Fluctuation of Conserved Quantities to look for Critical Point in Phase Diagram

TGSW2016

Toshihiro Nonaka
University of Tsukuba

筑波大学 University of Tsukuba

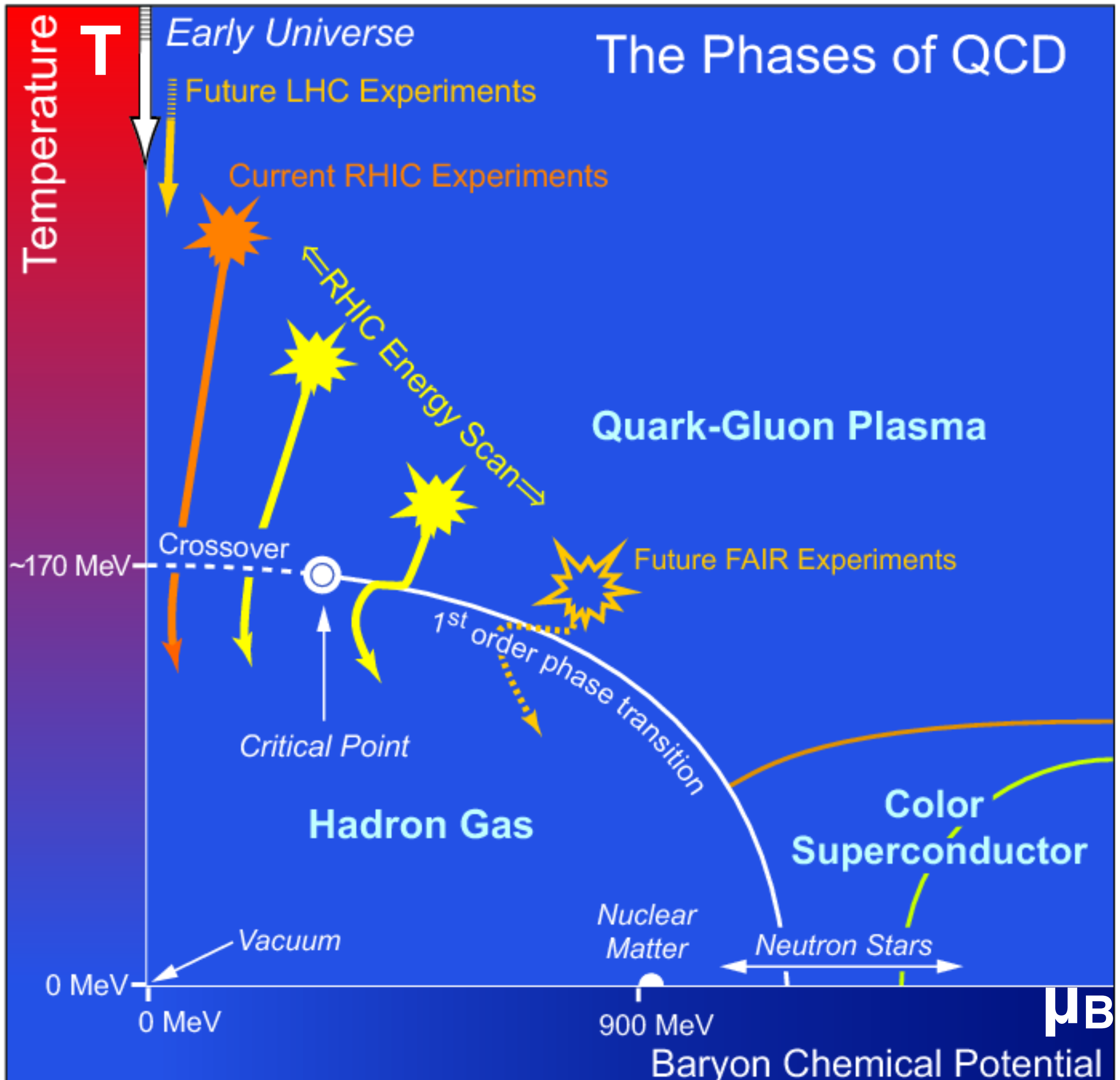
CiRfSE

STAR

Outline

- ✓ RHIC Beam Energy Scan Phase I
- ✓ Search for Critical Point with Higher order Fluctuations
- ✓ STAR Detector and Particle Identification
- ✓ Published results and Recent studies
- ✓ Beam Energy Scan Phase II

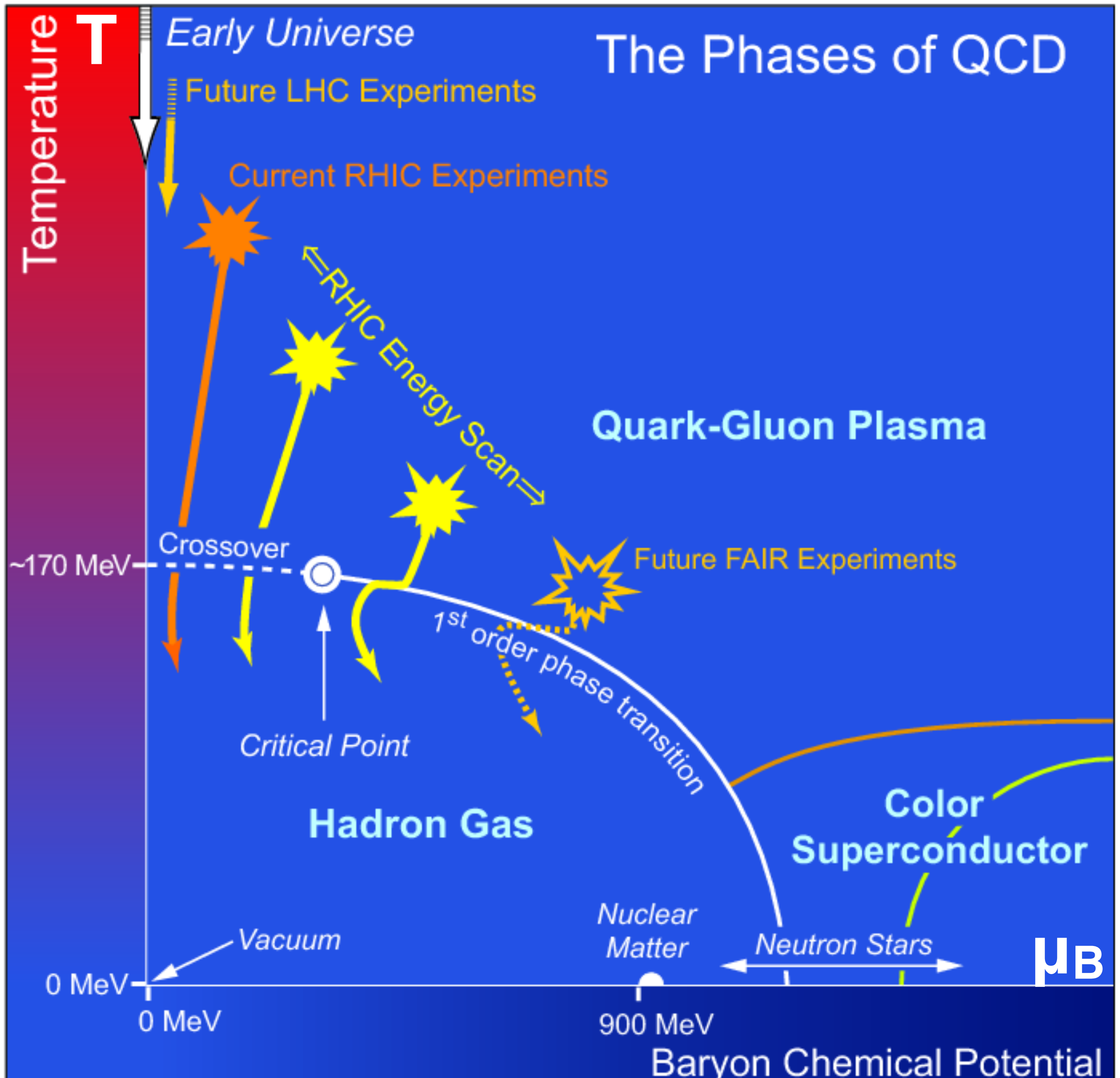
QCD phase diagram



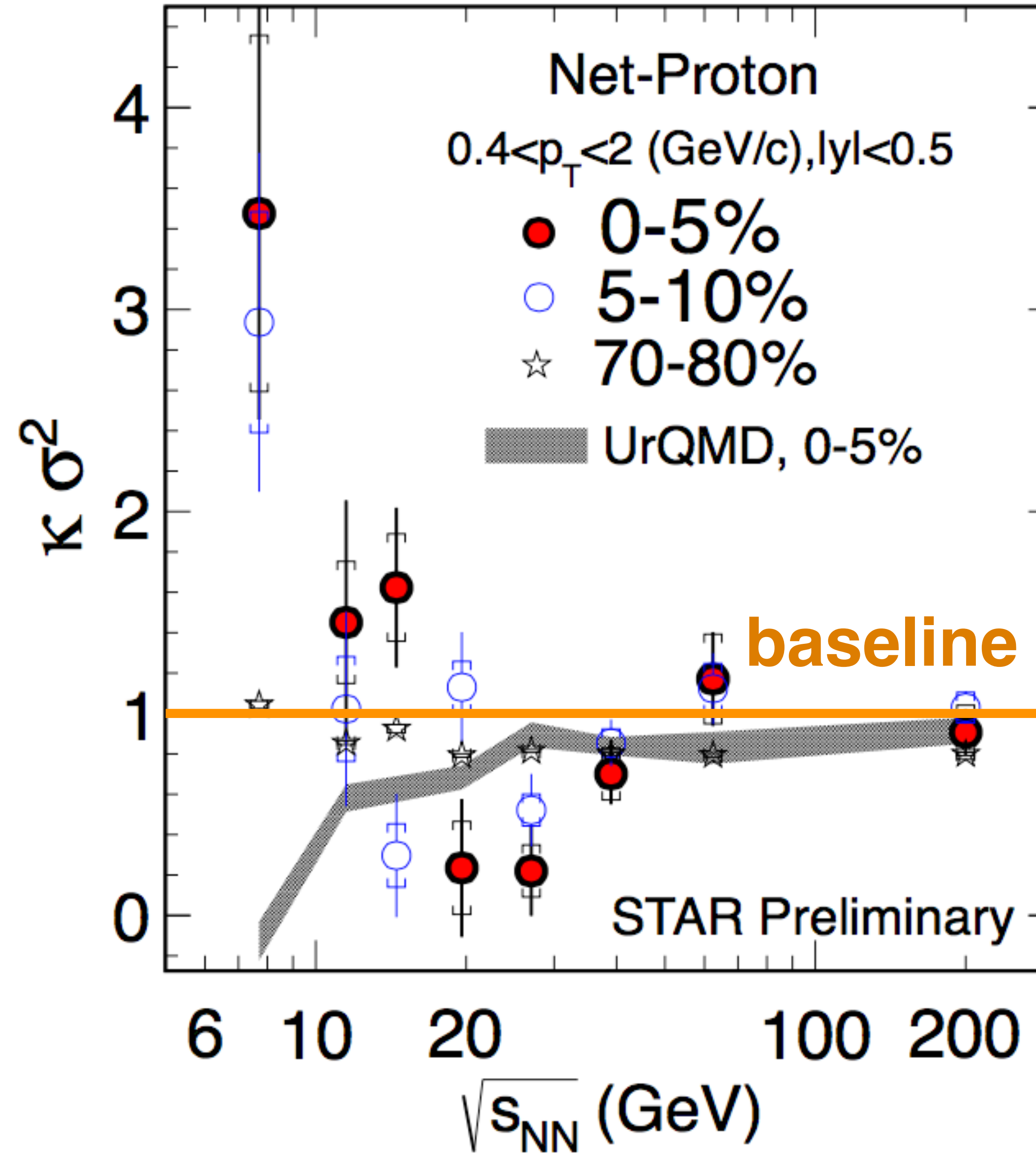
- ✓ **Crossover at μ_B=0**
- ✓ **1st order phase transition at large μ_B?**
- ✓ **Critical point?**
- ✓ **Beam Energy Scan Phase I at RHIC, $\sqrt{s_{NN}}$=7.7, 11.5, 14.5, 19.6, 27, 39, 62.4 and 200 GeV.**

QCD phase diagram

- ✦ We measure the higher order fluctuation of conserved quantities as a function of beam energy, and see "non-monotonic" behaviour with respect to the baseline.



X. Luo (STAR collaboration) arXiv:1503.02558v2



RHIC Beam Energy Scan Phase I

✓ **BES I was performed in 2010, 2011 and 2014.**

$\sqrt{s_{NN}}$ (GeV)	Year	Statistics(Million) 0-80%	μ_B (MeV)
7.7	2010	~3	422
11.5	2010	~6.6	316
14.5	2014	~13	266
19.6	2011	~15	206
27	2011	~32	156
39	2010	~86	112
62.4	2010	~45	73
200	2010	~238	24

μ_B , T ***: J. Cleymans et al., Phys. Rev. C 73, 034905 (2006)***

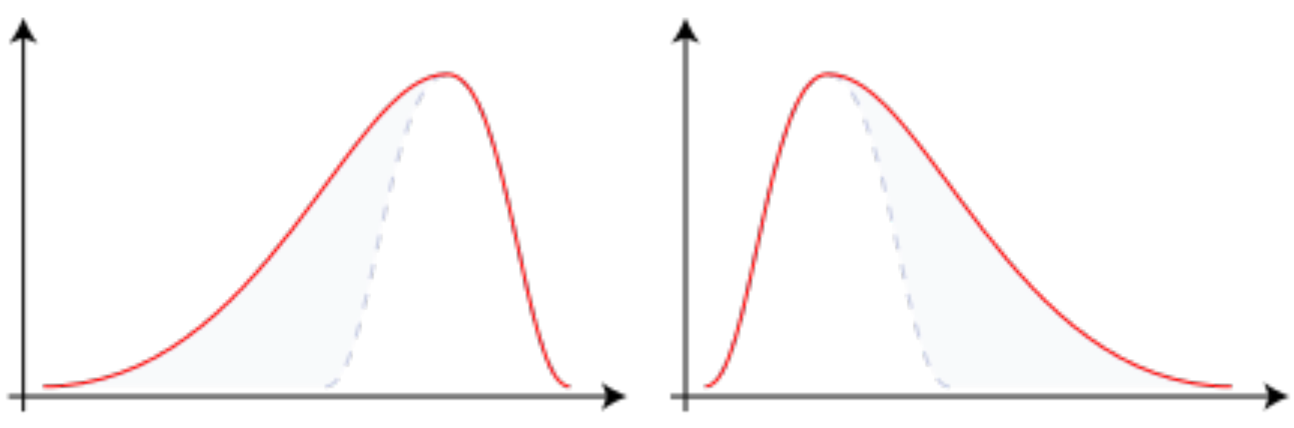
✓ **$\sqrt{s_{NN}}$=14.5 GeV in 2014 in order to fill in the large μ_B gap between 11.5 and 19.6 GeV.**

