Glauber、 粒子多重度、中心度、揺らぎなど

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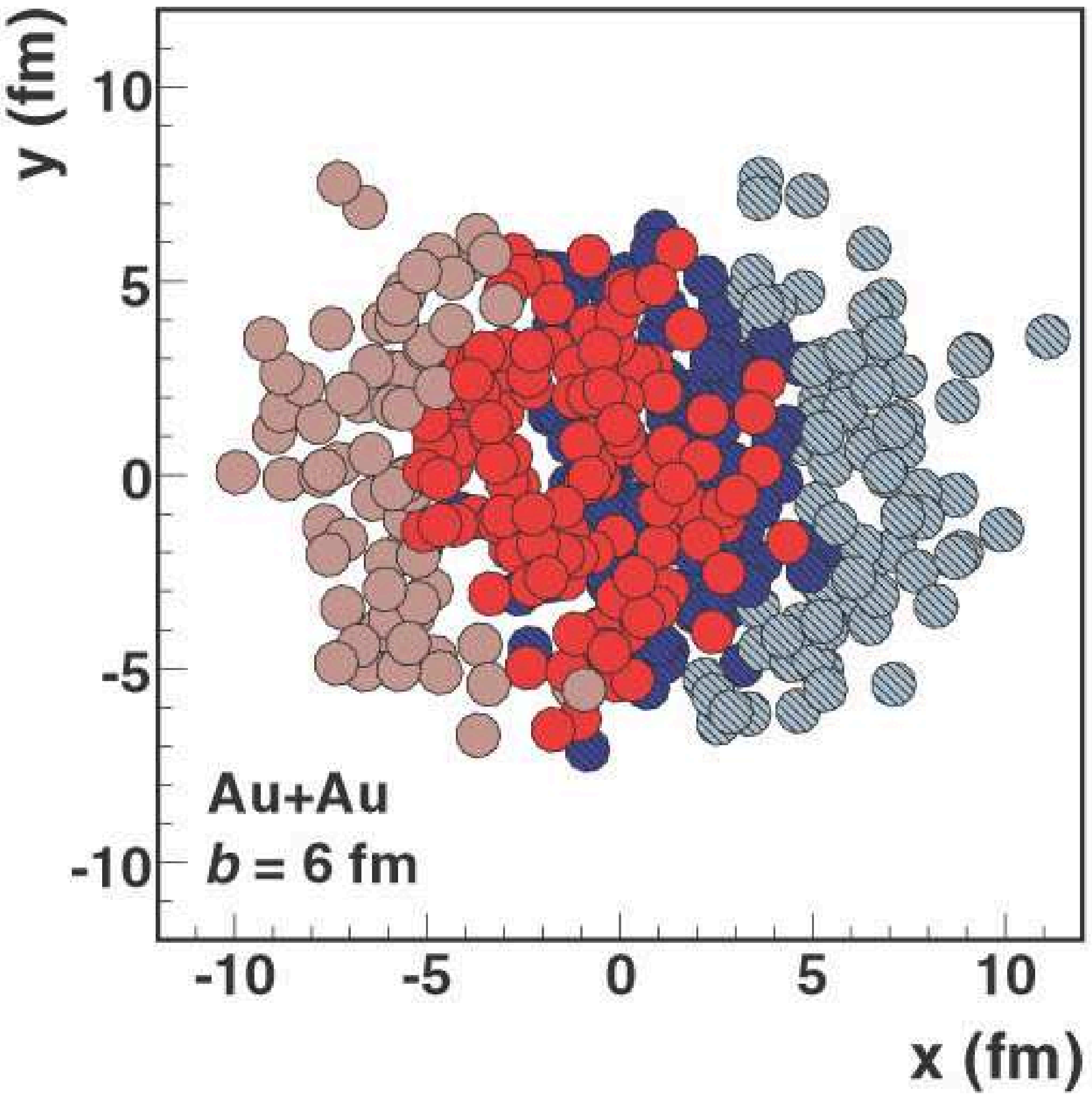
Outline

- Glauber model, multiplicity and centrality
  - Introduction of Glauber model in heavy ion collisions
    - more specifically, "Wounded nucleon model"
  - How to determine the centrality in experiment ?
    - multiplicity model, Negative Binomial Distribution
- Fluctuation
  - Why do we measure fluctuations in heavy ion collisions ?
  - Experimental results: particle ratio fluctuation (K/π), higher moments
- Summary

Introduction of Monte Carlo (MC) Glauber model

Glauber model



M. L. Miller et al, arXiv:nucl-ex/0701025

- The simplest approach to describe the initial condition of nucleus-nucleus collisions
- Widely used to determine centrality, and for initial conditions in hydrodynamical models, event generators

Monte Carlo (MC) Glauber model

- Basic assumptions
  - nucleons travel on straight line trajectories
  - independent binary nucleon-nucleon collisions
  - inelastic nucleon-nucleon cross section is independent of number of binary collisions of a nucleon underwent before
- Impact parameter is randomly sampled ($dN/db \sim b$)
- Nucleons are randomly distributed inside nuclei
- Collision occurred based on the transverse distance between nucleons, and on the measured nucleon-nucleon inelastic cross sections (from PDG)
- Model provides impact parameter (b), number of participants (N_{part}), number of binary collisions (N_{coll} or N_{bin}), and their correlations
  - also provides spatial anisotropy, so called “eccentricities”

How many parameters in Glauber model ?

- Nucleons
  - Density distribution of heavy nucleus is parameterized by Woods-saxon form (2)

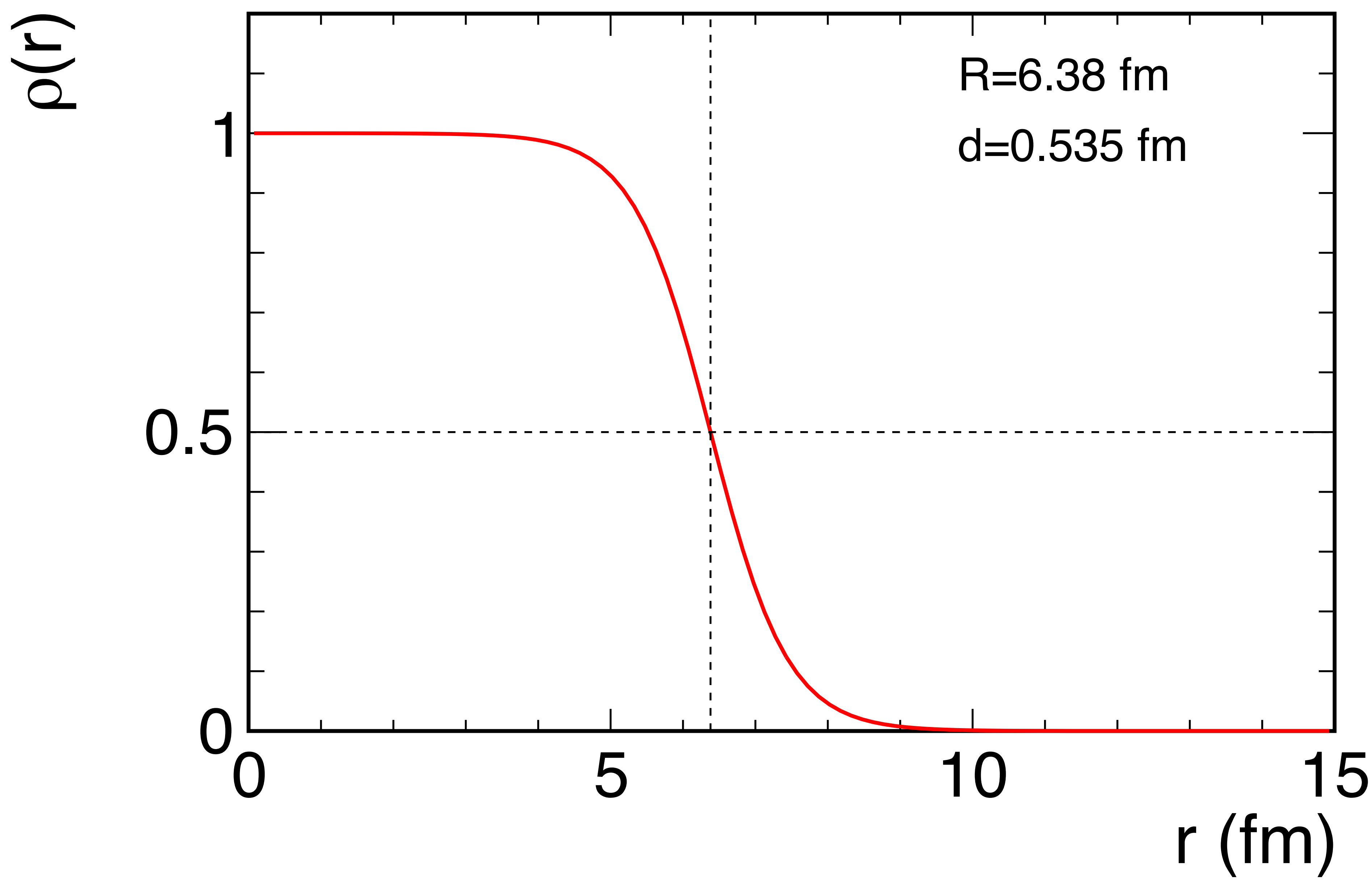
$$\rho(r) \propto \frac{1}{1+\exp\left((r-R)/d\right)}$$

    - radius of nucleus *R*, skin depth (or diffuseness parameter) *d*
  - Deformed nucleus needs additional parameters (1, 2, or maybe 3)
    - Au nucleus is deformed, Pb is spherical
  - Separation between two nucleons in a nucleus (1 or 2)
    - As far as I know, this option is not implemented by default at RHIC experiments
- Collision
  - Measured inelastic nucleon-nucleon cross section (1)
  - The simplest collision profile is box type

$$\sqrt{(x_1-x_2)^2+(y_1-y_2)^2} < \sqrt{\frac{\sigma_{pp}^{inel}}{\pi}}$$

    - additional parameters if one use non-box like collision profile
- We need additional parameters to calculate multiplicity
  - This is the place where Negative Binomial Distribution plays a role

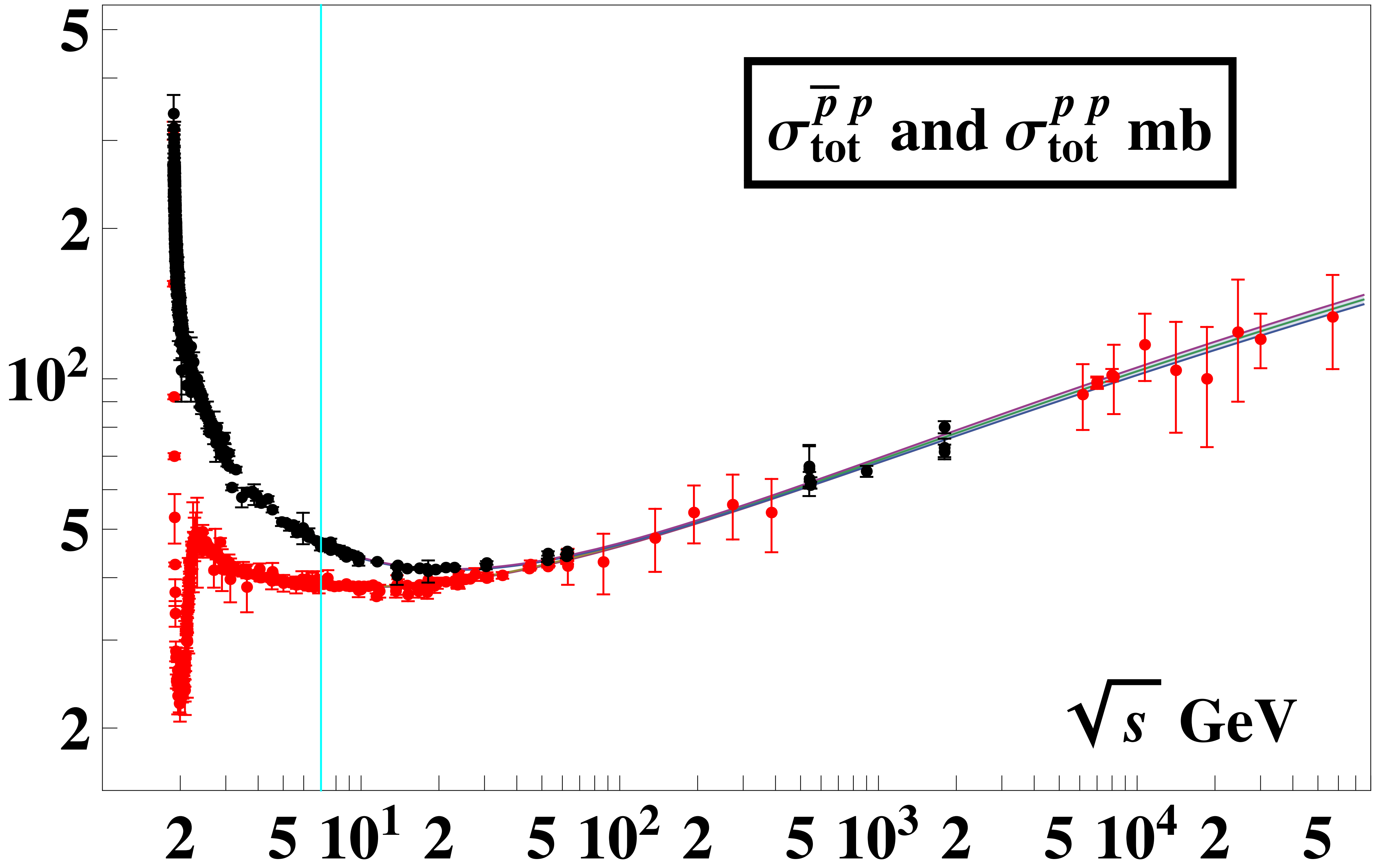
Woods-saxon distribution



- Constant up to r ~ 5 fm
- $\rho(r) = 1/2$ at r=R
- Finite probability in r>R due to the diffuseness parameter d

Total p+p cross section (PDG)

