Measurement of sixth order cumulant of net-proton multiplicity distribution in Au+Au collisions at $\sqrt{s_{NN}}$ = 200 GeV with the STAR detector

CiRfSE workshop
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Toshihiro Nonaka

STAR

CiRfSE

Motivation

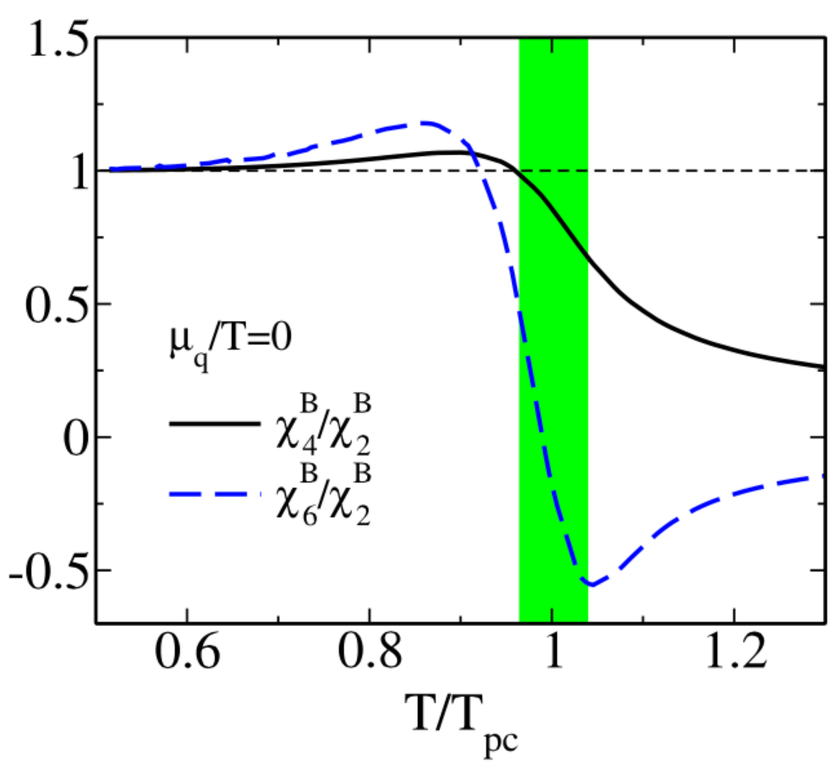
- ✓ **Lattice calculations predict a "smooth crossover" at μ_B=0.**

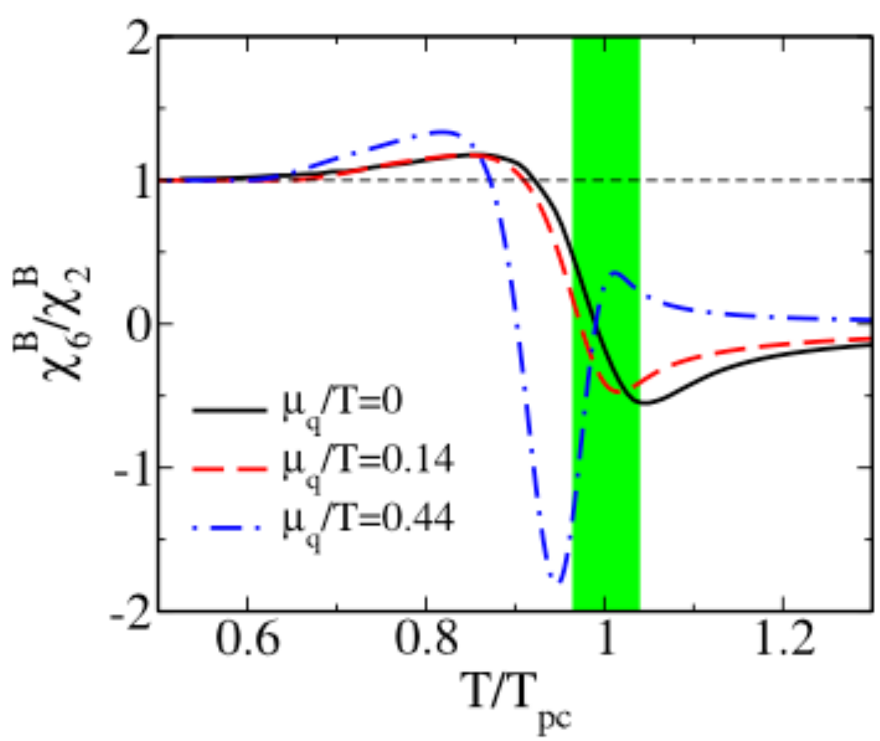
  Y. Aoki, Nature 443, 675(2006)

- ✓ **Theoretically sixth order cumulant of net-baryon and net-charge fluctuation change sign near the chiral phase transition.**

  Friman et al, Eur. Phys. J. C (2011) 71:1694

- ✓ **Find the evidence for crossover with measurement of the sixth order cumulant of net-proton multiplicity distribution.**





Friman et al, Eur. Phys. J. C (2011) 71:1694

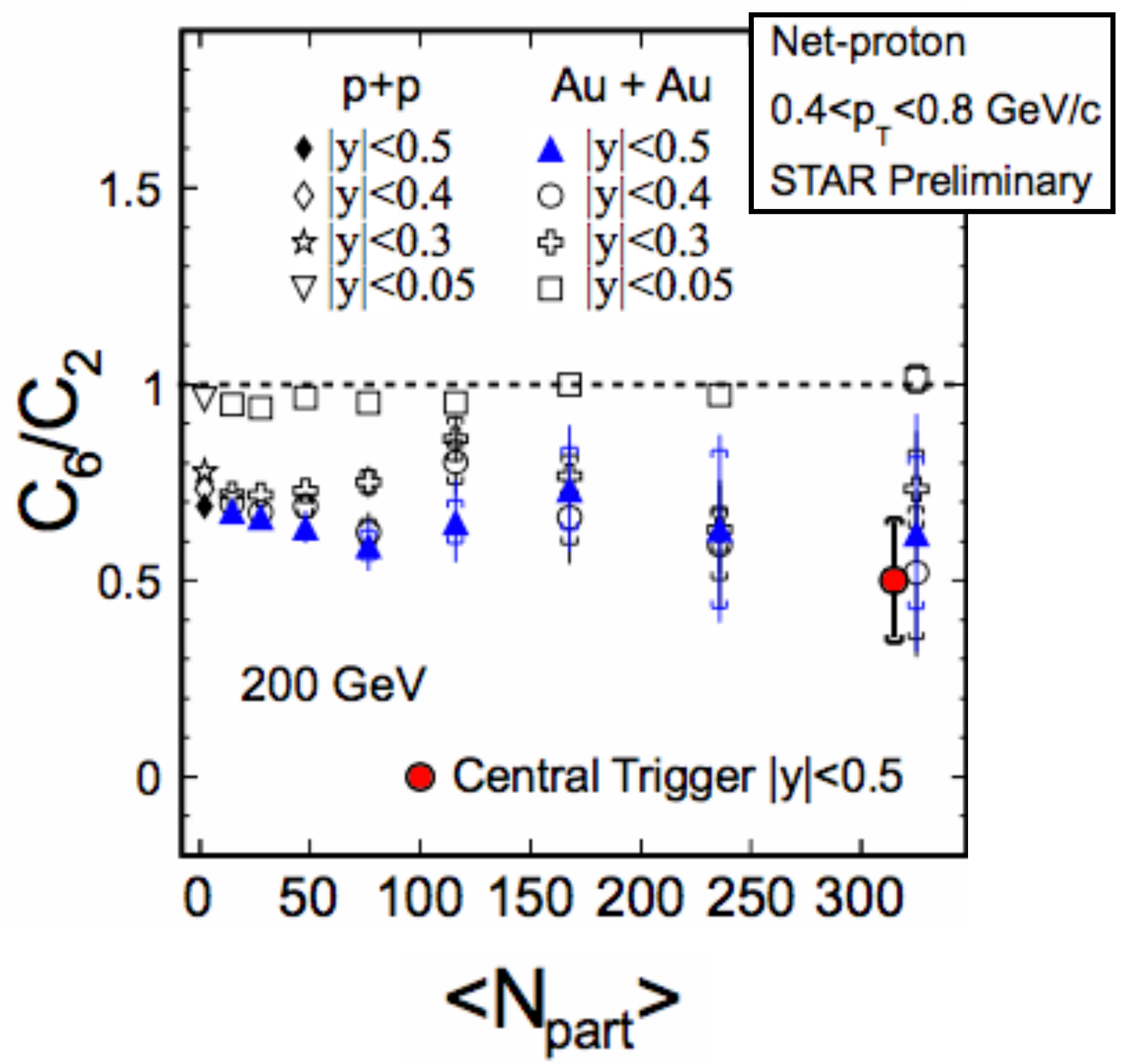
✦ **Can we observe the negative value?**

$$\frac{C_6}{C_2} = \frac{\chi_6}{\chi_2}$$

STAR results

- ✓ The STAR experiment measured C_6/C_2 at $0.4<p_T<0.8$ GeV/c without efficiency correction at $\sqrt{s_{NN}}$=200GeV of Run10 datasets (~240M events).

  L. Chen (STAR collabration), NPA 904-905(2013)
- ✓ Event statistics are very important for higher orders.
- ✓ We focus on $\sqrt{s_{NN}}$=200GeV at Run10 and Run11 datasets which have ~500M and ~250M events, and measure C_6/C_2 at $0.4<p_T<2.0$ GeV/c with efficiency correction.