Higher order fluctuations

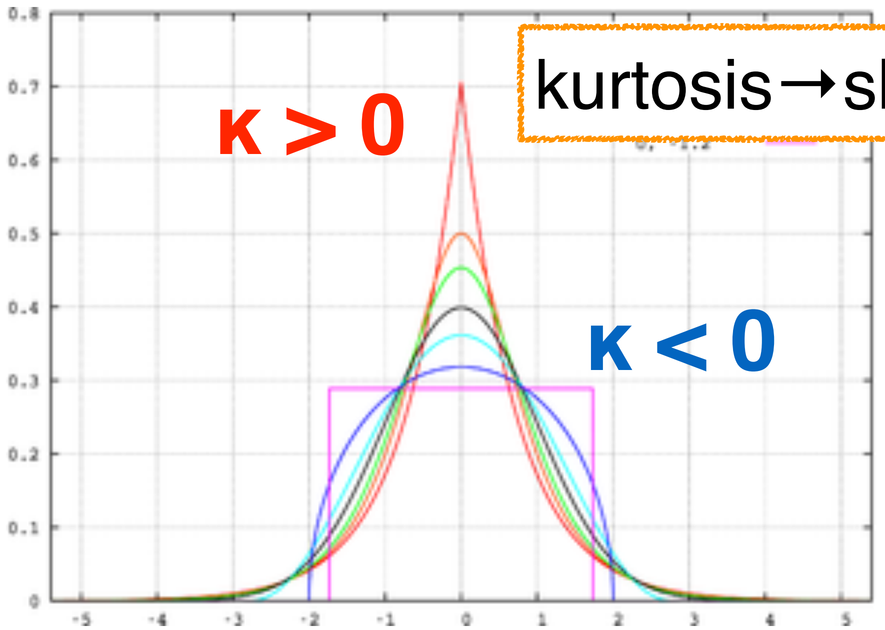
✦ **Moments and Cumulants are mathematical measures of “shape” of a histogram which probe the fluctuation of observables.**

- ✓ **Moments : Mean(*M*), sigma(*σ*), skewness(*S*) and kurtosis(κ).**
- ✓ ***S and κ are non-gaussian* fluctuations.**

skewness→asymmetry



kurtosis→shapness



from wikipedia

✓ **Cumulant ⇄ Moment**

$$< \delta N >= N - < N >$$
$$C_1 = M =< N >$$
$$C_2 = \sigma^2 =< (\delta N)^2 >$$
$$C_3 = S\sigma^3 =< (\delta N)^3 >$$
$$C_4 = \kappa\sigma^4 =< (\delta N)^4 > -3 < (\delta N)^2 >^2$$

✓ **Cumulant : additivity**

$$C_n(X+Y) = C_n(X) + C_n(Y)$$

➡ **Volume dependence**

Fluctuations of conserved quantities

✦ Net-baryon, net-charge and net-strangeness

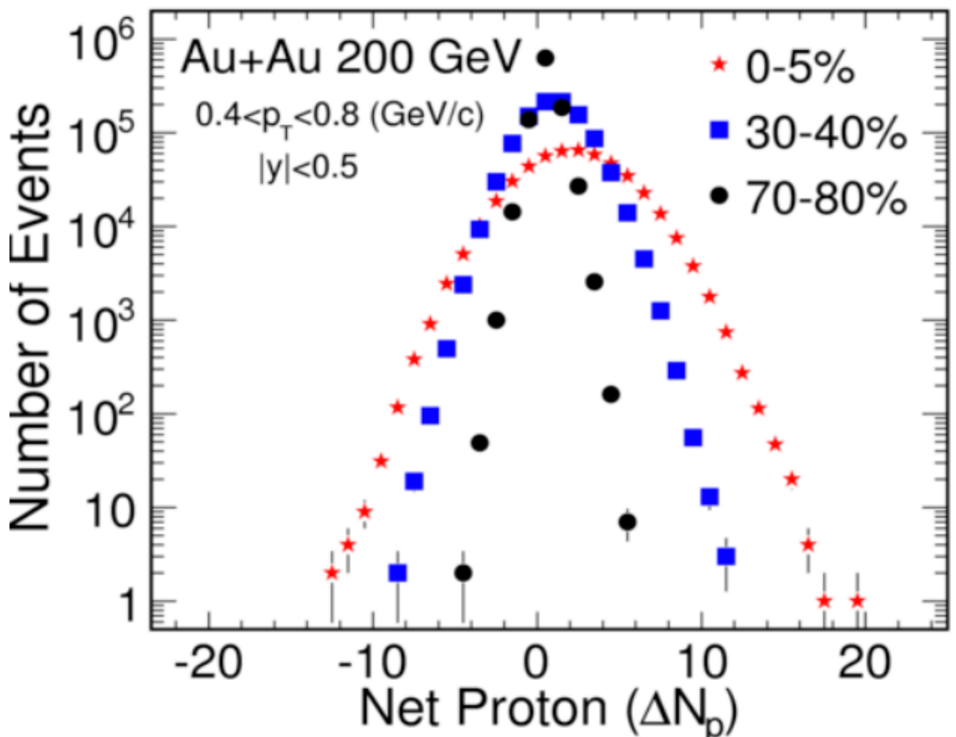
"Net" : positive - negative

$$\Delta N_q = N_q - N_{\bar{q}}, \quad q = B, Q, S$$

No. of "positively charged" particles in one collision

No. of "negatively charged" particles in one collision

Fill in histograms over many collisions



→neutrons cannot be measured

(1) Sensitive to correlation length

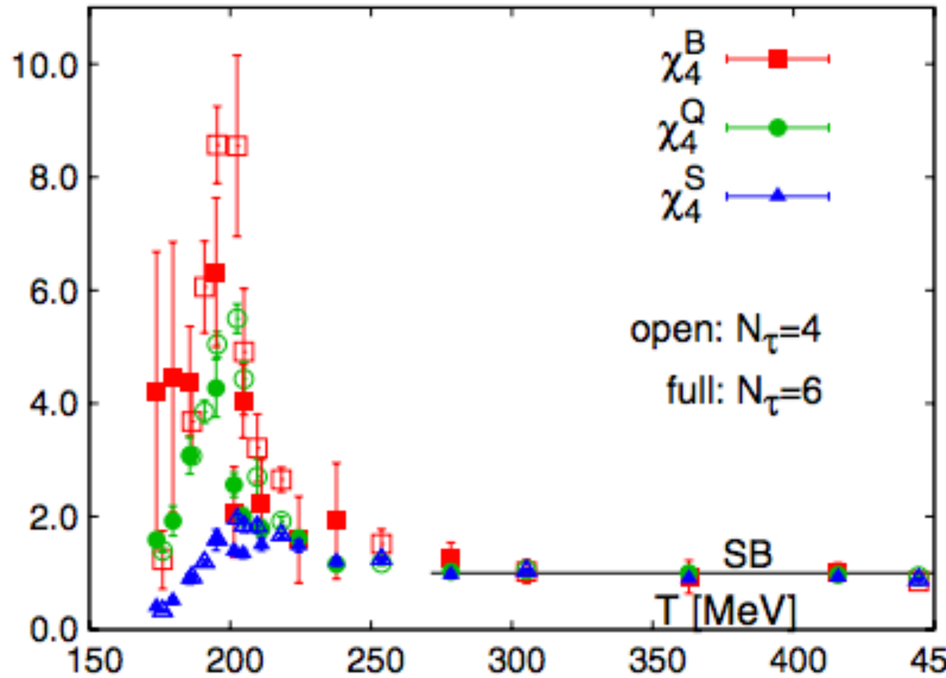
$$C_2 =< (\delta N)^2 >_c \approx \xi^2$$
$$C_3 =< (\delta N)^3 >_c \approx \xi^{4.5}$$
$$C_4 =< (\delta N)^4 >_c \approx \xi^7$$

(2) Direct comparison with susceptibilities.

M. Cheng et al, PRD 79, 074505 (2009)

$$S\sigma = \frac{C_3}{C_2} = \frac{\chi_3}{\chi_2} \qquad \kappa\sigma^2 = \frac{C_4}{C_2} = \frac{\chi_4}{\chi_2}$$

$$\chi_n^q = \frac{1}{VT^3} \times C_n^q = \frac{\partial^n p/T^4}{\partial \mu_q^n}, \quad q = B, Q, S$$



Volume **dependence can be canceled by taking ratio.**

Statistical baselines

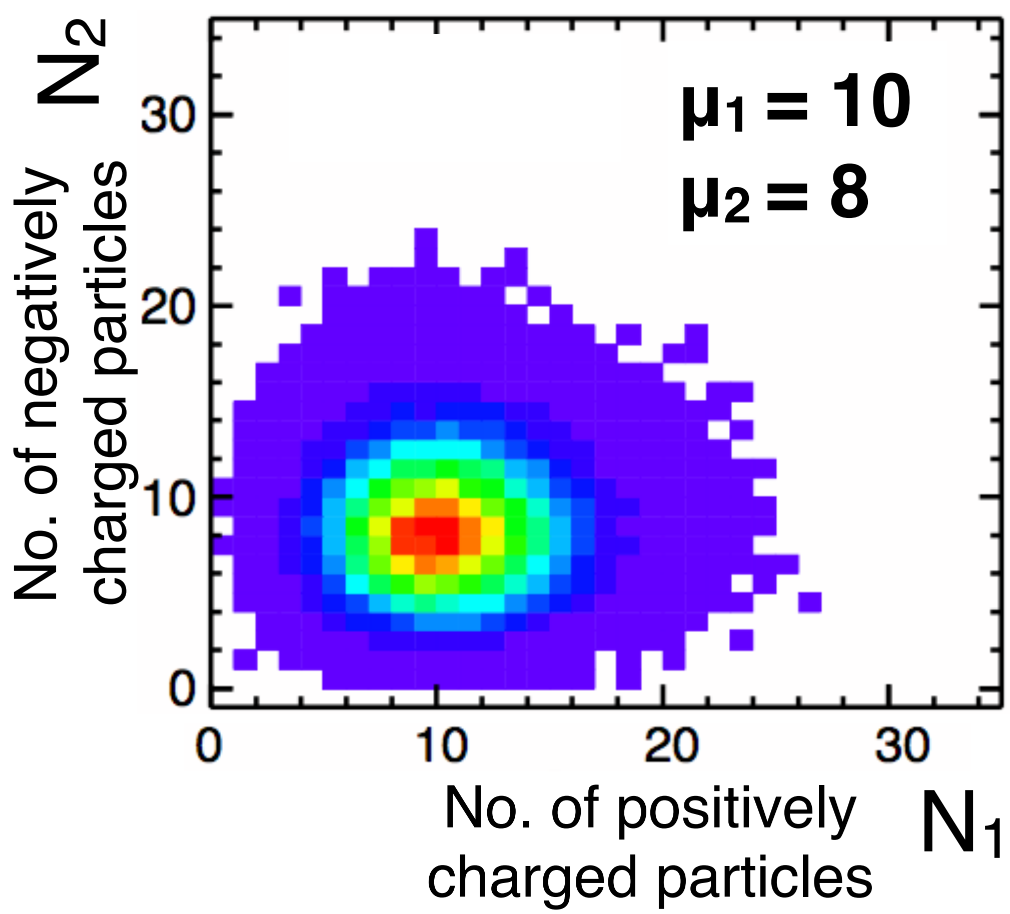
✓ **Poisson - Poisson = Skellam**

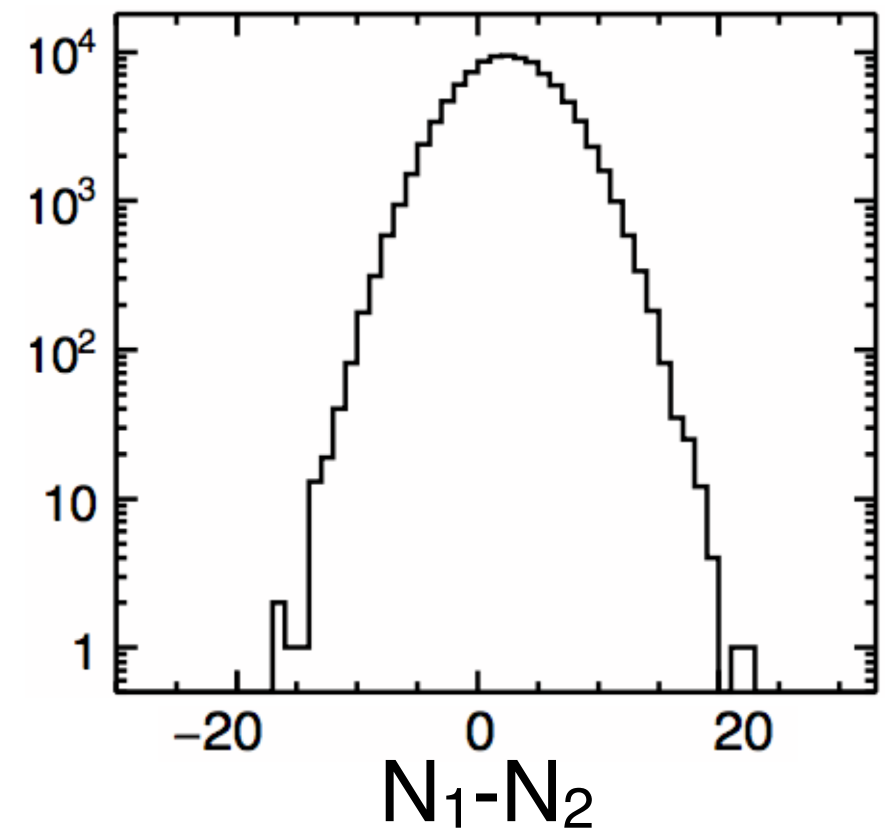
μ_1, μ_2 : mean parameter of Poisson

$$p(k;\mu_1,\mu_2) = \Pr\{K=k\} = e^{-(\mu_1+\mu_2)}\left(\frac{\mu_1}{\mu_2}\right)^{k/2} I_k(2\sqrt{\mu_1\mu_2})$$

✓ **Odd(even) order cumulant of Skellam distribution is difference(sum) between means of two Poissons.**

$$C_{odd} = \mu_1 - \mu_2 \qquad S\sigma = \frac{C_3}{C_2} = \frac{\mu_1-\mu_2}{\mu_1+\mu_2} \qquad \kappa\sigma^2 = \frac{C_4}{C_2} = 1$$
$$C_{even} = \mu_1 + \mu_2$$