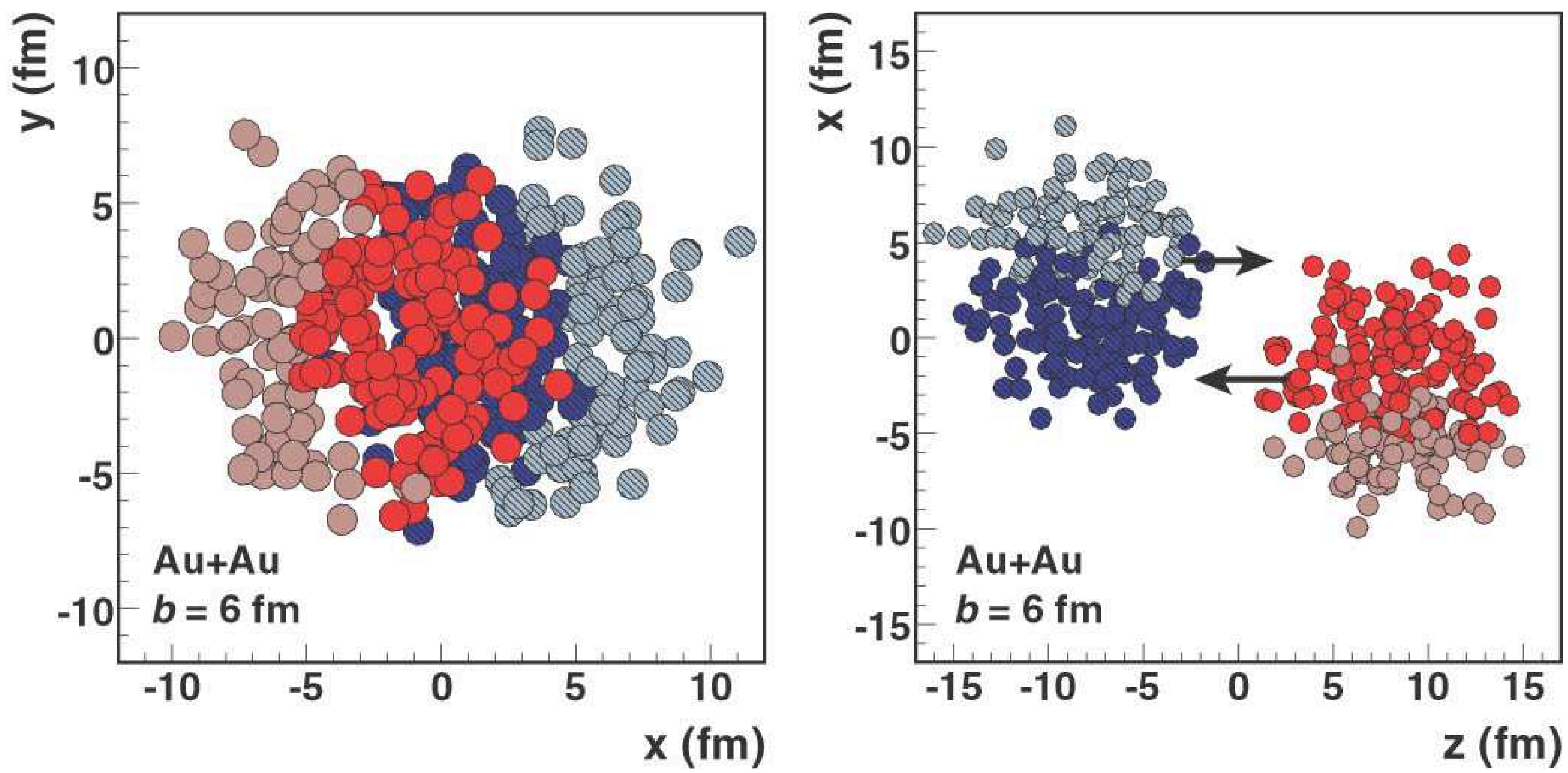
http://pdg.lbl.gov/2014/reviews/rpp2014-rev-cross-section-plots.pdf



- Total elastic cross sections are also available
  - ~42 mb is mostly used at RHIC

Snapshot of 1 collision at b=6 fm

Glauber Modeling in Nuclear Collisions 10

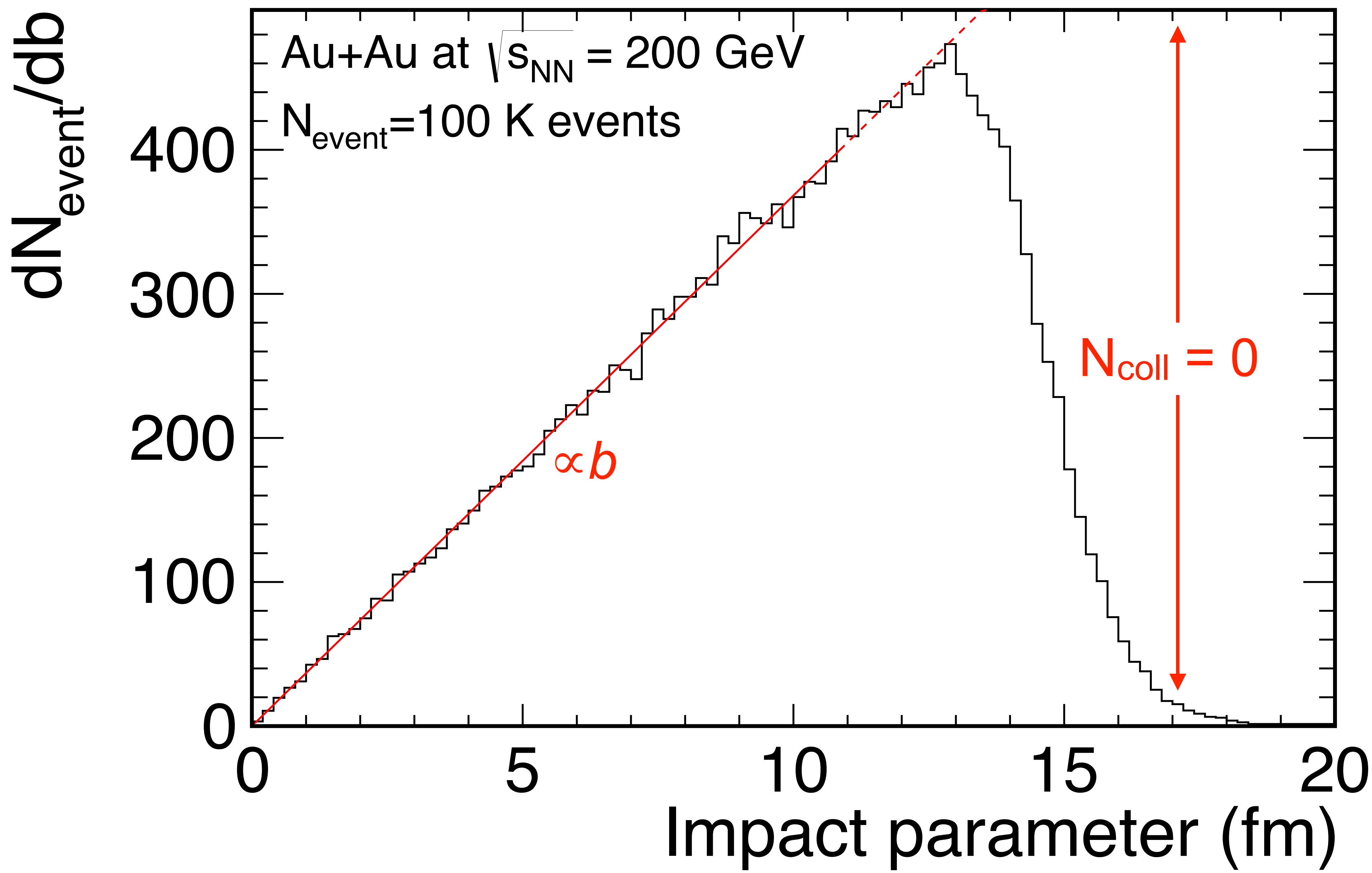


M. L. Miller et al, arXiv:nucl-ex/0701025

Figure 4: Glauber Monte Carlo event (Au+Au at $\sqrt{s_{\mathrm{NN}}} = 200$ GeV with impact parameter $b = 6$ fm) viewed in the transverse plane (left panel) and along the beam axis (right panel). The nucleons are drawn with a radius $\sqrt{\sigma_{\mathrm{inel}}^{\mathrm{NN}}/\pi}/2$. Darker disks represent participating nucleons.

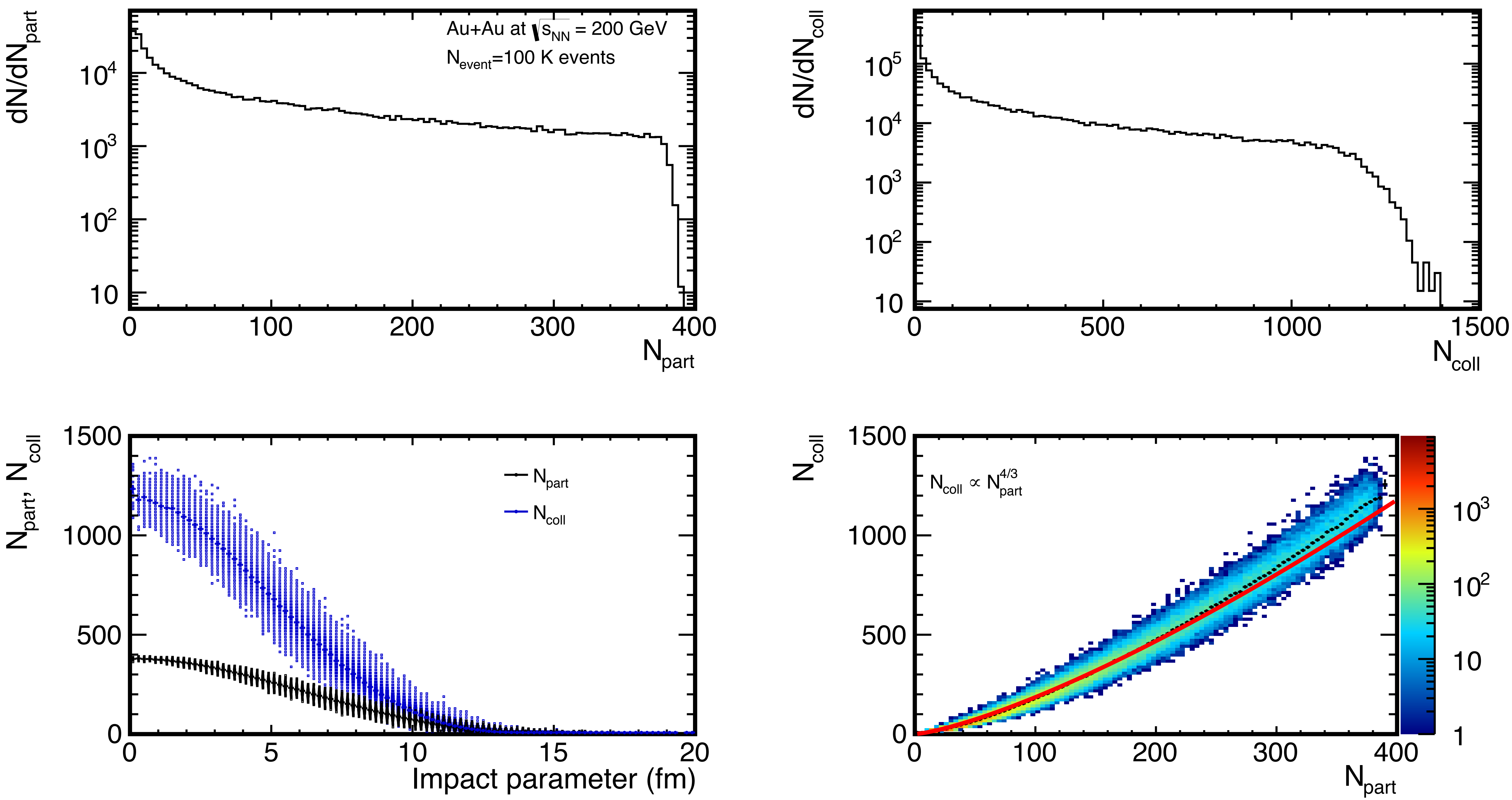
- Event display of 1 event (impact parameter b=6 fm)
- Positions of nucleon can be fluctuated event-by-event → N_{part} etc fluctuate even if we fix b in MC Glauber model

Impact parameter distribution



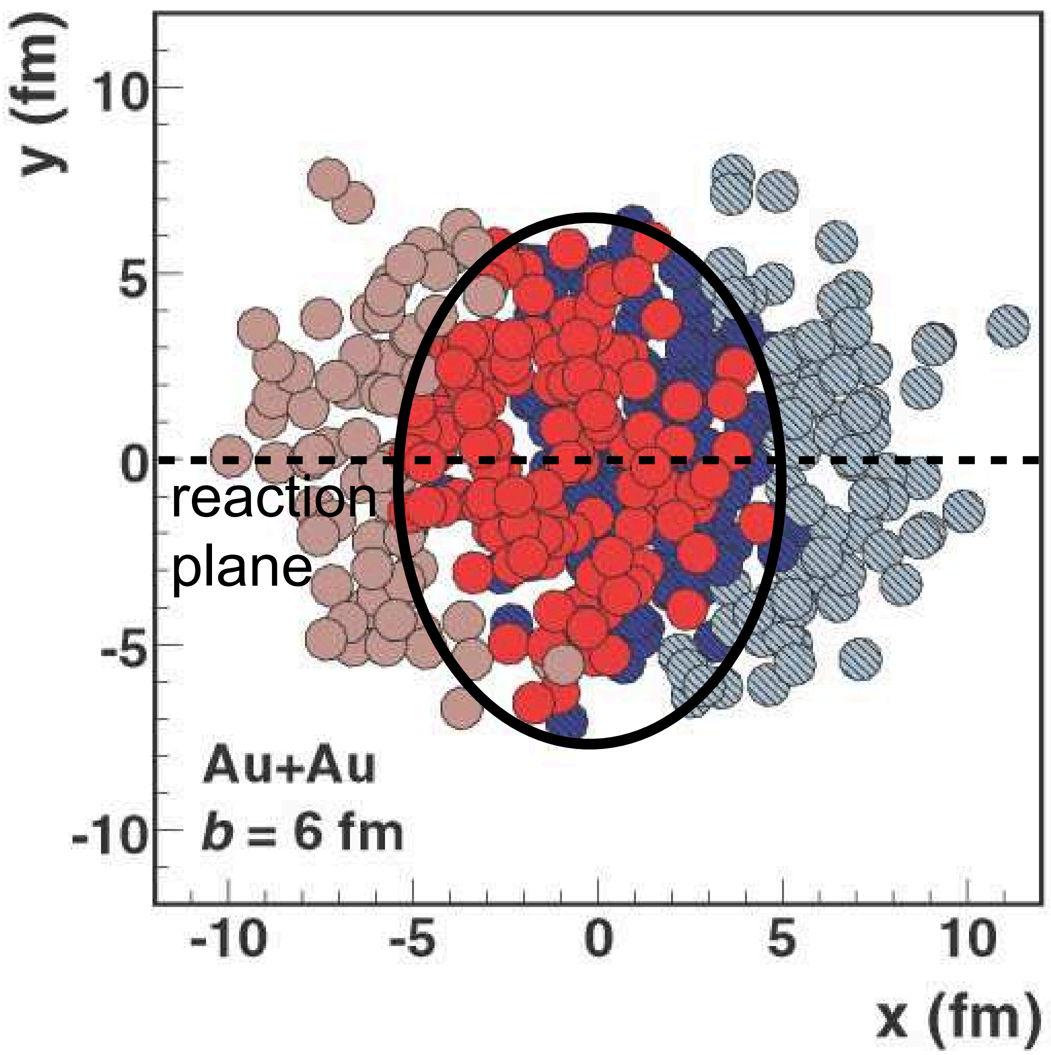
- We have collisions in b>2R because of Woods-saxon form
  - but collision (N_{coll}>0) isn't always occurred at high b

N_{part} & N_{coll} distributions



- Characteristic shape of N_{part}, N_{coll} (to be discussed later)
- $N_{coll} \propto N_{part}^{4/3}$

Spatial anisotropy (eccentricity)



$$\varepsilon_{RP} = \frac{\sigma_x^2 - \sigma_y^2}{\sigma_x^2 + \sigma_y^2},$$

standard eccentricity
(reaction plane eccentricity)

$$\sigma_x^2 = \langle x^2 \rangle - \langle x \rangle^2, \ \sigma_y^2 = \langle y^2 \rangle - \langle y \rangle^2,$$

$$\varepsilon_{PP} = \frac{\sqrt{(\sigma_x^2 - \sigma_y^2)^2 + 4(\sigma_{xy}^2)^2}}{\sigma_x^2 + \sigma_y^2},$$

