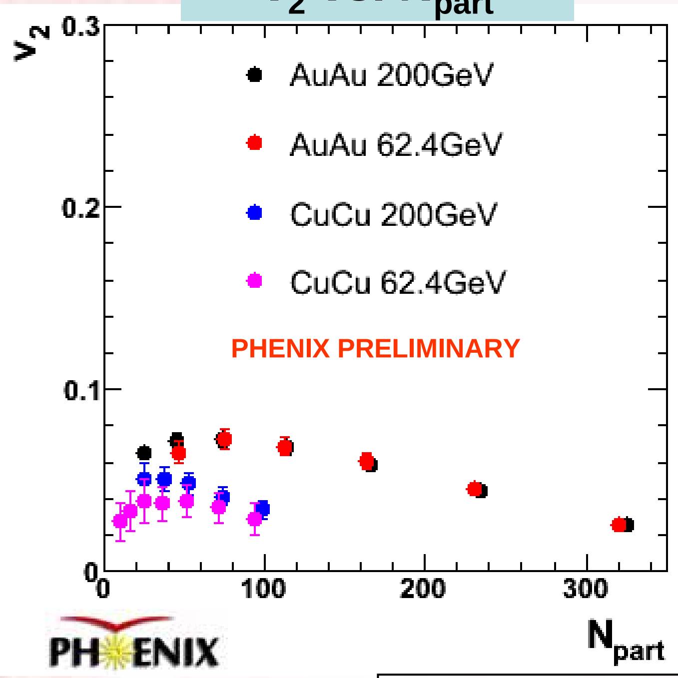


$0.2<p_T<1.0$ [GeV/c]

System size dependence

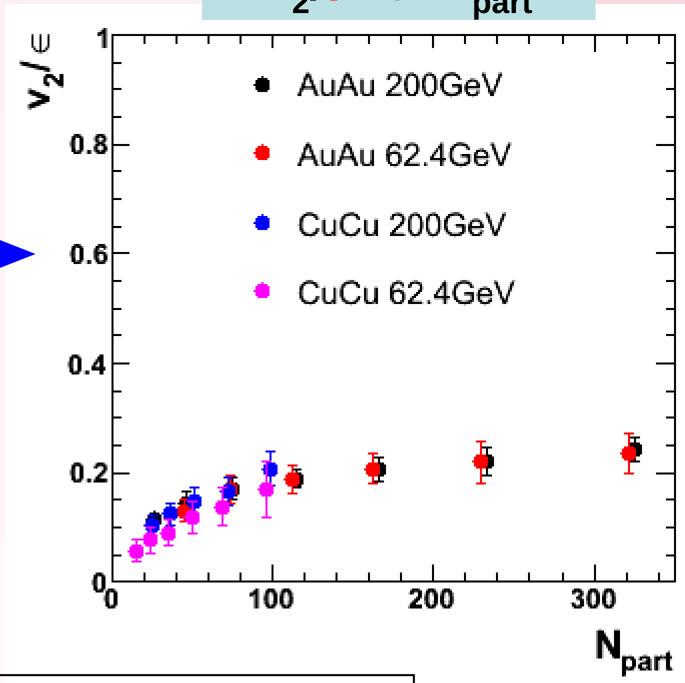
Compare v_2 normalized by eccentricity (ε) in the collisions of different size.

v_2 vs. N_{part}



0.2<p_T<1.0 [GeV/c]

v_2/ε vs. N_{part}



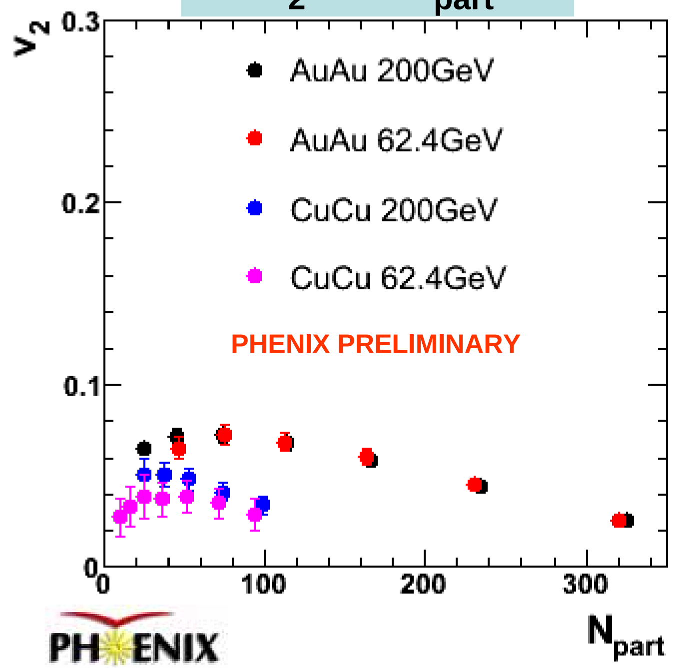
v_2/ε (Au+Au) = v_2/ε (Cu+Cu) !

Systematic errors from eccentricity is not included here.

System size dependence

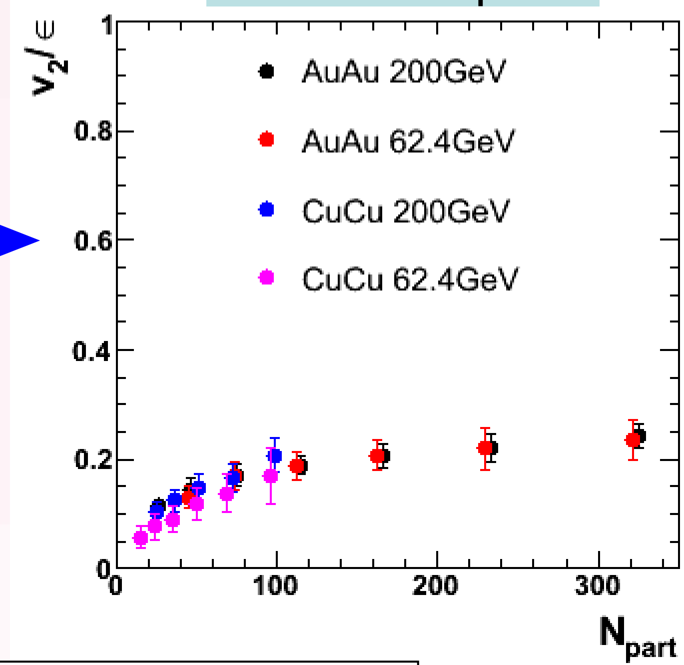
Compare v_2 normalized by eccentricity (ε) in the collisions of different size.

v_2 vs. N_{part}



$0.2<p_T<1.0$ [GeV/c]

v_2/ε vs. N_{part}



v_2/ε (Au+Au) = v_2/ε (Cu+Cu) !

Systematic errors from eccentricity is not included here.

but v_2/ε is not constant and it shades depending on N_{part} .

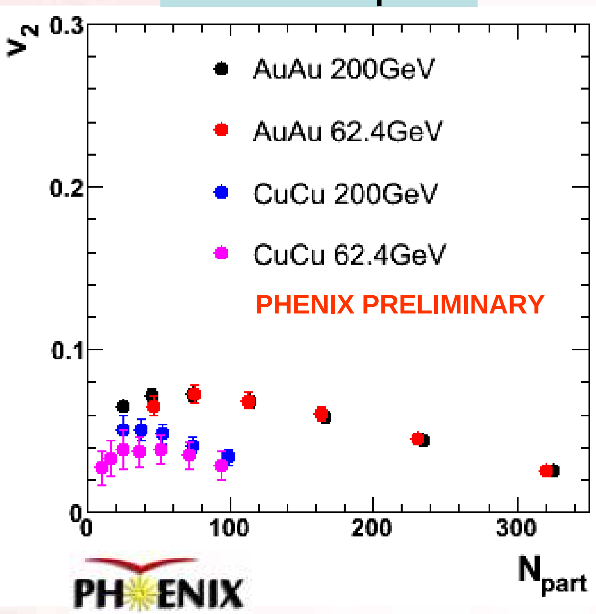
→v_2 <u>can be normalized by ε at same N_{part}</u> ,
but ε is not enough to determine v_2 .

System size dependence

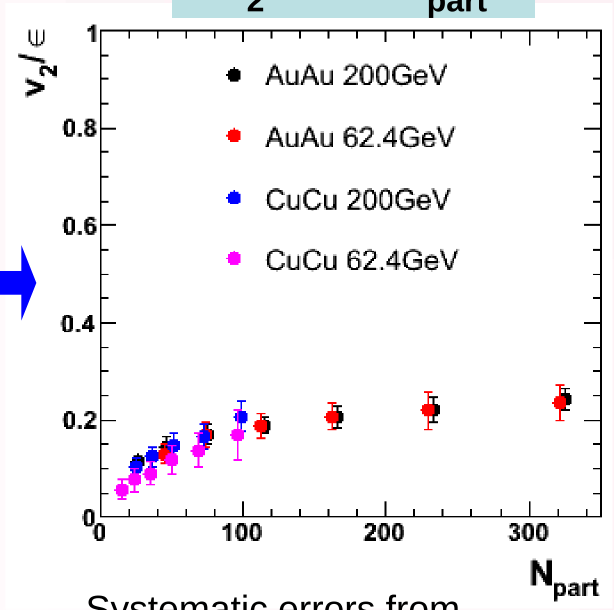
$0.2<p_T<1.0$ [GeV/c]

Dividing by $N_{part}^{1/3}$

v_2 vs. N_{part}

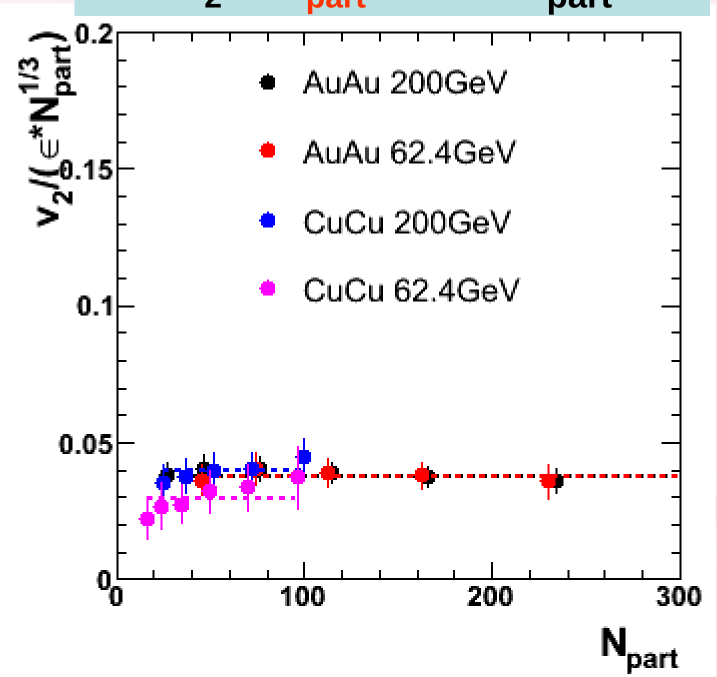


v_2/ε vs. N_{part}



Systematic errors from eccentricity is not included here.

$V_2/\varepsilon/N_{part}^{1/3}$ vs. N_{part}



v_2/ε (Au+Au) = v_2/ε (Cu+Cu)

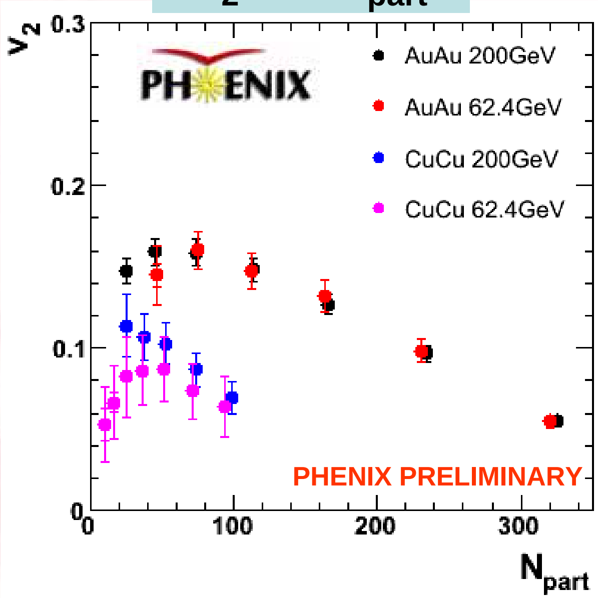
v_2/eccentricity is scaled by $N_{part}^{1/3}$ and not dependent on the collision system.