Statistical baselines

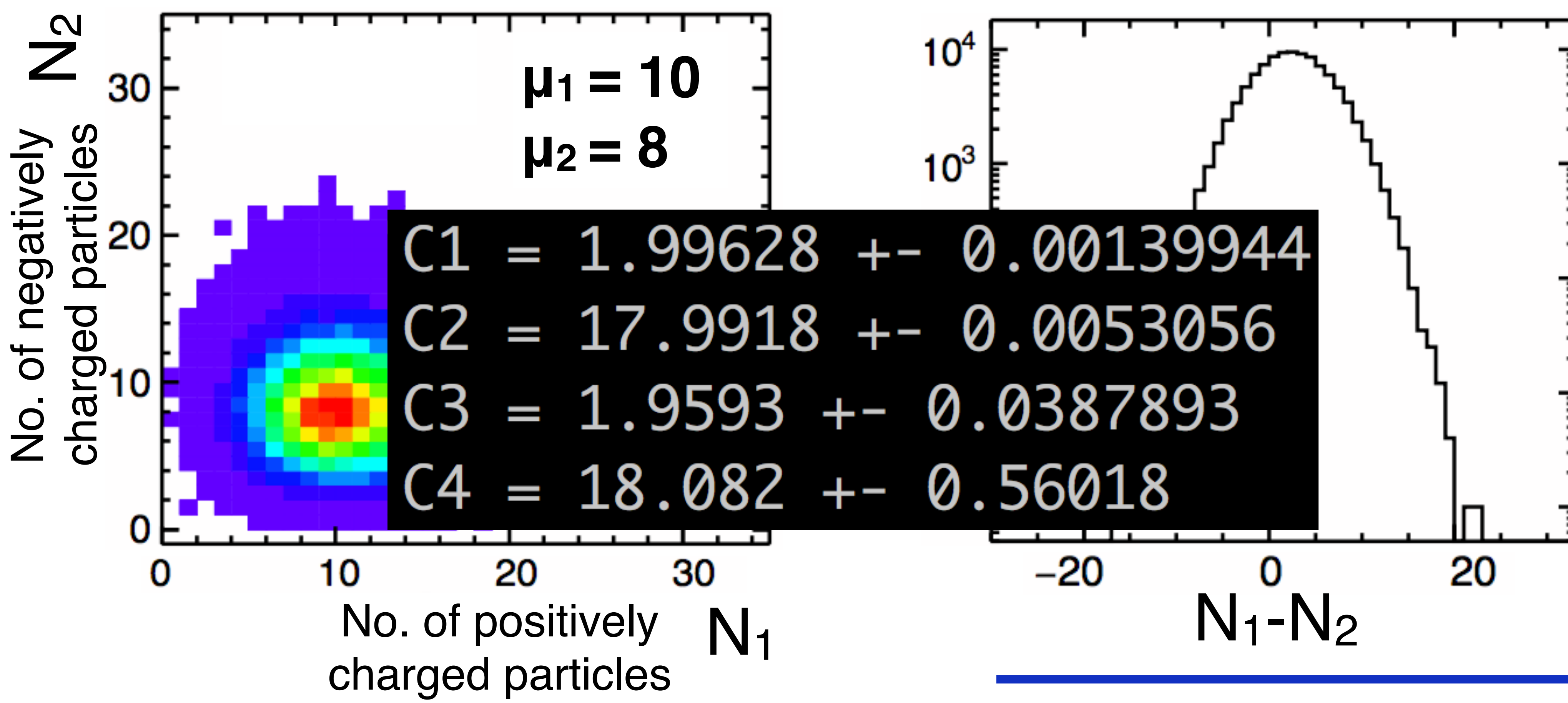
✓ **Poisson - Poisson = Skellam**

μ_1, μ_2 : mean parameter of Poisson

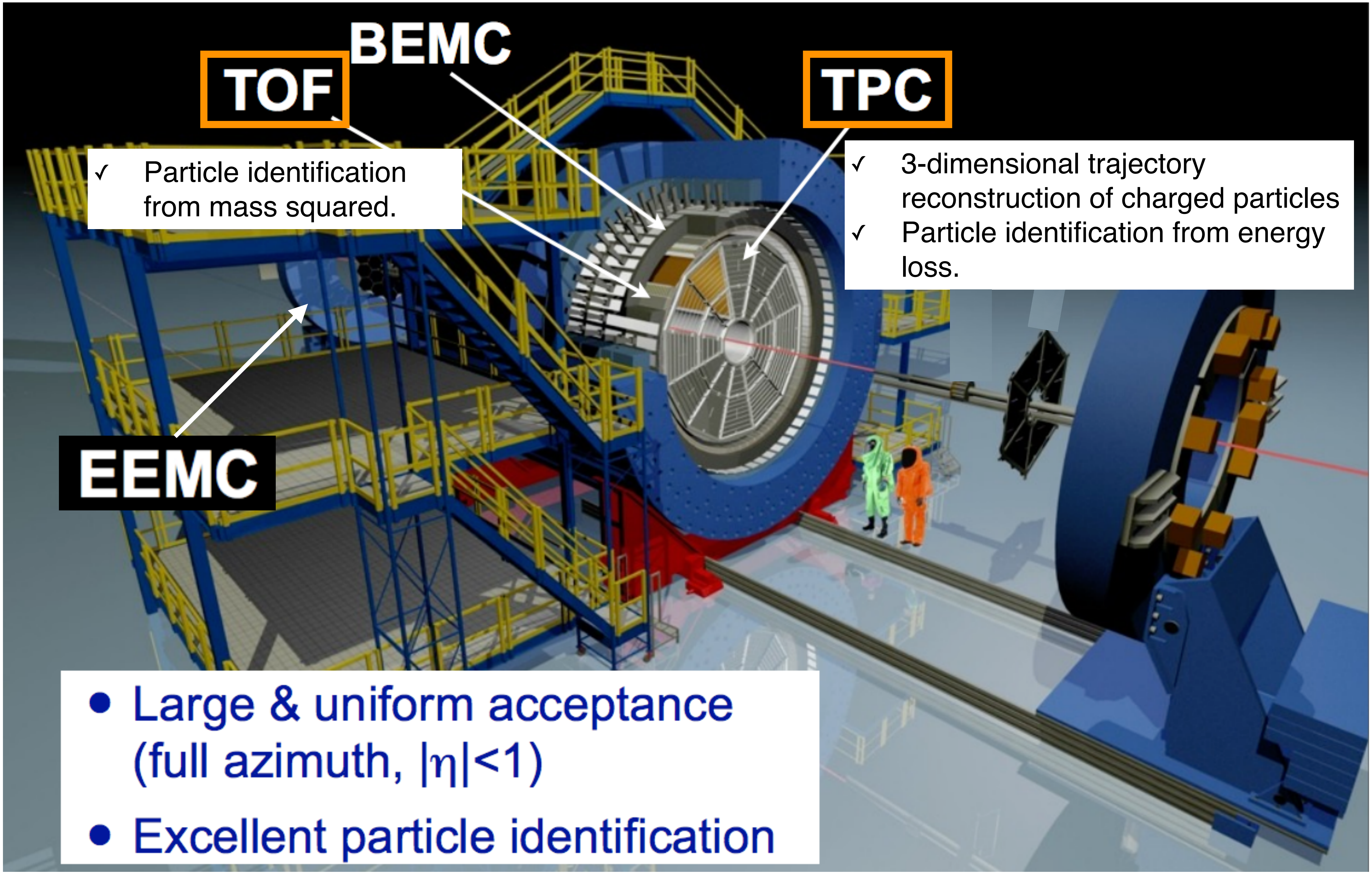
$$p(k;\mu_1,\mu_2) = \Pr\{K=k\} = e^{-(\mu_1+\mu_2)}\left(\frac{\mu_1}{\mu_2}\right)^{k/2} I_k(2\sqrt{\mu_1\mu_2})$$

✓ **Odd(even) order cumulant of Skellam distribution is difference(sum) between means of two Poissons.**

$$C_{odd} = \mu_1 - \mu_2 \qquad S\sigma = \frac{C_3}{C_2} = \frac{\mu_1-\mu_2}{\mu_1+\mu_2} \qquad \kappa\sigma^2 = \frac{C_4}{C_2} = 1$$
$$C_{even} = \mu_1 + \mu_2$$



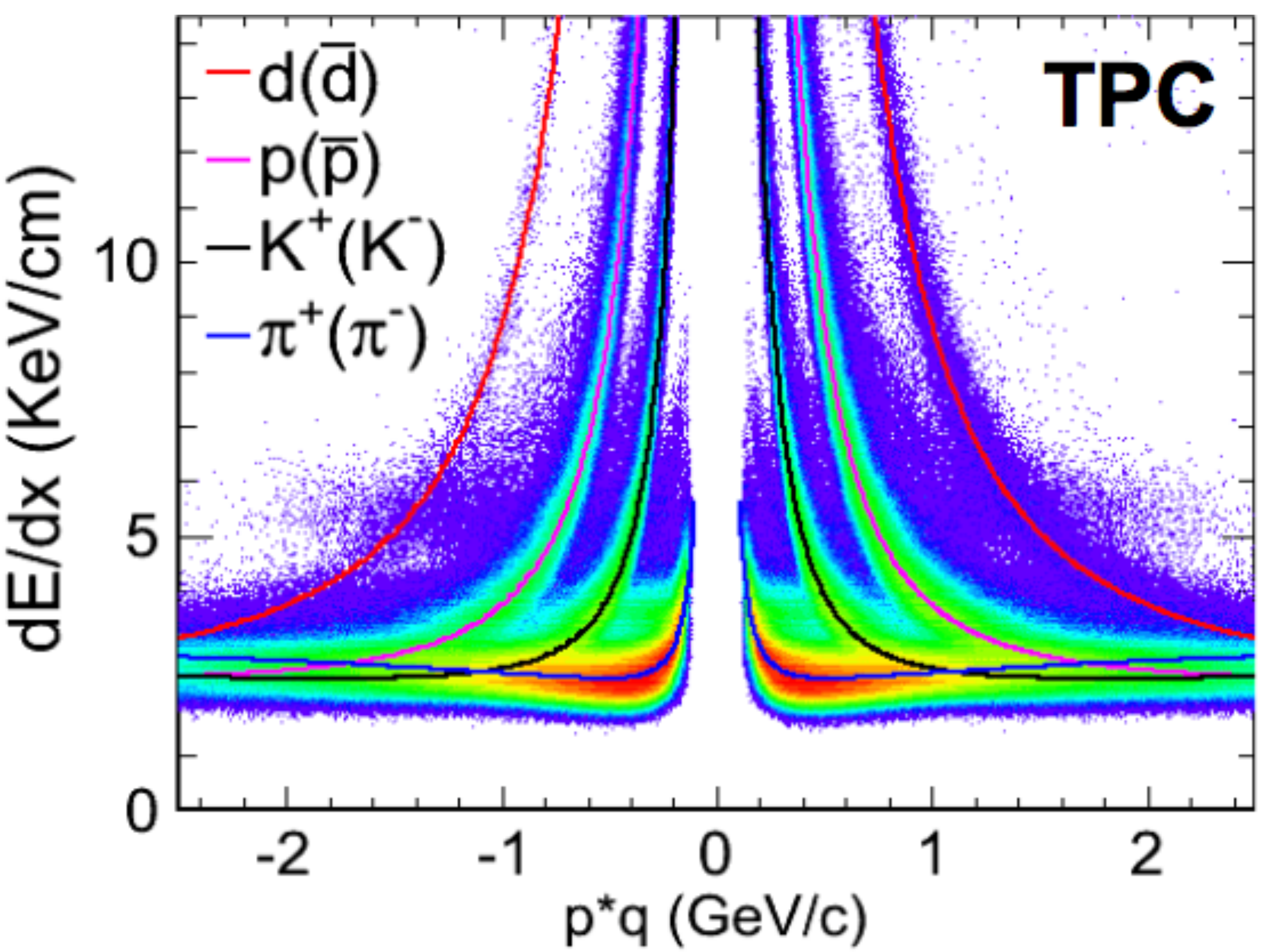
Solenoidal Tracker At RHIC



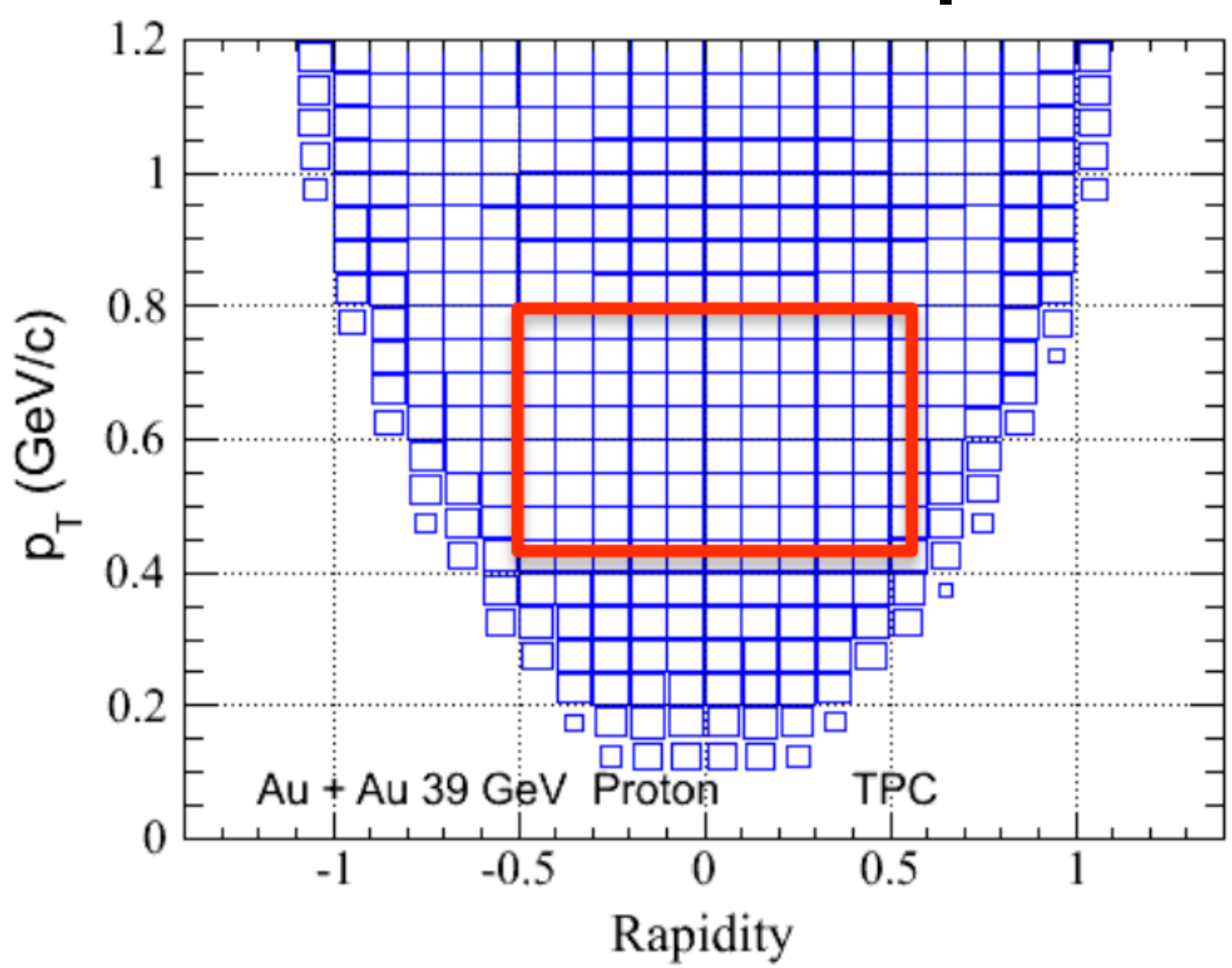
Particle Identification

- ✓ Charged particles are counted using the reconstructed tracks by TPC.
- ✓ Protons can be identified by using dE/dx from TPC.