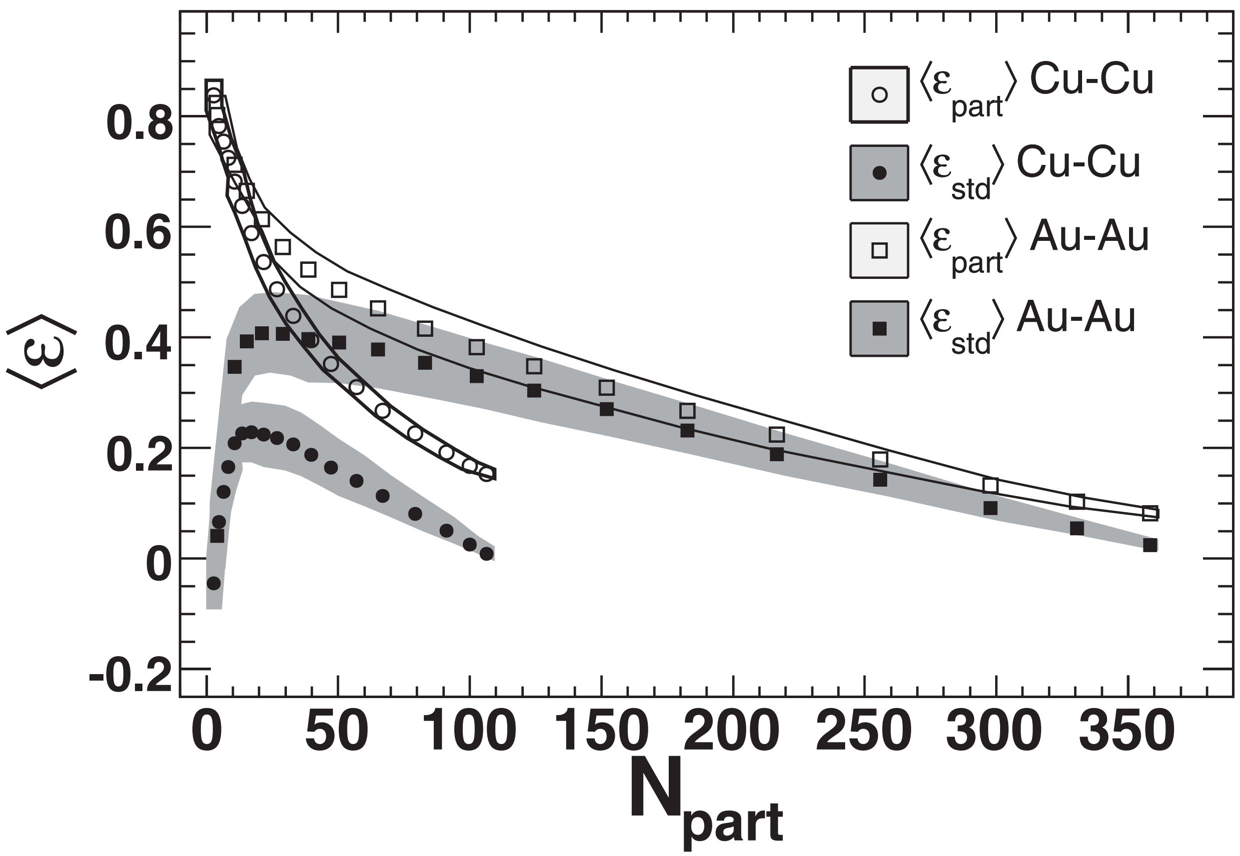
$$\sigma_{xy}^2 = \langle xy \rangle - \langle x \rangle \langle y \rangle$$

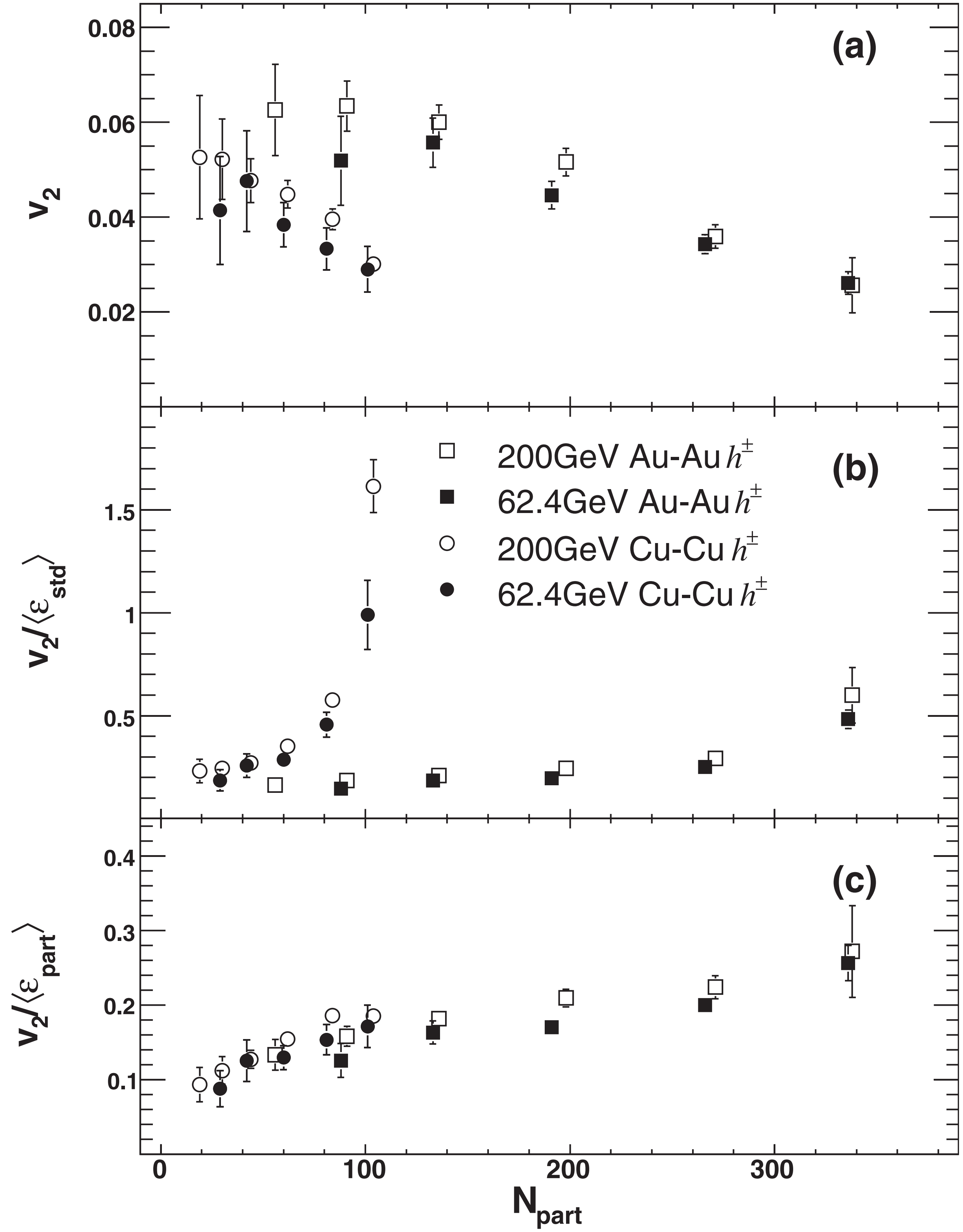
participant eccentricity

- Spatial anisotropy (eccentricity)
  - originally defined with respect to the reaction plane
- PHOBOS collaboration come up with better definition
  - takes into account the fluctuation of nucleon positions
  - “participant eccentricity” with respect to the “participant plane”

Fluctuations !

PHOBOS; **PRL98**, 242302 (2007)

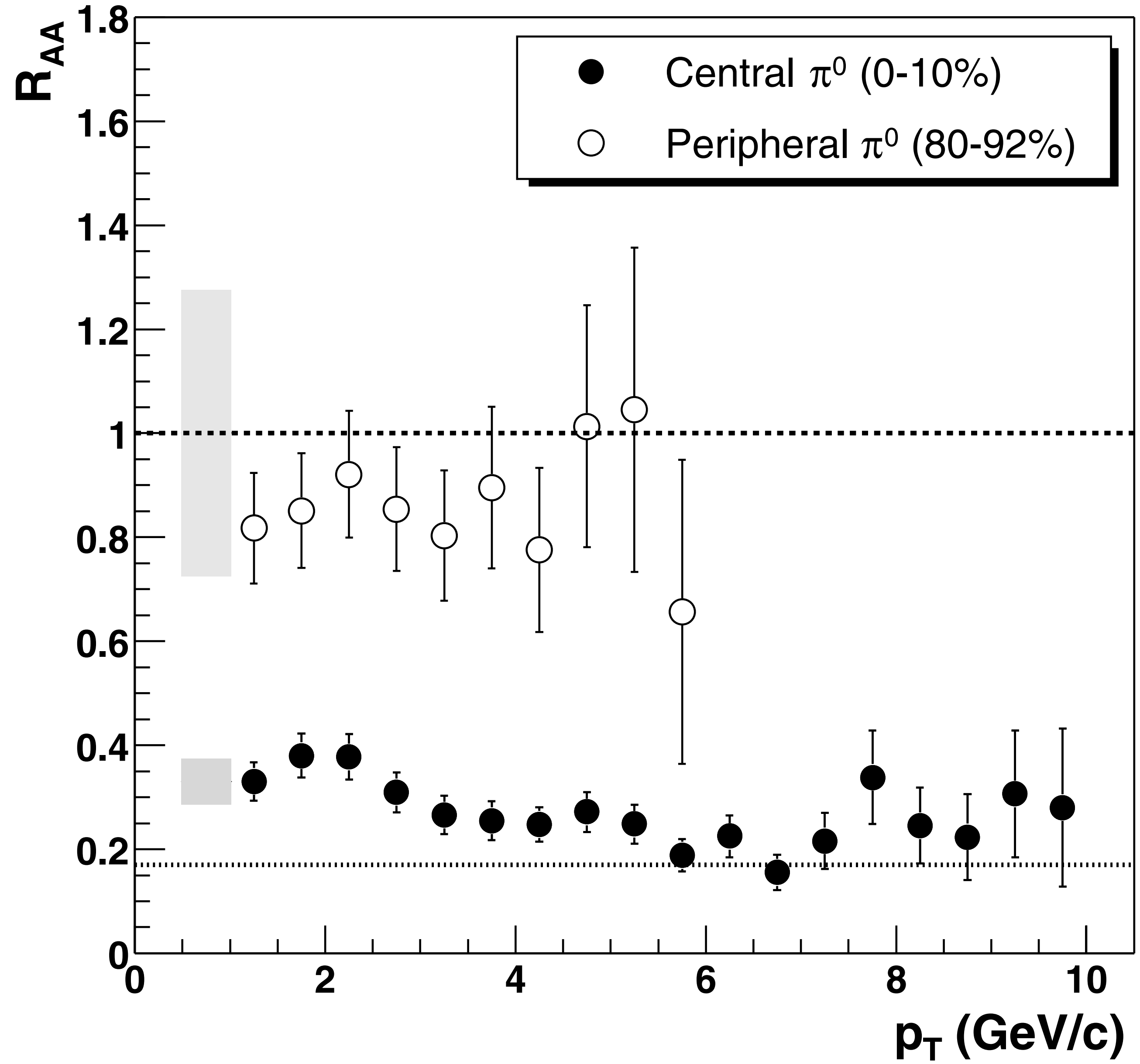




- Eccentricity increases by fluctuations
- Geometric v_2 scaling by participant eccentricity
  - fluctuation !

Applications: (1) R_{AA}

PHENIX; **PRL91**, 072301 (2003)



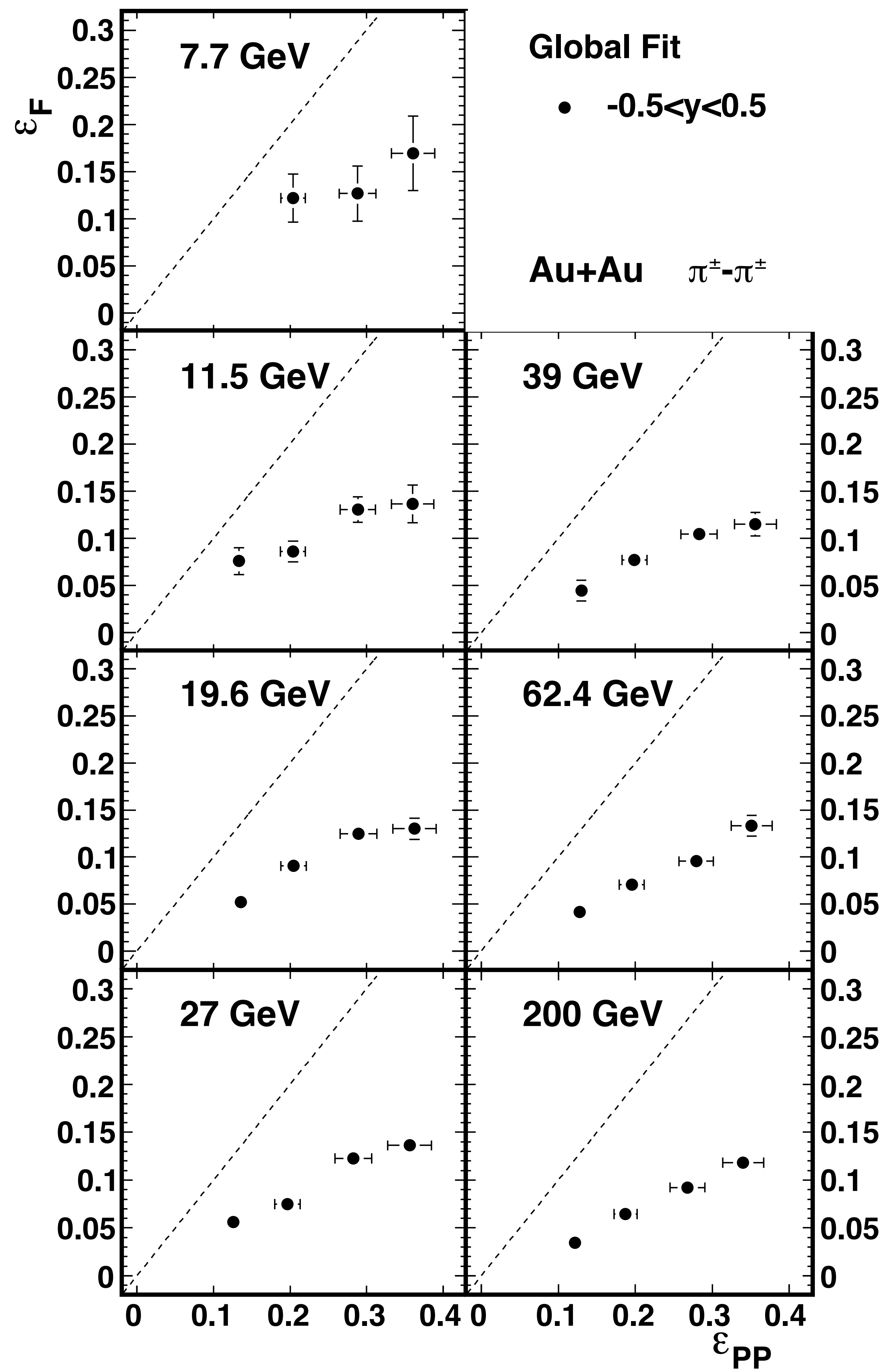
$$R_{AA}(p_T) = \frac{(1/N_{AA}^{\mathrm{evt}})d^2N_{AA}^{\pi^0}/dp_T dy}{\langle N_{\mathrm{coll}}\rangle/\sigma_{pp}^{\mathrm{inel}} \times d^2\sigma_{pp}^{\pi^0}/dp_T dy}, \quad (1)$$