System size dependence

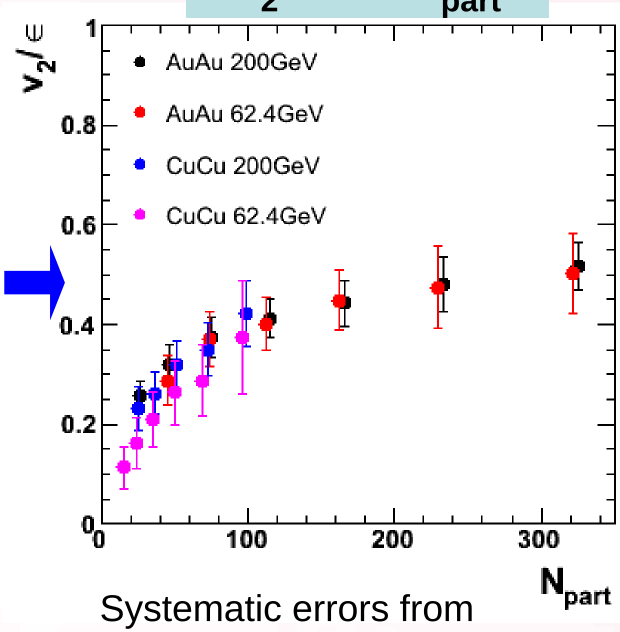
1.0<p_T<2.0 [GeV/c]

Dividing by $N_{part}^{1/3}$

v_2 vs. N_{part}

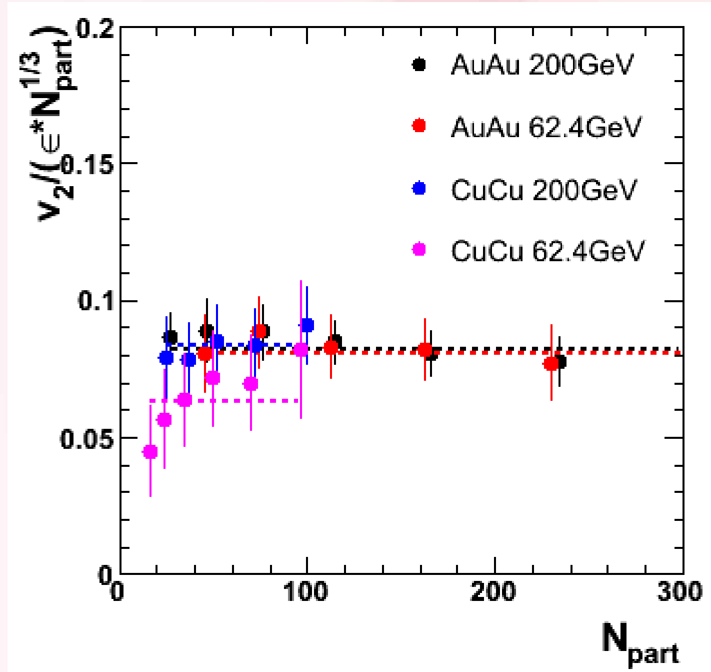


v_2/ε vs. N_{part}



Systematic errors from eccentricity is not included here.

$V_2/\varepsilon/N_{part}^{1/3}$ vs. N_{part}



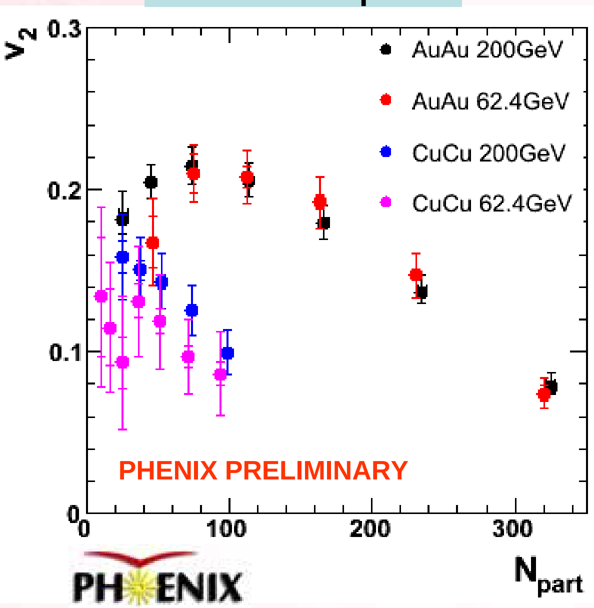
v_2/eccentricity is scaled by $N_{part}^{1/3}$ and not dependent on the collision system.

System size dependence

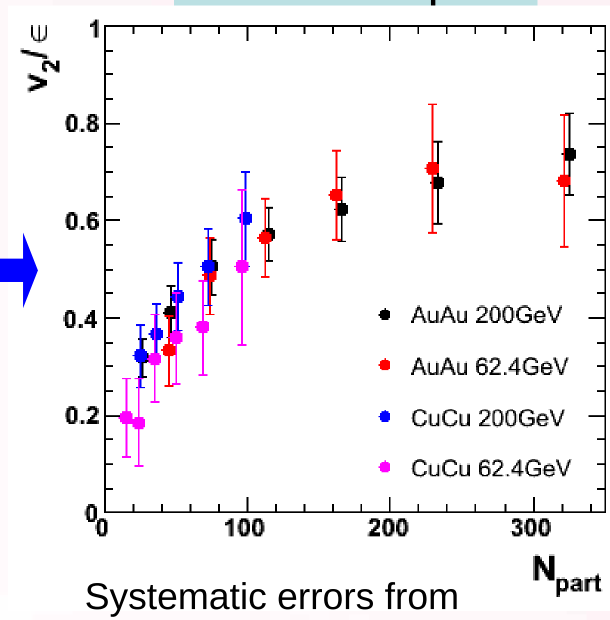
Dividing by $N_{part}^{1/3}$

$2.0<p_T<4.0$ [GeV/c]

v_2 vs. N_{part}

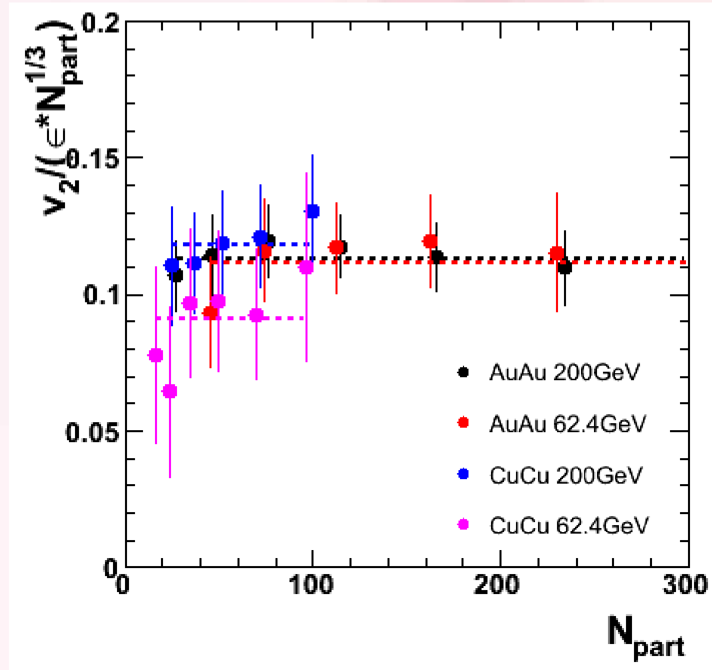


v_2/ε vs. N_{part}



Systematic errors from eccentricity is not included here.

$V_2/\varepsilon/N_{part}^{1/3}$ vs. N_{part}



v_2/eccentricity is scaled by $N_{part}^{1/3}$ and not dependent on the collision system.

Comparison Table

	Energy	Particle species	Size: System (CuCu, AuAu)	Size: Centrality
scaling	no change	n_q+K_{ET}	eccentricity	$N_{part}^{1/3}$
AuAu 200	●	●	●	●
AuAu 62			●	●
CuCu 200	●		●	●
CuCu 62			●	●

● **Already known**

● **Is going to check next**

● **checked**

Universal v_2

- Quark number + K_{ET} scaling
- Universal Scaling

Comparison Table

	Energy	Particle species	Size: System (CuCu, AuAu)	Size: Centrality
scaling	no change	$n_q + K_{ET}$	eccentricity	$N_{part}^{1/3}$
AuAu 200	● (checked, AuAu 200 and AuAu 62)	● (already known)	● (checked)	● (checked)
AuAu 62		● (going to check next)	● (checked)	● (checked)
CuCu 200	● (checked, CuCu 200 and CuCu 62)	● (going to check next)	● (checked)	● (checked)
CuCu 62			● (checked)	● (checked)

● **Already known**

● **Is going to check next**

● **checked**

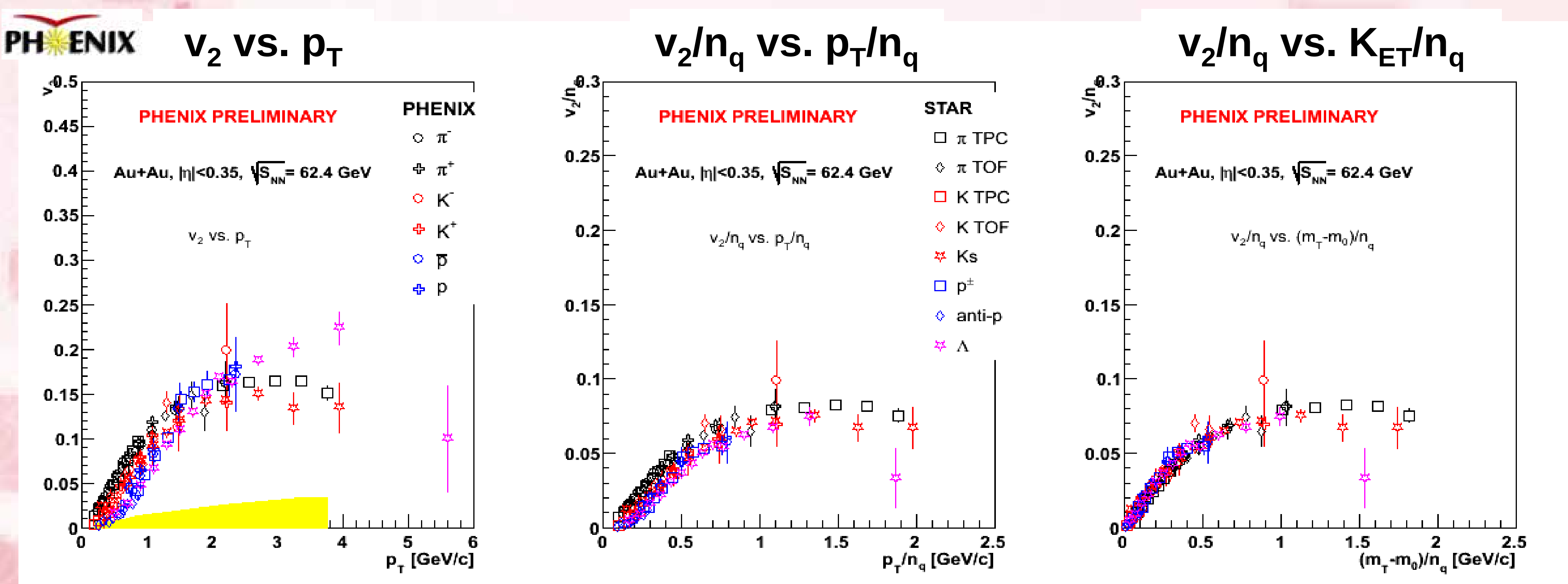
Quark number + K_{ET} scaling (AuAu 62.4GeV)

PHENIX: Error bars include both statistical and systematic errors.

STAR: Error bars include statistical errors. Yellow band indicates systematic errors.

