Measurement of the Sixth Order Cumulant of Net-Proton Multiplicity Distribution in Au+Au Collisions at $\sqrt{s_{NN}}$ = 200 GeV at the STAR Experiment

Toshihiro Nonaka

UTTAC seminar

2017/11/30

My work

✦ **2 papers and 5 international conference (oral)**

PRC. 94. 034909
T. Nonaka, S. Esumi, H. Masui, T. Sugiura, X. Luo
"Importance of separated efficiencies between positively and negatively charged particles for cumulant calculations"

PRC. 95. 064912
T. Nonaka, M. Kitazawa, S. Esumi
"More efficient formulas for efficiency correction of cumulants and effect of using averaged efficiency"

JPS : 2016 fall, 2017 spring, 2017 fall
QM2017 (poster)

Support MRPC developement

STAR shift taking (M2, D1, D2)

seminar @Osaka Univ. (last week)

CiRfSE workshop 2016
"Higher order cumulant of net-proton distribution"

ATHIC 2016
"Importance of separated efficiencies between positively and negatively charged particles for cumulant calculations"

TGSW 2016
"Fluctuation of Conserved Quantities to look for Critical Point in Phase Diagram"

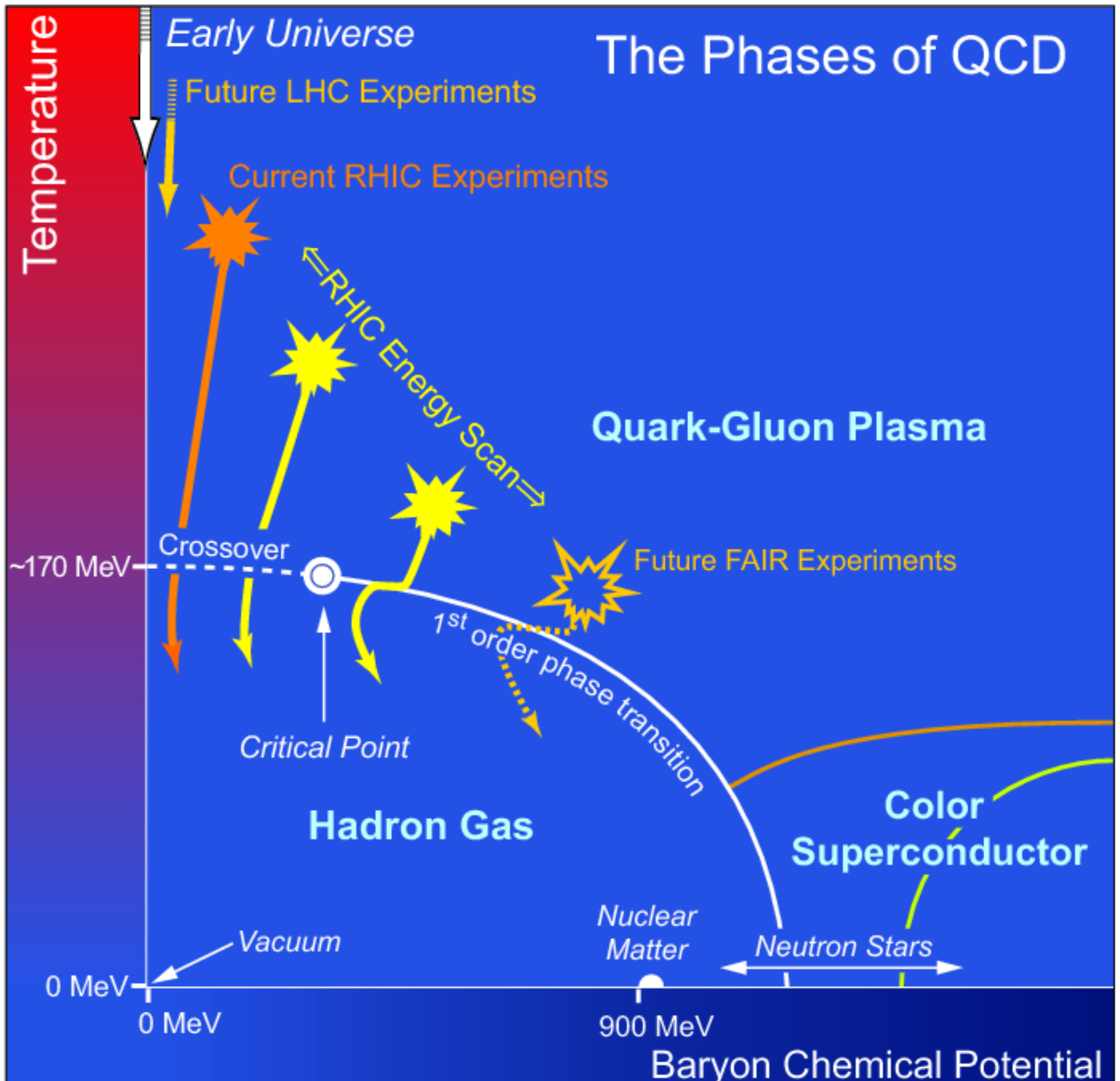
WPCF 2017
"Measurement of Sixth Order Cumulant of Net-Proton Multiplicity Distribution in Au+Au collisions at $\sqrt{s_{NN}}$ = 200 GeV from the STAR experiment"

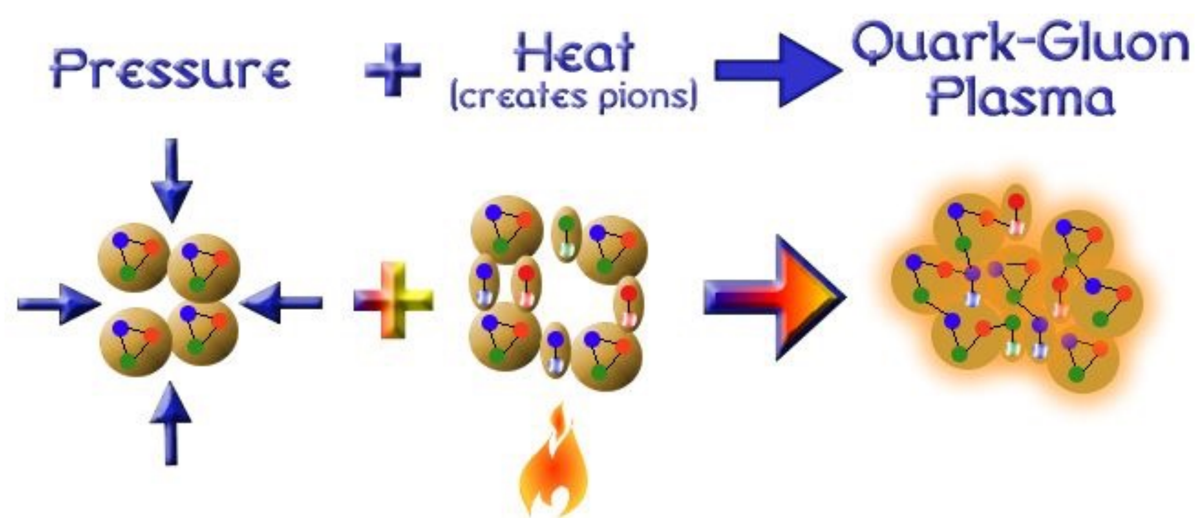
EMMI workshop
"Correction methods for detector effects on cumulants"

Introduction

QCD phase diagram

- Quark Gluon Plasma can be experimentally created by heavy ion collisions.
- Higher order fluctuations of conserved quantities can probe the QCD phase structure.





✓ Crossover at μ_B=0

Y. Aoki, Nature 443, 675(2006)

✓ 1st order phase transition at large μ_B?

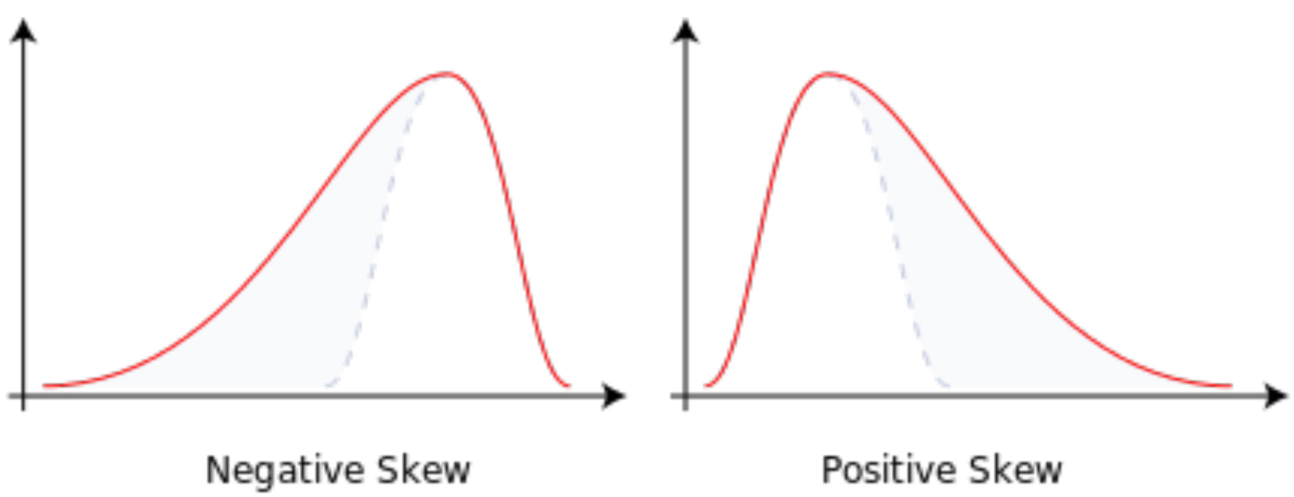
✓ Critical point?

Higher order fluctuations

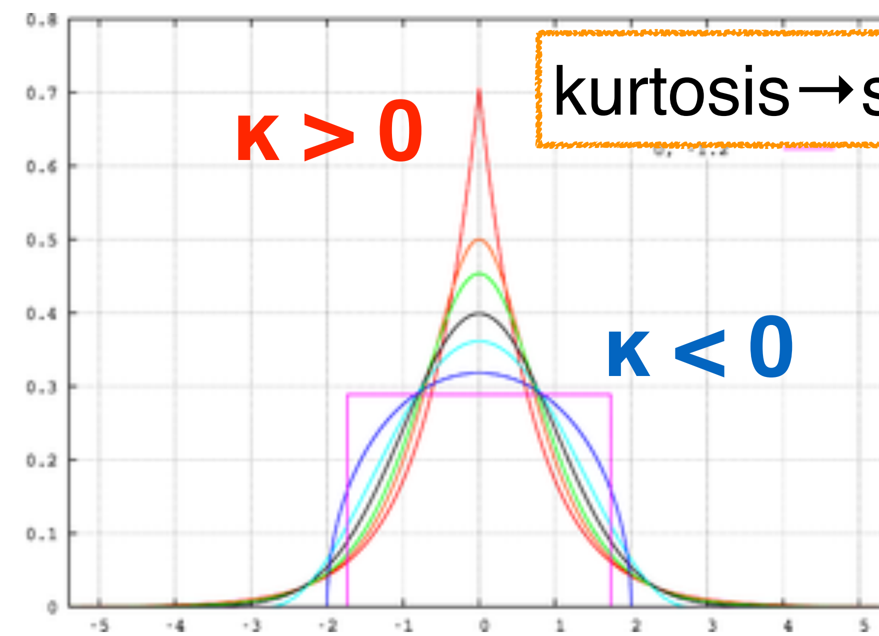
✦ **Moments and Cumulants are mathematical measures of “shape” of a histogram which probe the fluctuation of observables.**

- ✓ **Moments : Mean(*M*), sigma(*σ*), skewness(*S*) and kurtosis(κ).**
- ✓ ***S and κ are non-gaussian* fluctuations.**

skewness→asymmetry



kurtosis→shapness



from wikipedia

✓ **Cumulant ⇄ Moment**

$$< \delta N >= N - < N >$$

$$C_1 = M =< N >$$

$$C_2 = \sigma^2 =< (\delta N)^2 >$$

$$C_3 = S\sigma^3 =< (\delta N)^3 >$$

$$C_4 = \kappa\sigma^4 =< (\delta N)^4 > -3 < (\delta N)^2 >^2$$

✓ **Cumulant : additivity**

$$C_n(X+Y) = C_n(X) + C_n(Y)$$

→ **Volume dependence**

Fluctuations of conserved quantities

✦ Net-baryon, net-charge and net-strangeness

X. Luo, CiRfSE workshop 2016 @Tsukuba University

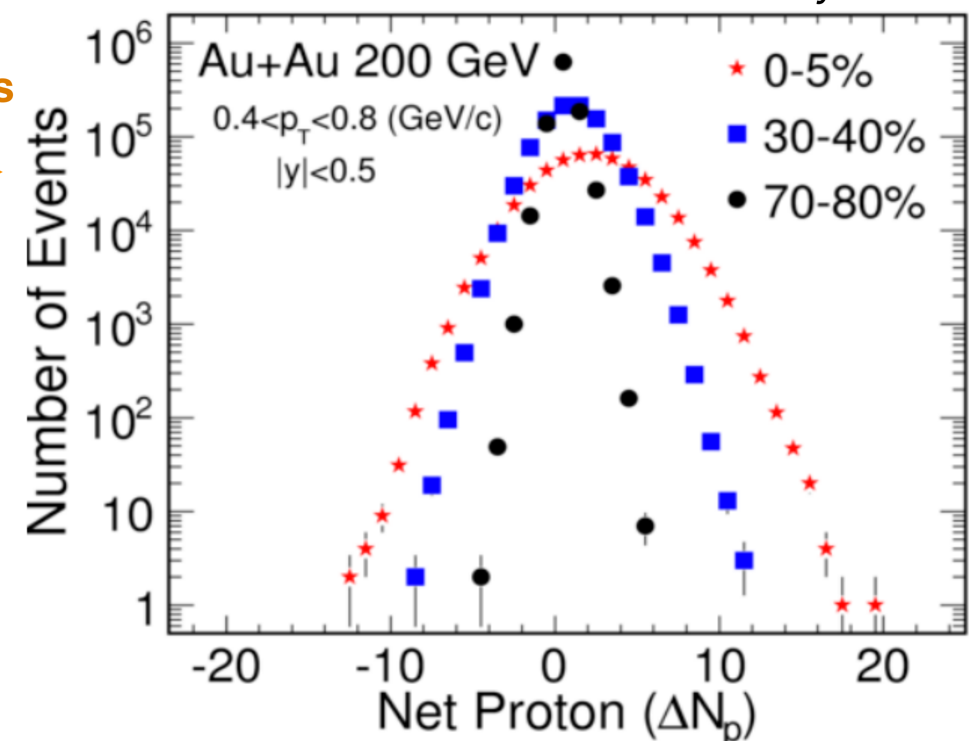
"Net" : positive - negative

$$\Delta N_q = N_q - N_{\bar{q}}, \quad q = B, Q, S$$

No. of positively charged particles in one collision

No. of negatively charged particles in one collision

Fill in histograms over many collisions



→neutrons cannot be measured

(1) Sensitive to correlation length

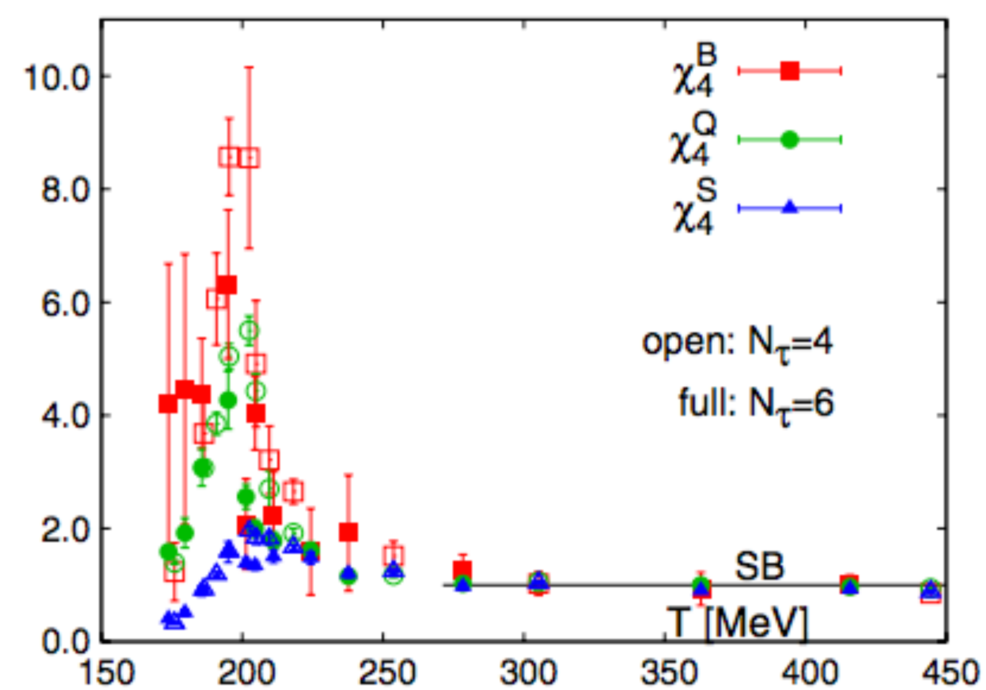
$$C_2 = <(\delta N)^2>_c \approx \xi^2 \qquad C_5 = <(\delta N)^5>_c \approx \xi^{9.5}$$
$$C_3 = <(\delta N)^3>_c \approx \xi^{4.5} \qquad C_6 = <(\delta N)^6>_c \approx \xi^{12}$$
$$C_4 = <(\delta N)^4>_c \approx \xi^7$$

(2) Direct comparison with susceptibilities.

M. Cheng et al, PRD 79, 074505 (2009)

$$S\sigma = \frac{C_3}{C_2} = \frac{\chi_3}{\chi_2} \qquad \kappa\sigma^2 = \frac{C_4}{C_2} = \frac{\chi_4}{\chi_2}$$

$$\chi_n^q = \frac{1}{VT^3} \times C_n^q = \frac{\partial^n p/T^4}{\partial \mu_q^n}, \quad q = B, Q, S$$



Volume **dependence can be canceled by taking ratio.**

Statistical baseline (Poisson)

- ✓ **Higher order fluctuations are compared to statistical baselines of the Poisson distribution.**
- ✓ **Poisson - Poisson = Skellam**

μ_1, μ_2 : mean parameter of Poisson

$$p(k;\mu_1,\mu_2)=\Pr\{K=k\}=e^{-(\mu_1+\mu_2)}\left(\frac{\mu_1}{\mu_2}\right)^{k/2}I_k(2\sqrt{\mu_1\mu_2})$$

- ✓ **Odd(even) order cumulant of Skellam distribution is difference(sum) between means of two Poissons.**

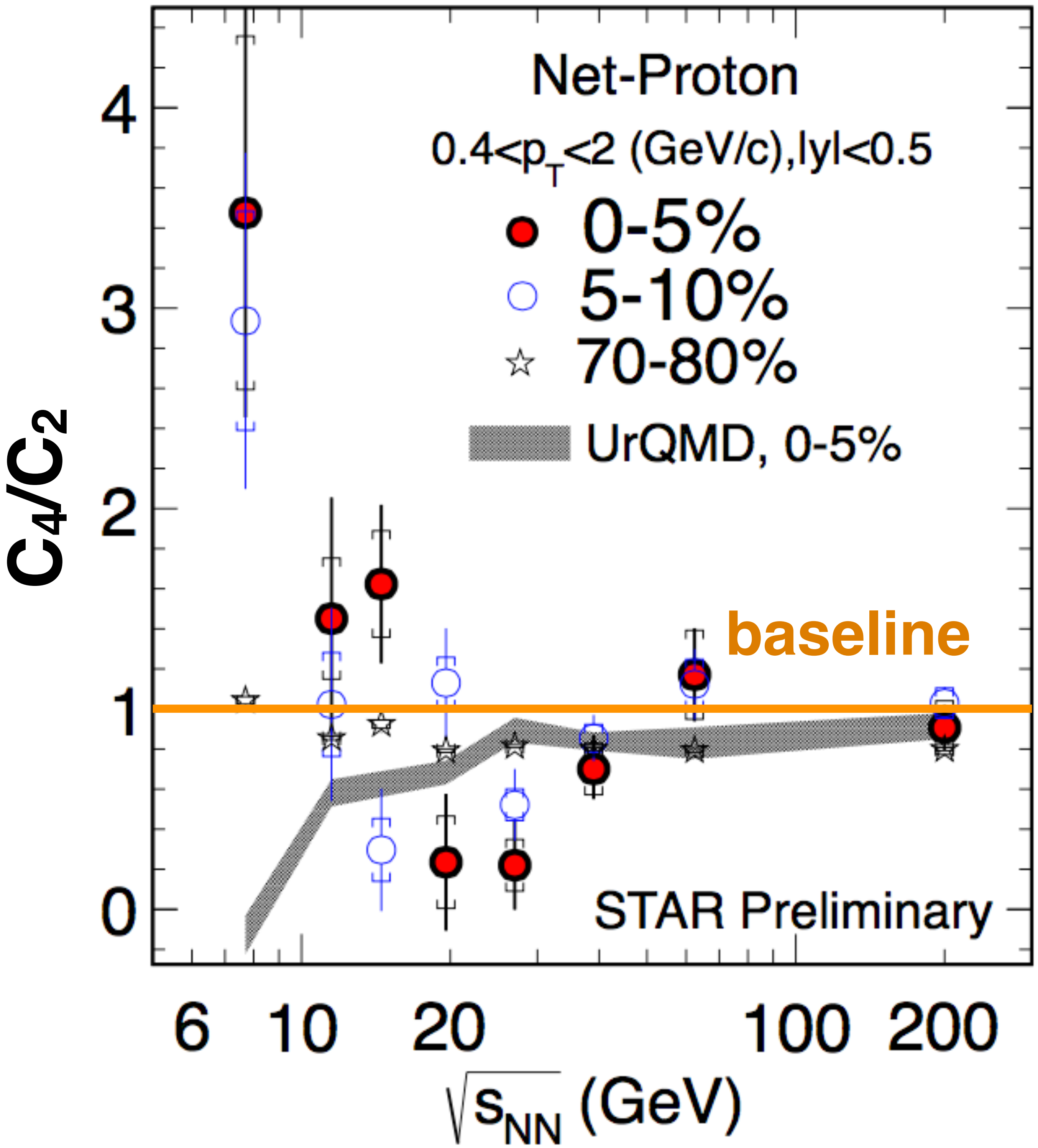
$$C_{odd}=\mu_1-\mu_2 \qquad S\sigma=\frac{C_3}{C_2}=\frac{\mu_1-\mu_2}{\mu_1+\mu_2} \qquad \kappa\sigma^2=\frac{C_4}{C_2}=1$$
$$C_{even}=\mu_1+\mu_2$$

$$\boxed{C_4/C_2 = C_6/C_2 = 1}$$

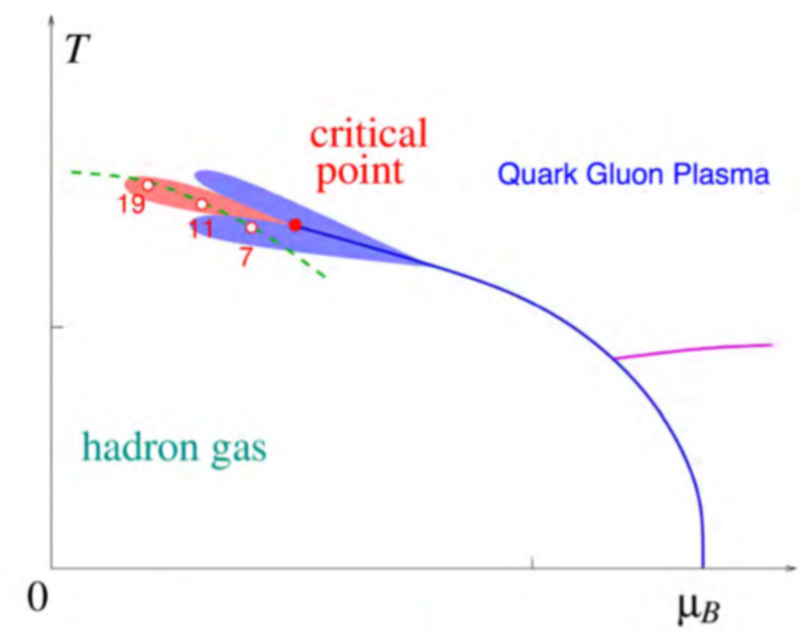
Critical end point?

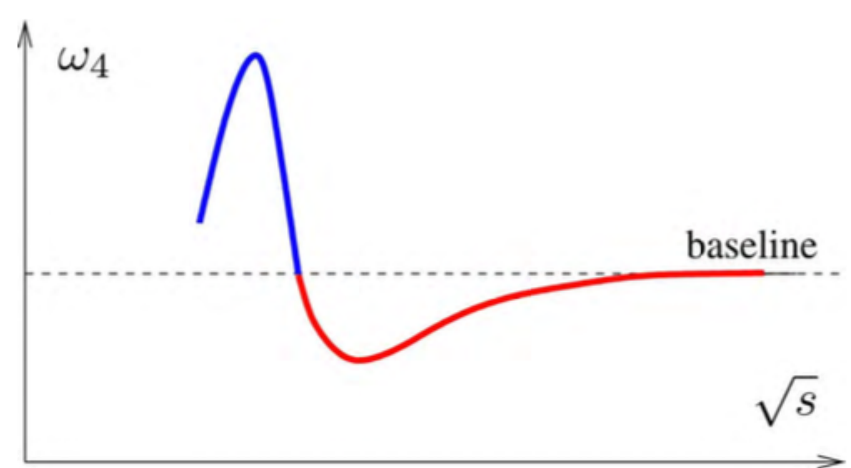
✓ **Measured non-momotonic behaviour of fourth order fluctuation of net-proton distribution might be a signal for the critical end point.**

X. Luo (STAR collaboration) arXiv:1503.02558v2



M. A. Stephanov, Phys. Rev. Lett. 107, 052301 (2011)





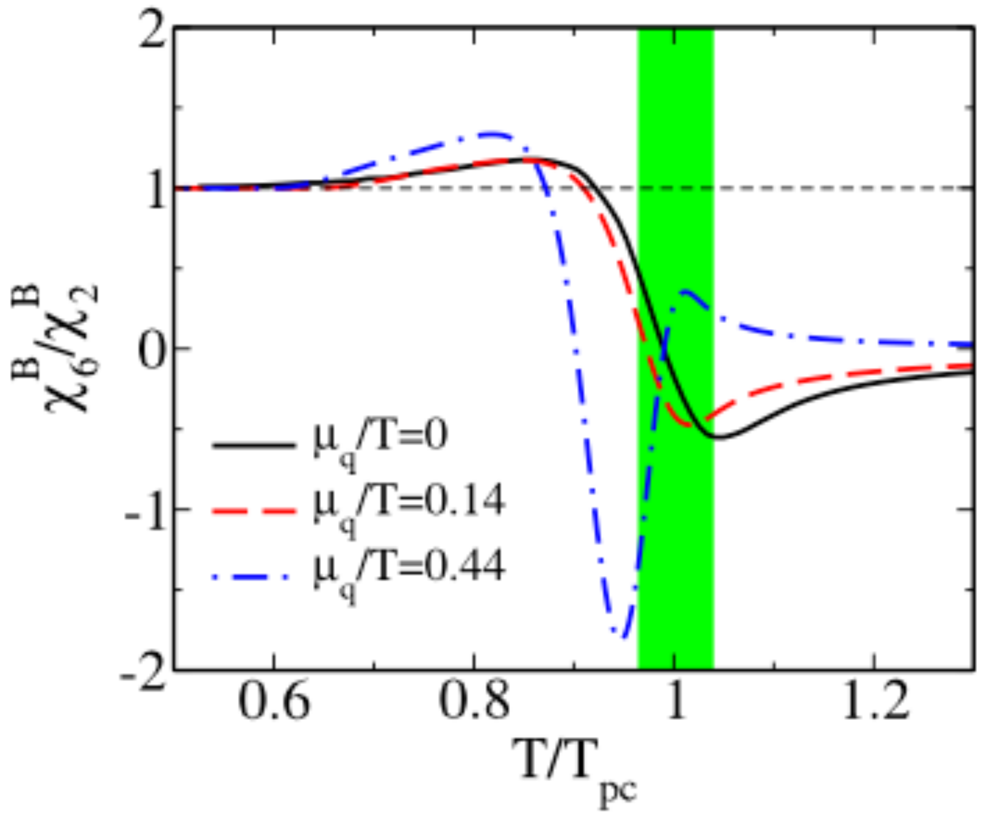
Crossover phase transition with C_6

- ✓ Lattice calculations predict a "smooth crossover" at μ_B=0.

  Y. Aoki, Nature 443, 675(2006)

- ✓ No experimental evidence for (crossover) phase transition.
- ✓ Any observable shows no discontinuity for this smooth crossover.
- ✓ Theoretically, the six order cumulant of net-baryon and net-charge fluctuation change sign if the chiral phase transition is close to the freeze-out line.

  Friman et al, Eur. Phys. J. C (2011) 71:1694

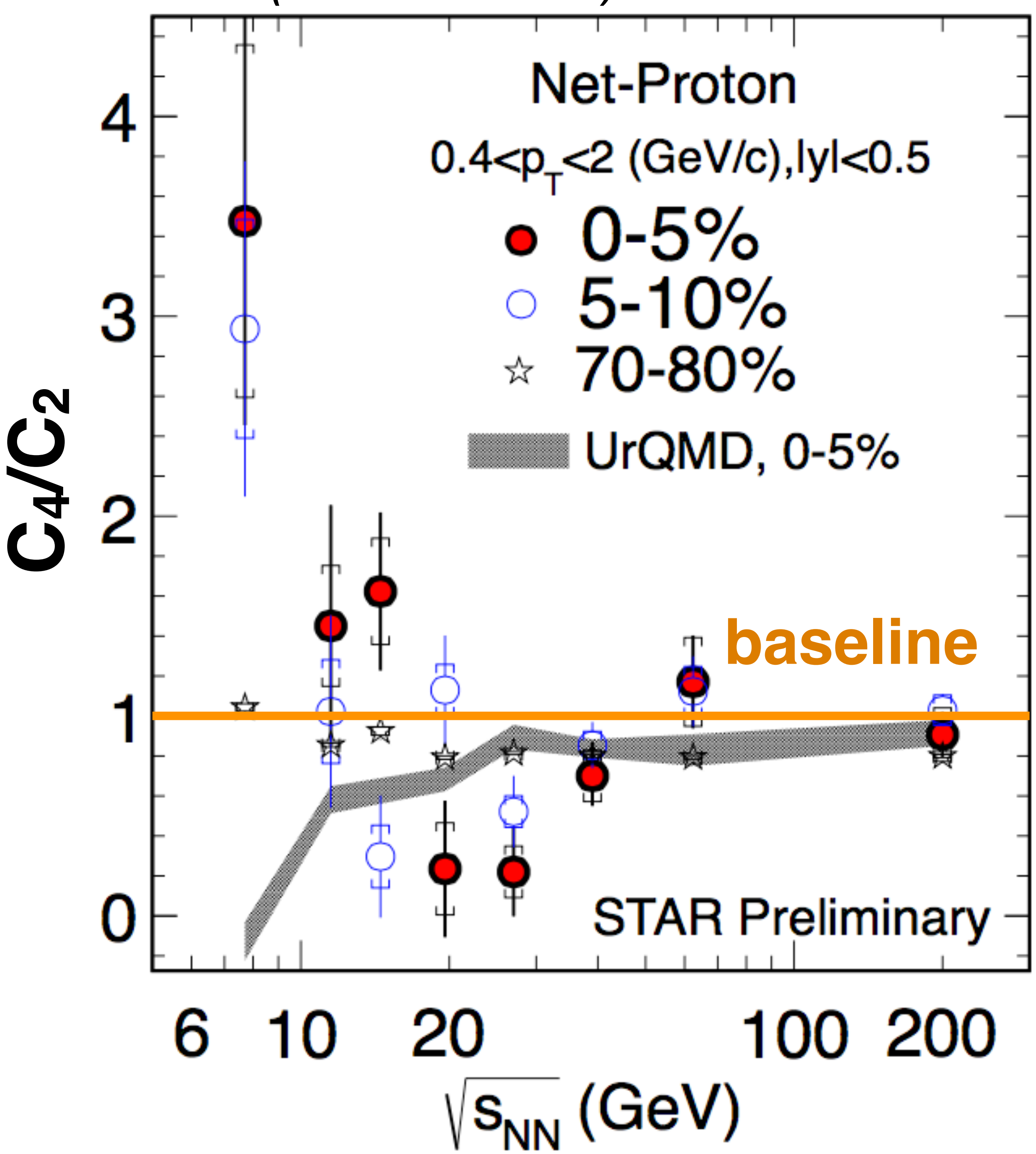


Freeze-out conditions	χ_6^B/χ_2^B
HRG	1
QCD: $T^{\mathrm{freeze}}/T_{pc} \lesssim 0.9$	$\gtrsim 1$
QCD: $T^{\mathrm{freeze}}/T_{pc} \simeq 1$	<0

Known issues?

✓ **Non-momotonic behaviour of fourth order fluctuation of net-proton distribution might be a signal for the critical end point.**

X. Luo (STAR collaboration) arXiv:1503.02558v2



Known issues

✓ **Huge calculation cost on binomial efficiency correction**

- T. Nonaka, M. Kitazawa, S. Esumi : PRC.95.064912

✓ **Non-binomial efficiency**

- A. Bzdak, R. Holtzmann, V. Koch : PRC.94.064907

✓ **Volume fluctuation**

- P. Braun-Munzinger, A. Rustamov, J. Stachel: *arXiv:1612.00702*

How can we solve these issues?

Known issues?

- ✓ **Those effects on C_6 are expected to be much larger than C_4 since the higher order cumulant consists of combinations of all the lower order cumulants.**

Even if the background effects on C_4 is small enough, it is dangerous to be blind for C_6…

$$
\begin{aligned}
C_1 &= \mu_1, \\
C_2 &= \mu_2 - \mu_1^2, \\
C_3 &= \mu_3 - 3\mu_2\mu_1 + 2\mu_1, \\
C_4 &= \mu_4 - 4\mu_3\mu_1 - 3\mu_2^2 + 12\mu_2\mu_1^2 - 6\mu_1^4, \\
C_5 &= \mu_5 - 5\mu_4\mu_1 - 10\mu_3\mu_2 + 20\mu_3\mu_1 + 30\mu_2^2\mu_1 - 60\mu_2\mu_1^3 + 24\mu_1^5, \\
C_6 &= \mu_6 - 6\mu_5\mu_1 - 15\mu_4\mu_2 + 30\mu_4\mu_1^2 - 10\mu_3^2 + 120\mu_3\mu_2\mu_1 - 120\mu_3\mu_1^3 \\
&\quad +30\mu_2^3 - 270\mu_2^2\mu_1^2 + 360\mu_2\mu_1^4 - 120\mu_1^6,
\end{aligned}
$$

μ_n : n-th order moment

Motivation

Find an experimental evidence for the phase transition with measurement of the sixth order cumulant of net-proton distribution in Au+Au collisions at $\sqrt{s_{NN}}$ = 200 GeV at the STAR experiment.

(The world's first measurement!)

Investigate(develop) existing(new) analysis techniques to derive "true" fluctuations

Outline

Experiment and Datasets

Detector Effect

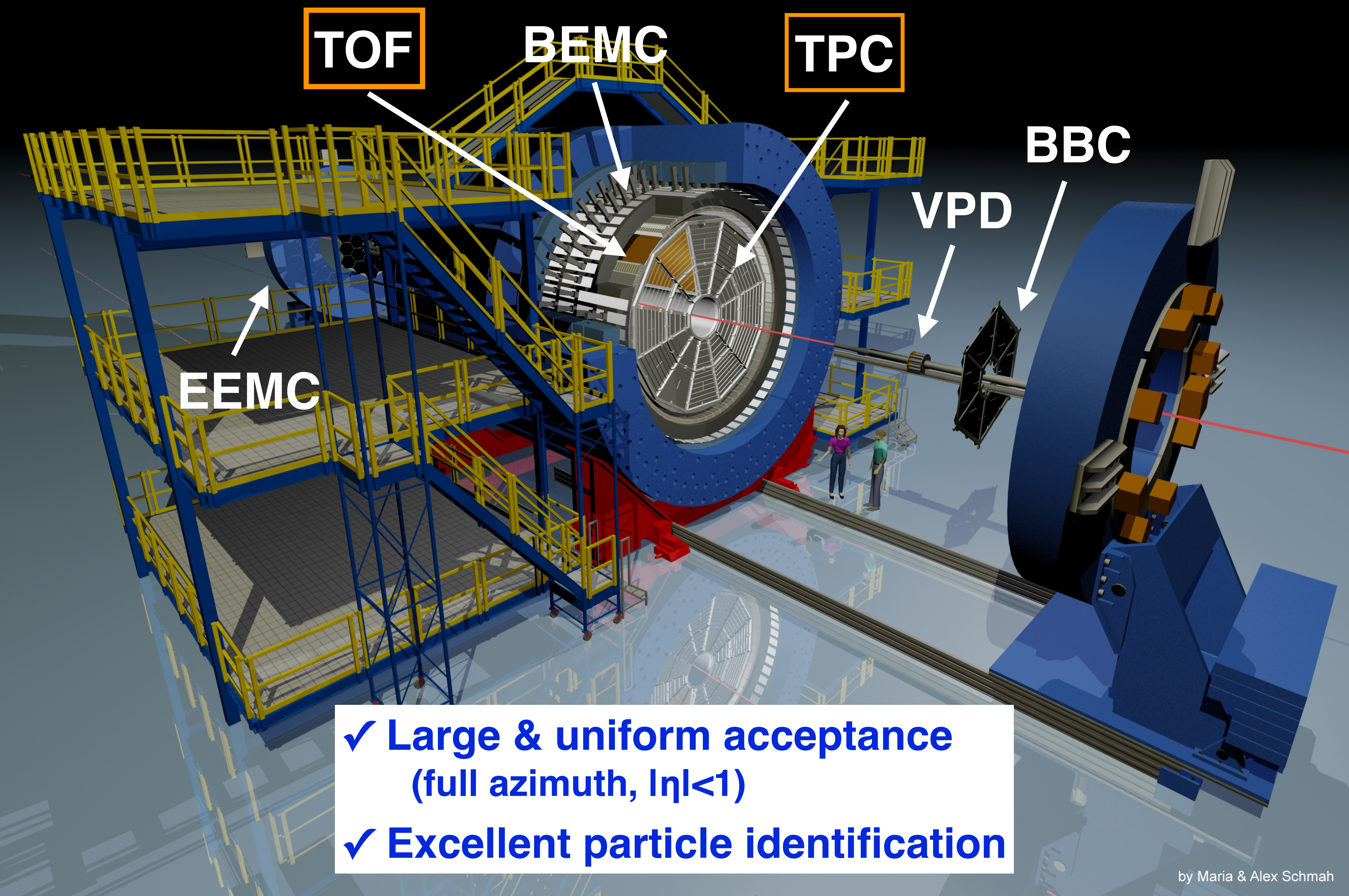
- ***Efficiency correction***
- ***Unfolding***

Volume Fluctuation

- ***Centrality Bin Width Correction***
- ***Volume Fluctuation Correction***

Results and Discussions

Solenoidal Tracker At RHIC



✓ **Large & uniform acceptance**
(full azimuth, $|\eta|<1$)

✓ **Excellent particle identification**

Datasets

✓ **Event selection**

|Vz|<30cm, |Vr|<2cm, |VpdVz-Vz|<3cm

Pileup rejection using tofmatched tracks

✓ **Track selection**

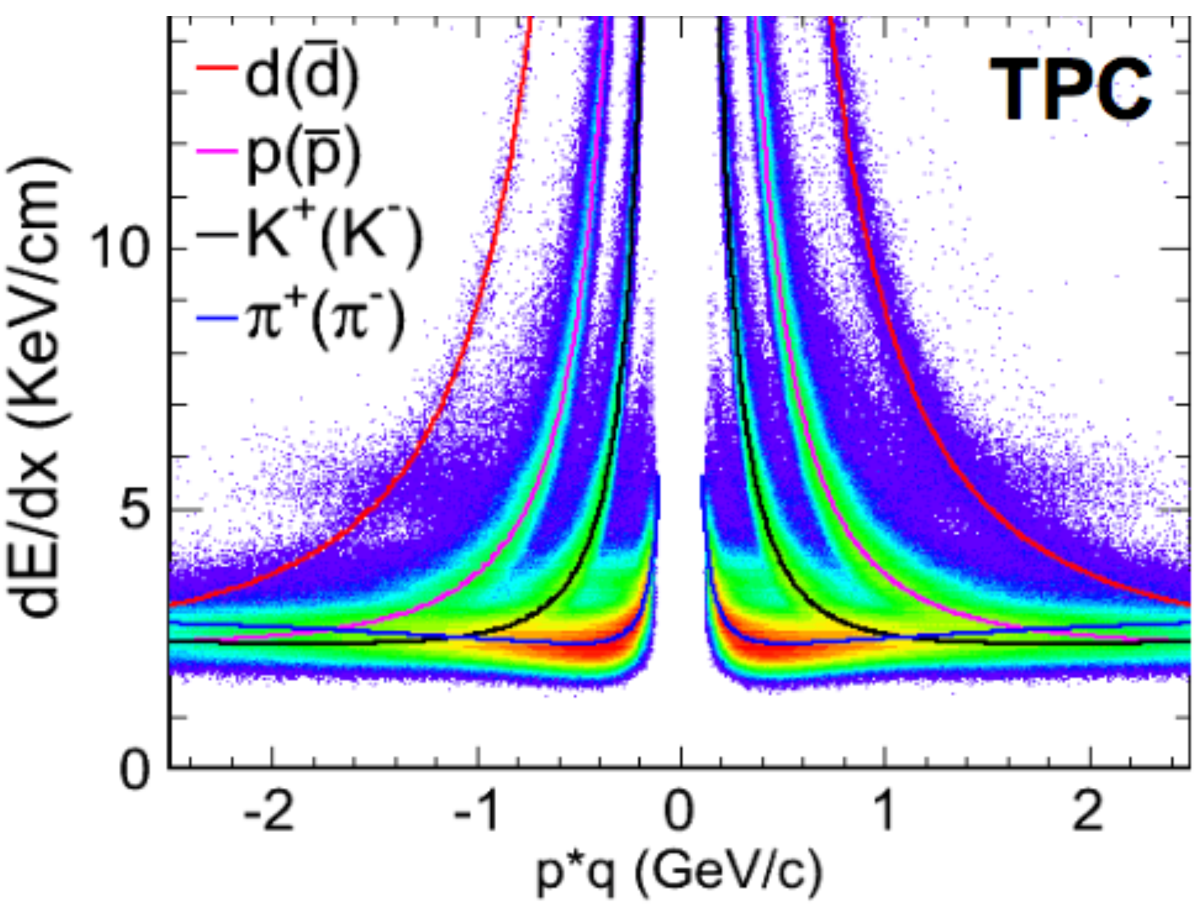
DCA<1cm, nHitsFit>20, nHitsFit/nFitPoss>0.52, nHitsDedx>5, |y|<0.5

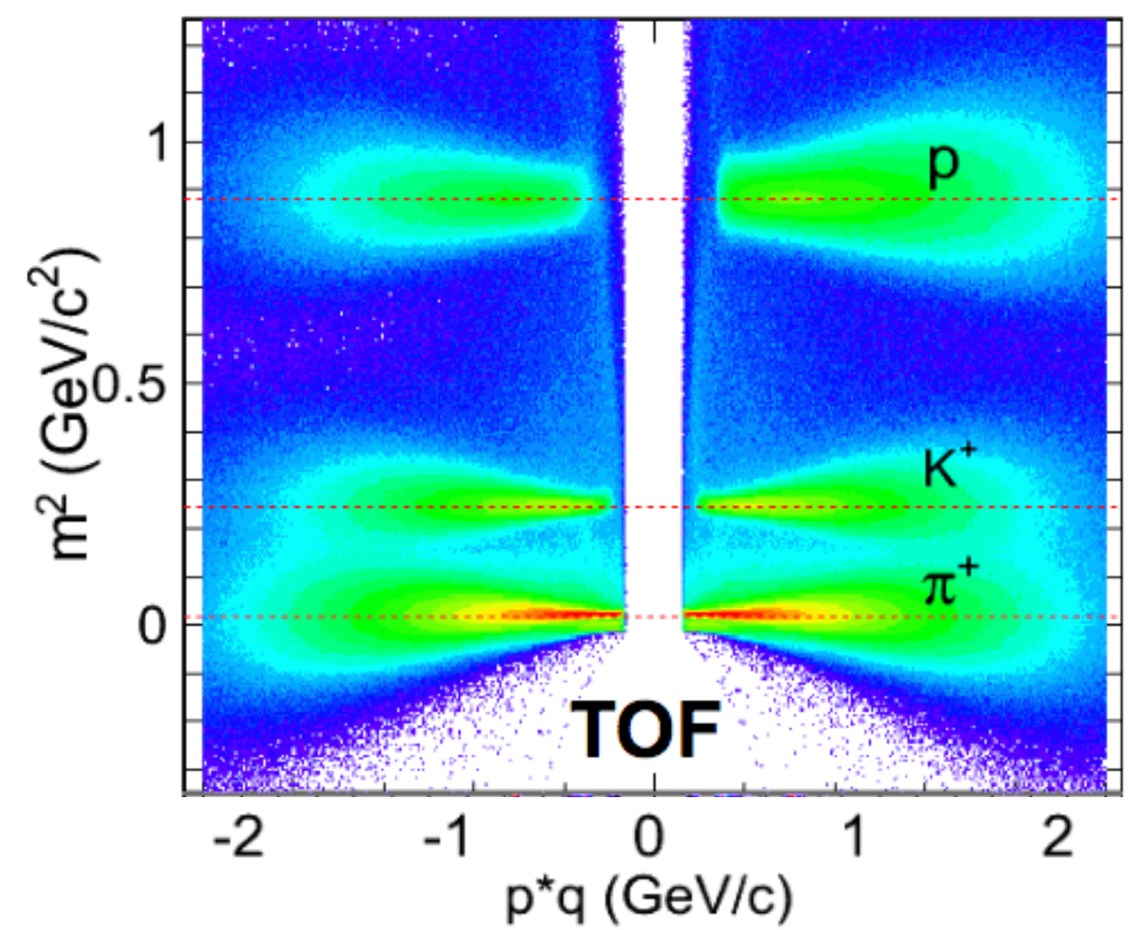
PID cut : $0.4<p_T<0.8$: $|n\sigma_{proton}|<2$

$0.8<p_T<2.0$: $|n\sigma_{proton}|<2$ && $0.6<m^2<1.2$

- ✦ **As C_6 is very statistical hungry, we focus on $\sqrt{s_{NN}}$ = 200 GeV datasets which have the largest statistics.**
- ✦ **Minimum bias trigger for Run10 and Run11, and central trigger in Run10 are analyzed separately, and combine them to reduce statistical errors.**

	0-10%	10-80%
Run10	~160M	~200M
Run11	~50M	~450M
Total	210M	650M





Experiment and Datasets

Detector Effect

- Efficiency correction
- Unfolding

Volume Fluctuation

- Centrality Bin Width Correction
- Volume Fluctuation Correction

Results and Discussions

Binomial model

- Efficiency follows binomial distribution.
- Factorial moments can be easily corrected.
  - M. Kitazawa : PRC.86.024904, M. Kitazawa and M. Asakawa : PRC.86.024904
  - A. Bzdak and V. Koch : PRC.86.044904, PRC.91.027901, X. Luo : PRC.91.034907
  - T. Nonaka, M. Kitazawa, S. Esumi : PRC.95.064912

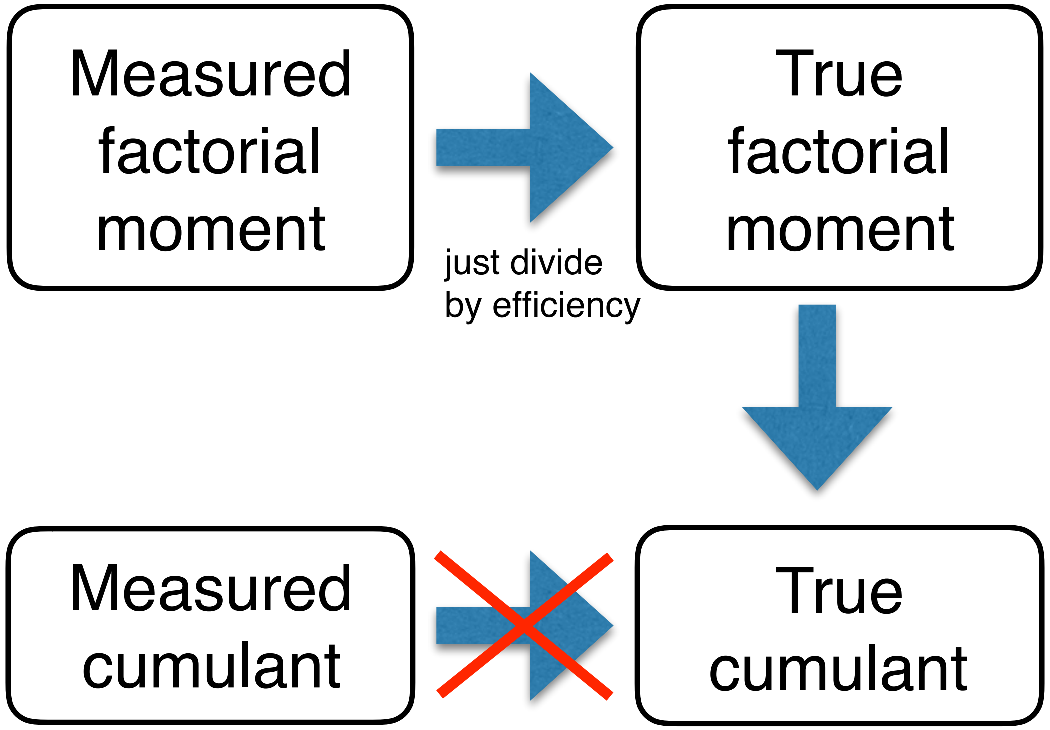
$$F_{ik} \equiv \left\langle \frac{N_1!}{(N_1-i)!} \frac{N_2!}{(N_2-k)!} \right\rangle$$

$$f_{ik} \equiv \left\langle \frac{n_1!}{(n_1-i)!} \frac{n_2!}{(n_2-k)!} \right\rangle$$

$$B(n,N;\varepsilon) = \frac{N!}{n!(N-n)!}\varepsilon^n(1-\varepsilon)^{N-n} \qquad f_{ik} = \varepsilon_p^i \varepsilon_{pbar}^k F_{ik}$$

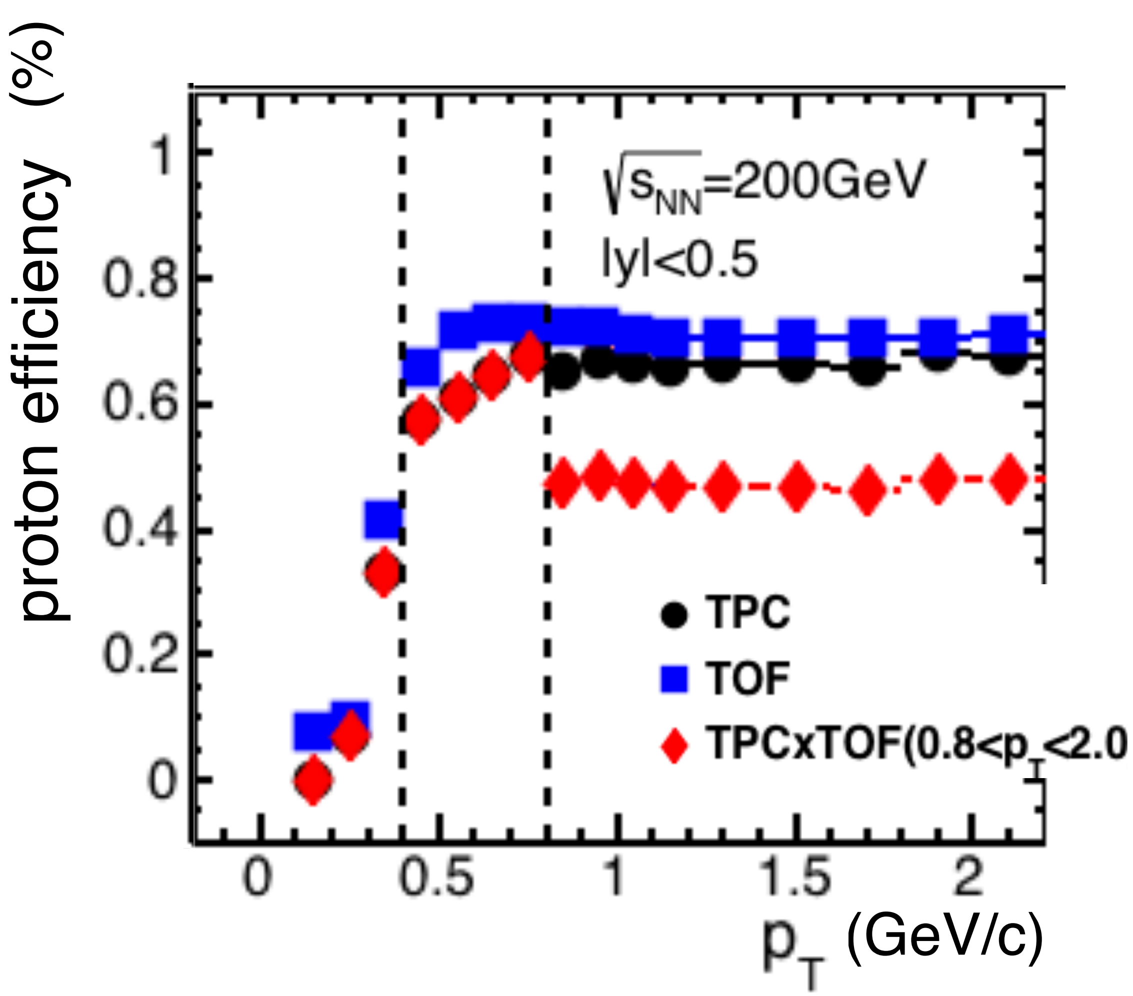
- Corrected cumulants are expressed in terms of measured factorial moments and efficiency.