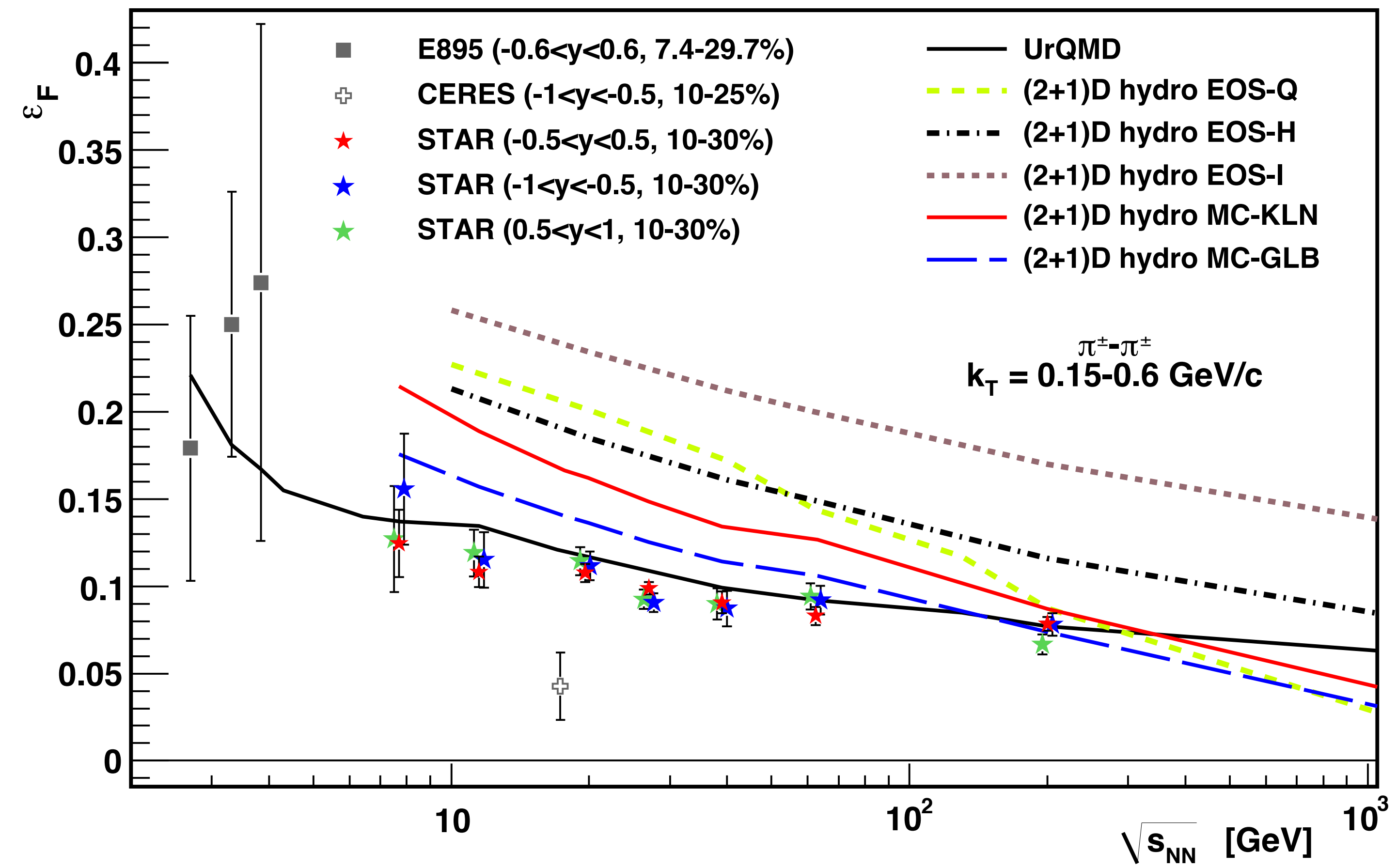
where the $\langle N_{\mathrm{coll}}\rangle/\sigma_{pp}^{\mathrm{inel}}$ is just the average Glauber nuclear overlap function, $\langle T_{\mathrm{AuAu}}\rangle$, in the centrality bin under consideration (Table II). $R_{AA}(p_T)$ measures the deviation of AA data from an incoherent superposition of NN

- Nuclear modification factor R_{AA} at high p_T
  - Test N_{coll} scaling; R_{AA} = 1 → A+A is superposition of p+p
  - Any deviations from R_{AA} = 1 → information at early stage of collisions

Applications: (2) flow, asHBT

STAR; arXiv:1403.4972v1 [nucl-ex]





- Eccentricity at freeze-out
  - measured by using azimuthal sensitive HBT
- Information for time evolution in heavy ion collisions
  - by comparing initial & freeze-out eccentricity

Possible improvements

- Proton distribution
  - consider point-like nucleons by default
- Effect of neutrons
  - Inelastic cross section ?
  - Radius of nucleus (and perhaps skin depth as well)
- Adjustment of radius of nuclei
  - relevant if one starts considering the nucleon distributions inside nuclei
  - deformation also affects
- …

  - NOTE: Comments above might not be relevant for experiments at LHC. Some of experiments might have already considered these kind of effects

Centrality determination

Centrality determination

- Centrality

$$\text{centrality } (\%) \equiv \left(1 - \frac{\sigma}{\sigma_{tot}}\right) \times 100$$

  - Fraction of events in terms of *total geometrical cross section*
    - 0% at most central, 100% at most peripheral
  - Impact parameter cannot be measured experimentally
- Basic assumption is monotonic relationship between impact parameter and multiplicity
  - Multiplicity monotonically decrease with b
- Centrality is determined by various ways (detectors)
  - the TPC at midrapidity (STAR)
  - the BBC (and/or the ZDC) at forward rapidity (PHENIX)
  - the V0 counter (ALICE)
- They essentially measure charged particles
  - not the case for the ZDC, and for the FCAL in ATLAS

Centrality determination (cartoon)

M. L. Miller et al,
arXiv:nucl-ex/0701025

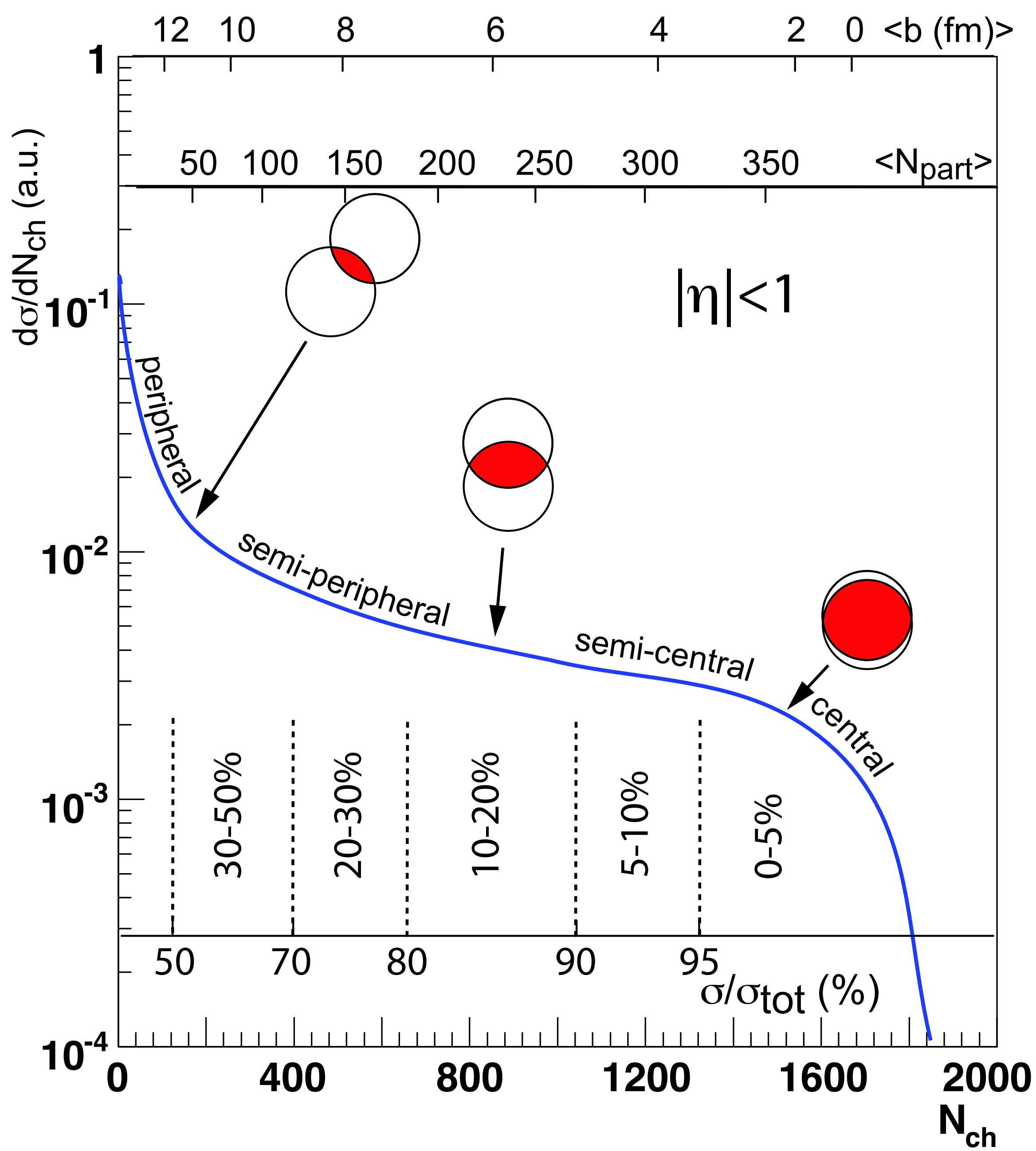


Figure 8: A cartoon example of the correlation of the final state observable N_{ch} with Glauber calculated quantities (b, N_{part}). The plotted distribution and various values are illustrative and not actual measurements (T. Ullrich, private communication).

How to model multiplicity distribution ?

- Two component model has been widely used

$$\frac{dN}{d\eta} = \mu\left[(1-x)\frac{N_{part}}{2} + xN_{coll}\right]$$

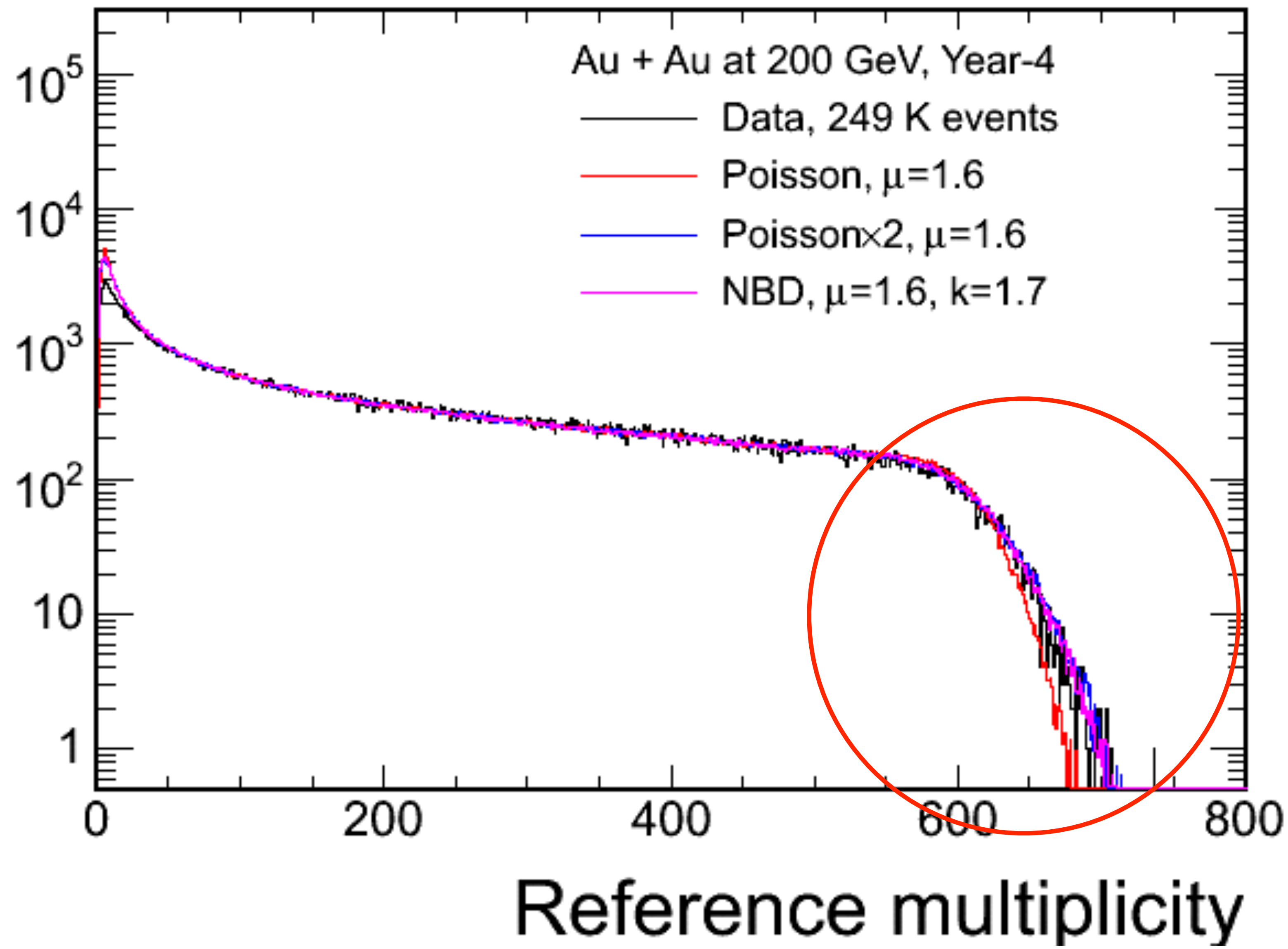
  - particles from initial hard scattering carry some fraction (x) of produced particles, x is *O*(0.1)
  - parameter μ controls the overall scale (or mean) of multiplicity

- PHENIX uses simple N_{part} scaling with power α

$$\frac{dN}{d\eta} = \mu\left(\frac{N_{part}}{2}\right)^{\alpha}$$

- Multiplicity can be calculable once N_{part} (and N_{coll}) values are obtained by MC Glauber model
  - Is this good enough to reproduce multiplicity in experiments ?
  - The answer is no. Let's take a look at the result

Need additional fluctuations



- Underestimate the tail even by using simple poisson fluctuation (compare black with red)
  - If one doesn't consider any additional fluctuations (not shown here), then results will be even worse, i.e. lower than red curve