STAR TPC dE/dx

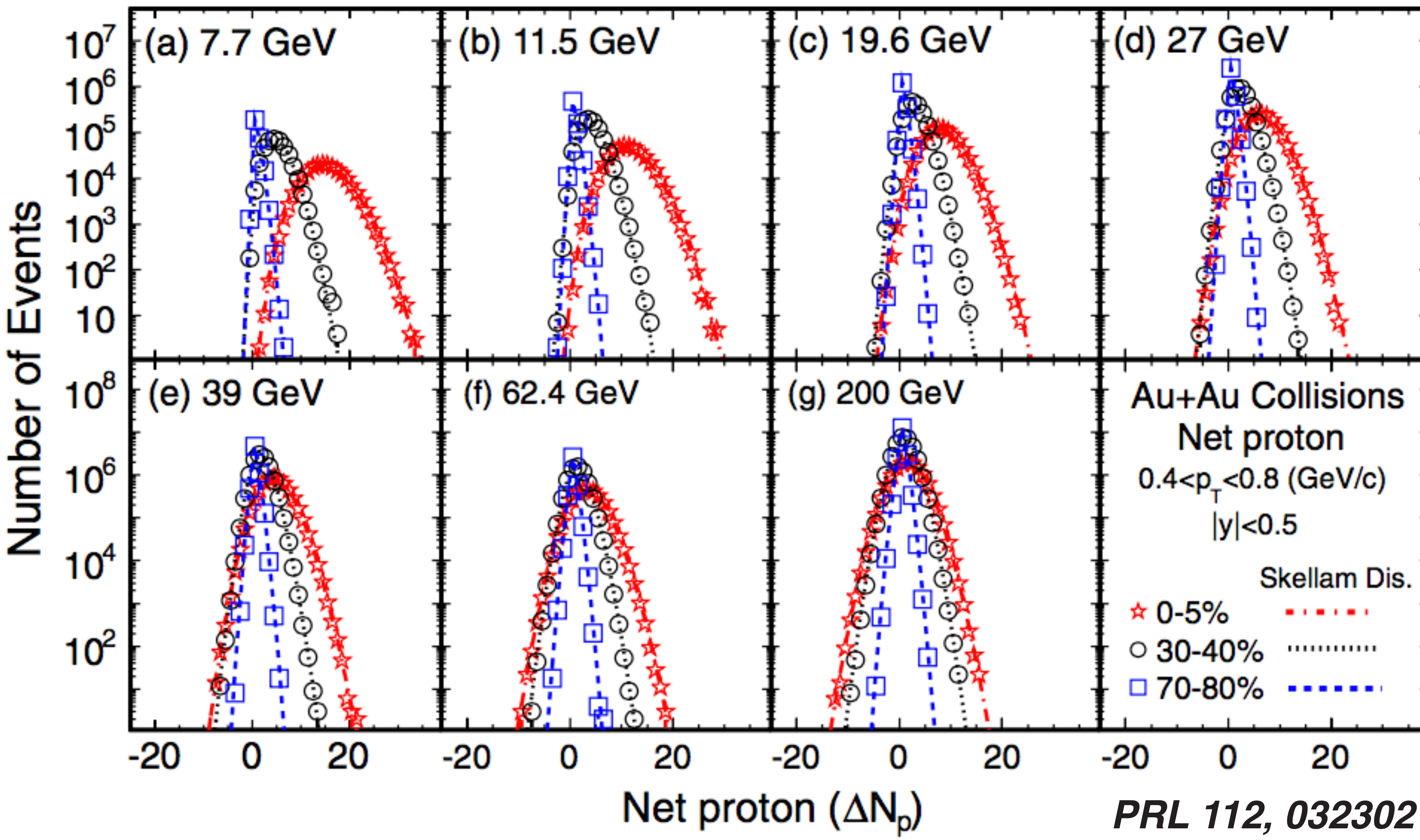


Proton Phase Space



Event by event distribution

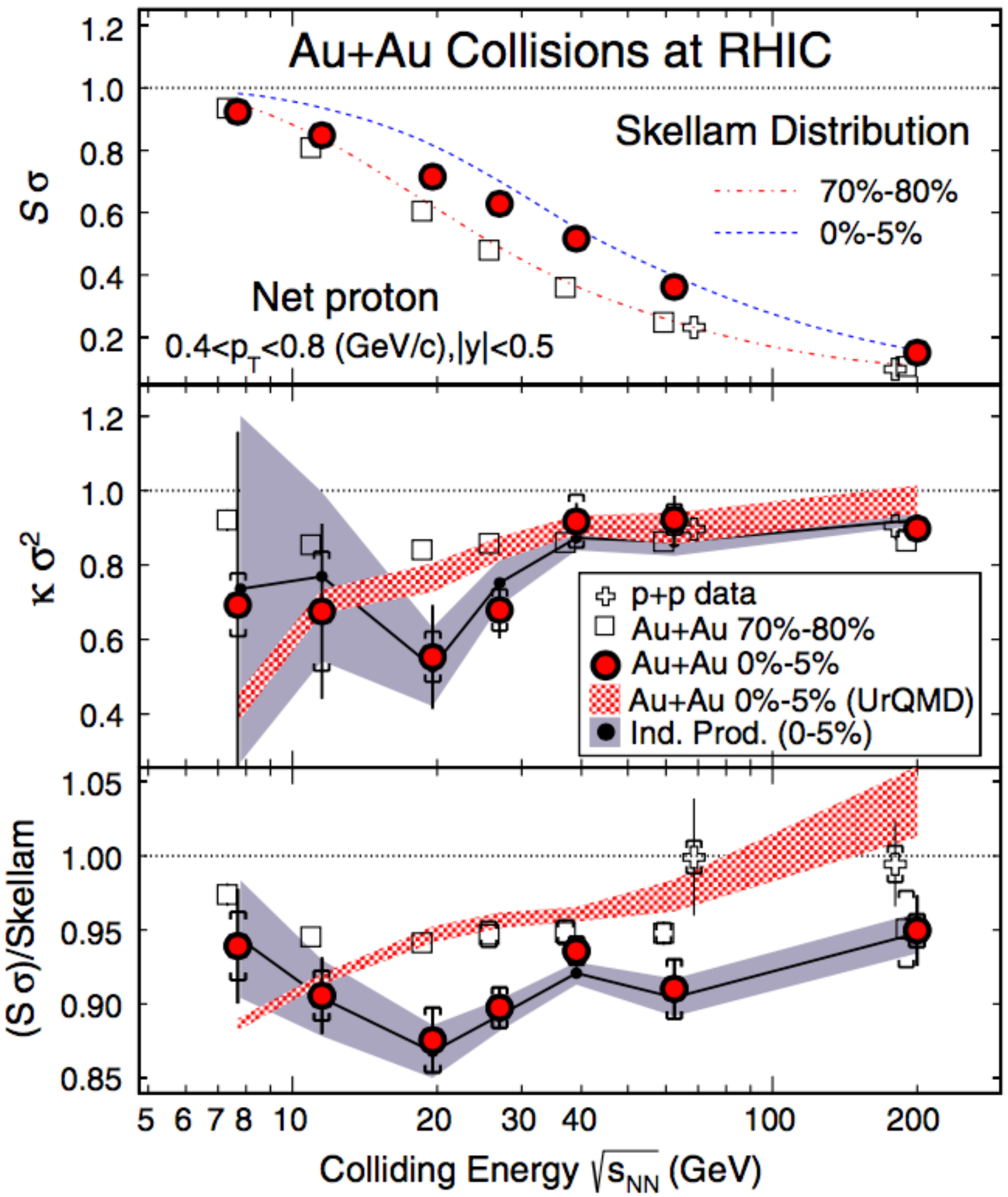
- ✓ **Event by event net-proton distribution.**
- ✓ **Low collision energy, small number of antiproton.**



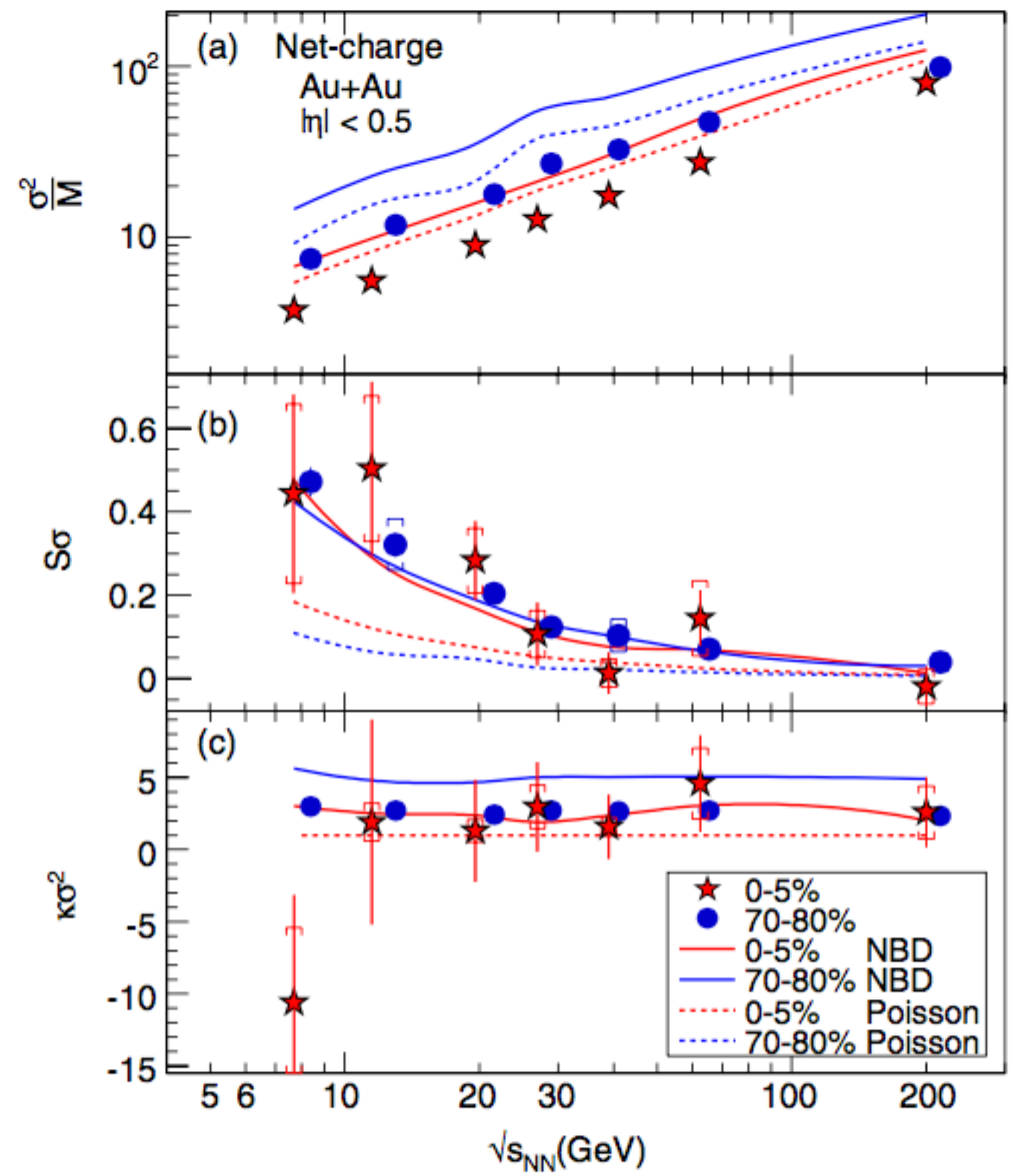
PRL 112, 032302 (2014)

Published results in 2014

- ✓ It seems to be interesting around 20 GeV for net-proton results.
- ✓ Net-charge results are consistent with the baseline due to large errors. → A wide distribution gives large statistical errors.



PRL 112, 032302 (2014)

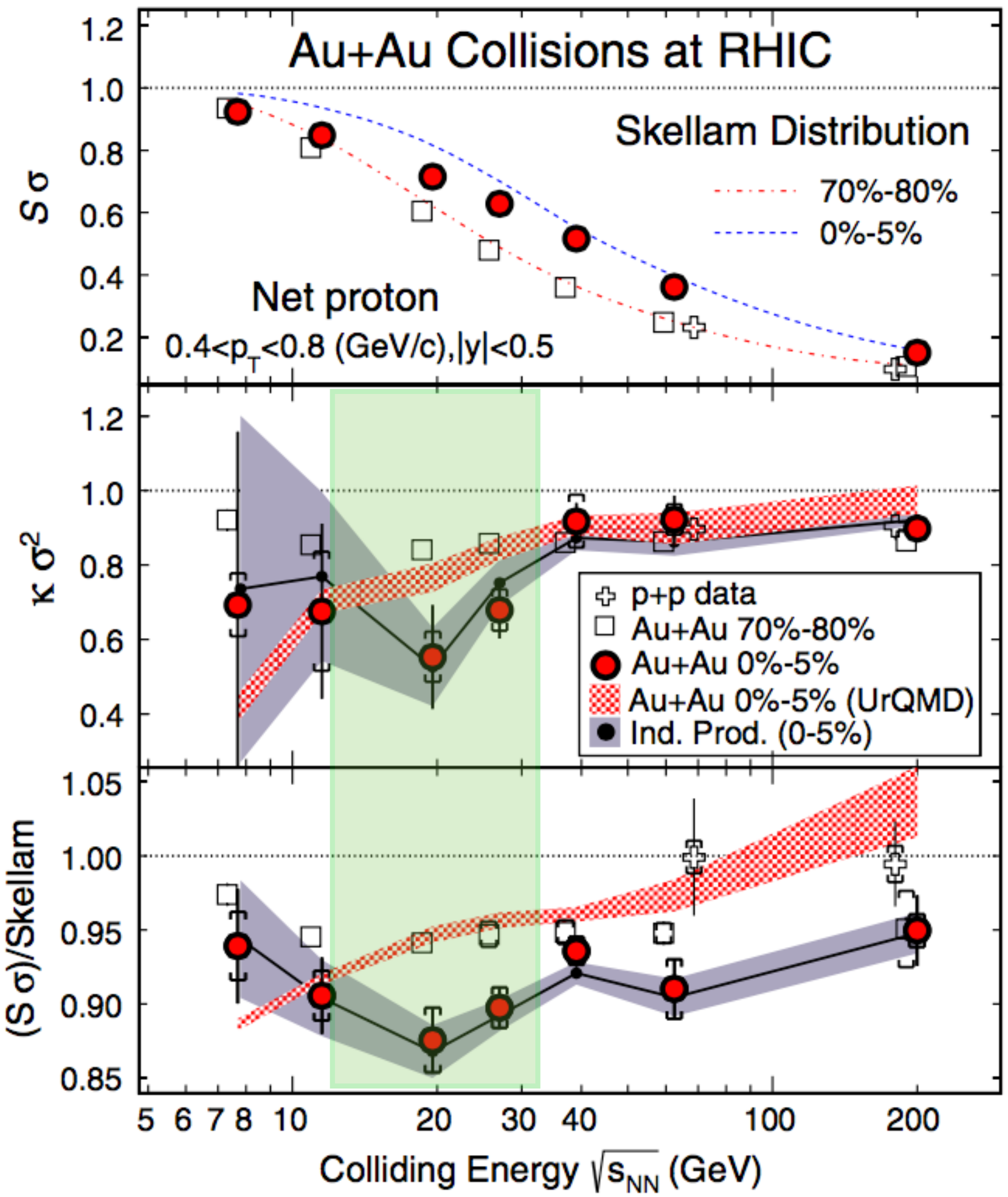


PRL 113, 092301 (2014)

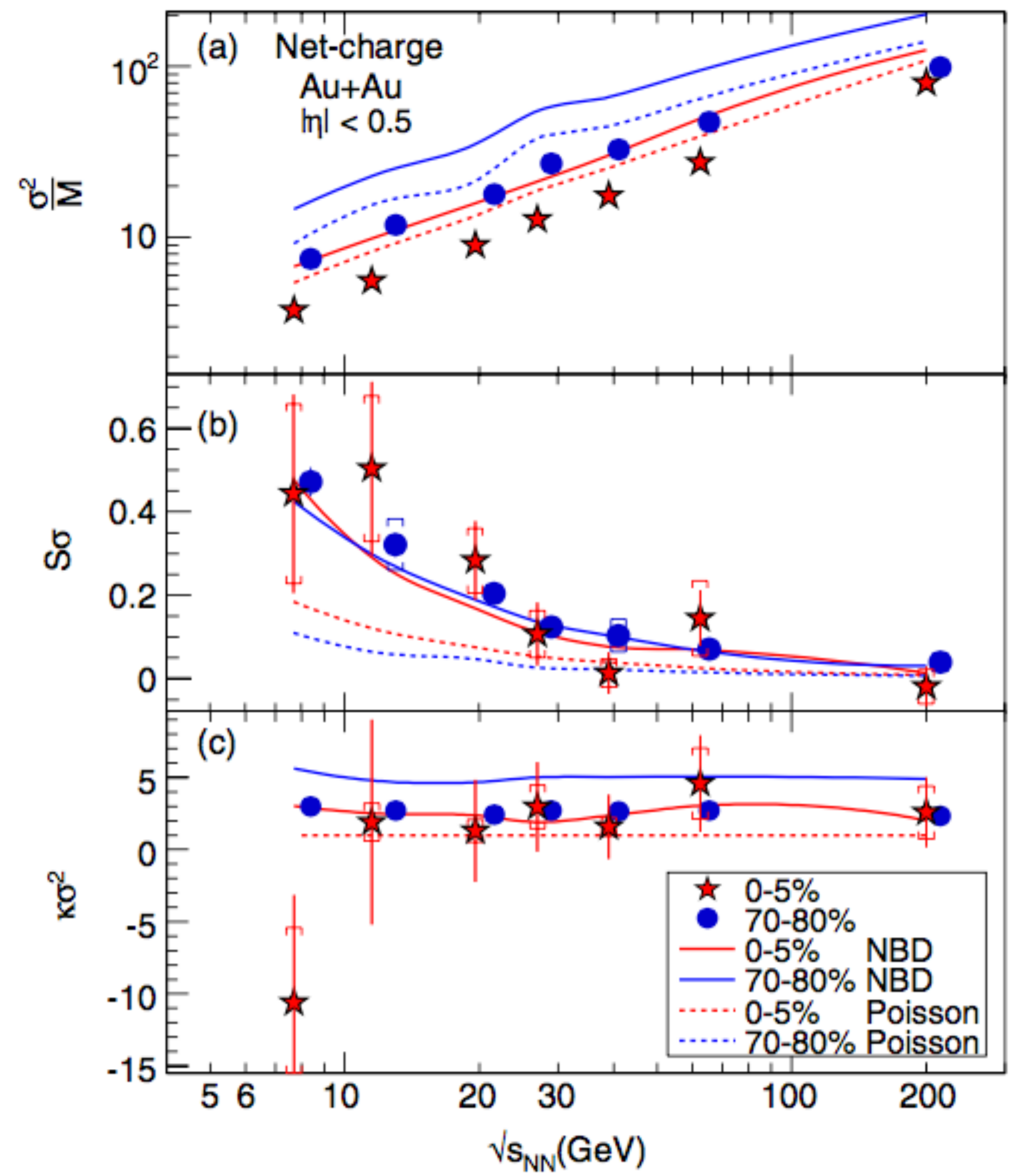
✦ Finite tracking efficiency is corrected.