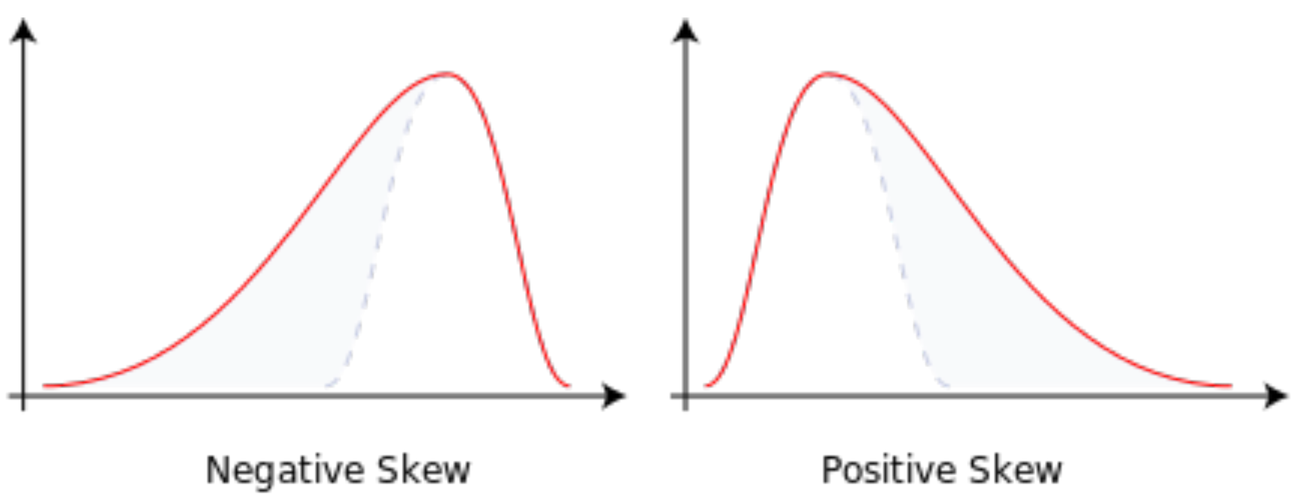
→ **Run10, $\sqrt{s_{NN}}$=200GeV, eff.uncorr**

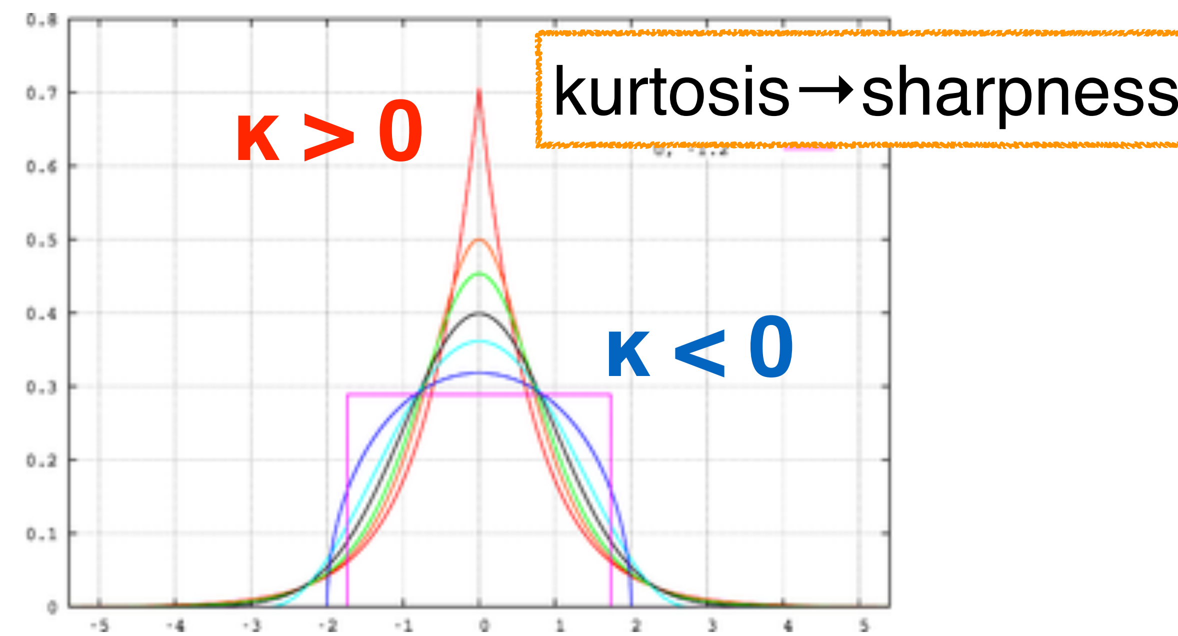
L. Chen (STAR collabration), NPA 904-905(2013)

Higher order fluctuation

- **Moments and Cumulants are mathematical measures of “shape” of a histogram which probe the fluctuation of observables.**
  - ✓ **Moments : Mean(*M*), sigma(*σ*), skewness(*S*) and kurtosis(κ).**
  - ✓ ***S and κ are non-gaussian* fluctuations.**

skewness→asymmetry





from wikipedia

✓ **Cumulant ⇄ Moment**

$$< \delta N >= N - < N >$$

$$C_1 = M =< N >$$

$$C_2 = \sigma^2 =< (\delta N)^2 >$$

$$C_3 = S\sigma^3 =< (\delta N)^3 >$$

$$C_4 = \kappa\sigma^4 =< (\delta N)^4 > -3 < (\delta N)^2 >^2$$

✓ **Cumulant : additivity**

$$C_n(X+Y) = C_n(X) + C_n(Y)$$

➡ **Volume dependence**

Fluctuations of conserved quantities

✦ Net-baryon, net-charge and net-strangeness

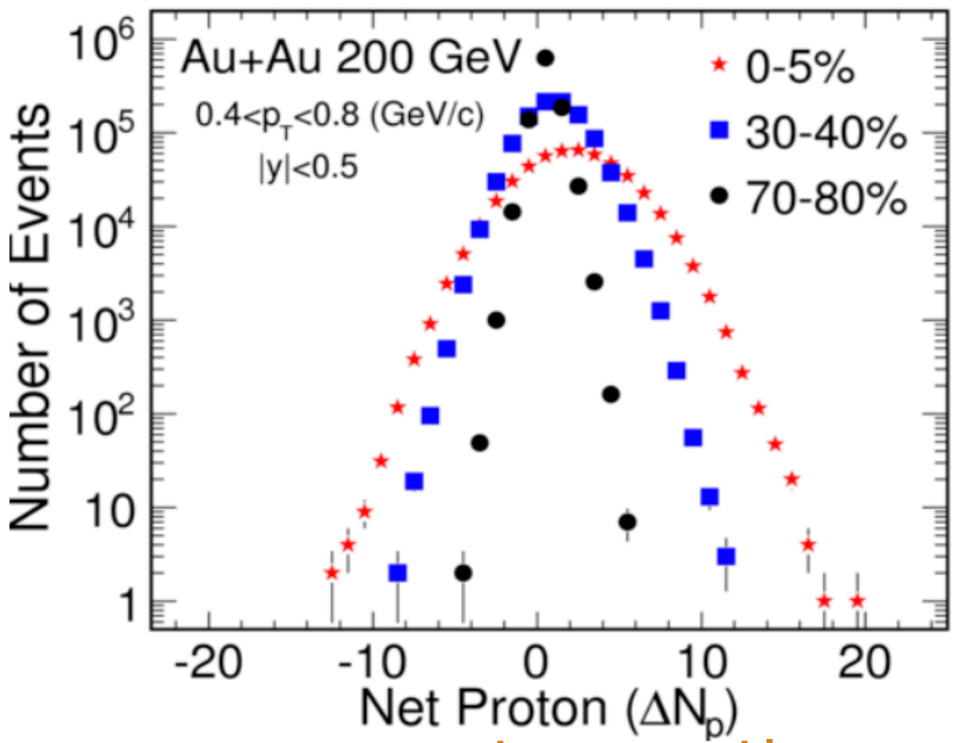
"Net" : positive - negative

$$\Delta N_q = N_q - N_{\bar{q}}, \quad q = B, Q, S$$

No. of "positively charged" particles in one collision

No. of "negatively charged" particles in one collision

Fill in histograms over many collisions



→neutrons cannot be measured

(1) Sensitive to correlation length

$$C_2 = <(\delta N)^2>_c \approx \xi^2$$

$$C_3 = <(\delta N)^3>_c \approx \xi^{4.5}$$

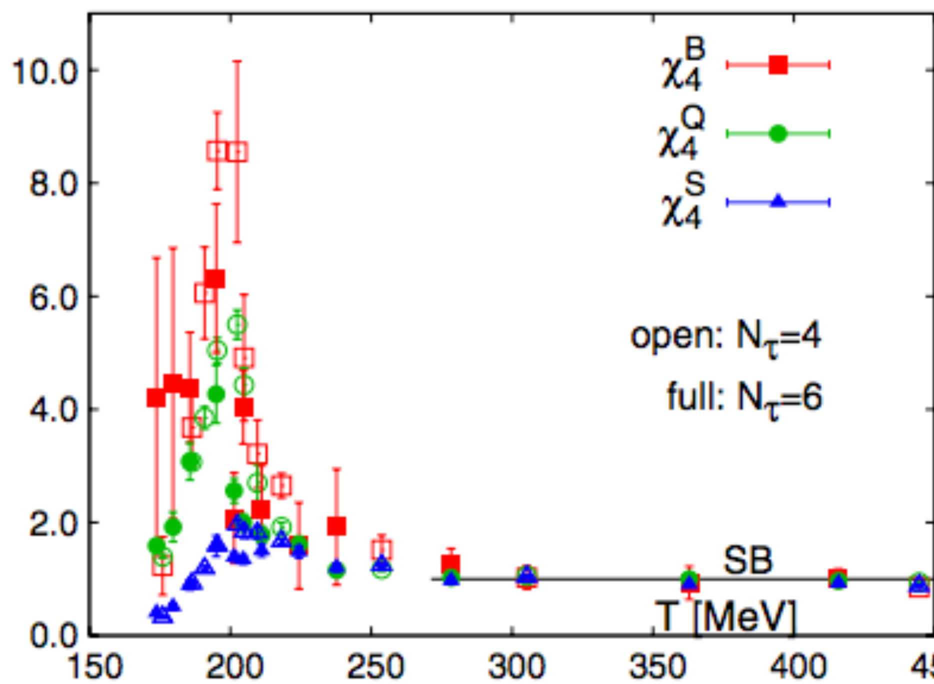
$$C_4 = <(\delta N)^4>_c \approx \xi^7$$

(2) Direct comparison with susceptibilities.