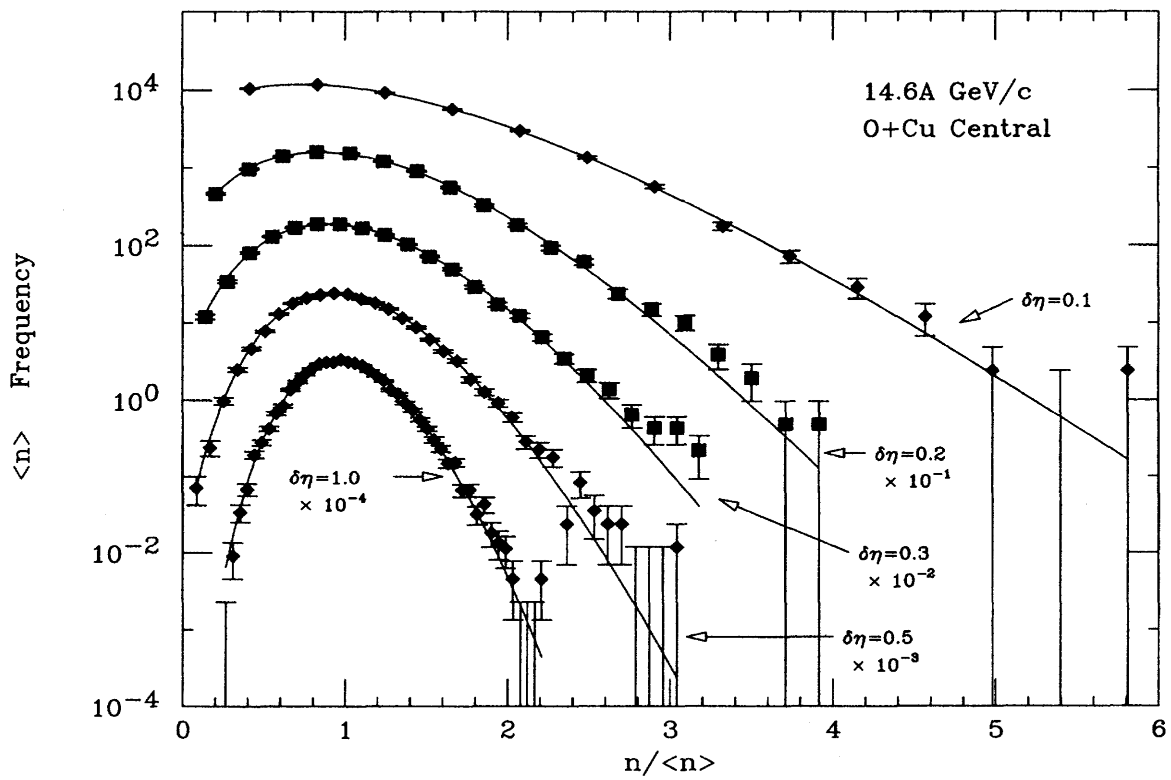
Basic idea

- Independent emission source
  - assume particles will be produced independently from each source
    - "source" would be participant pair, number of binary collisions or their mixture
  - mean particle number is determined by multiplicity models in previous slide
  - use some PDF (gaussian, NBD etc) to add fluctuations for number of produced particles
  - Negative Binomial Distribution (NBD) is mostly used to take into account additional fluctuations

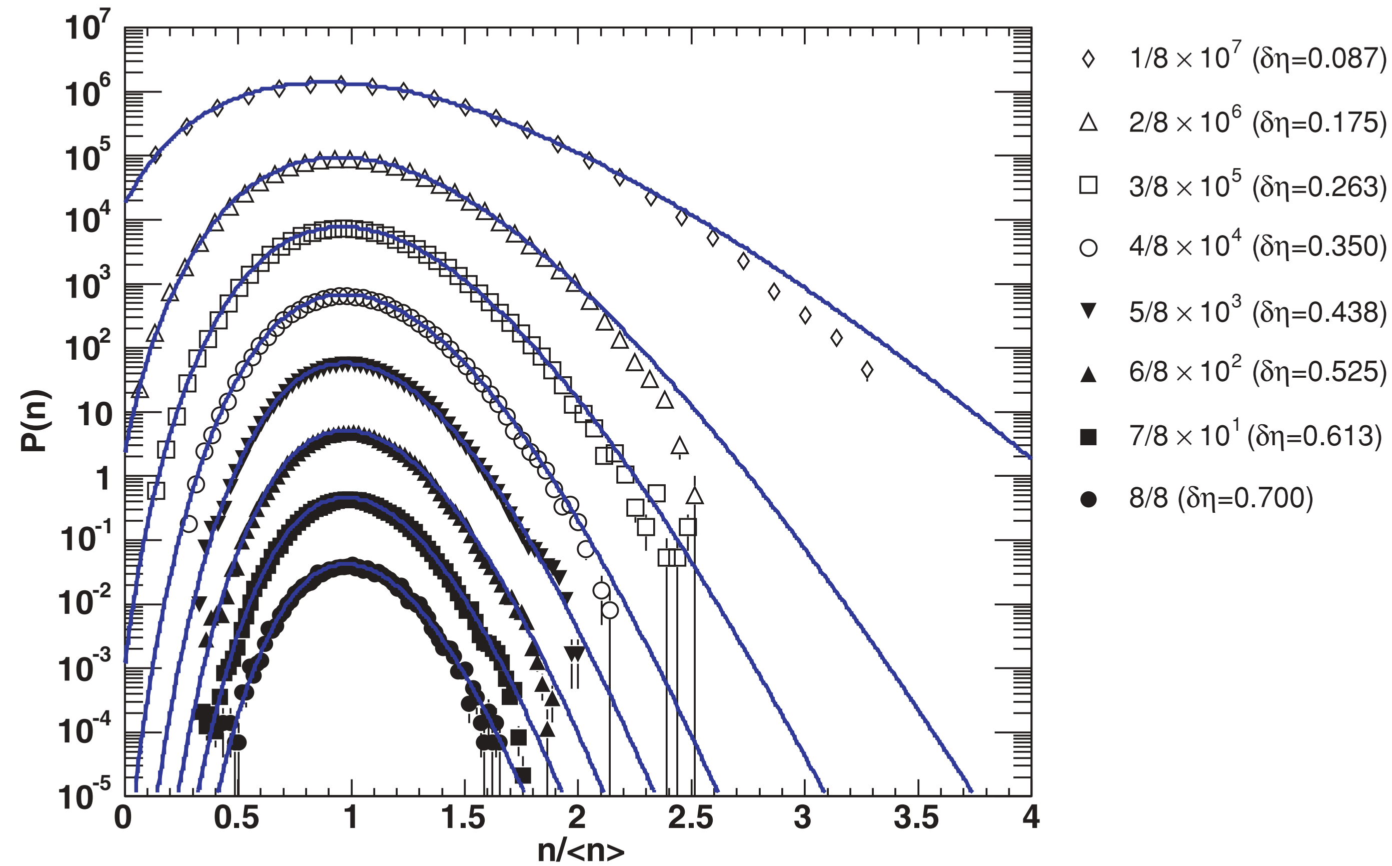
- Tune parameters in PDF (and multiplicity model) to reproduce the experimental data
  - "Fit" the data by multiplicity model + PDF

Negative Binomial Distribution

*E-802; **PRC52**, 2663 (1995)*



*PHENIX; **PRC76**, 034903 (2007)*

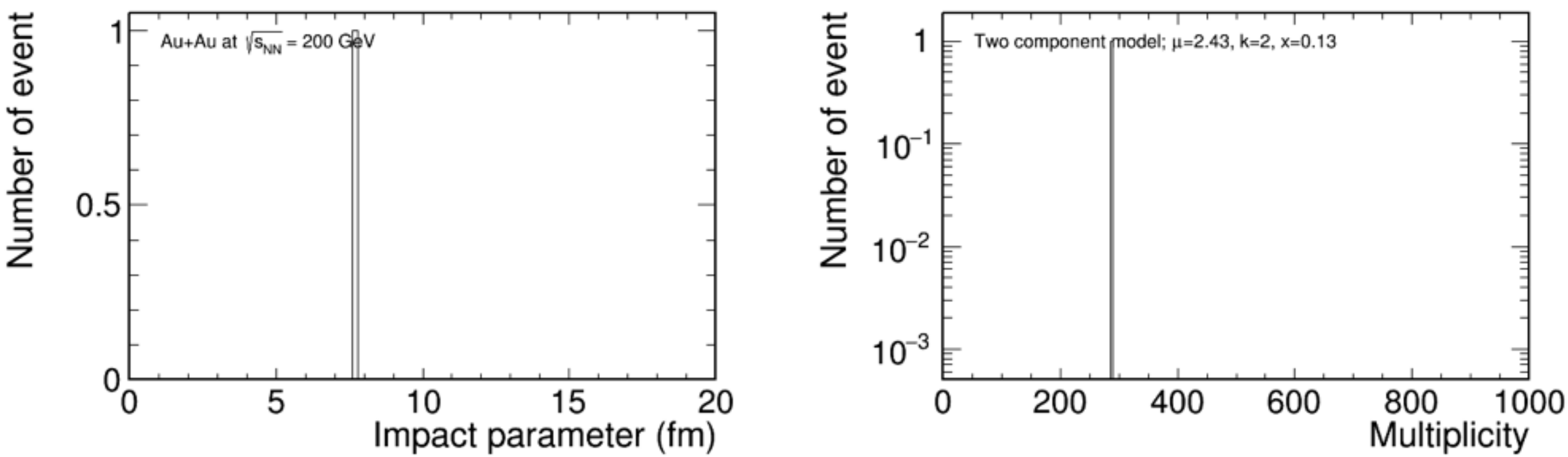


- Charged particle multiplicity distribution is empirically described well by Negative Binomial Distribution
  - in A+A, p+p, and $e^+ + e^-$

$$P(n;\mu,k) = \frac{\Gamma(n+k)}{\Gamma(n-1)\Gamma(k)} \left(\frac{\mu/k}{1+\mu/k}\right)^n \frac{1}{(1+\mu/k)^k}$$

- Poisson (k→∞), Binomial (k<0), Bose-Einstein (k=1)
  - k reflects the degree of correlation among particles

Construct multiplicity



- Demonstration with arbitrary parameters
- Multiplicity distribution
  - peak at peripheral, (relatively) flat region, tail at the most central
    - mostly driven by linear impact parameter dependence
    - additional NBD fluctuation increase the 'width' (tail)

What else ?

- Acceptance & tracking efficiency (particle-wise)
- Trigger (in)efficiency (event-wise)
  - we miss peripheral events, where number of produced particles is small so that trigger counters cannot observe any particles
  - this effect would be visible in the reduction of peripheral peak on multiplicity distribution
- Auto-correlation (or self-correlation)
  - correlation between centrality and physics observables if one determine the centrality at the same detector(s) which we carry out the analysis
    - see, e.g. PHOBOS white paper, Nucl. Phys. A757, 28 (2005)
  - significant effects on fluctuation observables
    - even at the most central (0-5%) collisions
- …

Fluctuations

Why do we measure fluctuations ?

- Good tool to study QCD phase diagram
  - information about the properties of the system (e.g. d.o.f.)
  - key signature for phase transition; susceptibilities (fluctuation) diverge at 2nd order phase transition
    - prominent example is CMB by COBE, WMAP, … → constrain important parameters for our universe
- Ultimate goal(s) in heavy ion collisions
  - search for QCD critical point (and 1st order phase transition)
  - ➡ Beam Energy Scan (BES), vary baryon chemical potential
    - extensive studies at SPS
    - recent RHIC BES phase-I
    - future RHIC BES phase-II, CBM at FAIR, J-PARC, …
- In this talk, focus on experimental results for K/π fluctuation, and higher moments for conserved charges

Strangeness enhancement, K/π fluctuation

- Proposed by J. Rafelski and R. Hagedorn (1981)

What we intend to show is that there are many more $\bar{s}$ quarks than antiquarks of each light flavour. Indeed:

$$\frac{\bar{s}}{\bar{q}} = \frac{1}{2}\left(\frac{m_s}{T}\right)^2 K_2\left(\frac{m_s}{T}\right) e^{\mu/3T} \qquad (28)$$

The function $x^2K^2(x)$ is, for example, tabulated in Ref. 15). For $x = m_s/T$ between 1.5 and 2, it varies between 1.3 and 1. Thus, we almost always have more $\bar{s}$ than $\bar{q}$ quarks and, in many cases of interest, $\bar{s}/\bar{q} \sim 5$. As $\mu \to 0$ there are about as many $\bar{u}$ and $\bar{q}$ quarks as there are $\bar{s}$ quarks.

Statistical Mechanics of Quarks and Hadrons, Edited by H. Satz @ North-Holland Publishing Company, p253-272, 1981

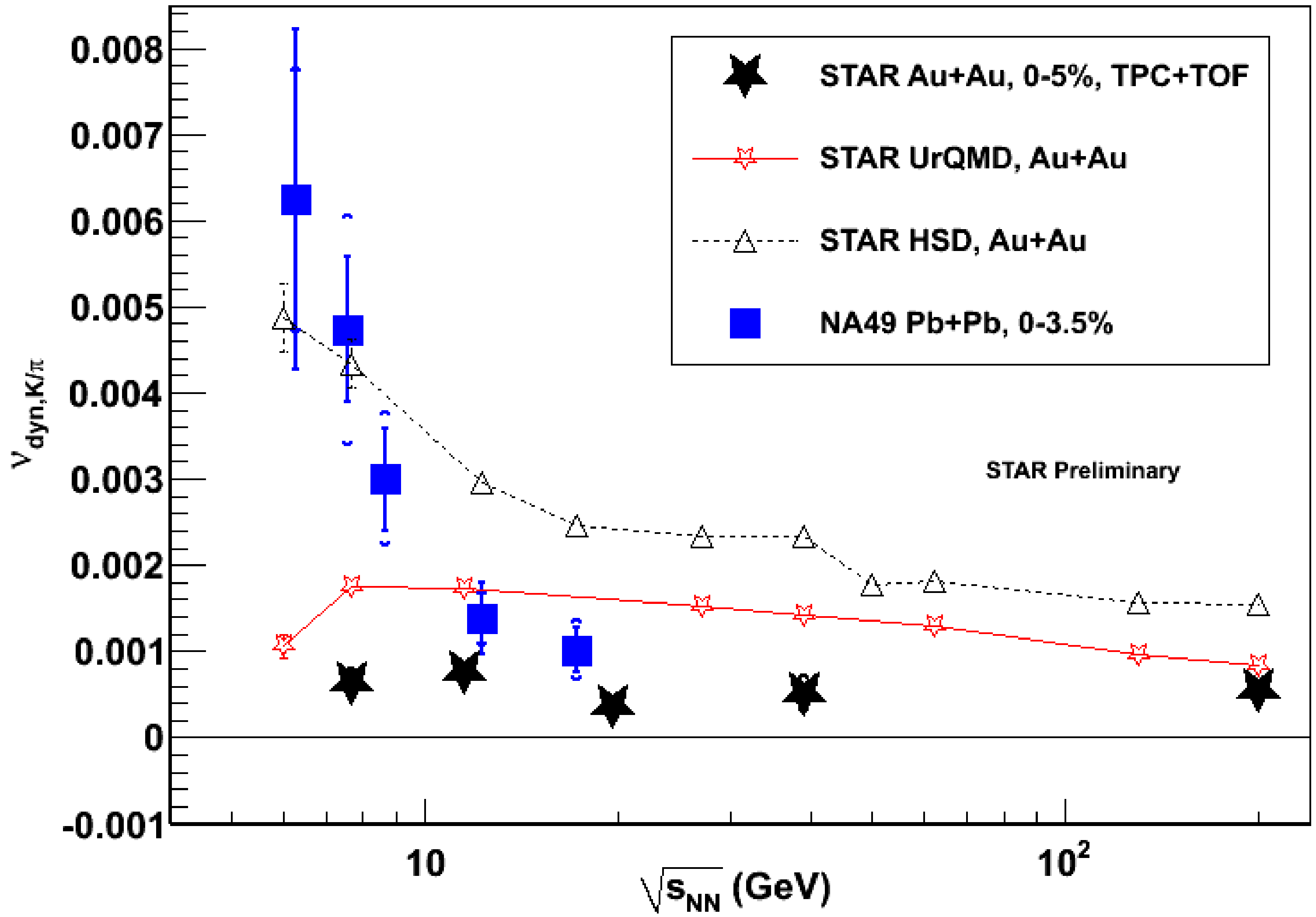
- s/q → K/π etc as signatures of QGP
- Study K/π ratio fluctuation to search for the phase transition
  - "*The measurement of K/π fluctuations by NA49 collaboration were the first event-by-event fluctuations measurement in heavy-ion experiment*" (V. Koch, arXiv:0810.2520v1 [nucl-th])
- Now, K/π fluctuation is also used to search for the QCD critical point
  - though it is not clear (to me) K/π fluctuation is really sensitive to CEP

Observable

$$\nu_{dyn,K\pi} = \frac{\langle N_K(N_K-1)\rangle}{\langle N_K\rangle^2} + \frac{\langle N_\pi(N_\pi-1)\rangle}{\langle N_\pi\rangle^2} - 2\frac{\langle N_K N_\pi\rangle}{\langle N_K\rangle\langle N_\pi\rangle}$$