Published results in 2014

- ✓ It seems to be interesting around 20 GeV for net-proton results.
- ✓ Net-charge results are consistent with the baseline due to large errors. → A wide distribution gives large statistical errors.



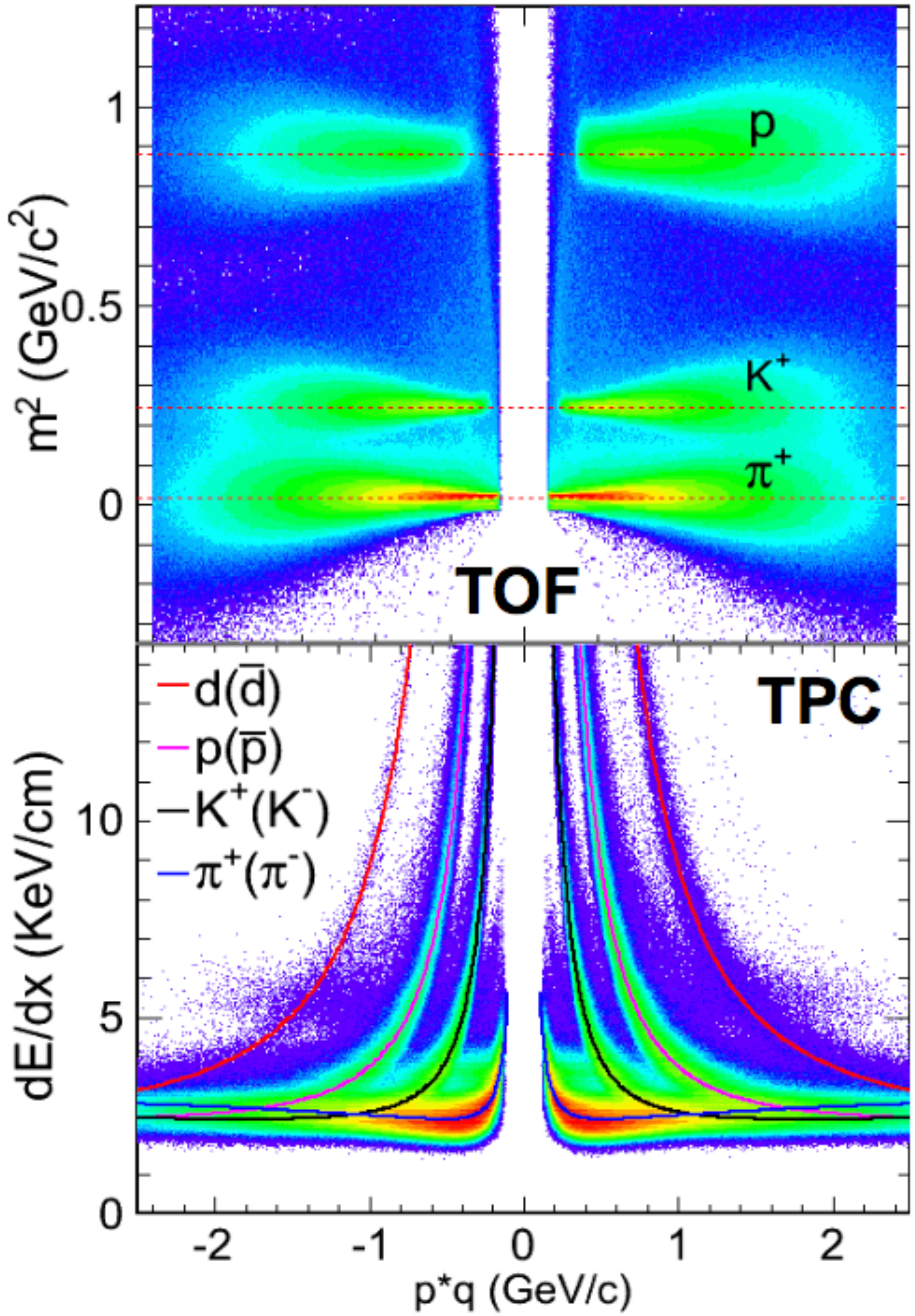
PRL 112, 032302 (2014)

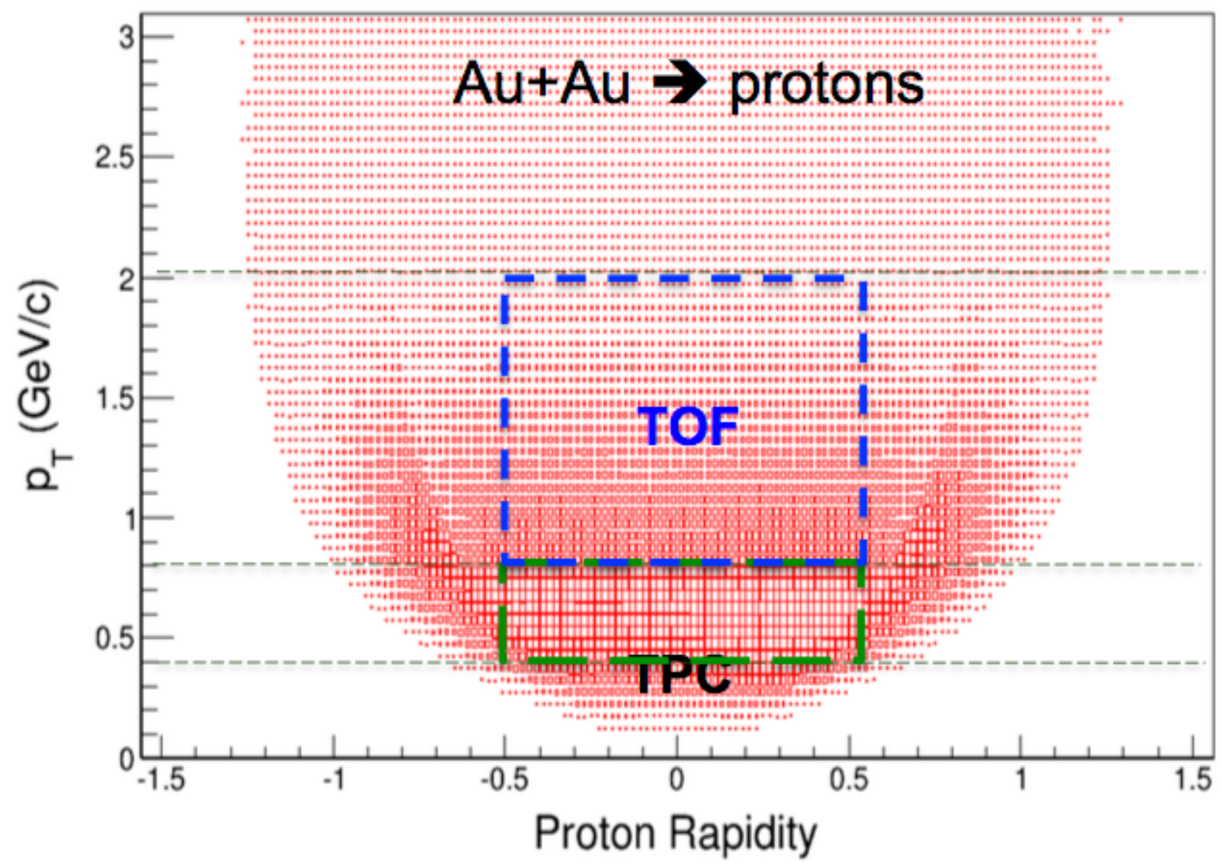


PRL 113, 092301 (2014)

✦ Finite tracking efficiency is corrected.

Extending p_T coverage

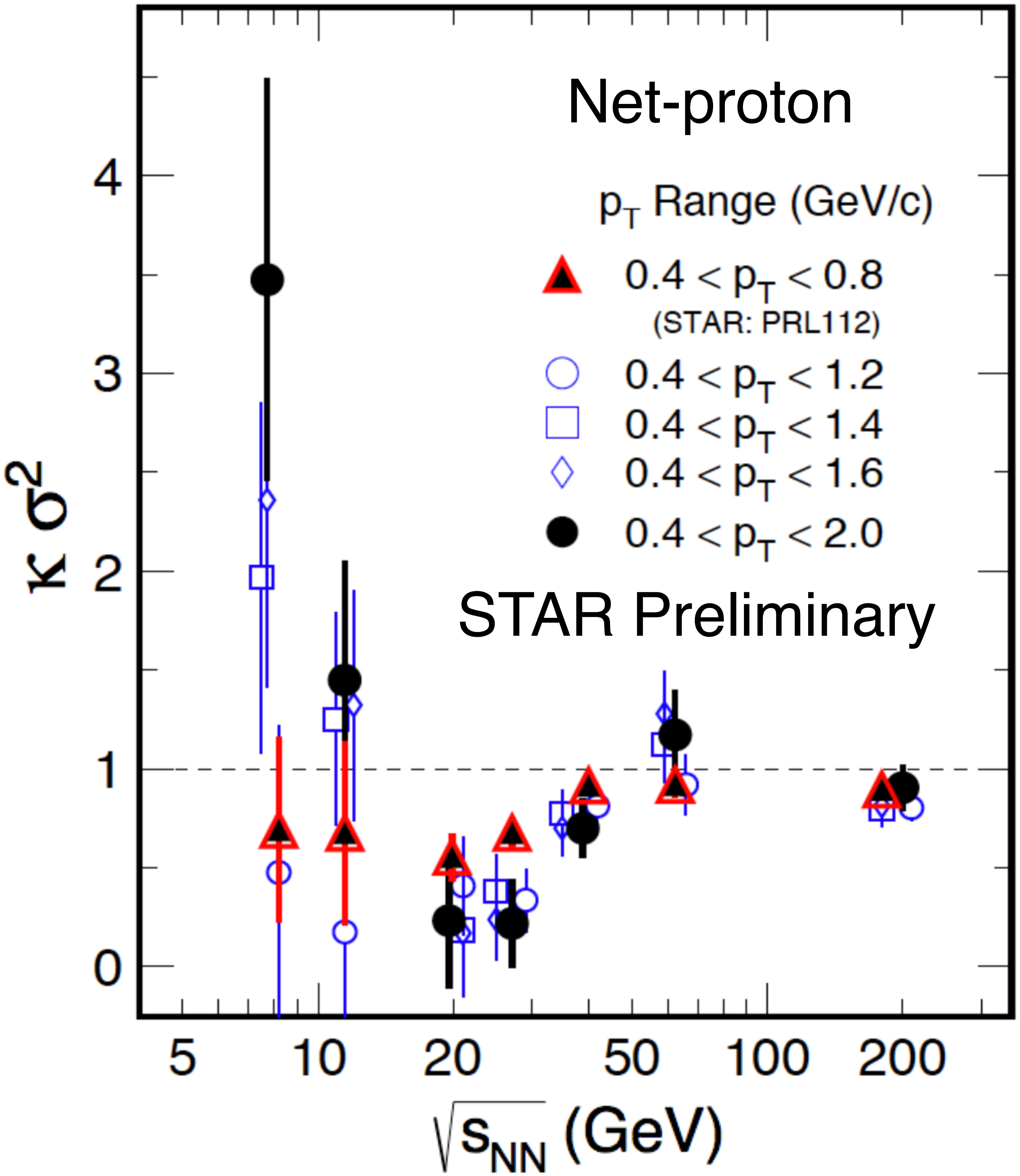




- ✓ p_T region can be extended up to 2.0 GeV by using m^2 cut from Time Of Flight detector.
- ✓ We gain factor two (anti)protons with respect to the published results.