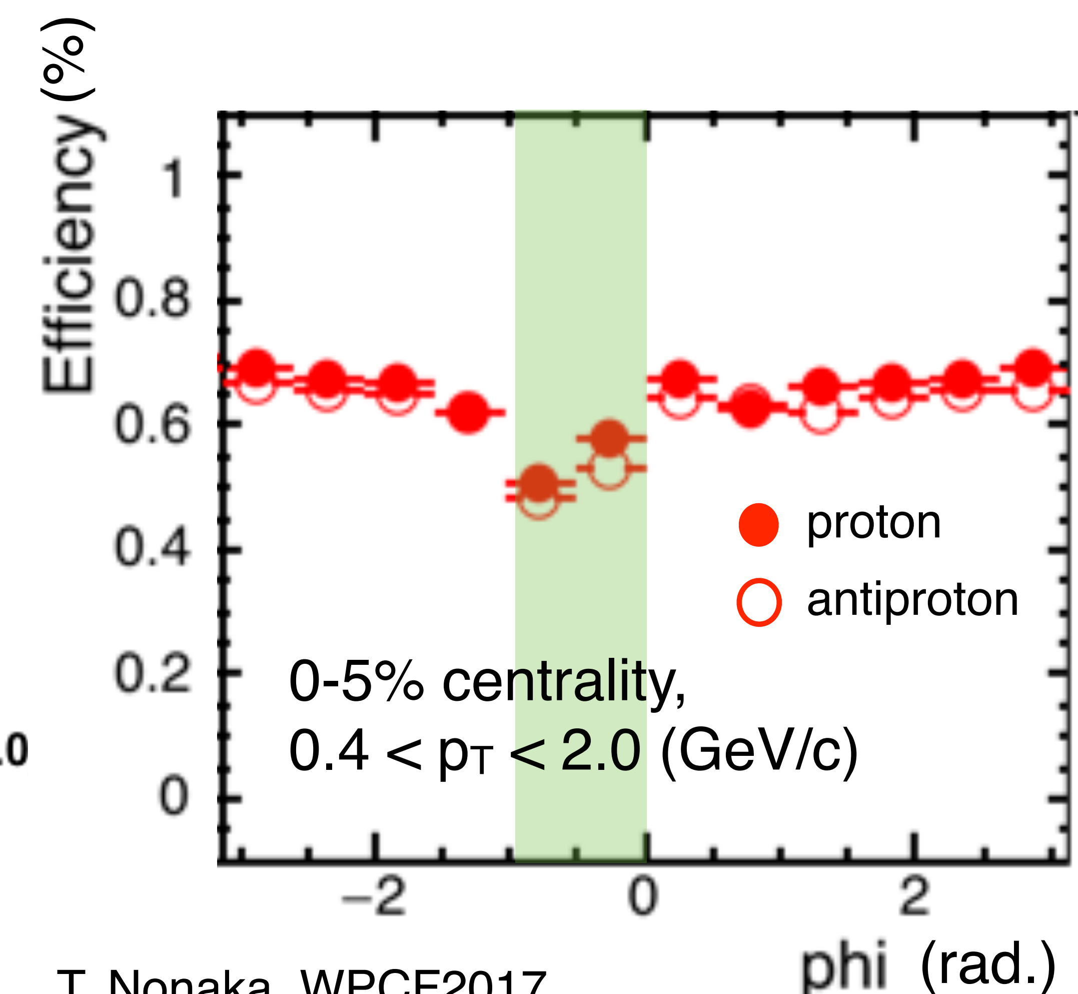
$$\begin{aligned}
\kappa_4(\Delta N) = {} & (((f_{10}/\varepsilon_1) + 7(f_{20}/\varepsilon_1^2) + 6(f_{30}/\varepsilon_1^3) + (f_{40}/\varepsilon_1^4) - 4(f_{10}/\varepsilon_1)^2 - \\
& 12(f_{20}/\varepsilon_1^2)(f_{10}/\varepsilon_1) - 4(f_{30}/\varepsilon_1^3)(f_{10}/\varepsilon_1) + 6(f_{10}/\varepsilon_1)^3 + 6(f_{20}/\varepsilon_1^2)(f_{10}/\varepsilon_1)^2 - 3(f_{10}/\varepsilon_1)^4) - \\
& 4((f_{11}/\varepsilon_1/\varepsilon_2) - (f_{10}/\varepsilon_1)(f_{01}/\varepsilon_2) + 3(f_{21}/\varepsilon_1^2/\varepsilon_2) - 3(f_{20}/\varepsilon_1^2)(f_{01}/\varepsilon_2) + (f_{31}/\varepsilon_1^3/\varepsilon_2) - \\
& (f_{30}/\varepsilon_1^3)(f_{01}/\varepsilon_2) - 3(f_{11}/\varepsilon_1/\varepsilon_2)(f_{10}/\varepsilon_1) + 3(f_{10}/\varepsilon_1)^2(f_{01}/\varepsilon_2) - 3(f_{21}/\varepsilon_1^2/\varepsilon_2)(f_{10}/\varepsilon_1) + \\
& 3(f_{20}/\varepsilon_1^2)(f_{10}/\varepsilon_1)(f_{01}/\varepsilon_2) + 3(f_{11}/\varepsilon_1/\varepsilon_2)(f_{10}/\varepsilon_1)^2 - 3(f_{10}/\varepsilon_1)^3(f_{01}/\varepsilon_2)) + 6((f_{11}/\varepsilon_1/\varepsilon_2) + \\
& (f_{12}/\varepsilon_1/\varepsilon_2^2) - 2(f_{11}/\varepsilon_1/\varepsilon_2)(f_{01}/\varepsilon_2) + (f_{10}/\varepsilon_1)(f_{01}/\varepsilon_2)^2 + (f_{21}/\varepsilon_1^2/\varepsilon_2) + (f_{22}/\varepsilon_1^2/\varepsilon_2^2) - \\
& 2(f_{21}/\varepsilon_1^2/\varepsilon_2)(f_{01}/\varepsilon_2) + (f_{20}/\varepsilon_1^2)(f_{01}/\varepsilon_2)^2 - 2(f_{11}/\varepsilon_1/\varepsilon_2)(f_{10}/\varepsilon_1) - 2(f_{12}/\varepsilon_1/\varepsilon_2^2)(f_{10}/\varepsilon_1) + \\
& 4(f_{11}/\varepsilon_1/\varepsilon_2)(f_{10}/\varepsilon_1)(f_{01}/\varepsilon_2) - 3(f_{10}/\varepsilon_1)^2(f_{01}/\varepsilon_2)^2 + (f_{10}/\varepsilon_1)^2(f_{01}/\varepsilon_2) + (f_{02}/\varepsilon_2^2)(f_{10}/\varepsilon_1)^2) - \\
& 4((f_{11}/\varepsilon_1/\varepsilon_2) + 3(f_{12}/\varepsilon_1/\varepsilon_2^2) + (f_{13}/\varepsilon_1/\varepsilon_2^3) - 3(f_{11}/\varepsilon_1/\varepsilon_2)(f_{01}/\varepsilon_2) - 3(f_{12}/\varepsilon_1/\varepsilon_2^2)(f_{01}/\varepsilon_2) + \\
& 3(f_{11}/\varepsilon_1/\varepsilon_2)(f_{01}/\varepsilon_2)^2 - 3(f_{10}/\varepsilon_1)(f_{01}/\varepsilon_2)^3 - (f_{10}/\varepsilon_1)(f_{01}/\varepsilon_2) - 3(f_{02}/\varepsilon_2^2)(f_{10}/\varepsilon_1) - \\
& (f_{03}/\varepsilon_2^3)(f_{10}/\varepsilon_1) + 3(f_{10}/\varepsilon_1)(f_{01}/\varepsilon_2)^2 + 3(f_{02}/\varepsilon_2^2)(f_{10}/\varepsilon_1)(f_{01}/\varepsilon_2)) + ((f_{01}/\varepsilon_2) + \\
& 7(f_{02}/\varepsilon_2^2) + 6(f_{03}/\varepsilon_2^3) + (f_{04}/\varepsilon_2^4) - 4(f_{01}/\varepsilon_2)^2 - 12(f_{02}/\varepsilon_2^2)(f_{01}/\varepsilon_2) - 4(f_{03}/\varepsilon_2^3)(f_{01}/\varepsilon_2) + \\
& 6(f_{01}/\varepsilon_2)^3 + 6(f_{02}/\varepsilon_2^2)(f_{01}/\varepsilon_2)^2 - 3(f_{01}/\varepsilon_2)^4)) - 3(((f_{10}/\varepsilon_1) + (f_{20}/\varepsilon_1^2) - (f_{10}/\varepsilon_1)^2) - \\
& 2((f_{11}/\varepsilon_1/\varepsilon_2) - (f_{10}/\varepsilon_1)(f_{01}/\varepsilon_2)) + ((f_{01}/\varepsilon_2) + (f_{02}/\varepsilon_2^2) - (f_{01}/\varepsilon_2)^2))^2
\end{aligned}$$



Efficiency bins

✓ **Experimentally, efficiency will depend on p_T ,rapidity and azimuthal angle, which needs to be implemented in the efficiency correction.**





T. Nonaka, WPCF2017

Efficiency correction with many efficiency bins

1 eff. bin

$$
\begin{aligned}
&\kappa_4(\Delta N) = (((f_{10}/\varepsilon_1) + 7(f_{20}/\varepsilon_1^2) + 6(f_{30}/\varepsilon_1^3) + (f_{40}/\varepsilon_1^4) - 4(f_{10}/\varepsilon_1)^2 - \\
&12(f_{20}/\varepsilon_1^2)(f_{10}/\varepsilon_1) - 4(f_{30}/\varepsilon_1^3)(f_{10}/\varepsilon_1) + 6(f_{10}/\varepsilon_1)^3 + 6(f_{20}/\varepsilon_1^2)(f_{10}/\varepsilon_1)^2 - 3(f_{10}/\varepsilon_1)^4) - \\
&4((f_{11}/\varepsilon_1/\varepsilon_2) - (f_{10}/\varepsilon_1)(f_{01}/\varepsilon_2) + 3(f_{21}/\varepsilon_1^2/\varepsilon_2) - 3(f_{20}/\varepsilon_1^2)(f_{01}/\varepsilon_2) + (f_{31}/\varepsilon_1^3/\varepsilon_2) - \\
&(f_{30}/\varepsilon_1^3)(f_{01}/\varepsilon_2) - 3(f_{11}/\varepsilon_1/\varepsilon_2)(f_{10}/\varepsilon_1) + 3(f_{10}/\varepsilon_1)^2(f_{01}/\varepsilon_2) - 3(f_{21}/\varepsilon_1^2/\varepsilon_2)(f_{10}/\varepsilon_1) + \\
&3(f_{20}/\varepsilon_1^2)(f_{10}/\varepsilon_1)(f_{01}/\varepsilon_2) + 3(f_{11}/\varepsilon_1/\varepsilon_2)(f_{10}/\varepsilon_1)^2 - 3(f_{10}/\varepsilon_1)^3(f_{01}/\varepsilon_2)) + 6((f_{11}/\varepsilon_1/\varepsilon_2) + \\
&(f_{12}/\varepsilon_1/\varepsilon_2^2) - 2(f_{11}/\varepsilon_1/\varepsilon_2)(f_{01}/\varepsilon_2) + (f_{10}/\varepsilon_1)(f_{01}/\varepsilon_2)^2 + (f_{21}/\varepsilon_1^2/\varepsilon_2) + (f_{22}/\varepsilon_1^2/\varepsilon_2^2) - \\
&2(f_{21}/\varepsilon_1^2/\varepsilon_2)(f_{01}/\varepsilon_2) + (f_{20}/\varepsilon_1^2)(f_{01}/\varepsilon_2)^2 - 2(f_{11}/\varepsilon_1/\varepsilon_2)(f_{10}/\varepsilon_1) - 2(f_{12}/\varepsilon_1/\varepsilon_2^2)(f_{10}/\varepsilon_1) + \\
&4(f_{11}/\varepsilon_1/\varepsilon_2)(f_{10}/\varepsilon_1)(f_{01}/\varepsilon_2) - 3(f_{10}/\varepsilon_1)^2(f_{01}/\varepsilon_2)^2 + (f_{10}/\varepsilon_1)^2(f_{01}/\varepsilon_2) + (f_{02}/\varepsilon_2^2)(f_{10}/\varepsilon_1)^2) - \\
&4((f_{11}/\varepsilon_1/\varepsilon_2) + 3(f_{12}/\varepsilon_1/\varepsilon_2^2) + (f_{13}/\varepsilon_1/\varepsilon_2^3) - 3(f_{11}/\varepsilon_1/\varepsilon_2)(f_{01}/\varepsilon_2) - 3(f_{12}/\varepsilon_1/\varepsilon_2^2)(f_{01}/\varepsilon_2) + \\
&3(f_{11}/\varepsilon_1/\varepsilon_2)(f_{01}/\varepsilon_2)^2 - 3(f_{10}/\varepsilon_1)(f_{01}/\varepsilon_2)^3 - (f_{10}/\varepsilon_1)(f_{01}/\varepsilon_2) - 3(f_{02}/\varepsilon_2^2)(f_{10}/\varepsilon_1) - \\
&(f_{03}/\varepsilon_2^3)(f_{10}/\varepsilon_1) + 3(f_{10}/\varepsilon_1)(f_{01}/\varepsilon_2)^2 + 3(f_{02}/\varepsilon_2^2)(f_{10}/\varepsilon_1)(f_{01}/\varepsilon_2)) + ((f_{01}/\varepsilon_2) + \\
&7(f_{02}/\varepsilon_2^2) + 6(f_{03}/\varepsilon_2^3) + (f_{04}/\varepsilon_2^4) - 4(f_{01}/\varepsilon_2)^2 - 12(f_{02}/\varepsilon_2^2)(f_{01}/\varepsilon_2) - 4(f_{03}/\varepsilon_2^3)(f_{01}/\varepsilon_2) + \\
&6(f_{01}/\varepsilon_2)^3 + 6(f_{02}/\varepsilon_2^2)(f_{01}/\varepsilon_2)^2 - 3(f_{01}/\varepsilon_2)^4)) - 3(((f_{10}/\varepsilon_1) + (f_{20}/\varepsilon_1^2) - (f_{10}/\varepsilon_1)^2) - \\
&2((f_{11}/\varepsilon_1/\varepsilon_2) - (f_{10}/\varepsilon_1)(f_{01}/\varepsilon_2)) + ((f_{01}/\varepsilon_2) + (f_{02}/\varepsilon_2^2) - (f_{01}/\varepsilon_2)^2))^2
\end{aligned}
$$

2 eff. bins : 412 terms

$$
\begin{aligned}
&\kappa_4(\Delta N) = (((f_{1000}/\varepsilon_1 + f_{0100}/\varepsilon_2) + 7(f_{2000}/\varepsilon_1^2 + f_{1100}/\varepsilon_1/\varepsilon_2 + f_{1100}/\varepsilon_1/\varepsilon_2 + f_{0200}/\varepsilon_2^2) + 6(f_{3000}/\varepsilon_1^3 + f_{2100}/\varepsilon_1^2/\varepsilon_2 + \\
&f_{2100}/\varepsilon_1^2/\varepsilon_2 + f_{1200}/\varepsilon_1/\varepsilon_2^2 + f_{2100}/\varepsilon_1^2/\varepsilon_2 + f_{1200}/\varepsilon_1/\varepsilon_2^2 + f_{1200}/\varepsilon_1/\varepsilon_2^2 + f_{0300}/\varepsilon_2^3) + (f_{4000}/\varepsilon_1^4 + f_{3100}/\varepsilon_1^3/\varepsilon_2 + f_{3100}/\varepsilon_1^3/\varepsilon_2 + \\
&f_{2200}/\varepsilon_1^2/\varepsilon_2^2 + f_{3100}/\varepsilon_1^3/\varepsilon_2 + f_{2200}/\varepsilon_1^2/\varepsilon_2^2 + f_{2200}/\varepsilon_1^2/\varepsilon_2^2 + f_{1300}/\varepsilon_1/\varepsilon_2^3 + f_{3100}/\varepsilon_1^3/\varepsilon_2 + f_{2200}/\varepsilon_1^2/\varepsilon_2^2 + f_{2200}/\varepsilon_1^2/\varepsilon_2^2 + f_{1300}/\varepsilon_1/\varepsilon_2^3 + \\
&f_{2200}/\varepsilon_1^2/\varepsilon_2^2 + f_{1300}/\varepsilon_1/\varepsilon_2^3 + f_{1300}/\varepsilon_1/\varepsilon_2^3 + f_{0400}/\varepsilon_2^4) - 4(f_{1000}/\varepsilon_1 + f_{0100}/\varepsilon_2)^2 - 12(f_{2000}/\varepsilon_1^2 + f_{1100}/\varepsilon_1/\varepsilon_2 + f_{1100}/\varepsilon_1/\varepsilon_2 + \\
&f_{0200}/\varepsilon_2^2)(f_{1000}/\varepsilon_1 + f_{0100}/\varepsilon_2) - 4(f_{3000}/\varepsilon_1^3 + f_{2100}/\varepsilon_1^2/\varepsilon_2 + f_{2100}/\varepsilon_1^2/\varepsilon_2 + f_{1200}/\varepsilon_1/\varepsilon_2^2 + f_{2100}/\varepsilon_1^2/\varepsilon_2 + f_{1200}/\varepsilon_1/\varepsilon_2^2 + \\
&f_{1200}/\varepsilon_1/\varepsilon_2^2 + f_{0300}/\varepsilon_2^3)(f_{1000}/\varepsilon_1 + f_{0100}/\varepsilon_2) + 6(f_{1000}/\varepsilon_1 + f_{0100}/\varepsilon_2)^3 + 6(f_{2000}/\varepsilon_1^2 + f_{1100}/\varepsilon_1/\varepsilon_2 + f_{1100}/\varepsilon_1/\varepsilon_2 + f_{0200}/\varepsilon_2^2)(f_{1000}/\varepsilon_1 + \\
&f_{0100}/\varepsilon_2)^2 - 3(f_{1000}/\varepsilon_1 + f_{0100}/\varepsilon_2)^4) - 4((f_{1010}/\varepsilon_1/\varepsilon_3 + f_{1001}/\varepsilon_1/\varepsilon_4 + f_{0110}/\varepsilon_2/\varepsilon_3 + f_{0101}/\varepsilon_2/\varepsilon_4) - (f_{1000}/\varepsilon_1 + f_{0100}/\varepsilon_2)(f_{0010}/\varepsilon_3 + \\
&f_{0001}/\varepsilon_4) + 3(f_{2010}/\varepsilon_1^2/\varepsilon_3 + f_{2001}/\varepsilon_1^2/\varepsilon_4 + f_{1110}/\varepsilon_1/\varepsilon_2/\varepsilon_3 + f_{1101}/\varepsilon_1/\varepsilon_2/\varepsilon_4 + f_{1110}/\varepsilon_1/\varepsilon_2/\varepsilon_3 + f_{1101}/\varepsilon_1/\varepsilon_2/\varepsilon_4 + f_{0210}/\varepsilon_2^2/\varepsilon_3 + \\
&f_{0201}/\varepsilon_2^2/\varepsilon_4) - 3(f_{2000}/\varepsilon_1^2 + f_{1100}/\varepsilon_1/\varepsilon_2 + f_{1100}/\varepsilon_1/\varepsilon_2 + f_{0200}/\varepsilon_2^2)(f_{0010}/\varepsilon_3 + f_{0001}/\varepsilon_4) + (f_{3010}/\varepsilon_1^3/\varepsilon_3 + f_{3001}/\varepsilon_1^3/\varepsilon_4 + \\
&f_{2110}/\varepsilon_1^2/\varepsilon_2/\varepsilon_3 + f_{2101}/\varepsilon_1^2/\varepsilon_2/\varepsilon_4 + f_{2110}/\varepsilon_1^2/\varepsilon_2/\varepsilon_3 + f_{2101}/\varepsilon_1^2/\varepsilon_2/\varepsilon_4 + f_{1210}/\varepsilon_1/\varepsilon_2^2/\varepsilon_3 + f_{1201}/\varepsilon_1/\varepsilon_2^2/\varepsilon_4 + f_{2110}/\varepsilon_1^2/\varepsilon_2/\varepsilon_3 + \\
&f_{2101}/\varepsilon_1^2/\varepsilon_2/\varepsilon_4 + f_{1210}/\varepsilon_1/\varepsilon_2^2/\varepsilon_3 + f_{1201}/\varepsilon_1/\varepsilon_2^2/\varepsilon_4 + f_{1210}/\varepsilon_1/\varepsilon_2^2/\varepsilon_3 + f_{1201}/\varepsilon_1/\varepsilon_2^2/\varepsilon_4 + f_{0310}/\varepsilon_2^3/\varepsilon_3 + f_{0301}/\varepsilon_2^3/\varepsilon_4) - \\
&(f_{3000}/\varepsilon_1^3 + f_{2100}/\varepsilon_1^2/\varepsilon_2 + f_{2100}/\varepsilon_1^2/\varepsilon_2 + f_{1200}/\varepsilon_1/\varepsilon_2^2 + f_{2100}/\varepsilon_1^2/\varepsilon_2 + f_{1200}/\varepsilon_1/\varepsilon_2^2 + f_{1200}/\varepsilon_1/\varepsilon_2^2 + f_{0300}/\varepsilon_2^3)(f_{0010}/\varepsilon_3 + \\
&f_{0001}/\varepsilon_4) - 3(f_{1010}/\varepsilon_1/\varepsilon_3 + f_{1001}/\varepsilon_1/\varepsilon_4 + f_{0110}/\varepsilon_2/\varepsilon_3 + f_{0101}/\varepsilon_2/\varepsilon_4)(f_{1000}/\varepsilon_1 + f_{0100}/\varepsilon_2) + 3(f_{1000}/\varepsilon_1 + f_{0100}/\varepsilon_2)^2(f_{0010}/\varepsilon_3 + \\
&f_{0001}/\varepsilon_4) - 3(f_{2010}/\varepsilon_1^2/\varepsilon_3 + f_{2001}/\varepsilon_1^2/\varepsilon_4 + f_{1110}/\varepsilon_1/\varepsilon_2/\varepsilon_3 + f_{1101}/\varepsilon_1/\varepsilon_2/\varepsilon_4 + f_{1110}/\varepsilon_1/\varepsilon_2/\varepsilon_3 + f_{1101}/\varepsilon_1/\varepsilon_2/\varepsilon_4 + f_{0210}/\varepsilon_2^2/\varepsilon_3 + \\
&f_{0201}/\varepsilon_2^2/\varepsilon_4)(f_{1000}/\varepsilon_1 + f_{0100}/\varepsilon_2) + 3(f_{2000}/\varepsilon_1^2 + f_{1100}/\varepsilon_1/\varepsilon_2 + f_{1100}/\varepsilon_1/\varepsilon_2 + f_{0200}/\varepsilon_2^2)(f_{1000}/\varepsilon_1 + f_{0100}/\varepsilon_2)(f_{0010}/\varepsilon_3 + \\
&f_{0001}/\varepsilon_4) + 3(f_{1010}/\varepsilon_1/\varepsilon_3 + f_{1001}/\varepsilon_1/\varepsilon_4 + f_{0110}/\varepsilon_2/\varepsilon_3 + f_{0101}/\varepsilon_2/\varepsilon_4)(f_{1000}/\varepsilon_1 + f_{0100}/\varepsilon_2)^2 - 3(f_{1000}/\varepsilon_1 + f_{0100}/\varepsilon_2)^3(f_{0010}/\varepsilon_3 + \\
&f_{0001}/\varepsilon_4)) + 6((f_{1010}/\varepsilon_1/\varepsilon_3 + f_{1001}/\varepsilon_1/\varepsilon_4 + f_{0110}/\varepsilon_2/\varepsilon_3 + f_{0101}/\varepsilon_2/\varepsilon_4) + (f_{1020}/\varepsilon_1/\varepsilon_3^2 + f_{1011}/\varepsilon_1/\varepsilon_3/\varepsilon_4 + f_{1011}/\varepsilon_1/\varepsilon_3/\varepsilon_4 + \\
&f_{1002}/\varepsilon_1/\varepsilon_4^2 + f_{0120}/\varepsilon_2/\varepsilon_3^2 + f_{0111}/\varepsilon_2/\varepsilon_3/\varepsilon_4 + f_{0111}/\varepsilon_2/\varepsilon_3/\varepsilon_4 + f_{0102}/\varepsilon_2/\varepsilon_4^2) - 2(f_{1010}/\varepsilon_1/\varepsilon_3 + f_{1001}/\varepsilon_1/\varepsilon_4 + f_{0110}/\varepsilon_2/\varepsilon_3 + \\
&f_{0101}/\varepsilon_2/\varepsilon_4)(f_{0010}/\varepsilon_3 + f_{0001}/\varepsilon_4) + (f_{1000}/\varepsilon_1 + f_{0100}/\varepsilon_2)(f_{0010}/\varepsilon_3 + f_{0001}/\varepsilon_4)^2 + (f_{2010}/\varepsilon_1^2/\varepsilon_3 + f_{2001}/\varepsilon_1^2/\varepsilon_4 + f_{1110}/\varepsilon_1/\varepsilon_2/\varepsilon_3 + \\
&f_{1101}/\varepsilon_1/\varepsilon_2/\varepsilon_4 + f_{1110}/\varepsilon_1/\varepsilon_2/\varepsilon_3 + f_{1101}/\varepsilon_1/\varepsilon_2/\varepsilon_4 + f_{0210}/\varepsilon_2^2/\varepsilon_3 + f_{0201}/\varepsilon_2^2/\varepsilon_4) + (f_{2020}/\varepsilon_1^2/\varepsilon_3^2 + f_{2011}/\varepsilon_1^2/\varepsilon_3/\varepsilon_4 + f_{2011}/\varepsilon_1^2/\varepsilon_3/\varepsilon_4 + \\
&f_{2002}/\varepsilon_1^2/\varepsilon_4^2 + f_{1120}/\varepsilon_1/\varepsilon_2/\varepsilon_3^2 + f_{1111}/\varepsilon_1/\varepsilon_2/\varepsilon_3/\varepsilon_4 + f_{1111}/\varepsilon_1/\varepsilon_2/\varepsilon_3/\varepsilon_4 + f_{1102}/\varepsilon_1/\varepsilon_2/\varepsilon_4^2 + f_{1120}/\varepsilon_1/\varepsilon_2/\varepsilon_3^2 + f_{1111}/\varepsilon_1/\varepsilon_2/\varepsilon_3/\varepsilon_4 + \\
&f_{1111}/\varepsilon_1/\varepsilon_2/\varepsilon_3/\varepsilon_4 + f_{1102}/\varepsilon_1/\varepsilon_2/\varepsilon_4^2 + f_{0220}/\varepsilon_2^2/\varepsilon_3^2 + f_{0211}/\varepsilon_2^2/\varepsilon_3/\varepsilon_4 + f_{0211}/\varepsilon_2^2/\varepsilon_3/\varepsilon_4 + f_{0202}/\varepsilon_2^2/\varepsilon_4^2) - 2(f_{2010}/\varepsilon_1^2/\varepsilon_3 + \\
&f_{2001}/\varepsilon_1^2/\varepsilon_4 + f_{1110}/\varepsilon_1/\varepsilon_2/\varepsilon_3 + f_{1101}/\varepsilon_1/\varepsilon_2/\varepsilon_4 + f_{1110}/\varepsilon_1/\varepsilon_2/\varepsilon_3 + f_{1101}/\varepsilon_1/\varepsilon_2/\varepsilon_4 + f_{0210}/\varepsilon_2^2/\varepsilon_3 + f_{0201}/\varepsilon_2^2/\varepsilon_4)(f_{0010}/\varepsilon_3 + \\
&f_{0001}/\varepsilon_4) + (f_{2000}/\varepsilon_1^2 + f_{1100}/\varepsilon_1/\varepsilon_2 + f_{1100}/\varepsilon_1/\varepsilon_2 + f_{0200}/\varepsilon_2^2)(f_{0010}/\varepsilon_3 + f_{0001}/\varepsilon_4)^2 - 2(f_{1010}/\varepsilon_1/\varepsilon_3 + f_{1001}/\varepsilon_1/\varepsilon_4 + \\
&f_{0110}/\varepsilon_2/\varepsilon_3 + f_{0101}/\varepsilon_2/\varepsilon_4)(f_{1000}/\varepsilon_1 + f_{0100}/\varepsilon_2) - 2(f_{1020}/\varepsilon_1/\varepsilon_3^2 + f_{1011}/\varepsilon_1/\varepsilon_3/\varepsilon_4 + f_{1011}/\varepsilon_1/\varepsilon_3/\varepsilon_4 + f_{1002}/\varepsilon_1/\varepsilon_4^2 + f_{0120}/\varepsilon_2/\varepsilon_3^2 + \\
&f_{0111}/\varepsilon_2/\varepsilon_3/\varepsilon_4 + f_{0111}/\varepsilon_2/\varepsilon_3/\varepsilon_4 + f_{0102}/\varepsilon_2/\varepsilon_4^2)(f_{1000}/\varepsilon_1 + f_{0100}/\varepsilon_2) + 4(f_{1010}/\varepsilon_1/\varepsilon_3 + f_{1001}/\varepsilon_1/\varepsilon_4 + f_{0110}/\varepsilon_2/\varepsilon_3 + f_{0101}/\varepsilon_2/\varepsilon_4)(f_{1000}/\varepsilon_1 + \\
&f_{0100}/\varepsilon_2)(f_{0010}/\varepsilon_3 + f_{0001}/\varepsilon_4) - 3(f_{1000}/\varepsilon_1 + f_{0100}/\varepsilon_2)^2(f_{0010}/\varepsilon_3 + f_{0001}/\varepsilon_4)^2 + (f_{1000}/\varepsilon_1 + f_{0100}/\varepsilon_2)^2(f_{0010}/\varepsilon_3 + f_{0001}/\varepsilon_4) + \\
&(f_{0020}/\varepsilon_3^2 + f_{0011}/\varepsilon_3/\varepsilon_4 + f_{0011}/\varepsilon_3/\varepsilon_4 + f_{0002}/\varepsilon_4^2)(f_{1000}/\varepsilon_1 + f_{0100}/\varepsilon_2)^2) - 4((f_{1010}/\varepsilon_1/\varepsilon_3 + f_{1001}/\varepsilon_1/\varepsilon_4 + f_{0110}/\varepsilon_2/\varepsilon_3 + \\
&f_{0101}/\varepsilon_2/\varepsilon_4) + 3(f_{1020}/\varepsilon_1/\varepsilon_3^2 + f_{1011}/\varepsilon_1/\varepsilon_3/\varepsilon_4 + f_{1011}/\varepsilon_1/\varepsilon_3/\varepsilon_4 + f_{1002}/\varepsilon_1/\varepsilon_4^2 + f_{0120}/\varepsilon_2/\varepsilon_3^2 + f_{0111}/\varepsilon_2/\varepsilon_3/\varepsilon_4 + f_{0111}/\varepsilon_2/\varepsilon_3/\varepsilon_4 + \\
&f_{0102}/\varepsilon_2/\varepsilon_4^2) + (f_{1030}/\varepsilon_1/\varepsilon_3^3 + f_{1021}/\varepsilon_1/\varepsilon_3^2/\varepsilon_4 + f_{1021}/\varepsilon_1/\varepsilon_3^2/\varepsilon_4 + f_{1012}/\varepsilon_1/\varepsilon_3/\varepsilon_4^2 + f_{1021}/\varepsilon_1/\varepsilon_3^2/\varepsilon_4 + f_{1012}/\varepsilon_1/\varepsilon_3/\varepsilon_4^2 + \\
&f_{1012}/\varepsilon_1/\varepsilon_3/\varepsilon_4^2 + f_{1003}/\varepsilon_1/\varepsilon_4^3 + f_{0130}/\varepsilon_2/\varepsilon_3^3 + f_{0121}/\varepsilon_2/\varepsilon_3^2/\varepsilon_4 + f_{0121}/\varepsilon_2/\varepsilon_3^2/\varepsilon_4 + f_{0112}/\varepsilon_2/\varepsilon_3/\varepsilon_4^2 + f_{0121}/\varepsilon_2/\varepsilon_3^2/\varepsilon_4 + f_{0112}/\varepsilon_2/\varepsilon_3/\varepsilon_4^2 + \\
&f_{0112}/\varepsilon_2/\varepsilon_3/\varepsilon_4^2 + f_{0103}/\varepsilon_2/\varepsilon_4^3) - 3(f_{1010}/\varepsilon_1/\varepsilon_3 + f_{1001}/\varepsilon_1/\varepsilon_4 + f_{0110}/\varepsilon_2/\varepsilon_3 + f_{0101}/\varepsilon_2/\varepsilon_4)(f_{0010}/\varepsilon_3 + f_{0001}/\varepsilon_4) - 3(f_{1020}/\varepsilon_1/\varepsilon_3^2 + \\
&f_{1011}/\varepsilon_1/\varepsilon_3/\varepsilon_4 + f_{1011}/\varepsilon_1/\varepsilon_3/\varepsilon_4 + f_{1002}/\varepsilon_1/\varepsilon_4^2 + f_{0120}/\varepsilon_2/\varepsilon_3^2 + f_{0111}/\varepsilon_2/\varepsilon_3/\varepsilon_4 + f_{0111}/\varepsilon_2/\varepsilon_3/\varepsilon_4 + f_{0102}/\varepsilon_2/\varepsilon_4^2)(f_{0010}/\varepsilon_3 + \\
&f_{0001}/\varepsilon_4) + 3(f_{1010}/\varepsilon_1/\varepsilon_3 + f_{1001}/\varepsilon_1/\varepsilon_4 + f_{0110}/\varepsilon_2/\varepsilon_3 + f_{0101}/\varepsilon_2/\varepsilon_4)(f_{0010}/\varepsilon_3 + f_{0001}/\varepsilon_4)^2 - 3(f_{1000}/\varepsilon_1 + f_{0100}/\varepsilon_2)(f_{0010}/\varepsilon_3 + \\
&f_{0001}/\varepsilon_4)^3 - (f_{1000}/\varepsilon_1 + f_{0100}/\varepsilon_2)(f_{0010}/\varepsilon_3 + f_{0001}/\varepsilon_4) - 3(f_{0020}/\varepsilon_3^2 + f_{0011}/\varepsilon_3/\varepsilon_4 + f_{0011}/\varepsilon_3/\varepsilon_4 + f_{0002}/\varepsilon_4^2)(f_{1000}/\varepsilon_1 + \\
&f_{0100}/\varepsilon_2) - (f_{0030}/\varepsilon_3^3 + f_{0021}/\varepsilon_3^2/\varepsilon_4 + f_{0021}/\varepsilon_3^2/\varepsilon_4 + f_{0012}/\varepsilon_3/\varepsilon_4^2 + f_{0021}/\varepsilon_3^2/\varepsilon_4 + f_{0012}/\varepsilon_3/\varepsilon_4^2 + f_{0012}/\varepsilon_3/\varepsilon_4^2 + f_{0003}/\varepsilon_4^3)(f_{1000}/\varepsilon_1 + \\
&f_{0100}/\varepsilon_2) + 3(f_{1000}/\varepsilon_1 + f_{0100}/\varepsilon_2)(f_{0010}/\varepsilon_3 + f_{0001}/\varepsilon_4)^2 + 3(f_{0020}/\varepsilon_3^2 + f_{0011}/\varepsilon_3/\varepsilon_4 + f_{0011}/\varepsilon_3/\varepsilon_4 + f_{0002}/\varepsilon_4^2)(f_{1000}/\varepsilon_1 + \\
&f_{0100}/\varepsilon_2)(f_{0010}/\varepsilon_3 + f_{0001}/\varepsilon_4)) + ((f_{0010}/\varepsilon_3 + f_{0001}/\varepsilon_4) + 7(f_{0020}/\varepsilon_3^2 + f_{0011}/\varepsilon_3/\varepsilon_4 + f_{0011}/\varepsilon_3/\varepsilon_4 + f_{0002}/\varepsilon_4^2) + 6(f_{0030}/\varepsilon_3^3 + \\
&f_{0021}/\varepsilon_3^2/\varepsilon_4 + f_{0021}/\varepsilon_3^2/\varepsilon_4 + f_{0012}/\varepsilon_3/\varepsilon_4^2 + f_{0021}/\varepsilon_3^2/\varepsilon_4 + f_{0012}/\varepsilon_3/\varepsilon_4^2 + f_{0012}/\varepsilon_3/\varepsilon_4^2 + f_{0003}/\varepsilon_4^3) + (f_{0040}/\varepsilon_3^4 + f_{0031}/\varepsilon_3^3/\varepsilon_4 + \\
&f_{0031}/\varepsilon_3^3/\varepsilon_4 + f_{0022}/\varepsilon_3^2/\varepsilon_4^2 + f_{0031}/\varepsilon_3^3/\varepsilon_4 + f_{0022}/\varepsilon_3^2/\varepsilon_4^2 + f_{0022}/\varepsilon_3^2/\varepsilon_4^2 + f_{0013}/\varepsilon_3/\varepsilon_4^3 + f_{0031}/\varepsilon_3^3/\varepsilon_4 + f_{0022}/\varepsilon_3^2/\varepsilon_4^2 + f_{0022}/\varepsilon_3^2/\varepsilon_4^2 + \\
&f_{0013}/\varepsilon_3/\varepsilon_4^3 + f_{0022}/\varepsilon_3^2/\varepsilon_4^2 + f_{0013}/\varepsilon_3/\varepsilon_4^3 + f_{0013}/\varepsilon_3/\varepsilon_4^3 + f_{0004}/\varepsilon_4^4) - 4(f_{0010}/\varepsilon_3 + f_{0001}/\varepsilon_4)^2 - 12(f_{0020}/\varepsilon_3^2 + f_{0011}/\varepsilon_3/\varepsilon_4 + \\
&f_{0011}/\varepsilon_3/\varepsilon_4 + f_{0002}/\varepsilon_4^2)(f_{0010}/\varepsilon_3 + f_{0001}/\varepsilon_4) - 4(f_{0030}/\varepsilon_3^3 + f_{0021}/\varepsilon_3^2/\varepsilon_4 + f_{0021}/\varepsilon_3^2/\varepsilon_4 + f_{0012}/\varepsilon_3/\varepsilon_4^2 + f_{0021}/\varepsilon_3^2/\varepsilon_4 + \\
&f_{0012}/\varepsilon_3/\varepsilon_4^2 + f_{0012}/\varepsilon_3/\varepsilon_4^2 + f_{0003}/\varepsilon_4^3)(f_{0010}/\varepsilon_3 + f_{0001}/\varepsilon_4) + 6(f_{0010}/\varepsilon_3 + f_{0001}/\varepsilon_4)^3 + 6(f_{0020}/\varepsilon_3^2 + f_{0011}/\varepsilon_3/\varepsilon_4 + f_{0011}/\varepsilon_3/\varepsilon_4 + \\
&f_{0002}/\varepsilon_4^2)(f_{0010}/\varepsilon_3 + f_{0001}/\varepsilon_4)^2 - 3(f_{0010}/\varepsilon_3 + f_{0001}/\varepsilon_4)^4)) - 3(((f_{1000}/\varepsilon_1 + f_{0100}/\varepsilon_2) + (f_{2000}/\varepsilon_1^2 + f_{1100}/\varepsilon_1/\varepsilon_2 + f_{1100}/\varepsilon_1/\varepsilon_2 + \\
&f_{0200}/\varepsilon_2^2) - (f_{1000}/\varepsilon_1 + f_{0100}/\varepsilon_2)^2) - 2((f_{1010}/\varepsilon_1/\varepsilon_3 + f_{1001}/\varepsilon_1/\varepsilon_4 + f_{0110}/\varepsilon_2/\varepsilon_3 + f_{0101}/\varepsilon_2/\varepsilon_4) - (f_{1000}/\varepsilon_1 + f_{0100}/\varepsilon_2)(f_{0010}/\varepsilon_3 + \\
&f_{0001}/\varepsilon_4)) + ((f_{0010}/\varepsilon_3 + f_{0001}/\varepsilon_4) + (f_{0020}/\varepsilon_3^2 + f_{0011}/\varepsilon_3/\varepsilon_4 + f_{0011}/\varepsilon_3/\varepsilon_4 + f_{0002}/\varepsilon_4^2) - (f_{0010}/\varepsilon_3 + f_{0001}/\varepsilon_4)^2))^2
\end{aligned}
$$

P. Tribedy

Efficiency correction with many efficiency bins

$$\kappa_4(\Delta N) =$$

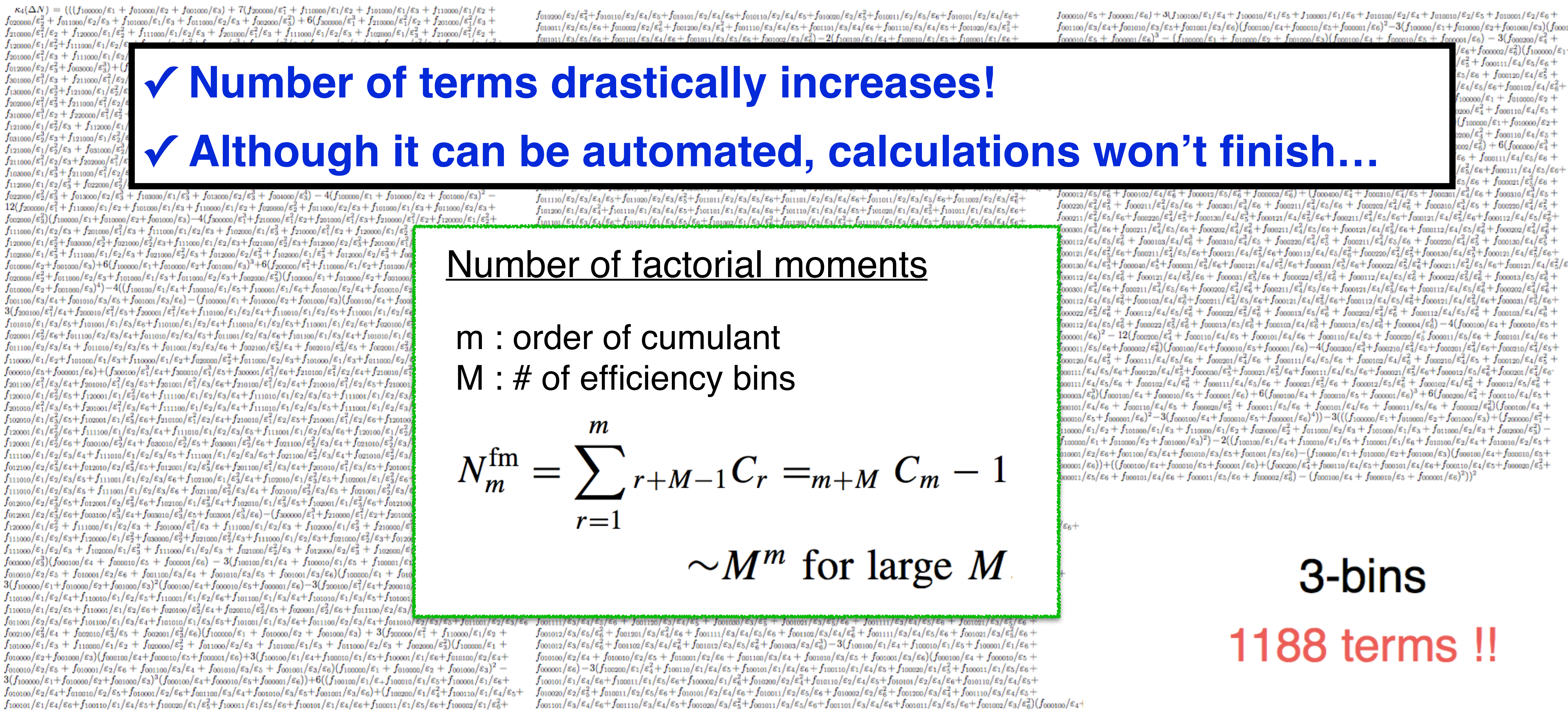
3-bins

1188 terms !!

P. Tribedy

Efficiency correction with many efficiency bins

$$\kappa_4(\Delta N) =$$



✓ **Number of terms drastically increases!**

✓ **Although it can be automated, calculations won't finish…**

<u>Number of factorial moments</u>

m : order of cumulant
M : # of efficiency bins

$$N_m^{\mathrm{fm}} = \sum_{r=1}^{m} {}_{r+M-1}C_r = {}_{m+M}C_m - 1$$

$$\sim M^m \text{ for large } M$$

3-bins

1188 terms !!

P. Tribedy

More efficient formulas

- ✓ **Derivation using factorial cumulants.**
- ✓ **For more details, see** PRC.95.064912.

measured cumulant → measured **factorial cumulant**

↓ just divide by efficiency

corrected cumulants ← corrected **factorial cumulant**

$$q_{(r,s)} = q_{(a^r/p^s)} = \sum_{i=1}^{M} (a_i^r/p_i^s) n_i$$

M : # of efficiency bins
n : # of particles
p : efficiency
a : electric charge

$$\langle Q \rangle_c = \langle q_{(1,1)} \rangle_c, \tag{62}$$

$$\langle Q^2 \rangle_c = \langle q_{(1,1)}^2 \rangle_c + \langle q_{(2,1)} \rangle_c - \langle q_{(2,2)} \rangle_c, \tag{63}$$

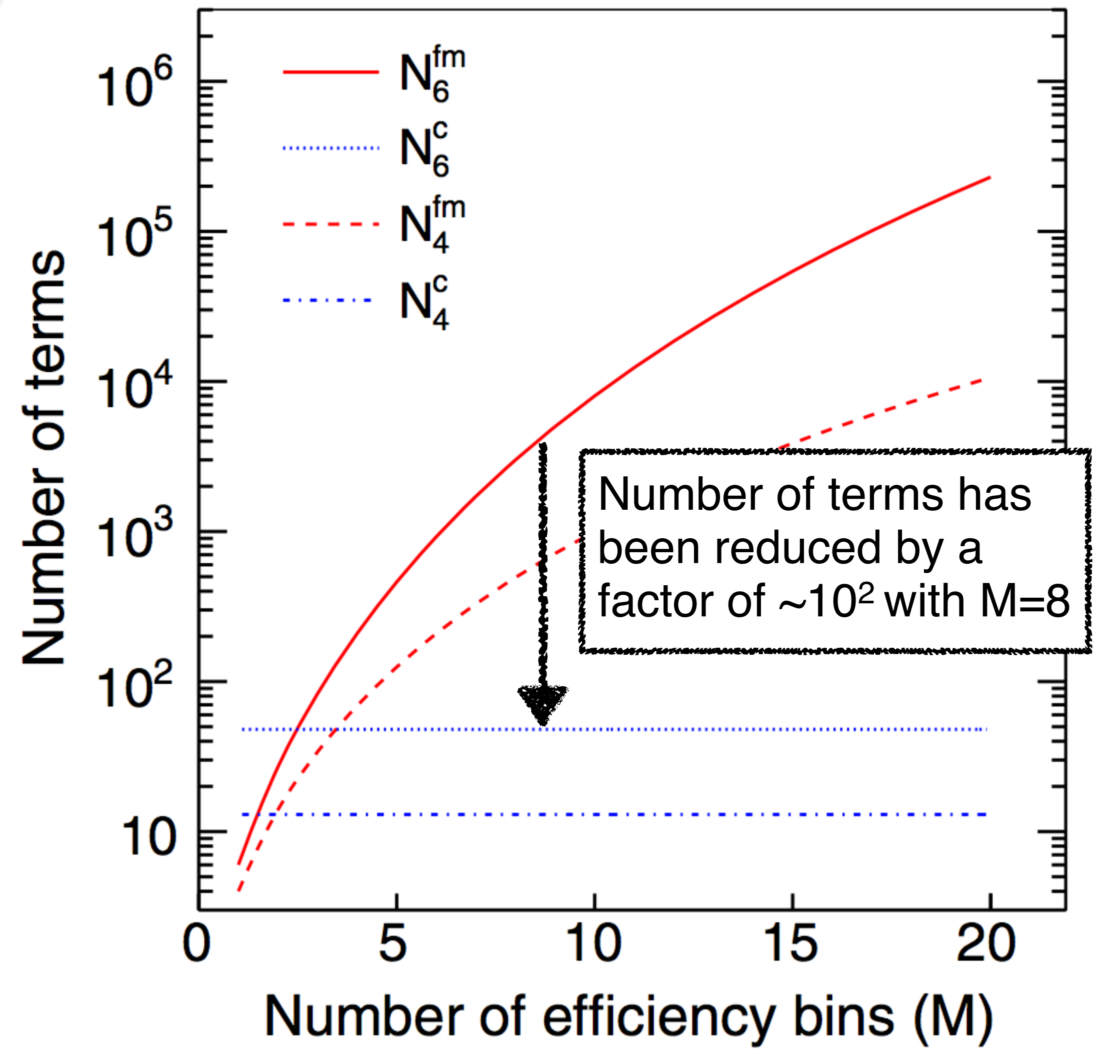
$$\langle Q^3 \rangle_c = \langle q_{(1,1)}^3 \rangle_c + 3\langle q_{(1,1)} q_{(2,1)} \rangle_c - 3\langle q_{(1,1)} q_{(2,2)} \rangle_c + \langle q_{(3,1)} \rangle_c - 3\langle q_{(3,2)} \rangle_c + 2\langle q_{(3,3)} \rangle_c, \tag{64}$$

$$\begin{aligned} \langle Q^4 \rangle_c = {} & \langle q_{(1,1)}^4 \rangle_c + 6\langle q_{(1,1)}^2 q_{(2,1)} \rangle_c - 6\langle q_{(1,1)}^2 q_{(2,2)} \rangle_c + 4\langle q_{(1,1)} q_{(3,1)} \rangle_c + 3\langle q_{(2,1)}^2 \rangle_c + 3\langle q_{(2,2)}^2 \rangle_c - 12\langle q_{(1,1)} q_{(3,2)} \rangle_c \\ & + 8\langle q_{(1,1)} q_{(3,3)} \rangle_c - 6\langle q_{(2,1)} q_{(2,2)} \rangle_c + \langle q_{(4,1)} \rangle_c - 7\langle q_{(4,2)} \rangle_c + 12\langle q_{(4,3)} \rangle_c - 6\langle q_{(4,4)} \rangle_c, \end{aligned} \tag{65}$$

$$\begin{aligned} \langle Q^5 \rangle_c = {} & \langle q_{(1,1)}^5 \rangle_c + 10\langle q_{(1,1)}^3 q_{(2,1)} \rangle_c - 10\langle q_{(1,1)}^3 q_{(2,2)} \rangle_c + 10\langle q_{(1,1)}^2 q_{(3,1)} \rangle_c - 30\langle q_{(1,1)}^2 q_{(3,2)} \rangle_c + 20\langle q_{(1,1)}^2 q_{(3,3)} \rangle_c + 15\langle q_{(2,2)}^2 q_{(1,1)} \rangle_c \\ & + 15\langle q_{(2,1)}^2 q_{(1,1)} \rangle_c - 30\langle q_{(1,1)} q_{(2,1)} q_{(2,2)} \rangle_c + 5\langle q_{(1,1)} q_{(4,1)} \rangle_c - 35\langle q_{(1,1)} q_{(4,2)} \rangle_c + 60\langle q_{(1,1)} q_{(4,3)} \rangle_c - 30\langle q_{(1,1)} q_{(4,4)} \rangle_c \\ & + 10\langle q_{(2,1)} q_{(3,1)} \rangle_c - 30\langle q_{(2,1)} q_{(3,2)} \rangle_c + 20\langle q_{(2,1)} q_{(3,3)} \rangle_c - 10\langle q_{(2,2)} q_{(3,1)} \rangle_c + 30\langle q_{(2,2)} q_{(3,2)} \rangle_c - 20\langle q_{(2,2)} q_{(3,3)} \rangle_c + \langle q_{(5,1)} \rangle_c \\ & - 15\langle q_{(5,2)} \rangle_c + 50\langle q_{(5,3)} \rangle_c - 60\langle q_{(5,4)} \rangle_c + 24\langle q_{(5,5)} \rangle_c, \end{aligned} \tag{66}$$

$$\begin{aligned} \langle Q^6 \rangle_c = {} & \langle q_{(1,1)}^6 \rangle_c + 15\langle q_{(1,1)}^4 q_{(2,1)} \rangle_c - 15\langle q_{(1,1)}^4 q_{(2,2)} \rangle_c + 20\langle q_{(1,1)}^3 q_{(3,1)} \rangle_c - 60\langle q_{(1,1)}^3 q_{(3,2)} \rangle_c + 40\langle q_{(1,1)}^3 q_{(3,3)} \rangle_c - 90\langle q_{(1,1)}^2 q_{(2,2)} q_{(2,1)} \rangle_c \\ & + 45\langle q_{(1,1)}^2 q_{(2,1)}^2 \rangle_c + 45\langle q_{(1,1)}^2 q_{(2,2)}^2 \rangle_c + 15\langle q_{(2,1)}^3 \rangle_c - 15\langle q_{(2,2)}^3 \rangle_c + 15\langle q_{(1,1)}^2 q_{(4,1)} \rangle_c - 105\langle q_{(1,1)}^2 q_{(4,2)} \rangle_c + 180\langle q_{(1,1)}^2 q_{(4,3)} \rangle_c \\ & - 90\langle q_{(1,1)}^2 q_{(4,4)} \rangle_c - 45\langle q_{(2,1)}^2 q_{(2,2)} \rangle_c + 45\langle q_{(2,2)}^2 q_{(2,1)} \rangle_c + 60\langle q_{(1,1)} q_{(2,1)} q_{(3,1)} \rangle_c - 180\langle q_{(1,1)} q_{(2,1)} q_{(3,2)} \rangle_c \\ & + 120\langle q_{(1,1)} q_{(2,1)} q_{(3,3)} \rangle_c - 60\langle q_{(1,1)} q_{(2,2)} q_{(3,1)} \rangle_c + 180\langle q_{(1,1)} q_{(2,2)} q_{(3,2)} \rangle_c - 120\langle q_{(1,1)} q_{(2,2)} q_{(3,3)} \rangle_c + 6\langle q_{(1,1)} q_{(5,1)} \rangle_c \\ & - 90\langle q_{(1,1)} q_{(5,2)} \rangle_c + 300\langle q_{(1,1)} q_{(5,3)} \rangle_c - 360\langle q_{(1,1)} q_{(5,4)} \rangle_c + 144\langle q_{(1,1)} q_{(5,5)} \rangle_c + 15\langle q_{(2,1)} q_{(4,1)} \rangle_c - 105\langle q_{(2,1)} q_{(4,2)} \rangle_c \\ & + 180\langle q_{(2,1)} q_{(4,3)} \rangle_c - 90\langle q_{(2,1)} q_{(4,4)} \rangle_c - 15\langle q_{(2,2)} q_{(4,1)} \rangle_c + 105\langle q_{(2,2)} q_{(4,2)} \rangle_c - 180\langle q_{(2,2)} q_{(4,3)} \rangle_c + 90\langle q_{(2,2)} q_{(4,4)} \rangle_c \\ & + 10\langle q_{(3,1)}^2 \rangle_c - 60\langle q_{(3,1)} q_{(3,2)} \rangle_c + 40\langle q_{(3,1)} q_{(3,3)} \rangle_c + 90\langle q_{(3,2)}^2 \rangle_c - 120\langle q_{(3,2)} q_{(3,3)} \rangle_c + 40\langle q_{(3,3)}^2 \rangle_c + \langle q_{(6,1)} \rangle_c - 31\langle q_{(6,2)} \rangle_c \\ & + 180\langle q_{(6,3)} \rangle_c - 390\langle q_{(6,4)} \rangle_c + 360\langle q_{(6,5)} \rangle_c - 120\langle q_{(6,6)} \rangle_c, \end{aligned} \tag{67}$$

- ✓ **Number of terms does not depend on efficiency bins.**
- ✓ **Calculation cost has been drastically suppressed.**



Analytical calculation

- ✓ Assume two distributions which have the same cumulants ($C_m+C_m=2C_m$) with different efficiencies.
- ✓ Apply correction using the averaged efficiency and see the deviation.

$$\bar{\varepsilon} = (\varepsilon_{\rm A}+\varepsilon_{\rm B})/2 \qquad \Delta\varepsilon = \varepsilon_{\rm A}-\varepsilon_{\rm B}$$

$$\Delta K_m = K_m - K_m^{\rm (ave)} = 2C_m - K_m^{\rm (ave)}$$

- ✓ The 1st order cumulant can be recovered by averaged efficiency.