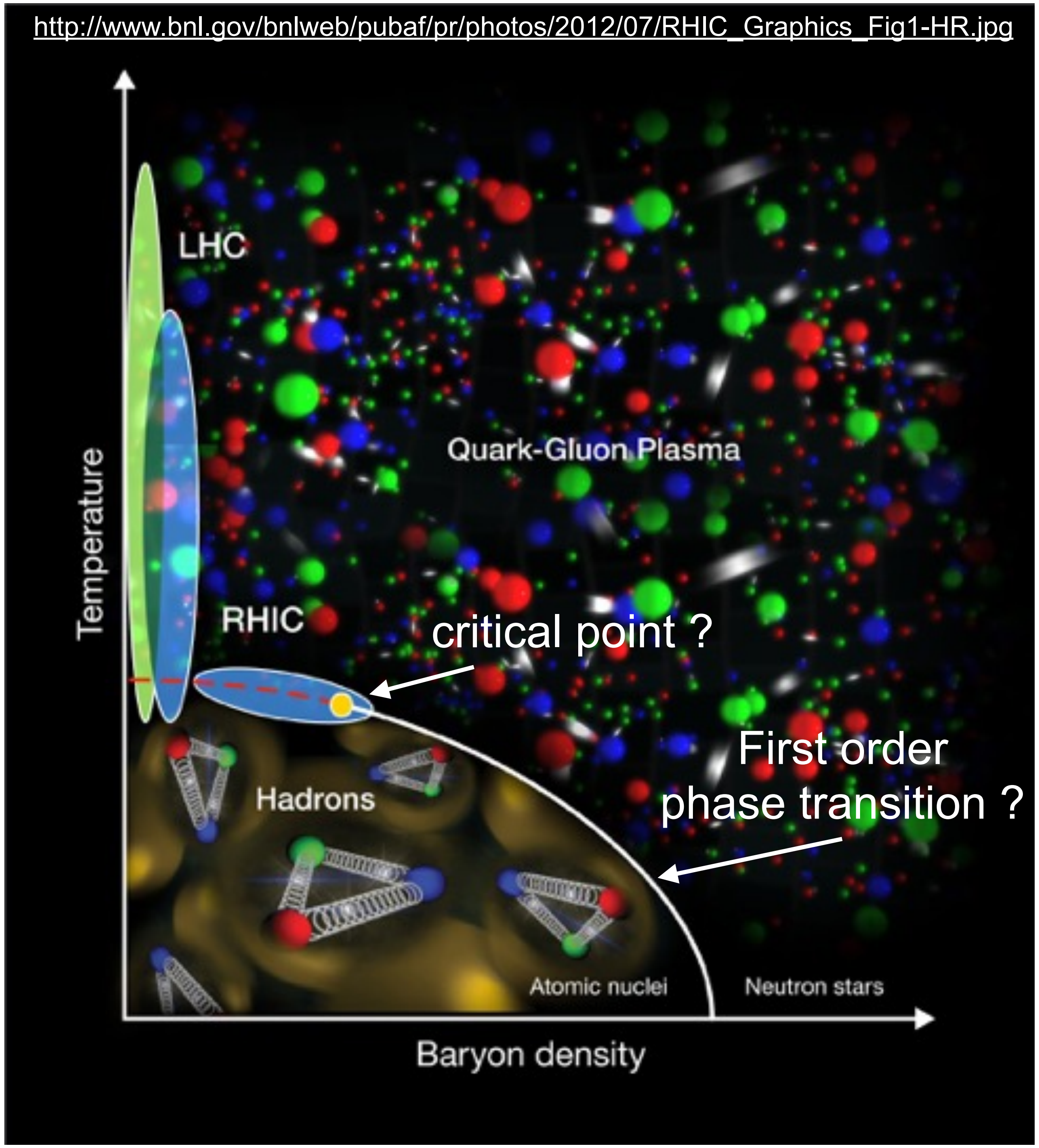
- ν_{dyn} = 0 for the lack of dynamical correlation
- NA49 uses different definition σ_{dyn}
  - see e.g. PRC79, 044910 (2009)
    - σ_{dyn} is the same with ν_{dyn} if statistical fluctuation $\nu_{stat} = \frac{1}{\langle N_K\rangle} + \frac{1}{\langle N_\pi\rangle}$ is small
- Advantage of ν_{dyn}
  - insensitive to efficiency corrections (factorial moment)
  - no mixed events

NA49 vs STAR



- NA49 shows rapid increase with decreasing beam energy
- STAR shows constant down to 7.7 GeV
- Discrepancy between NA49 and STAR
  - not resolved yet
  - acceptance ? particle identification ?

Search for QCD critical point



- QCD critical point search is one of the main goals in heavy-ion experiments
- Theoretical approach (lattice QCD) is valid in small μ_B at this point
  - probably valid up to $\mu_B/T \sim 1$
  - ➡ experimental search
- In order to explore the QCD phase diagram, we need to vary baryon density (baryon chemical potential)
  - ➡ Beam Energy Scan

Observables ?

- What is the best observable to search for the QCD critical point ? → Fluctuation !
- Why ?
  - correlation length and susceptibilities diverge at critical point
  - but they are not direct observables in experiments
- What are actual observables ?
  - Moments (cumulants) of conserved charges (e.g. net-baryons)
  - Before RHIC BES, we mostly focused on 2nd moment (width)
- Why conserved charges ?
  - Direct connection to susceptibilities (calculable in lattice QCD)

$$\kappa_2 = \left\langle (\delta N)^2 \right\rangle \sim \xi^2, \kappa_3 = \left\langle (\delta N)^3 \right\rangle \sim \xi^{4.5}, \kappa_4 = \left\langle (\delta N)^4 \right\rangle - 3 \left\langle (\delta N) \right\rangle^2 \sim \xi^7$$

$$S\sigma = \frac{\kappa_3}{\kappa_2} \sim \frac{\chi_3}{\chi_2}, \; K\sigma^2 = \frac{\kappa_4}{\kappa_2} \sim \frac{\chi_4}{\chi_2}$$

Observables ?

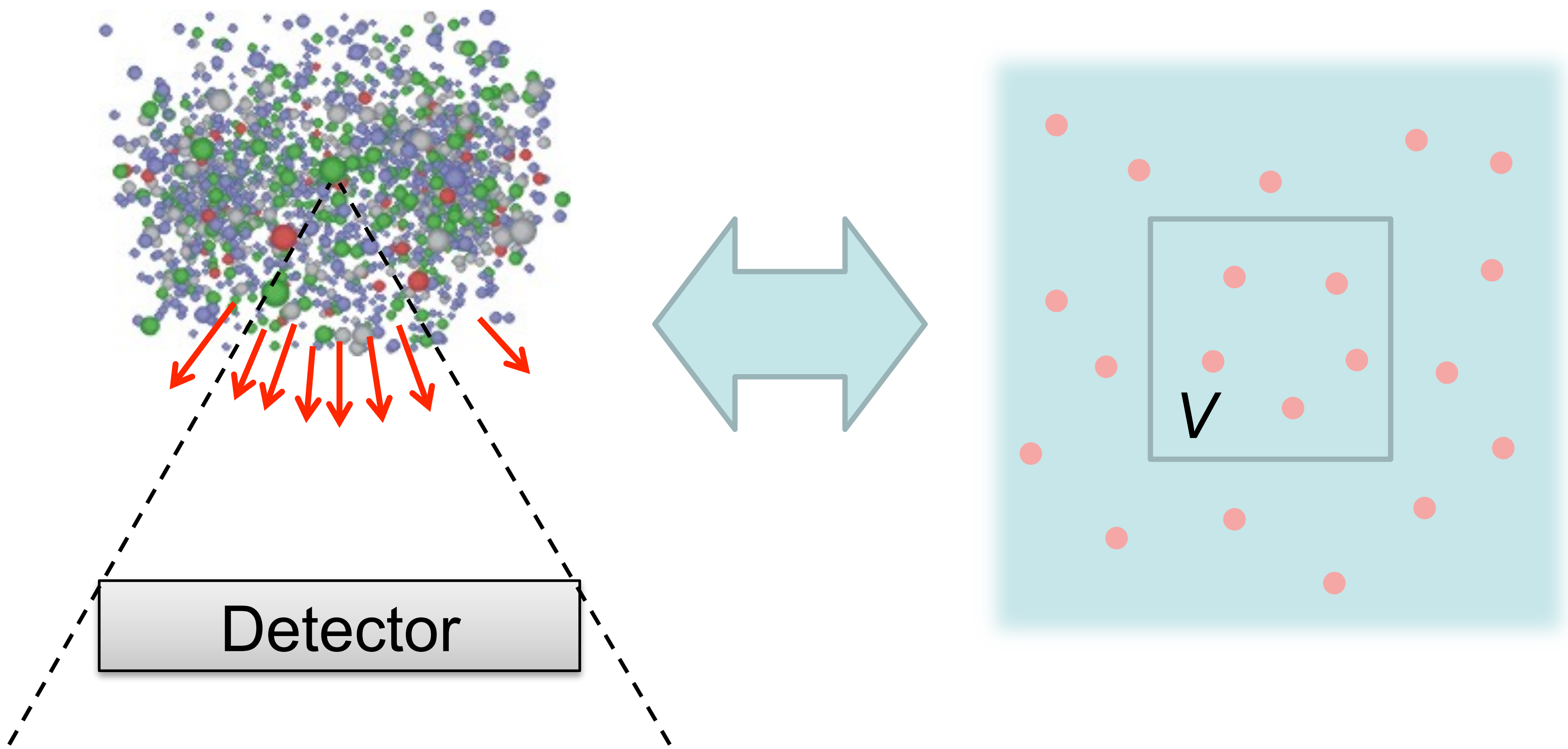
$$\kappa_2 = \left\langle (\delta N)^2 \right\rangle \sim \xi^2, \kappa_3 = \left\langle (\delta N)^3 \right\rangle \sim \xi^{4.5}, \kappa_4 = \left\langle (\delta N)^4 \right\rangle - 3 \left\langle (\delta N) \right\rangle^2 \sim \xi^7$$

$$S\sigma = \frac{\kappa_3}{\kappa_2} \sim \frac{\chi_3}{\chi_2}, \ K\sigma^2 = \frac{\kappa_4}{\kappa_2} \sim \frac{\chi_4}{\chi_2}$$

- Higher order moments (cumulants) are more sensitive to correlation length (see power)
- Product of moments (ratio of cumulants) ↔ ratio of susceptibilities
  - by taking the ratio volume effect is canceled out (good for experiment since we cannot measure volume of the system)
- What is the signal of critical point ?
  - Non-monotonic energy dependence of the product of moments (ratio of cumulants) for conserved charges

Fluctuation of "conserved" charge ?

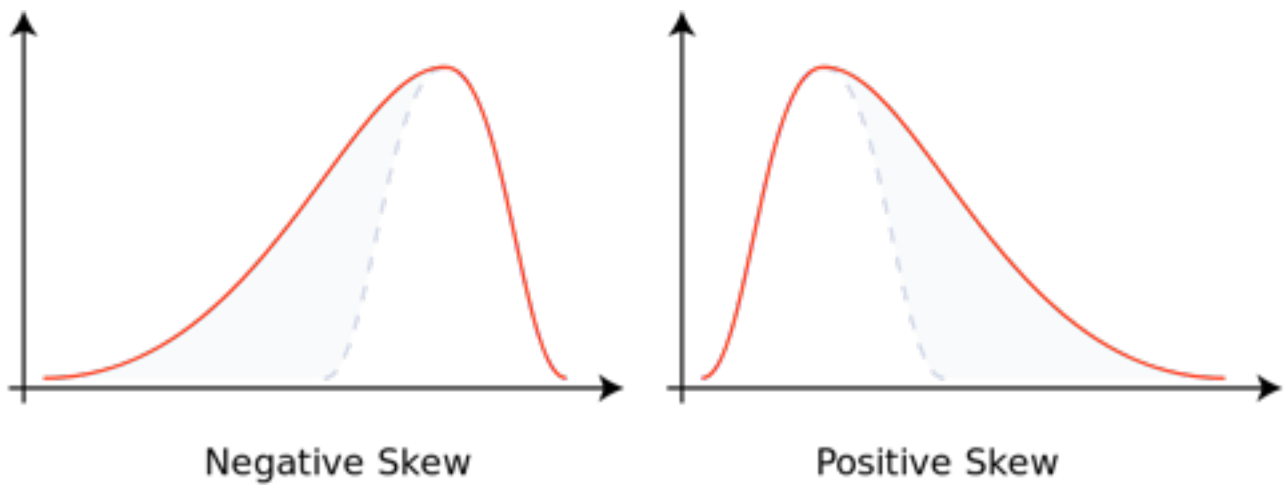
cartoon from
Kitazawa-san's slide

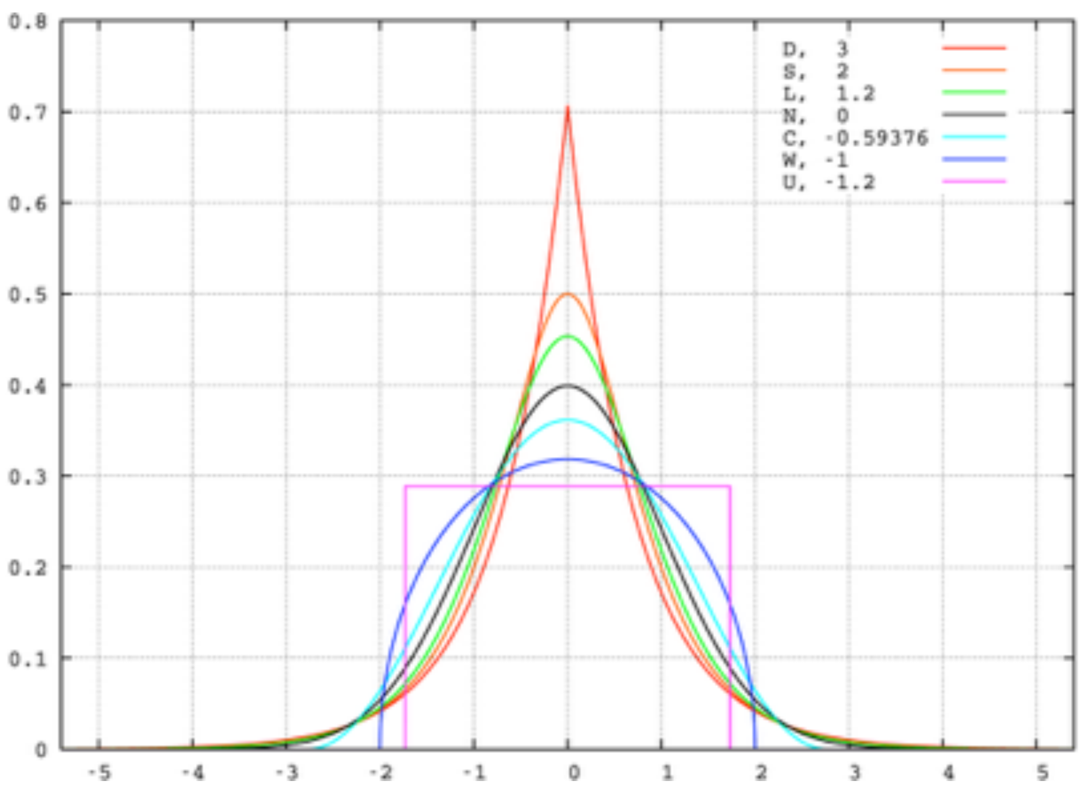


- Fluctuation should be 0 if we are able to measure all particles
- Measure event-by-event fluctuation in the (limited) detector acceptance
  - in pseudorapidity range $\pm O(1)$

Non-gaussian fluctuation

From Wikipedia





- 3rd moment = skewness
  - asymmetry
- 4th moment = kurtosis
  - peakedness
- Both moments = 0 for gaussian distribution
- Critical point search → non-gaussian fluctuations