Centrality 10-40 %

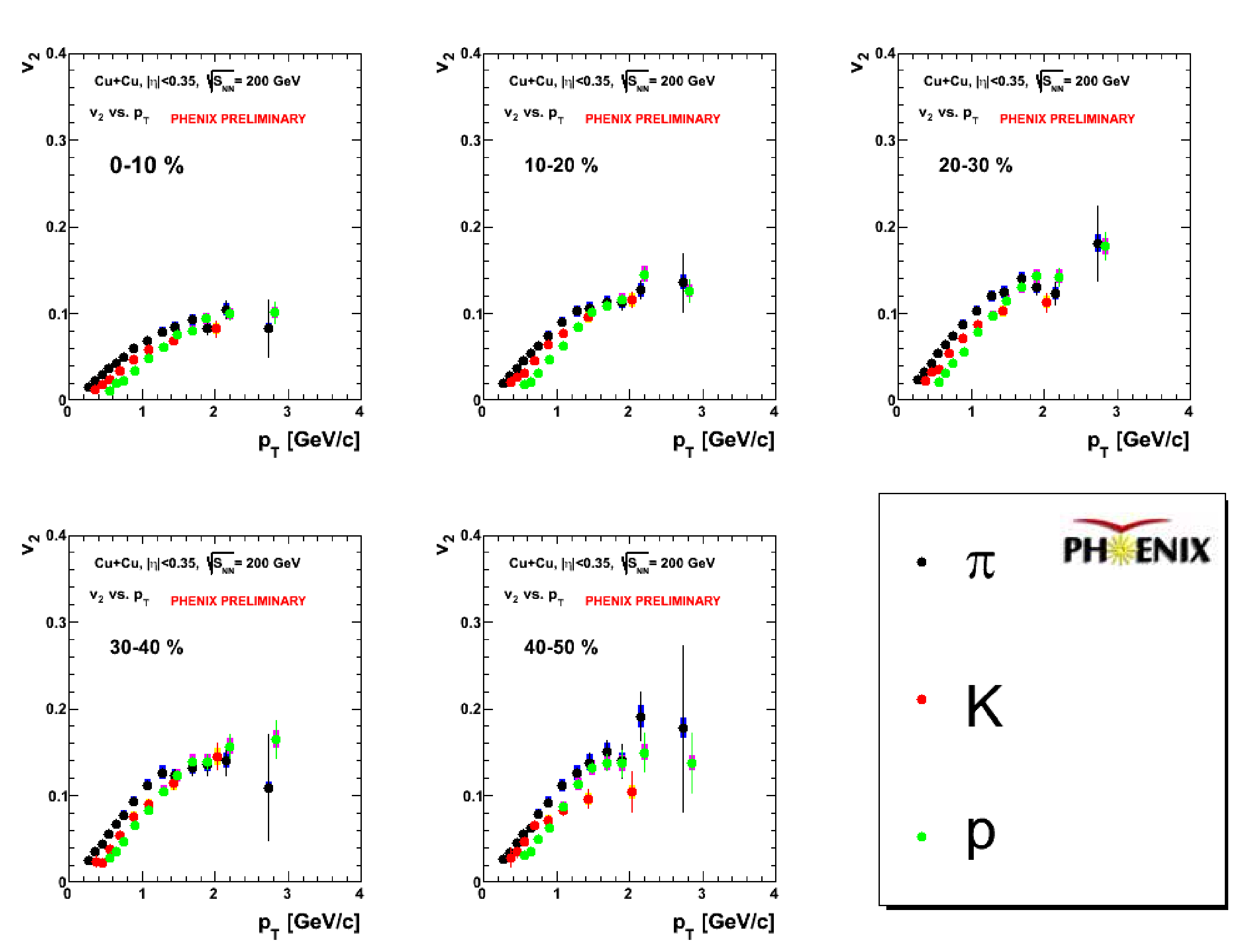
Star results : Phys. Rev. C 75



quark number + K_{ET} scaling is OK at 62.4 GeV, too!

$v_2(p_T)$ /n_{quark} vs. K_{ET}/n_{quark} is the universal curve independent on particle species.

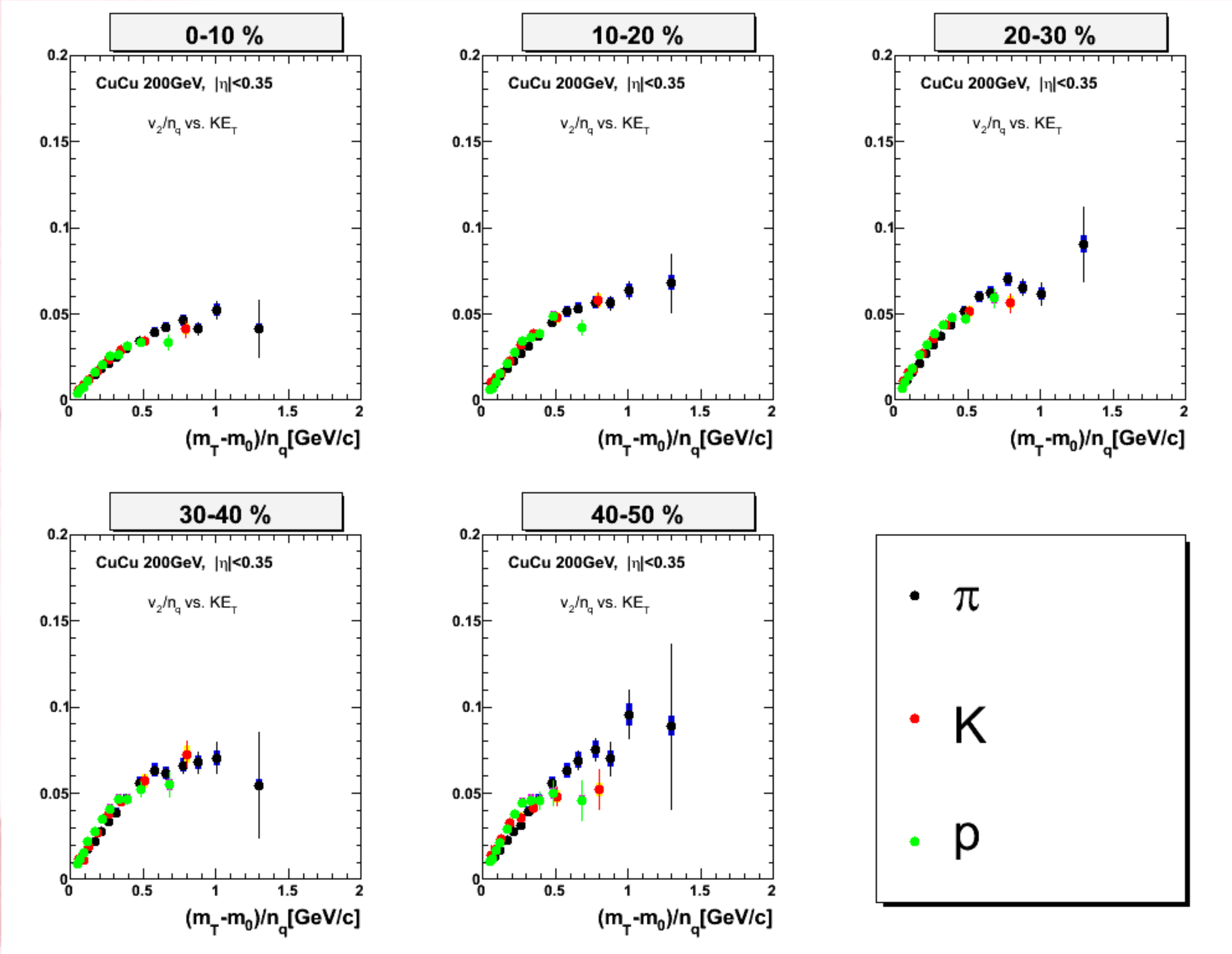
v2 vs. pT at Cu+Cu in 200GeV collision



Centrality dependence of PID v2 vs. pT for Cu+Cu 200GeV is measured.

Quark number + K_{ET} scaling

Cu+Cu √s = 200GeV



At all centrality, (between 0- 50 %) v_2 of π/K/p is consistent to quark number + K_{ET} scaling.

quark number + K_{ET} scaling seems to works out at Cu+Cu 200GeV.

Summary of Scaling

- Collision energy → no change
- Eccentricity of participants → eccentricity scaling
- Particle species → n_q +K_{ET} scaling
- Number of participants → $N_{part}^{1/3}$ scaling

Comparison Table

	Energy	Particle species	Size	
			System (CuCu, AuAu)	Centrality
scaling	no change	n_q+K_{ET}	eccentricity	$N_{part}^{1/3}$
AuAu 200	●	🔵	●	●
AuAu 62		●	●	●
CuCu 200	●	●	●	●
CuCu 62			●	●

🔵 **Already known**

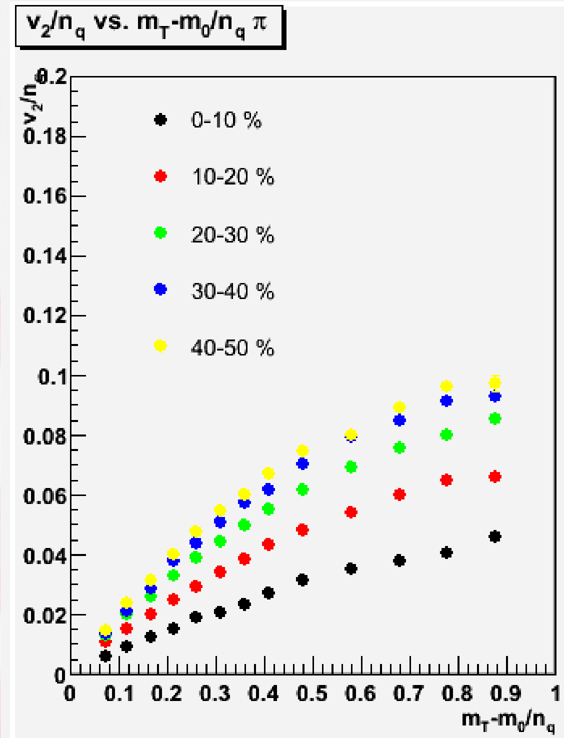
● **Is going to check next**

● **checked**

Universal Scaling

ex. Au+Au 200GeV π

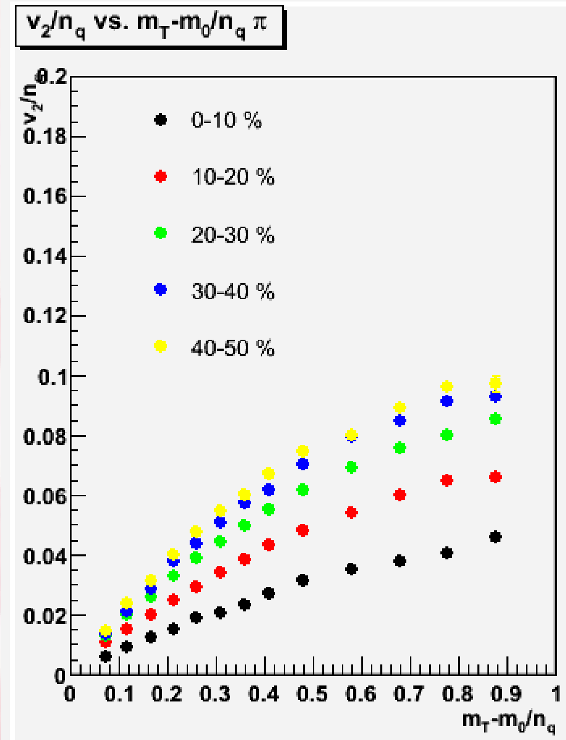
quark number + K_{ET} scaling.



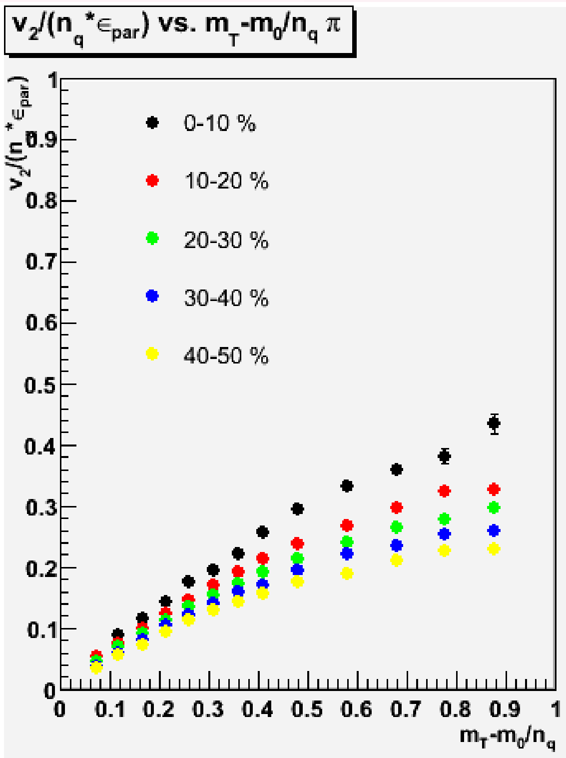
Universal Scaling

ex. Au+Au 200GeV π

quark number + K_{ET} scaling.