M. Cheng et al, PRD 79, 074505 (2009)

$$S\sigma = \frac{C_3}{C_2} = \frac{\chi_3}{\chi_2} \qquad \kappa\sigma^2 = \frac{C_4}{C_2} = \frac{\chi_4}{\chi_2}$$

$$\chi_n^q = \frac{1}{VT^3} \times C_n^q = \frac{\partial^n p/T^4}{\partial \mu_q^n}, \quad q = B, Q, S$$



Volume **dependence can be canceled by taking ratio.**

Event and track selection

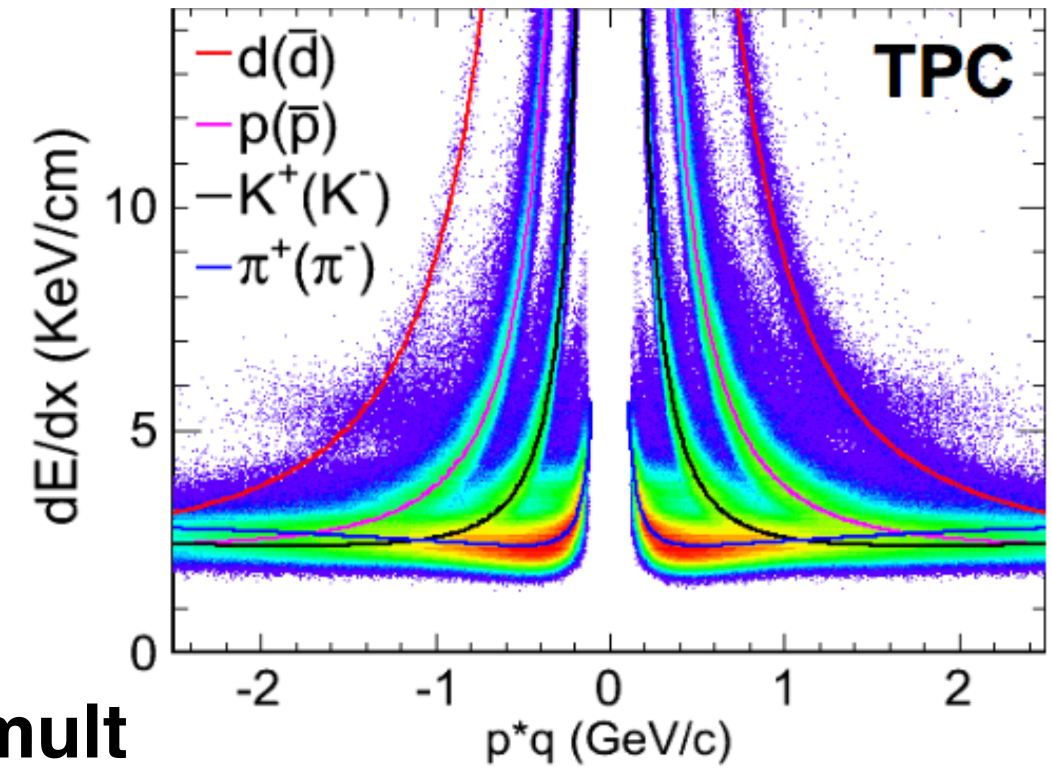
✓ Dataset

Au+Au, $\sqrt{s_{NN}}$=200GeV, mb trigger, Run10 and Run11

✓ Event selection

|Vz|<30cm, |Vr|<2cm, |VpdVz-Vz|<3cm

Pileup rejection from tofmatched vs refmult



✓ Track selection

DCA<1cm, nHitsFit>20,

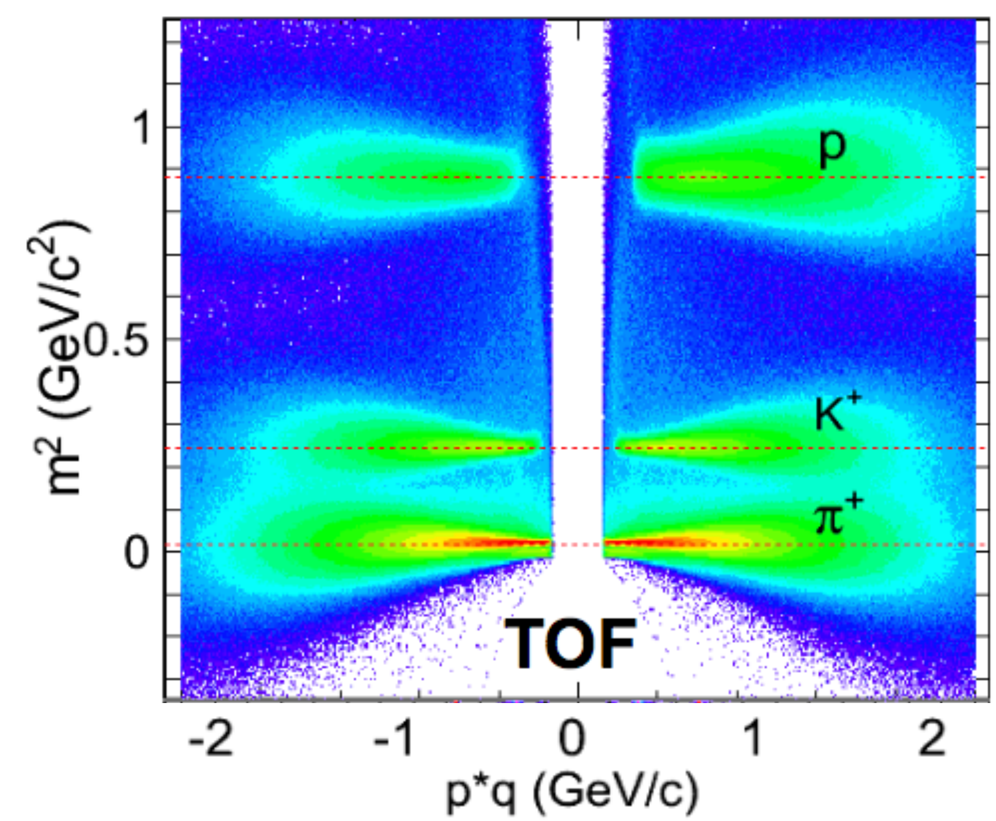
nHitsFit/nFitPoss>0.52, nHitsDedx>5,

|y|<0.5

<u>PID cut</u>

$0.4<p_T<0.8$: $|n\sigma_{proton}|<2$

$0.8<p_T<2.0$: $|n\sigma_{proton}|<2$ && $0.6<m^2<1.2$



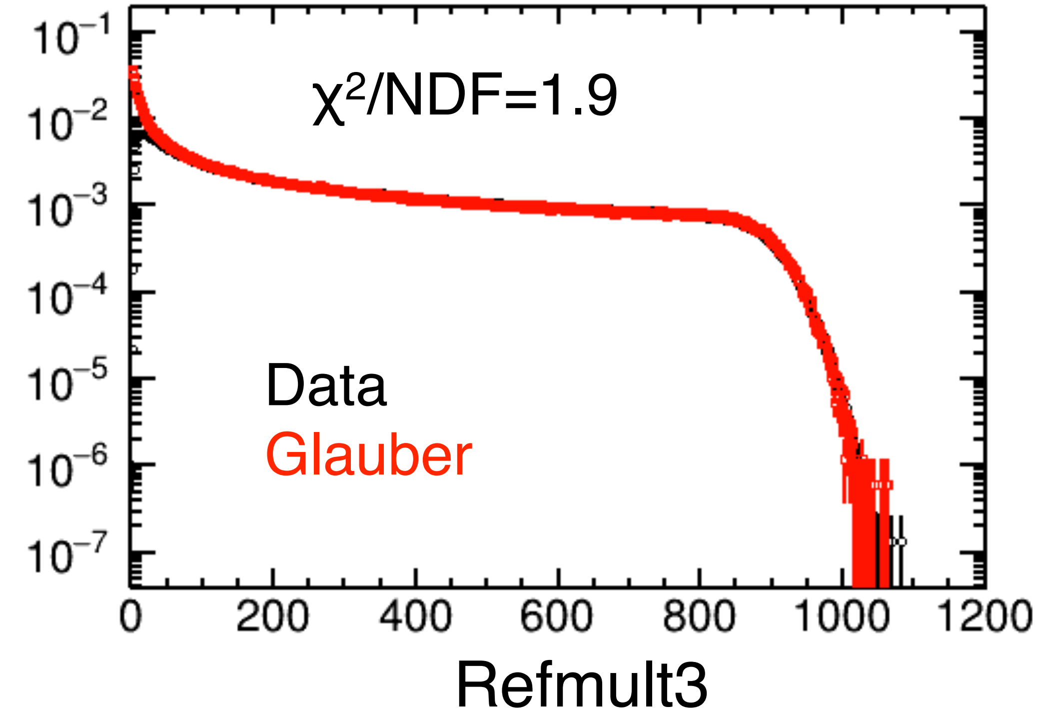
Analysis techniques

1. Centrality determination

Use charged particles except protons in order to avoid the auto correlation.