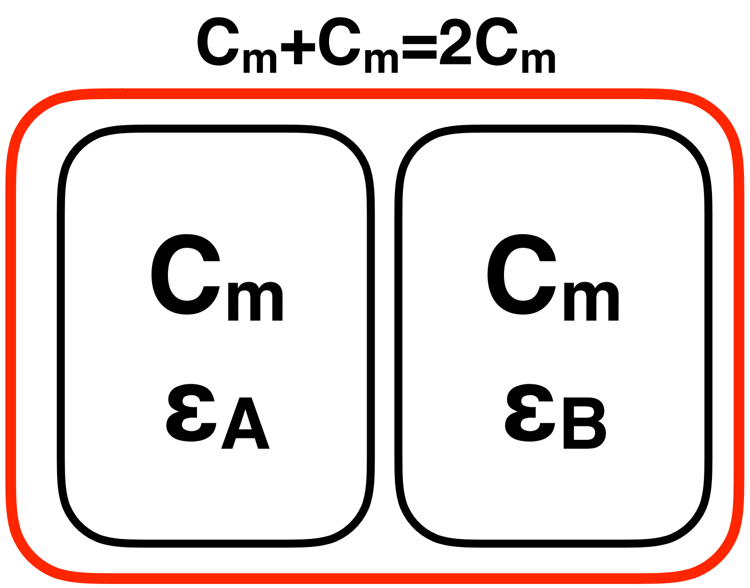
$$K_1^{\rm ave} = \langle N_A\rangle + \langle N_B\rangle = \frac{\langle n_A\rangle}{\bar{\varepsilon}} + \frac{\langle n_B\rangle}{\bar{\varepsilon}}$$
$$= \frac{\varepsilon_A C_1}{\bar{\varepsilon}} + \frac{\varepsilon_B C_1}{\bar{\varepsilon}} = 2C_1$$

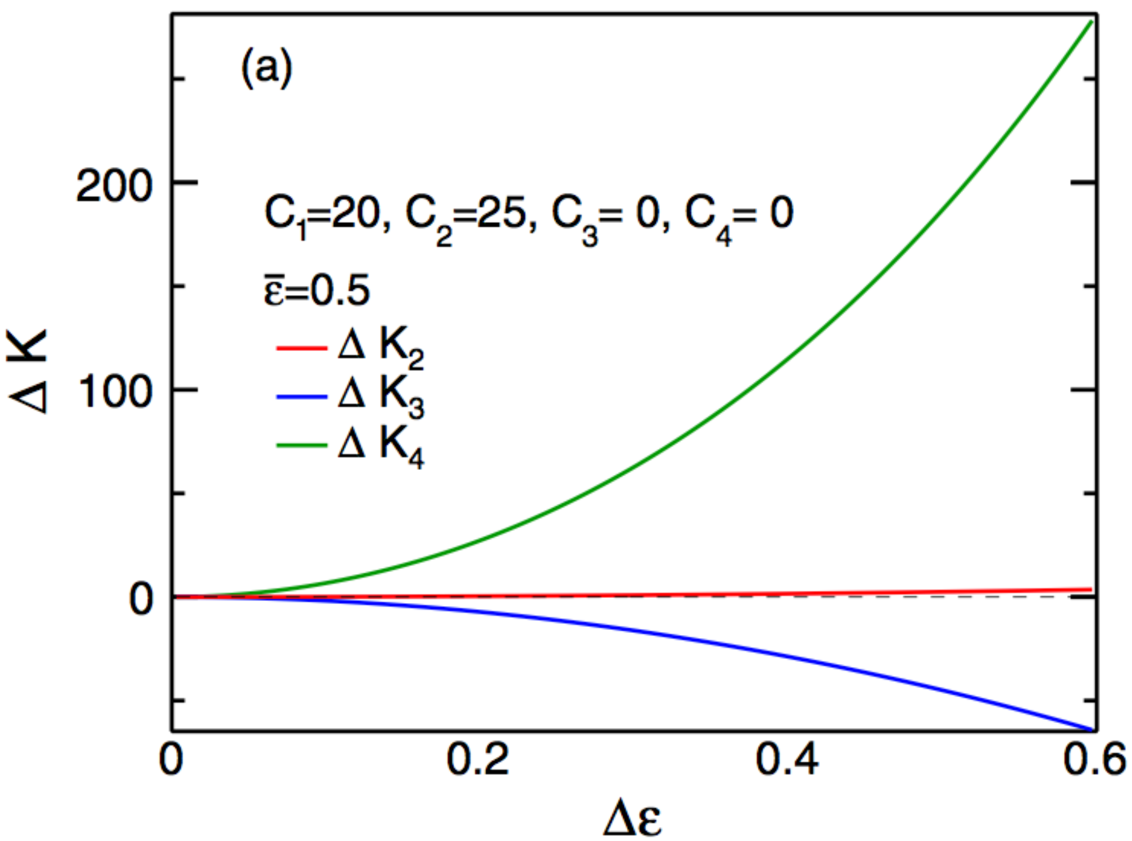
- ✓ Higher the order of cumulant is, larger deviation appears.
- ✓ Interestingly, deviation becomes zero if both distributions are Poisson ($C_m=C_1$).

$$\Delta K_2 = \frac{1}{2}\left(\frac{\Delta\varepsilon}{\bar{\varepsilon}}\right)^2 (C_2 - C_1),$$

$$\Delta K_3 = \frac{3}{2}\left(\frac{\Delta\varepsilon}{\bar{\varepsilon}}\right)^2 (C_3 - 2C_2 + C_1),$$

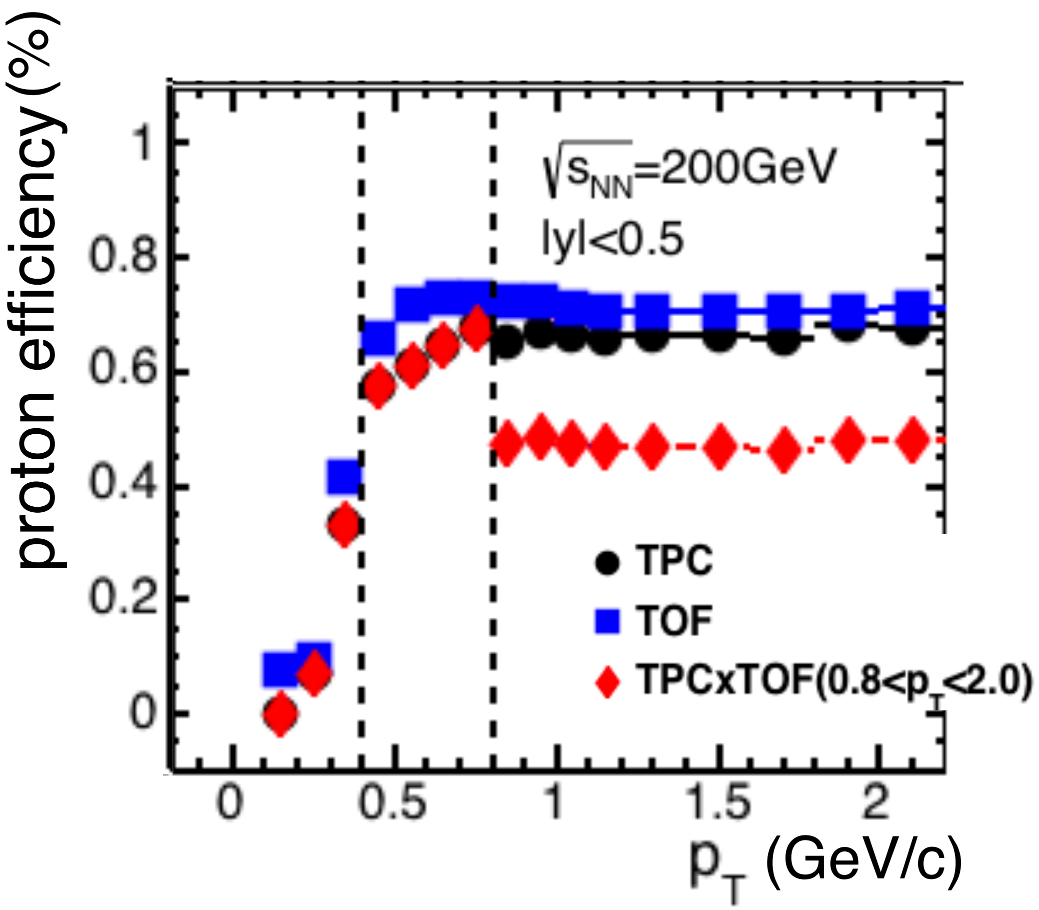
$$\Delta K_4 = \frac{1}{2}\left(\frac{\Delta\varepsilon}{\bar{\varepsilon}}\right)^2 (6C_4 - 18C_3 + 19C_2 - 7C_1) + \frac{1}{8}\left(\frac{\Delta\varepsilon}{\bar{\varepsilon}}\right)^4 (C_4 - 6C_3 + 11C_2 - 6C_1),$$

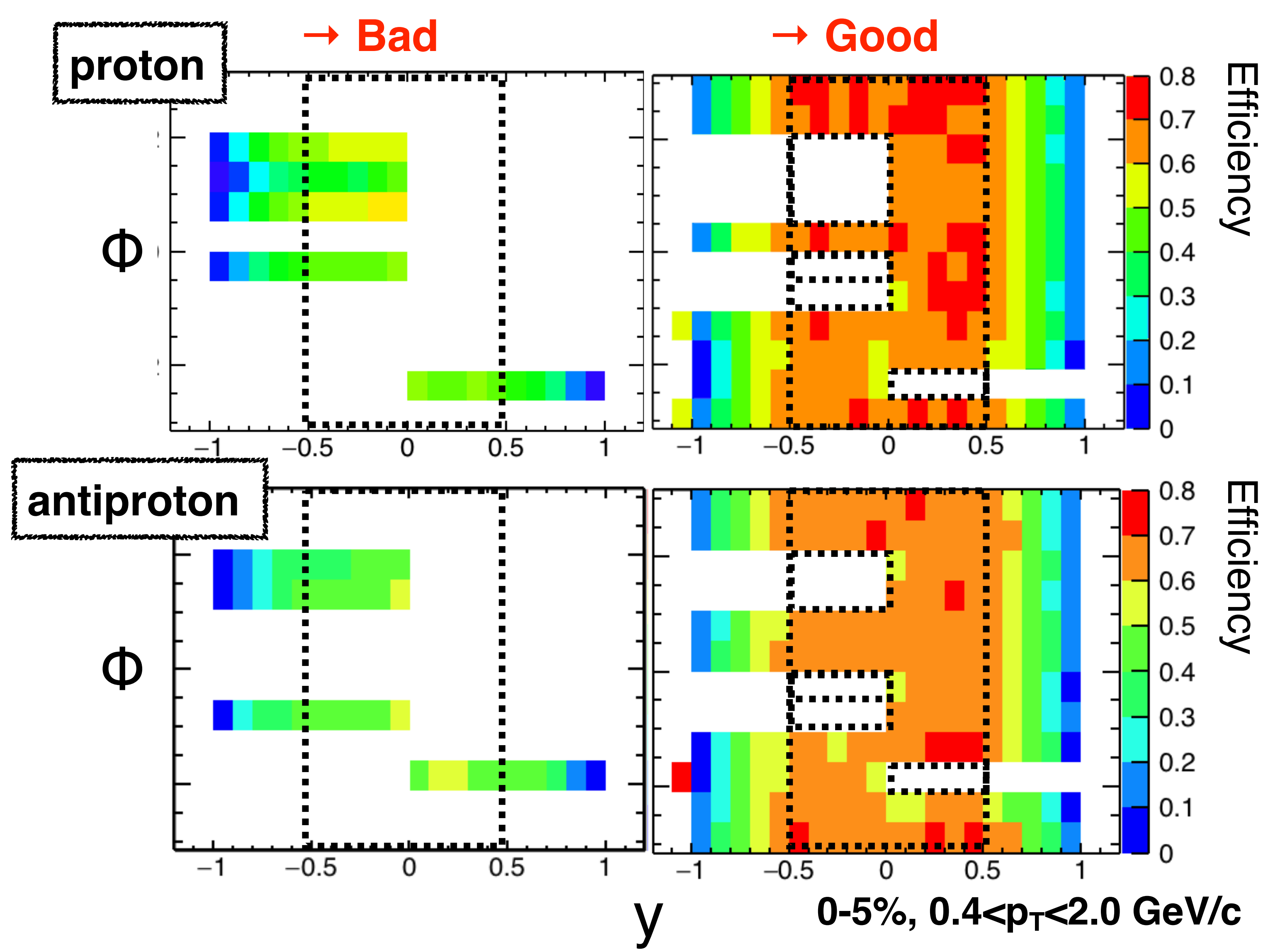




Efficiency bins

- ✓ **Number of efficiency bins = 8 = (charge) x (pT) x (TPC sector)**
- ✓ **It will take more than 1 year to calculate C_6 as a function of centrality by using conventional correction formulas, while it has been reduced to less than 2 days by using new formulas.**





Experiment and Datasets

Detector Effect

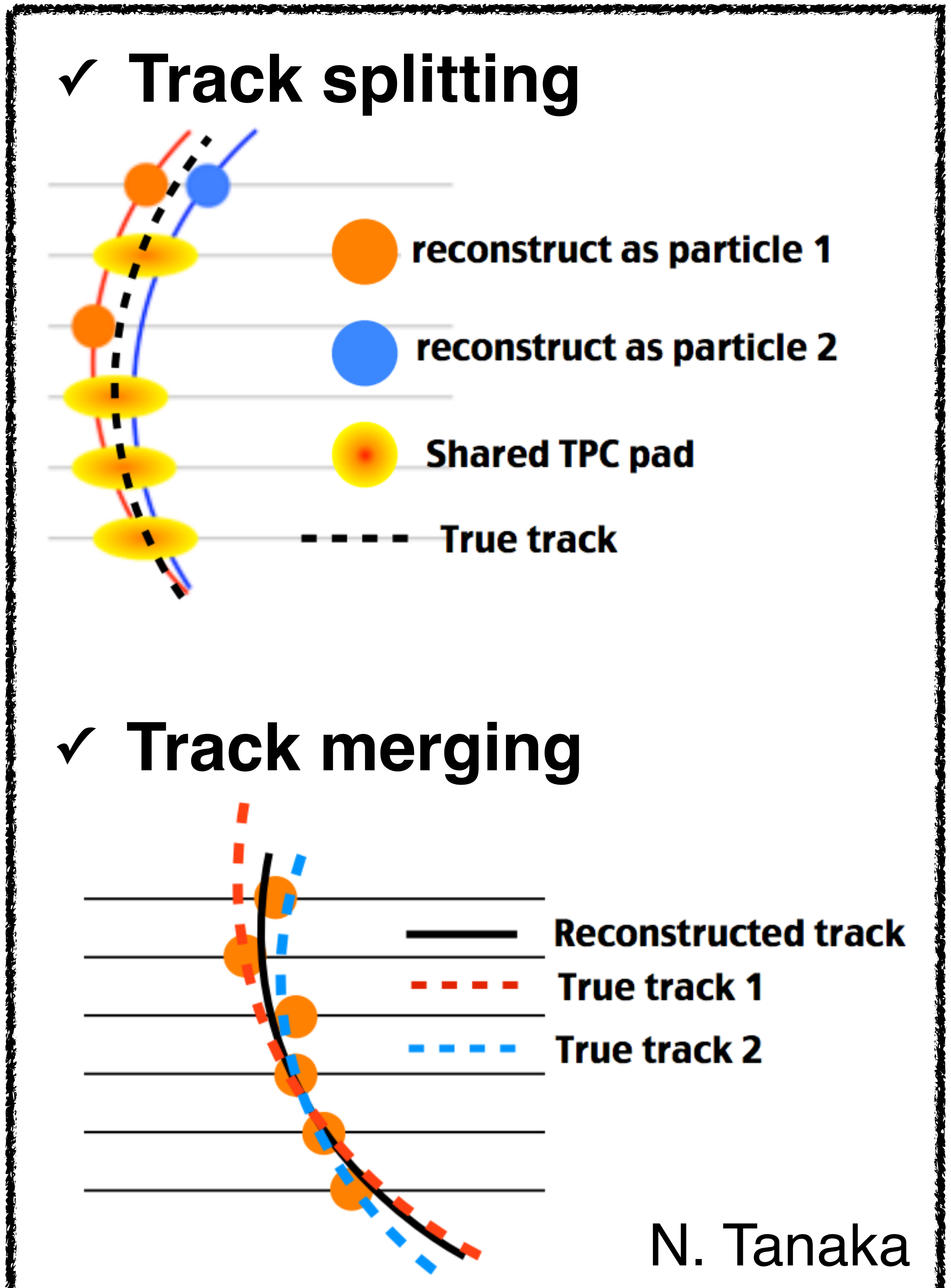
- Efficiency correction
- Unfolding

Volume Fluctuation

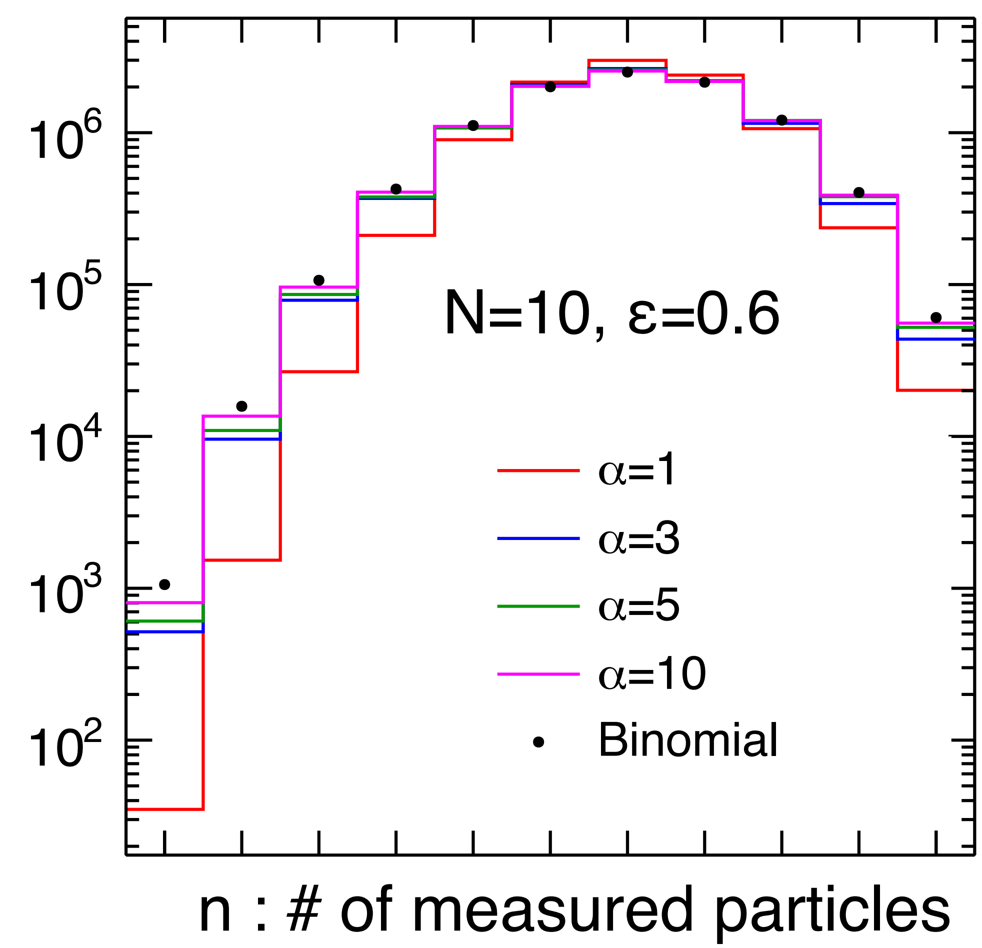
- Centrality Bin Width Correction
- Volume Fluctuation Correction

Results and Discussions

Non-binomial efficiency

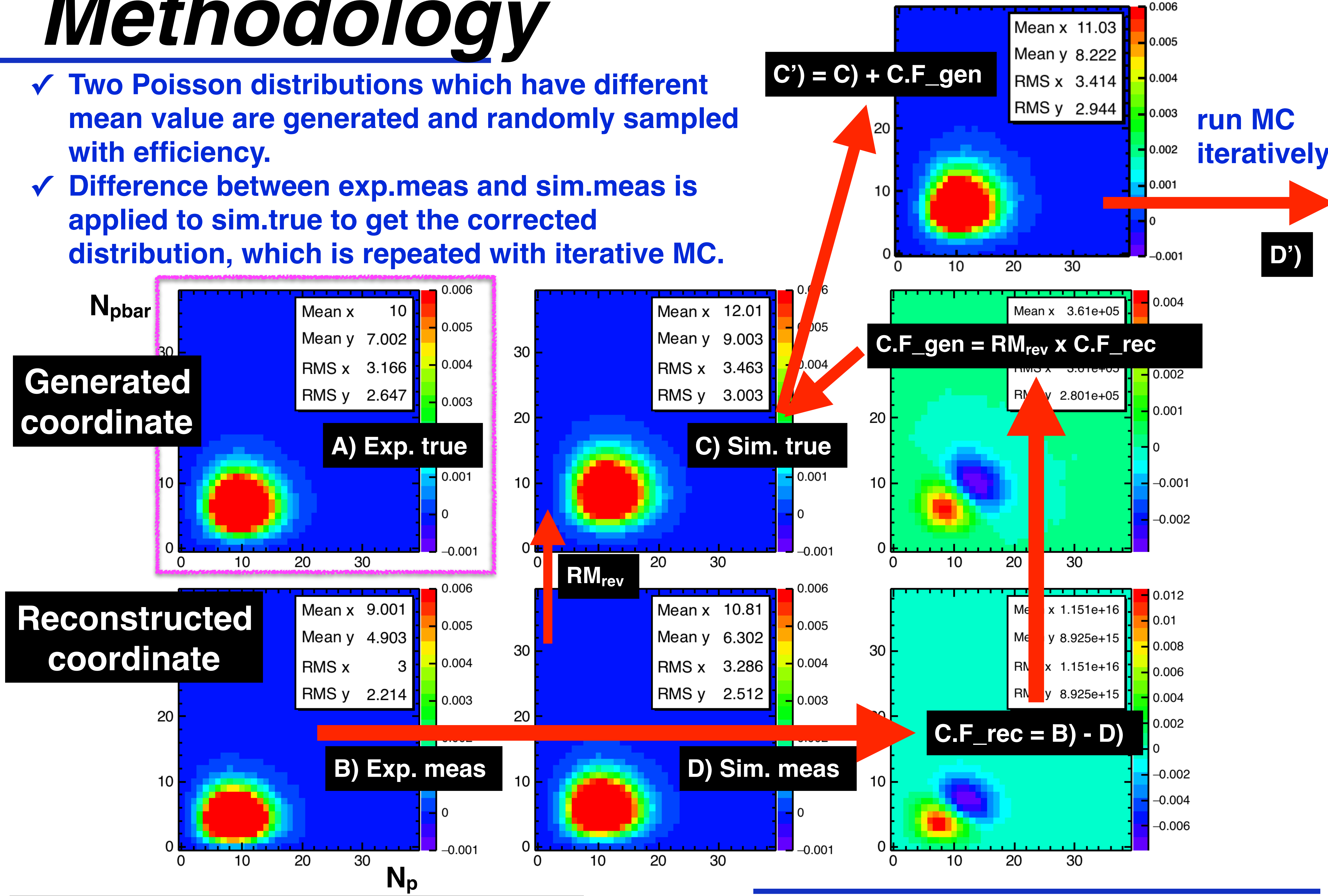


- **Efficiency correction does not work in the case of non-binomial efficiency.**
  - A. Bzdak et al : PRC.94.064907
- **Unfolding is necessary.**
  - Reconstruct the distribution itself by using well-described detector response functions.



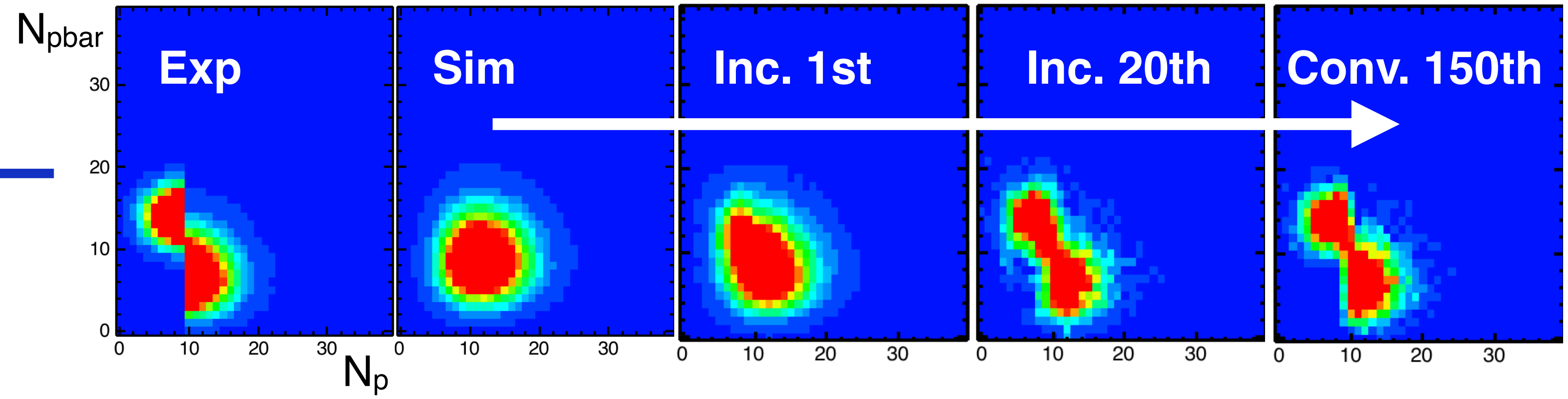
Methodology

- ✓ Two Poisson distributions which have different mean value are generated and randomly sampled with efficiency.
- ✓ Difference between exp.meas and sim.meas is applied to sim.true to get the corrected distribution, which is repeated with iterative MC.

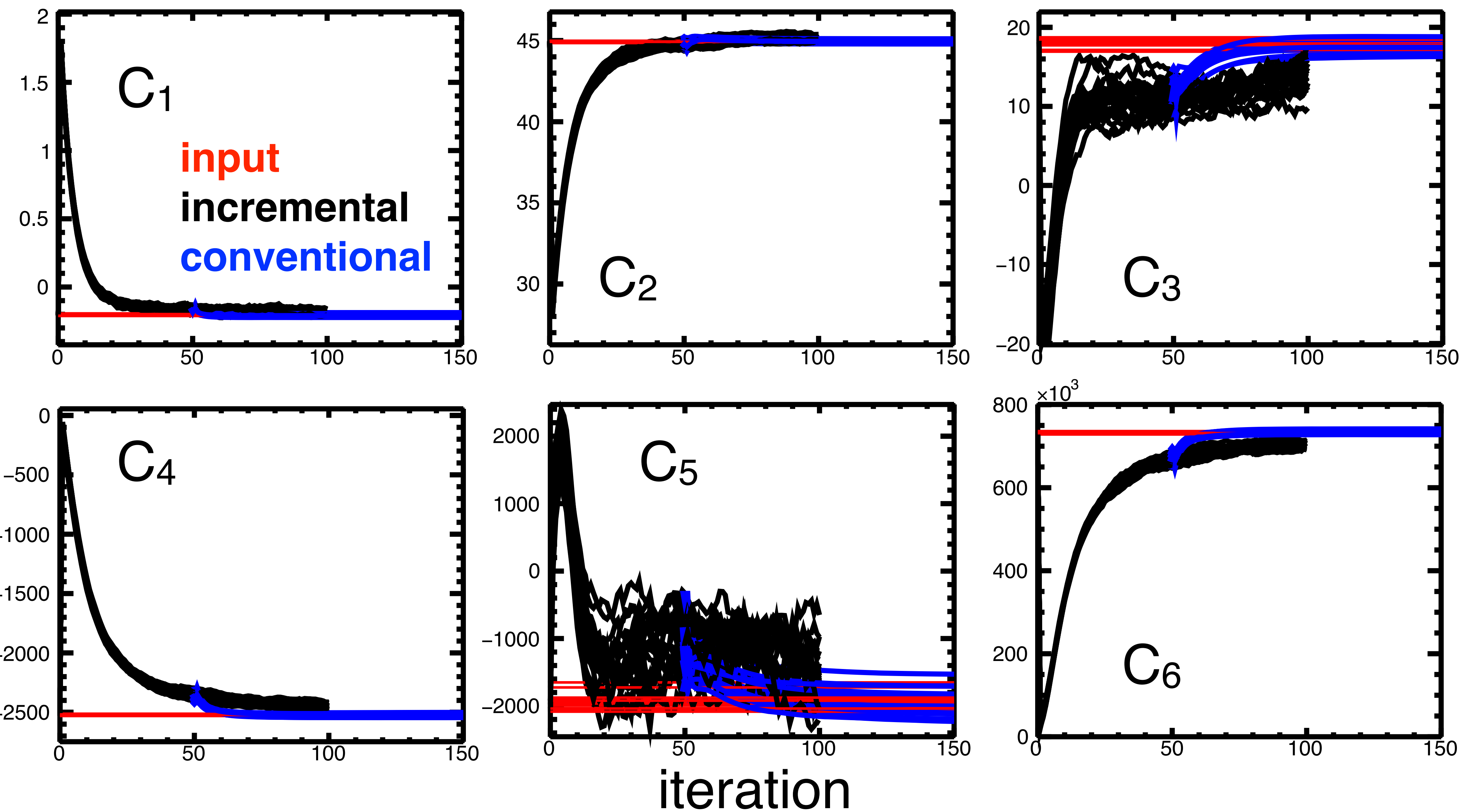


MC filter : binomial efficiency $\varepsilon_p = 0.9$, $\varepsilon_{pbar} = 0.7$

Toy model



✦ **Cumulants up to 6th order have been recovered.**



MC filter : binomial efficiency $\varepsilon_p = 0.8$, $\varepsilon_{pbar} = 0.6$

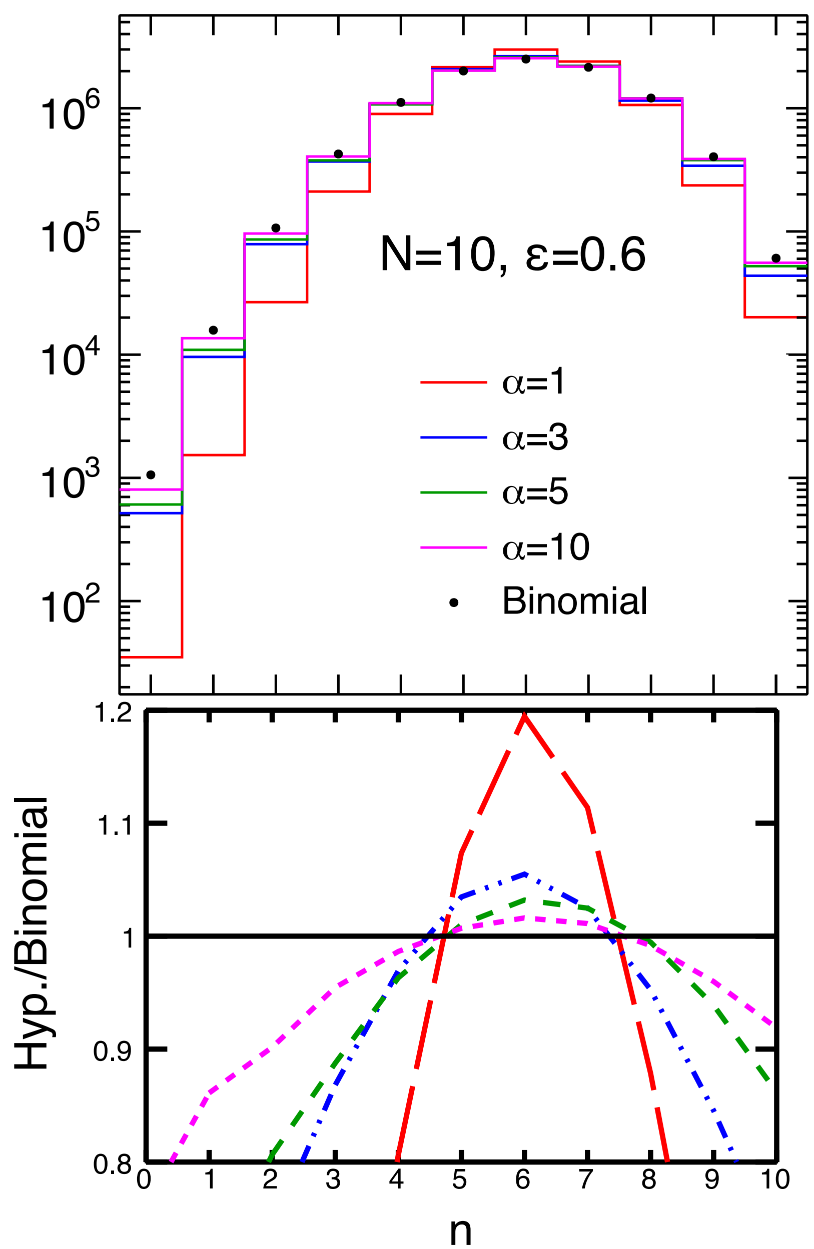
Non-binomial distribution

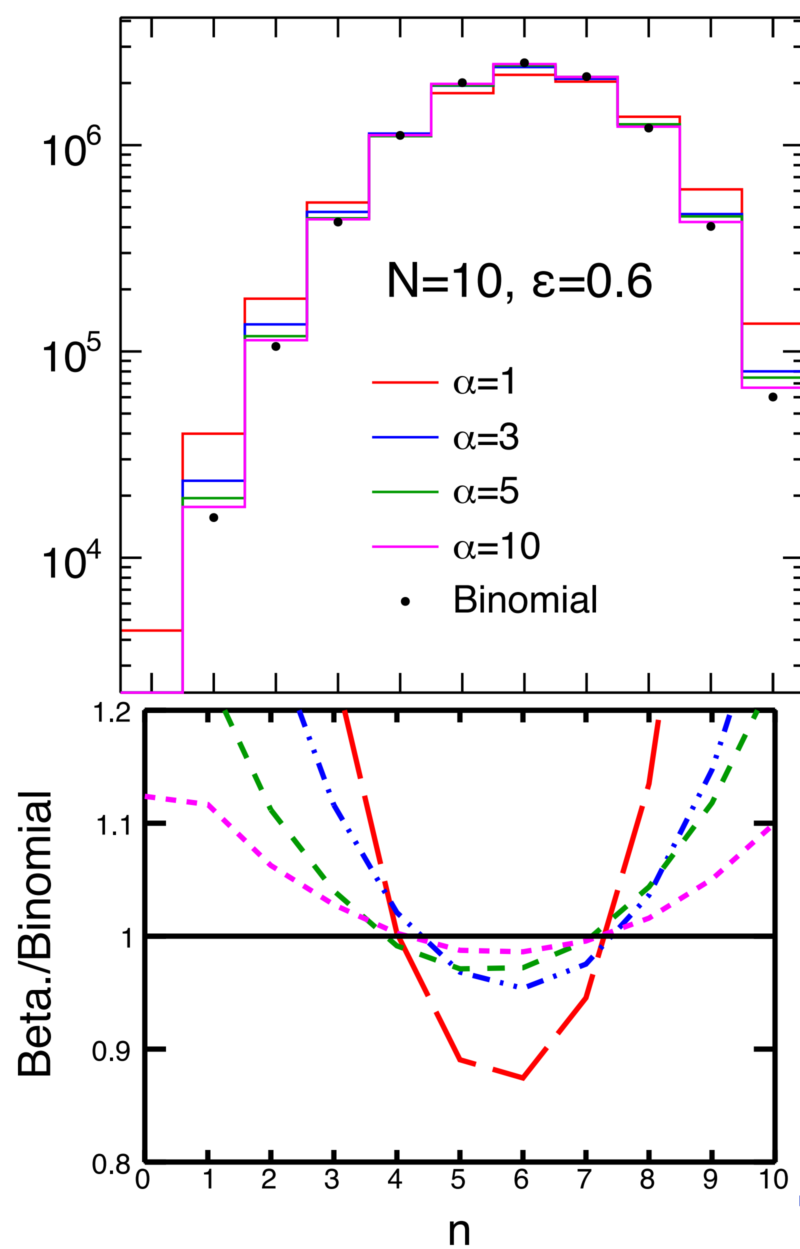
N_w : white balls, N_b: black balls

$$\frac{1-\varepsilon}{\varepsilon} = \frac{N_b}{N_w}$$ I set $N_w = 2\alpha N$ $\xrightarrow{N_w=\mathrm{const}}$ $\alpha \propto \frac{1}{N}$

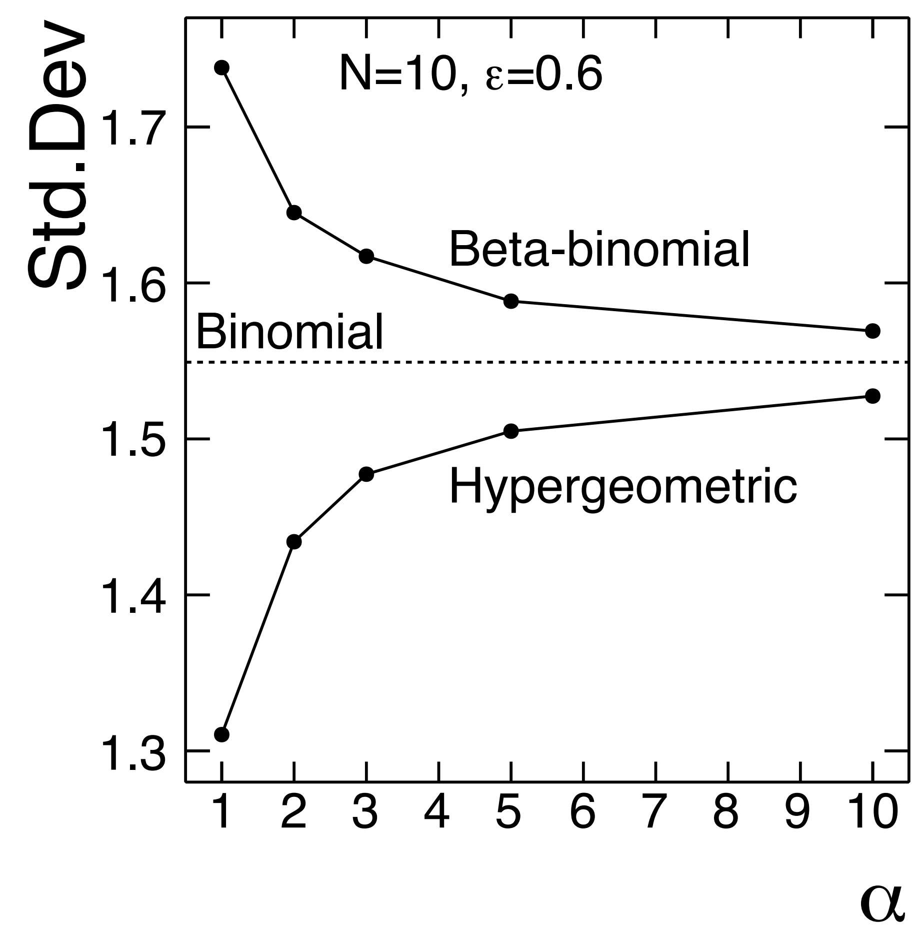
Hypergeometric distribution
Draw a ball from urn, if it is white, count particle. This is repeated without replacement.

Beta-binomial distribution
Draw a ball from urn, if it is white, count particle. And return two white balls to urn (similar for black balls).

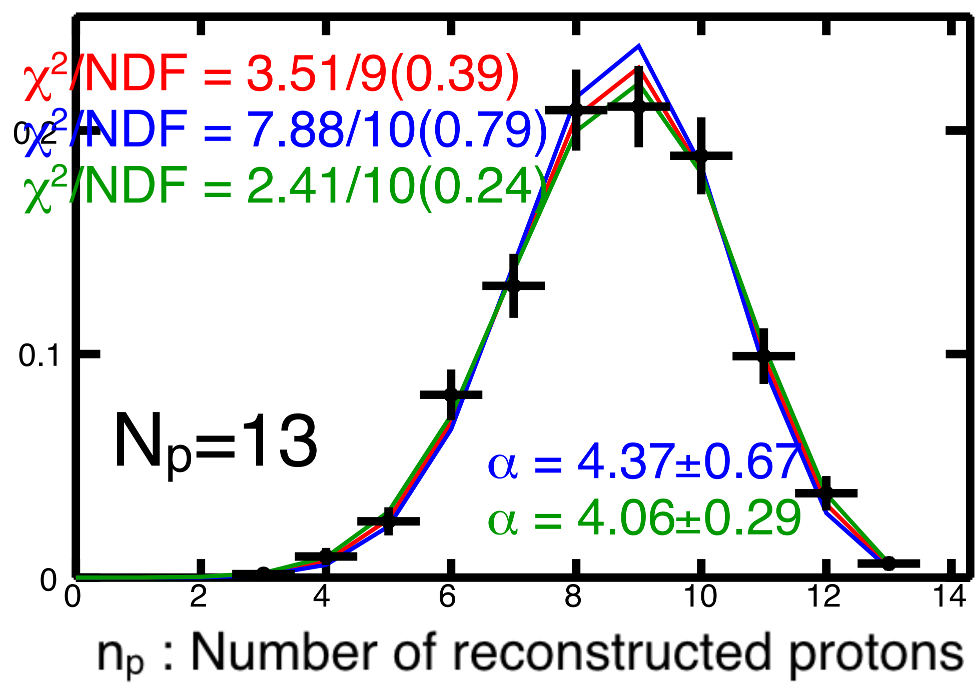




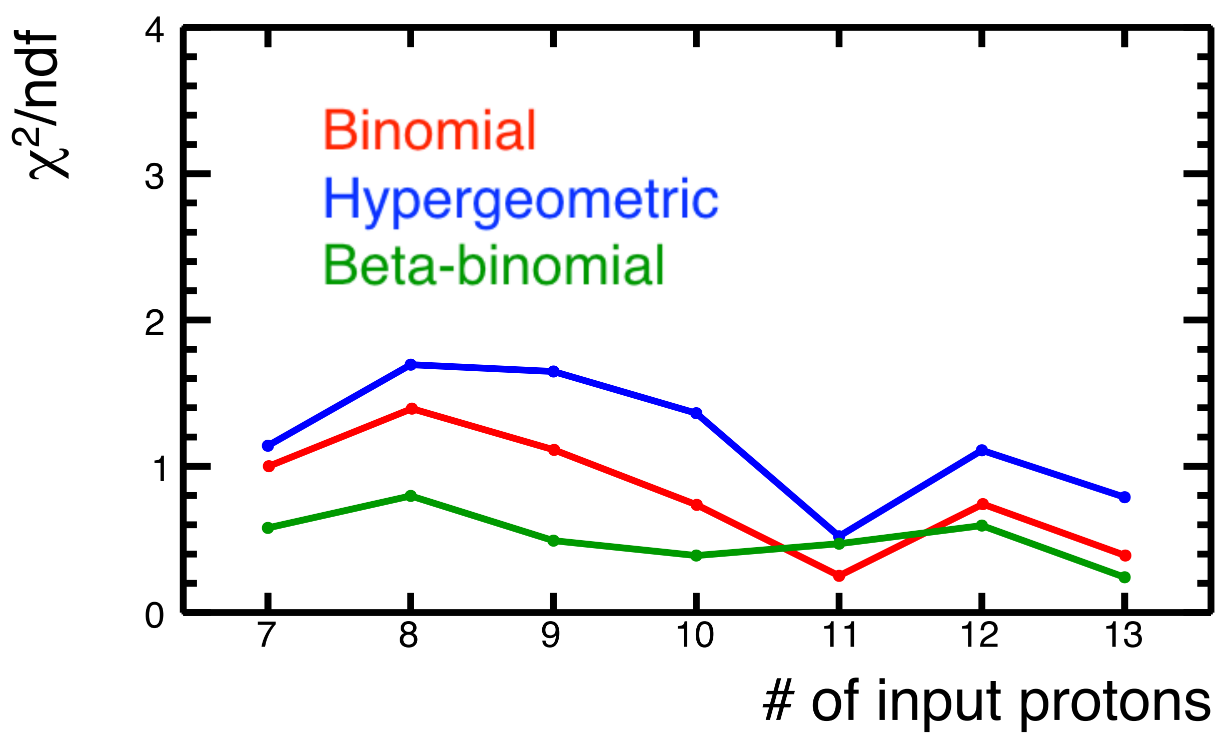
- ✓ **Smaller α for *Hypergeometric* distribution, becomes narrower than binomial distribution.**
- ✓ **Smaller α for *Beta-binomial* distribution, becomes wider than binomial distribution.**
- ✓ **Both non-binomial distributions become close to the binomial with large α.**

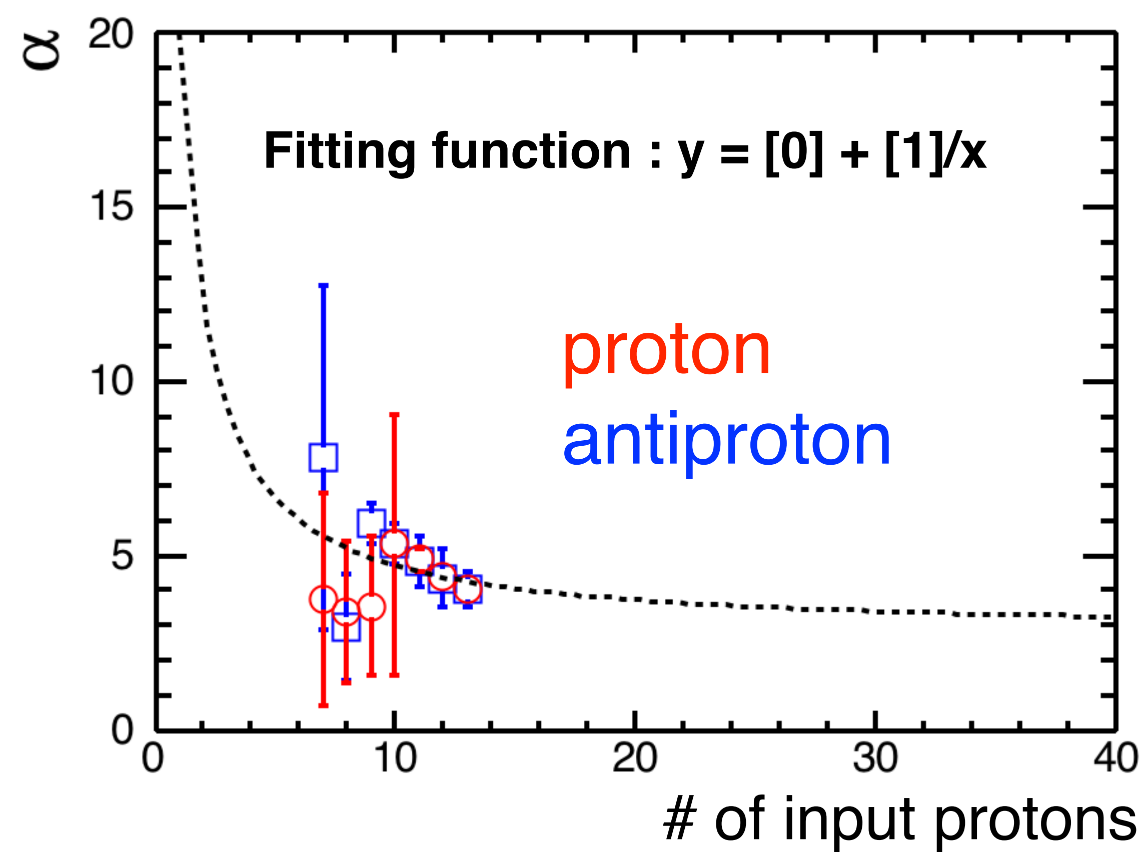


Embedding datasets



- ✓ **Beta-binomial distribution is the best function to describe the experimental data.**
- ✓ **Results of unfolding with beta-binomial model will be included in systematic uncertainties.**





Embedding samples, $\sqrt{s_{NN}}$ = 200 GeV, 0-5% centrality, 1.0<p_T<2.0 (GeV/c)

Summary for the detector effect

- More efficient formulas for efficiency correction are derived, which enables us to apply efficiency correction on C_6 with reasonable CPU time.
- Unfolding approach is establised to correct non-binomial detector effect.
- A non-binomial distribution is inferred with embedding simulations, which will be included in systematic uncertainties.

Experiment and Datasets

Detector Effect

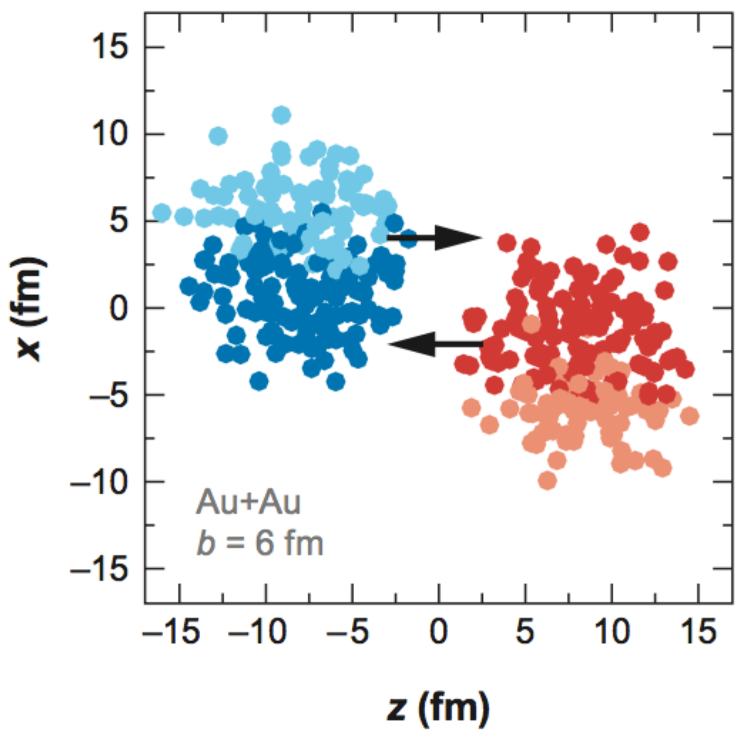
- *Efficiency correction*
- *Unfolding*

Volume Fluctuation

- ***Centrality Bin Width Correction***
- ***Volume Fluctuation Correction***

Results and Discussions

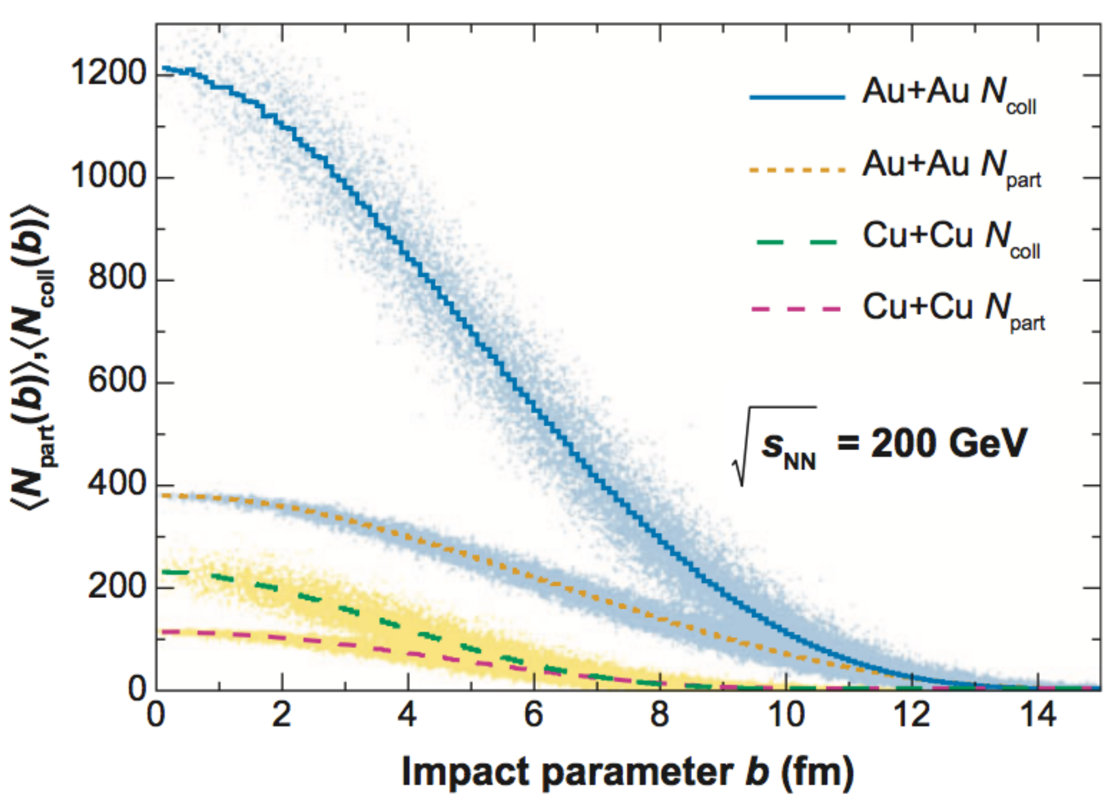
Volume fluctuation?