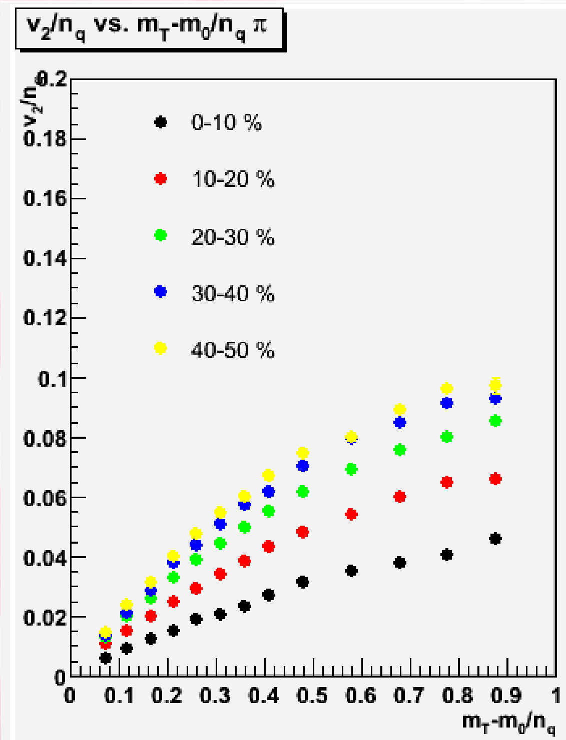
+ eccentricity scaling



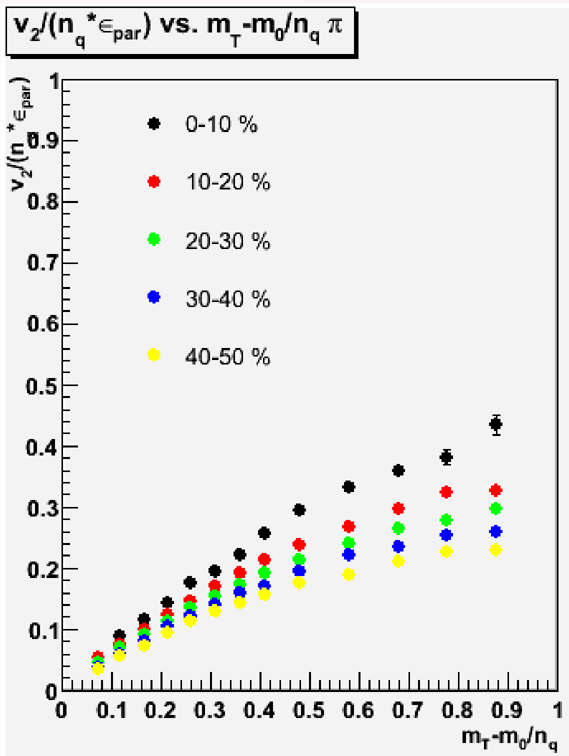
Universal Scaling

ex. Au+Au 200GeV π

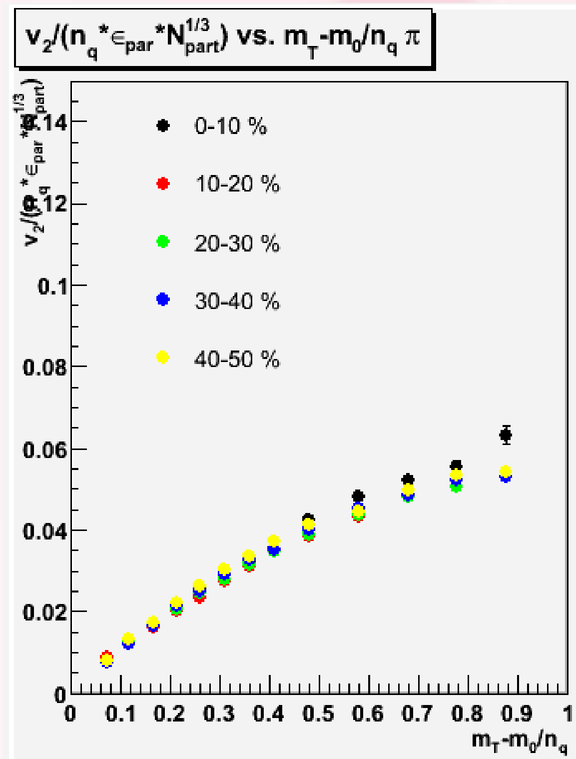
quark number + K_{ET} scaling.



+ eccentricity scaling

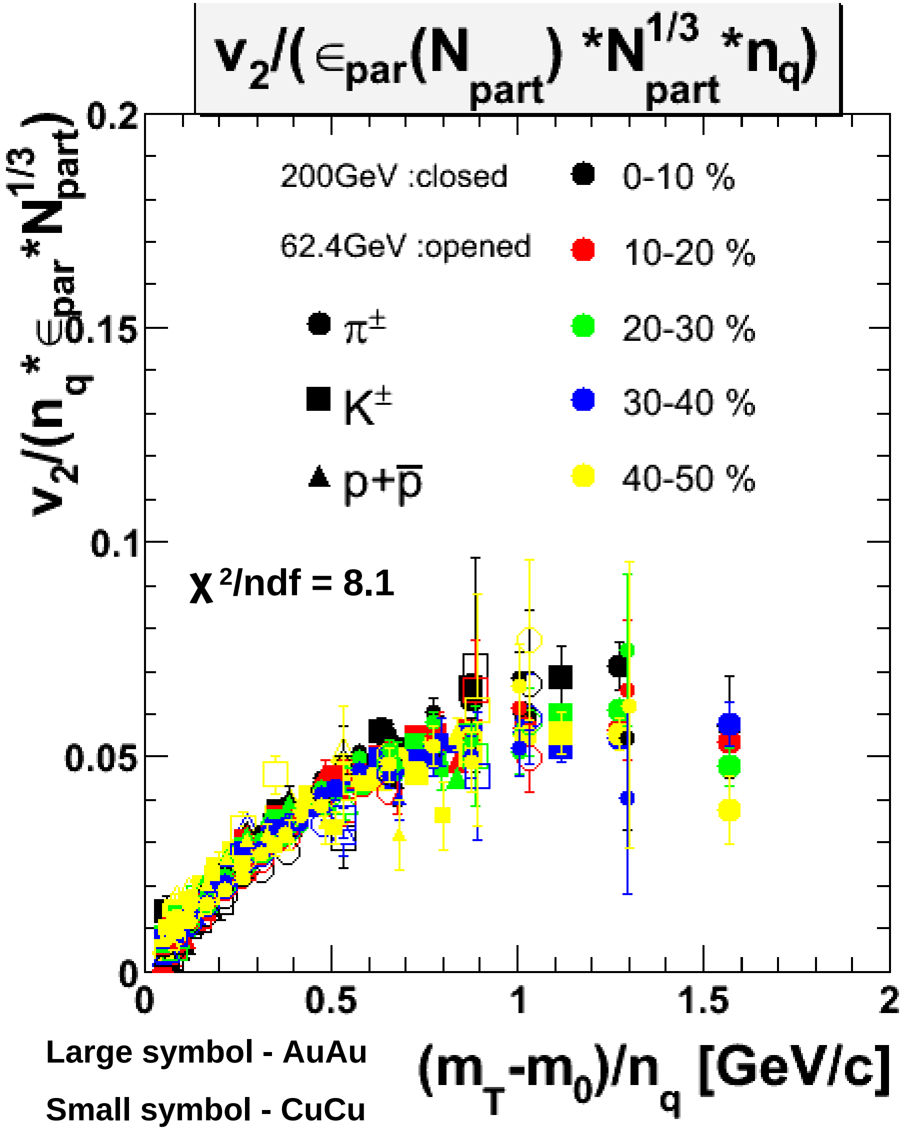


+ Npart$^{1/3}$ scaling



$v_2(K_{ET}/n_q)/n_q/\varepsilon_{par}/N_{part}^{1/3}$ is consistent at 0-50% centralities.

Universal Scaling



Large symbol - AuAu

Small symbol - CuCu

- Different System (Au+Au, Cu+Cu)
- Different Energy (200GeV - 62.4GeV)
- Different Centrality (0-50%)
- Different particles (π/ K /p)

$$\frac{v_2(K_{ET} / n_q)}{n_q \times \varepsilon \times N_{part}^{1/3}}$$

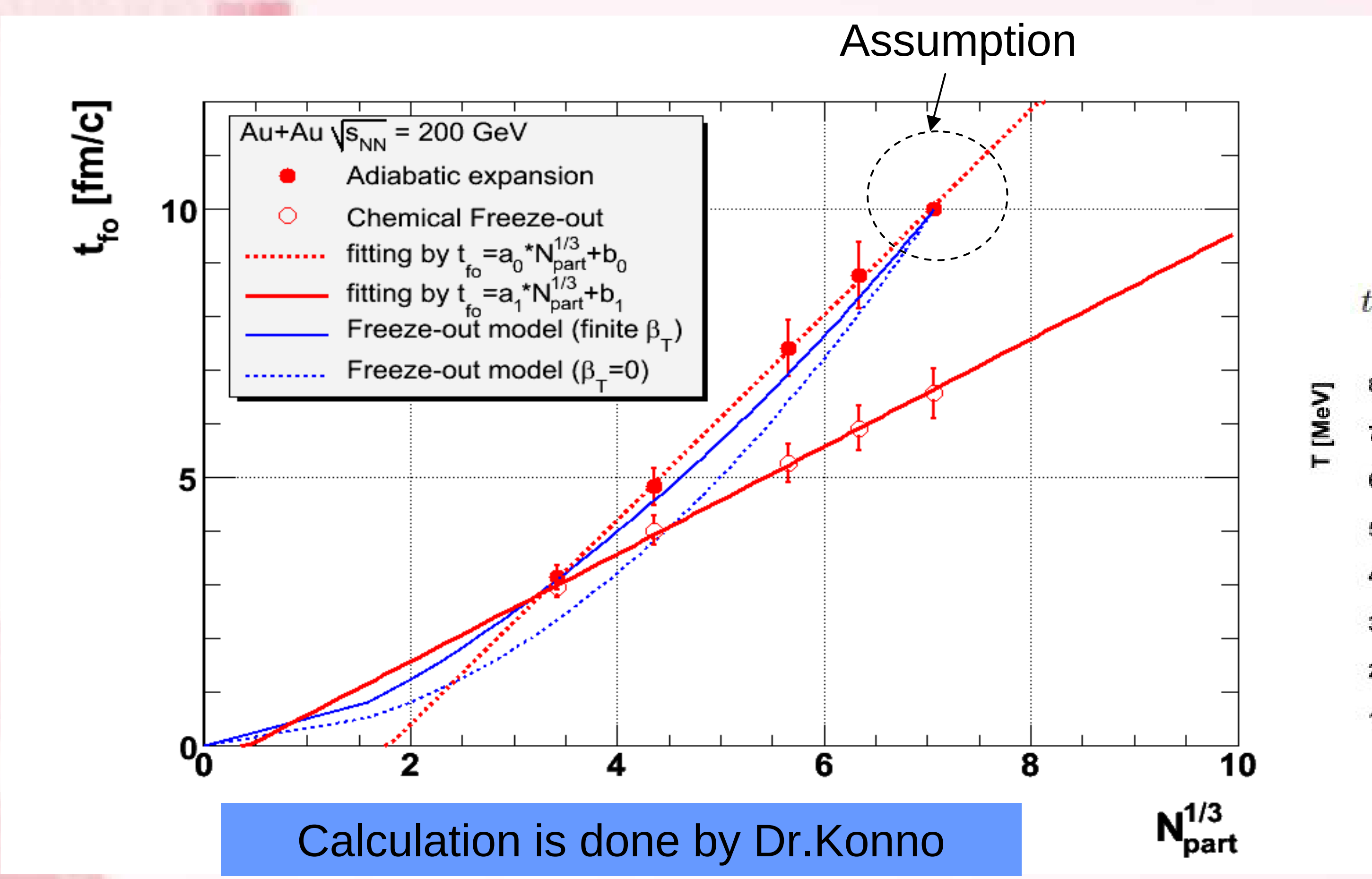
Universal Curve !!

Conclusion

- v_2 were measured at 4 systems.
  - (Au+Au, Cu+Cu) x (62.4GeV, 200GeV)
- Same $v_2(p_T)$ are obtained in different collision energies ($\sqrt{s}$ = 62.4 - 200GeV)
- $v_2(p_T)$ of various hadron species are scaled by quark number + K_{ET} scaling at these three systems. (no results for Cu+Cu 62.4GeV)
- $v_2(N_{part})$ scaled by participant Eccentricity are consistent between Au+Au and Cu+Cu collisions
- $v_2(p_T)/\varepsilon_{par}$ are scaled by $N_{part}^{1/3}$.
- $v_2(K_{ET}/n_q)/n_q/\varepsilon_{par}/N_{part}^{1/3}$ has Universal Curve.

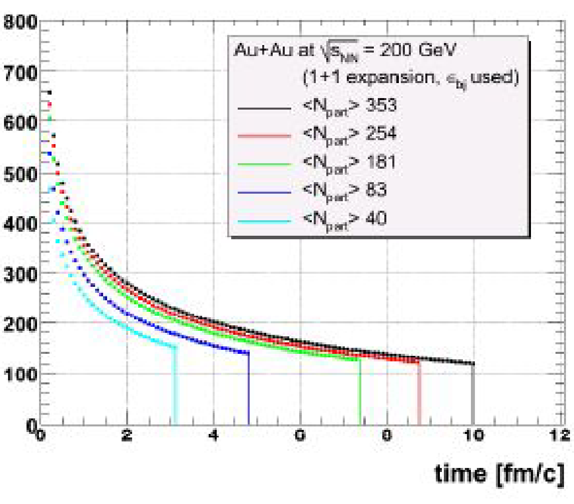
  → This indicates v_2 are determined by the initial geometrical anisotropy and its time evolution effect depending on the initial volume.