Analysis : |y|<0.5, p and pbar

Centrality : |η|<1.0, exclude p and pbar



2. Centrality Bin Width Correction

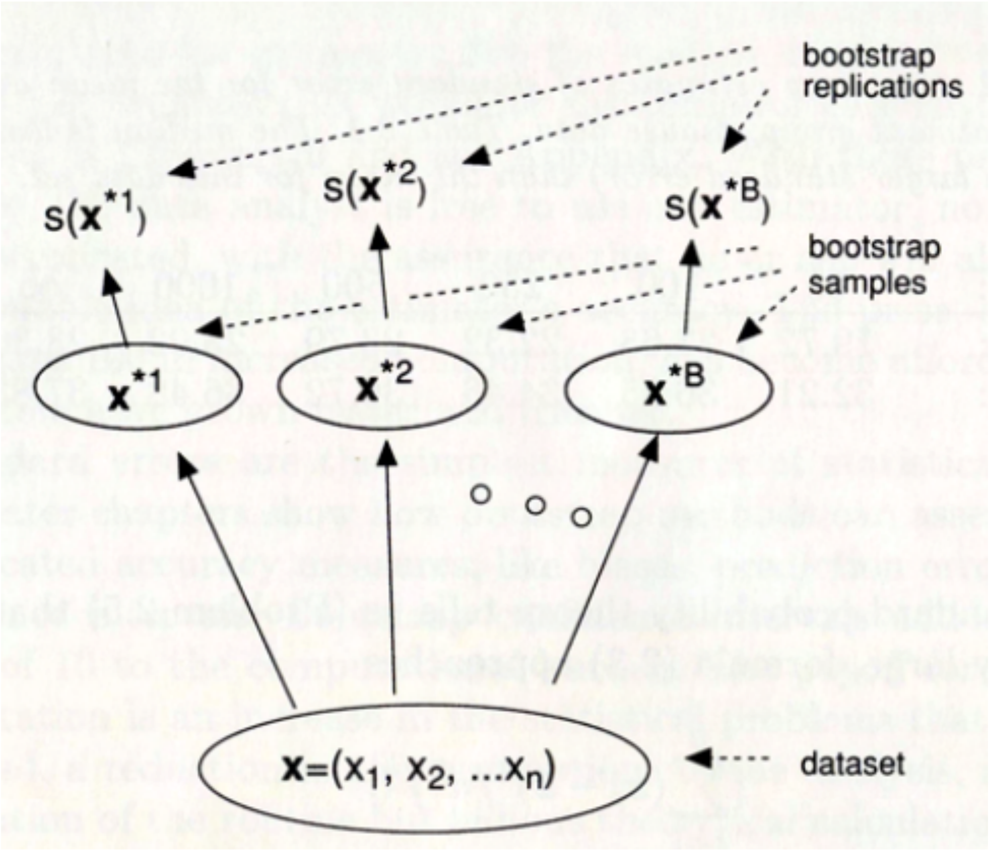
Calculate cumulants at each multiplicity bin in order to suppress the volume fluctuation.

X.Luo et al. J. Phys.G40,105104(2013)

3. Statistical error calculation : Bootstrap

- ✓ **Bootstrap**
- ✓ Delta theorem

4. Efficiency correction



B. Efron,R. Tibshirani, An introduction to the bootstrap, Chapman & Hall (1993).

Efficiency correction

✓ **Based on the assumption of binomial efficiency.**

$$p(n_1,n_2) = \sum_{N_1=n_1}^{\infty} \sum_{N_2=n_2}^{\infty} P(N_1,N_2) \frac{N_1!}{n_1!(N_1-n_1)!} p_1^{n_1}(1-p_1)^{N_1-n_1} \times \frac{N_2!}{n_2!(N_2-n_2)!} p_2^{n_2}(1-p_2)^{N_2-n_2}.$$

A.Bzdak and V. Koch PRC.86.044904

M.Kitazawa PRC.86.024904

✓ **Simple relationship between measured and true factorial moments.**

$$f_{ik} = p_1^i \cdot p_2^k \cdot F_{ik}.$$

$$F_{ik} \equiv \left\langle \frac{N_1!}{(N_1-i)!} \frac{N_2!}{(N_2-k)!} \right\rangle = \sum_{N_1=i}^{\infty} \sum_{N_2=k}^{\infty} P(N_1,N_2) \frac{N_1!}{(N_1-i)!} \frac{N_2!}{(N_2-k)!},$$

$$f_{ik} \equiv \left\langle \frac{n_1!}{(n_1-i)!} \frac{n_2!}{(n_2-k)!} \right\rangle = \sum_{n_1=i}^{\infty} \sum_{n_2=k}^{\infty} p(n_1,n_2) \frac{n_1!}{(n_1-i)!} \frac{n_2!}{(n_2-k)!}.$$

✓ **It can be extended to the case of multi-number of phase spaces.**

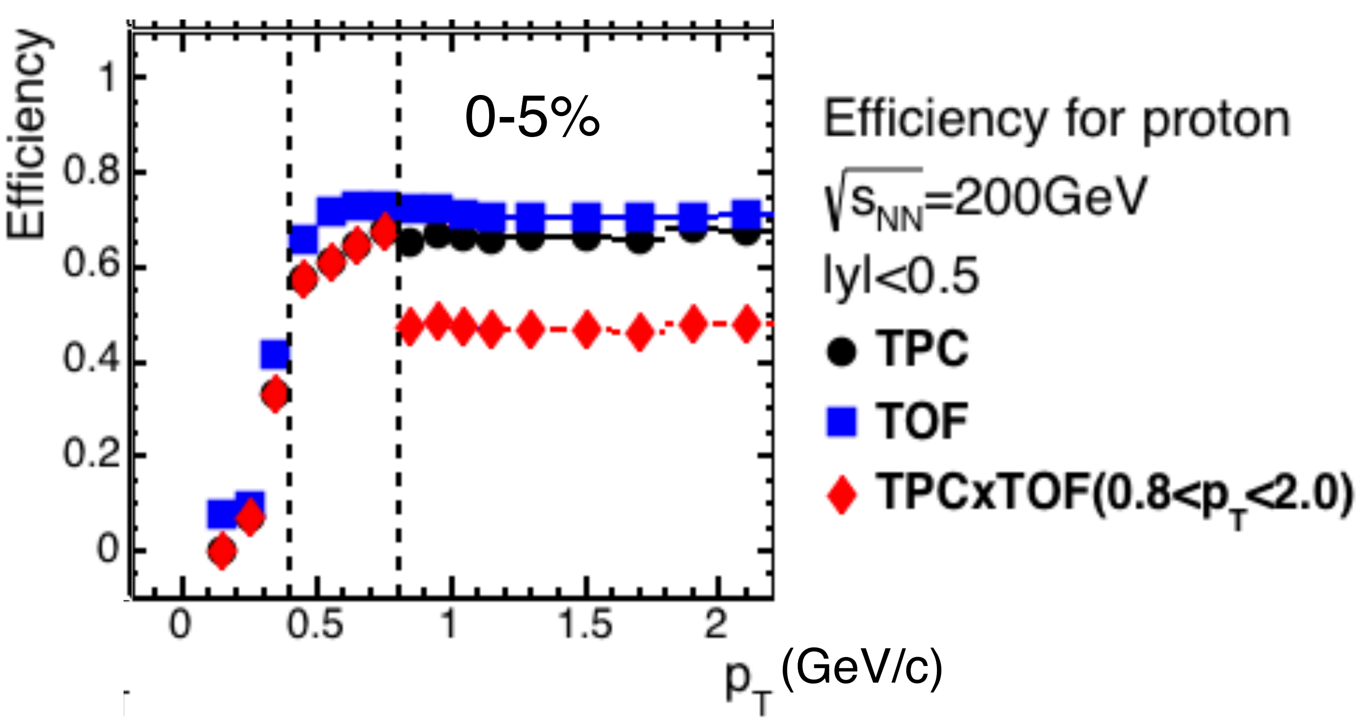
A.Bzdak and V. Koch PRC.91.027901

X. Luo PRC.91.034907

$$\begin{aligned}
F_{r_1,r_2}(N_p,N_{\bar{p}}) &= F_{r_1,r_2}(N_{p_1}+N_{p_2},N_{\bar{p}_1}+N_{\bar{p}_2}) \\
&= \sum_{i_1=0}^{r_1}\sum_{i_2=0}^{r_2} s_1(r_1,i_1)s_1(r_2,i_2)\langle (N_{p_1}+N_{p_2})^{i_1}(N_{\bar{p}_1}+N_{\bar{p}_2})^{i_2}\rangle \\
&= \sum_{i_1=0}^{r_1}\sum_{i_2=0}^{r_2} s_1(r_1,i_1)s_1(r_2,i_2)\left\langle \sum_{s=0}^{i_1}\binom{i_1}{s} N_{p_1}^{i_1-s}N_{p_2}^{s} \sum_{t=0}^{i_2}\binom{i_2}{t} N_{\bar{p}_1}^{i_2-t}N_{\bar{p}_2}^{t}\right\rangle \\
&= \sum_{i_1=0}^{r_1}\sum_{i_2=0}^{r_2}\sum_{s=0}^{i_1}\sum_{t=0}^{i_2}\sum_{u=0}^{i_1-s}\sum_{v=0}^{s}\sum_{j=0}^{i_2-t}\sum_{k=0}^{t} s_1(r_1,i_1)s_1(r_2,i_2)\binom{i_1}{s}\binom{i_2}{t} \\
&\quad \times s_2(i_1-s,u)s_2(s,v)s_2(i_2-t,j)s_2(t,k) \times F_{u,v,j,k}(N_{p_1},N_{p_2},N_{\bar{p}_1},N_{\bar{p}_2}).
\end{aligned}$$