Baseline - Skellam distribution

$$\text{Poisson distribution}: \; p(k) = \frac{\lambda^k}{k!} e^{-\lambda}, \; (k = 0, 1, 2, \cdots)$$

$$\text{Skellam distribution}: \; p(k) = e^{-(\mu_1+\mu_2)} \left(\frac{\mu_1}{\mu_2}\right)^{k/2} I_k(2\sqrt{\mu_1\mu_2}), \; (k = \cdots, -2, -1, 0, 1, 2, \cdots)$$

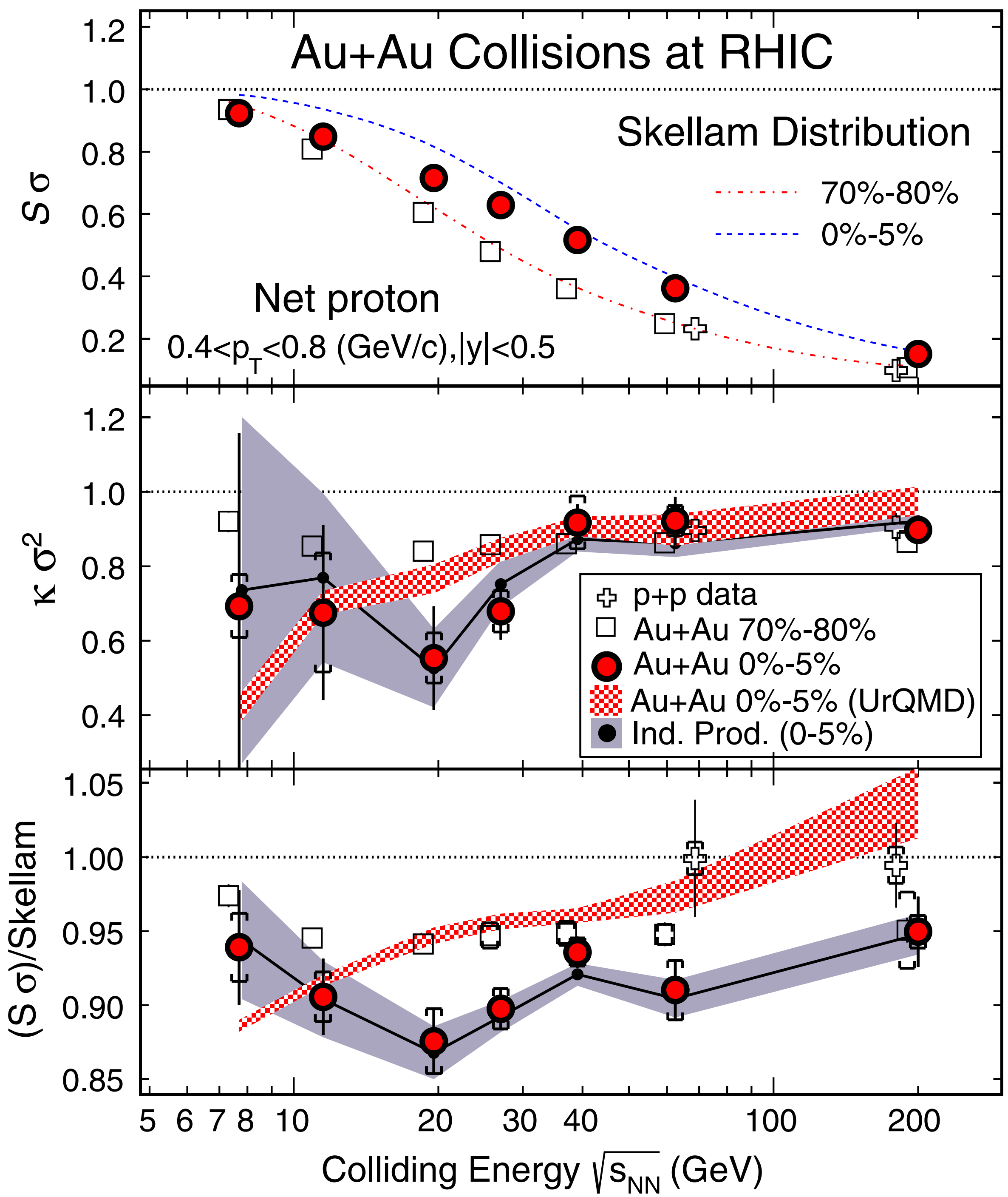
$$\text{mean} \; = \mu_1 - \mu_2, \; \text{variance} \; = \mu_1 + \mu_2, \; \text{skewness} \; = \frac{\mu_1 - \mu_2}{(\mu_1 + \mu_2)^{3/2}}, \; \text{kurtosis} \; = \frac{1}{\mu_1 + \mu_2}$$

$$\rightarrow S\sigma = \frac{\mu_1 - \mu_2}{(\mu_1 + \mu_2)}, \; \kappa\sigma^2 = 1$$

- What would be the baseline we compare with ?
  - Skellam distribution - difference of two statistically independent random variables, each having Poisson distribution with different expected values μ_1 and μ_2
- If particle and anti-particle distributions are Poisson, then the difference of them (net-charge, net-protons etc) follow Skellam distribution
  - product of even (or odd) order cumulants will be 1

Net-proton fluctuations

*STAR: **PRL112**, 032302 (2014)*

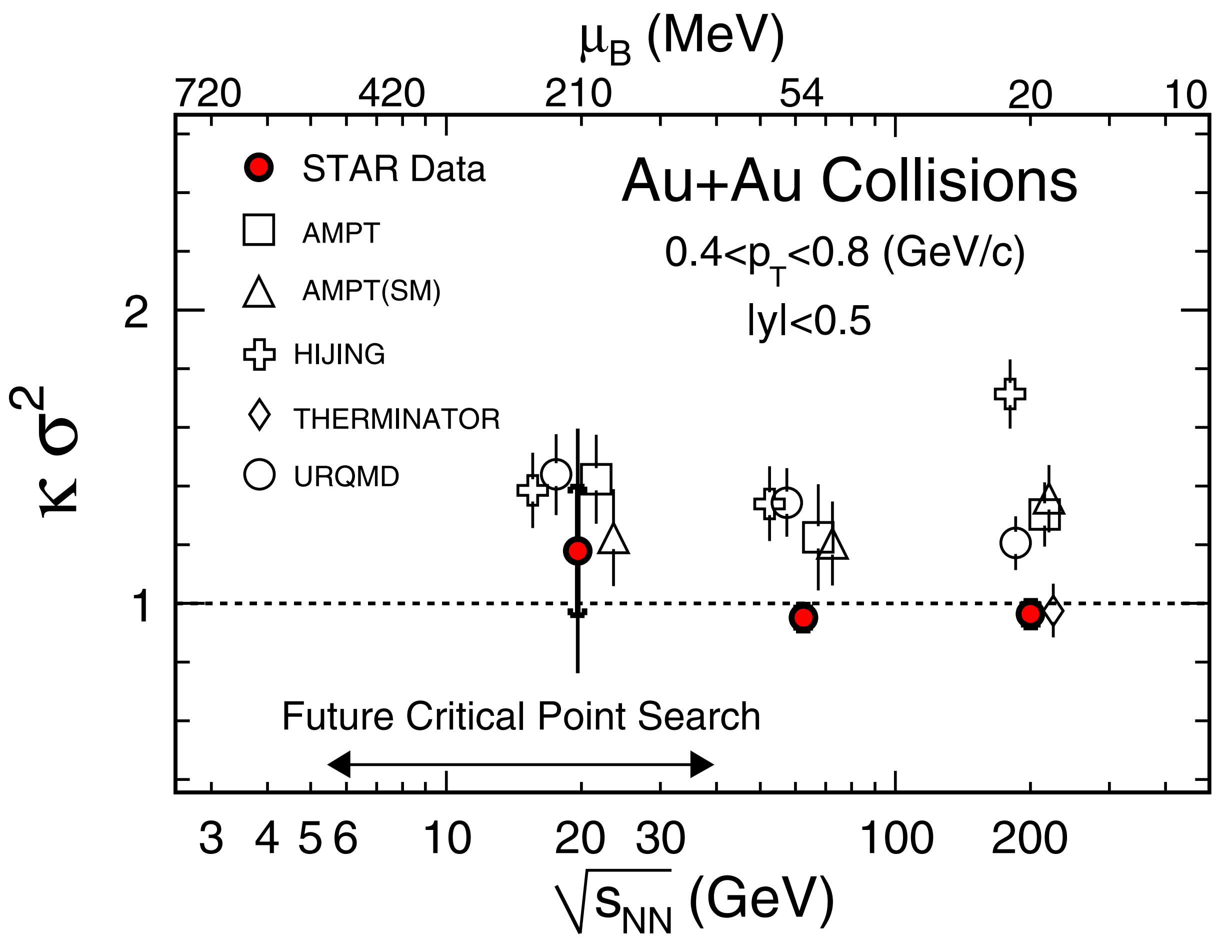


- Latest published results
- Compare with the baseline 1 (middle & bottom panels)
- Interesting structure around 19.6 GeV
- A lot of experimental developments to reach here
  - auto-correlation effect
  - statistical error calculation
  - efficiency corrections
- Caveat
  - Net-protons ≠ net-baryons

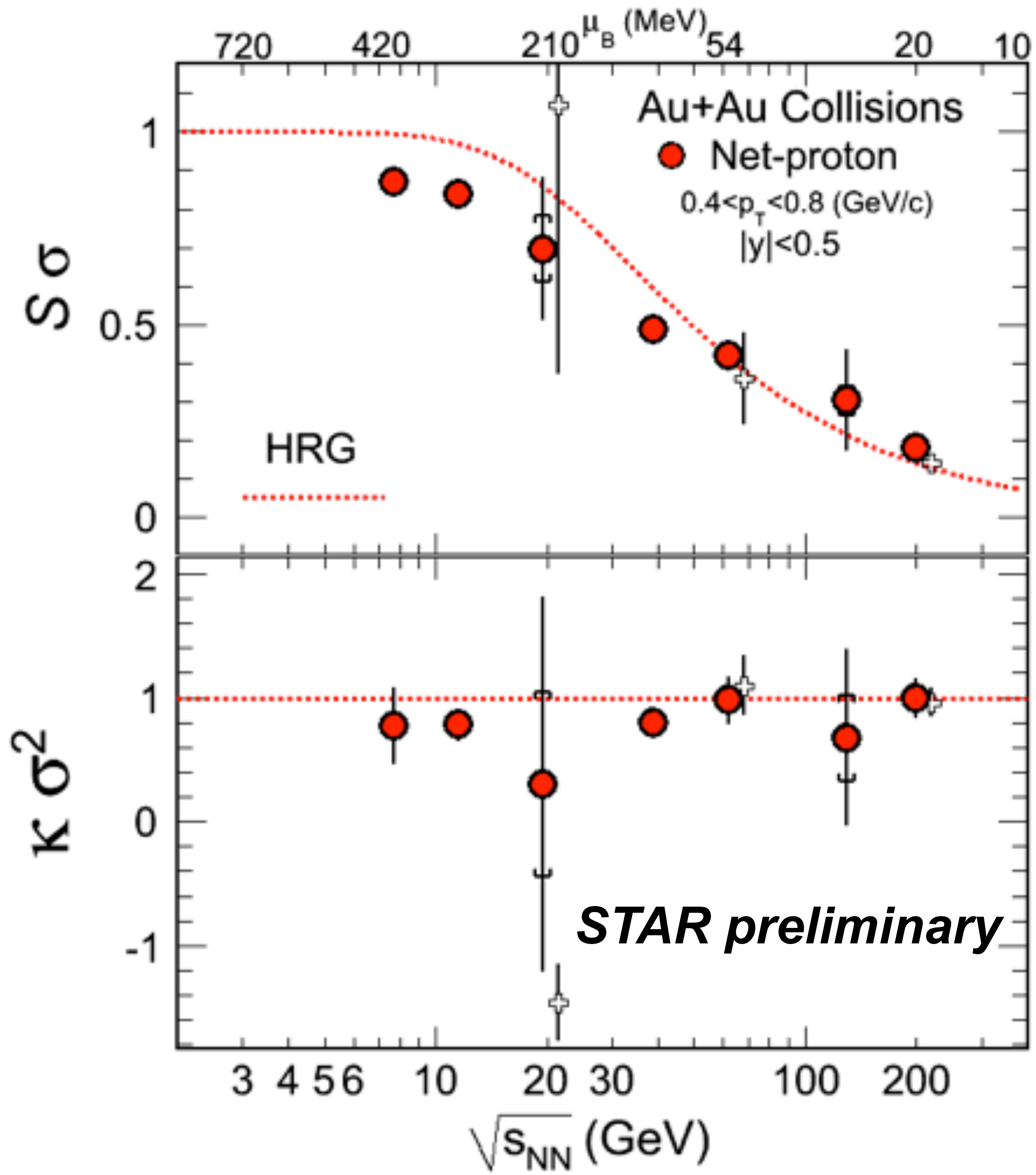
*M. Kitazawa, M. Asakawa; **PRC86**, 024904 (2012), **PRC86**, 069902 (2012)*