Calculation by simple expansion model



$$T(t) = T_0\left(\frac{t_0(R_0+\beta_T t_0)^2}{t(R_0+\beta_T t)^2}\right)^{1/3}$$

$$t_{fo} = (\sqrt{R_0^2 + 4\beta_T K N_p} - R_0)/2\beta_T$$



Time until chemical freeze-out is proportional to $N_{part}^{1/3}$.

Summary of v_2 production and development

Low to mid p_T

Time t

collision

Determine initial geometrical eccentricity, ε, with the participant.

thermal equilibrium

Determine pressure gradient from ε.

expanding

v_2 is expanding during finite time.

Not depending on the kind of quarks.

hadronization

This finite time becomes longer with larger collision system, and the v_2 increases proportionally.

radial flow depending on each mass expands.

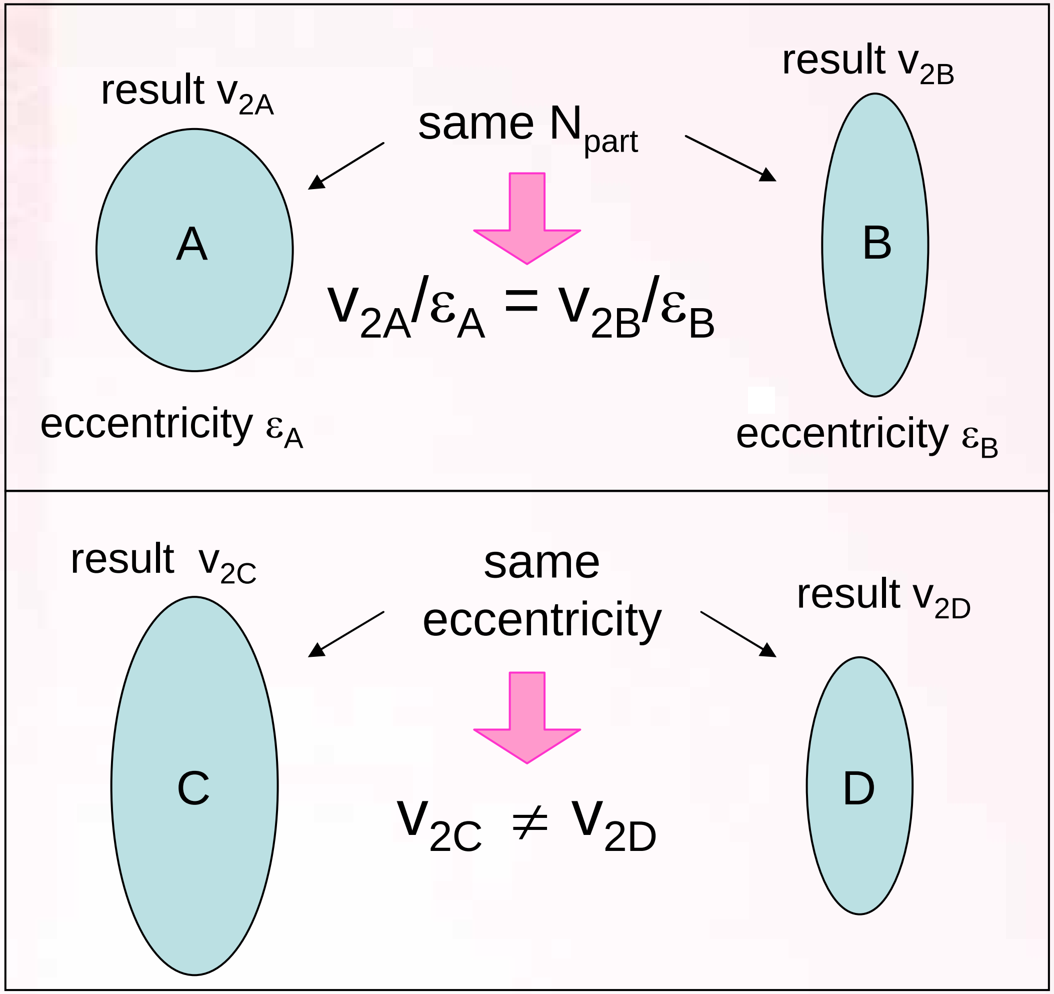
freeze out

No change

Measurement

Summary (1)

When the systems have same N_{part}, v_2 is scaled by ε of paricipant geometry.

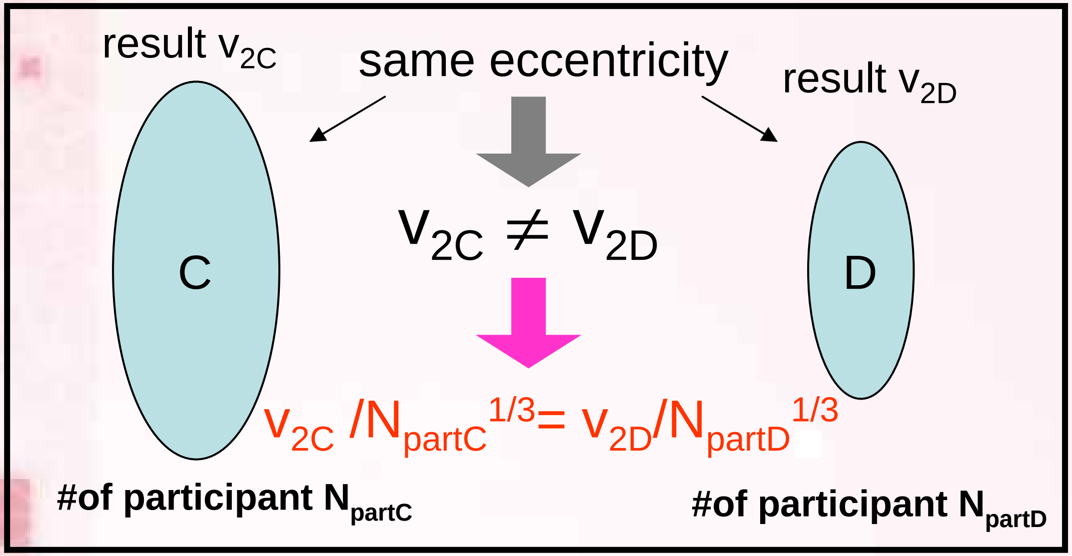


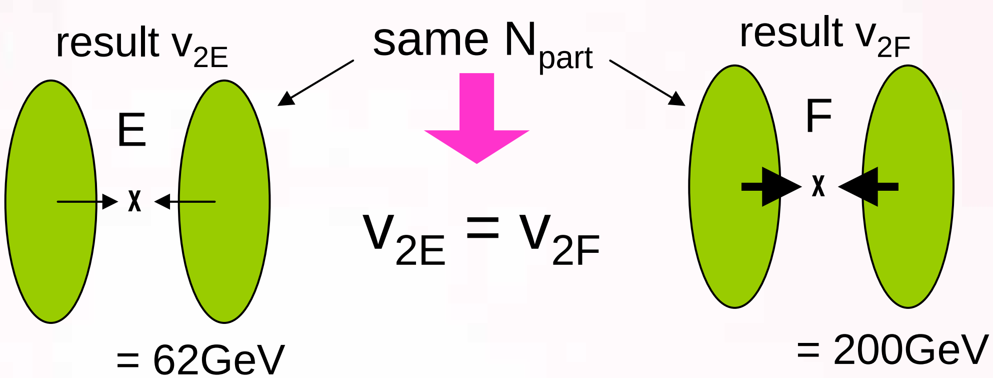
If v_2 only depends on eccentricity of initial participant geometry, v_2/ε should be constant at any N_{part}, but it is not.

Therefore, to explain v_2, in addition to the initial geometrical eccentricity, there are something related to N_{part}.

Summary (2)

With same eccentricity, v_2 is scaled by (number of participants)$^{1/3}$.





v_2 becomes consistent after scaled by not only ε but also $N_{part}^{1/3}$.

Is it because of thickness increasing along beam axis then energy per unit area increasing ?

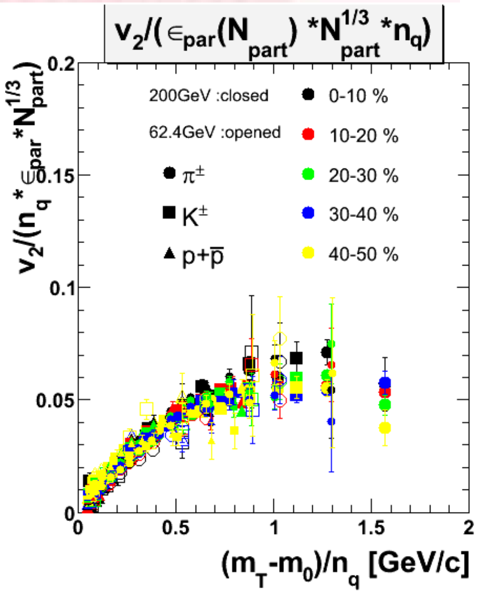
$v_2(200\text{GeV}) = v_2(62.4\text{GeV})$

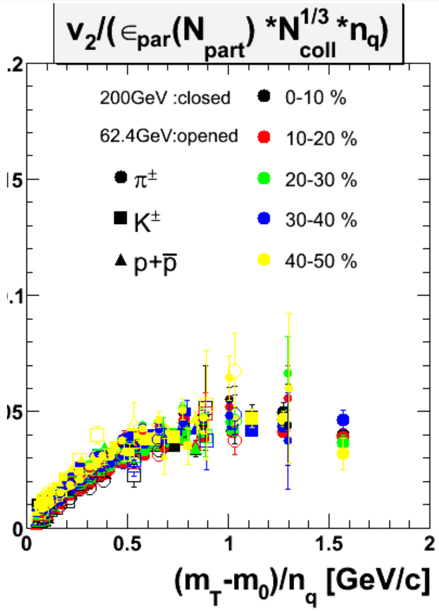
This concludes that increasing dN/dy doesn't change v_2 at RHIC energy.

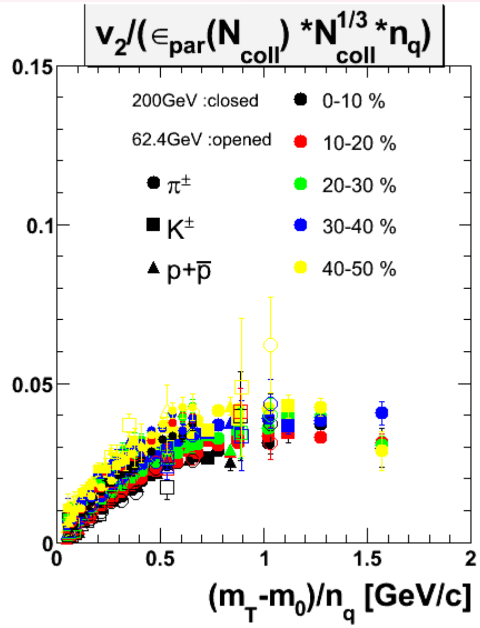
It might be because that number of participant to 1/3 (like length) is proportional to the time period taken to freeze out v_2 , and v_2 expands proportional to that period.