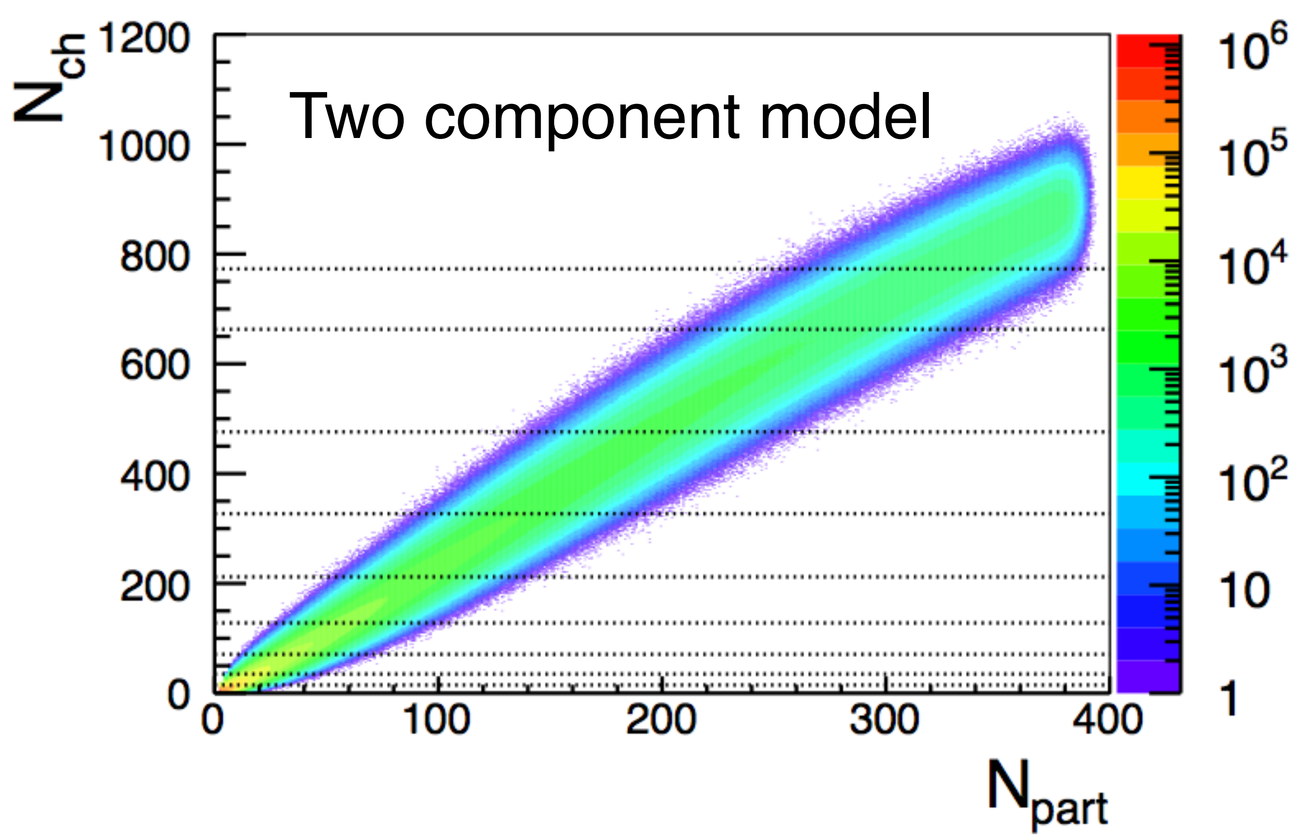


✓ **We have two kinds of "geometry" fluctuations.**

1. **Number of participant nucleons fluctuates event by event even at fixed impact parameter.**
2. **Number of produced particles fluctuates event by event even at fixed number of participant nucleons.**

→ **need to be removed**



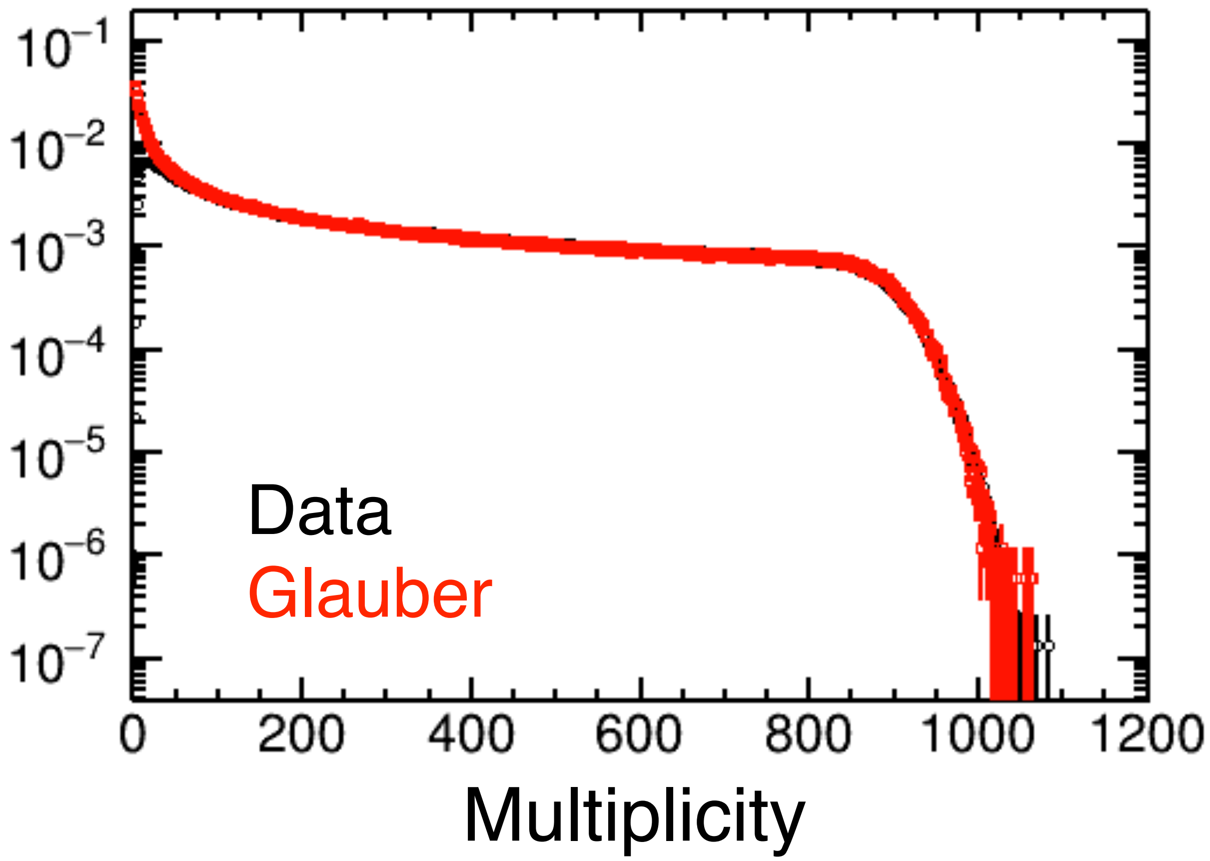


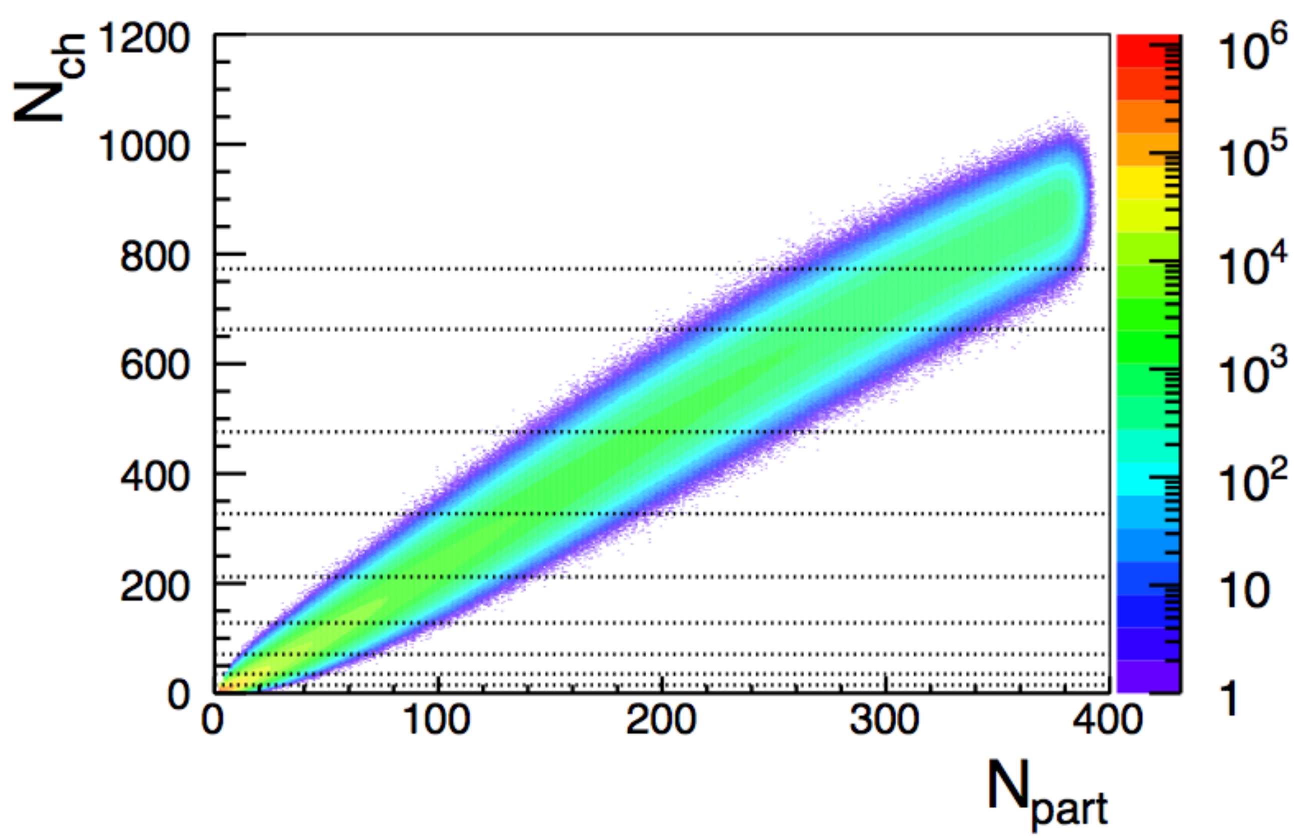
Centrality Bin Width Correction (CBWC)

- ✓ Calculate cumulants in each multiplicity bin, and average them in one centrality.
- ✓ Strongly depend on the centrality resolution.

$$C_r = \sum_i \omega_i K_r^i, \qquad \omega_i = \frac{n_i}{\sum_i n_i},$$

K_r^i : r-th order cumulant in i-th multiplicity bin
n^i : # of events in i-th multiplicity bin





Correction formulas

- ✓ **Derived based on the assumption of independent particle production from source of N_{part}.**
- ✓ **Volume fluctuations can be completely eliminated with some model inputs.**

Measured cumulant

True cumulant

Additional terms appears from the event by event participant fluctuation

$$
\begin{aligned}
\boxed{\kappa_1(\Delta N)} &= \boxed{\langle N_W\rangle\,\kappa_1(\Delta n)} \\
\boxed{\kappa_2(\Delta N)} &= \boxed{\langle N_W\rangle\,\kappa_2(\Delta n)} + \langle \Delta n\rangle^2\,\kappa_2(N_W), \\
\boxed{\kappa_3(\Delta N)} &= \boxed{\langle N_W\rangle\,\kappa_3(\Delta n)} + 3\,\langle \Delta n\rangle\,\kappa_2(\Delta n)\kappa_2(N_W) + \langle \Delta n\rangle^3\,\kappa_3(N_W), \\
\boxed{\kappa_4(\Delta N)} &= \boxed{\langle N_W\rangle\,\kappa_4(\Delta n)} + 4\,\langle \Delta n\rangle\,\kappa_3(\Delta n)\kappa_2(N_W) \\
&\quad + 3\kappa_2^2(\Delta n)\kappa_2(N_W) + 6\,\langle \Delta n\rangle^2\,\kappa_2(\Delta n)\kappa_3(N_W) + \langle \Delta n\rangle^4\,\kappa_4(N_W).
\end{aligned}
$$

P. Braun-Munzinger, A. Rustamov, J. Stachel: *arXiv:1612.00702*

Δn : net-proton per Nw
ΔN : net-proton

Up to 6th order

✓ **By using cumulant expansion techinique, correction formulas up tp 6th order cumulant has been derived.**

Measured cumulant

True cumulant

Additional terms appears from the event by event participant fluctuation

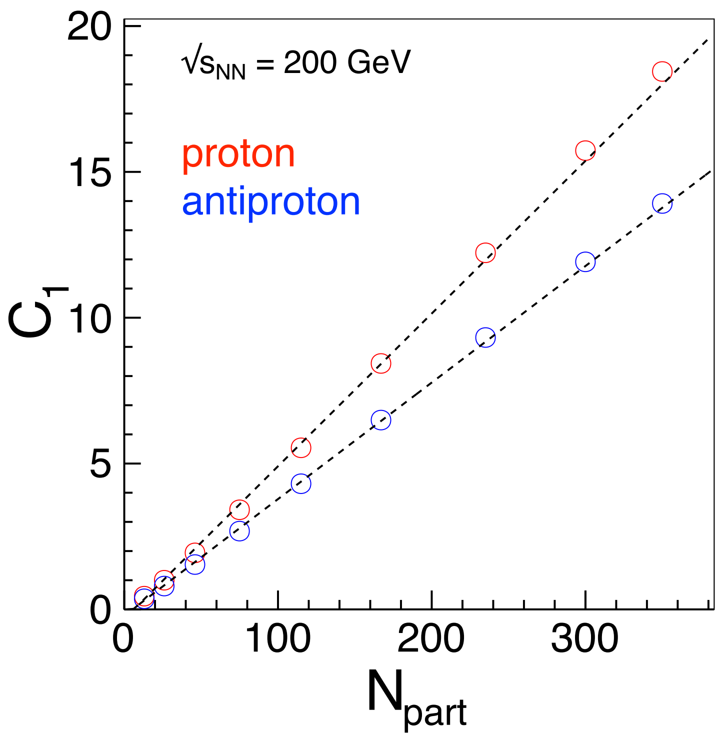
$$\boxed{\kappa_5(\Delta N)} = \boxed{\langle N_W\rangle \kappa_5(\Delta n)} + \left[5\kappa_4(\Delta n)\kappa_1(\Delta n) + 10\kappa_3(\Delta n)\kappa_2(\Delta n)\right]\kappa_2(N_W)$$
$$+\left[10\kappa_3(\Delta n)\kappa_1^2(\Delta n) + 15\kappa_2^2(\Delta n)\kappa_1(\Delta n)\right]\kappa_3(N_W)$$
$$+ 10\kappa_2(\Delta n)\kappa_1^3(\Delta n)\kappa_4(N_W) + \kappa_1^5(\Delta n)\kappa_5(N_W)$$

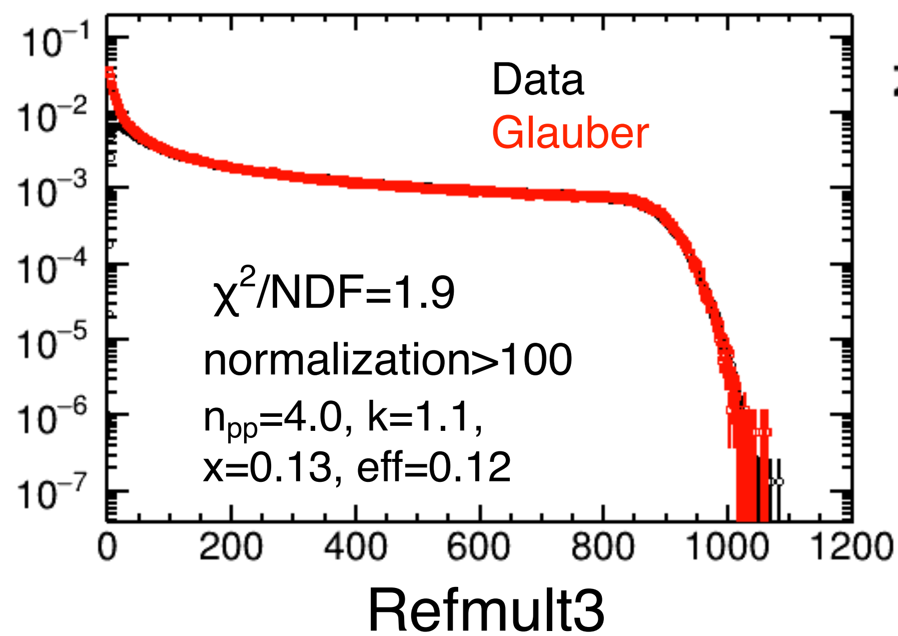
$$\boxed{\kappa_6(\Delta N)} = \boxed{\langle N_W\rangle \kappa_6(\Delta n)} + \left[6\kappa_5(\Delta n)\kappa_1(\Delta n) + 15\kappa_4(\Delta n)\kappa_2(\Delta n) + 10\kappa_3^2(\Delta n)\right]\kappa_2(N_W)$$
$$+\left[15\kappa_4(\Delta n)\kappa_1^2(\Delta n) + 60\kappa_3(\Delta n)\kappa_2(\Delta n)\kappa_1(\Delta n) + 15\kappa_2^3(\Delta n)\right]\kappa_3(N_W)$$
$$+\left[20\kappa_3(\Delta n)\kappa_1^3(\Delta n) + 45\kappa_2^2(\Delta n)\kappa_1^2(\Delta n)\right]\kappa_4(N_W)$$
$$+ 15\kappa_2(\Delta n)\kappa_1^4(\Delta n)\kappa_5(N_W) + \kappa_1^6(\Delta n)\kappa_6(N_W)$$

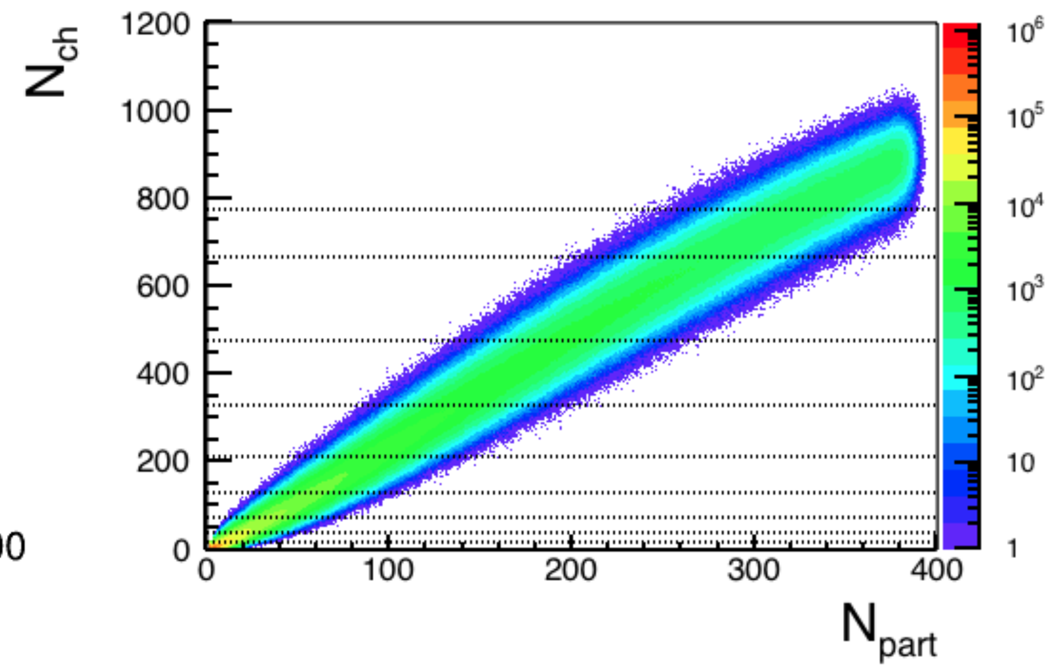
Toy model (Glauber + two-component model)

- ✓ Generate p and pbar from Npart according to the Poisson distribution.
- ✓ Cumulants from each source are independent.

- ✓ For centrality definition, charged particles are generated from source.







✦ **True cumulants can be expressed by superposition of cumulants from each N_{part}.**

$$\kappa_m(\Delta N) = \langle N_{part} \rangle \kappa_m(\Delta n)$$

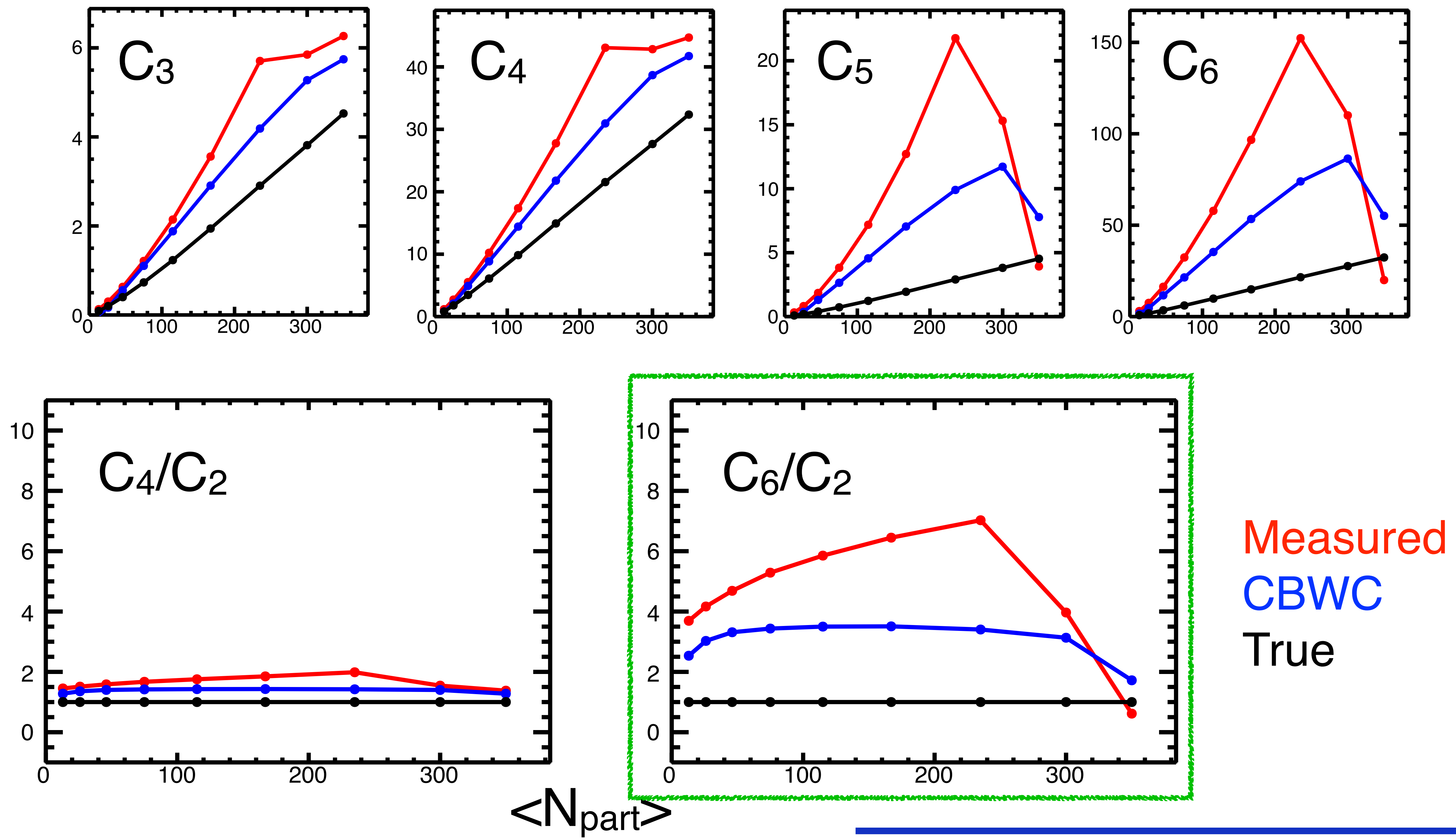
✦ **Two component model and NBD fluctuations.**

$$N_{ch} = n_{pp}\left[\frac{1-x}{2}N_{part} + xN_{coll}\right]$$

$$P_{n_{pp},k}(m) = \frac{\Gamma(m+K)}{\Gamma(m+1)\Gamma(k)}\frac{(n_{pp}/k)^m}{(n_{pp}/k+1)^{m+k}}$$

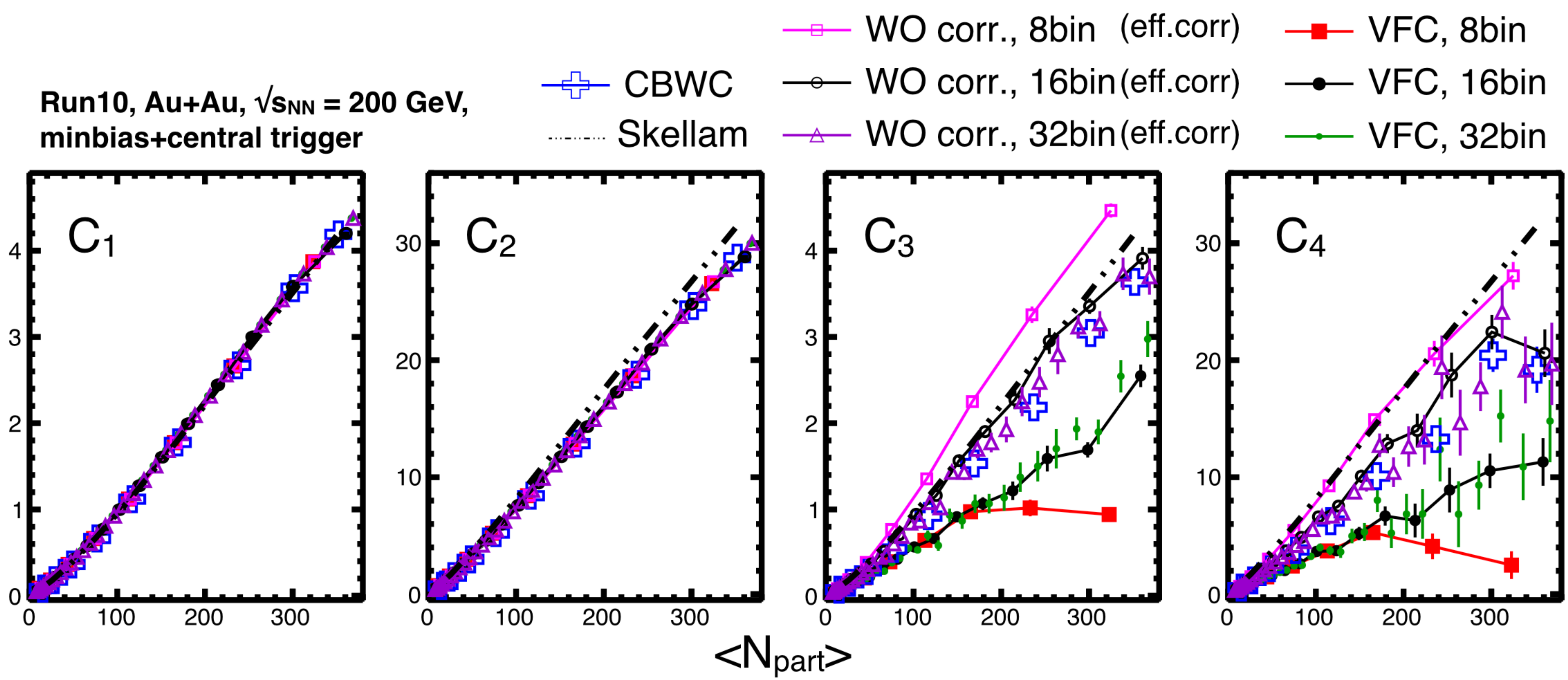
Effect of volume fluctuation

✦ **Effect of volume fluctuation on C_6 is much larger than C_4.**



Centrality bin width dependence?

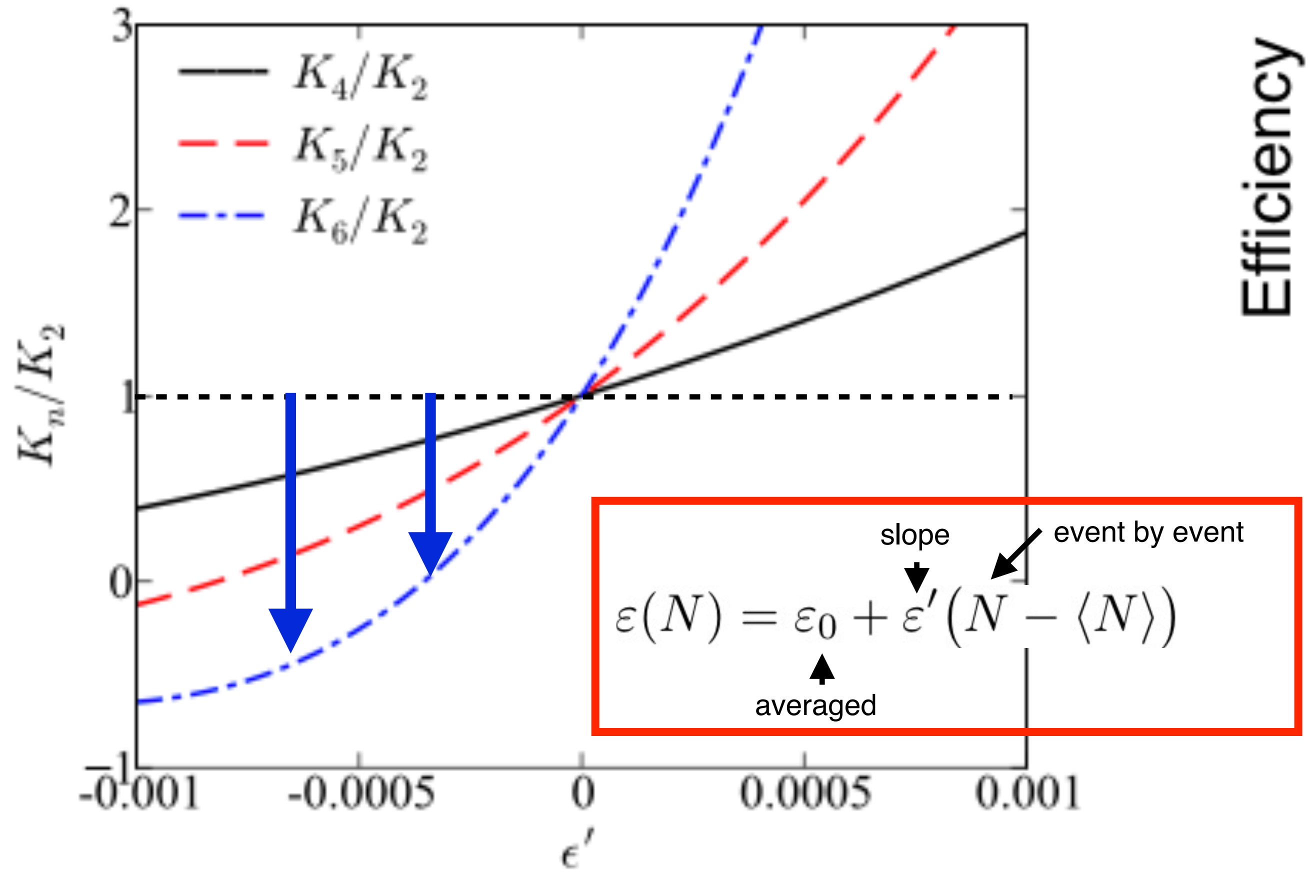
- ✓ It was found that VFC results on experimentl data up to 4th order cumulant depend on the centrality bin width.
- ✓ Results converge with small bin width.

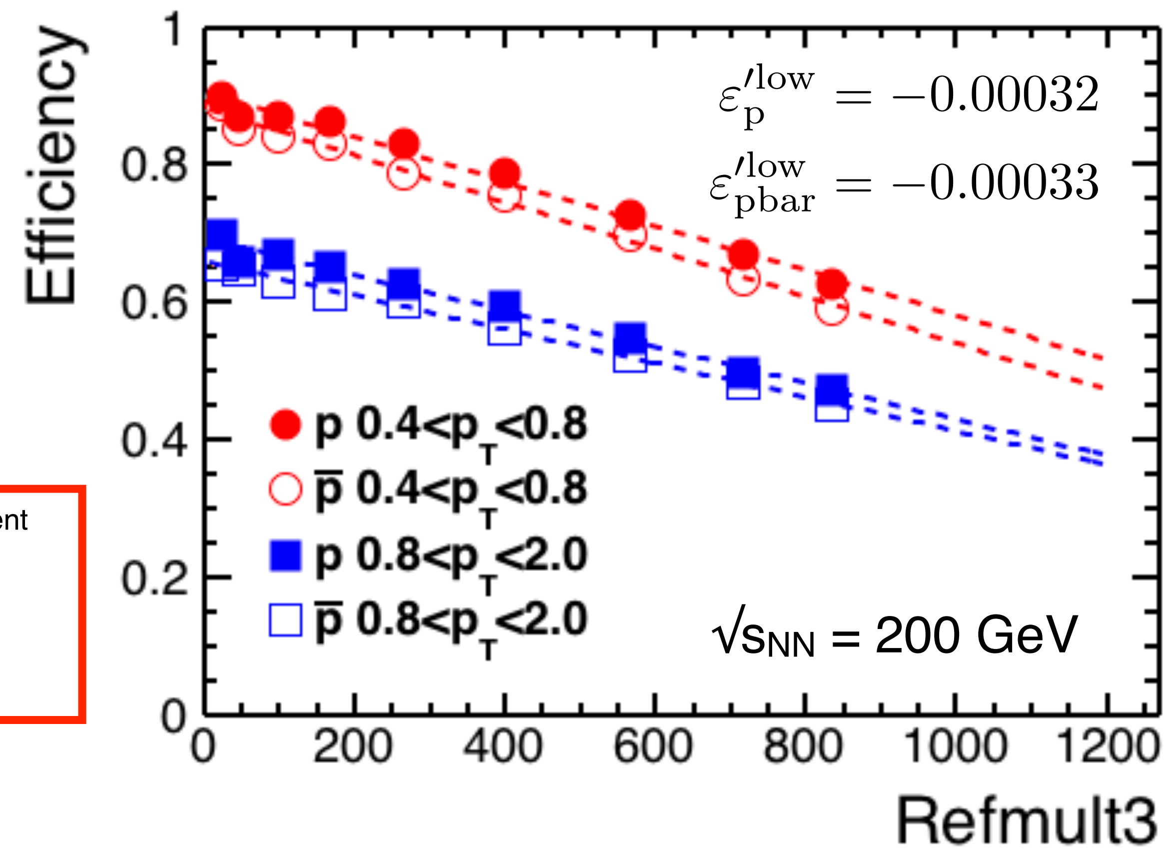


Multiplicity dependent efficiency

- **Centrality bin width dependence for VFC can be explained by multiplicity dependent efficiency.**
- **For VFC : Efficiency correction is applied using the averaged efficiency in one centrality, then apply VFC. This result is suppressed by multiplicity dependent efficiency** *PRC.94.064907*.
- **This effect will be small by reducing the centrality bin width (efficiency variation).**

PRC.94.064907





Summary for the volume fluctuation

- Correction formulas up to 6th order cumulant are derived.
- Effect of volume fluctuation on C_6 is estimated with the model, which is much larger than C_4 and cannot be eliminated by CBWC.
- VFC needs to be applied with narrow centrality bin width in order to avoid the effect of multiplicity dependent efficiency.
- However, VFC needs some model assumptions, independent particle production model, glauber model, and two-component model.
- Both CBWC and VFC will be applied to C_6/C_2.

Systematic uncertainties

variable	default	cut	
$\|n\sigma_p\|$	<2.0	<2.5	worsen purity
mass^2	$0.6<m^2<1.2$	$0.7<m^2<1.3$ $0.8<m^2<1.4$	decreas kaon contamination
nHitsFit	>20	>15	increases the fraction of track splitting
DCA	<1.0	<1.5	increases secondory protons
efficiency	$(\varepsilon_{lowpt}, \varepsilon_{highpt})$	$(1.05^*\varepsilon_{lowpt}, 1.05^*\varepsilon_{highpt})$ $(0.95^*\varepsilon_{lowpt}, 0.95^*\varepsilon_{highpt})$ $(1.05^*\varepsilon_{lowpt}, 0.95^*\varepsilon_{highpt})$ $(0.95^*\varepsilon_{lowpt}, 1.05^*\varepsilon_{highpt})$	
Detector effect correction	efficiency correcction with binomial model	unfolding with beta-binomial model	

Experiment and Datasets

Detector Effect

- *Efficiency correction*
- *Unfolding*

Volume Fluctuation

- *Centrality Bin Width Correction*
- *Volume Fluctuation Correction*

Results

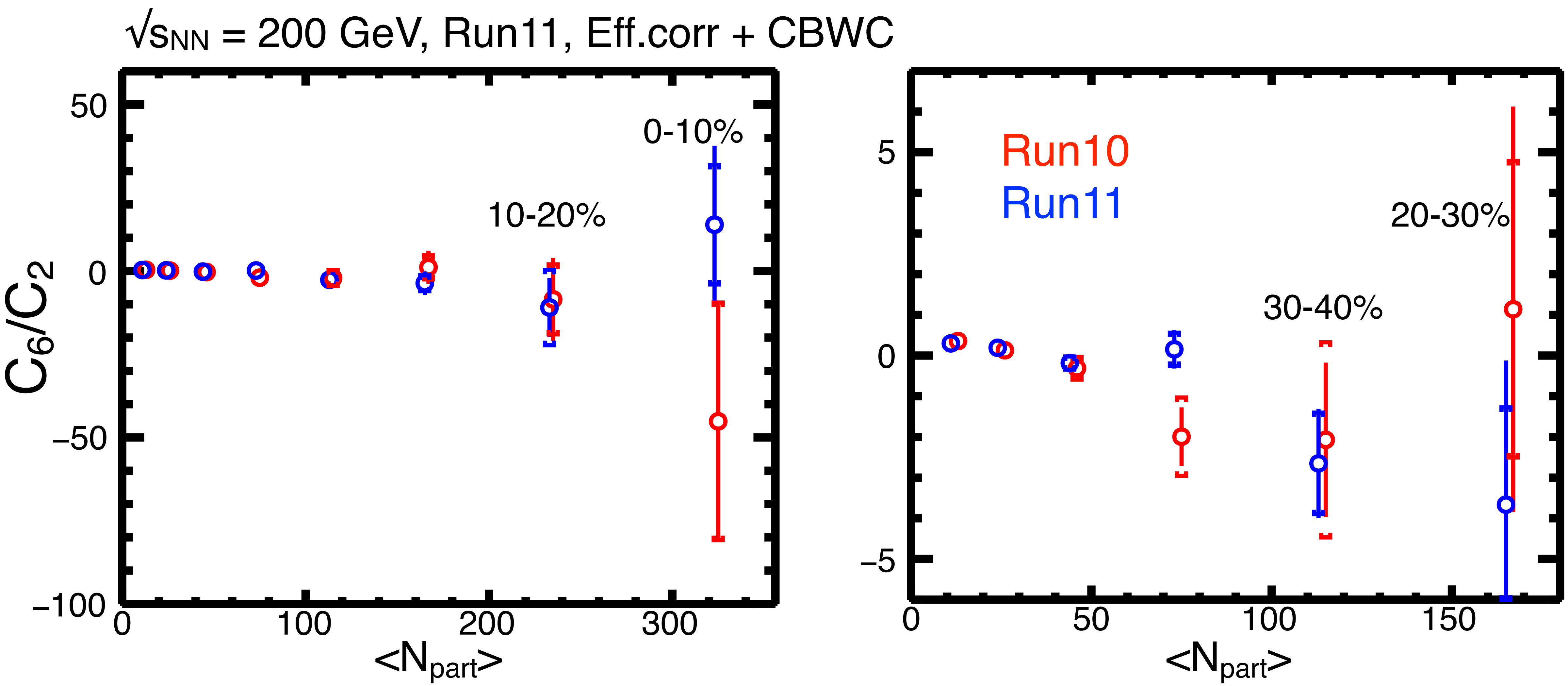
Results

✓ **Three sets of results will be shown with different combination of correction methods.**

Detector effect	Volume fluctuation	
Eff. corr	CBWC	standard in STAR
Eff. corr	VFC	new method
Unfolding with beta-binomial model	with 2.5 % bin	new method (systematic check)

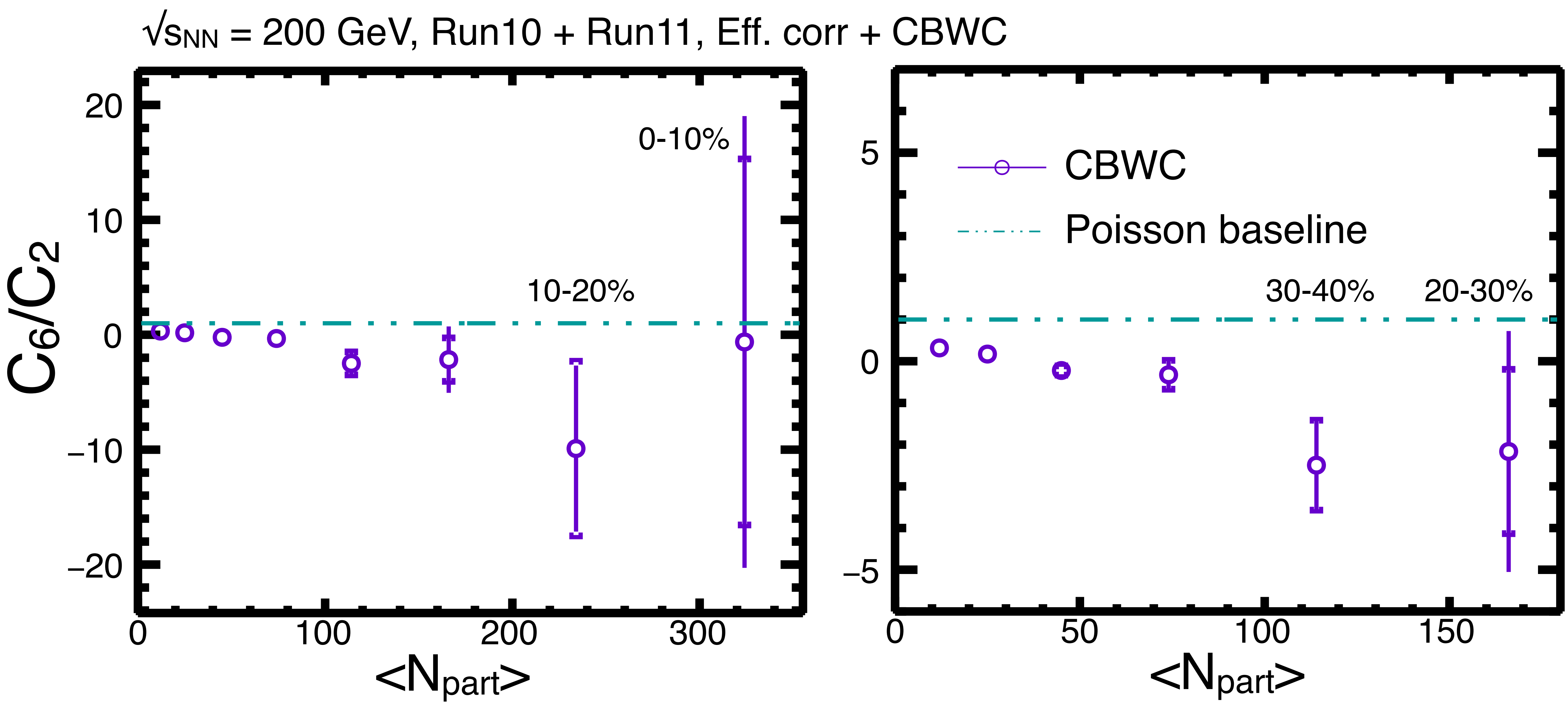
Run10 and Run11

- ✓ **Results from run10 and run11 are consistent within 3σ.**
- ✓ **Results from run10 central trigger are not shown here.**



Run10 + Run11

- ✓ **Results from run10 and run11 have been merged to reduce errors.**
- ✓ **Results are systematically suppressed compared to the Poisson baseline.**



Run10 + Run11

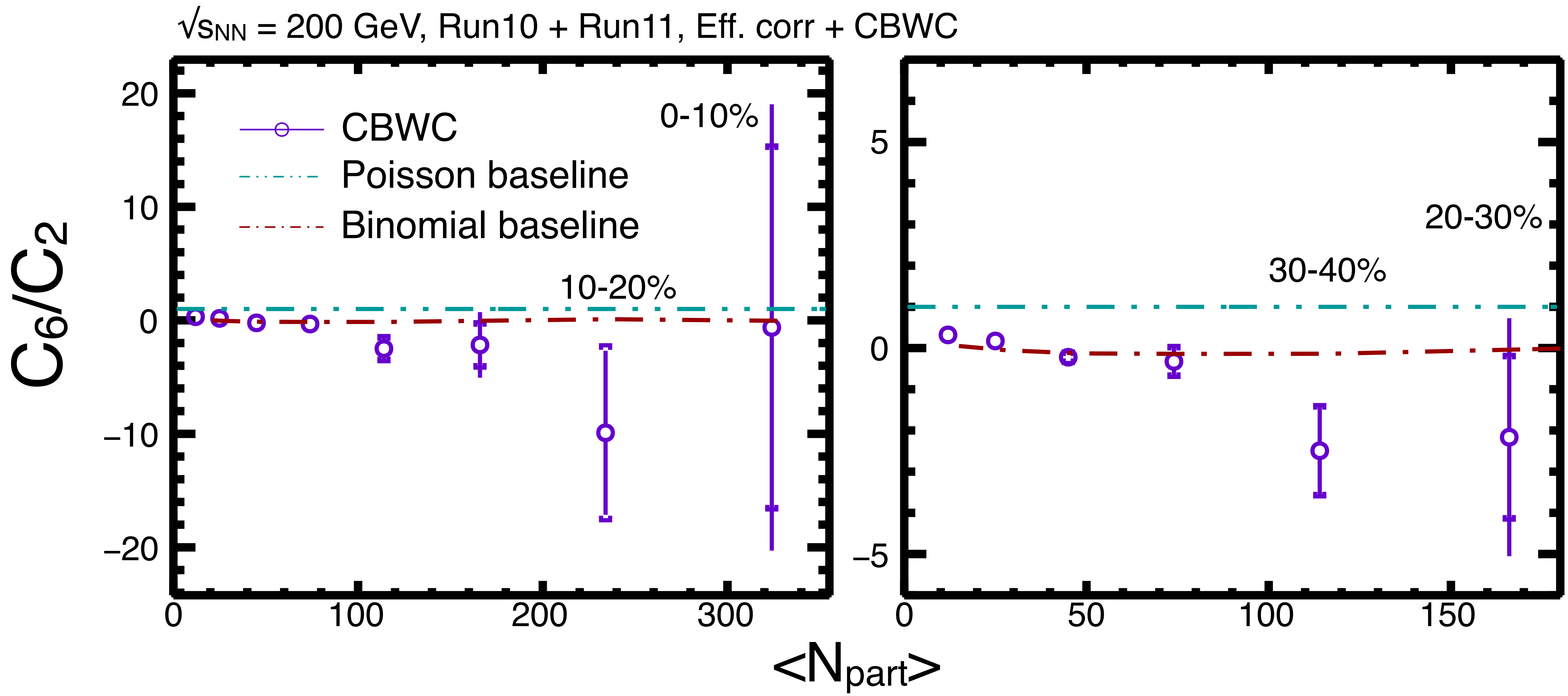
- ✓ **Binomial distributions are compared as statistical baseline by using the width as well as mean parameter.**

$$C_n^{\mathrm{net-p}} = C_n^{\mathrm{p}} + C_n^{\mathrm{pbar}}$$
$$C_2^x = \mu_x \varepsilon_x$$
$$C_6^x = \mu_x \varepsilon_x \left[120\varepsilon_x^4 - 240\varepsilon_x^3 + 150\varepsilon_x^2 - 30\varepsilon_x + 1\right]$$

μ : measured mean

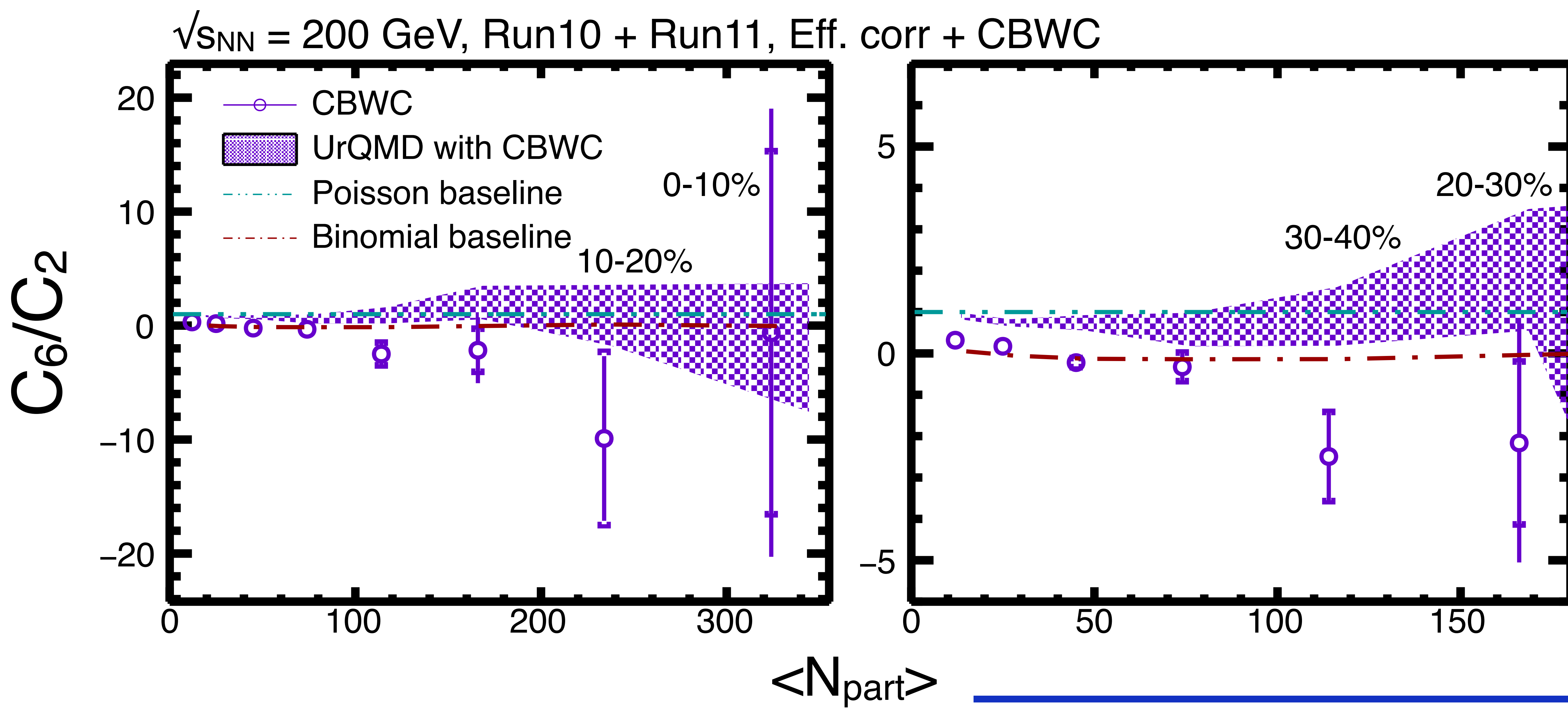
$\varepsilon = \dfrac{\sigma^2}{\mu}$: measured scaled variance

- ✓ **Results can be described well by the binomial distribution.**



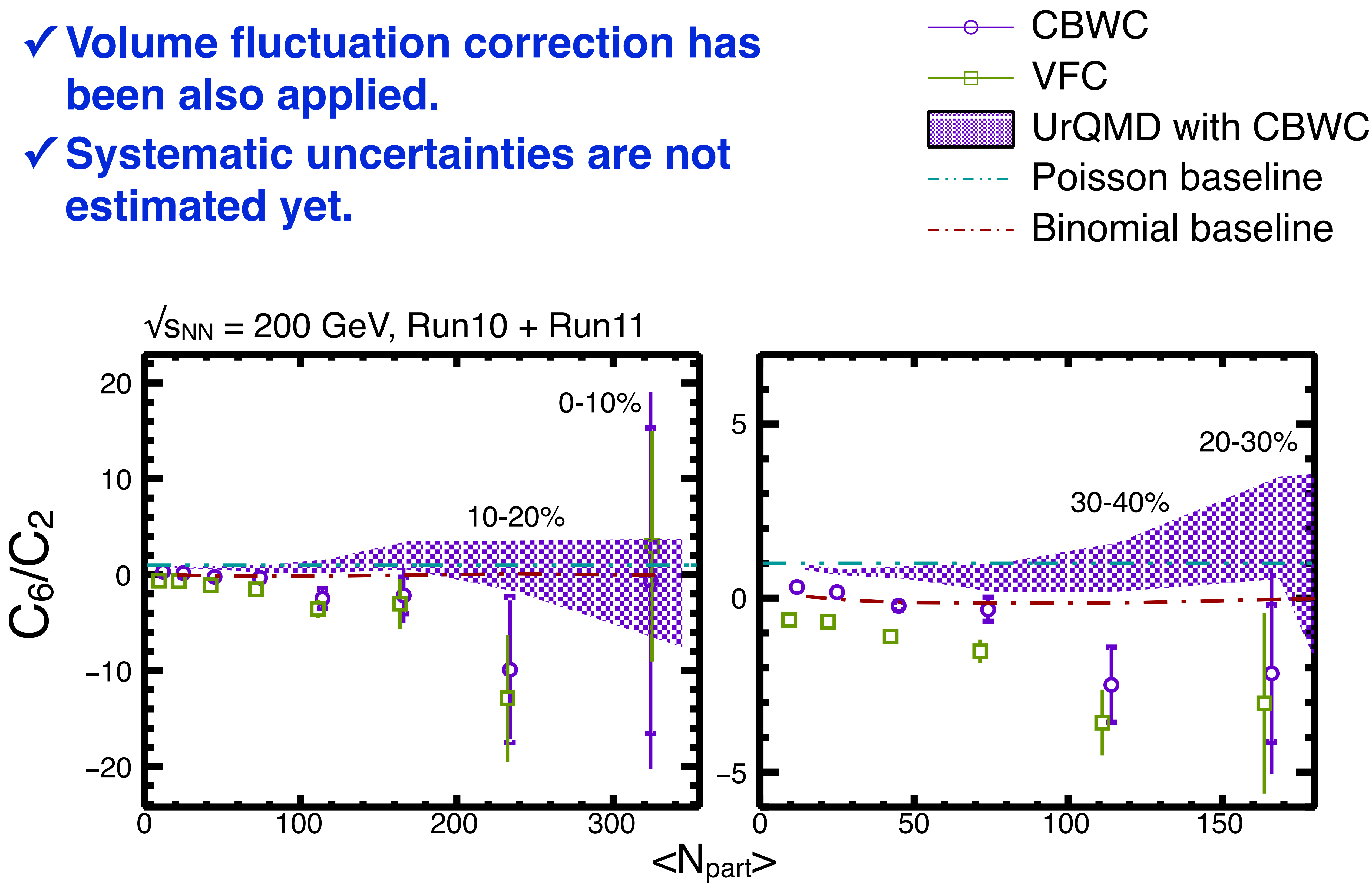
Run10 + Run11 vs UrQMD

- ✓ UrQMD data has been analyzed with ~40 M events.
- ✓ UrQMD shows smaller results compared to the Poisson baseline in peripheral collisions, which might be due to the global baryon number conservation.
- ✓ Experimental data are systematically smaller than UrQMD.