Recent results

X. Luo, CPOD2014, Bielefeld, Germany

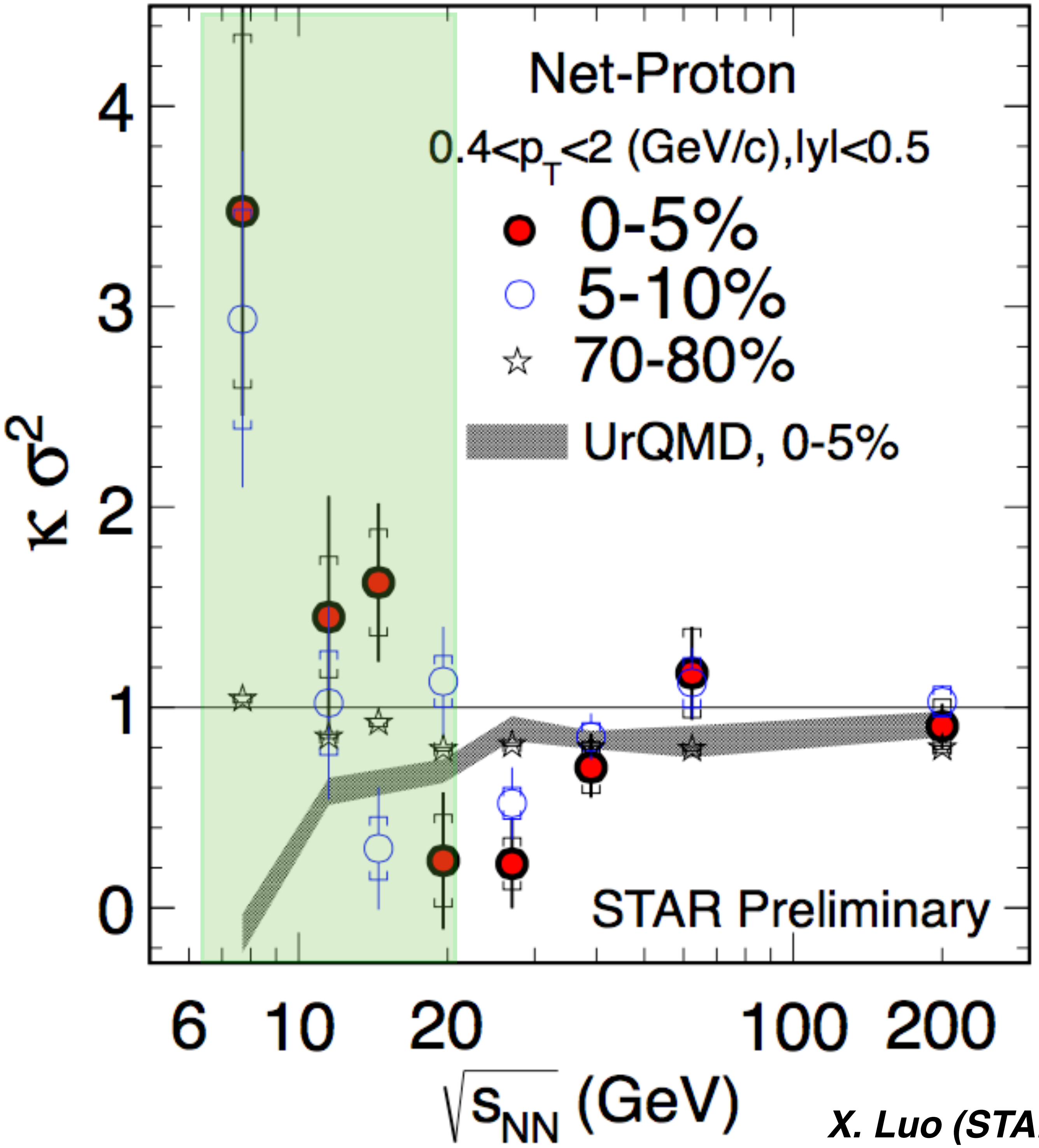


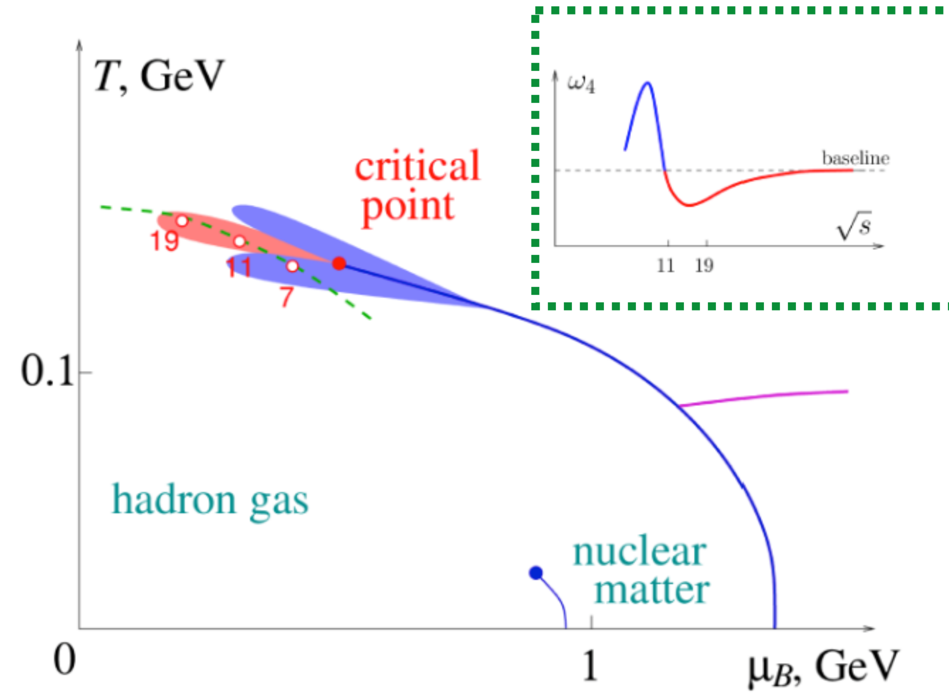
- ✓ **We can obtain larger signals with larger acceptance.**
- ✓ **Acceptance is crucial.**

✦ **Finite tracking efficiency is corrected.**

Recent results

σ model, M.A. Stephanov, PRL107, 052301 (2011)





Signal from the critical point?

- ✓ **$\kappa\sigma^2$ (C_4/C_2) shows a non-monotonic behaviour. The trend is consistent with the theoretical calculation.**
- ✓ **Measurement at the lower energy is important.**

X. Luo (STAR collaboration) arXiv:1503.02558v2

✦ **Finite tracking efficiency is corrected.**

Beam Energy Scan PhaseII

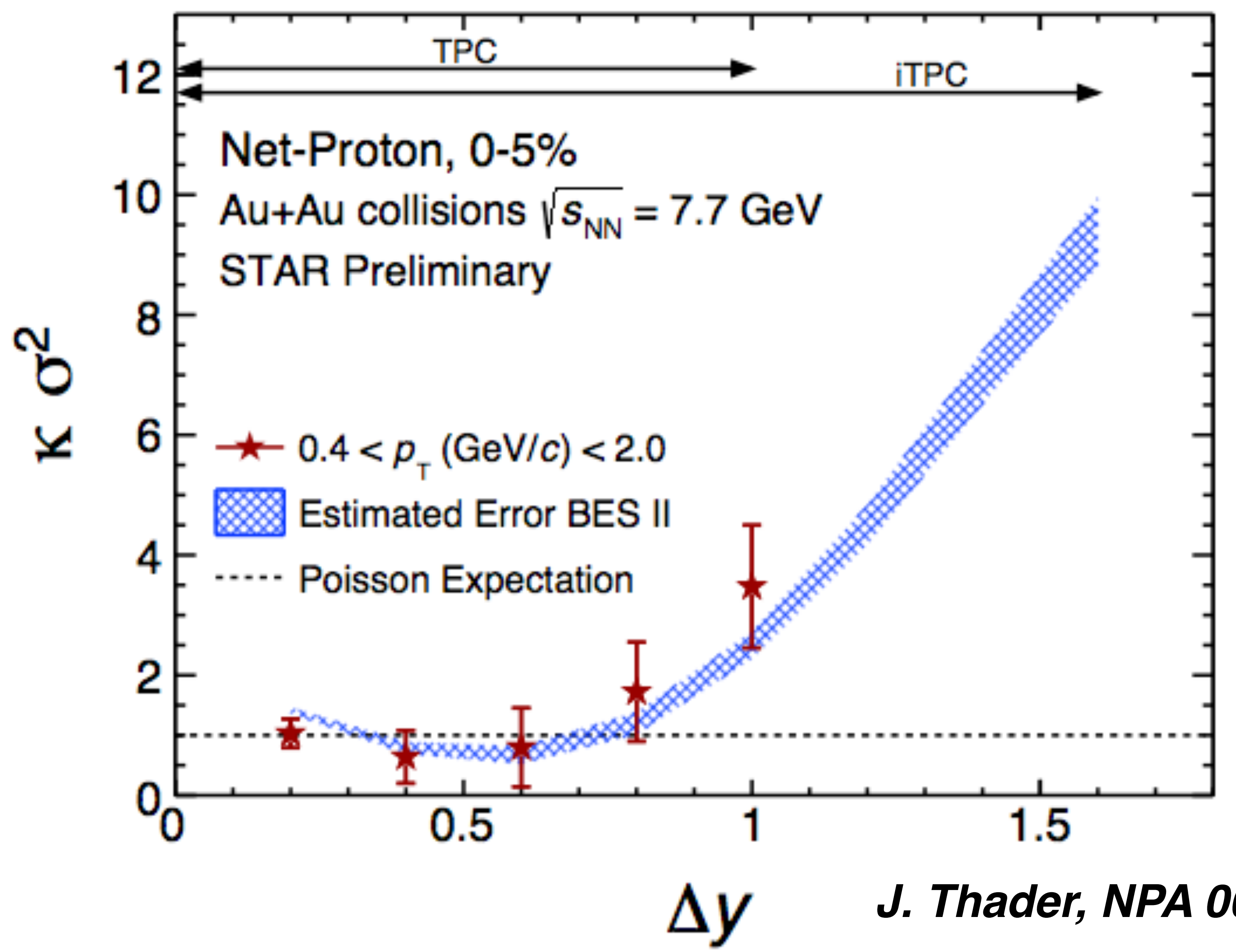
- ✓ BES II is planned in 2019 and 2020.
- ✓ Luminosity will be improved with electron cooling system.
- ✓ Some detector upgrades will be done by BESII. Pseudo-rapidity coverage will be extended from 1.0 to 1.5.
- ✓ Higher order fluctuation measurement with small errors and large acceptance.

$\sqrt{s_{NN}}$ (GeV)	7.7	9.1	11.5	14.5	19.6
μ_B (MeV)	420	370	315	250	205
BES I (MEvts)	4.3	---	11.7	24	36
Rate(MEvts/day)	0.25		1.7	2.4	4.5
BES I $\mathcal{L}$ (1×10^{25}/cm^{2}sec)	0.13		1.5	2.1	4.0
BES II (MEvts)	100	160	230	300	400
eCooling (Factor)	4	4	4	3	3
Beam Time (weeks)	**12**	**9.5**	**5.0**	**5.5**	**4.5**

D. Cebra, 34th Reimei workshop, J-PARC, Tokai, Japan

Beam Energy Scan PhaseII

- ✓ BES II is planned in 2019 and 2020.
- ✓ Luminosity will be improved with electron cooling system.
- ✓ Some detector upgrades will be done by BESII. Pseudo-rapidity coverage will be extended from 1.0 to 1.5.
- ✓ Higher order fluctuation measurement with small errors and large acceptance.



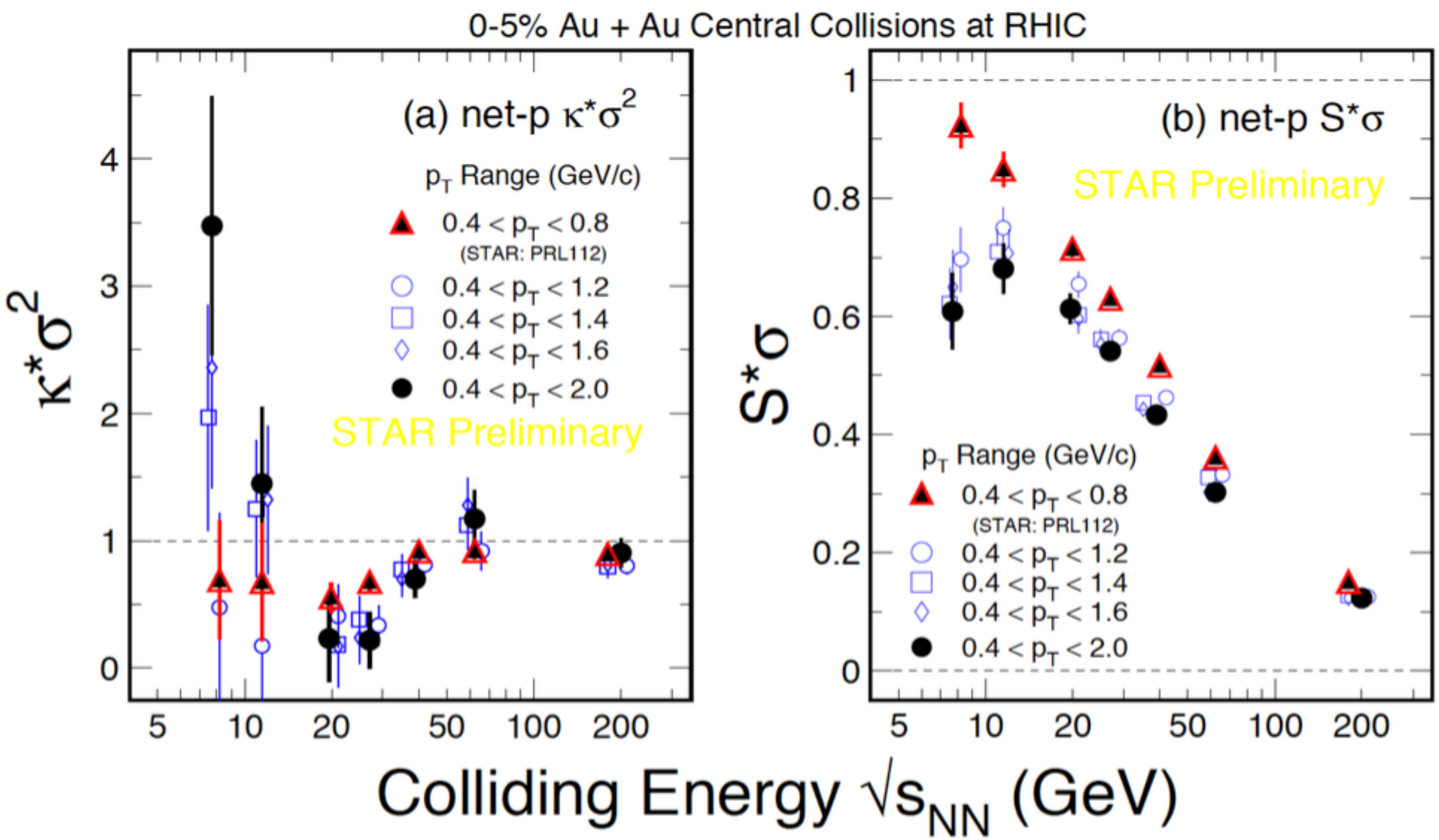
J. Thader, NPA 00(2016)

Summary

- ✓ **Beam Energy Scan Phase I was carried out at $\sqrt{s_{NN}}$ = 7.7, 11.5, 14.5, 19.6, 27, 39, 62.4 and 200 GeV in 2010, 2011 and 2014.**
- ✓ **STAR experiment has measured up to 4th order fluctuation of net-charge and net-proton multiplicity distributions for searching the critical point.**
- ✓ **Net-proton results with extended p_T region show the non-monotonic behaviour. However there is still large errors at low beam energies.**
- ✓ **Beam Energy Scan Phase II is planned in 2019 and 2020 focusing on low energy region.**

Back up

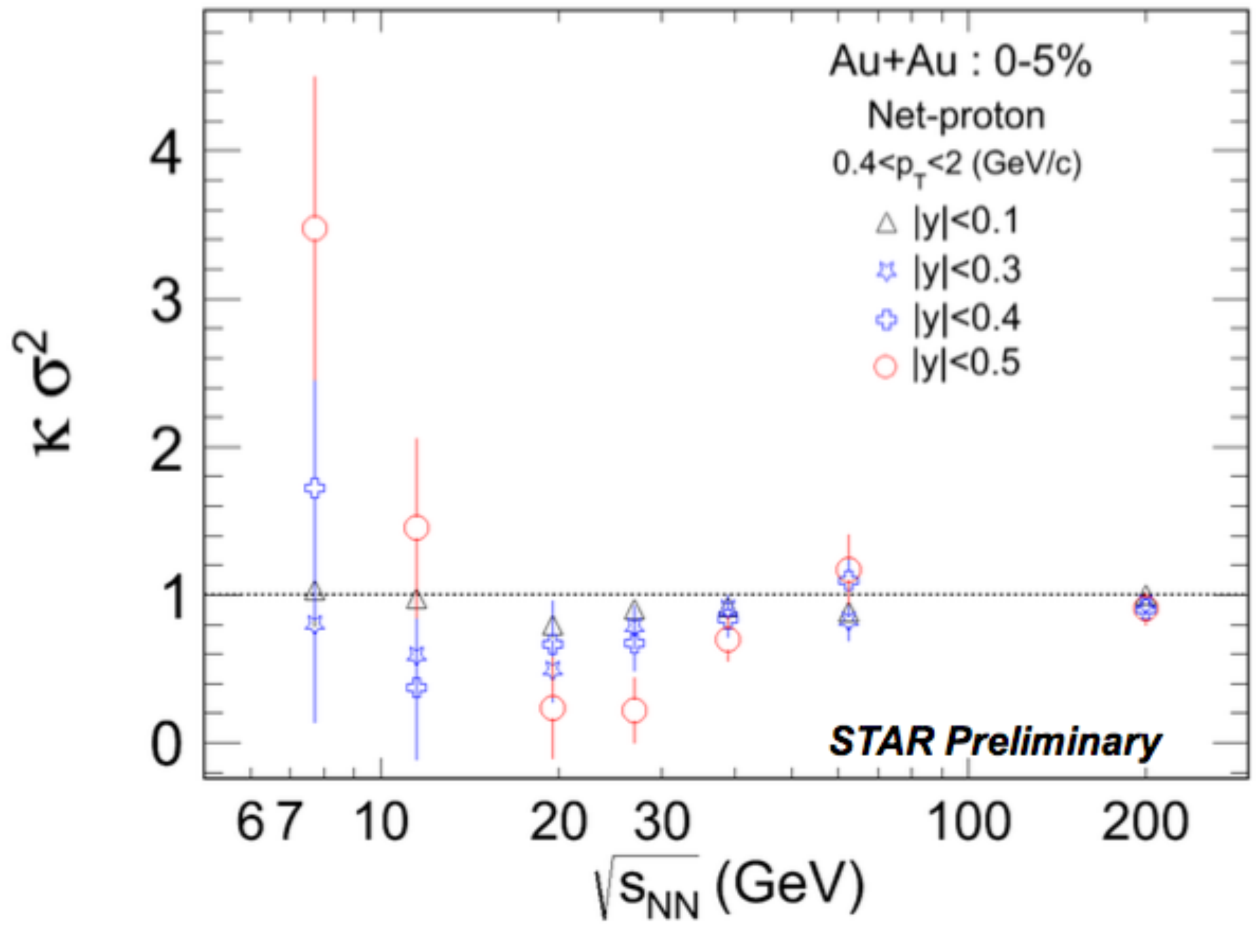
Acceptance dependence (p_T)