Calculation cost

- ✓ **Efficiency drops at $p_T = 0.8$ Gev/c region where TOF is included in proton identification.**
- ✓ **The number of efficiency bin is four. (p and pbar, low and high pT)**
- ✓ **Calculation cost become ~4 times larger than fourth order.**



- **# of terms to be calculated per event for efficiency correction**
- **increase by power of efficiency bins**

	C_4	C_6
2+2	225	784
4+4	4900	44100

New development

- ✦M.Kitazawa and I developed a new correction formula via factorial cumulants, which can drastically reduce the calculation time.
- ✦It has been checked that the formulas below give exactly the same value with Bzdak-Koch.
- ✦About ~2000 times faster than usual Bzdak-Koch formulas in the case of 8bins.

$$q_{(r,s)} \equiv \sum_{i}^{M} \frac{a_i^r}{p_i^s} n_i,$$ Loops just increase with efficiency bin

$$\begin{aligned}
\langle Q^6\rangle_c \;=\;& \langle q^6_{(1,1)}\rangle_c - 15\langle q^4_{(1,1)}q_{(2,2)}\rangle_c + 15\langle q^4_{(1,1)}q_{(2,1)}\rangle_c - 90\langle q^2_{(1,1)}q_{(2,2)}q_{(2,1)}\rangle_c\\
&+40\langle q^3_{(1,1)}q_{(3,3)}\rangle_c + 45\langle q^2_{(2,2)}q^2_{(1,1)}\rangle_c + 20\langle q^3_{(1,1)}q_{(3,1)}\rangle_c - 60\langle q^3_{(1,1)}q_{(3,2)}\rangle_c\\
&+45\langle q^2_{(1,1)}q^2_{(2,1)}\rangle_c - 90\langle q^2_{(1,1)}q_{(4,4)}\rangle_c - 120\langle q_{(3,3)}q_{(2,2)}q_{(1,1)}\rangle_c - 15\langle q^3_{(2,2)}\rangle_c\\
&+15\langle q^2_{(1,1)}q_{(4,1)}\rangle_c - 60\langle q_{(1,1)}q_{(2,2)}q_{(3,1)}\rangle_c - 60\langle q^2_{(1,1)}q_{(4,2)}\rangle_c\\
&+120\langle q_{(1,1)}q_{(2,1)}q_{(3,3)}\rangle_c + 180\langle q^2_{(1,1)}q_{(4,3)}\rangle_c + 45\langle q^2_{(2,2)}q_{(2,1)}\rangle_c\\
&+180\langle q_{(1,1)}q_{(2,2)}q_{(3,2)}\rangle_c - 45\langle q^2_{(1,1)}q_{(4,2)}\rangle_c - 45\langle q^2_{(2,1)}q_{(2,2)}\rangle_c\\
&-180\langle q_{(1,1)}q_{(2,1)}q_{(3,2)}\rangle_c + 60\langle q_{(1,1)}q_{(2,1)}q_{(3,1)}\rangle_c + 15\langle q_{(2,1)}\rangle_c\\
&+144\langle q_{(5,5)}q_{(1,1)}\rangle_c + 90\langle q_{(4,4)}q_{(2,2)}\rangle_c + 40\langle q^2_{(3,3)}\rangle_c + 6\langle q_{(5,1)}q_{(1,1)}\rangle_c\\
&+15\langle q_{(4,1)}q_{(2,1)}\rangle_c + 10\langle q^2_{(3,1)}\rangle_c - 30\langle q_{(5,2)}q_{(1,1)}\rangle_c - 15\langle q_{(4,1)}q_{(2,2)}\rangle_c\\
&+300\langle q_{(5,3)}q_{(1,1)}\rangle_c + 40\langle q_{(3,3)}q_{(3,1)}\rangle_c + 60\langle q_{(4,2)}q_{(2,2)}\rangle_c - 360\langle q_{(5,4)}q_{(1,1)}\rangle_c\\
&-90\langle q_{(4,4)}q_{(2,1)}\rangle_c - 120\langle q_{(3,3)}q_{(3,2)}\rangle_c - 180\langle q_{(4,3)}q_{(2,2)}\rangle_c\\
&+180\langle q_{(4,3)}q_{(2,1)}\rangle_c + 45\langle q_{(4,2)}q_{(2,2)}\rangle_c + 90\langle q^2_{(3,2)}\rangle_c\\
&-60\langle q_{(3,2)}q_{(3,1)}\rangle_c - 60\langle q_{(4,2)}q_{(2,1)}\rangle_c - 60\langle q_{(5,2)}q_{(1,1)}\rangle_c - 45\langle q_{(4,2)}q_{(2,1)}\rangle_c\\
&-120\langle q_{(6,6)}\rangle_c + 360\langle q_{(6,5)}\rangle_c - 390\langle q_{(6,4)}\rangle_c + 180\langle q_{(6,3)}\rangle_c - 31\langle q_{(6,2)}\rangle_c + \langle q_{(6,1)}\rangle_c.
\end{aligned}$$

Nonaka, Kitazawa, Esumi : paper in preparation

Analytical calculation

- ✓ **Assume two distribution that have exactly the same shape (C_m) but different efficiencies.**
- ✓ **Efficiency correction using averaged efficiency does not give a correct value for any probability distributions other than Poisson.**

$$K_m = 2C_m + \Delta K_m \qquad \bar{p} = \frac{p_1+p_2}{2} \qquad \Delta p = p_1 - p_2$$

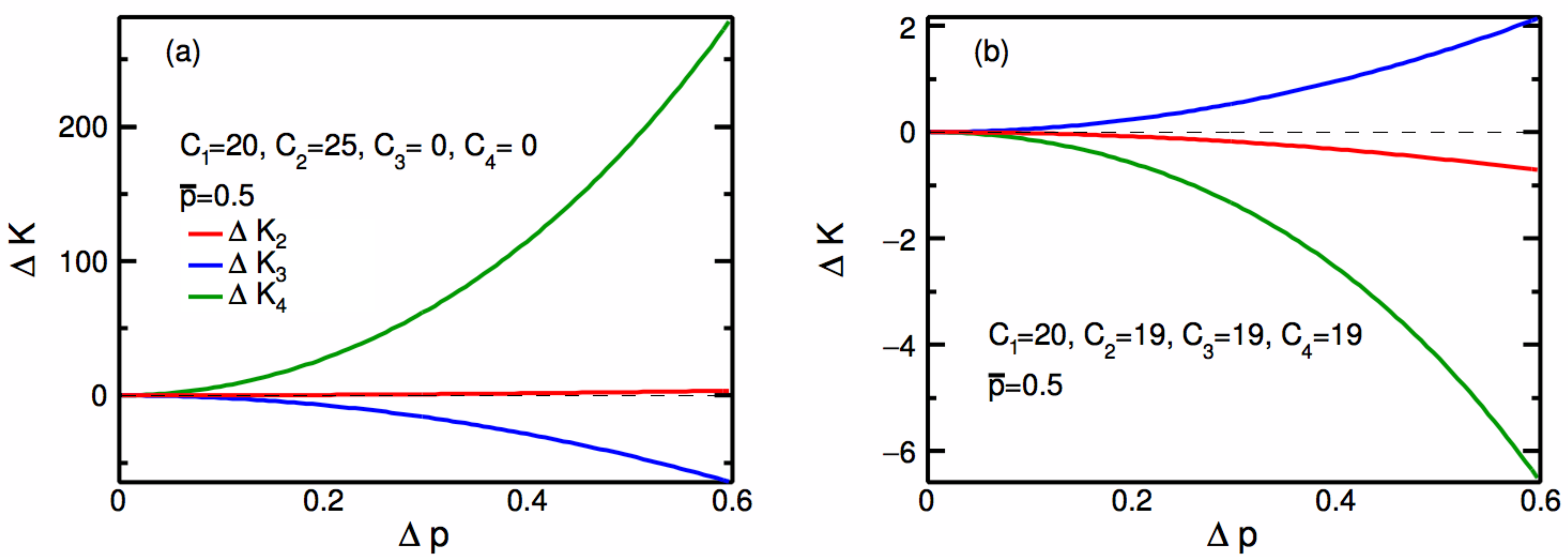
$$\Delta K_2 = \frac{1}{2}\left(\frac{\Delta p}{\bar{p}}\right)^2 (C_2 - C_1),$$

$$\Delta K_3 = \frac{3}{2}\left(\frac{\Delta p}{\bar{p}}\right)^2 (C_3 - 2C_2 + C_1),$$

$$\Delta K_4 = \frac{1}{2}\left(\frac{\Delta p}{\bar{p}}\right)^2 (6C_4 - 18C_3 + 19C_2 - 7C_1) + \frac{1}{8}\left(\frac{\Delta p}{\bar{p}}\right)^4 (C_4 - 6C_3 + 11C_2 - 6C_1),$$