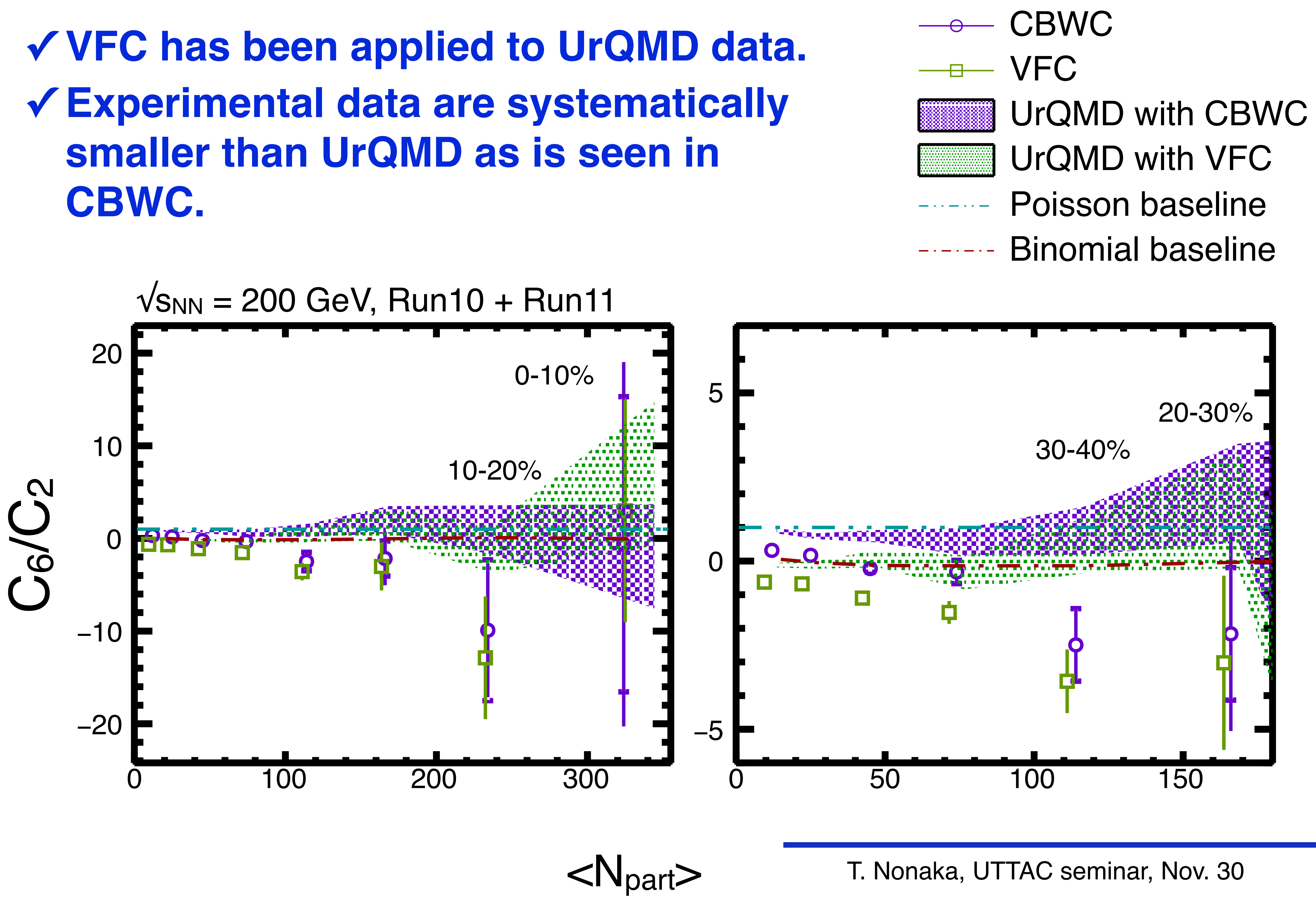
CBWC vs VFC

- ✓ **Volume fluctuation correction has been also applied.**
- ✓ **Systematic uncertainties are not estimated yet.**



CBWC vs VFC

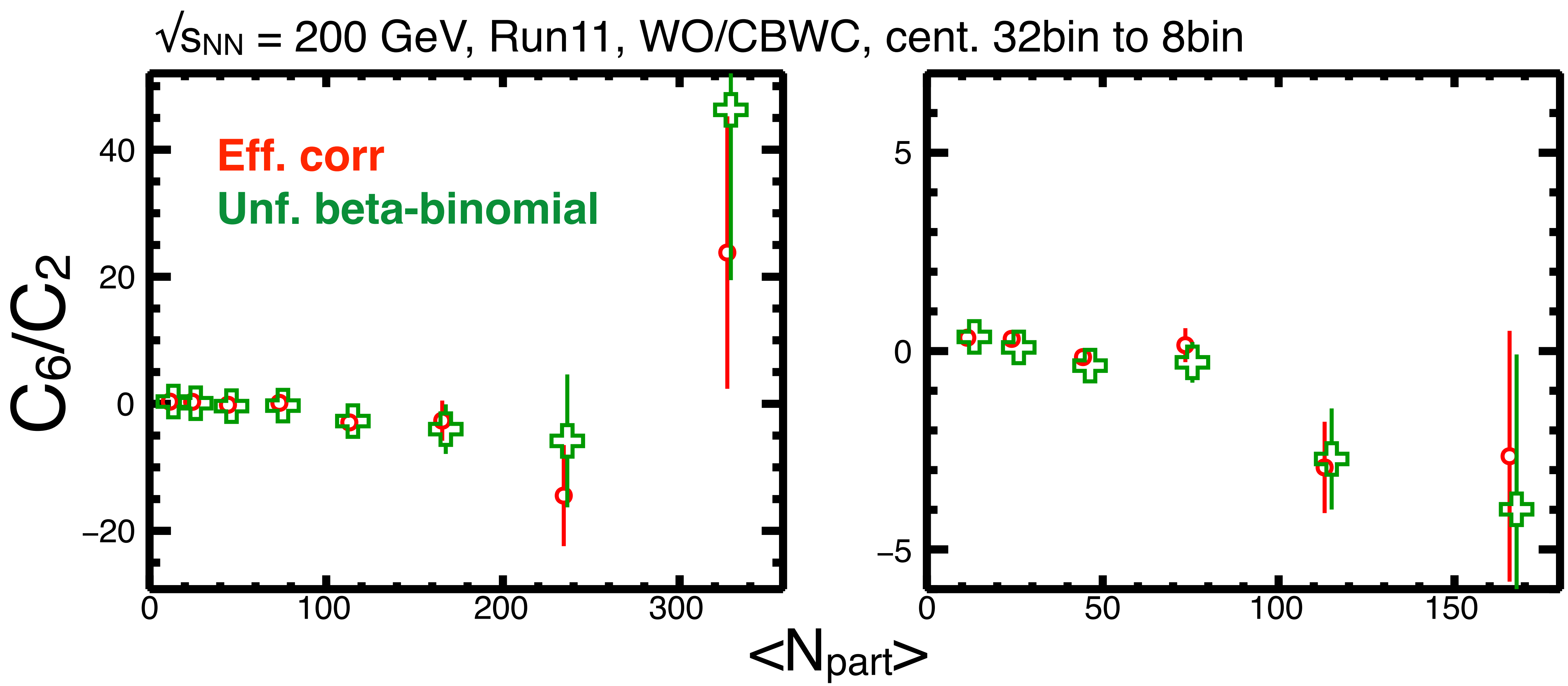
- ✓ **VFC has been applied to UrQMD data.**
- ✓ **Experimental data are systematically smaller than UrQMD as is seen in CBWC.**



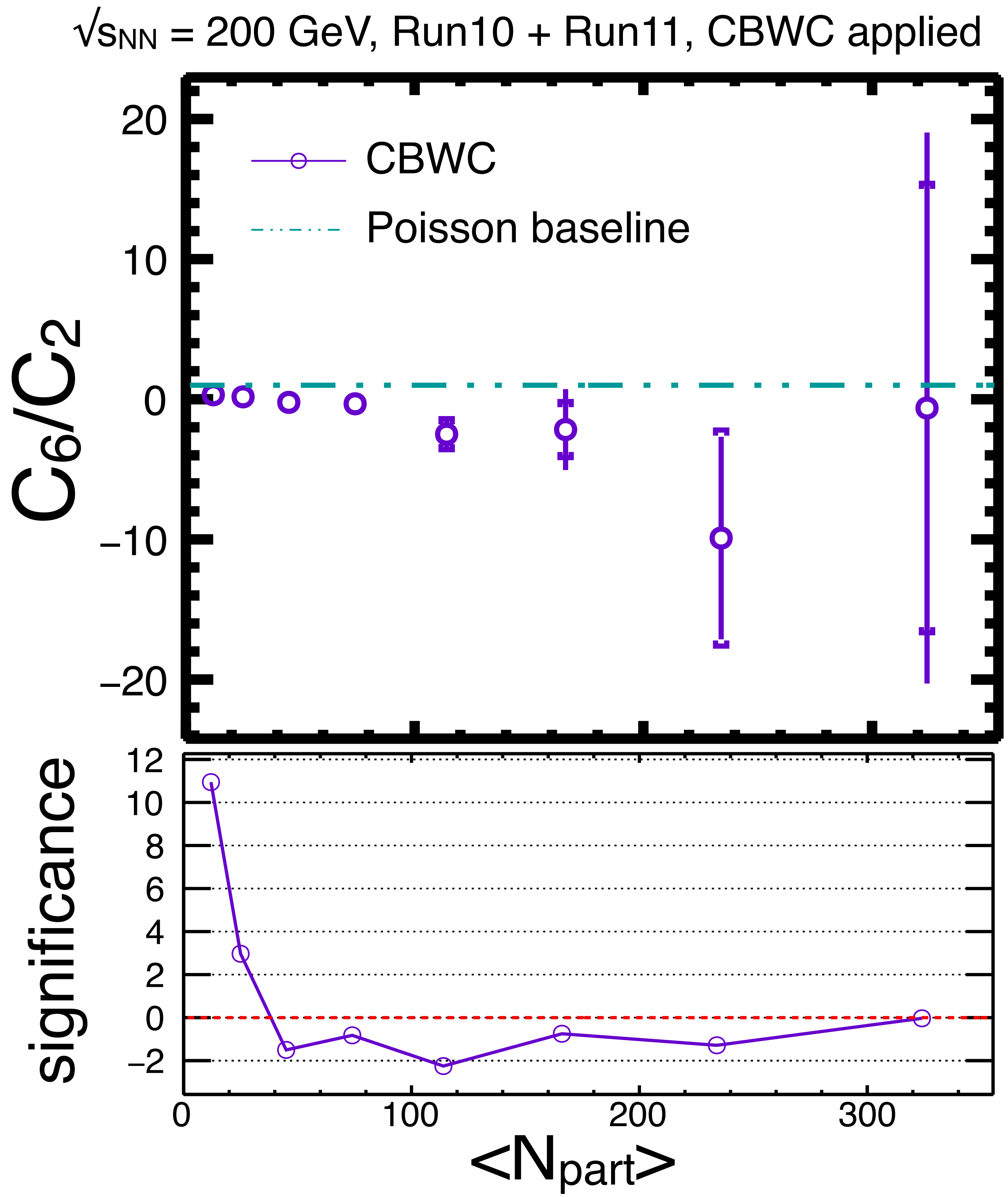
Non-binomial effect

✓ **Unfolding has been applied with beta-binomial model.**

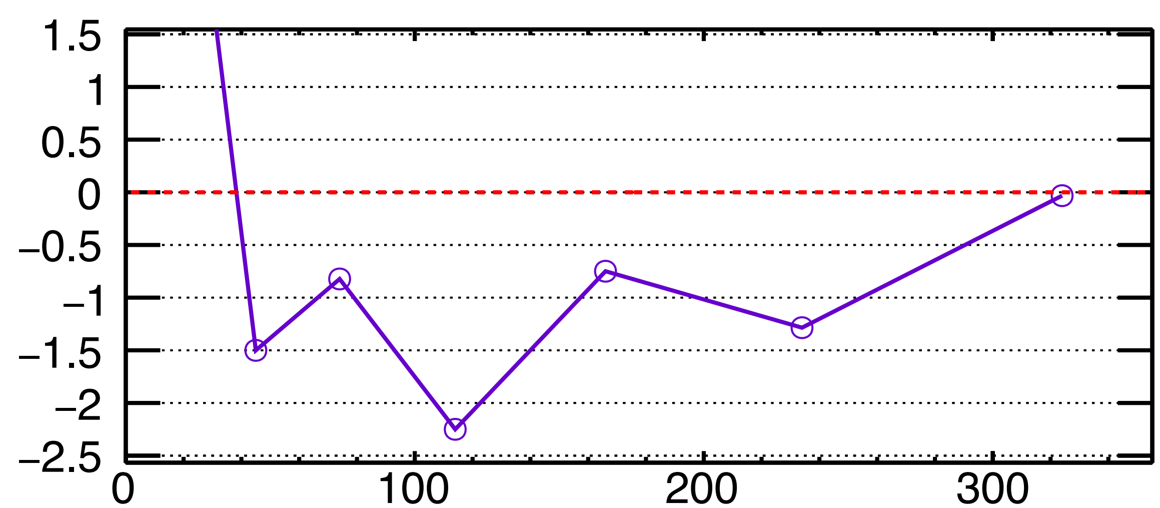
✓ **Corrections are within statistical errors.**



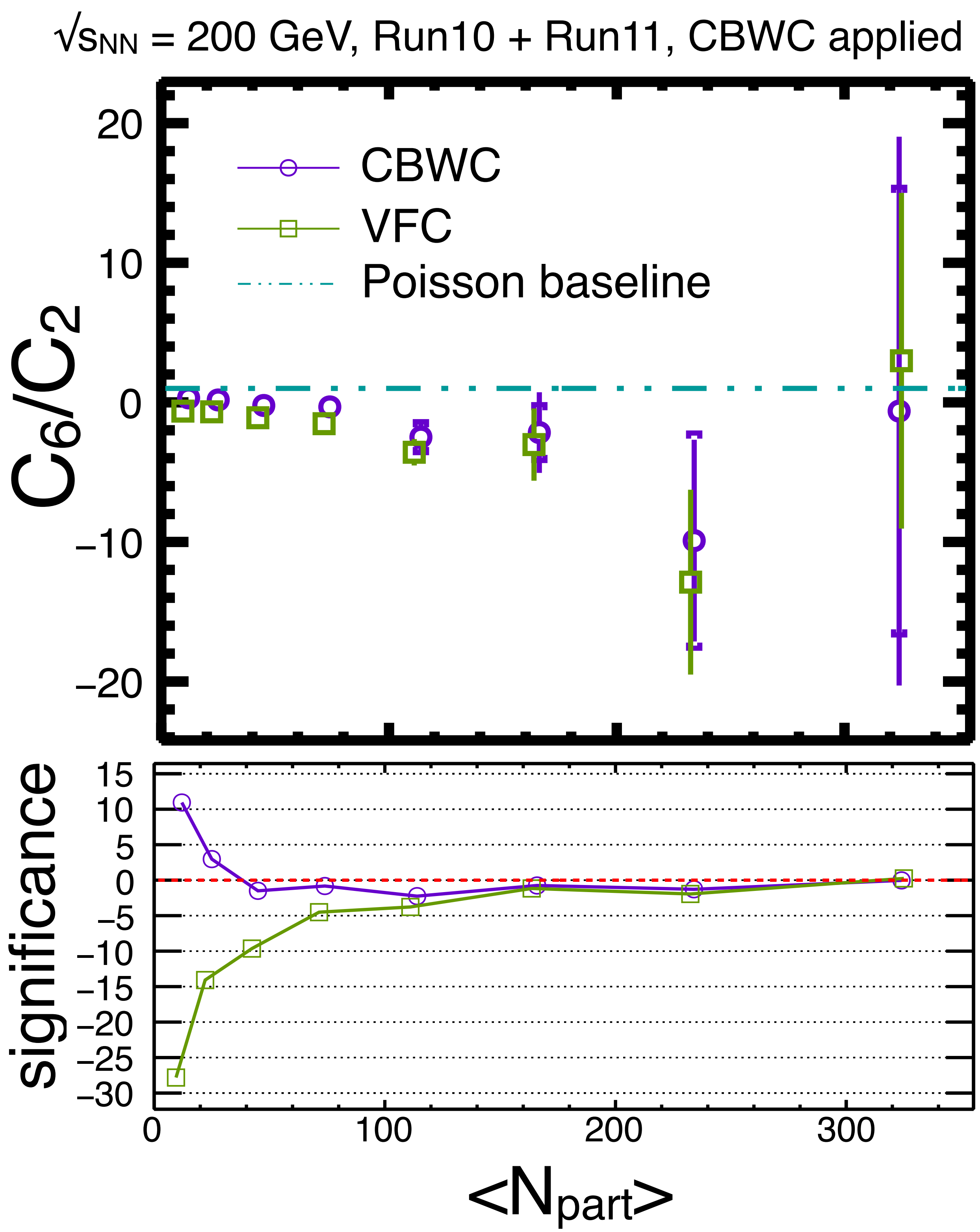
Signal of phase transition?



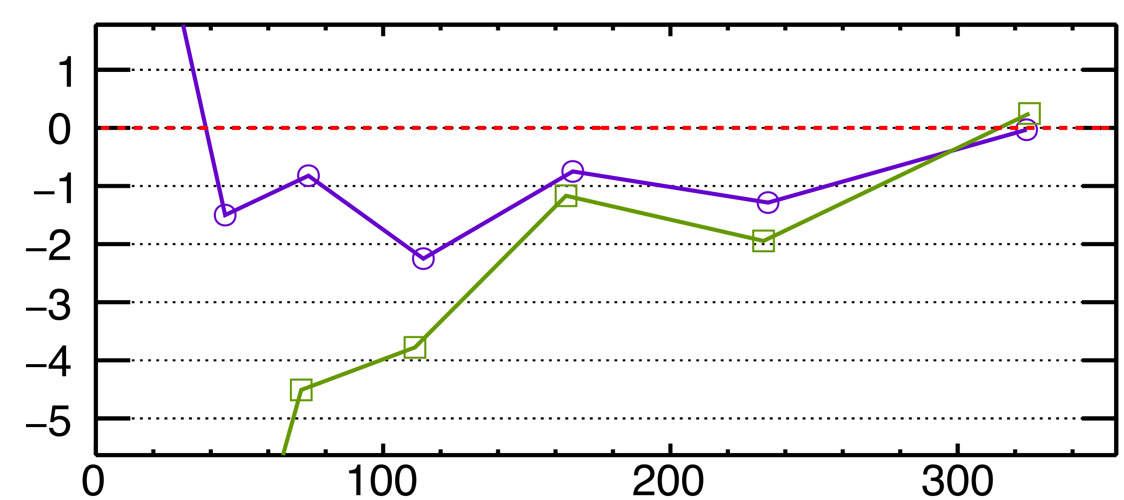
- ✓ **Positive values at 60-70 and 70-80% centralities. → no QGP? or finite size effect?**
- ✓ **Negative values at -60% centralities, where the significance is the best at 30-40% centrality with 2.3σ.**



Signal of phase transition?



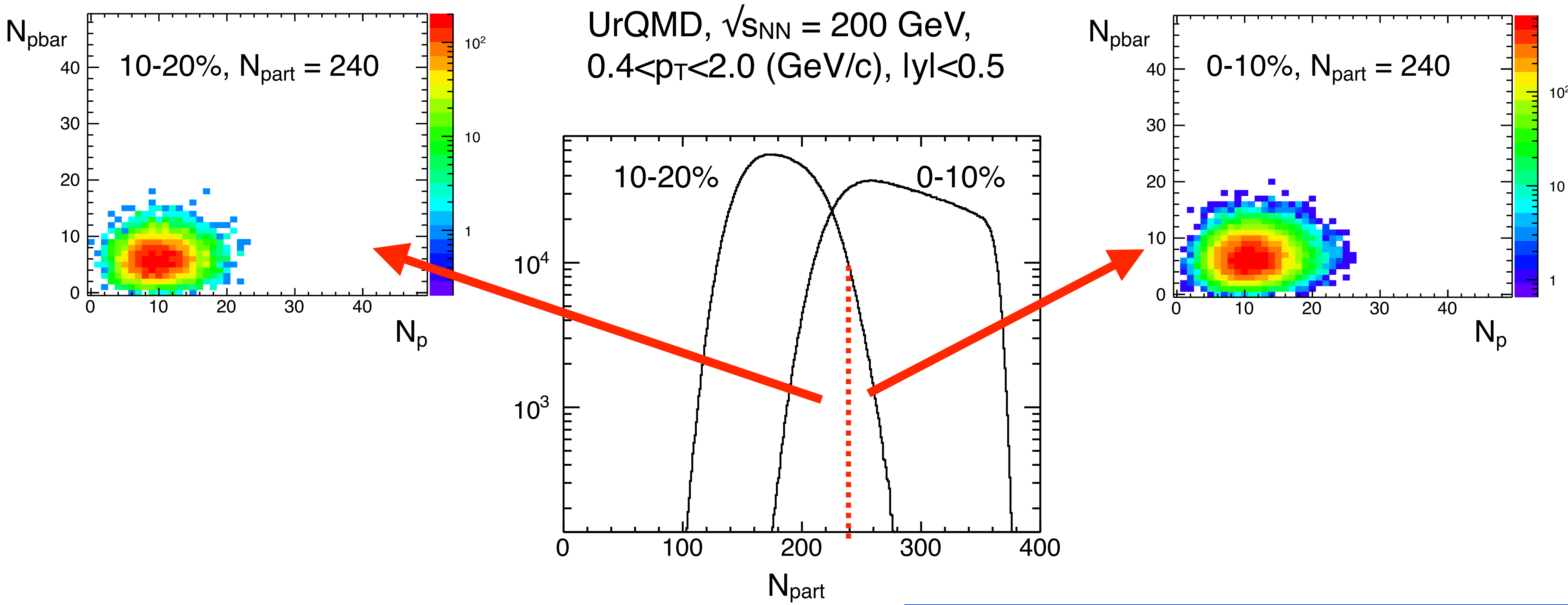
- ✓ VFC results show negative values except the most central collisions.
- ✓ Negative at peripheral collisions??
- ✓ Conclusions will depend on the correction methods on volume fluctuation → Which is better correction method, CBWC or VFC?



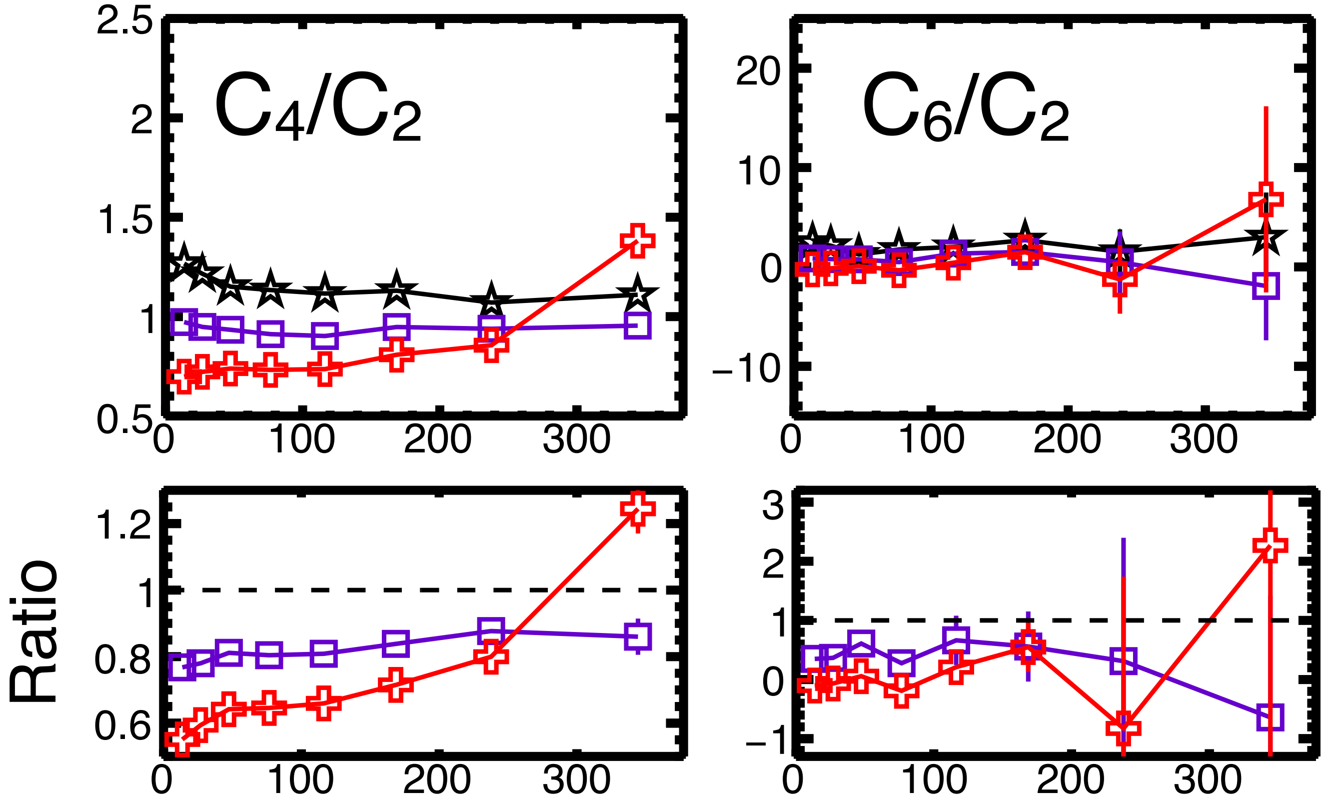
How to define "true" fluctuation in UrQMD model ?

✦ **In view of the participant fluctuation, "true" fluctuations can be defined by calculating cumulants at each N_{part} value given by UrQMD.**

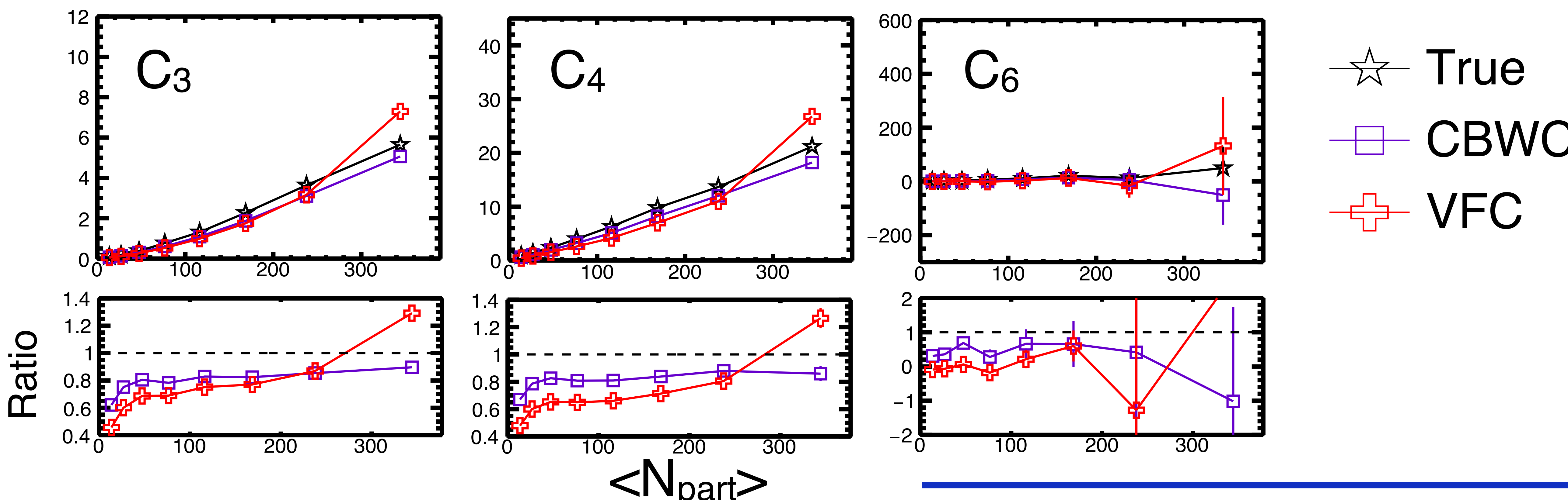
✓ **Centrality is defined by using multiplicity distributions as is done in experimental data.**



Comparison with “true” fluctuation in UrQMD model



- ✓ **Both methods don’t reproduce the true cumulants.**
- ✓ **CBWC results are systematically and qualitatively closer to true cumulants than VFC.**
  - **IPP model breaks in UrQMD.**
  - **“True” fluctuation is partly killed by CBWC.**
- ✓ **I decide to pick up CBWC as final results.**



Conclusions

- More efficiency formulas for efficiency correction and unfolding methods have been developed.
- Two correction methods are compared with some models. Based on the study using UrQMD model, I decided to pick up CBWC in final results. Note that it is still unknow which method is correct in real experiment.
- **In 30-40% centrality, C_6/C_2 shows negative value with 2.2σ significance, which is consistent with the theoretical prediction, and might be an experimental evidence of phase transition.**
- **In that sense, positive values observed in 60-70 and 70-80% centralities might be due to the absence of QGP or due to the finite size effect.**
- **In central collisions, statistical errors are huge and consistent with statistical baselines.**
- STAR experiment need to collect more statistics in order to derive more definite physics messages.

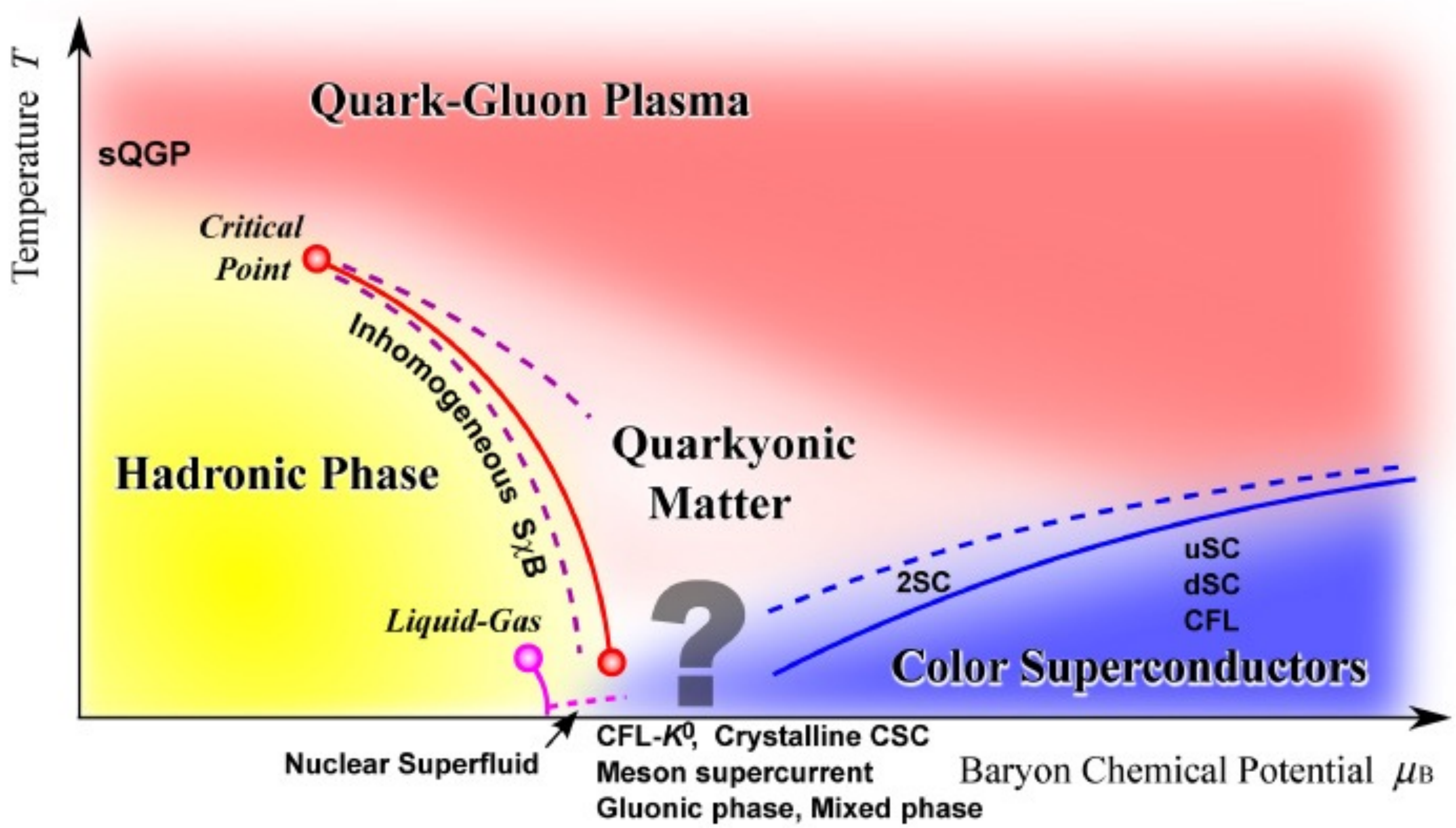
Thank you!

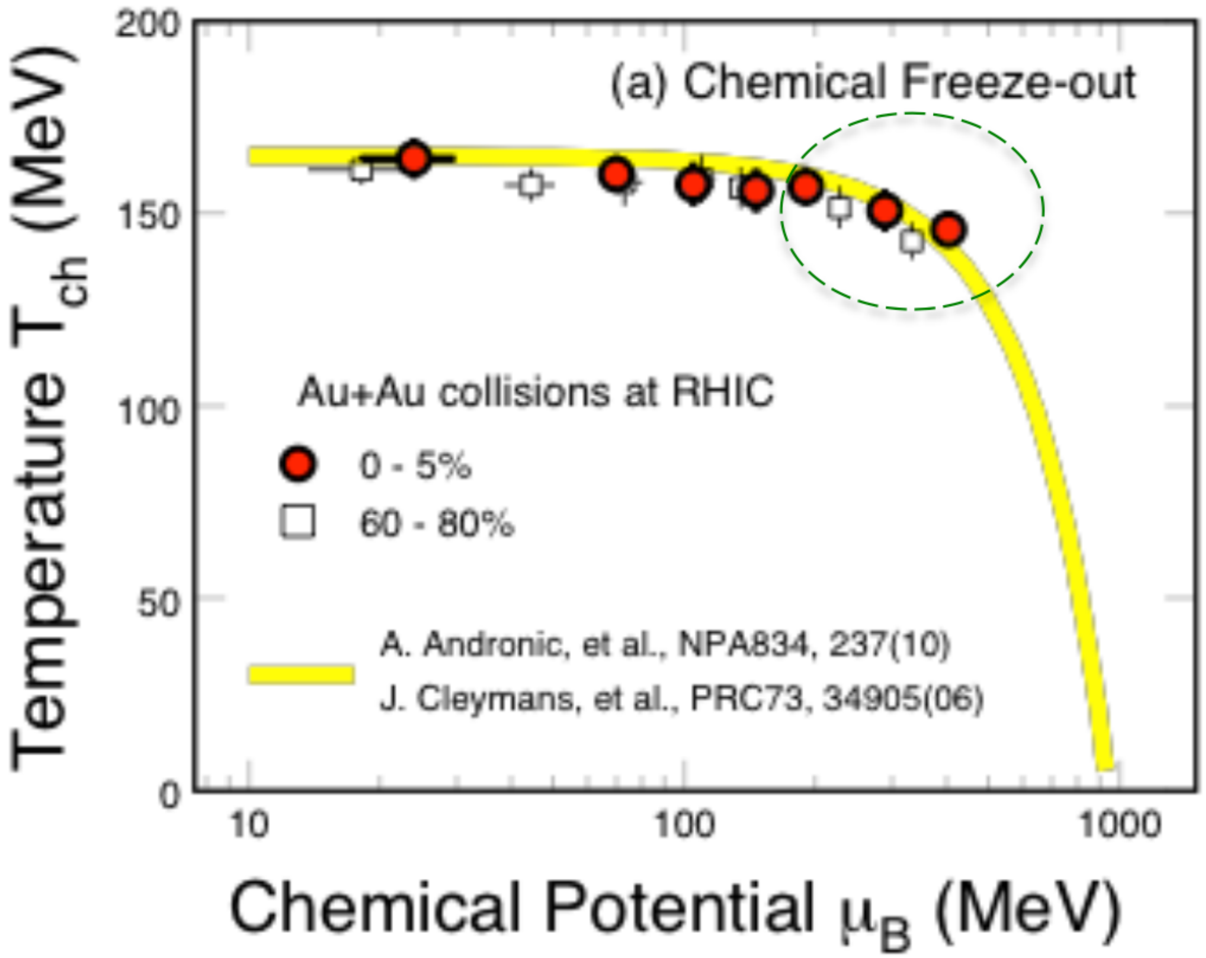
Backup

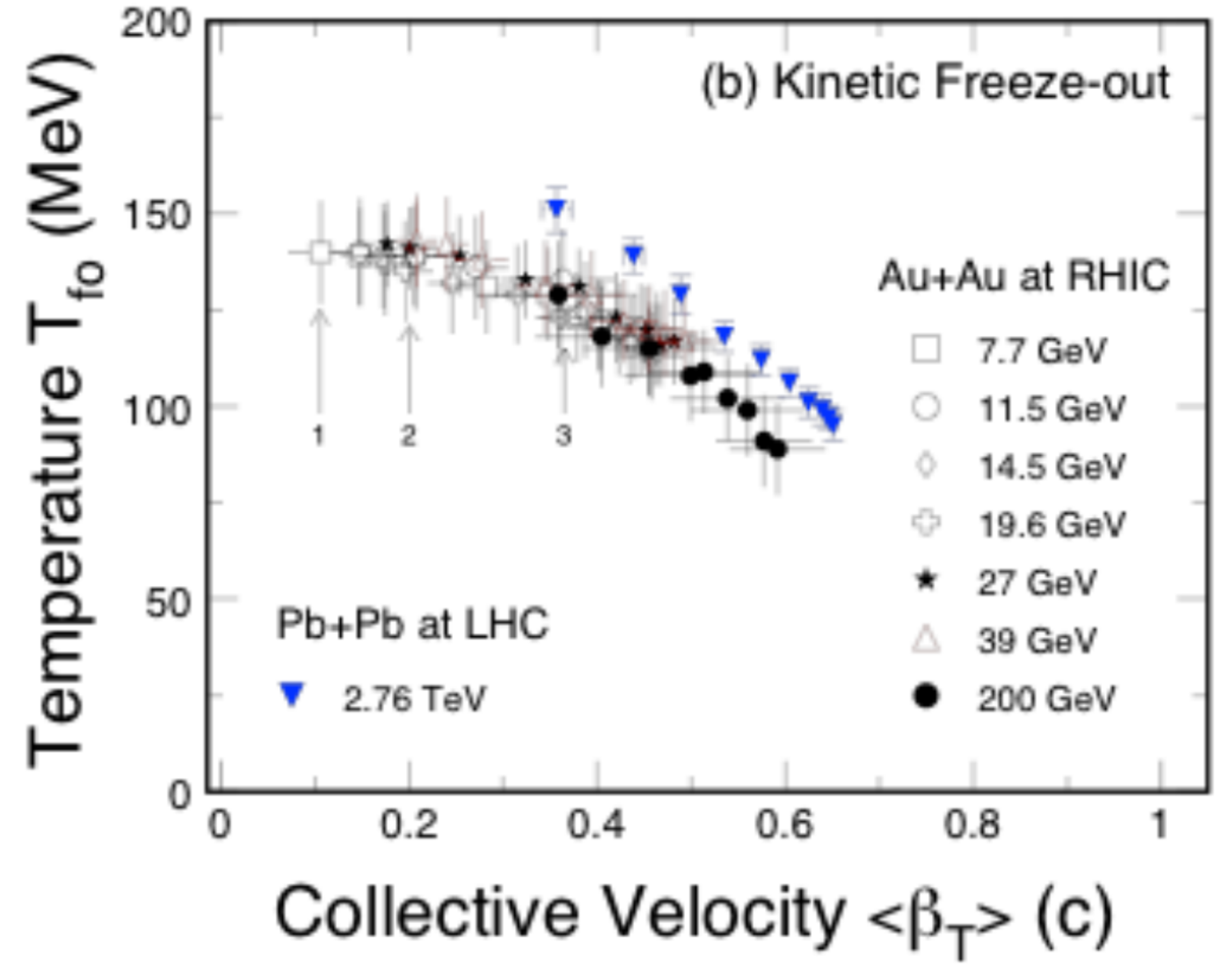
QCD phase diagram

- ✓ Ultimate goal is to elucidate the QCD phase structrue.
  - ✓ There will be QGP phase and hadronic phase.
  - ✓ Where is the phase transition line?
  - ✓ What kind of phase transition?
  - ✓ Critical end point?

Higher order fluctuations of **conserved quantities**
can probe the QCD phase structure







Chemical Freeze-out: (GCE)
- Weak temperature dependence
- Centrality dependence μ_B!
- Lattice prediction on CP around $\mu_B \sim 300 - 400$ MeV

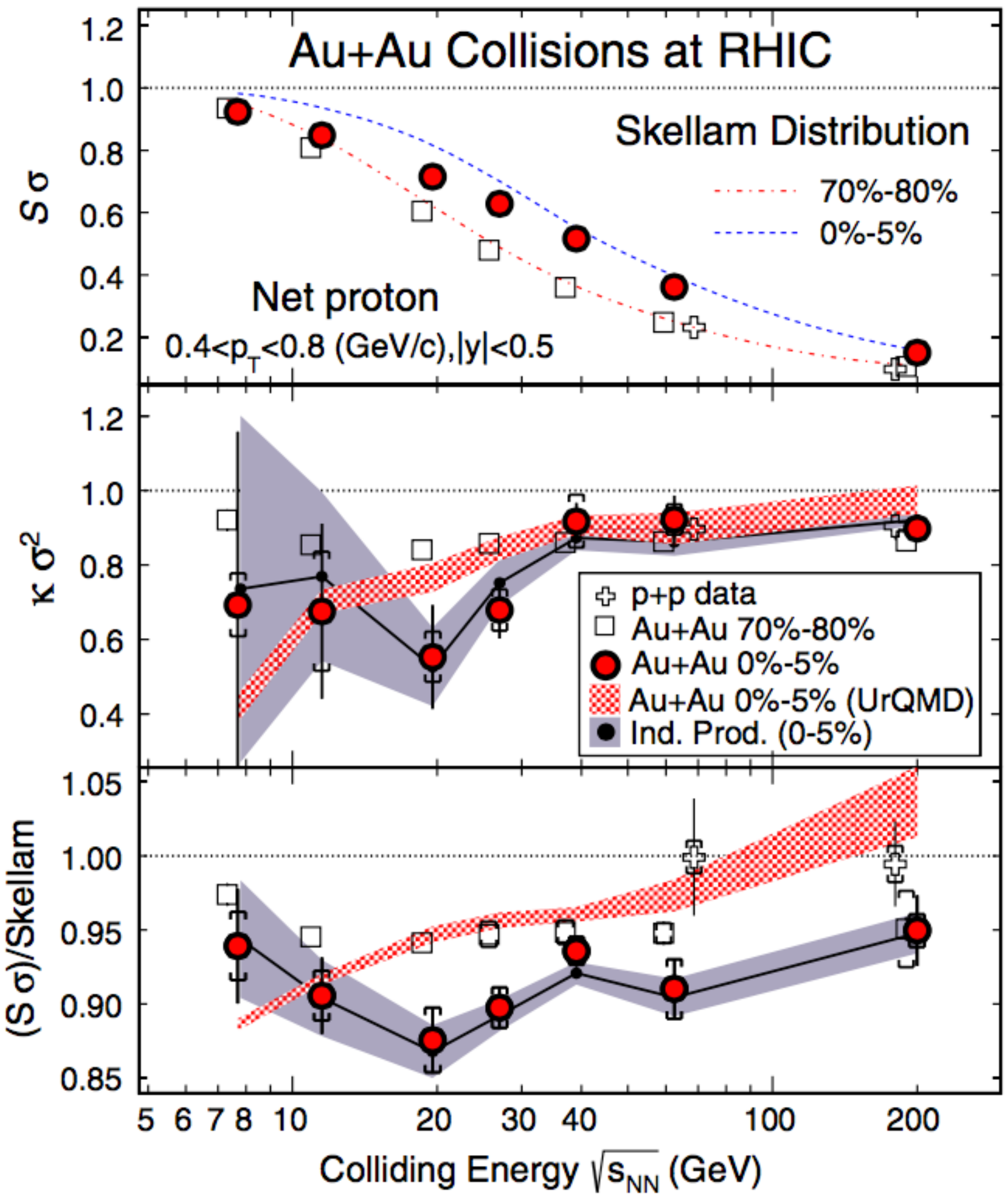
Kinetic Freeze-out:
- Central collisions => lower value of T_{fo} and larger collectivity β_T
- Stronger collectivity at higher energy, even for peripheral collisions

ALICE: B.Abelev et al., PRL109, 252301(12); PRC88, 044910(2013).
STAR: J. Adams, et al., NPA757, 102(05); X.L. Zhu, NPA931, c1098(14); L. Kumar, NPA931, c1114(14)

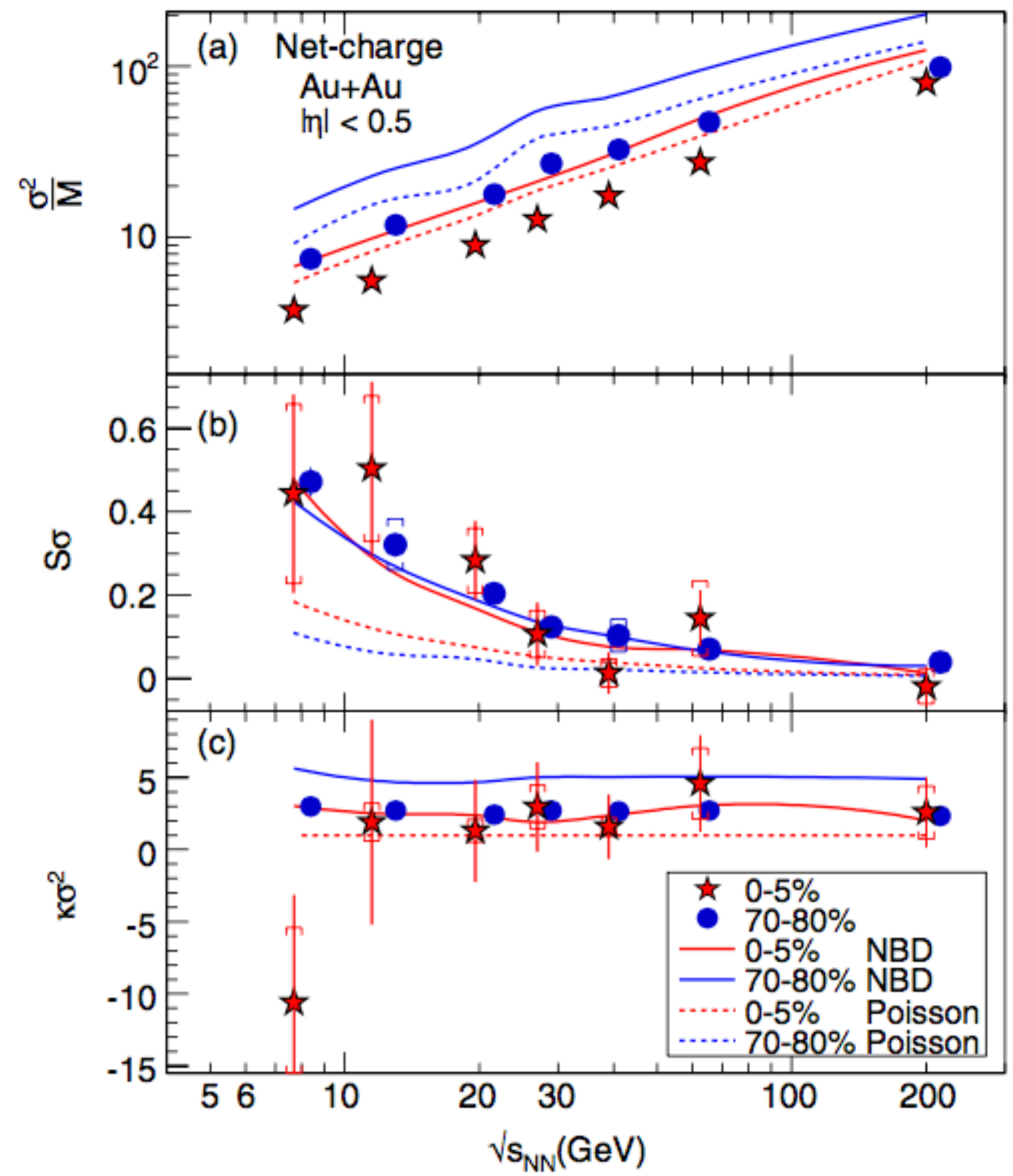
N. Xu, WPCF2015

Published results in 2014

- ✓ It seems to be interesting around 20 GeV for net-proton results.
- ✓ Net-charge results are consistent with the baseline due to large errors. → A wide distribution gives large statistical errors.



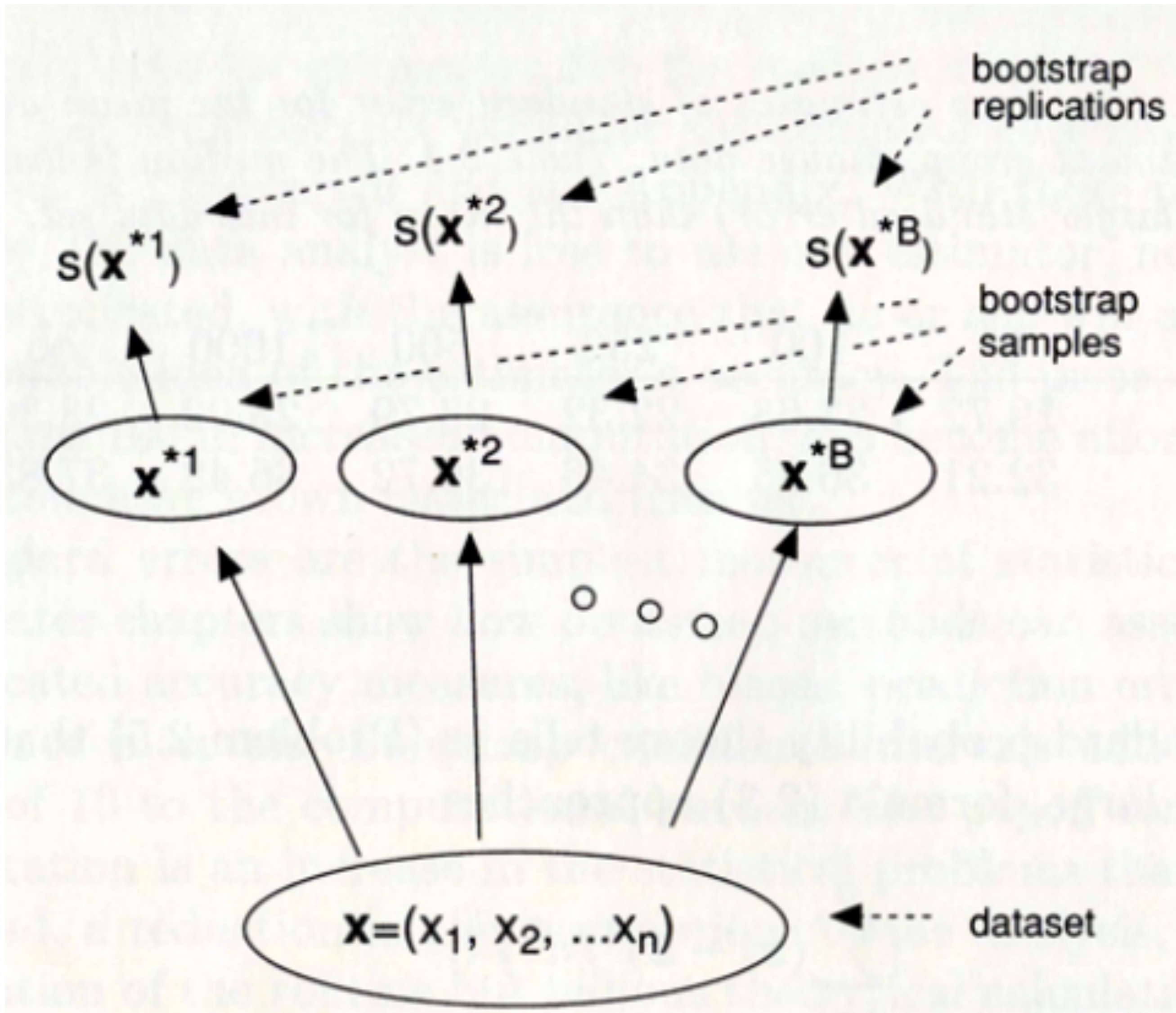
PRL 112, 032302 (2014)



PRL 113, 092301 (2014)

✦ Finite tracking efficiency is corrected.

Bootstrap



B. Efron,R. Tibshirani, An introduction to the bootstrap, Chapman & Hall (1993).

Bootstrap

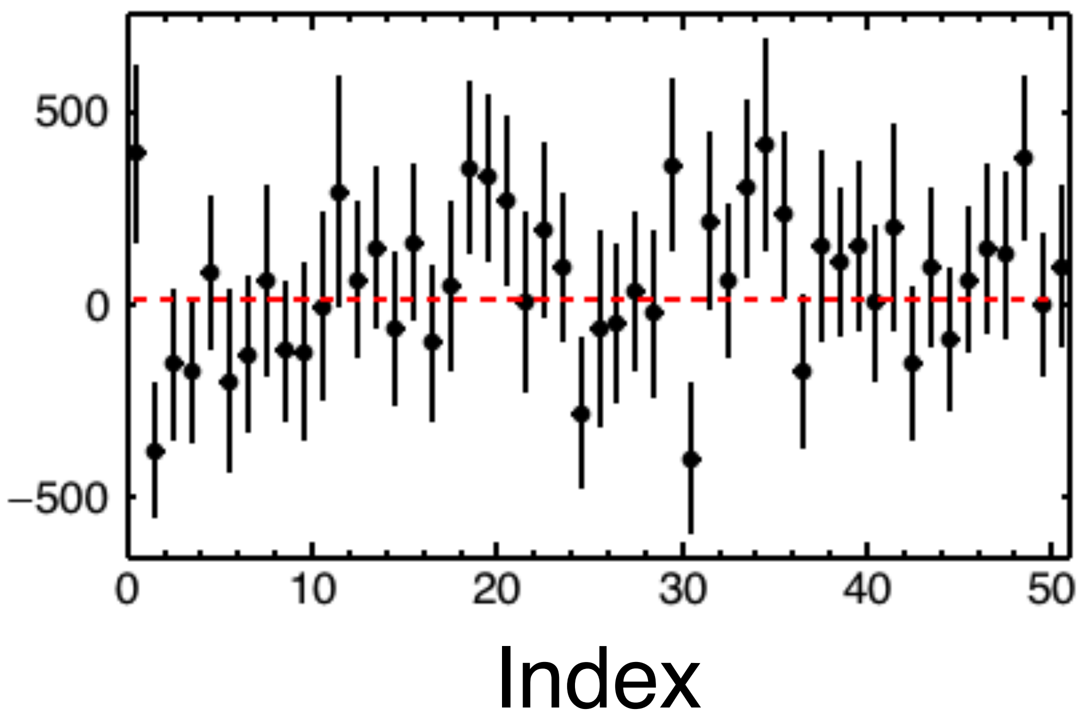
Input : Poisson - Poisson = Skellam

```
const double μp[2] = { 5, 4 };
const double μpbar[2] = { 3, 1 };
const double εp[2] = { 0.8, 0.6 };
const double εpbar[2] = { 0.7, 0.9 };
```

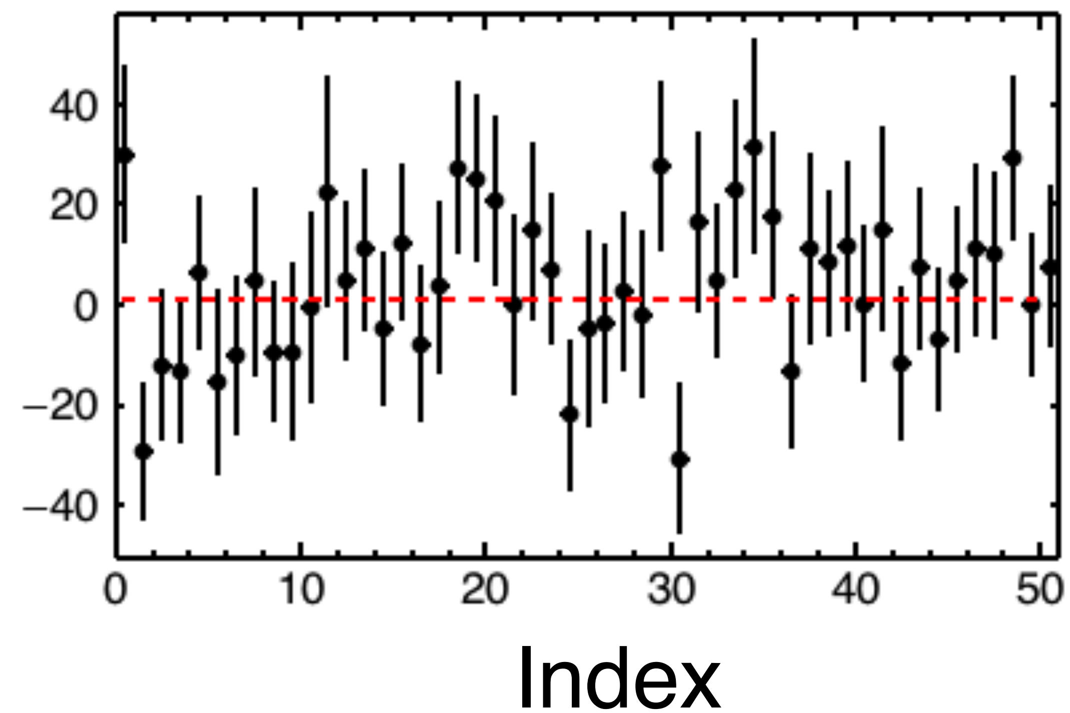
- ✓ Bootstrap (300 resampling) are performed with 50 independent trials.
- ✓ Efficiency correction in case of 2+2 phase space.
- ✓ Bootstrap works well for C_6 and C_6/C_2.