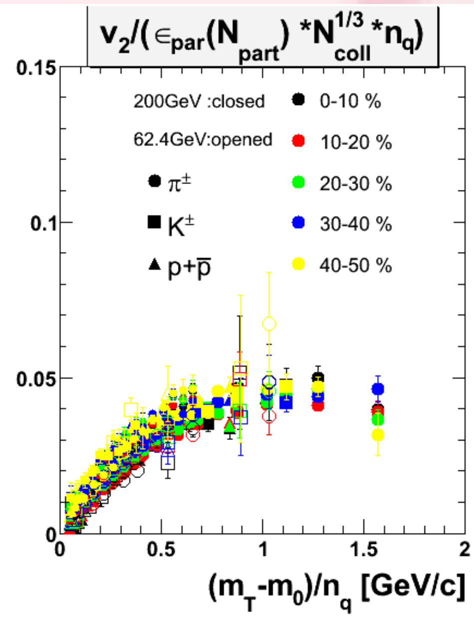
Back Up

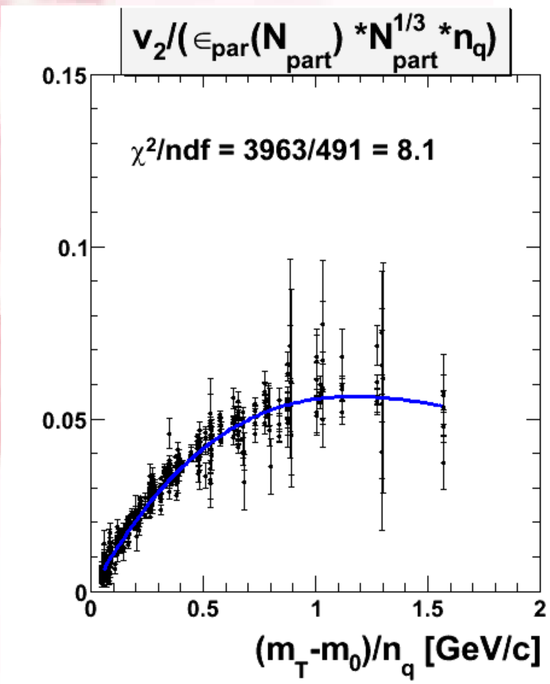
3 systemes comparison

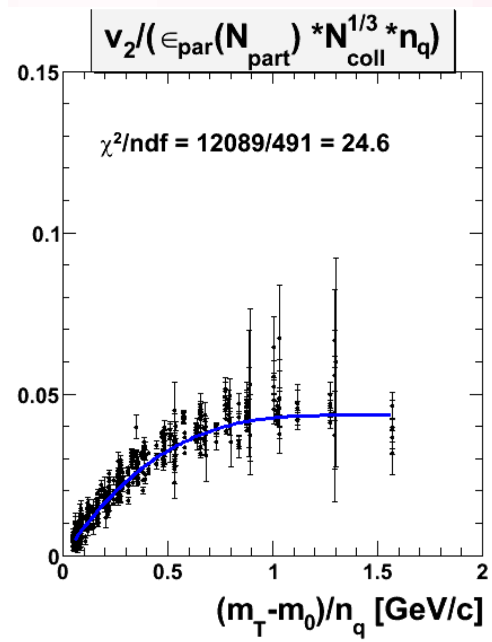










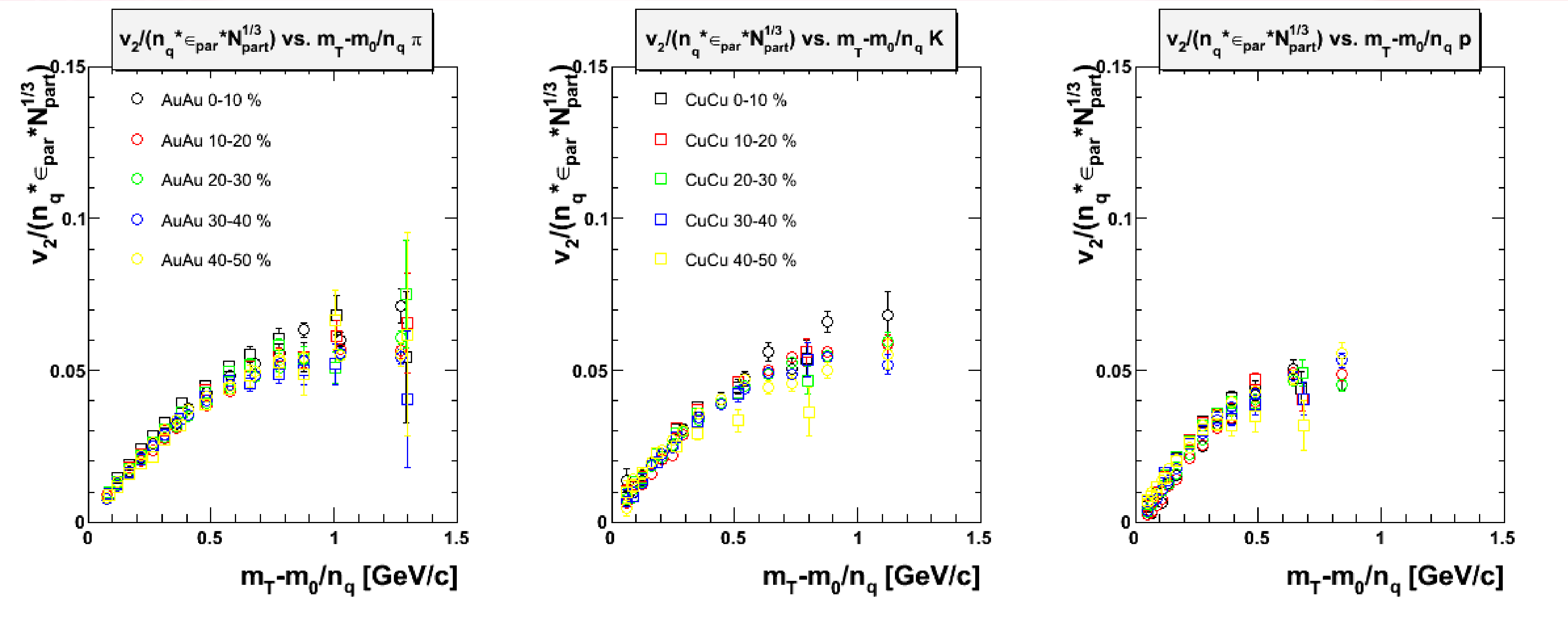


Various scalings.
Eccentricity of N_{part} and $N_{part}^{1/3}$ looks best.

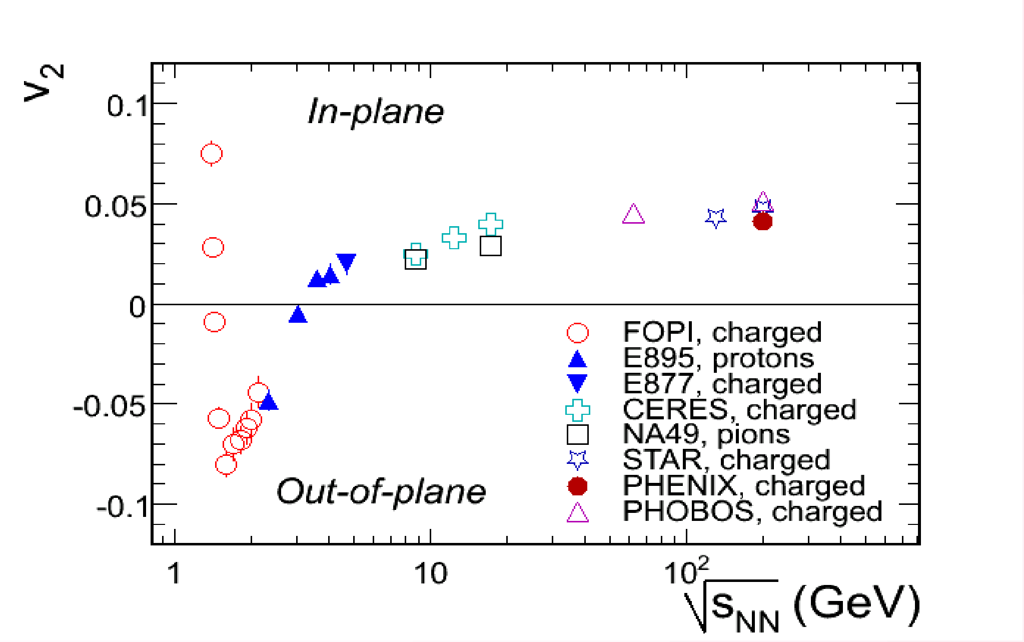
Comparison of AuAu to CuCu

Cu+Cu and Au+Au, 200GeV, PID by EMC

Apply quark number + KET scaling, eccentricity scaling and Npart$^{1/3}$ scaling.

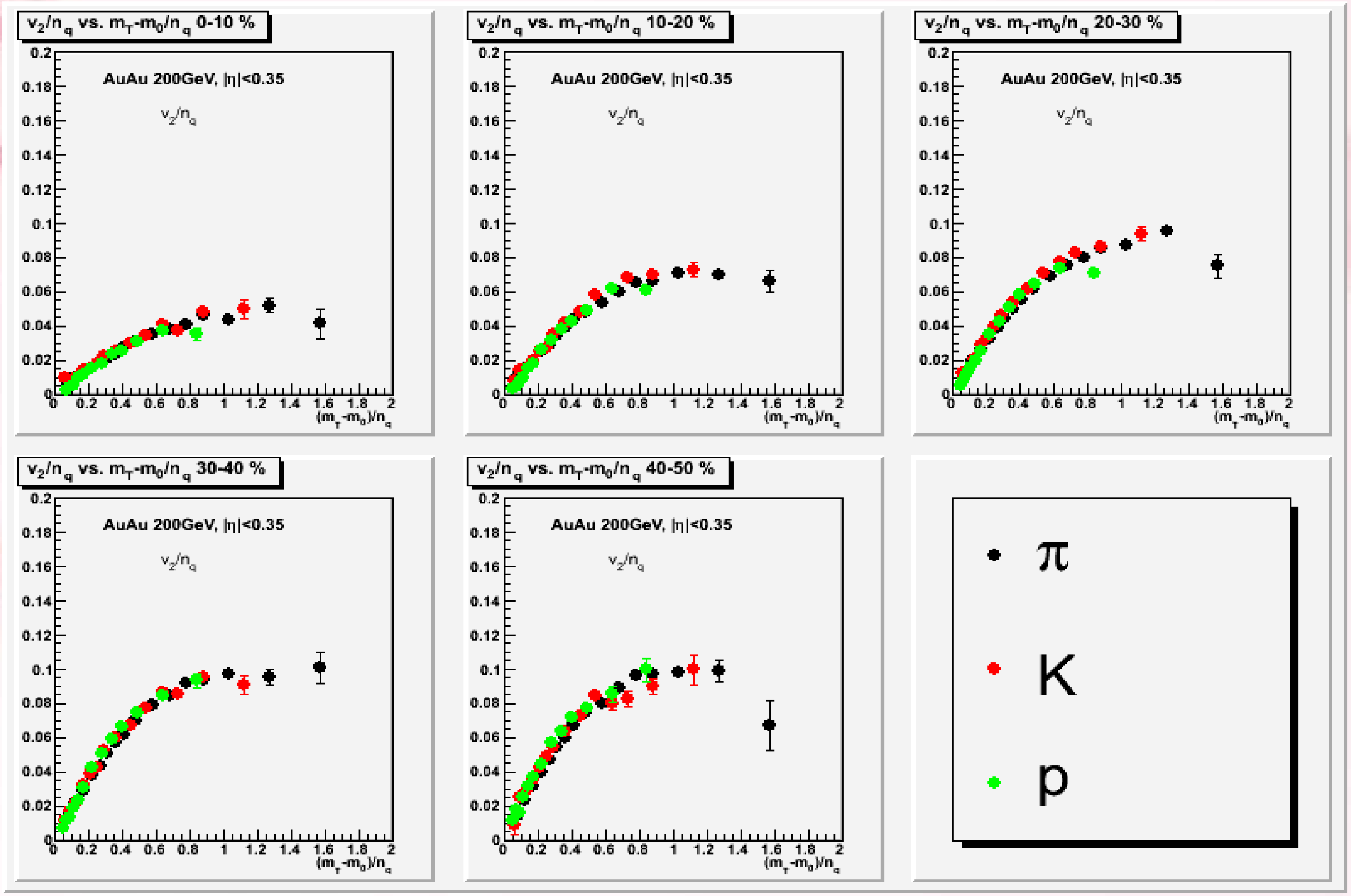


Energy dependence



FOPI : Phys. Lett. B612, 713 (2005). E895 : Phys. Rev. Lett. 83, 1295 (1999)
CERES : Nucl. Phys. A698, 253c (2002). NA49 : Phys. Rev. C68, 034903 (2003)
STAR : Nucl. Phys. A715, 45c, (2003). PHENIX : Preliminary.
PHOBOS : nucl-ex/0610037 (2006)

Quark number + K_{ET} scaling (AuAu 200GeV)

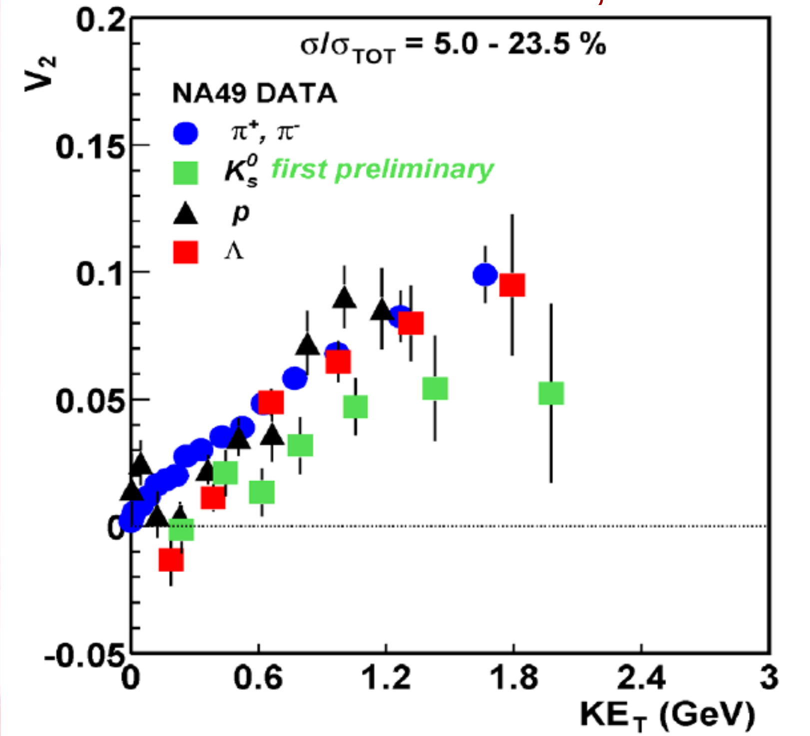


- Quark number + K_{ET} scaling exists.