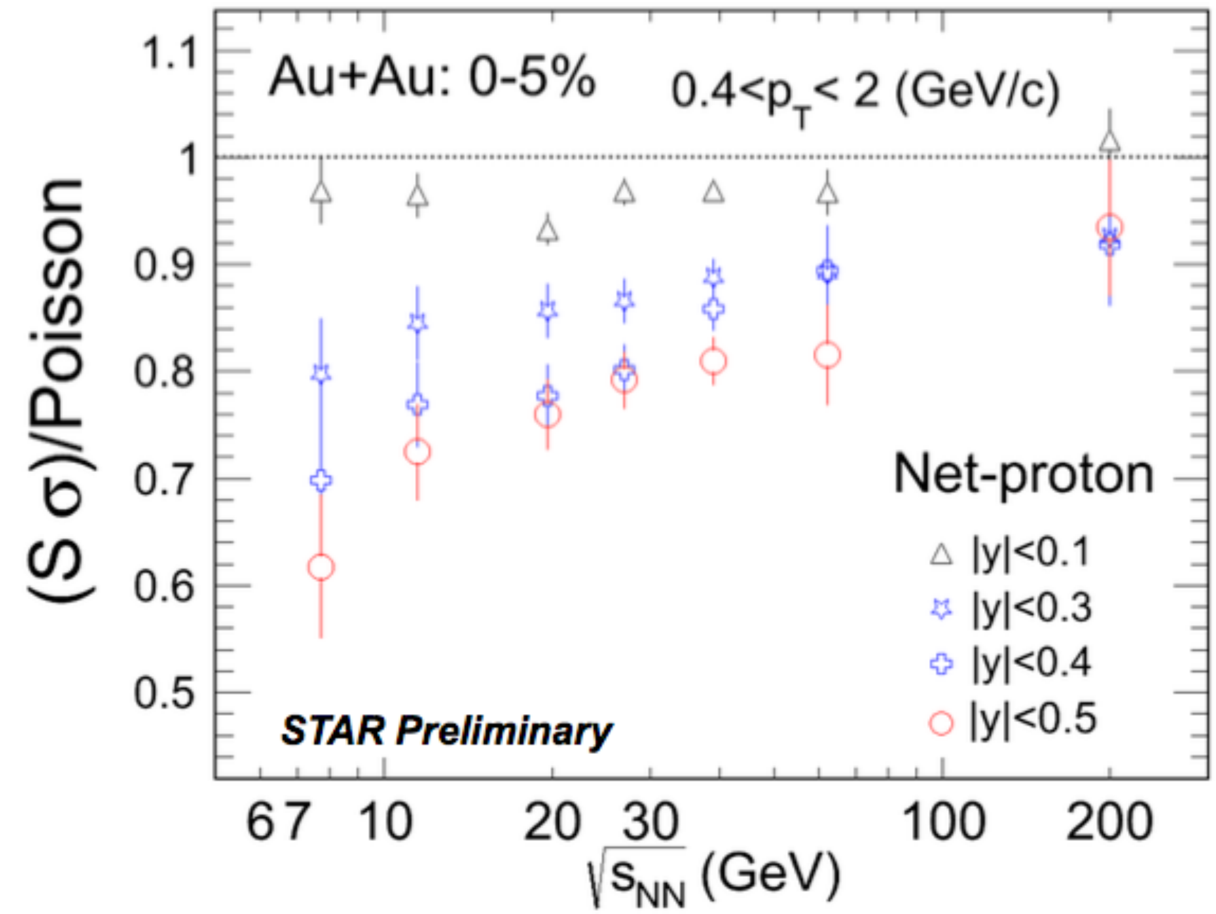


- $K*\sigma^2$: the energy dependence tends to be more pronounced with wider p_T acceptance, relative to published results.
- $S*\sigma$: the values are smaller for wider p_T acceptance.

From the talk by X. Luo at CPOD2014, Bielefeld, Germany

Acceptance dependence (y)

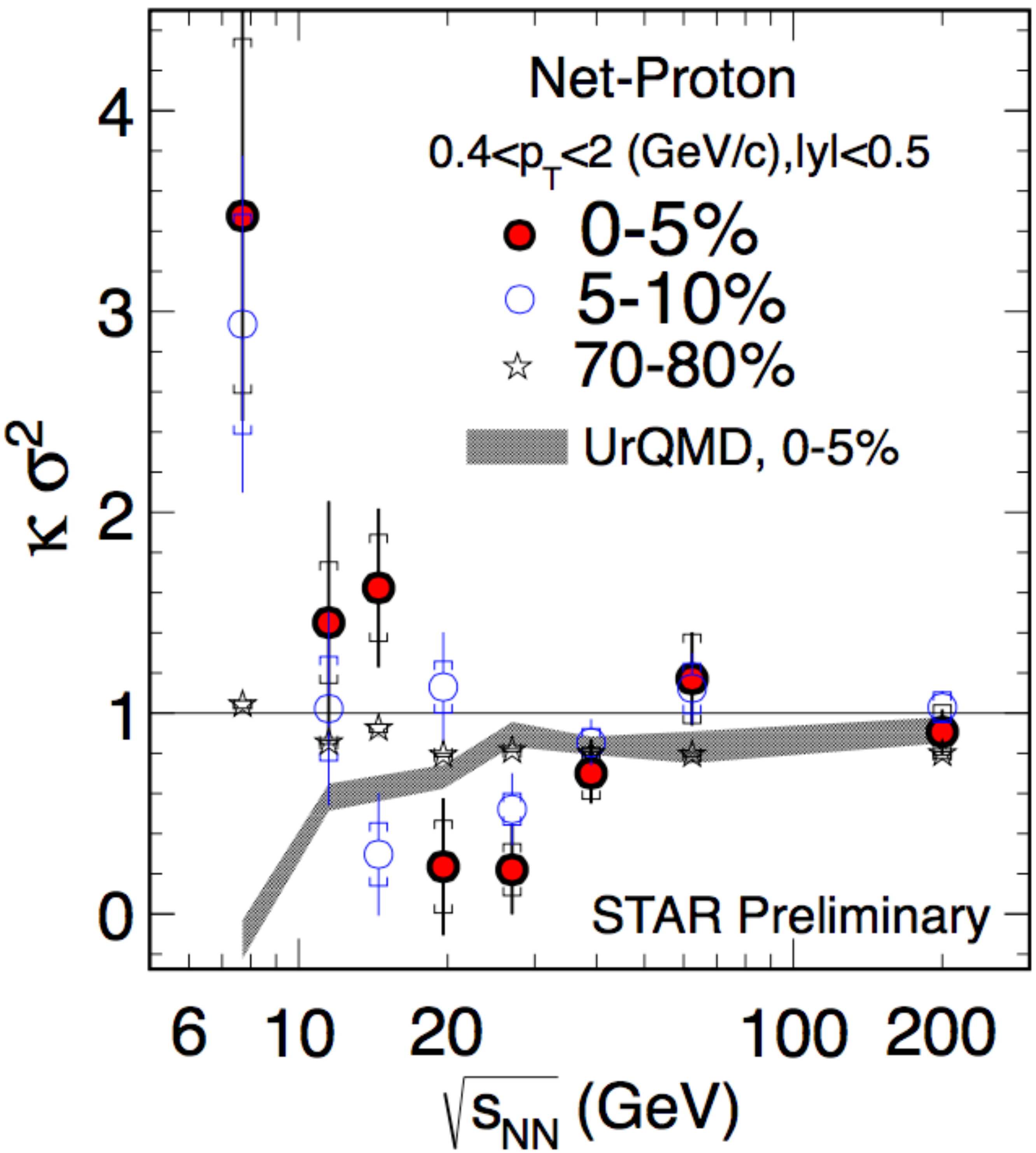




- The smaller the rapidity window the closer to the Poisson values.
- The studies indicate that the acceptance, both in p_T and y, will impact the values of moments. The acceptance needs to be large enough to capture the dynamical fluctuations. The related systematic errors should be carefully addressed.

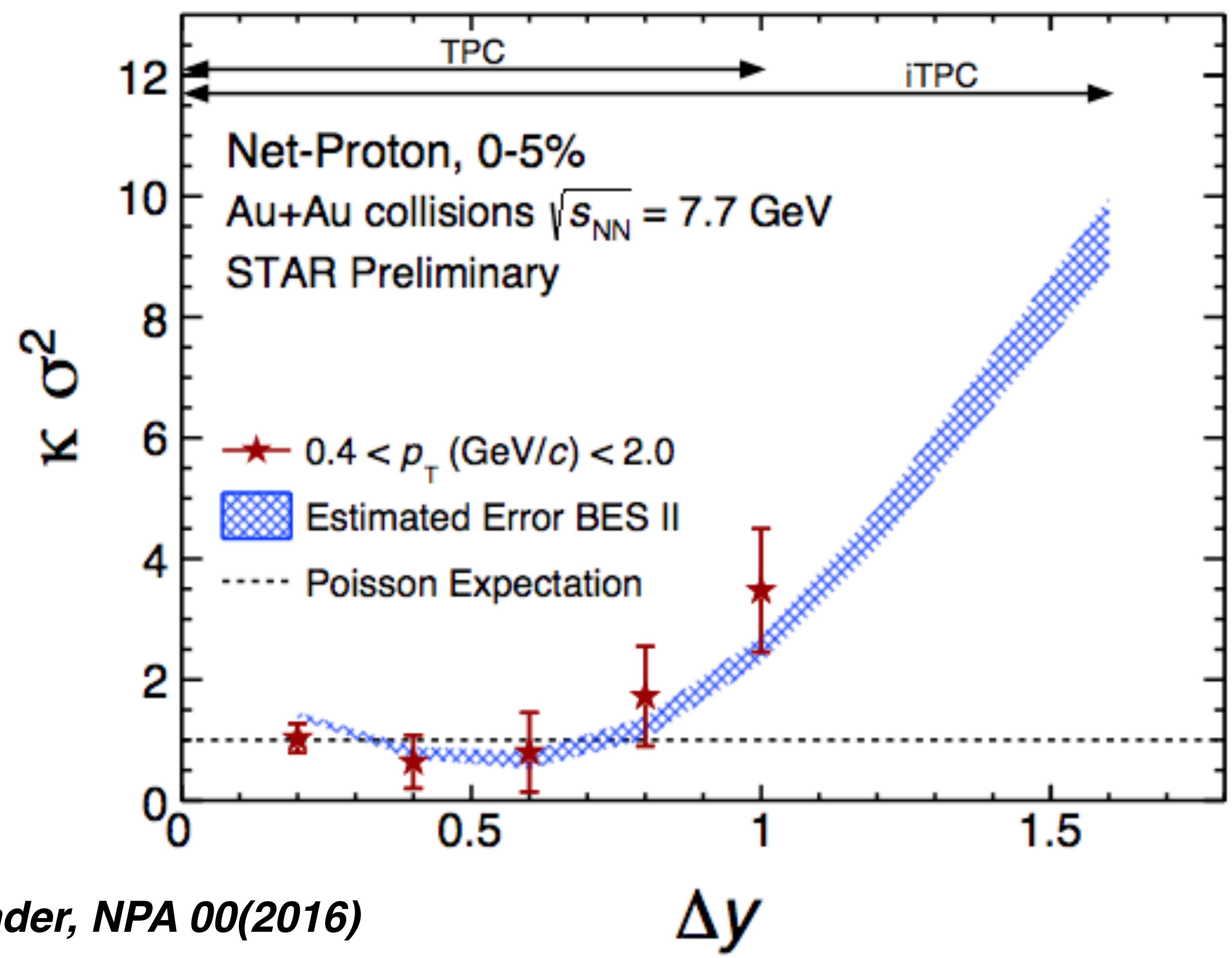
Centrality dependence

X. Luo (STAR collaboration) arXiv:1503.02558v2



Beam Energy Scan PhaseII

- ✓ Some detector upgrades will be done by BESII. Pseudo-rapidity coverage will be extended from 1.0 to 1.5.
- ✓ Higher order fluctuation measurement with small errors and large signals.