Poisson : $C_1 = C_2 = C_3 \ldots = C_m$
→ $\Delta K_m = 0$

✓ **Gaussian (C_m=0, m>=3)** ✓ **Poisson — 5% deviation**



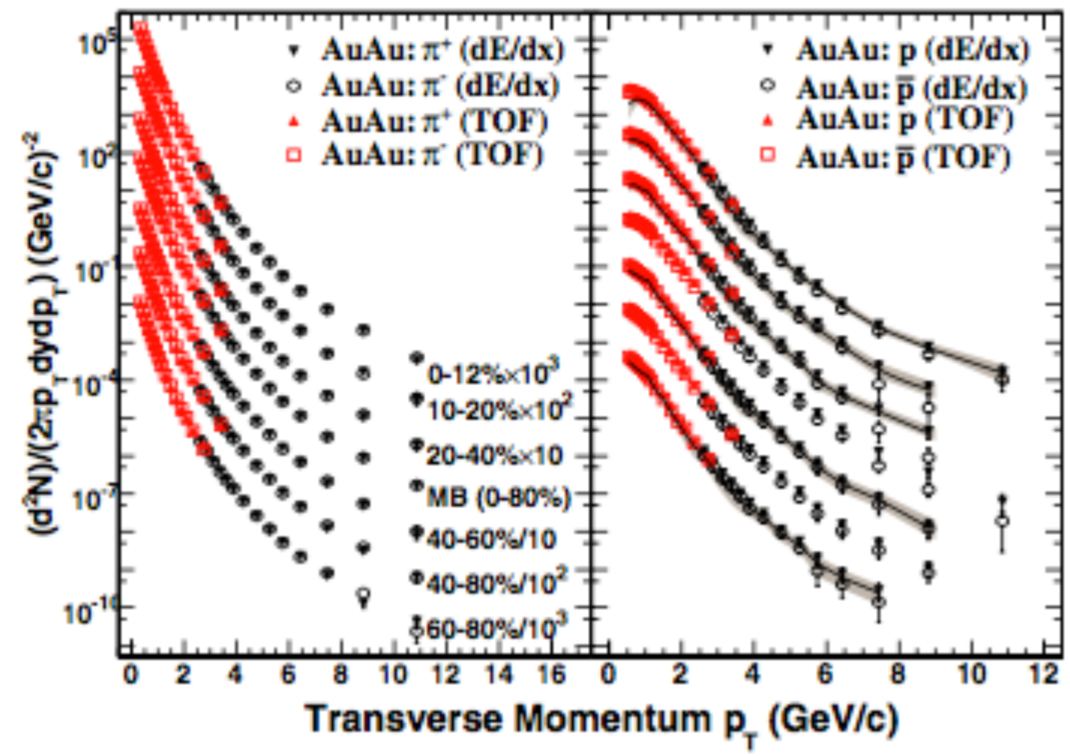
Nonaka, Kitazawa, Esumi : paper in preparation

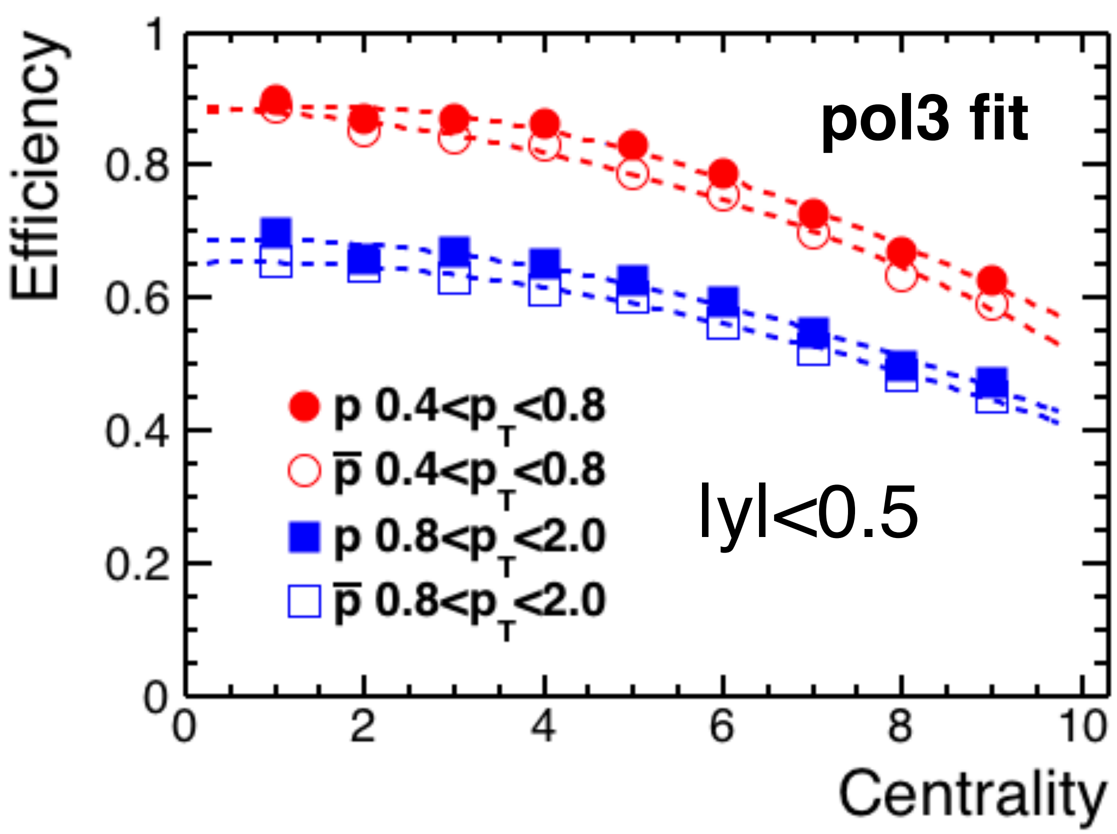
p_T integrated efficiency

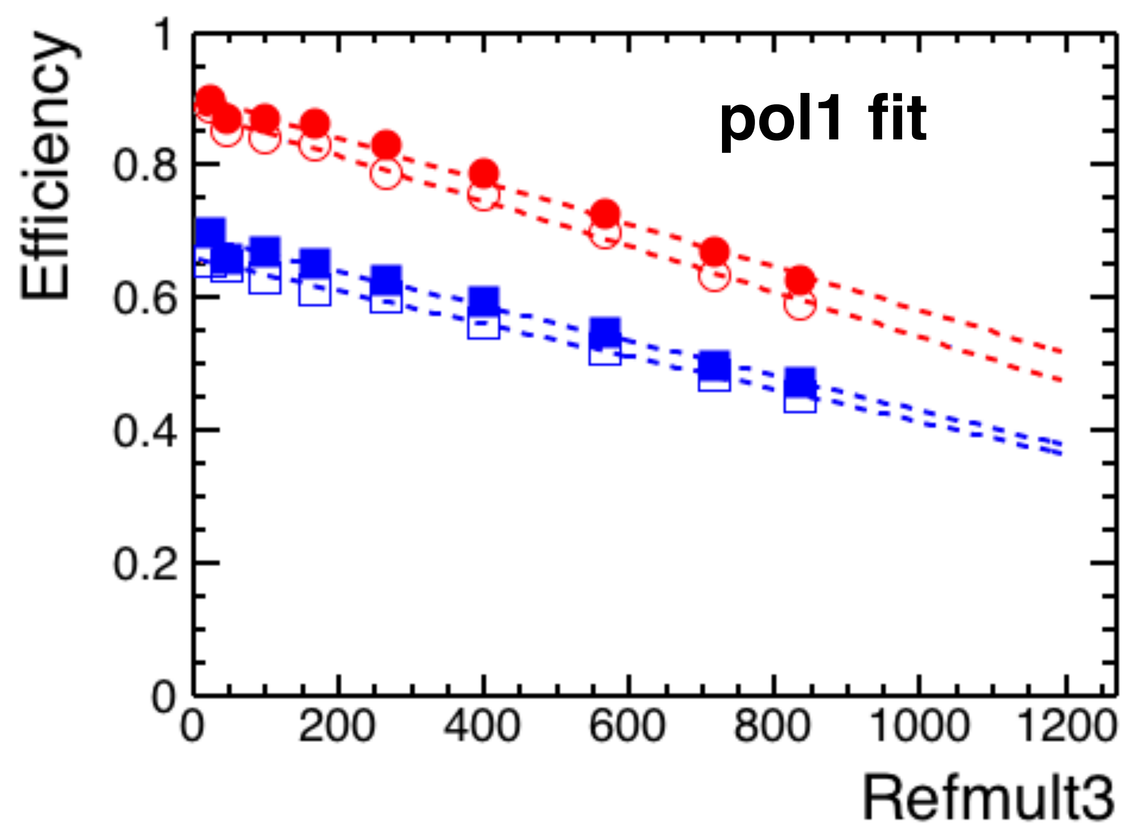
PRL. 97. 152301 (2006)

- Integrated using corrected p_T spectra.
- One can obtain efficiency vs centrality.
- Translate centrality into <refmult3>.

$$\varepsilon = \frac{\int \varepsilon(p_T) f(p_T) p_T dp_T}{\int f(p_T) p_T dp_T}$$