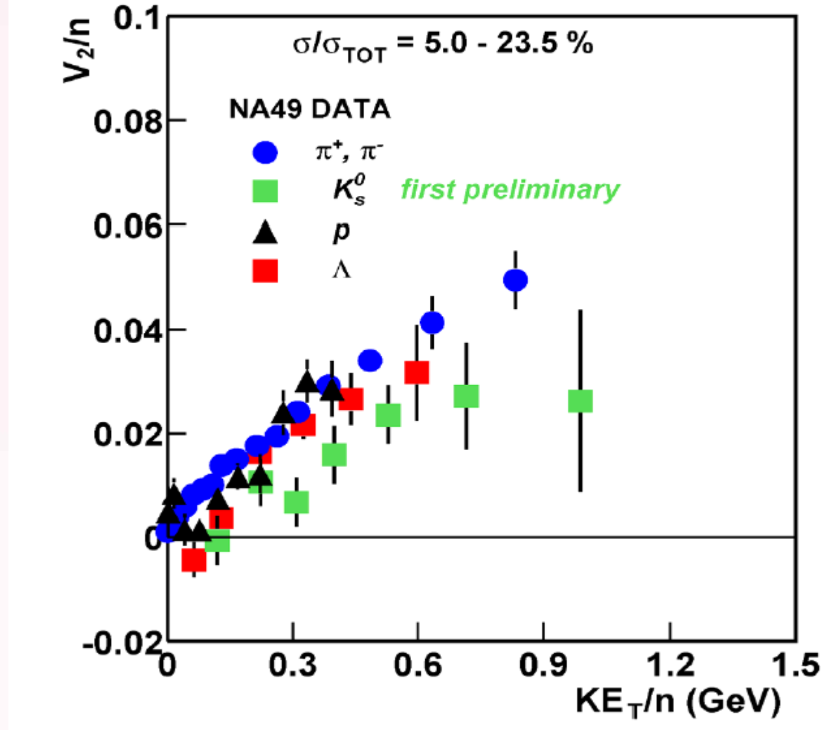
Additional quark number + K_{ET} scaling (PbPb 17.2GeV)

v_2 of p, **π**, **Λ** - C. Alt et al (NA49 collaboration) nucl-ex/0606026 submitted to PRL
v_2 of K^0 (preliminary) - G. Stefanek for NA49 collaboration (nucl-ex/0611003)

Pb+Pb at 158A GeV, NA49



Taken from A. Tranenko's talk at QM 2006

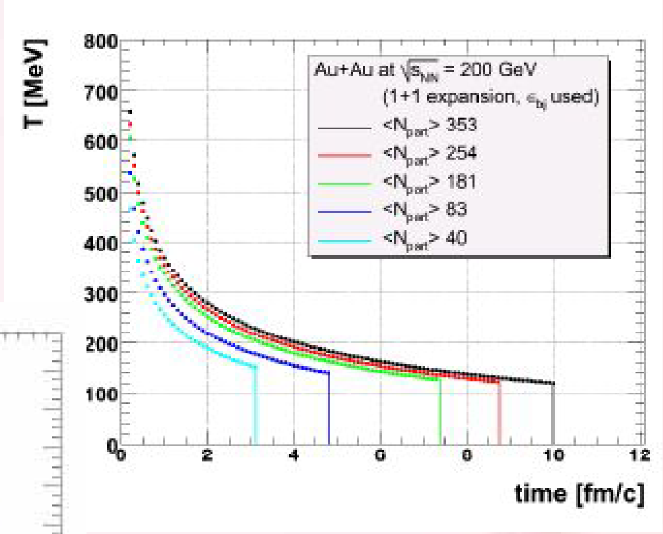


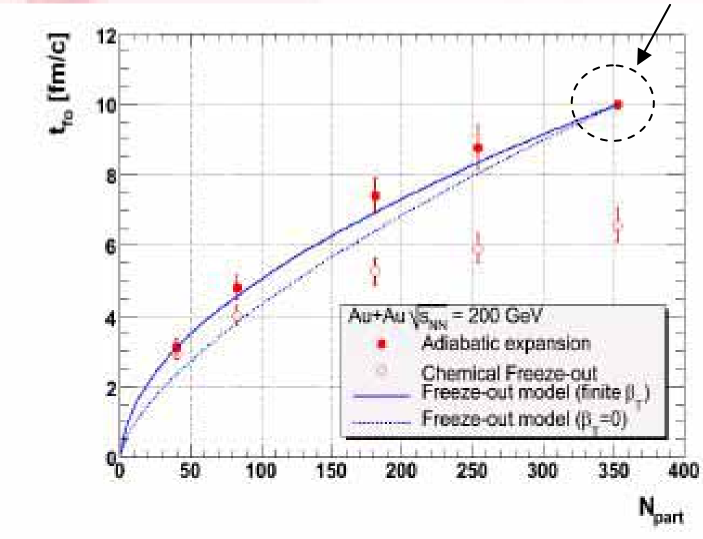
- Quark number + K_{ET} scaling doesn't seem to work out at SPS.
- No flow at partonic level due to nonexistence of QGP ?
- Errors are to big to conclude it.

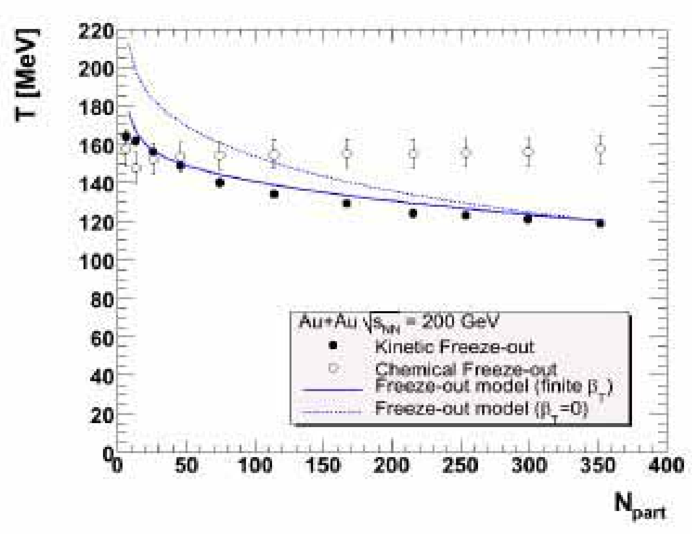
t_{f0} vs. N_{part}

$$T(t) = T_0 \left(\frac{t_0 (R_0 + \beta_T t_0)^2}{t (R_0 + \beta_T t)^2} \right)^{1/3}$$

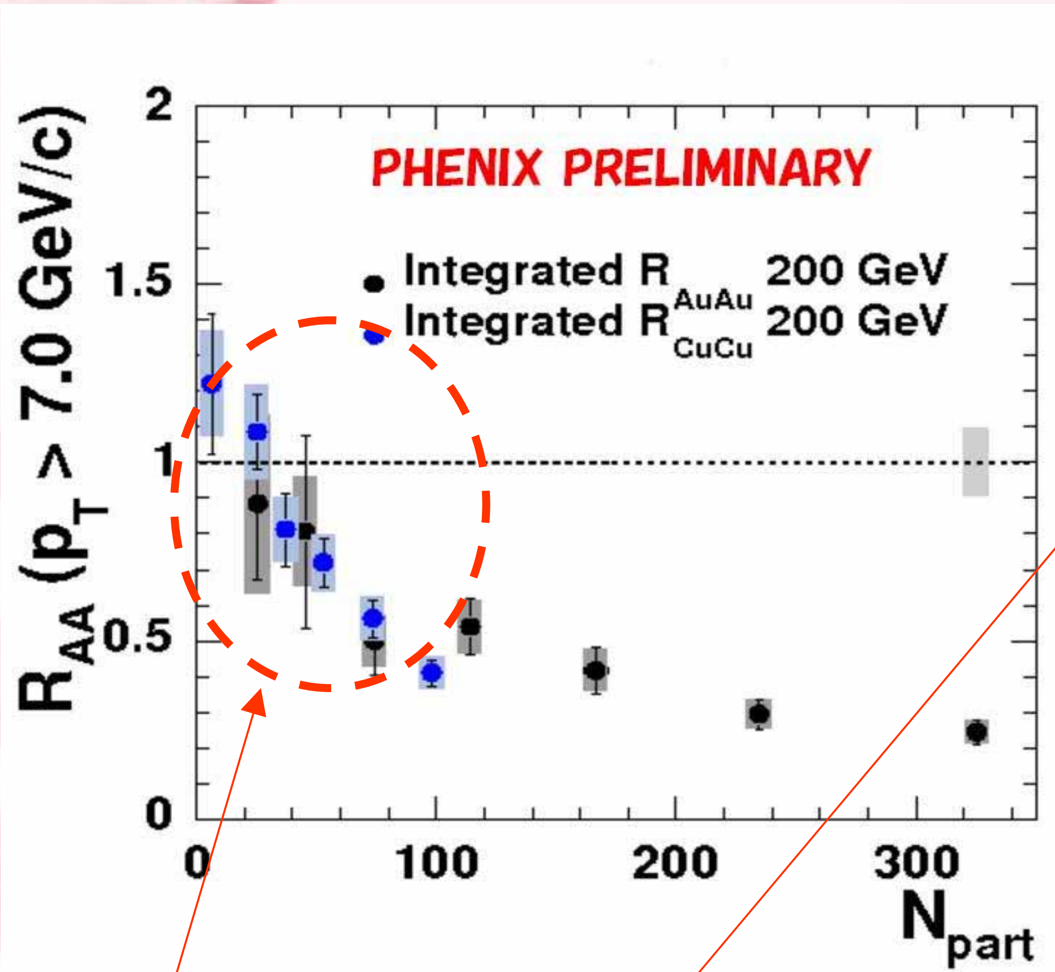
$$t_{fo} = \left(\sqrt{R_0^2 + 4\beta_T K N_p} - R_0 \right) / 2\beta_T$$