$C_6 \rightarrow 1\sigma$: 74%

1M events

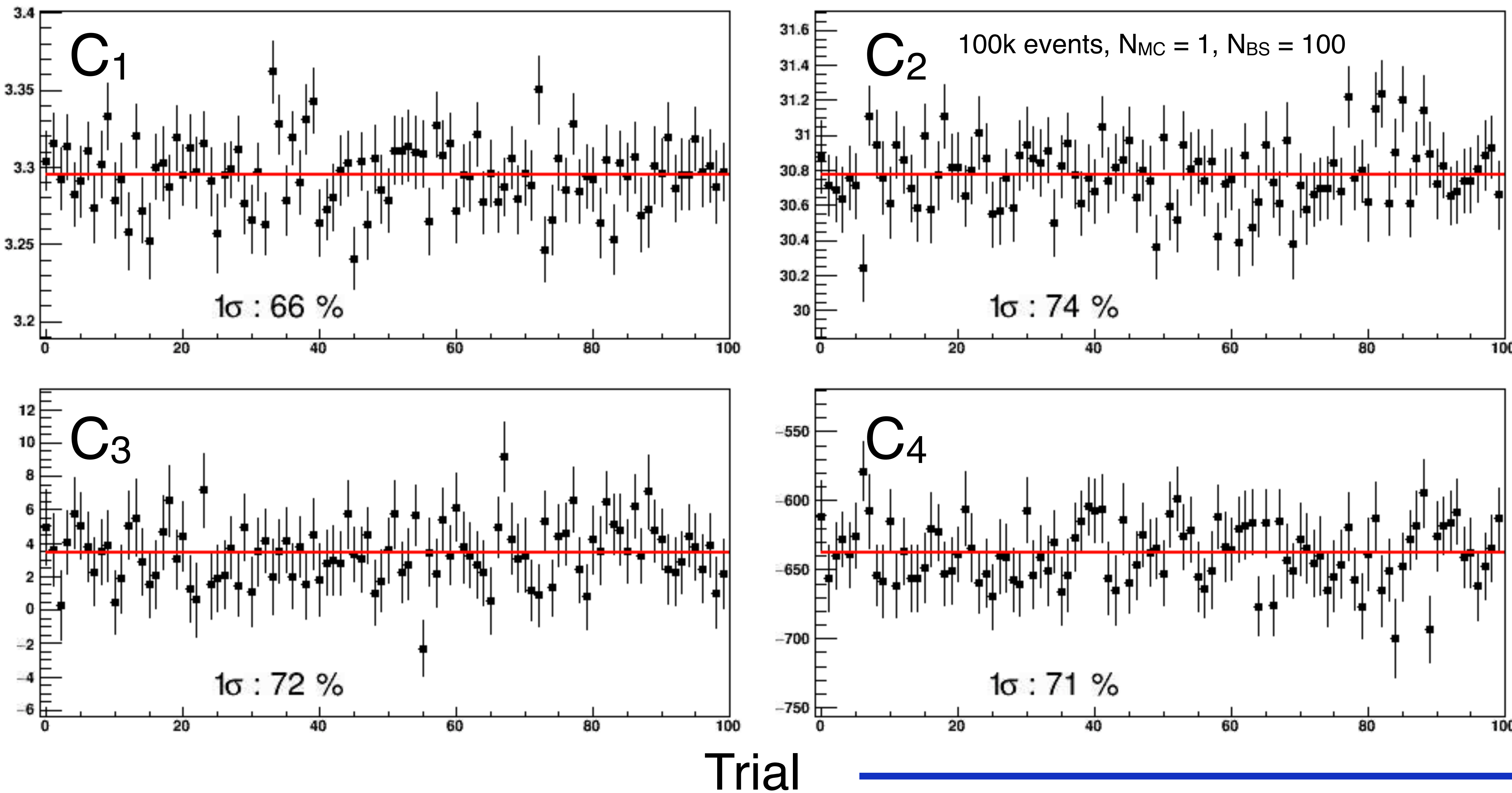


$C_6/C_2 \rightarrow 1\sigma$: 64%



Bootstrap application in unfolding

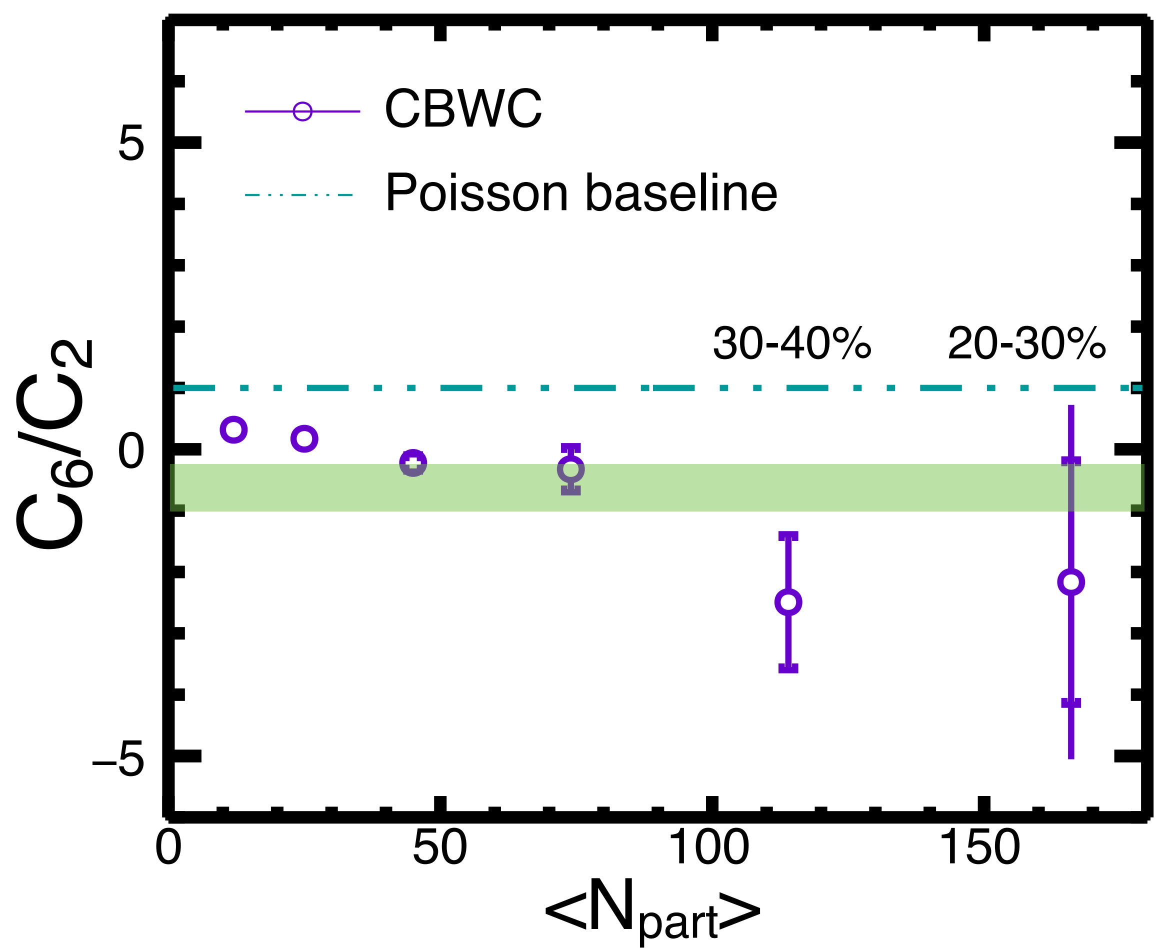
- ✦ Errors for each point has been calculated from 100 bootstrap samplings with one MC.
- ✦ 100 independent trials have been tested.
- ✦ Bootstrap works well.

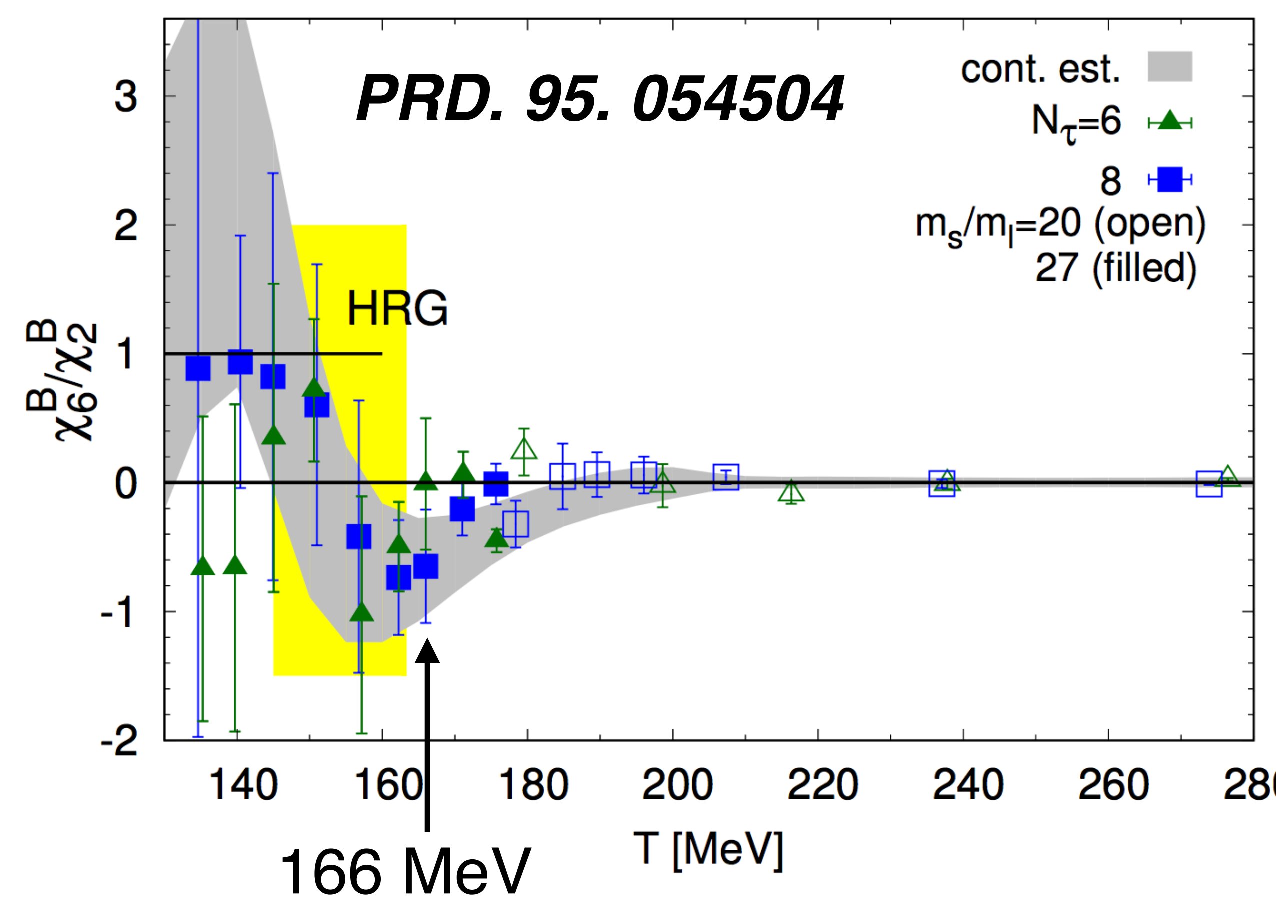


Comparison with LQCD results

- ✓ $\mu B \sim 20$ MeV $\neq 0$ at $\sqrt{s_{NN}} = 200$ GeV.
- ✓ Finite size effect, volume fluctuation and baryon number conservation will dilute the experimental results.

Caution

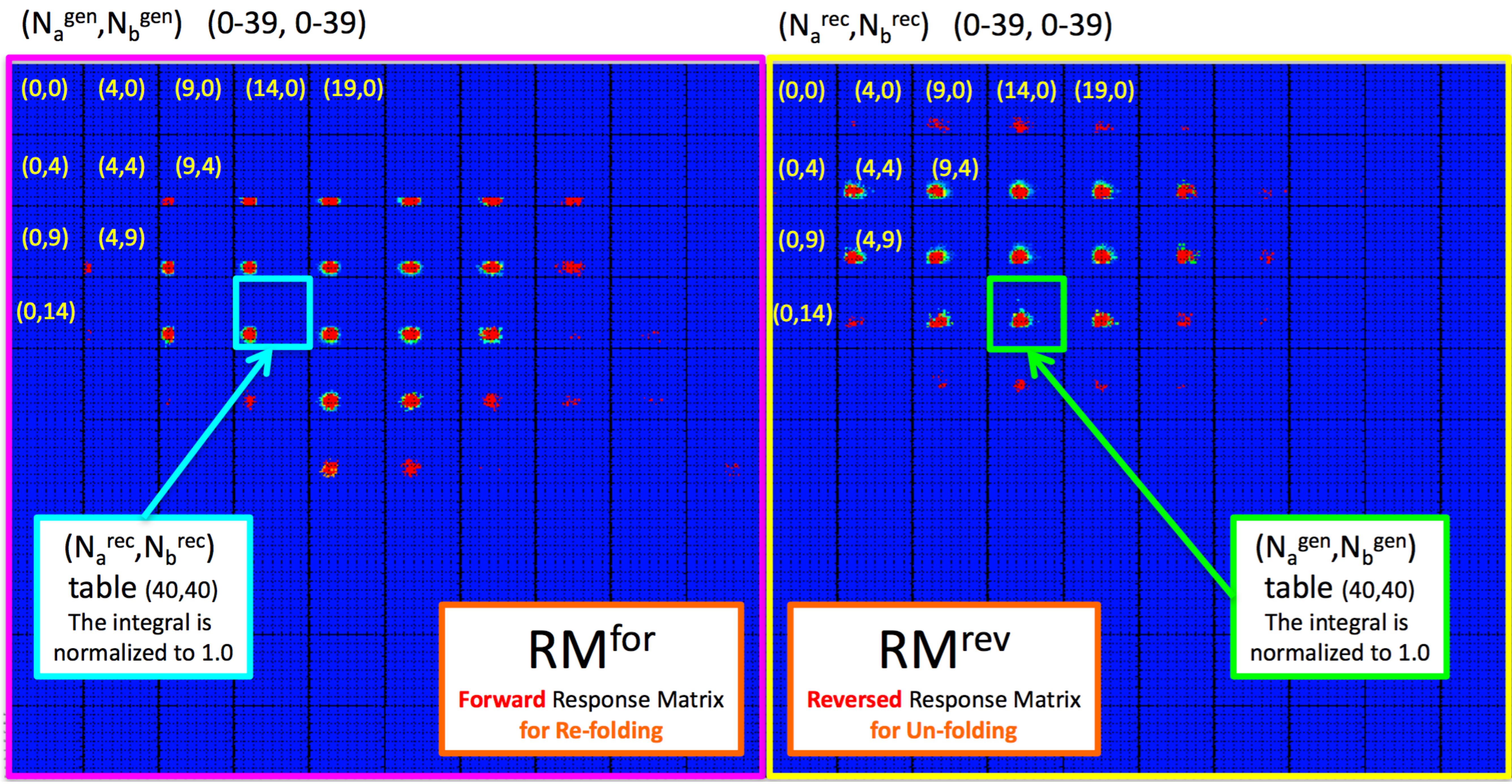




Response matrix

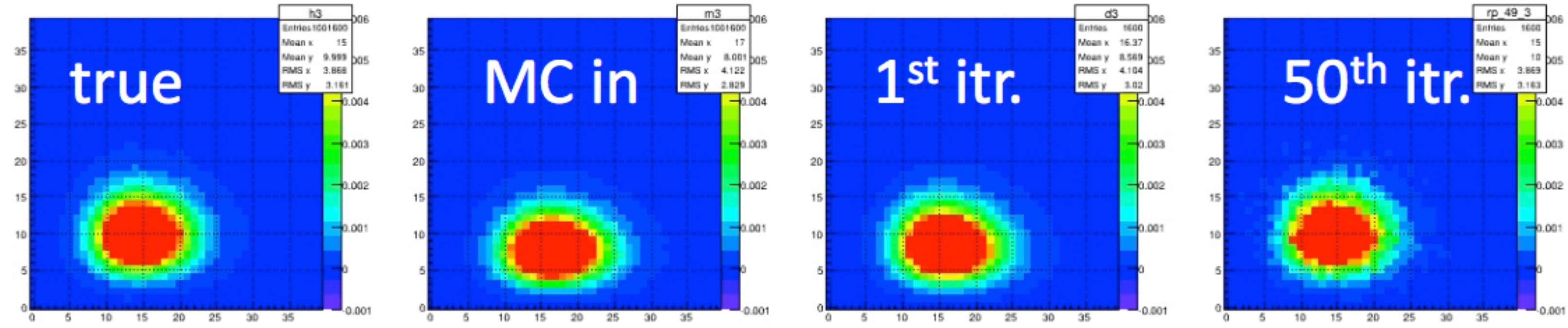
Forward Matrix (MC^{gen} ---> MC^{rec})

Reversed Matrix (MC^{rec} ---> MC^{gen})

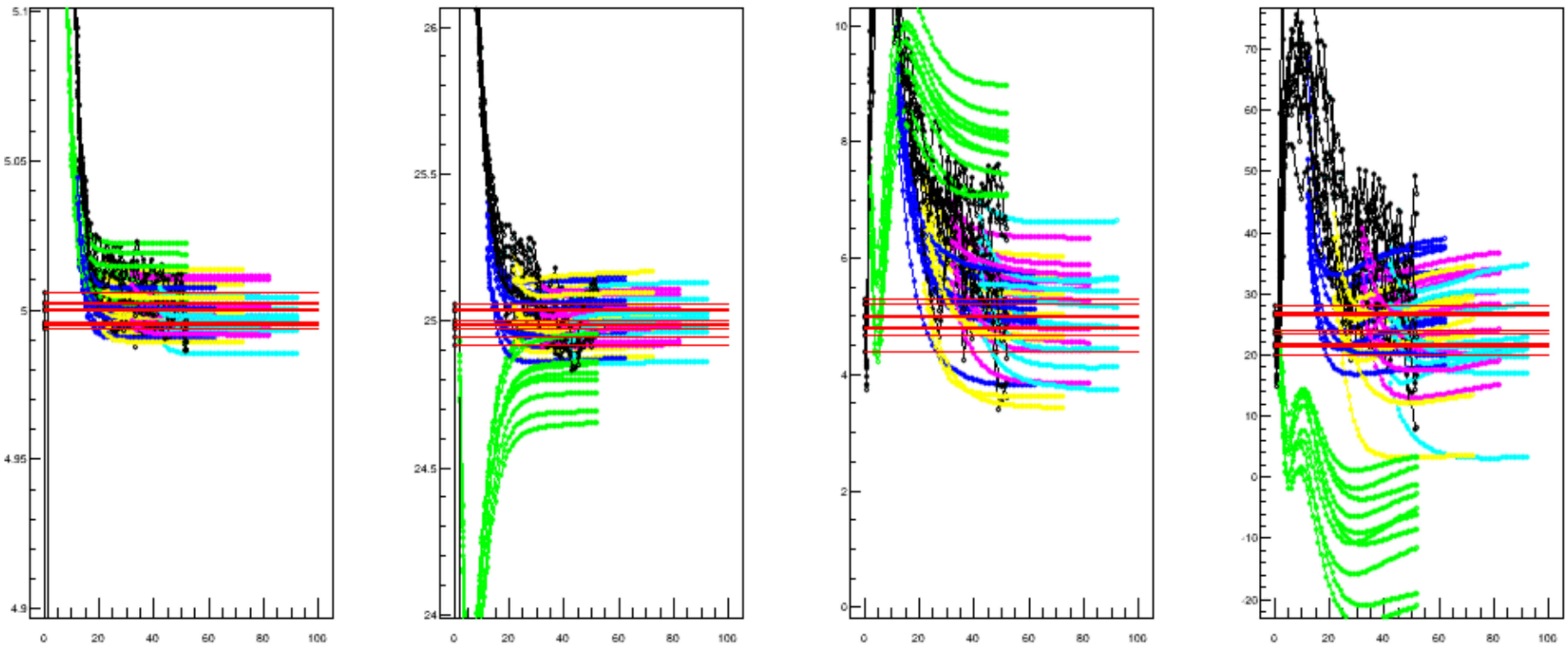


Hybrid iteration

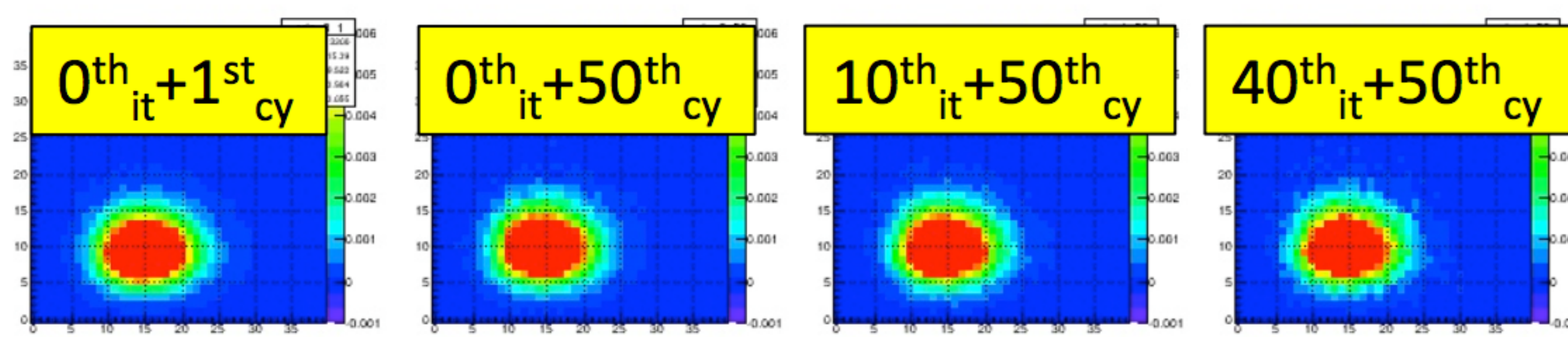
(ε_x=0.7, ε_y=0.65)



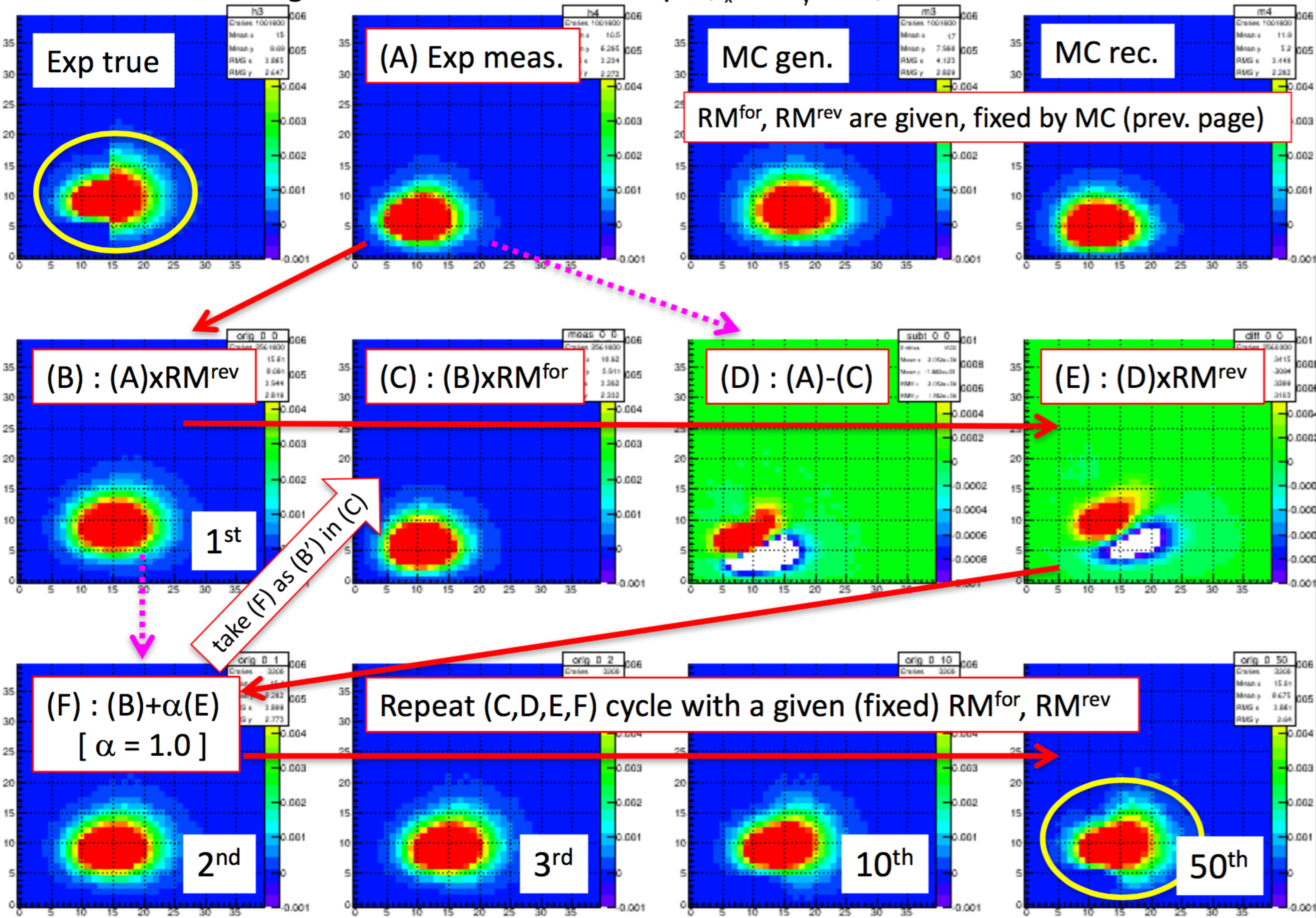
C_1 = 5+/- 0.01 C_2 = 25 +/- 0.1 C_3 = 5 +/- 0.5 C_4 = 25 +/- 5



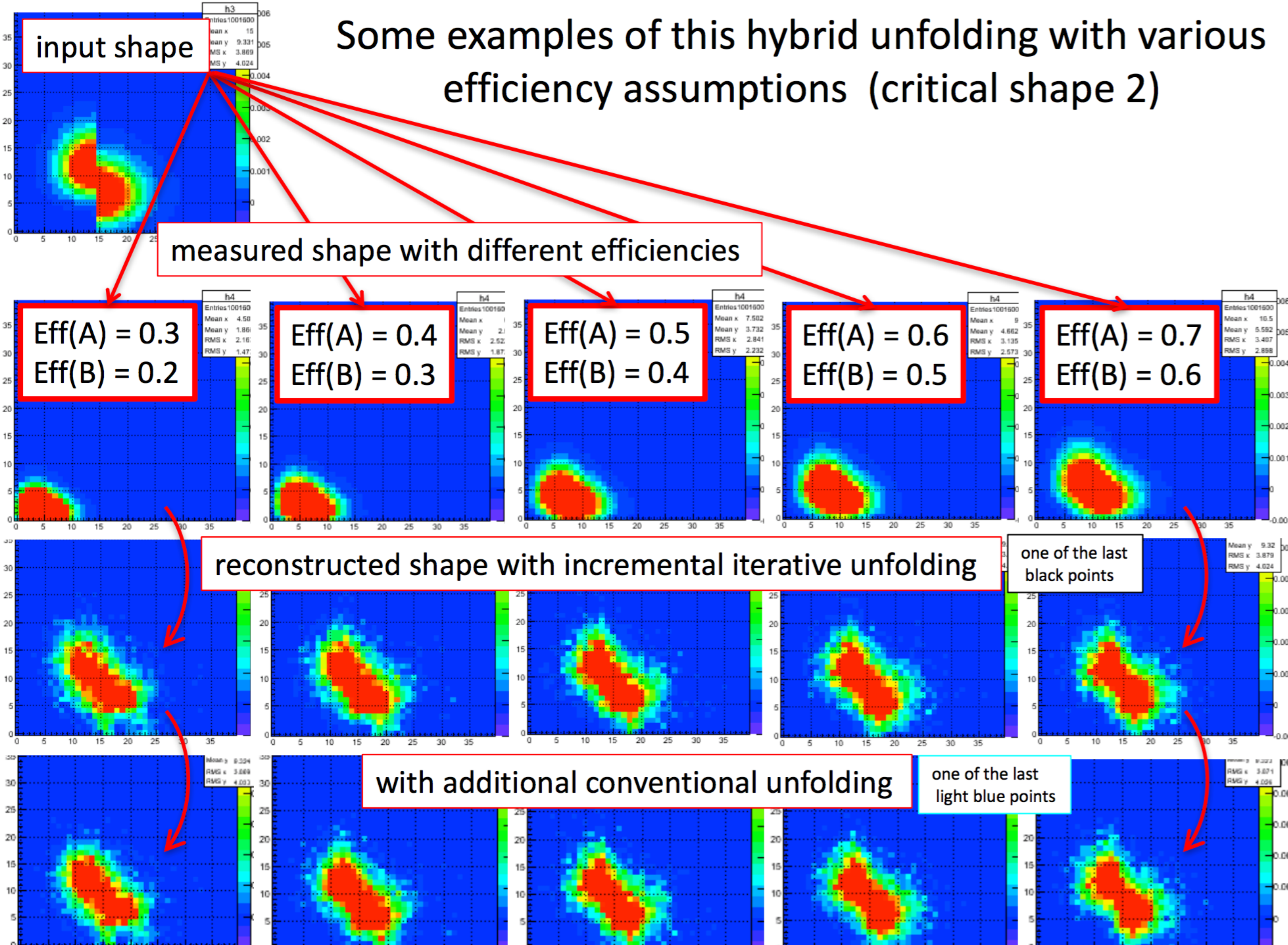
original, 10th, 20th, 30th and 40th MC^{gen}-MC^{rec} matrix



Conventional unfolding method with a critical shape (ε_x=0.7, ε_y=0.65)

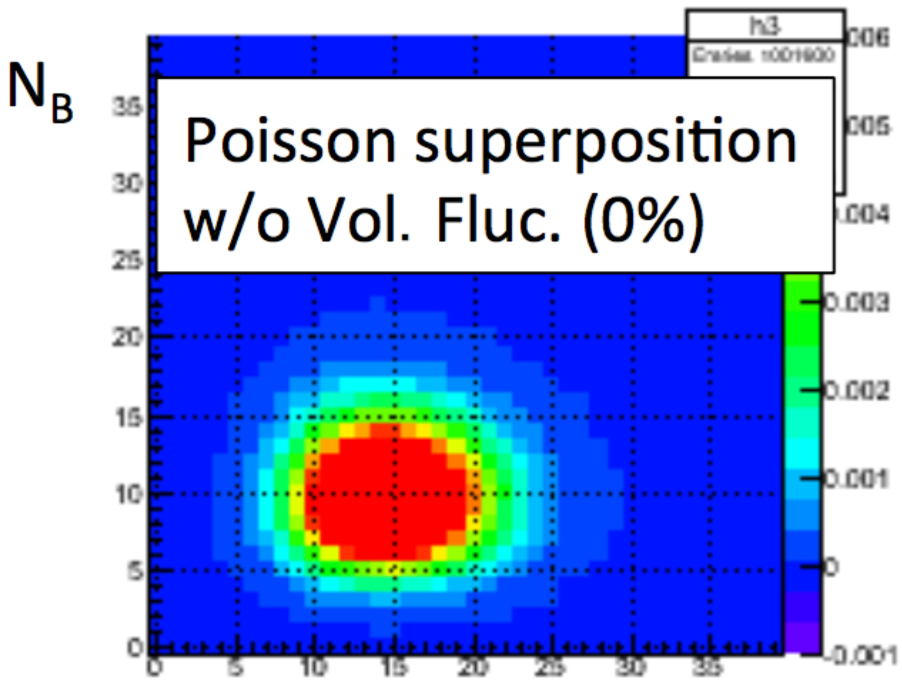


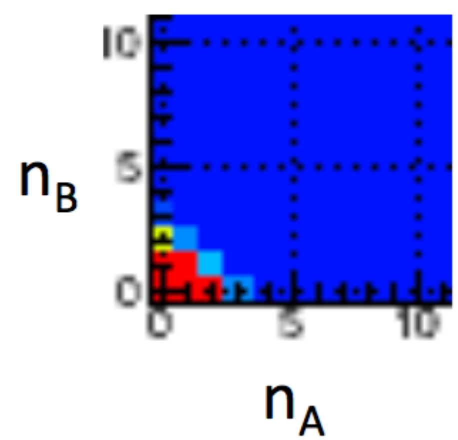
Some examples of this hybrid unfolding with various efficiency assumptions (critical shape 2)



Unfolding with Volume Fluctuation (V.F.)

Volume fluctuation is assumed to be known precisely according to Glauber (or any other initial) model. V.F. naturally induces a positive correlation between N_A vs N_B. Gaussian fluctuation is used in this toy model simulation.



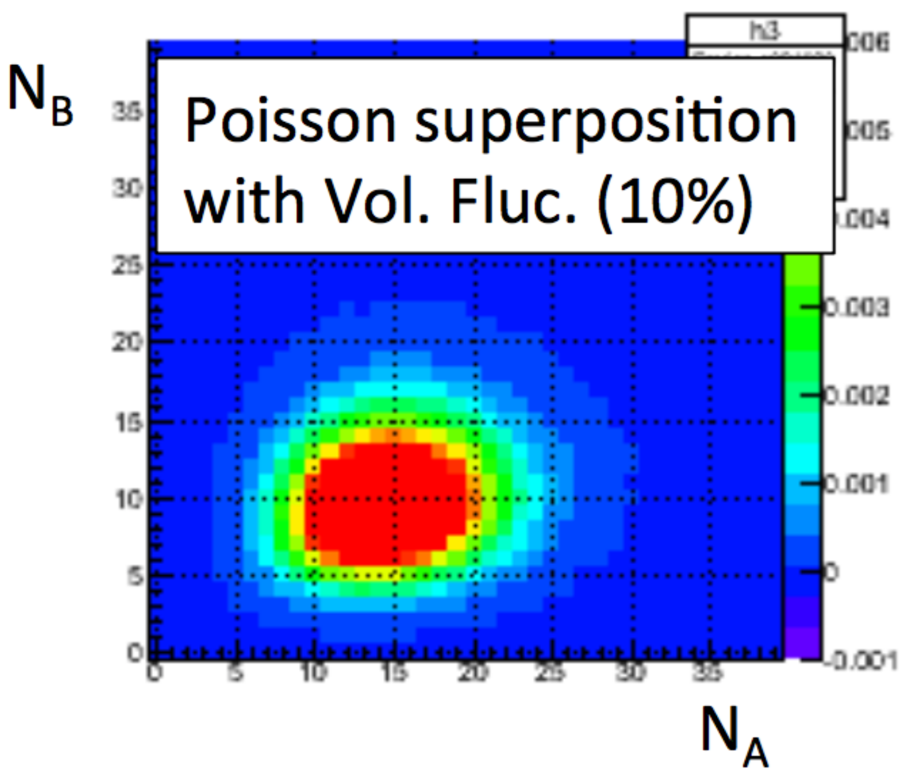


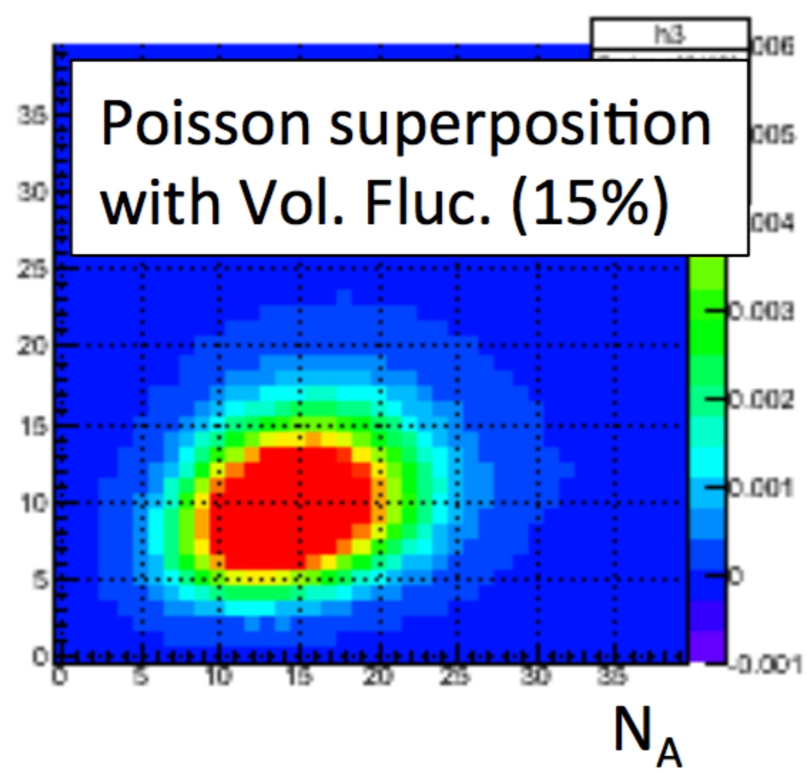
"n_A vs n_B" distribution per source will be extracted, where the number of sources "n_S" has well known fluctuation from VF.

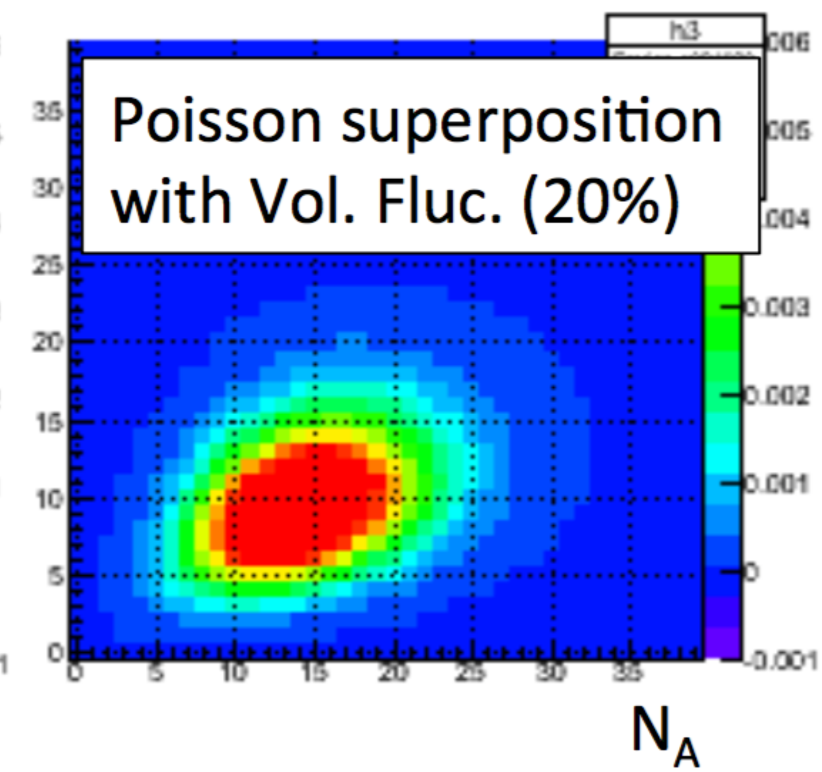
Poisson source : $\langle n_A \rangle = 0.15$, $\langle n_B \rangle = 0.1$

Gaussian fluc. : $\langle n_S \rangle = 100$ +/- V.F.(%)

$N_A = n_A \times n_S$, $N_B = n_B \times n_S$





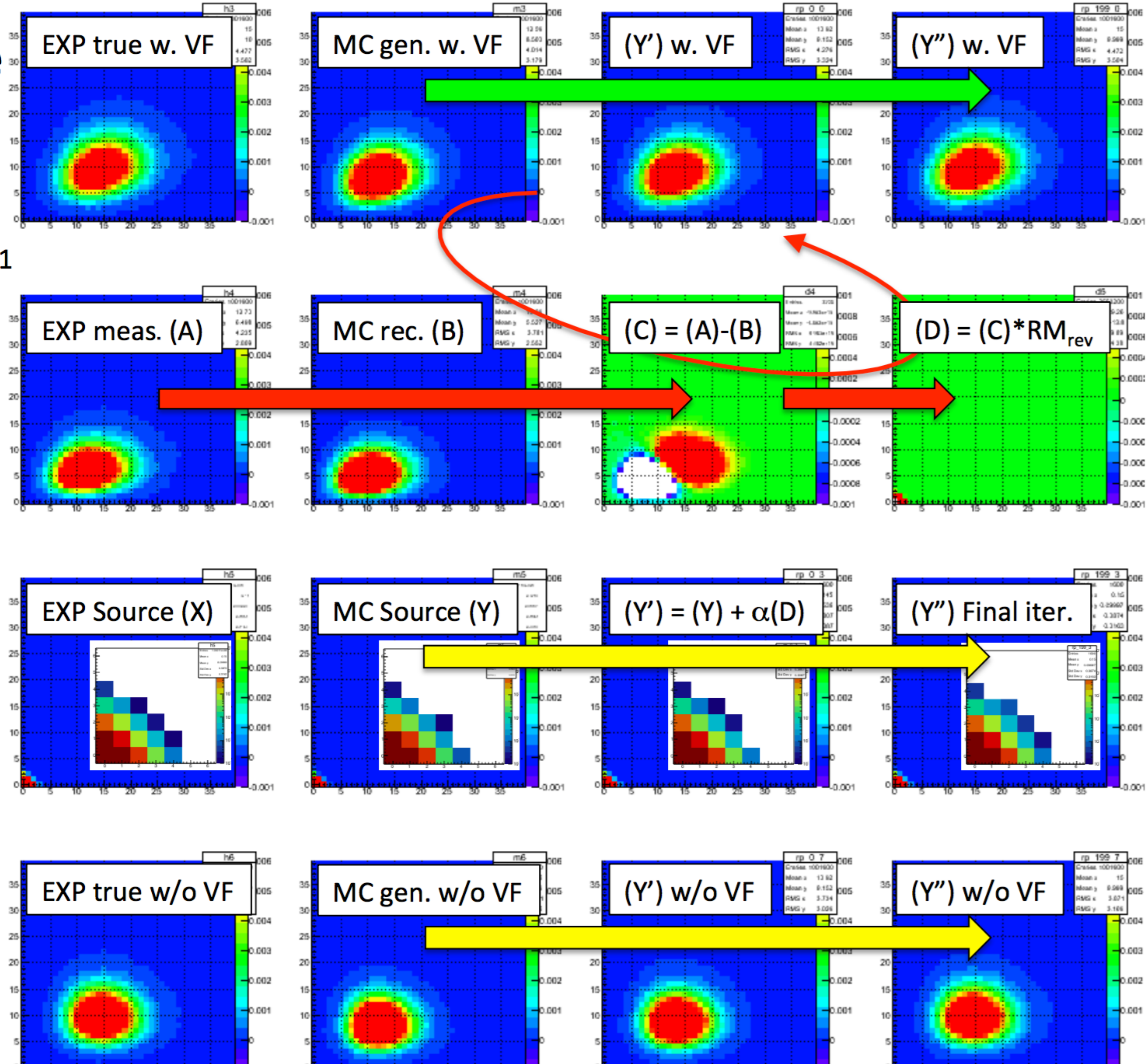


Poisson Source with V.F.

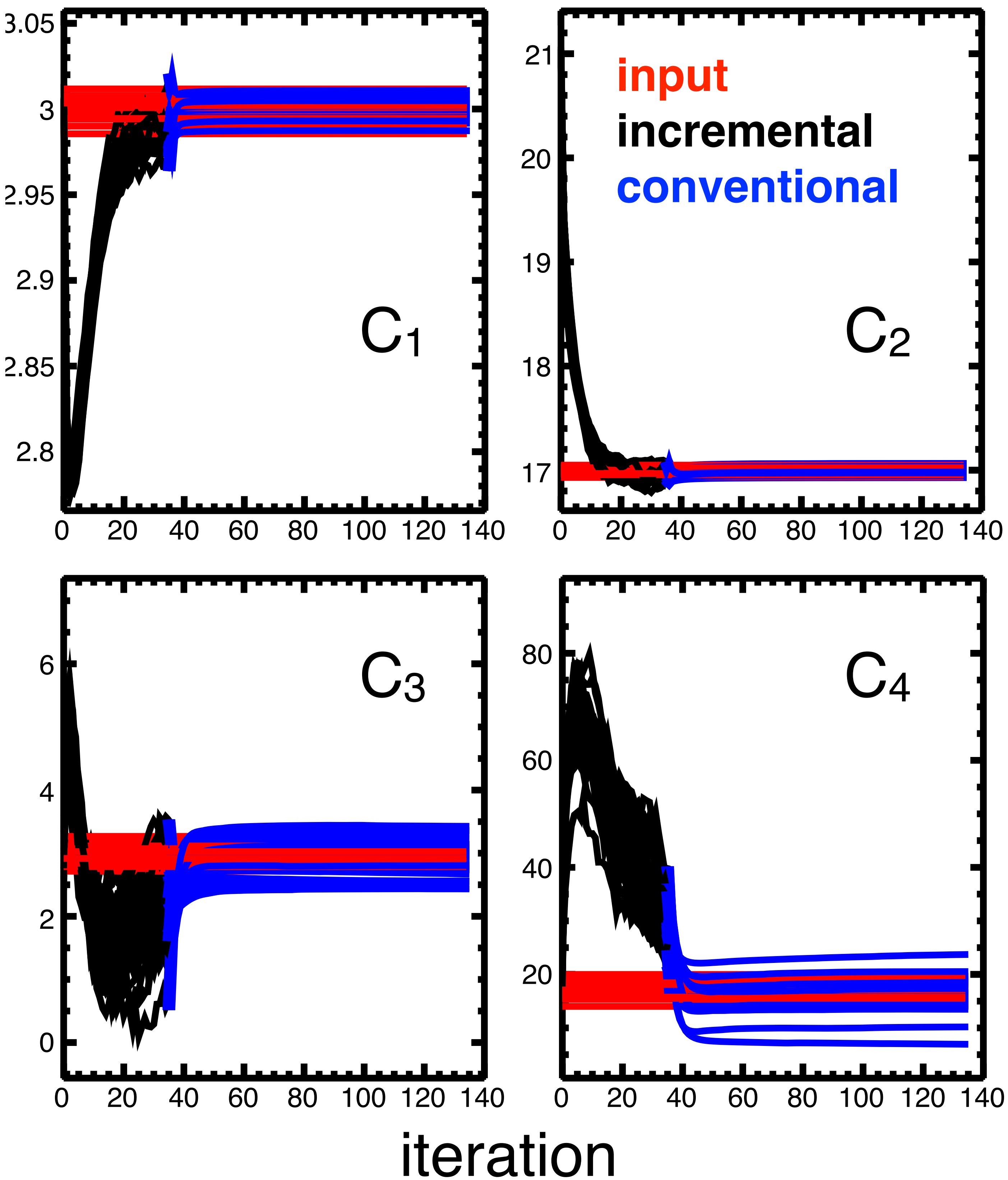
- Poisson Source : $\langle n_A \rangle = 0.15$, $\langle n_B \rangle = 0.1$
- Gaussian Vol. fluc. : $\langle n_S \rangle = 100 \pm 15(\%)$
- $\varepsilon_x=0.85$, $\varepsilon_y=0.65$

Recovery of Source distribution

Superimposed fixed # of Sources



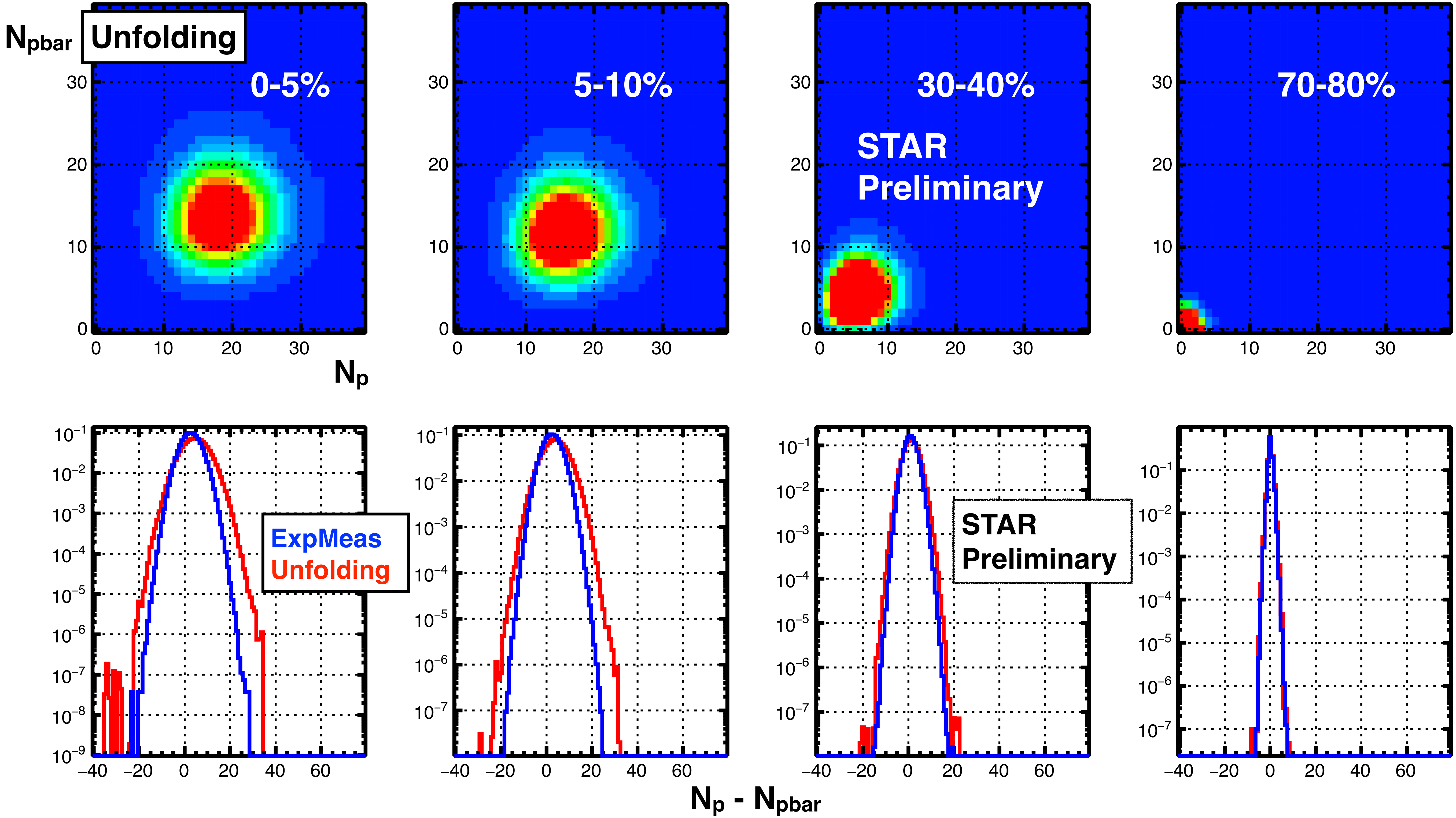
Poisson test : Cumulants



- ✓ **We can stop iterations once cumulants don’t change with iterations.**
- ✓ **Incremental unfolding is effective way to recover bins that don’t exist in simulation, but seems difficult to get higher order cumulants converged.**
- ✓ **Conventional unfolding (not updating the response matrix) is also implemented to get cumulants converged.**

Unfolding with binomial model

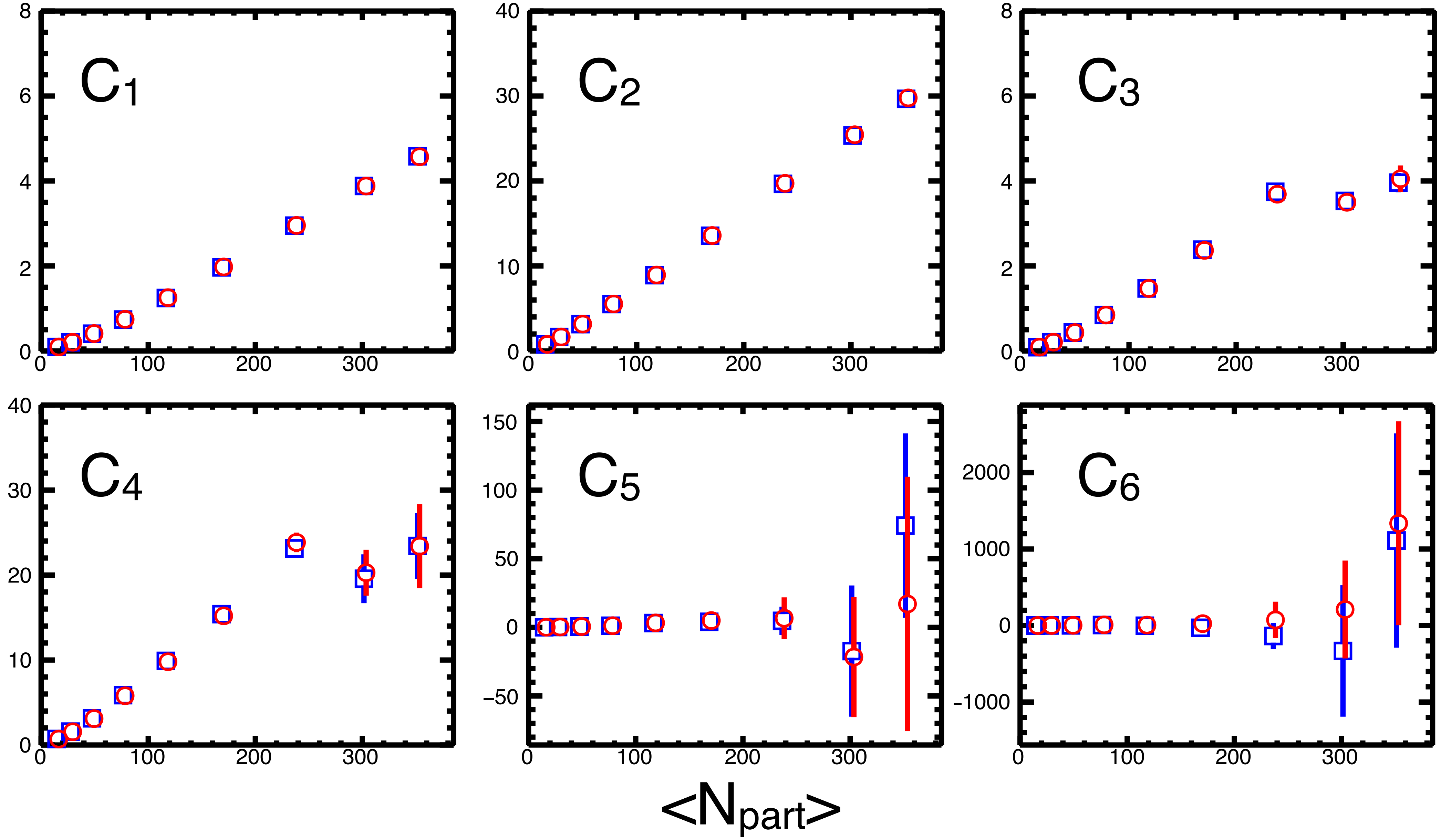
$\sqrt{s_{NN}}$ = 200 GeV, net-proton, |y| < 0.5, 0.4 < p_T < 2.0 (GeV/c), without CBWC, binomial model