History of net-proton fluctuations

STAR: ***PRL*** *105, 022302 (2010)*



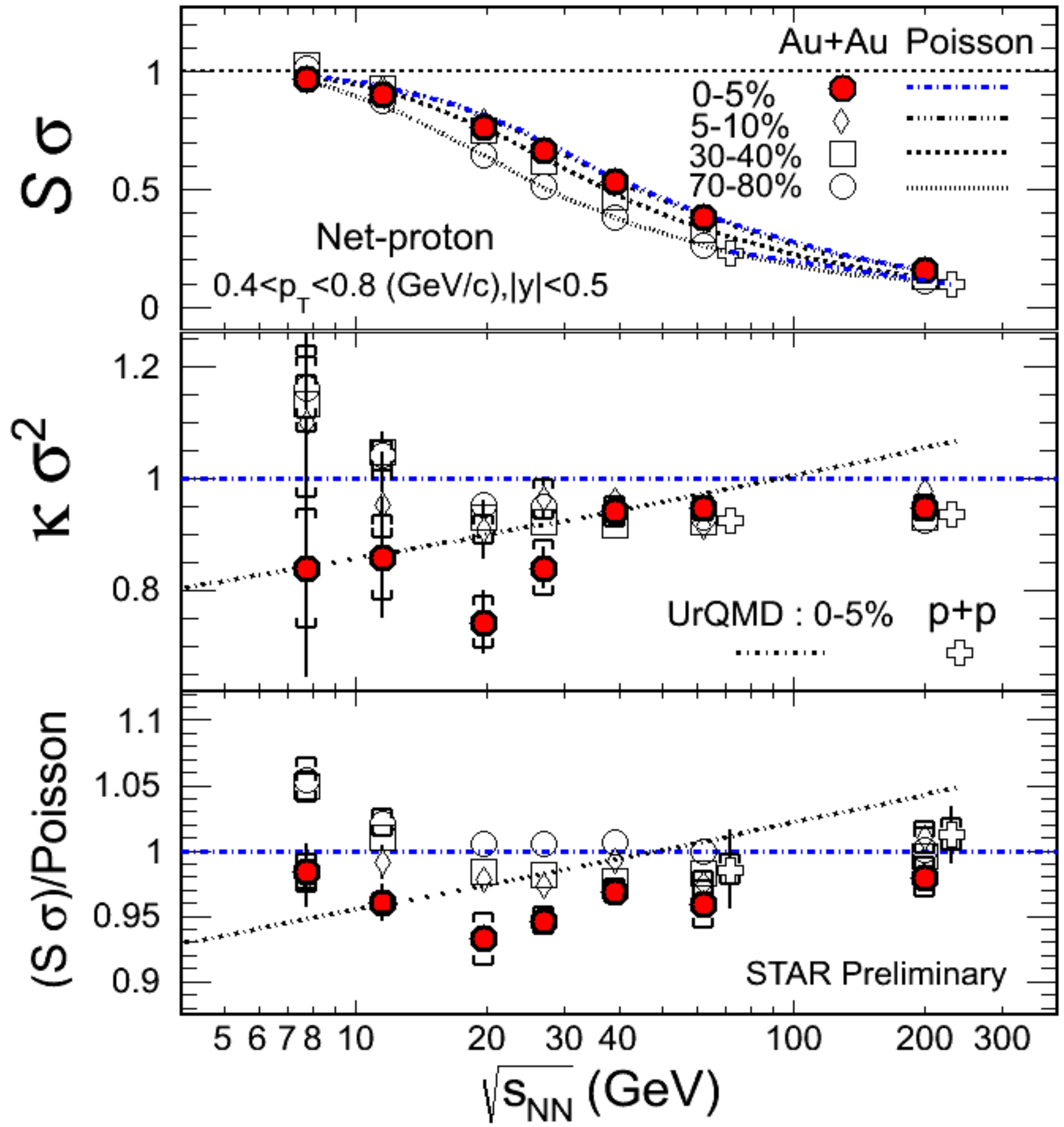
first result



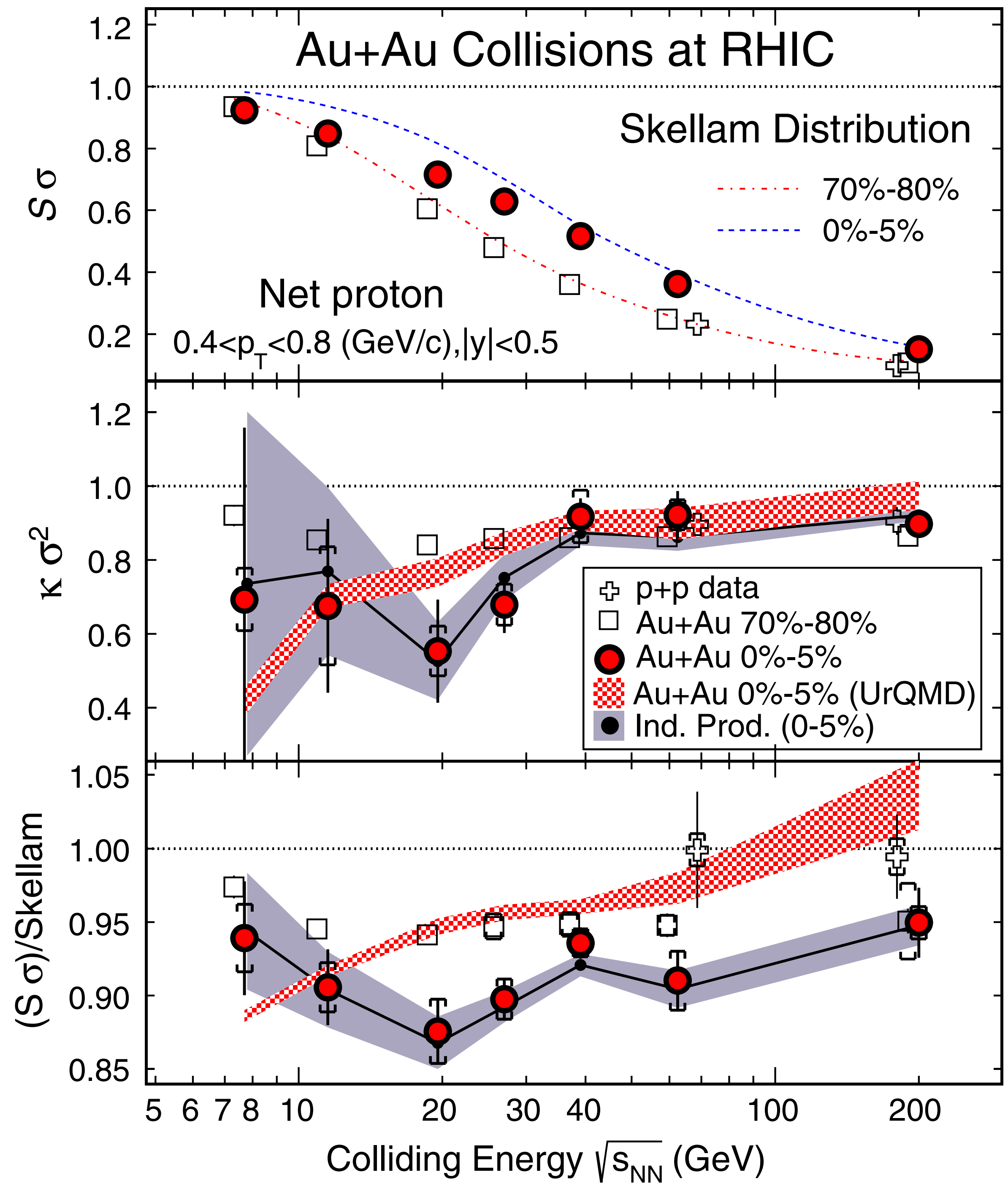
first BES result (QM2010)

History of net-proton fluctuations

second BES result (QM2012)



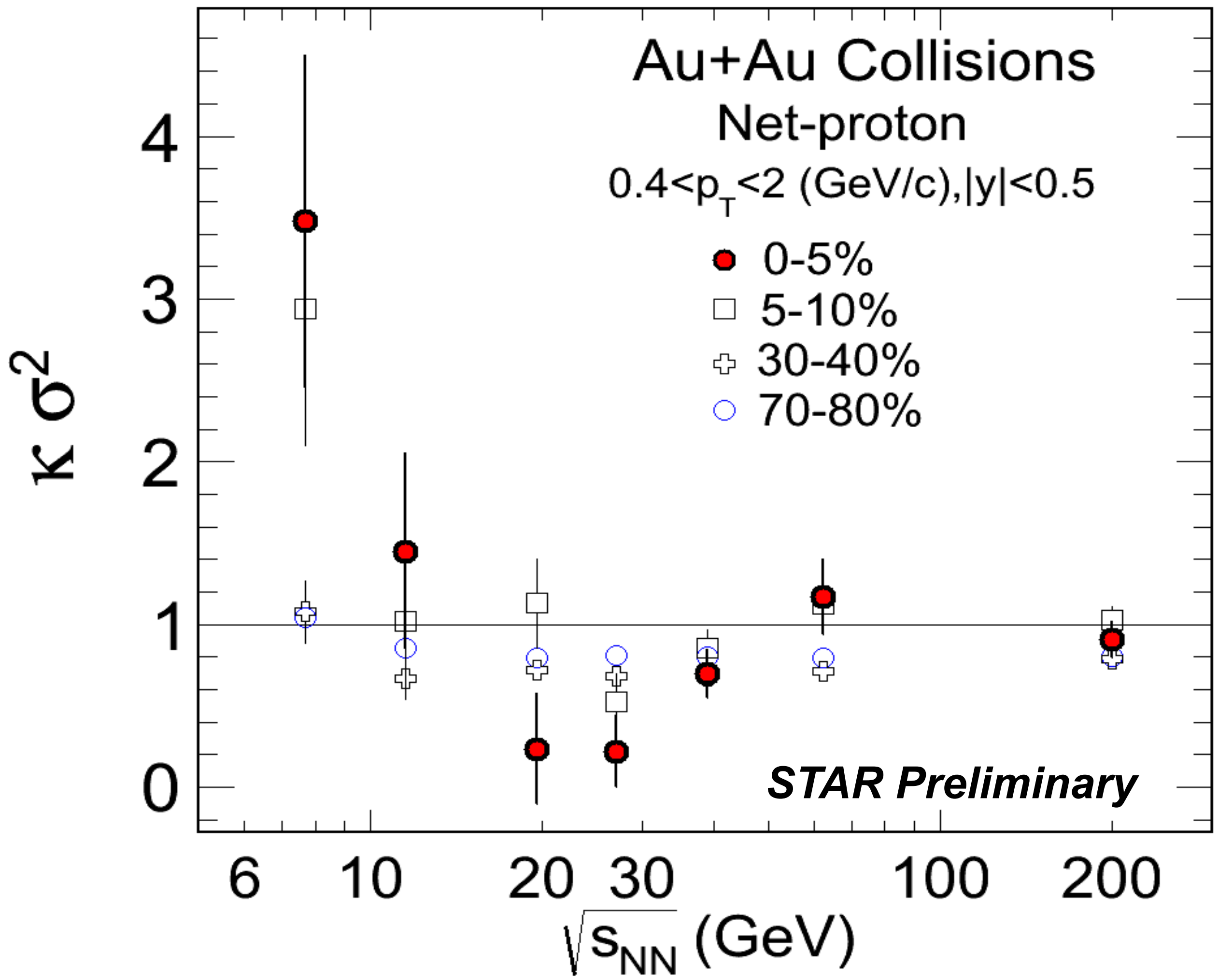
STAR: **PRL112**, 032302 (2014)

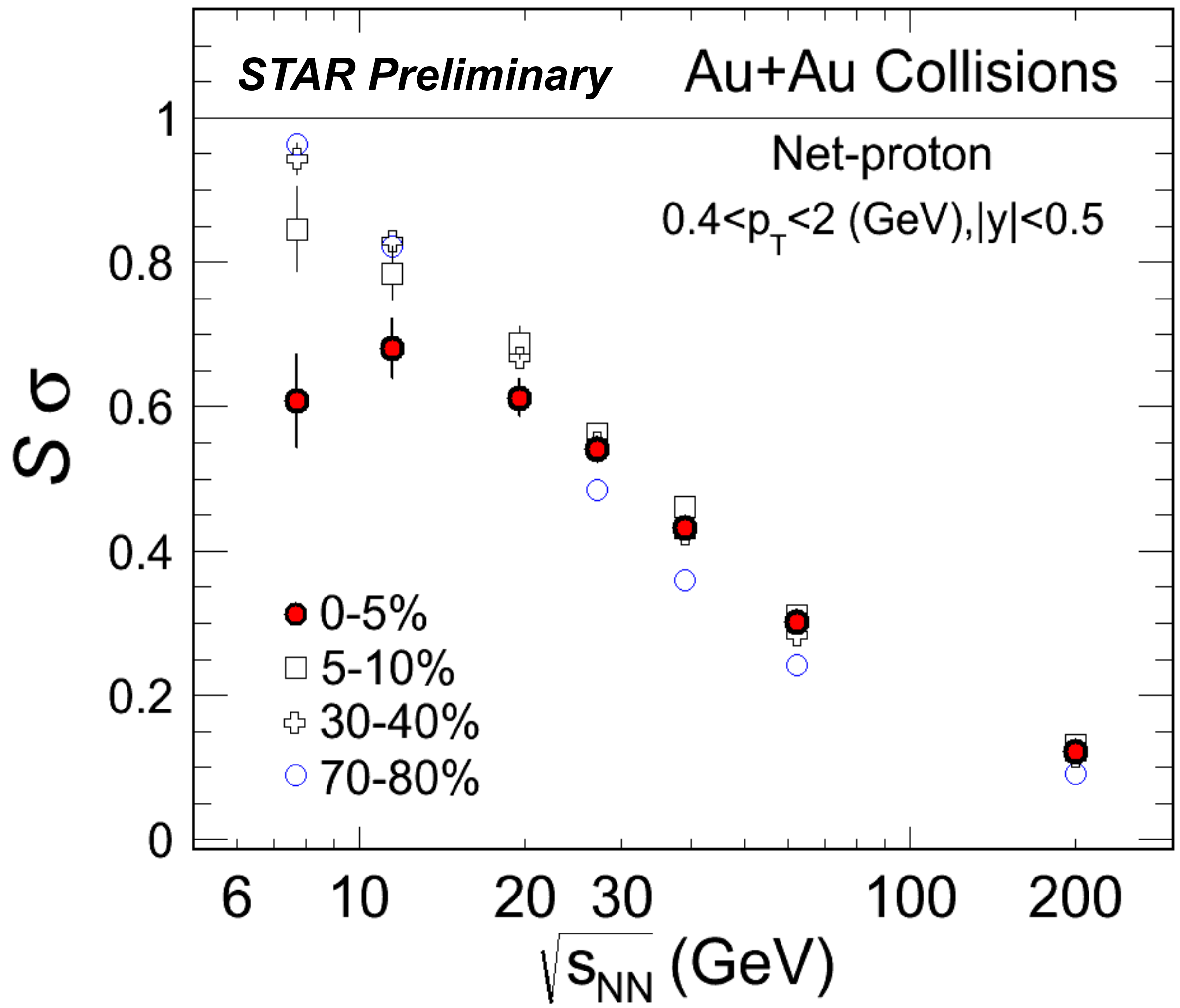


- Independent centrality determination, correct statistical error calculation (+ efficiency correction for final results)

History of net-proton fluctuations

STAR: CPOD2014



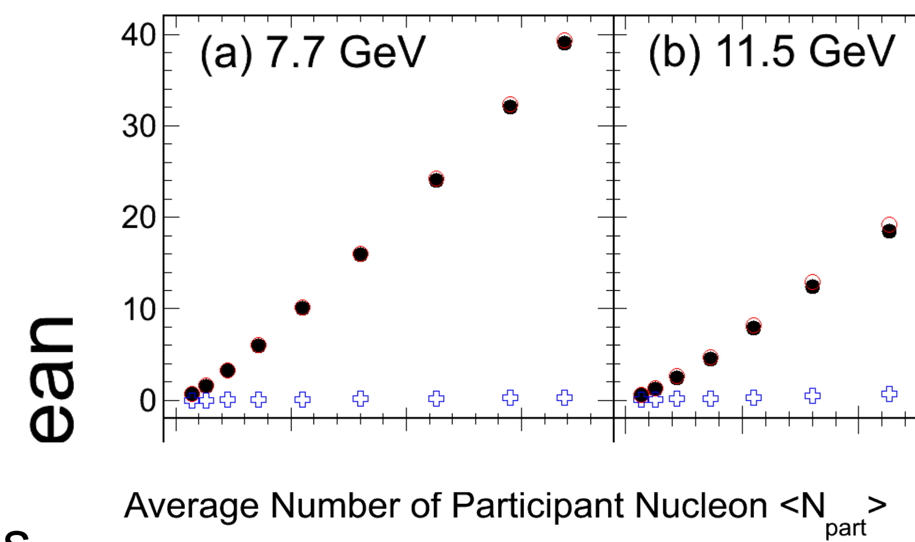


- Independent centrality determination, correct statistical error calculation, efficiency correction
- What has been changed from "published" results ?
  - p_T cut: $0.4 < p_T < 0.8$ GeV/c (TPC) → $0.4 < p_T < 2$ GeV/c (TPC+TOF)

So what ?

- We need differential study in phase space (p_T, η)
  - We don't know what would be the optimal window of p_T, η
  - Naively, focus on low p_T (bulk) would be better, while we miss ~50% of protons with p_T < 1 GeV/c cut off

- Efficiency correction is important
  - for both cumulants and their statistical error
    - Lower efficiency with the TOF → correction factor is large



- We need to measure conserved charge
  - net-charge is better ?
  - neutrons → Hadron calorimeter (J-PARC ?)
    - we essentially measure protons only at low beam energies

Summary

- MC Glauber model is convenient tool (for experimentalists) to study initial conditions, determine centrality, and relate it with initial geometry (N_{part} etc)
  - Possible improvements can be done to make the centrality determination to be more precise
- Fluctuation observables are important to search for the QCD critical point
- Future experiments should provide precise measurements below 20 GeV
  - Future BES phase-II, starting from 2018 or 2019, CBM at FAIR, J-PARC
    - neutron detection would provide more precise measurements on net-baryon fluctuations at low energies