J. Thader, NPA 00(2016)

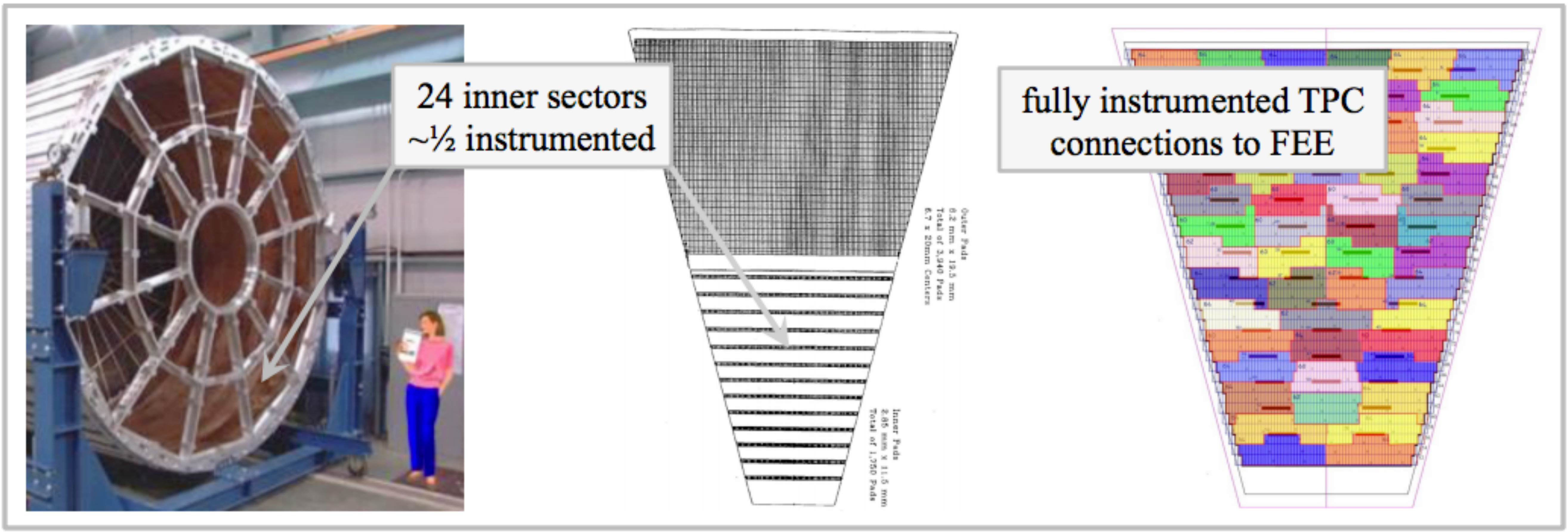
iTPC upgrade

iTPC – fully instrument the inner padrows of the Time Projection Chamber

will extend pseudorapidity acceptance from $|\eta|<1$ to $|\eta|<1.5$

& transverse momentum acceptance from $P_T>125$ MeV to $P_T>60$ MeV/c

while improving track dE/dx resolution from 7.5% to 6.2%



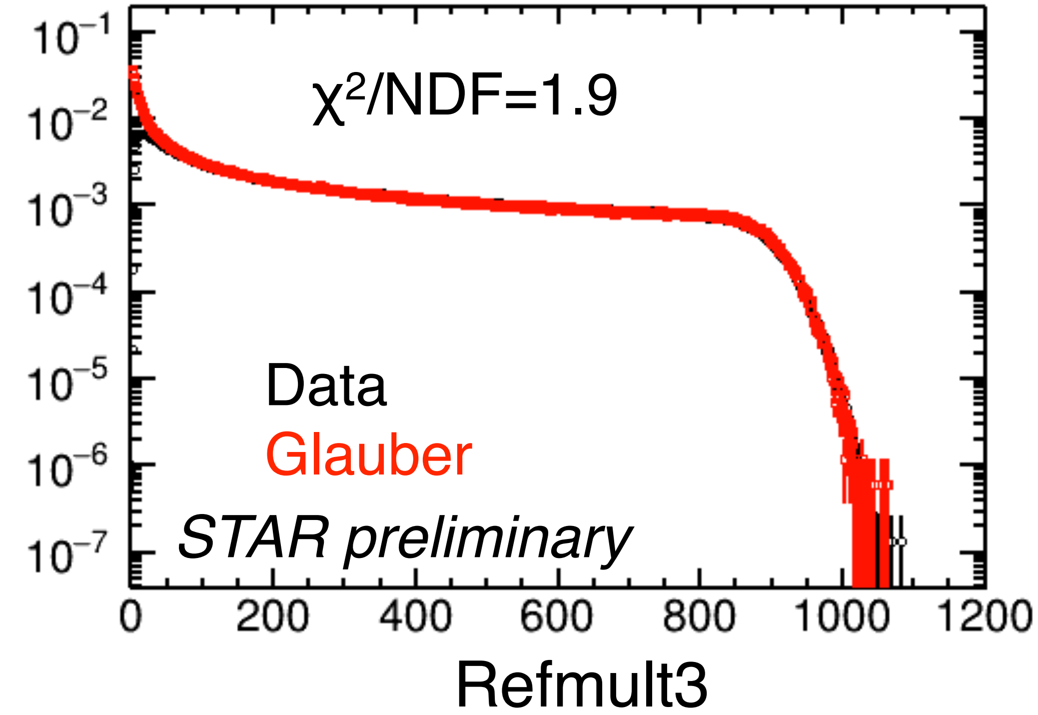
From the talk by W. J. Llope at AGS/RHIC Annual User's Meeting, BNL, June 7, 2016

Analysis technique

1. Centrality determination

Use charged particles except protons in order to avoid the auto correlation.

Analysis : $|y|<0.5$, p and pbar
Centrality : $|\eta|<1.0$, exclude p and pbar



2. Centrality Bin Width Correction

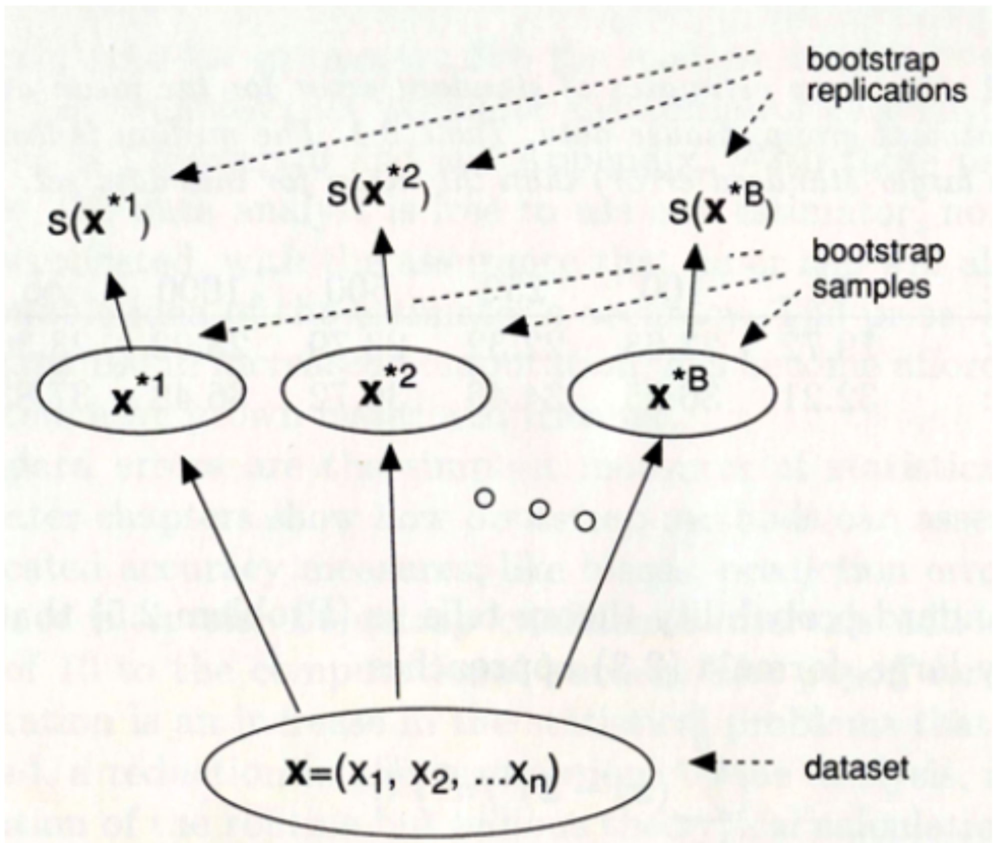
Calculate cumulants at each multiplicity bin in order to suppress the volume fluctuation.

X.Luo et al. J. Phys.G40,105104(2013)

3. Statistical error calculation

✓ **Bootstrap**

✓ Delta theorem



B. Efron,R. Tibshirani, An introduction to the bootstrap, Chapman & Hall (1993).

Efficiency correction

✓ **Based on the assumption of binomial efficiency.**

$$p(n_1,n_2) = \sum_{N_1=n_1}^{\infty}\sum_{N_2=n_2}^{\infty} P(N_1,N_2)\frac{N_1!}{n_1!(N_1-n_1)!}p_1^{n_1}(1-p_1)^{N_1-n_1} \times \frac{N_2!}{n_2!(N_2-n_2)!}p_2^{n_2}(1-p_2)^{N_2-n_2}.$$

A.Bzdak and V. Koch PRC.86.044904

M.Kitazawa PRC.86.024904

✓ **Simple relationship between measured and true factorial moments.**

$$f_{ik} = p_1^i \cdot p_2^k \cdot F_{ik}.$$

$$F_{ik} \equiv \left\langle \frac{N_1!}{(N_1-i)!}\frac{N_2!}{(N_2-k)!}\right\rangle = \sum_{N_1=i}^{\infty}\sum_{N_2=k}^{\infty} P(N_1,N_2)\frac{N_1!}{(N_1-i)!}\frac{N_2!}{(N_2-k)!},$$

$$f_{ik} \equiv \left\langle \frac{n_1!}{(n_1-i)!}\frac{n_2!}{(n_2-k)!}\right\rangle = \sum_{n_1=i}^{\infty}\sum_{n_2=k}^{\infty} p(n_1,n_2)\frac{n_1!}{(n_1-i)!}\frac{n_2!}{(n_2-k)!}.$$

✓ **It can be extended to the case of multi-number of phase spaces.**

$$\begin{aligned}
F_{r_1,r_2}(N_p,N_{\bar{p}}) &= F_{r_1,r_2}(N_{p_1}+N_{p_2},N_{\bar{p}_1}+N_{\bar{p}_2}) \\
&= \sum_{i_1=0}^{r_1}\sum_{i_2=0}^{r_2} s_1(r_1,i_1)s_1(r_2,i_2)\langle (N_{p_1}+N_{p_2})^{i_1}(N_{\bar{p}_1}+N_{\bar{p}_2})^{i_2}\rangle \\
&= \sum_{i_1=0}^{r_1}\sum_{i_2=0}^{r_2} s_1(r_1,i_1)s_1(r_2,i_2)\left\langle \sum_{s=0}^{i_1}\binom{i_1}{s} N_{p_1}^{i_1-s}N_{p_2}^{s}\sum_{t=0}^{i_2}\binom{i_2}{t} N_{\bar{p}_1}^{i_2-t}N_{\bar{p}_2}^{t}\right\rangle \\
&= \sum_{i_1=0}^{r_1}\sum_{i_2=0}^{r_2}\sum_{s=0}^{i_1}\sum_{t=0}^{i_2}\sum_{u=0}^{i_1-s}\sum_{v=0}^{s}\sum_{j=0}^{i_2-t}\sum_{k=0}^{t} s_1(r_1,i_1)s_1(r_2,i_2)\binom{i_1}{s}\binom{i_2}{t} \\
&\quad \times s_2(i_1-s,u)s_2(s,v)s_2(i_2-t,j)s_2(t,k) \times F_{u,v,j,k}(N_{p_1},N_{p_2},N_{\bar{p}_1},N_{\bar{p}_2}).
\end{aligned}$$

A.Bzdak and V. Koch PRC.91.027901

X. Luo PRC.91.034907

Baryon # vs Proton

✦ **Net-proton cumulants can be corrected to the net-baryon cumulants by assuming the binomial distribution function.**

$$\boxed{N_{\mathrm{B}} \to N_p}$$

$$\begin{aligned}
\langle(\delta N_p^{(\mathrm{net})})^3\rangle =& \frac{1}{8}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^3\rangle + \frac{3}{8}\langle\delta N_{\mathrm{B}}^{(\mathrm{net})}\delta N_{\mathrm{B}}^{(\mathrm{tot})}\rangle, \\
\langle(\delta N_p^{(\mathrm{net})})^4\rangle_c =& \frac{1}{16}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^4\rangle_c + \frac{3}{8}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^2\delta N_{\mathrm{B}}^{(\mathrm{tot})}\rangle \\
&+ \frac{3}{16}\langle(\delta N_{\mathrm{B}}^{(\mathrm{tot})})^2\rangle - \frac{1}{8}\langle N_{\mathrm{B}}^{(\mathrm{tot})}\rangle,
\end{aligned}$$

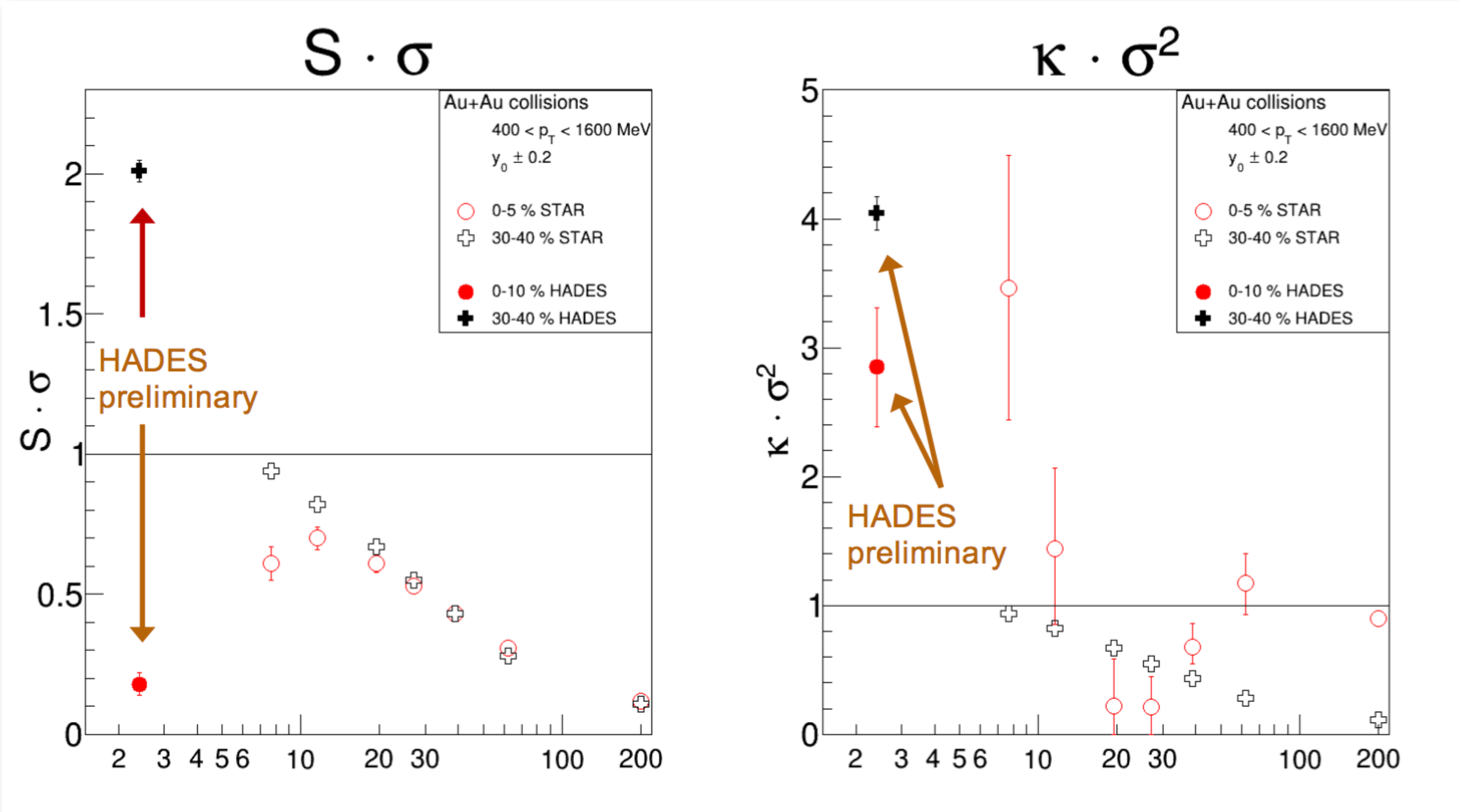
$$\boxed{N_p \to N_{\mathrm{B}}}$$

$$\begin{aligned}
\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^3\rangle =& 8\langle(\delta N_p^{(\mathrm{net})})^3\rangle - 12\langle\delta N_p^{(\mathrm{net})}\delta N_p^{(\mathrm{tot})}\rangle \\
&+ 6\langle N_p^{(\mathrm{net})}\rangle, \\
\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^4\rangle_c =& 16\langle(\delta N_p^{(\mathrm{net})})^4\rangle_c - 48\langle(\delta N_p^{(\mathrm{net})})^2\delta N_p^{(\mathrm{tot})}\rangle \\
&+ 48\langle(\delta N_p^{(\mathrm{net})})^2\rangle + 12\langle(\delta N_p^{(\mathrm{tot})})^2\rangle - 26\langle N_p^{(\mathrm{tot})}\rangle,
\end{aligned}$$

M. Kitazawa, M. Asakawa
PRC. 86. 024904

HADES results

HADES preliminary STAR analysis: X. Luo et al., PoS (CPOD2014) 01!



STAR: $y_0 \pm 0.5$ HADES: $y_0 \pm 0.2$

Additional effects to consider? fragments, stopping flucts., centrality resolution, etc...

R. Holzmann, Strangeness in Quark Matter 2016, @Barkley