Binomial model

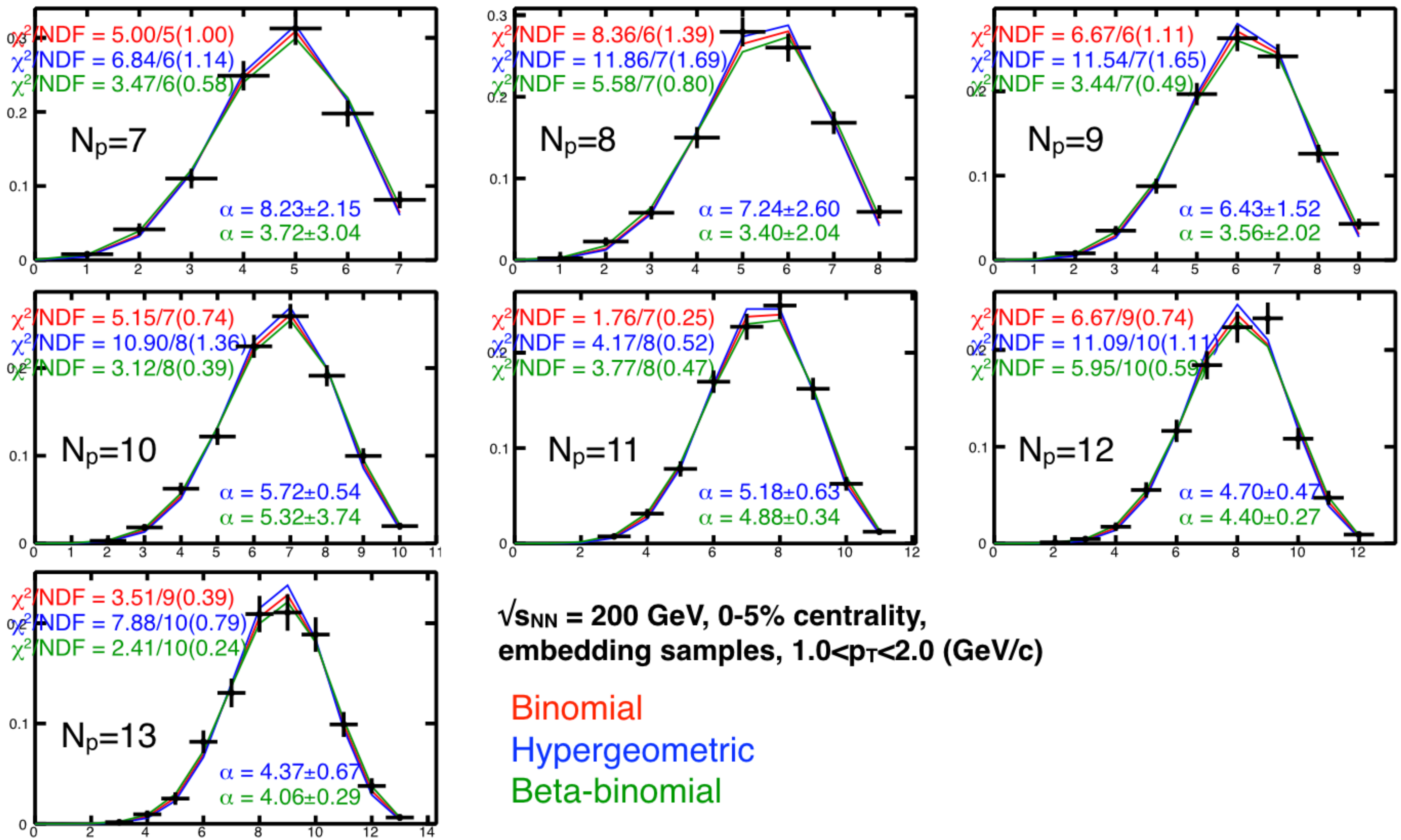
- **Unfolding gives consistent results with efficiency correction by assuming binomial model in MC filter.**

$\sqrt{s_{NN}}$ = 200 GeV, Run11, net-proton, |y| < 0.5, 0.4 < p_T < 2.0 (GeV/c), without CBWC, binomial model



(Non)binomial fitting

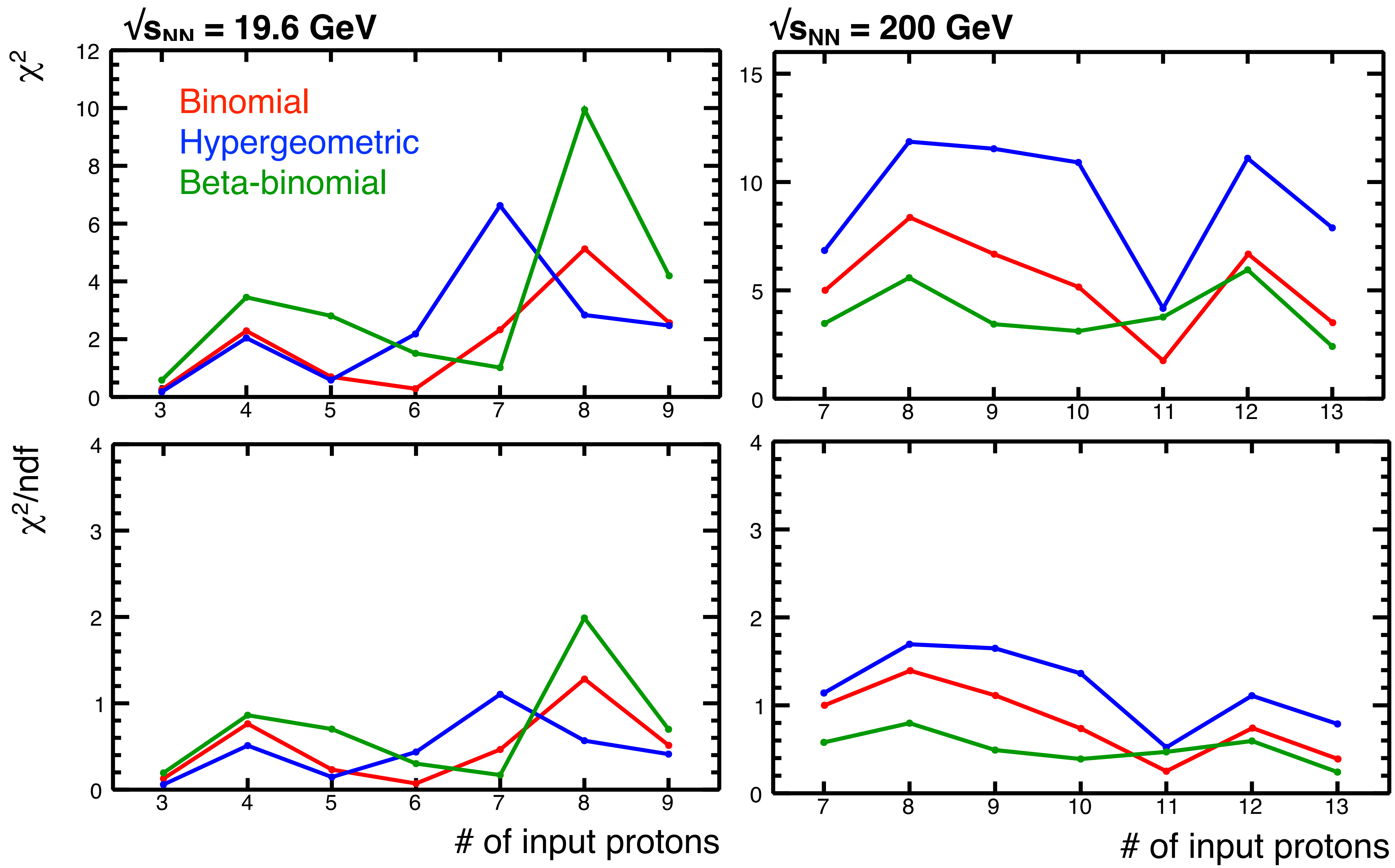
- ✦ **Extracted α parameter will be implemented in unfolding to see how C_6 is affected by non-binomial model.**



n_p : Number of reconstructed protons

Comparison of χ^2/ndf

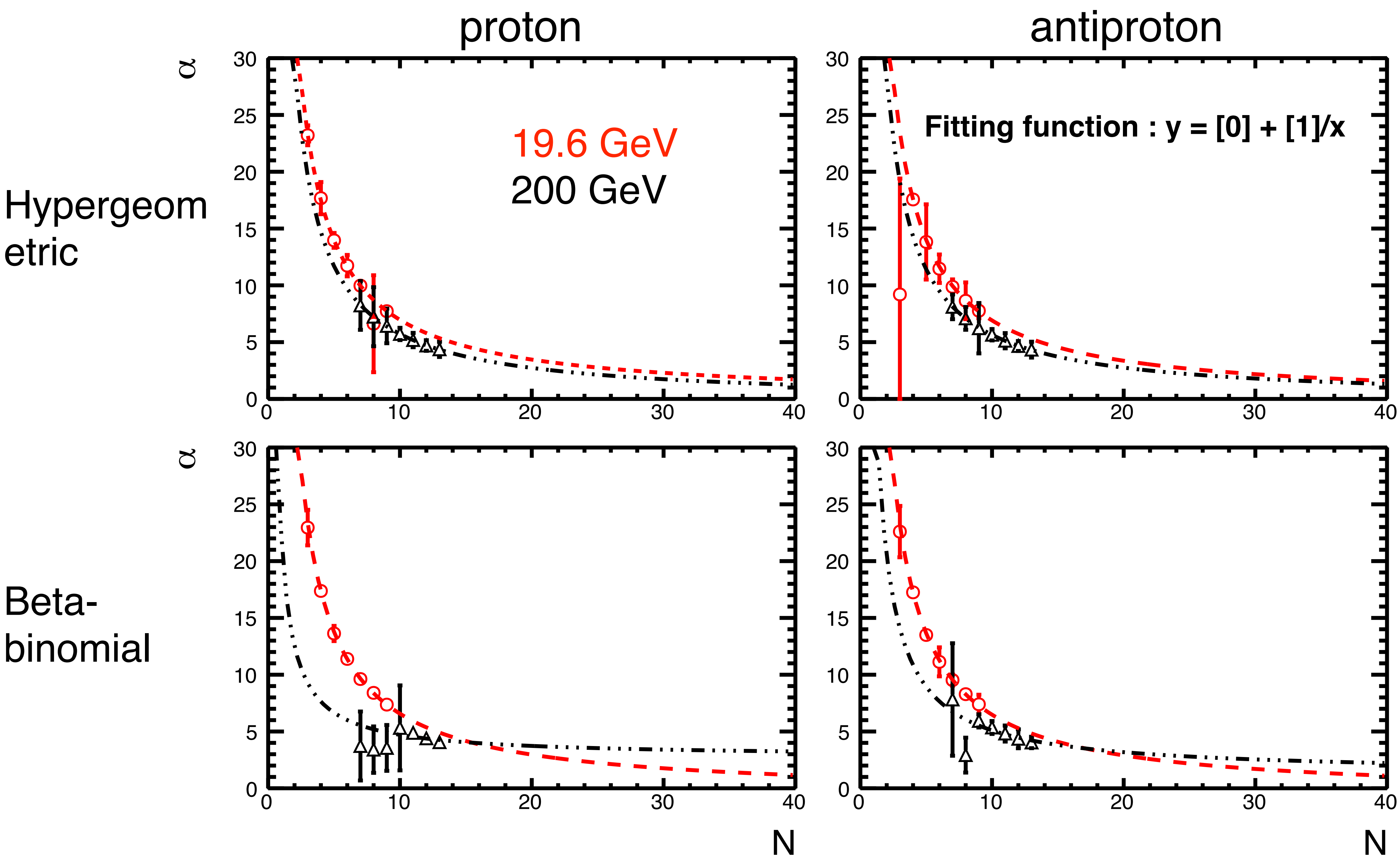
- ✓ At $\sqrt{s_{NN}}$ = 200 GeV, beta-binomial seems to close to data, which might be due to the superposition of different refmult3.
- ✓ Mostly χ^2/ndf<1, more embedding statistics are necessary!



Embedding samples, 0-5% centrality, 1.0<p_T<2.0 (GeV/c)

α vs N

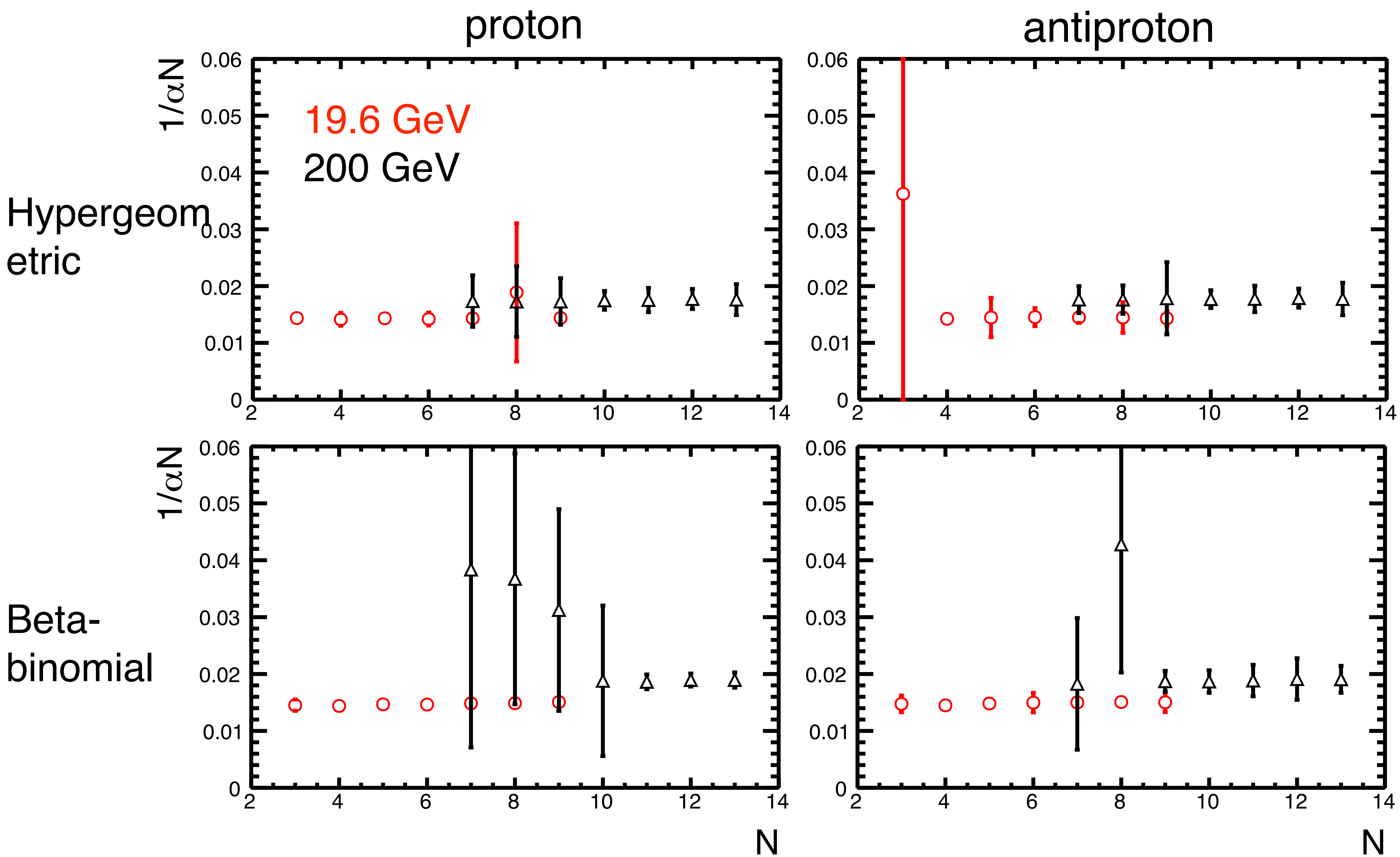
✓ **$\alpha \propto 1/N$ is nature of hypergeometric(beta-binomial) distributions when the deviation from binomial doesn't depend on N.**



Embedding samples, 0-5% centrality, $1.0<p_T<2.0$ (GeV/c)

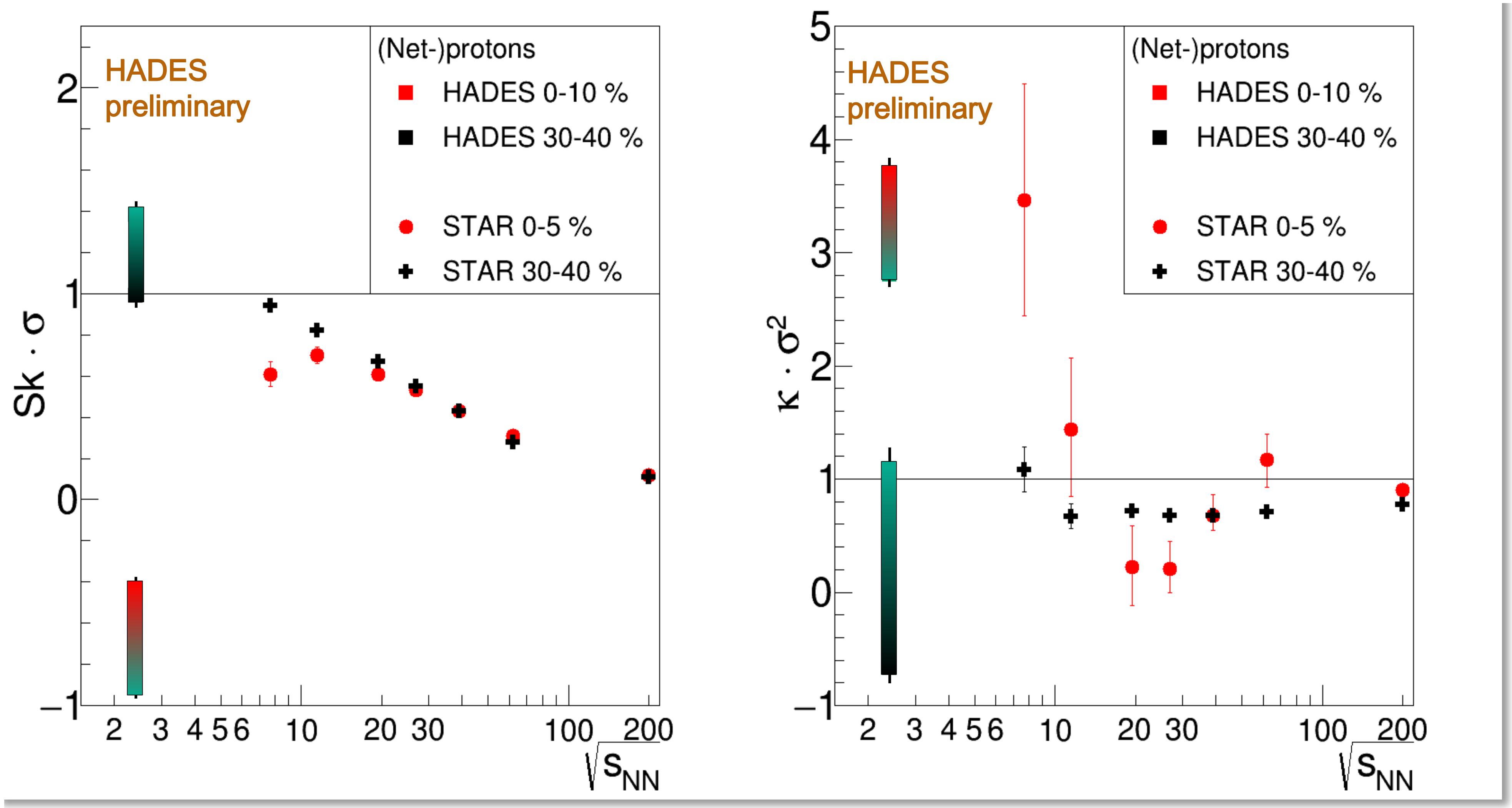
1/αN vs N

- ✓ **Binomial baseline ~ 0.**
- ✓ **Deviation from binomial does not depend on number of input protons, and it is less than 6%.**



Embedding samples, 0-5% centrality, $1.0<p_T<2.0$ (GeV/c)

R. Holtzman, QM2017



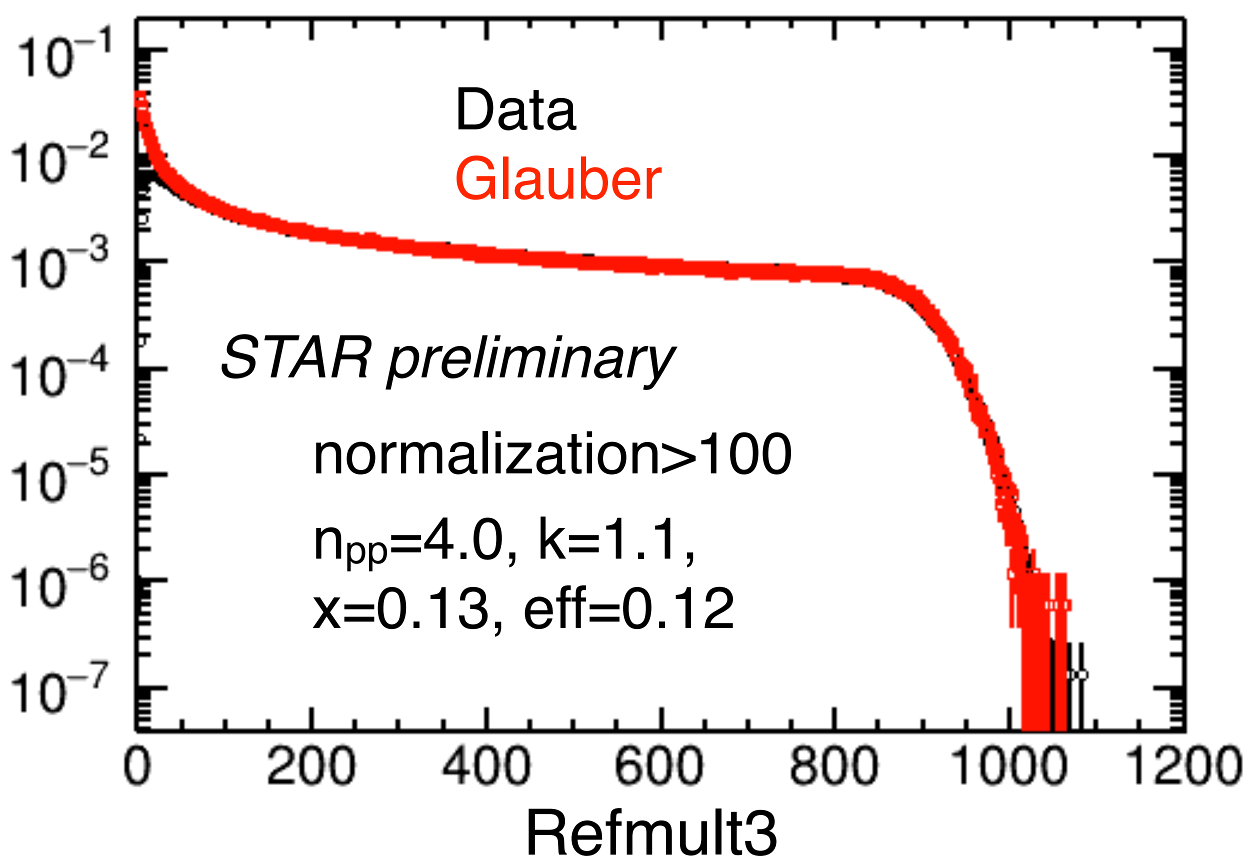
red/black = unfolding (preferred method) + vol. flucs. corr.

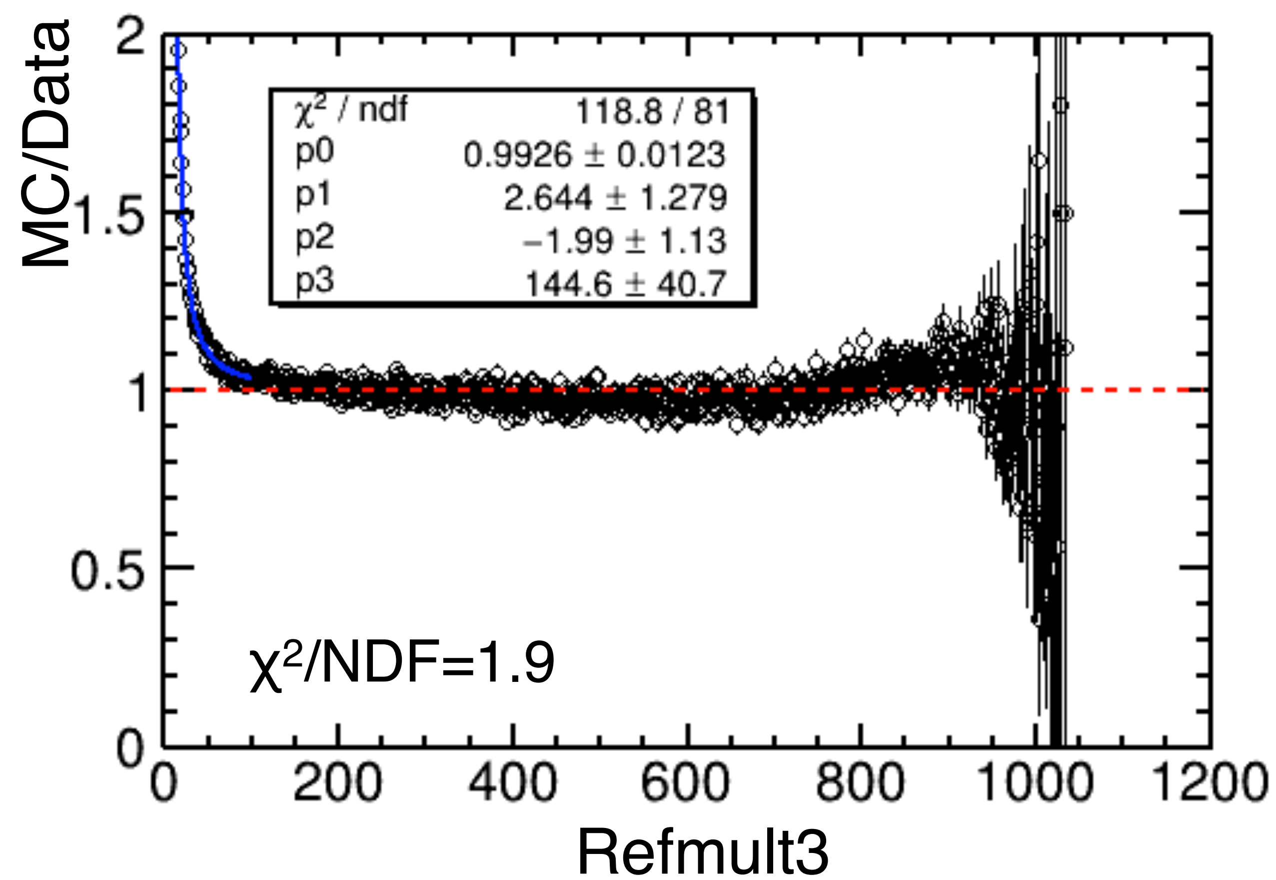
green = evt-by-evt eff correction of factorial moments + vol. flucs. corr.

Centrality determination

1. Use charged particles except protons in order to suppress the auto-correlation.
2. Those particles are counted at the wide eta range $|\eta|<1.0$ to increase the centrality resolution.

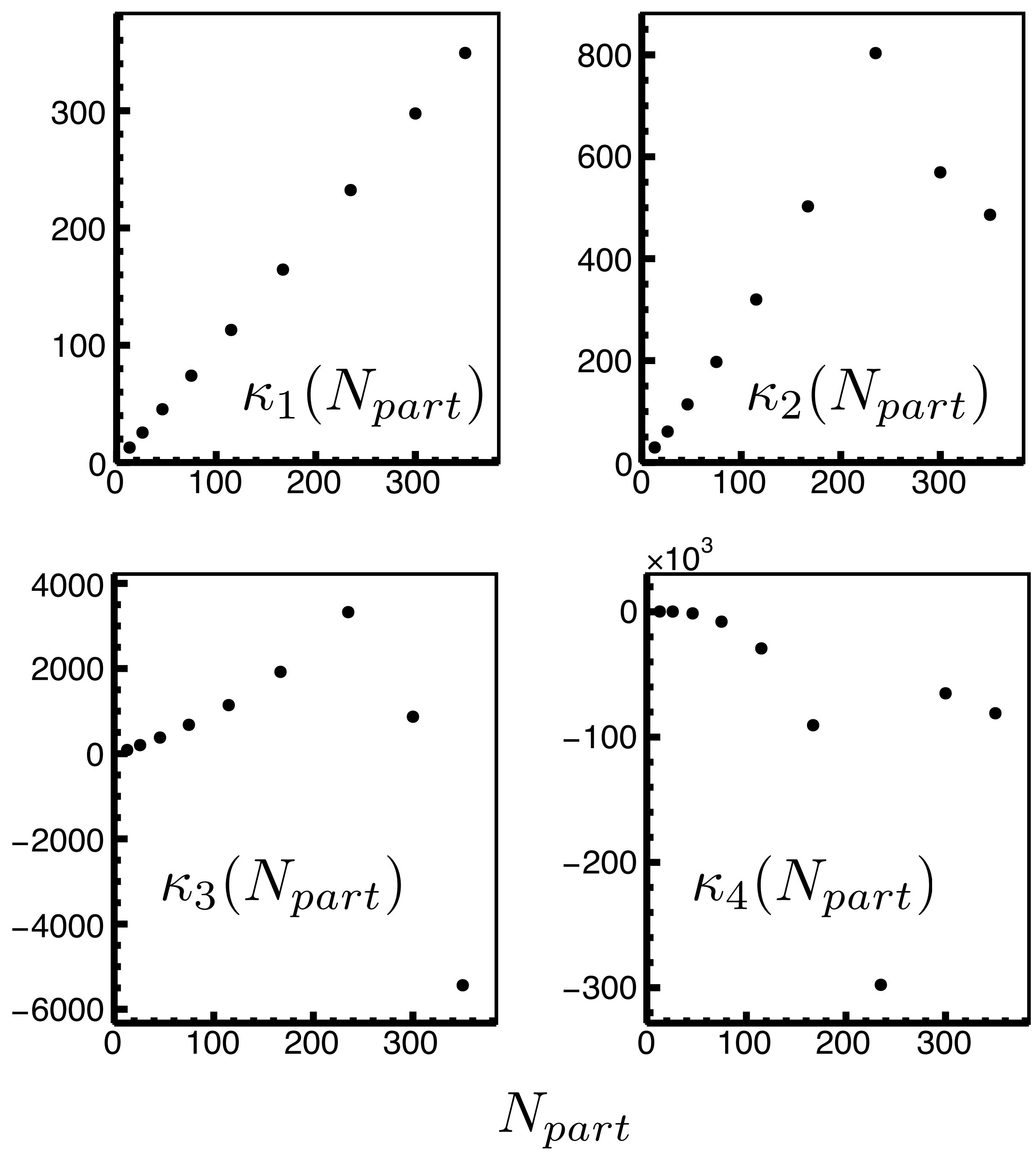
✦ Glauber and two-component model (with NBD fluctuaiton) are tuned to reproduce the measured multiplicity distribution.

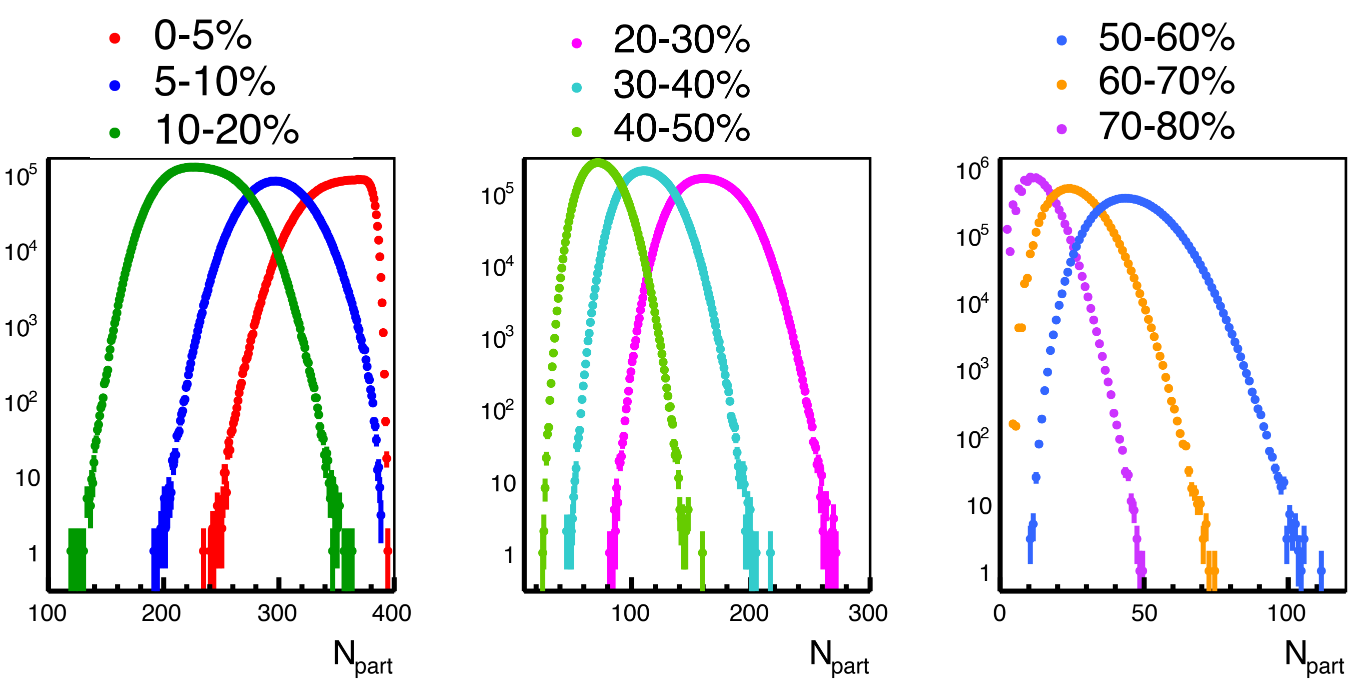




N_{part} cumulants

✦ N_{part} cumulants have the extreme value at 10-20% (N_{part}~230).





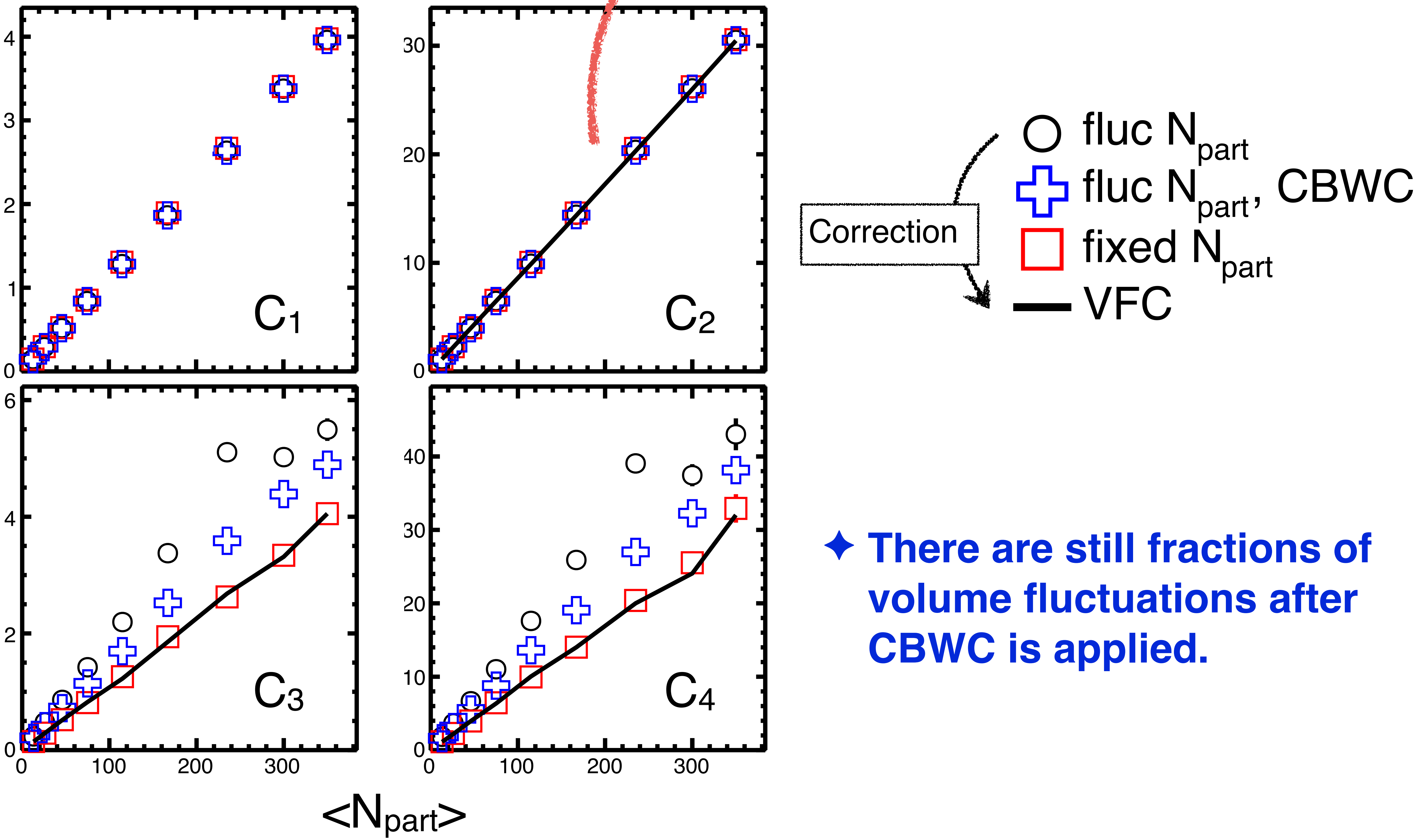
Toy model

✓ Correction of C_2 is small at high energies

$$\kappa_2(\Delta N) = \langle N_W \rangle \kappa_2(\Delta n) \boxed{+ \underline{\langle \Delta n \rangle}^2 \kappa_2(N_W)}$$

net-proton

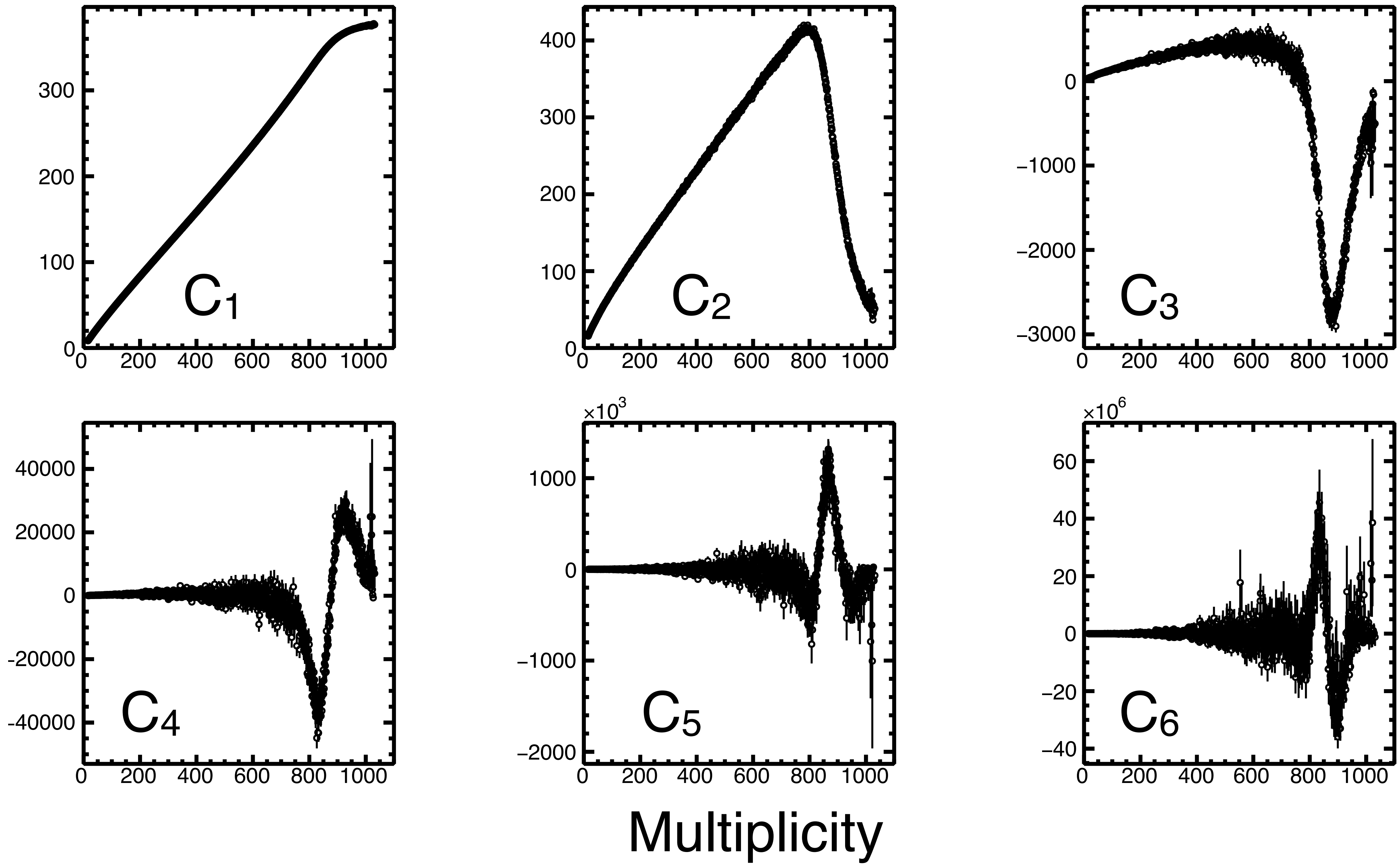
Glauber+two component model : Au+Au √sNN = 200 GeV



✦ **There are still fractions of volume fluctuations after CBWC is applied.**

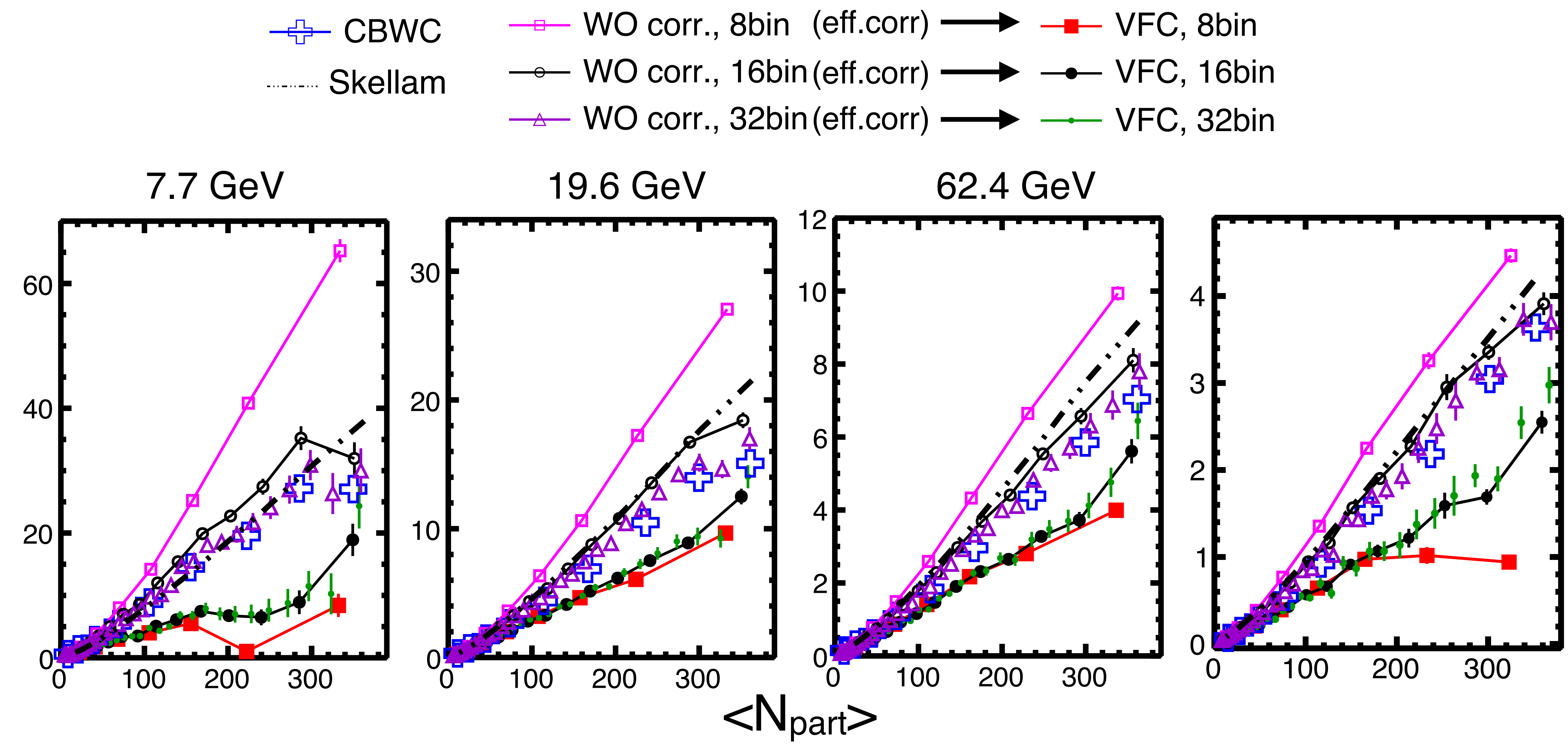
N_{part} cumulants for one multiplicity bin

- N_{part} cumulants have been calculated at each multiplicity bin in order to estimate the effect of participant fluctuations on CBWC.



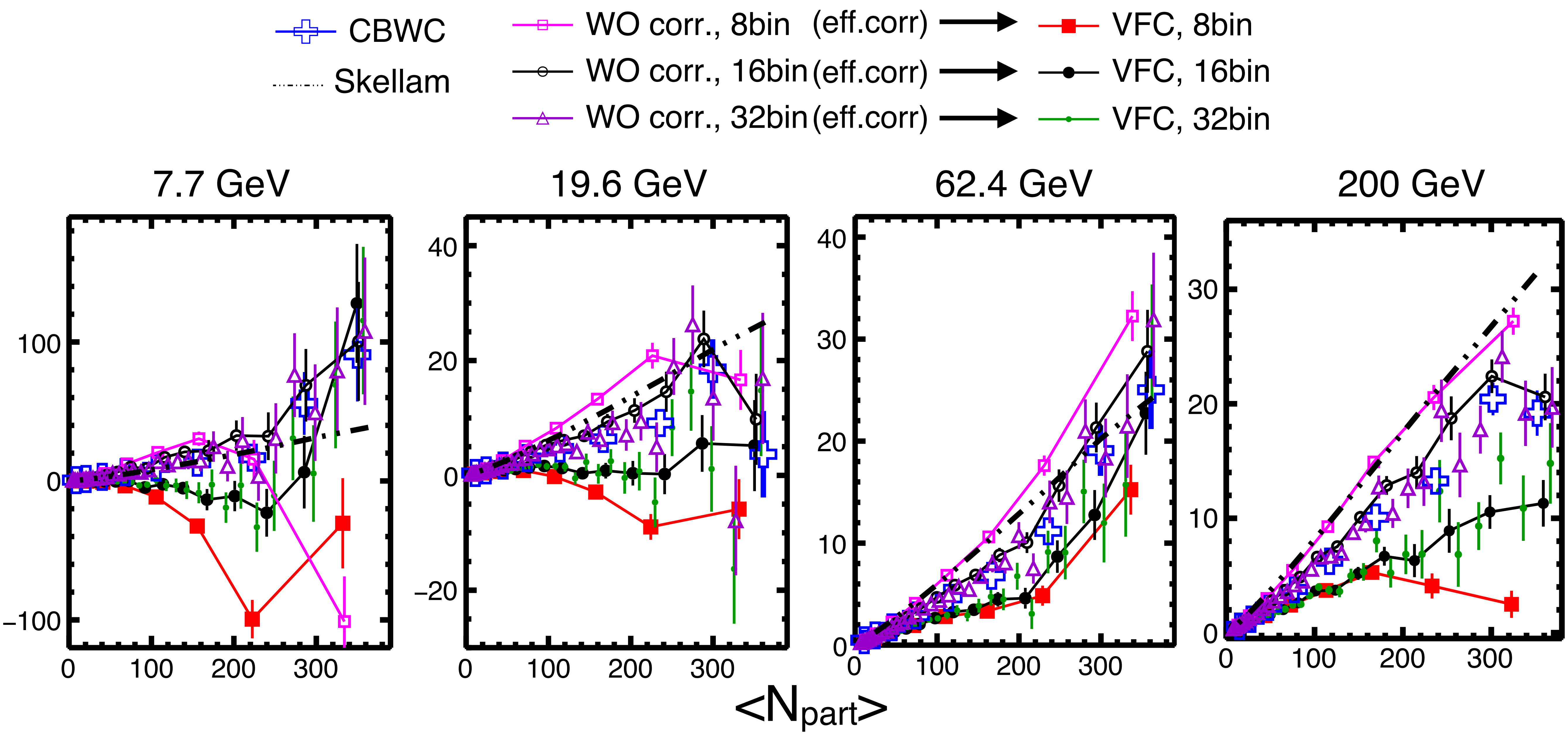
Beam energy dependence : C_3

- Centrality bin width dependence is observed in VFC at all beam energies, and they seem to converge to certain value with narrow bin width.
- C_3 is enhanced by volume fluctuation but ⊕ would be larger if multiplicity dependent efficiency is corrected.