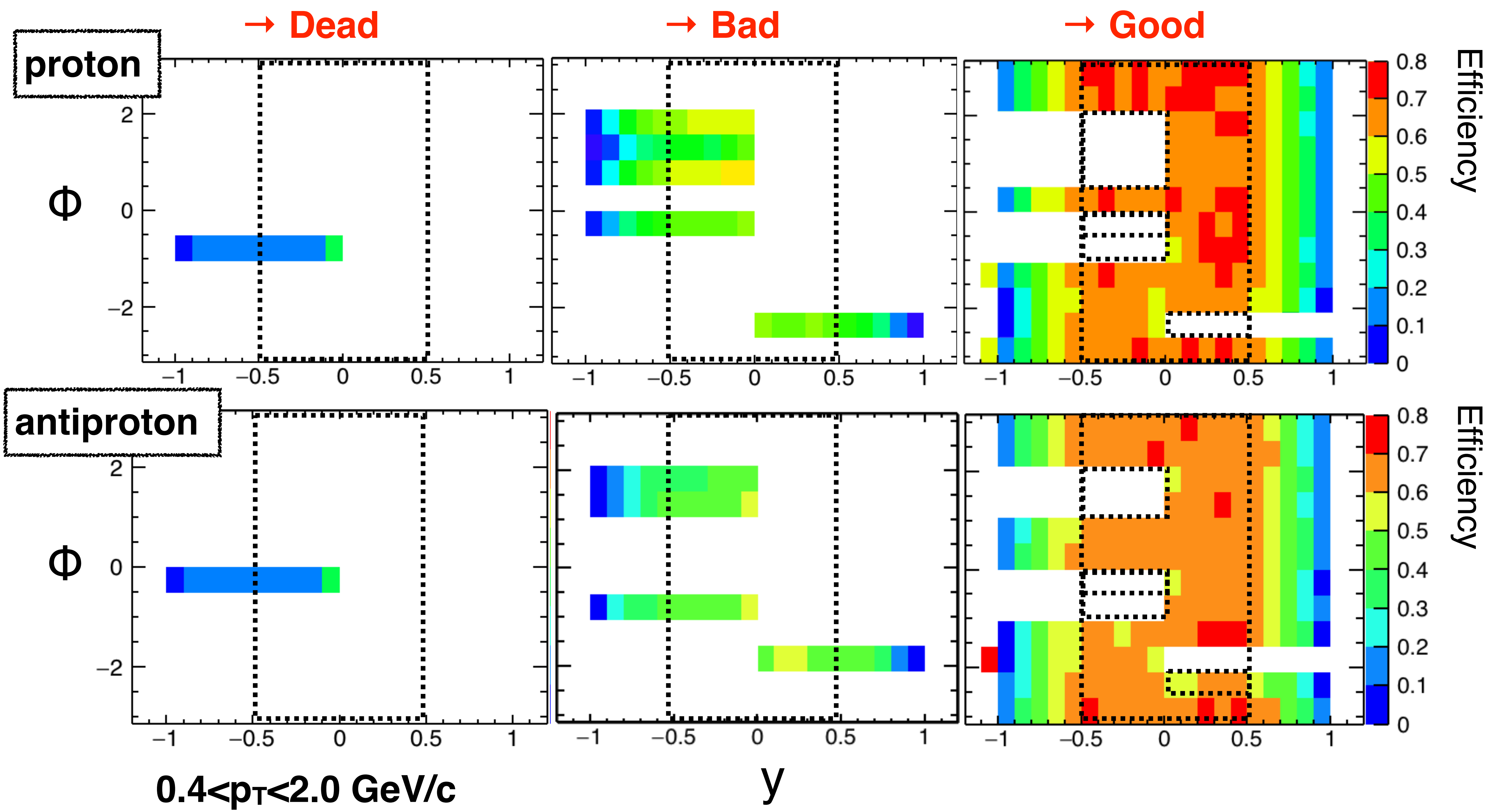






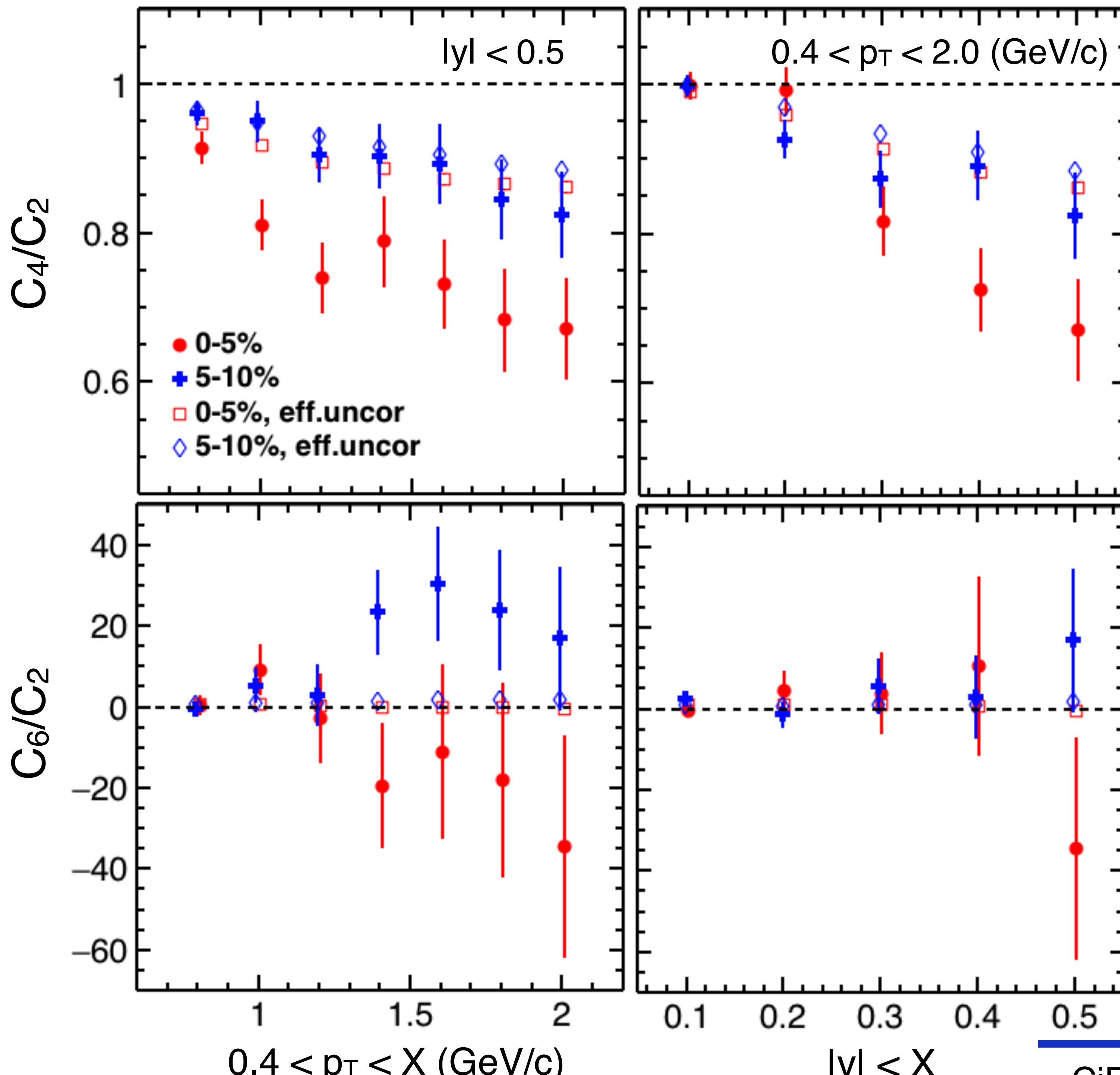
phi dependent efficiency

- TPC sectors are divided into 3 regions, dead, bad and good sectors.
- p_T integrated efficiencies are calculated for each case.



Acceptance dependence

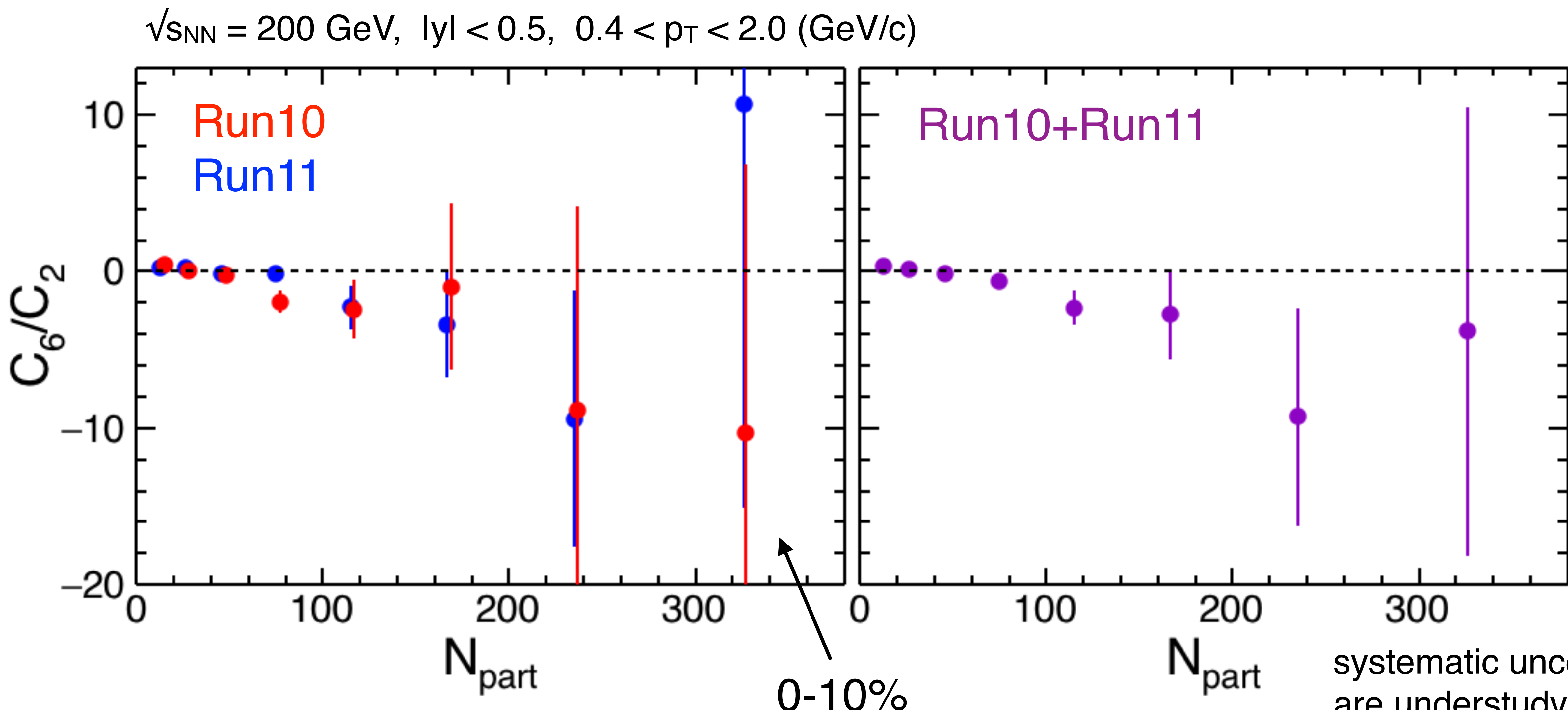
- ✦ p_T and rapidity dependence at central collisions of C_4/C_2 and C_6/C_2
- ✦ Large errors exist for C_6/C_2. More statistics are necessary.