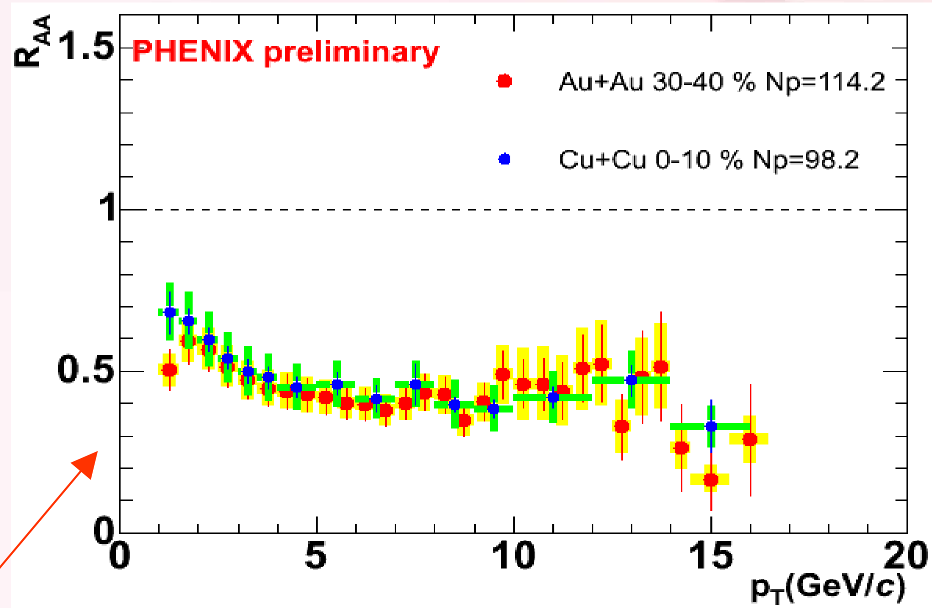


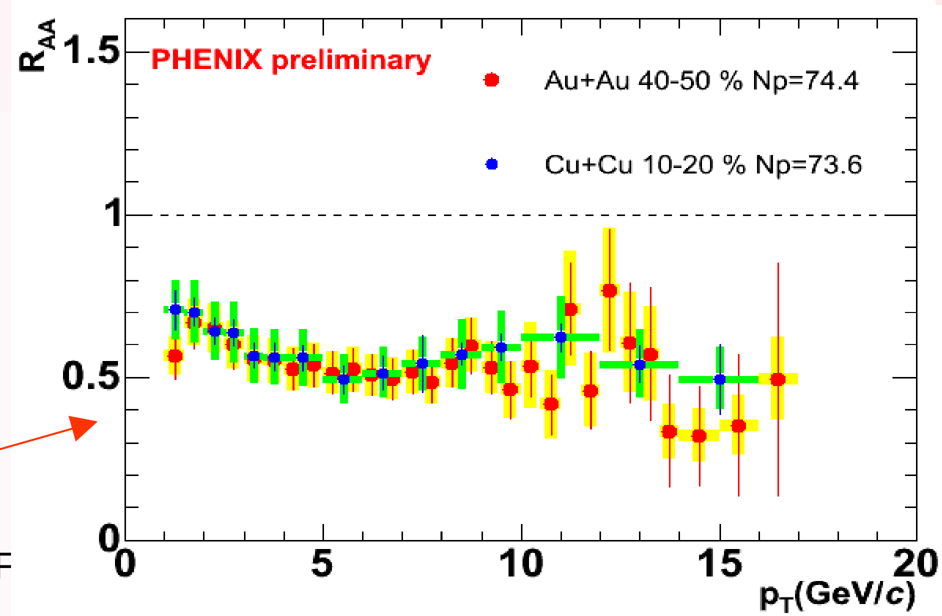




Comparison between Au+Au and Cu+Cu







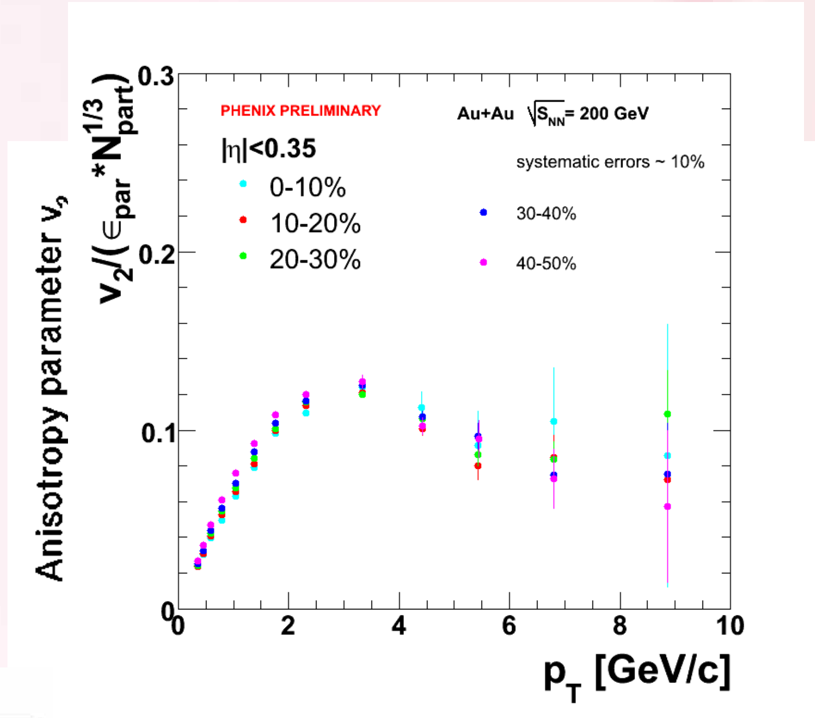
Both behave same at mid central.

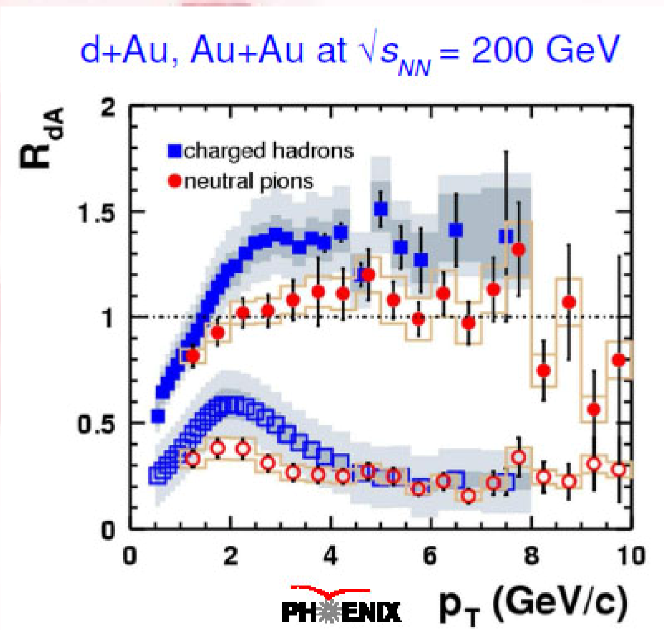
Additional N_{part} scaling

Au+Au 200GeV

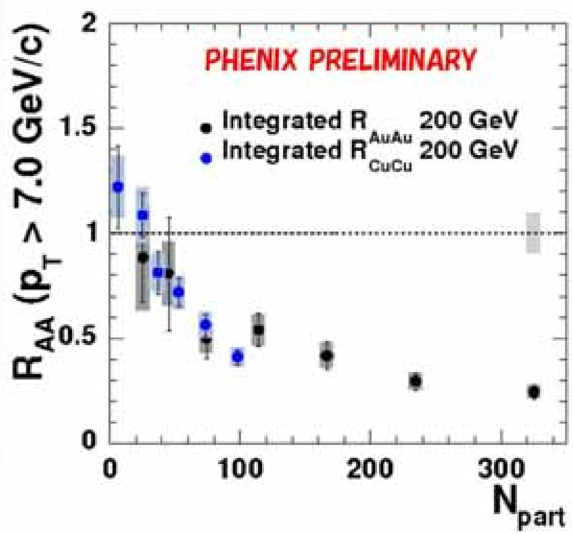
- At high p_T (> 6GeV/c), scaling might work out but errors are too large to conclude.

→need to analyze the data with higher statistics. (ex.Run7)





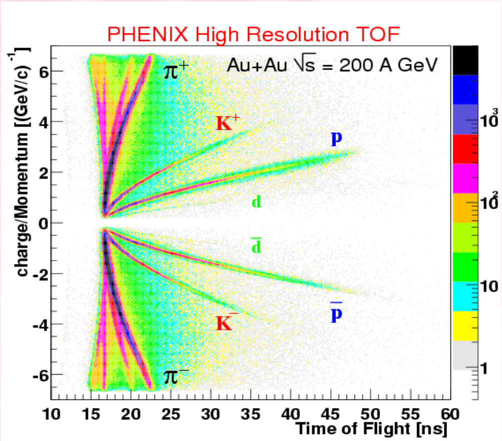
***PRL 91 072303 (2003)**



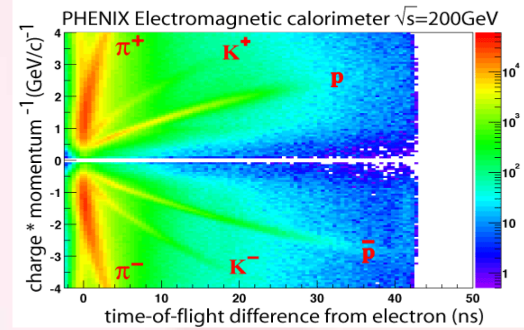
R_{AA} – yield normalized by p+p superposition.
It would be 1 without suppression.

By spectra analysis, the strength of the suppression seems to be consistent at same N_{part}. It doesn't depend on the nucleus species of collision system (Au+Au, Cu+Cu).

→R_{AA} can be scaled by N_{part}.



Analysis



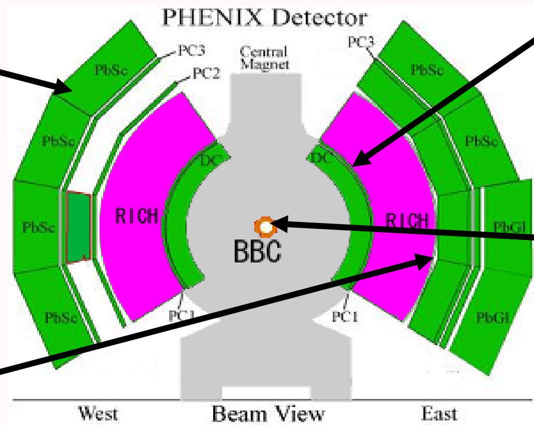
<Data set for this analysis>

- Au+Au Cu+Cu collision
- taken in 2003-2005 at RHIC-PHENIX
- Collision energy ：200, 62.4 GeV/2 nucleons

<PHENIX detectors>

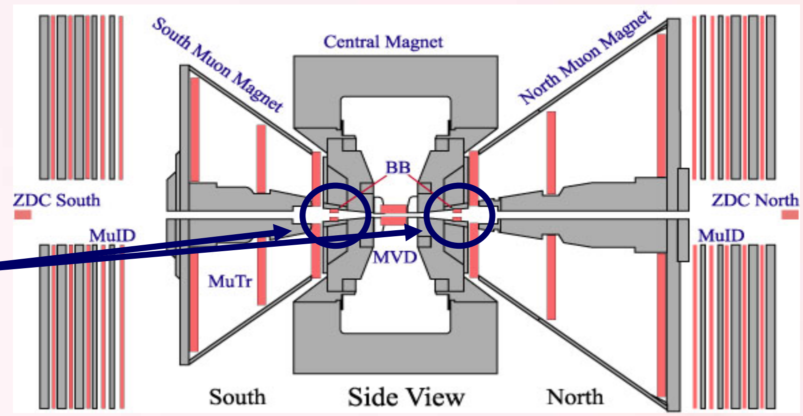
EMCAL
for Particle Identification
resolution=380ps

TOF for Particle Identification
resolution=120ps



DC + PC1
for good track selection and to determine p

BBC
to determine reaction plane and vertex



<PID by TOF measurement>

Using TOF or EMC with BBC, the flight time of the particles is obtained. Mass of the particle is calculated by the flight time and the momentum measured by DC.

<Reaction Plane determination>

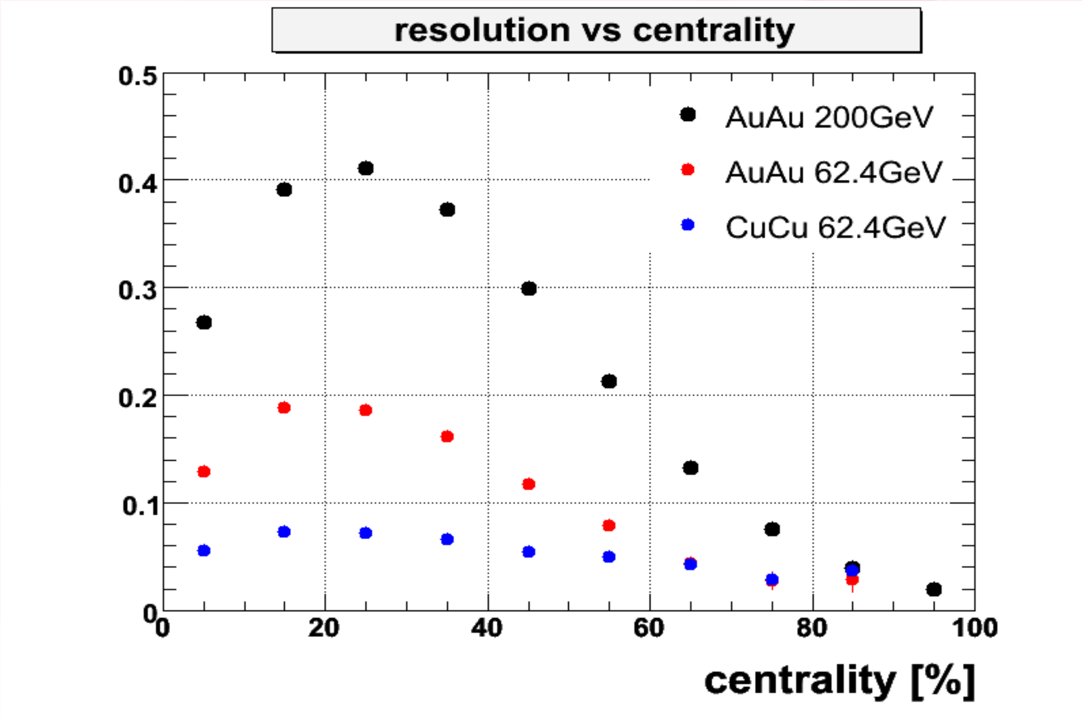
The reaction plane is obtained by measurement of the anisotropic distribution for the produced particles with north and south BBCs located at $|\eta| \sim 3 – 4$.

Resolution Calculation of Reaction Plane

$$resolution = \langle \cos[2(\Psi_{measured} - \Psi_{true})] \rangle \sim \sqrt{\langle \cos[2(\Psi_A - \Psi_B)] \rangle}$$

ΨA,B : reaction plane determined for each sub sample.

$$v_2^{real} = \frac{v_2^{measured}}{resolution}$$



- BBC North + South combined