systematic uncertainties are understudy

Centrality dependence

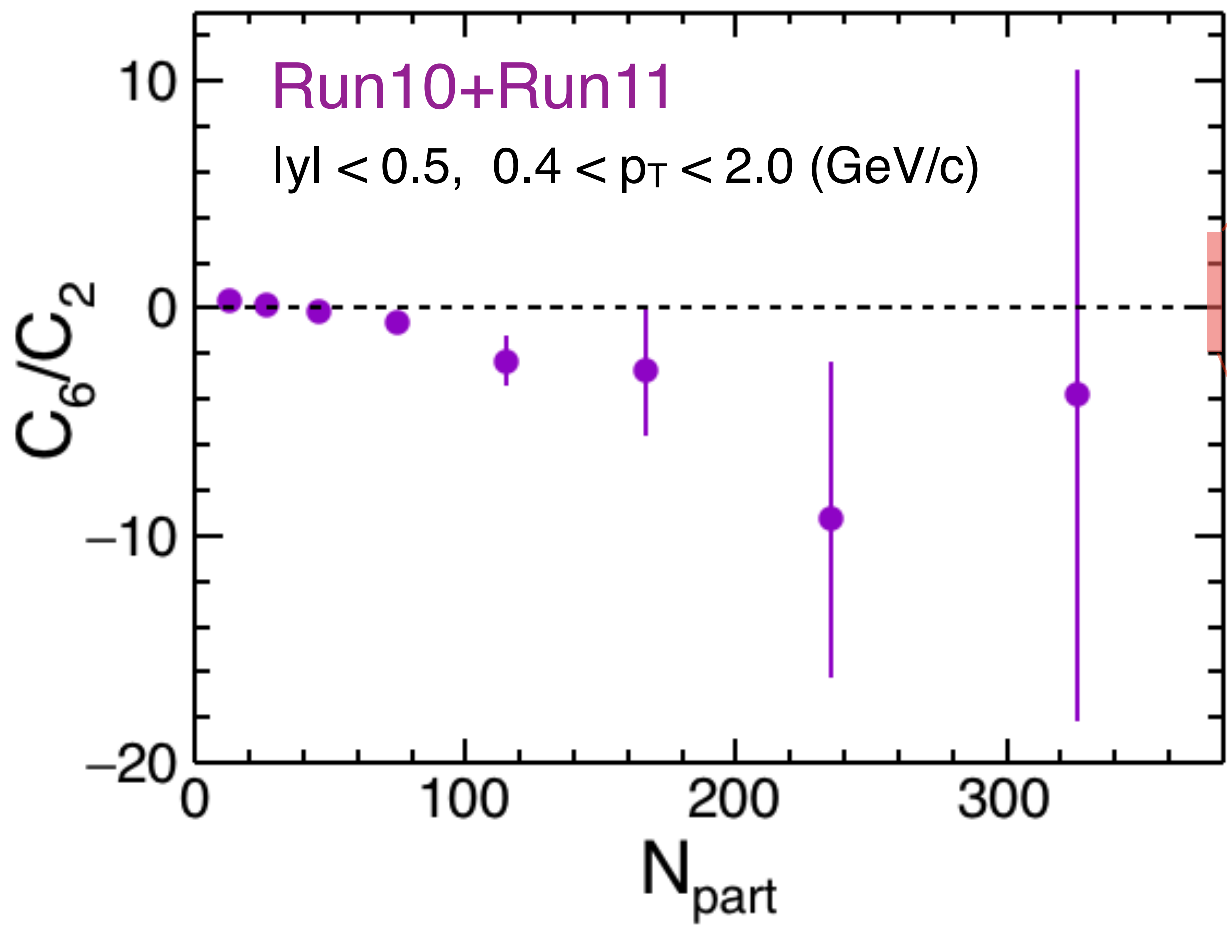
- Central trigger is used for Run10 results at 0-10%.
- Other points are from minimum bias trigger.
- All the results except 40-50% (~3sigma) are consistent within 1~2sigma between Run10 and Run11.

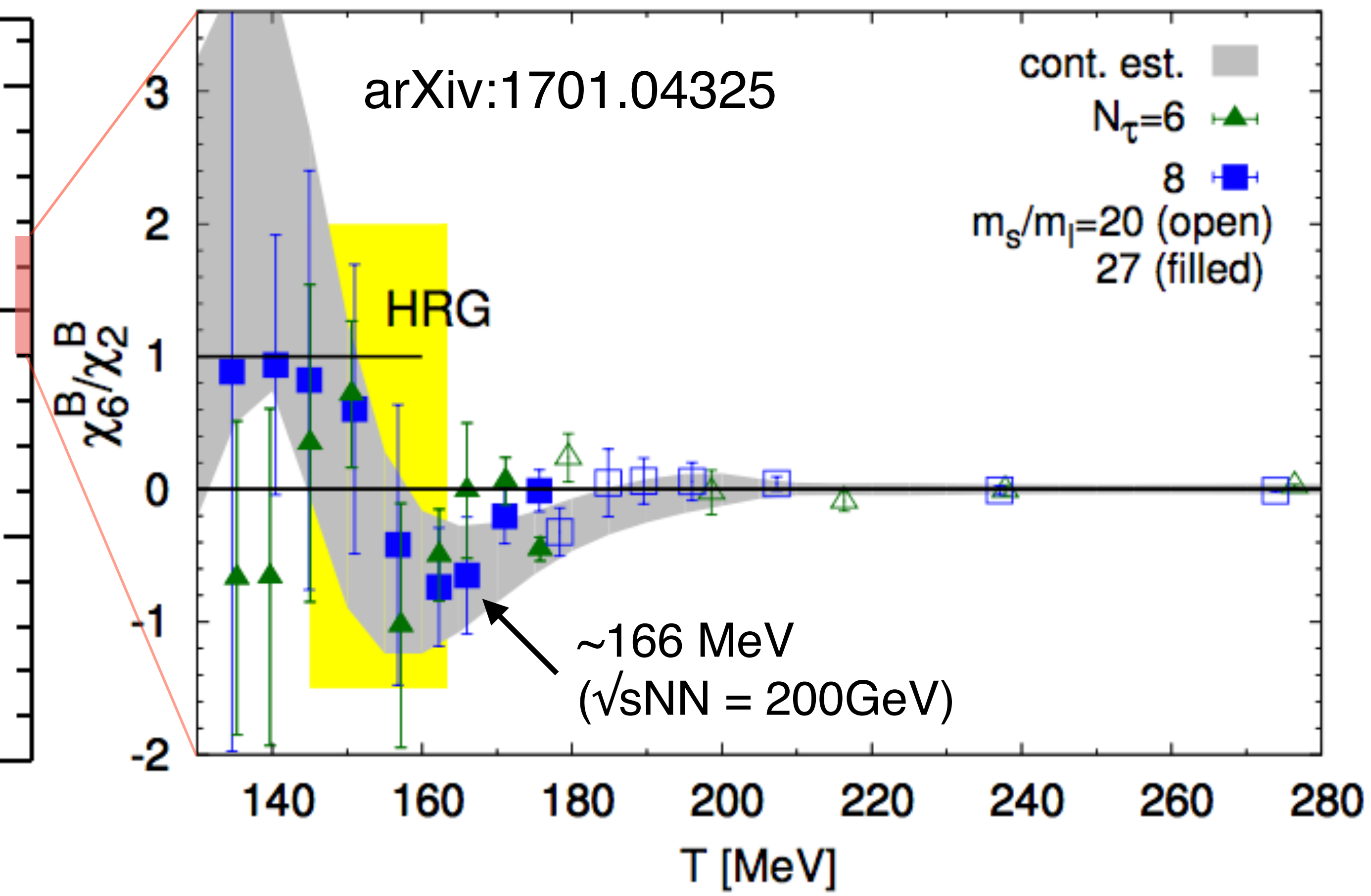


systematic uncertainties are understudy

Comparison with theory

- Lattice results predict -1~0.5 for C_6/C_2 at $\sqrt{s_{NN}}$=200GeV (~166MeV)
- Large errors exist for experiment and theory.





More statistics are needed !!

Summary and Outlook

- ✓ **Development of more efficient formulas for efficiency correction up to sixth order cumulant.**
- ✓ **C_6/C_2 of net-proton multiplicity distribution has been measured in Au+Au collisions at $\sqrt{s_{NN}}$ = 200 GeV.**
- ✓ **Use more Au+Au data.**