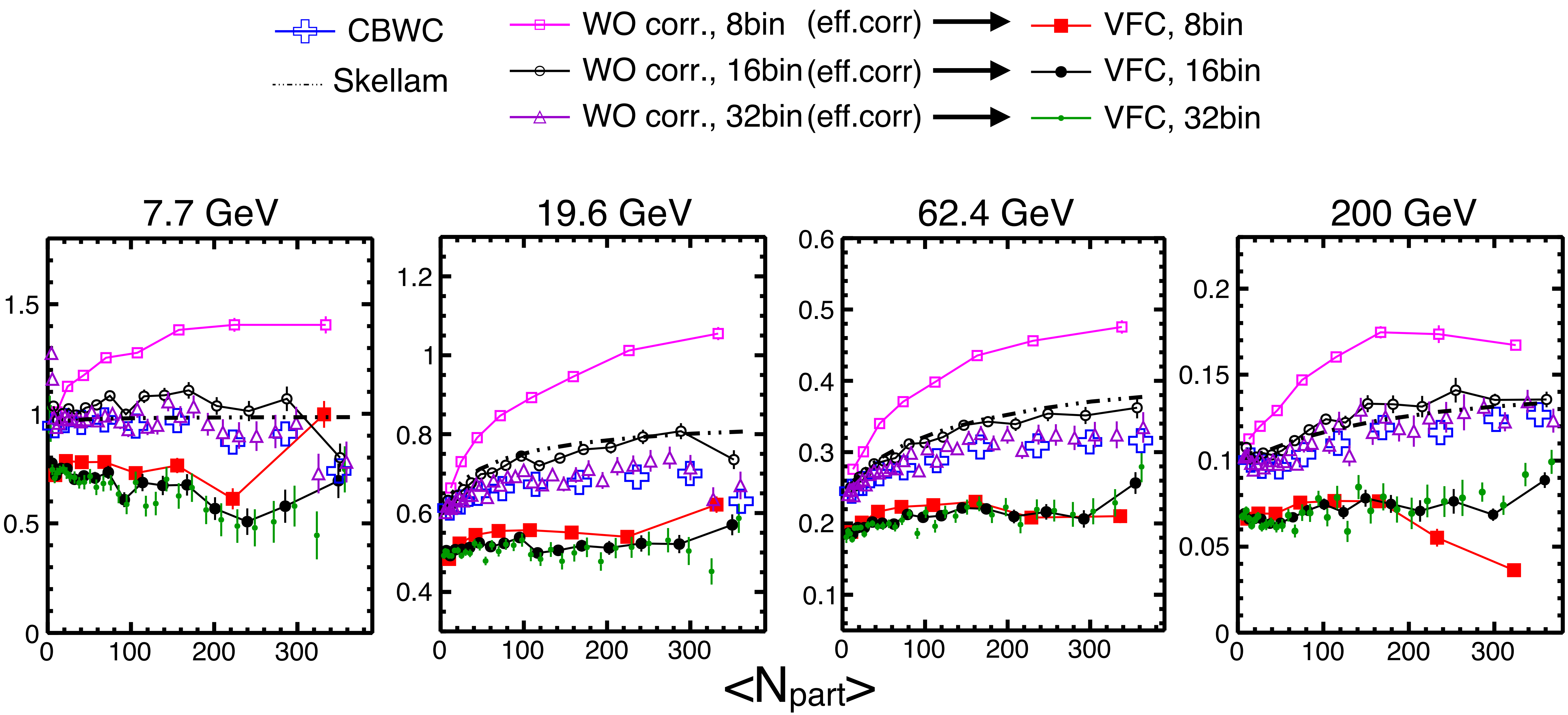
Net-proton, |y| < 0.5, 0.4 < p_T < 2.0 (GeV/c)

Beam energy dependence : C_4



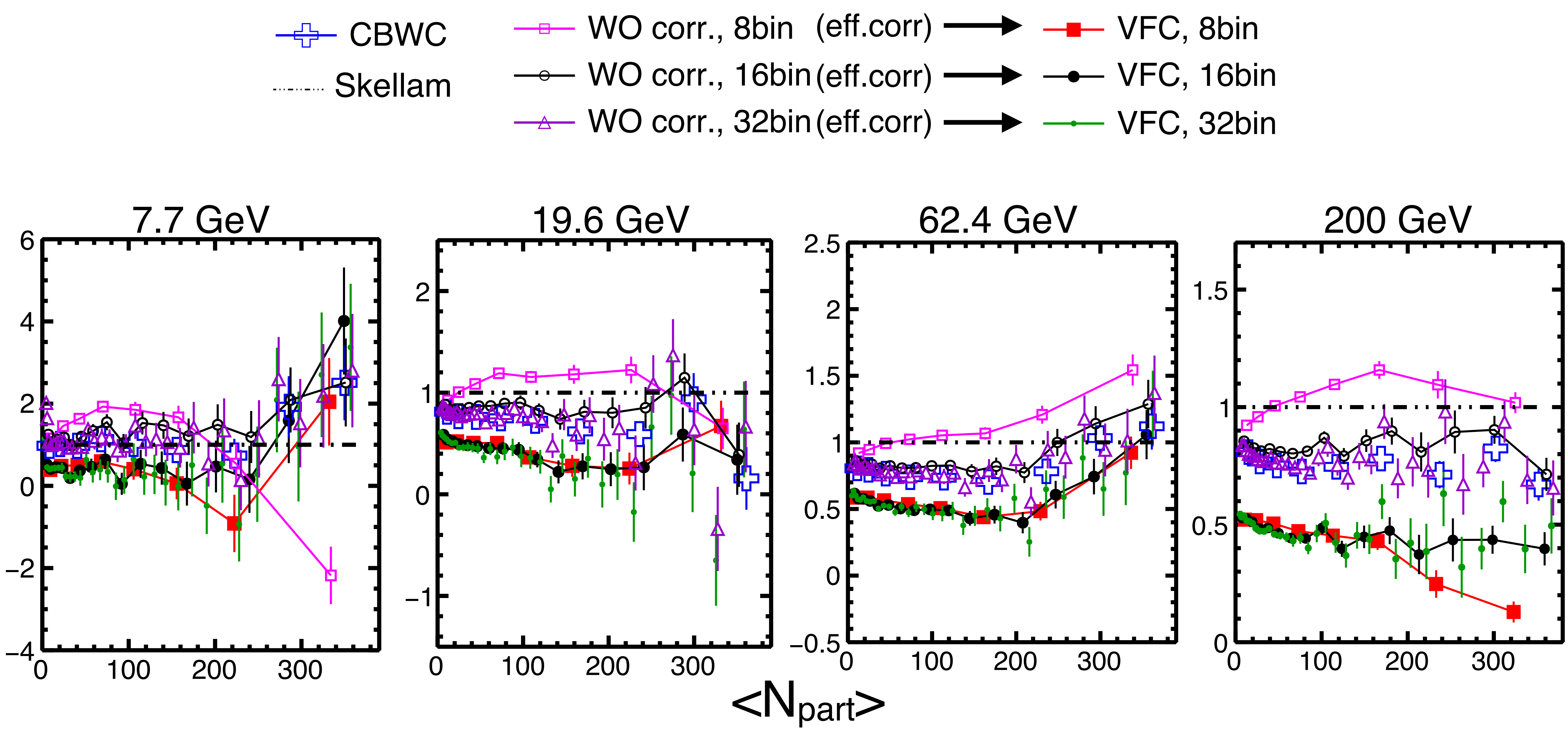
Net-proton, |y| < 0.5, 0.4 < p_T < 2.0 (GeV/c)

Beam energy dependence : C_3/C_2



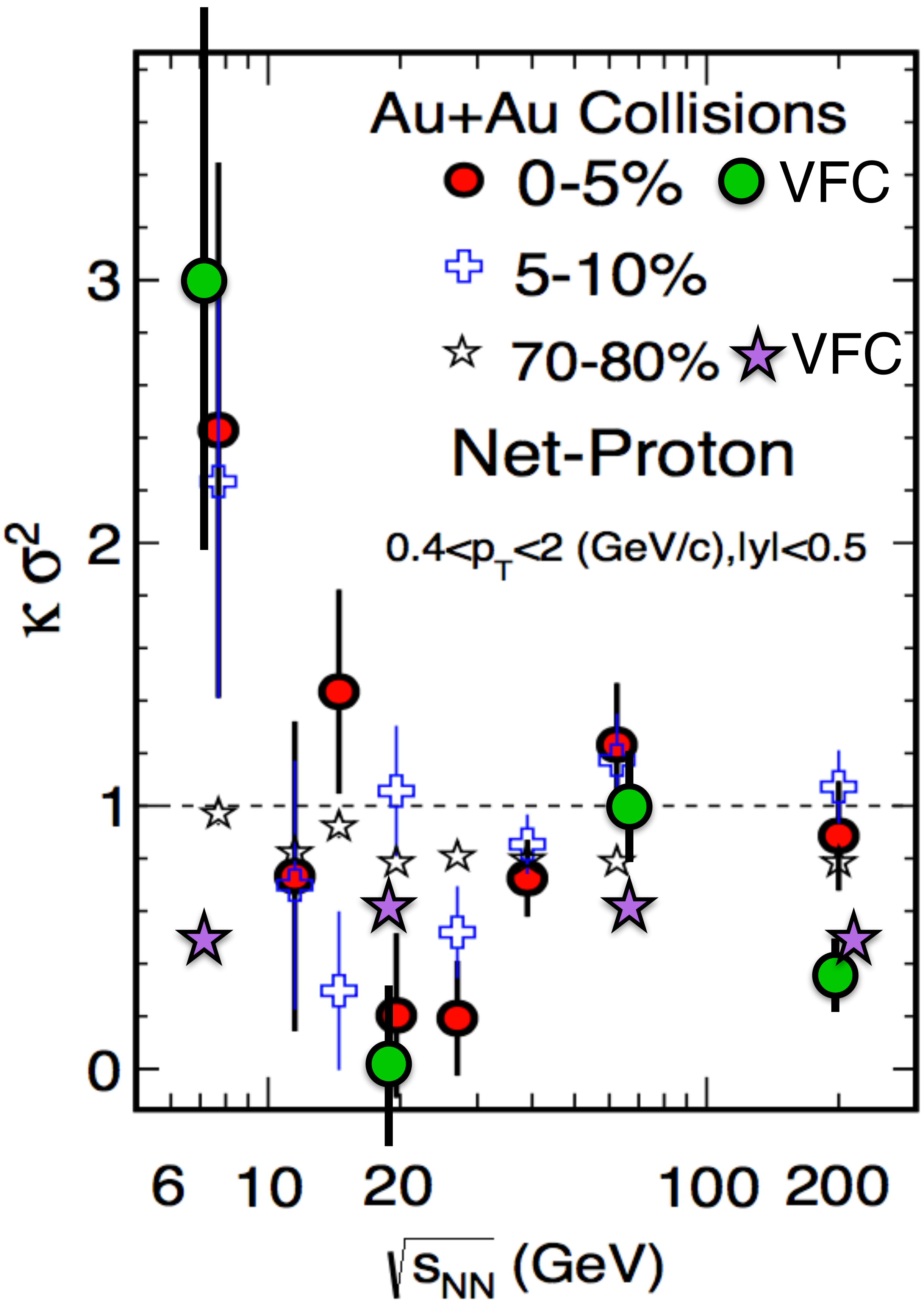
Net-proton, |y| < 0.5, 0.4 < p_T < 2.0 (GeV/c)

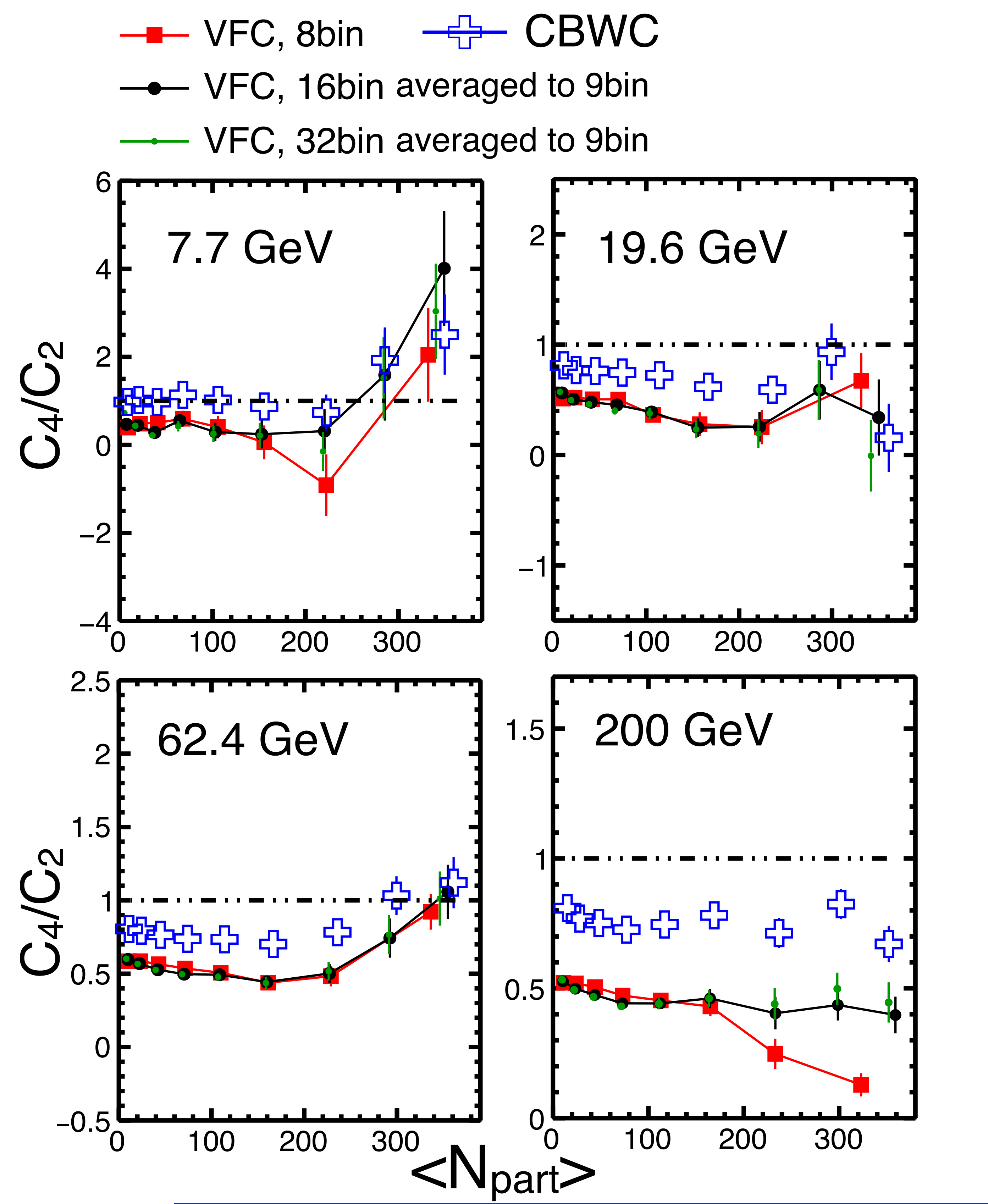
Beam energy dependence : C_4/C_2



Net-proton, |y| < 0.5, 0.4 < p_T < 2.0 (GeV/c)

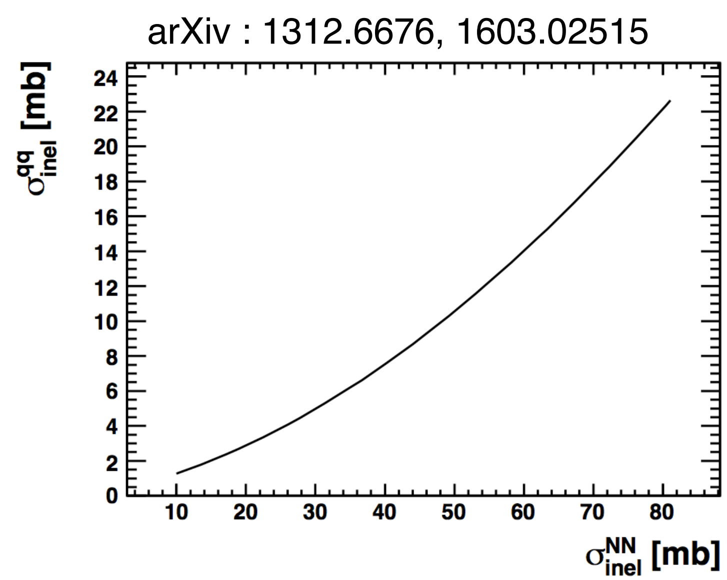
Comparison with preliminary results

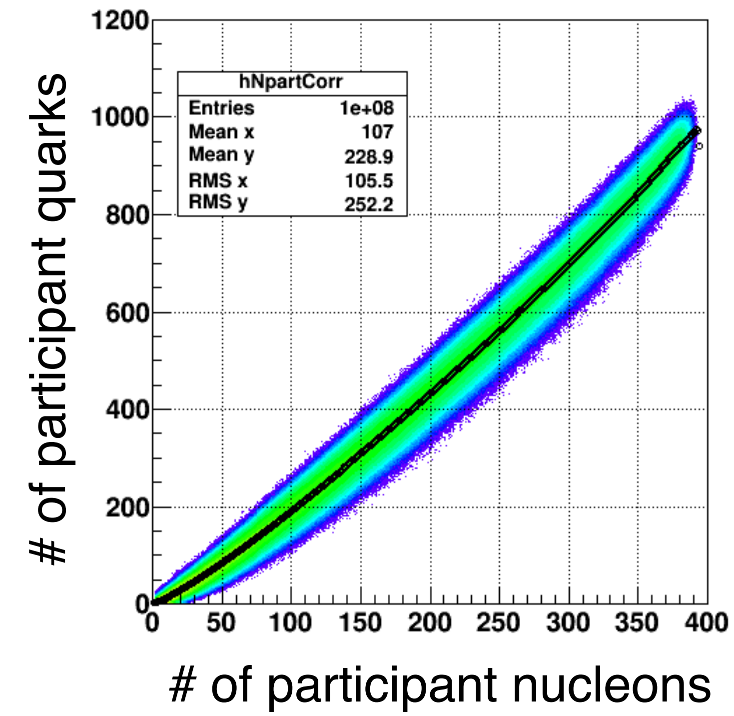




Quark participant model

- ✦ Thanks to Jamie for suggestion.
- ✦ Assumption that particles are generated from independent source of quark participant.
- ✦ Fitting with quark participant model to the STAR data at 200 GeV refmult3 distribution to extract correction factors.





Nw (# of participant nucleons)
→ # of participant quarks

$$\kappa_1(\Delta N) = \langle N_W \rangle \, \kappa_1(\Delta n),$$

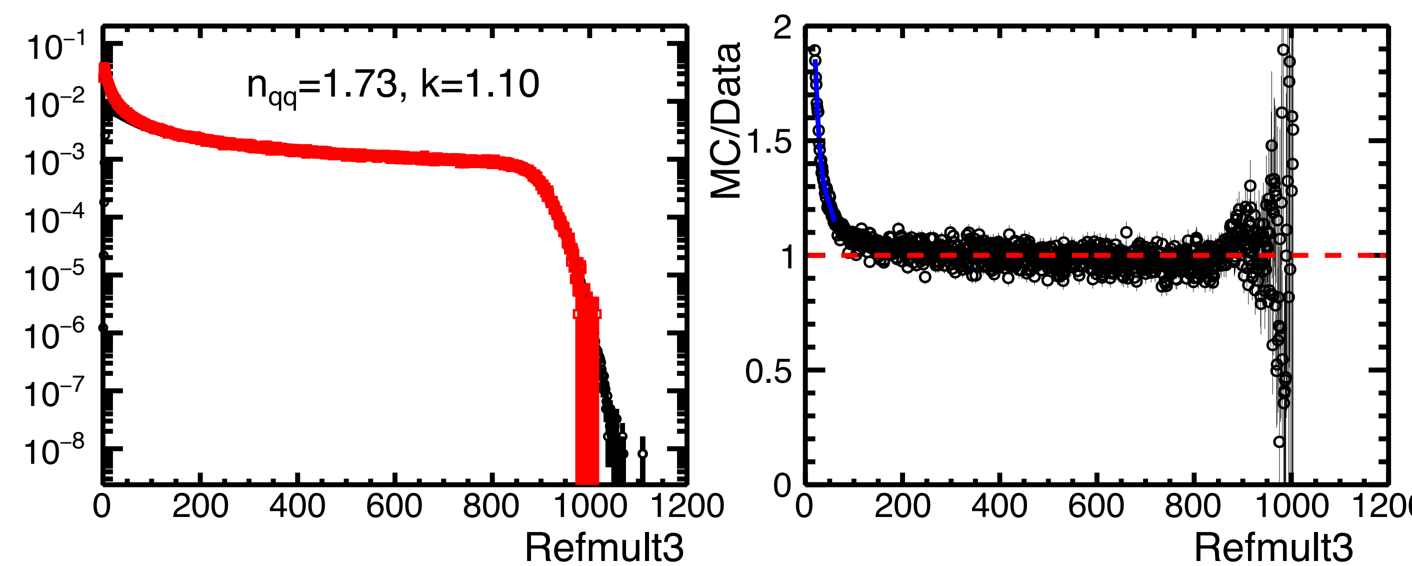
$$\kappa_2(\Delta N) = \langle N_W \rangle \, \kappa_2(\Delta n) + \langle \Delta n \rangle^2 \, \kappa_2(N_W),$$

$$\kappa_3(\Delta N) = \langle N_W \rangle \, \kappa_3(\Delta n) + 3 \langle \Delta n \rangle \, \kappa_2(\Delta n)\kappa_2(N_W) + \langle \Delta n \rangle^3 \, \kappa_3(N_W),$$

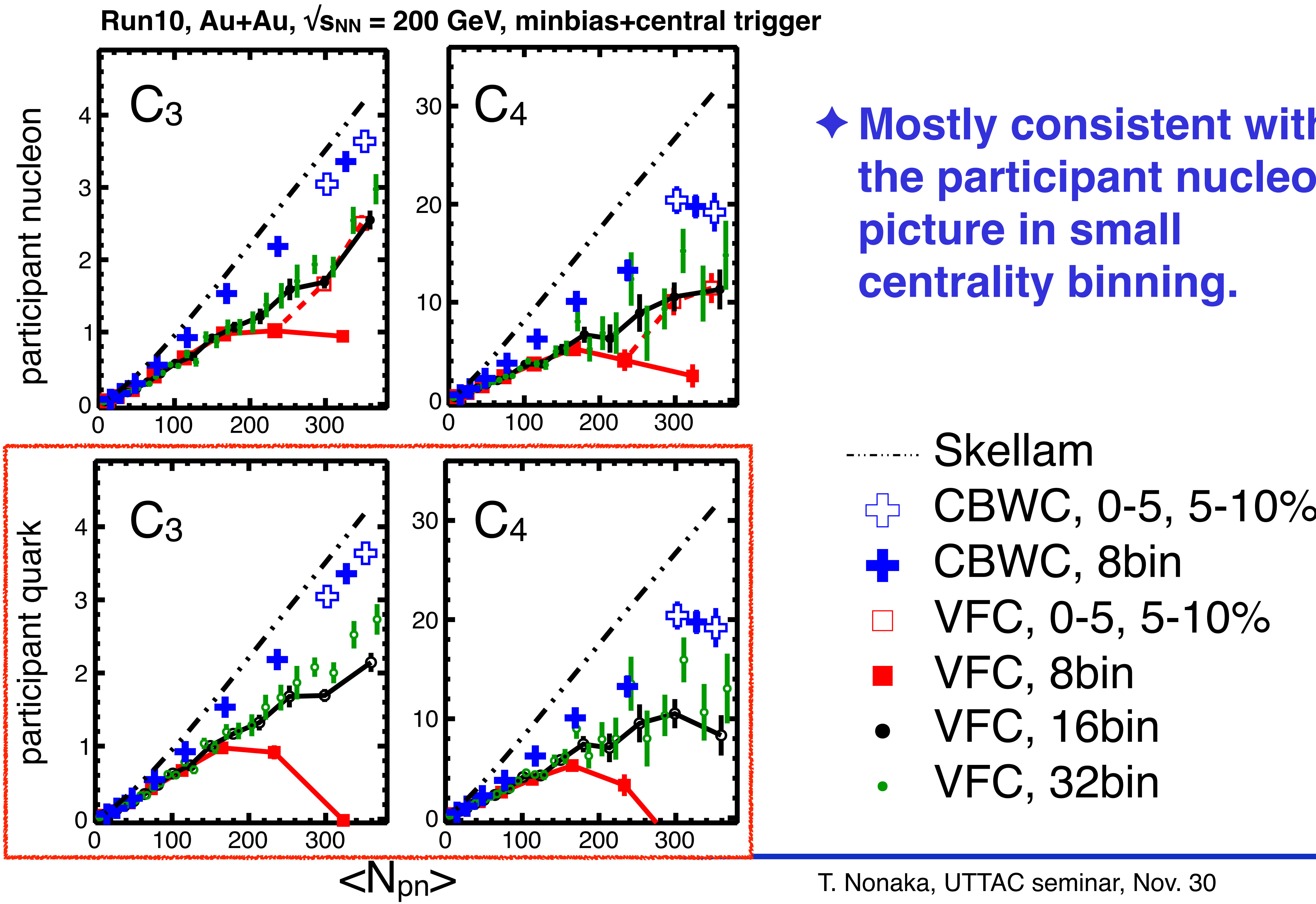
$$\kappa_4(\Delta N) = \langle N_W \rangle \, \kappa_4(\Delta n) + 4 \langle \Delta n \rangle \, \kappa_3(\Delta n)\kappa_2(N_W)$$

$$+ 3\kappa_2^2(\Delta n)\kappa_2(N_W) + 6 \langle \Delta n \rangle^2 \, \kappa_2(\Delta n)\kappa_3(N_W) + \langle \Delta n \rangle^4 \, \kappa_4(N_W).$$

Run10, Au+Au, $\sqrt{s_{NN}}$ = 200 GeV



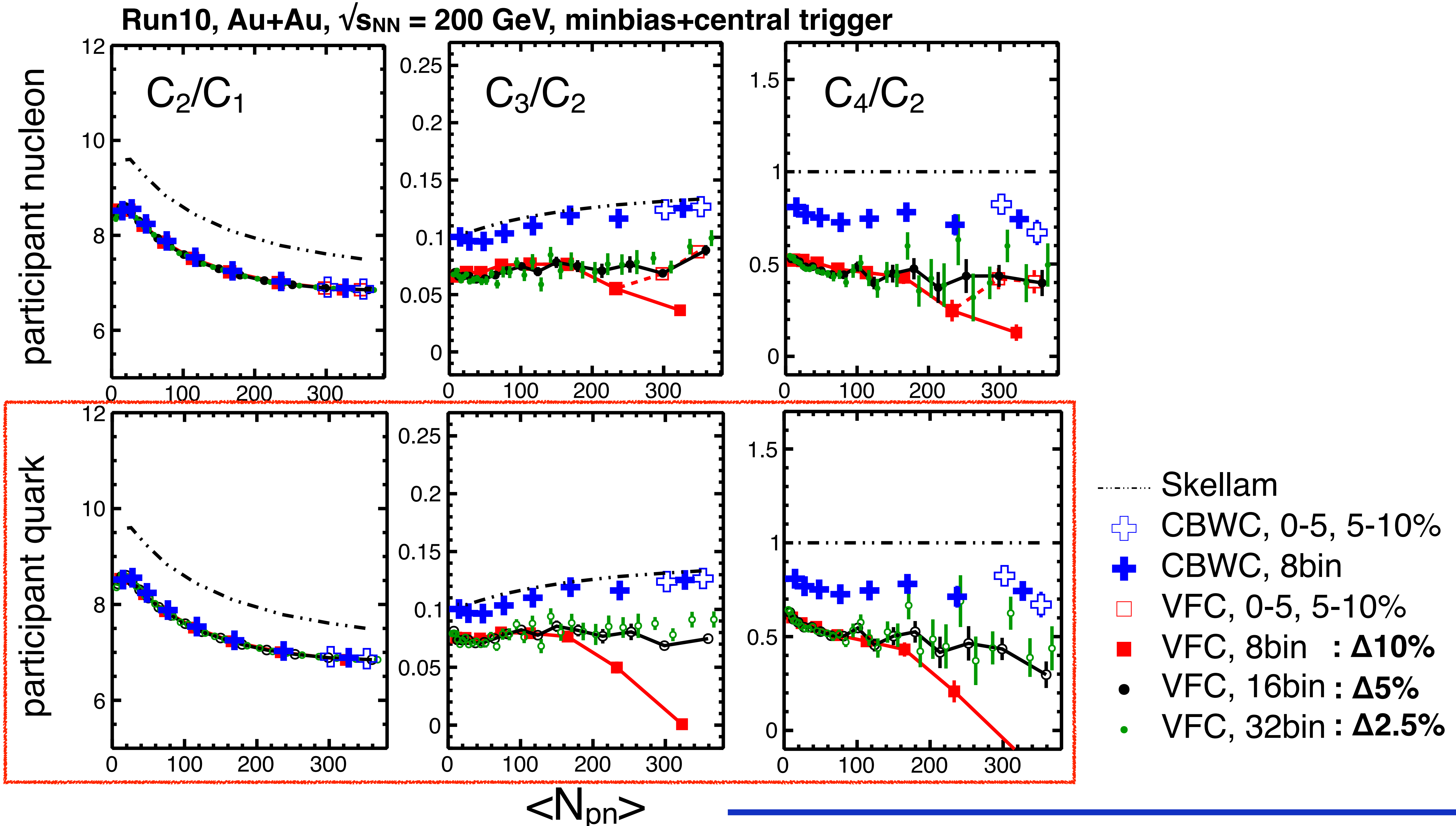
Quark participant model



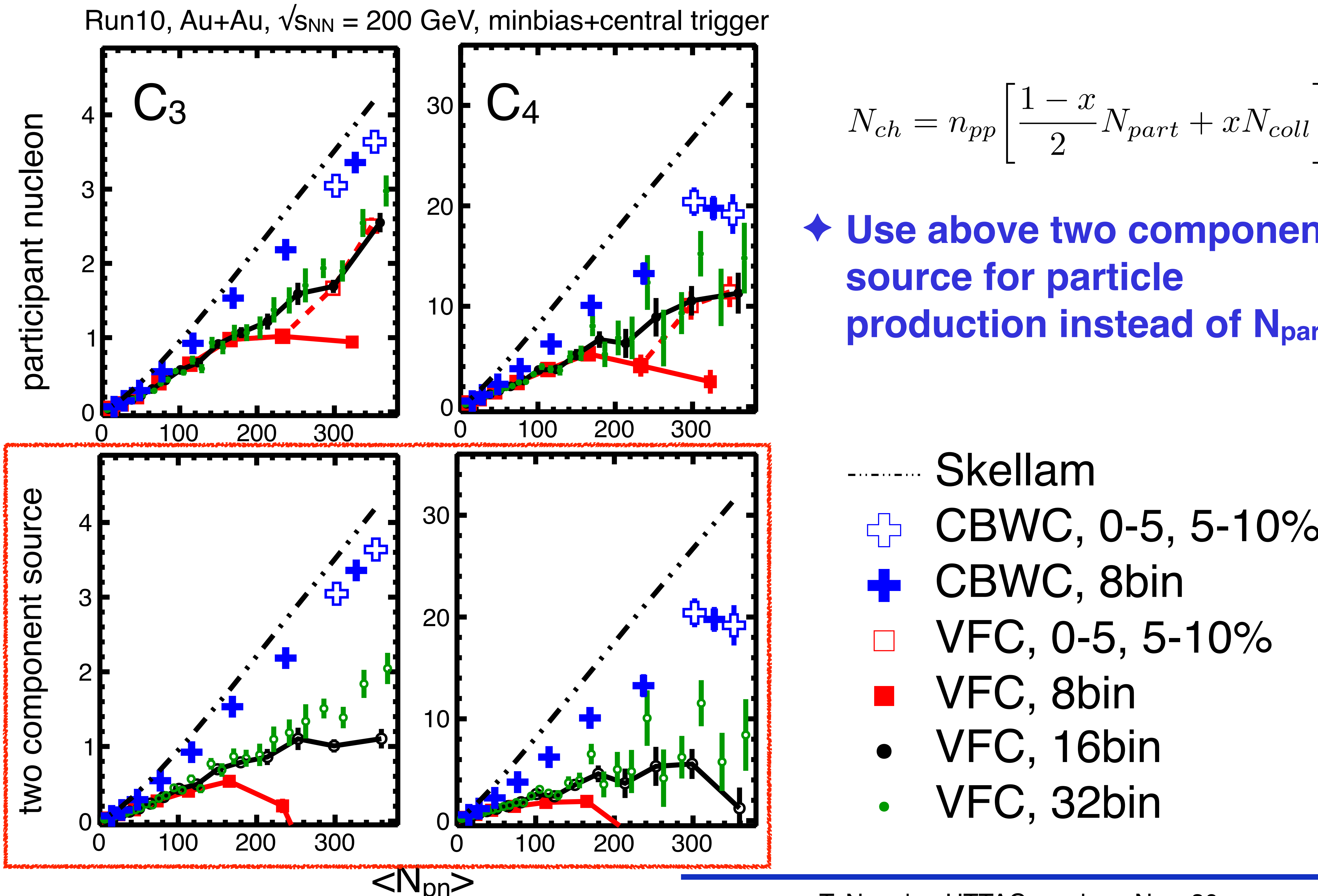
✦ **Mostly consistent with the participant nucleon picture in small centrality binning.**

Quark participant model

- ✦ **Mostly consistent with the participant nucleon picture in small centrality binning.**



Additional check with two component source

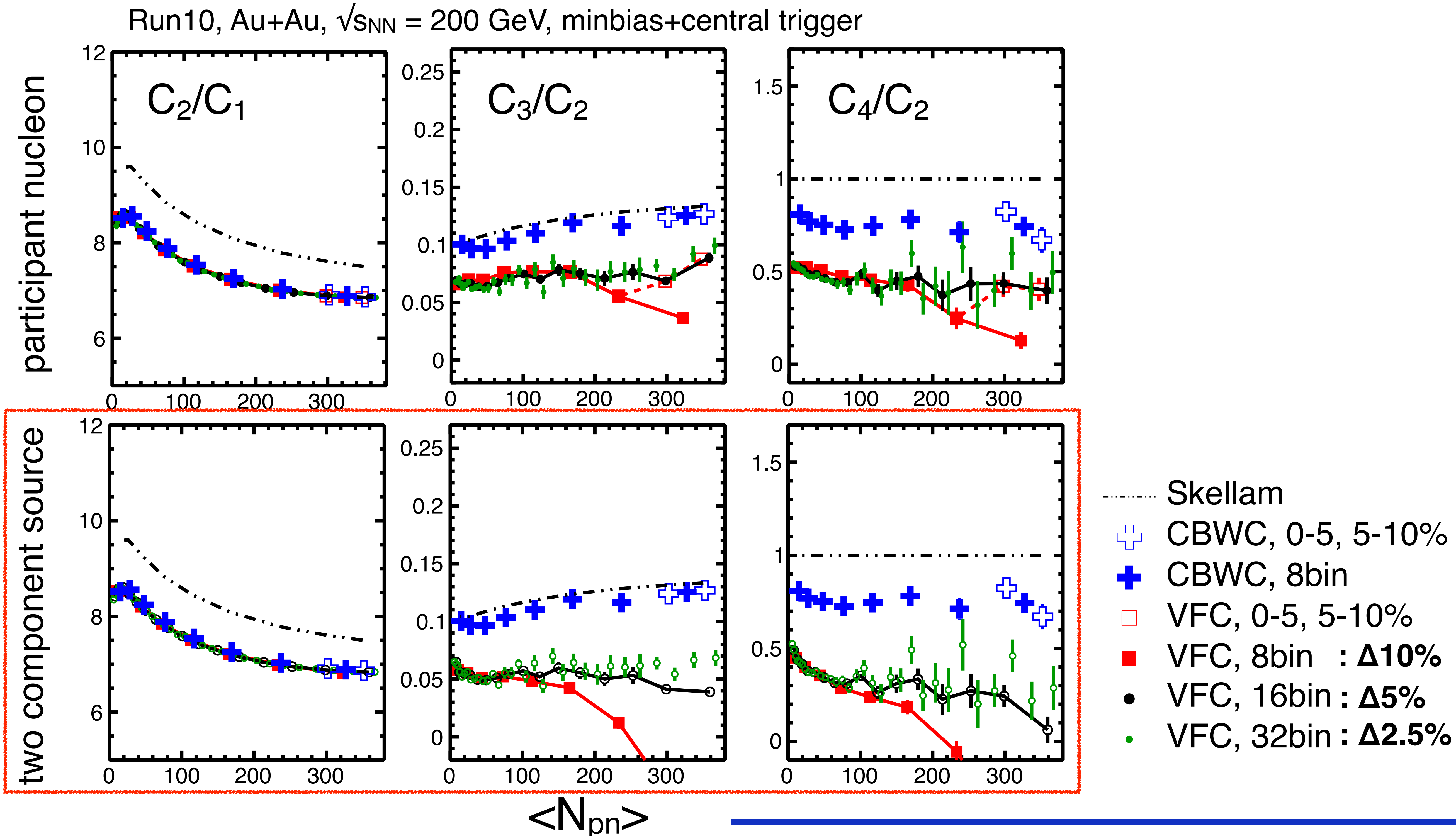


$$N_{ch} = n_{pp}\left[\frac{1-x}{2}N_{part} + xN_{coll}\right]$$

✦ **Use above two component source for particle production instead of N_{part} .**

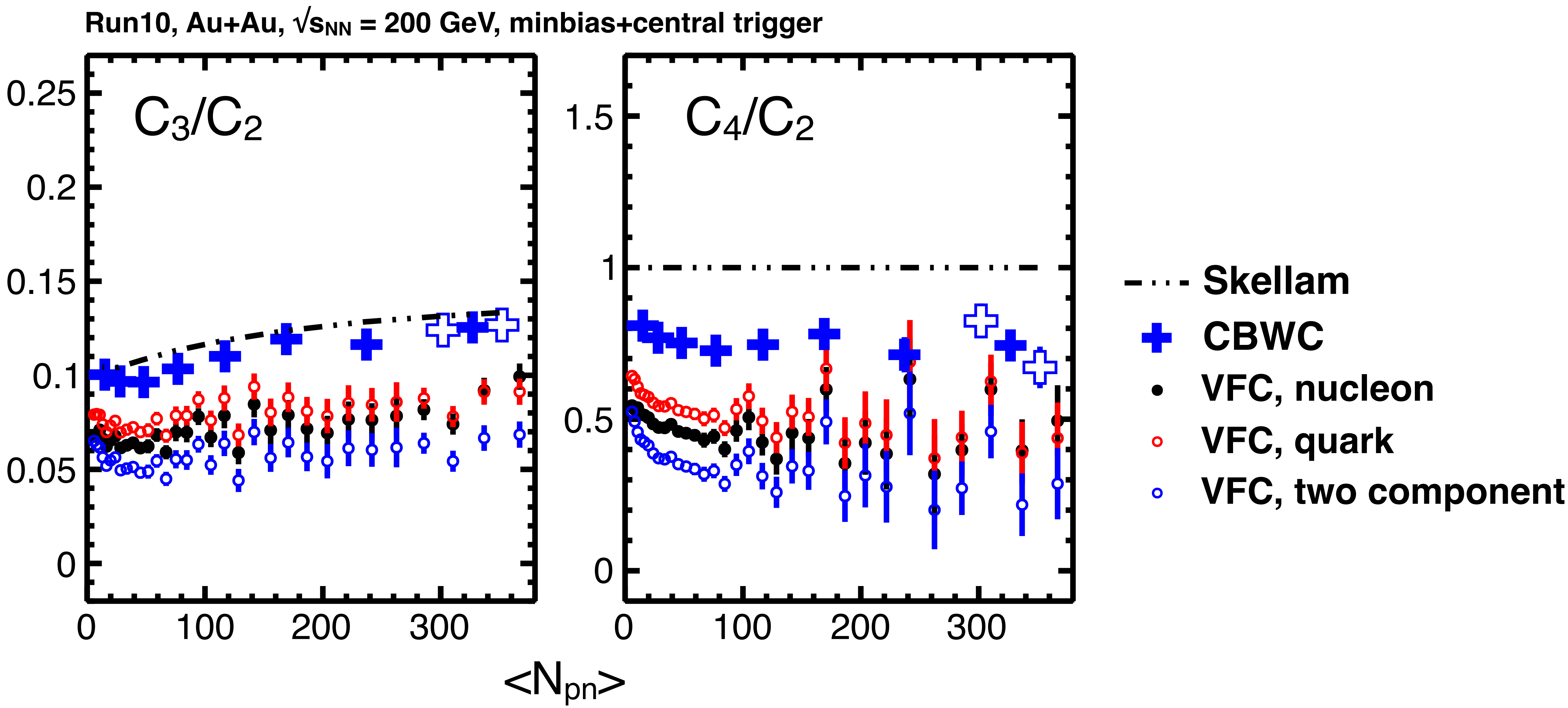
Additional check with two component source

✦ Mostly consistent in small centrality binning.

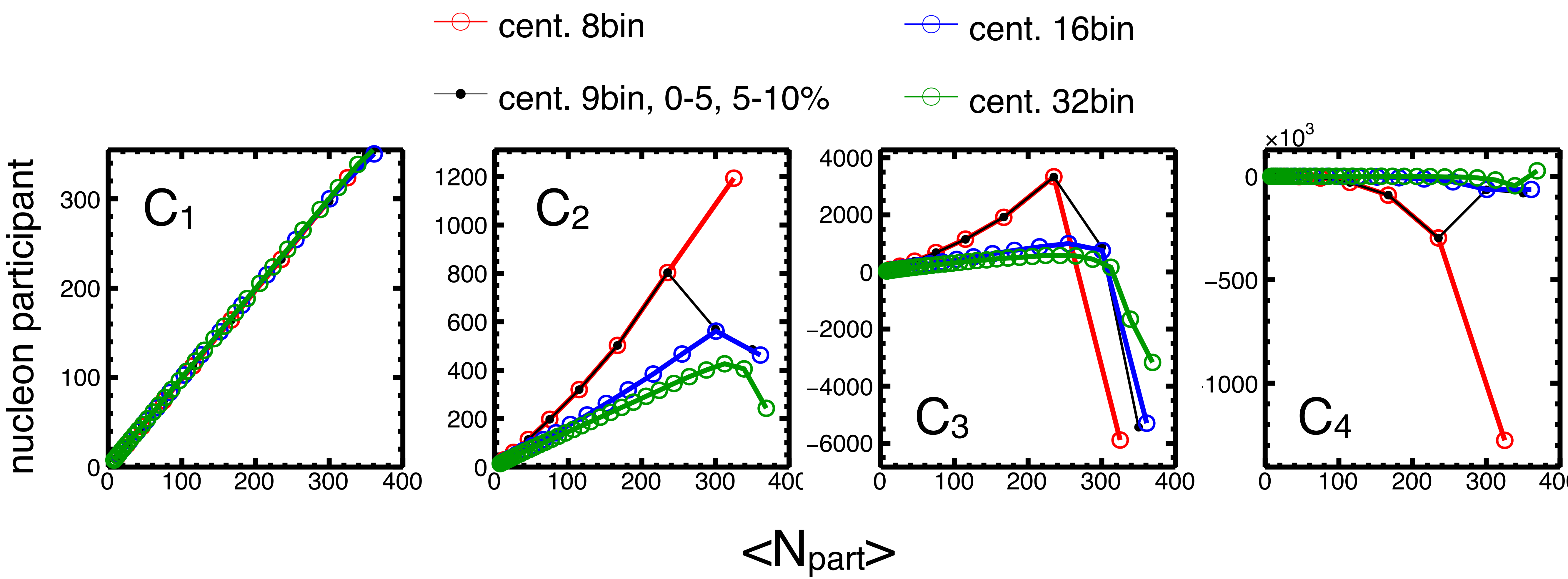


Results with difference source assumptions

- Another model assuming proton production from two component source as is used for centrality determination.
- Mostly consistent with the participant nucleon picture in small centrality binning.

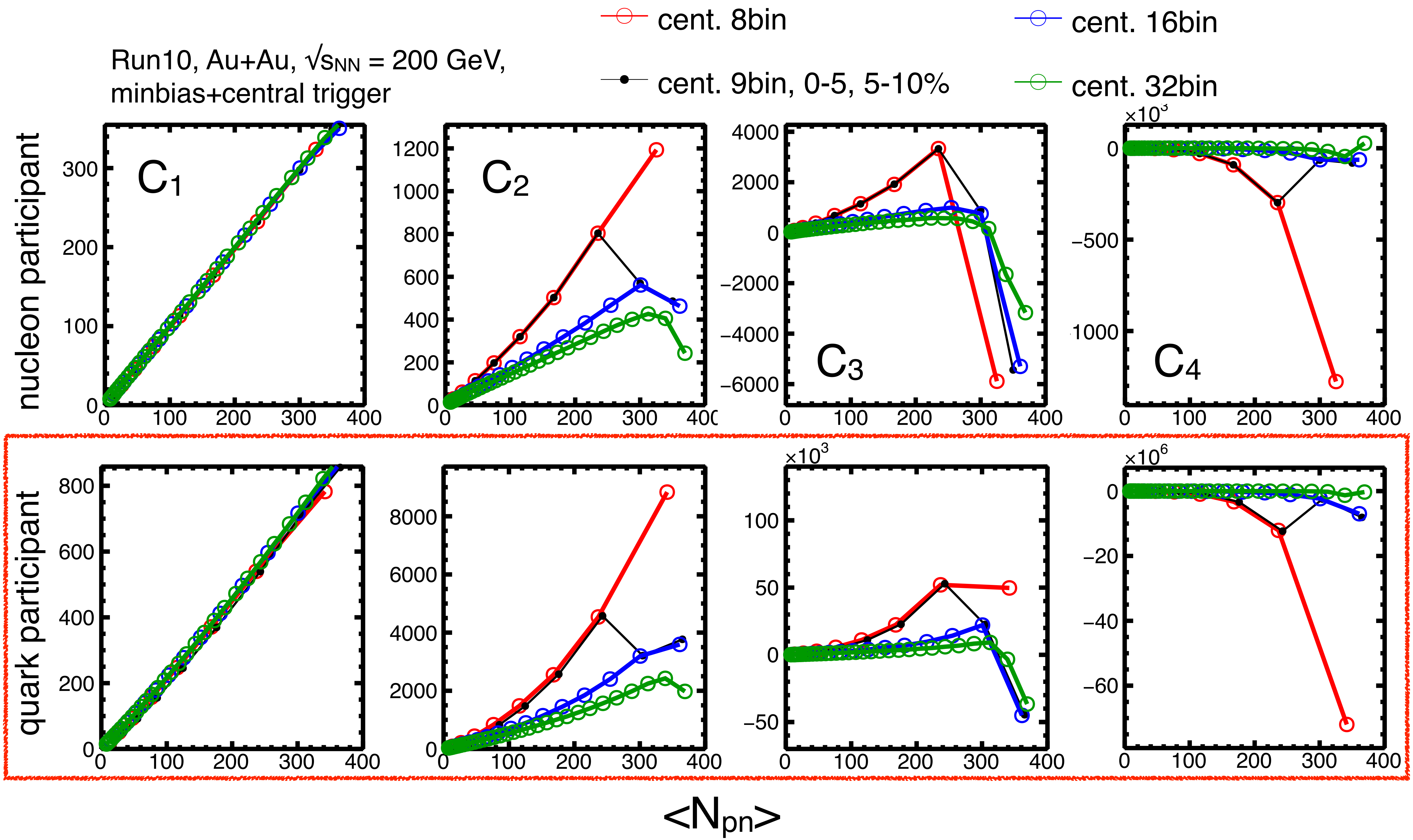


N_{part} cumulants with different centrality bin width



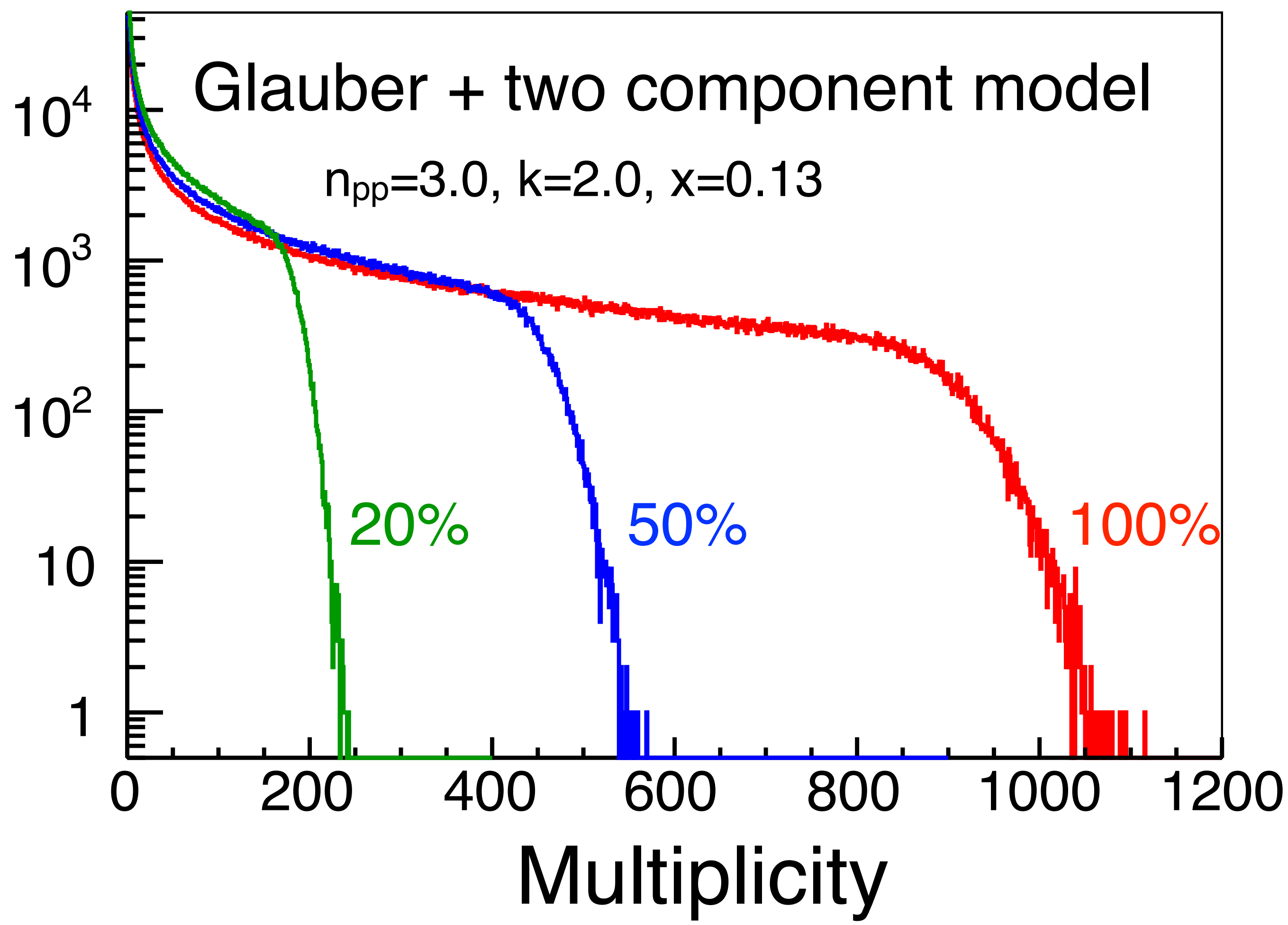
$\sqrt{s_{NN}}$ = 200 GeV, Glauber + Two-component model

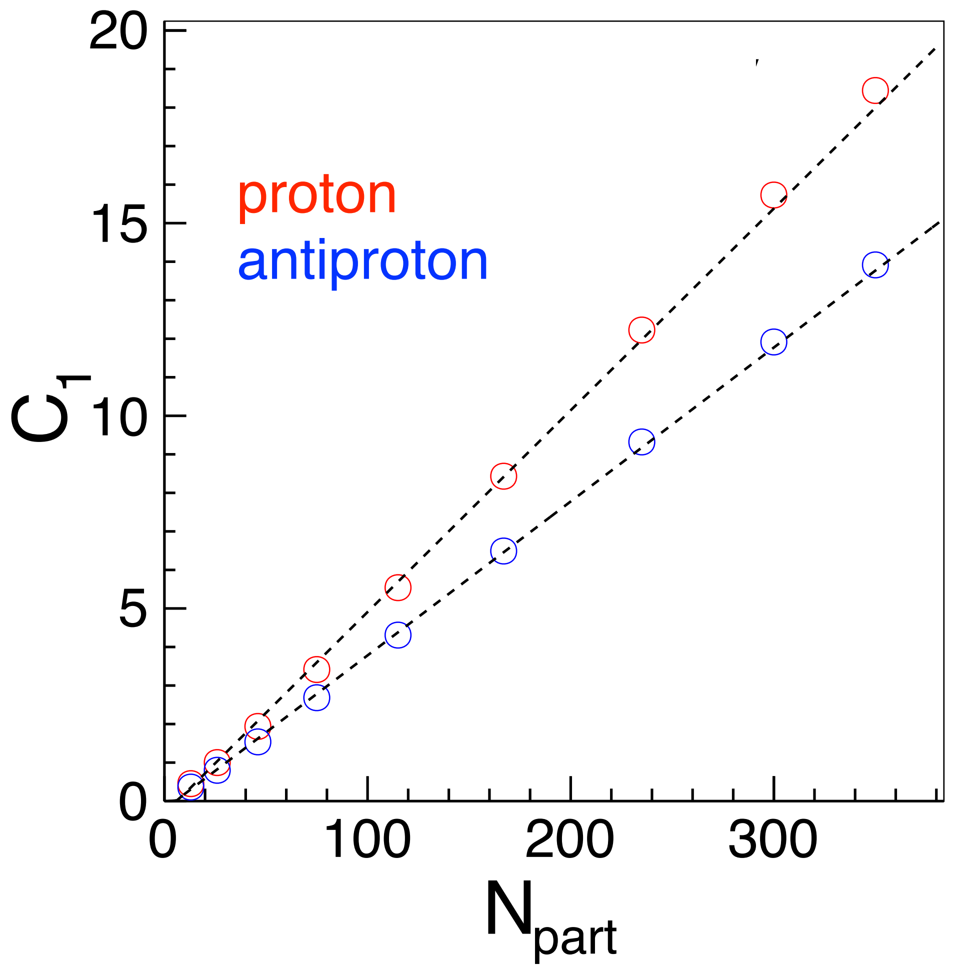
Quark participant model



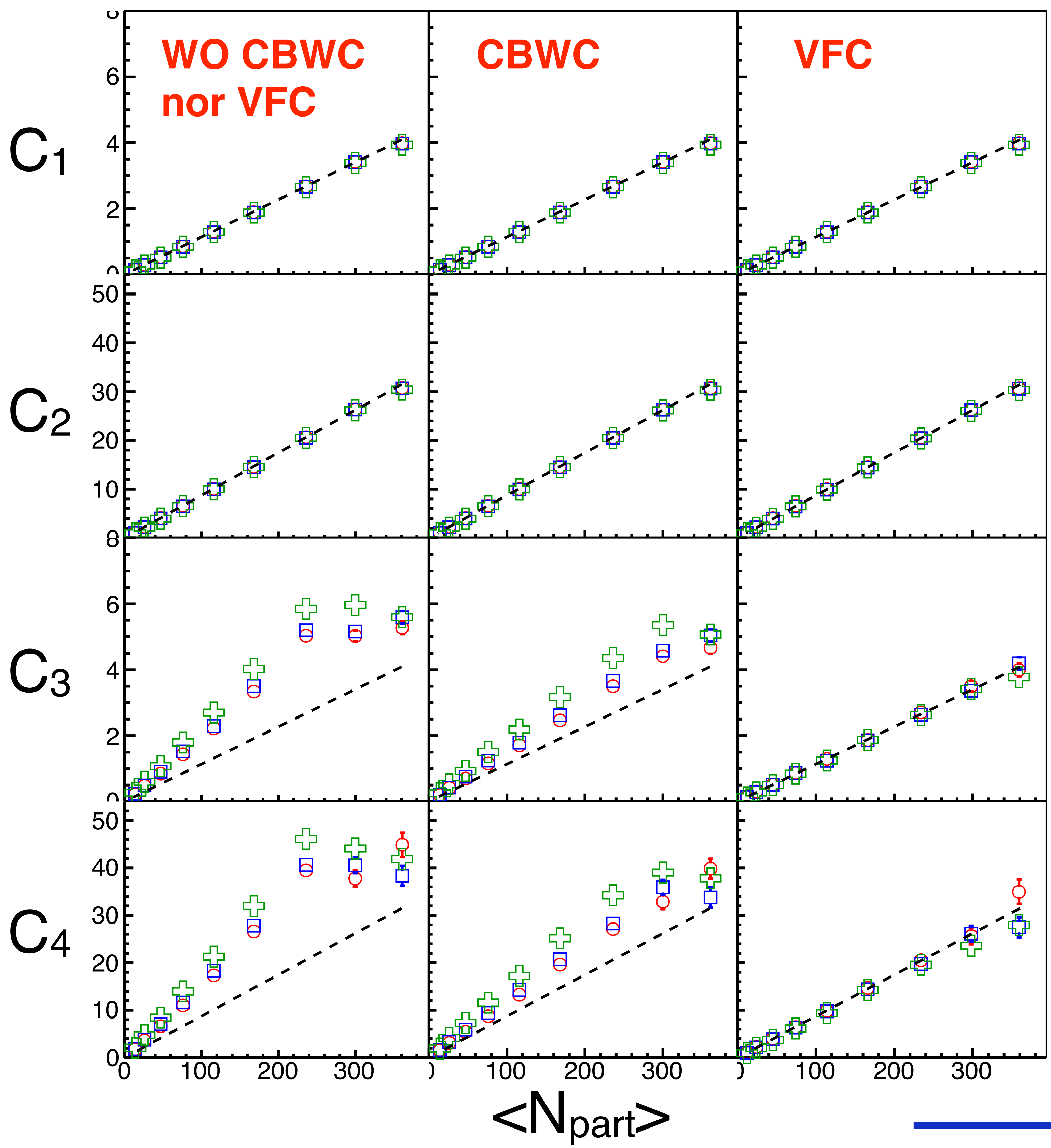
Centrality resolution

- ✓ **Multiplicity is determined by two component model with negative binomial fluctuation.**
- ✓ **(Anti)Protons are generated from event by event Npart source according to Poisson.**
- ✓ **Look at the effect of centrality resolution on the volume fluctuation.**





Results

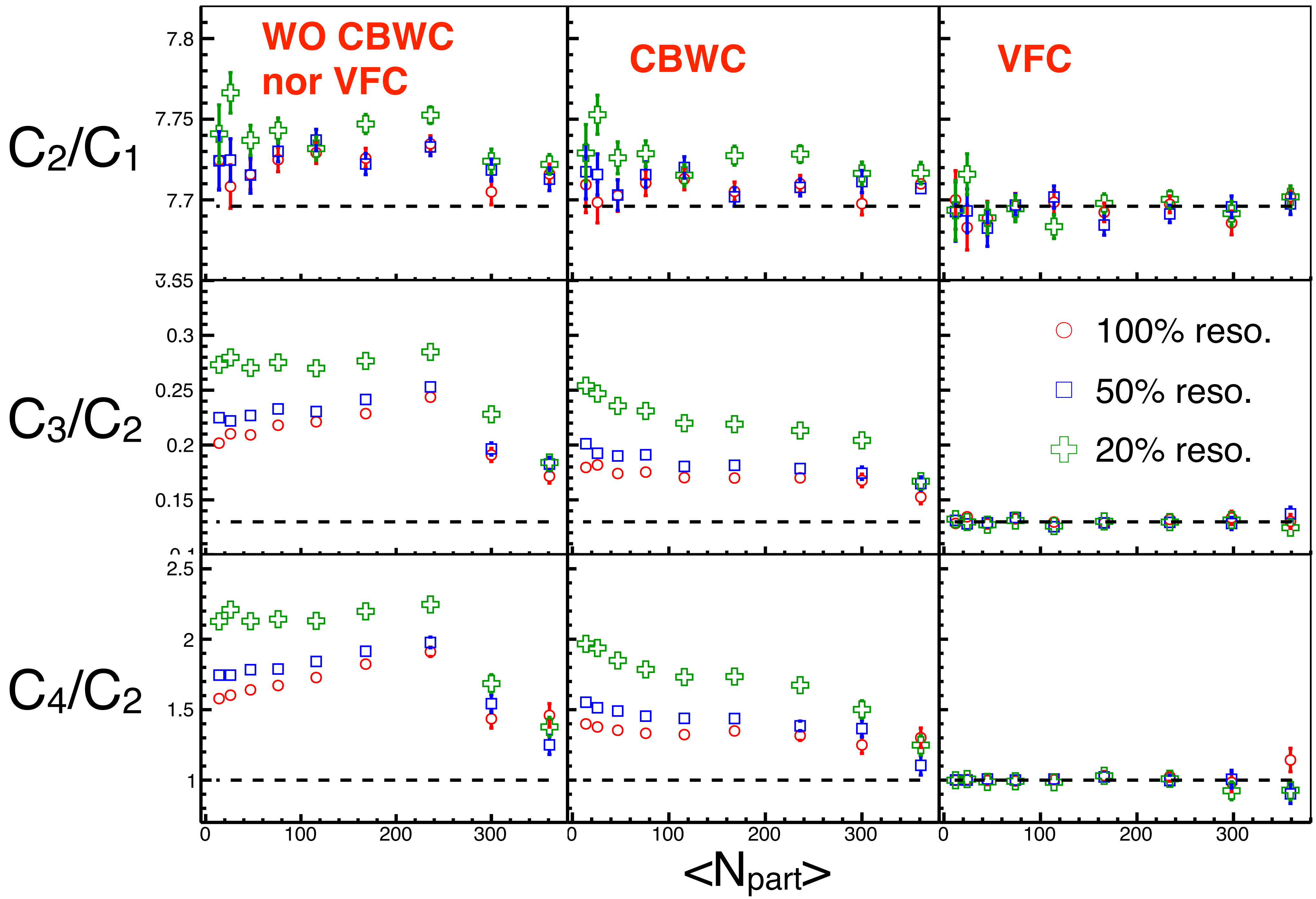


- ✓ CBWC strongly depends on the centrality resolution.
- ✓ VFC is independent on the centrality resolution.

Results

✓ CBWC strongly depends on the centrality resolution.

✓ VFC is independent on the centrality resolution.



Systematic uncertainties

	$n\sigma_p$	$mass^2$	DCA	nHitsFit	efficiency
0-10%	23.6	44.0	9.35	21.2	1.94
10-20%	2.31	23.9	27.9	40.5	5.36
20-30%	31.5	6.60	17.0	32.0	12.9
30-40%	51.4	21.3	4.07	7.47	15.8
40-50%	29.9	4.89	65.1	0.06	0.05
50-60%	8.06	12.3	62.6	7.30	9.76
60-70%	0.960	9.02	75.7	7.50	6.78
70-80%	48.1	4.13	0.10	29.1	18.6

Statistical errors

- Simple toy model to estimate the statistical errors assuming 0-10% centrality at 200 GeV.
- Statiscical errors strongly depends on efficiency
  - Statistical errors with HFT will become 100 times larger than without HFT (e.g. eff : 50%->10%).
  - C_6 analysis with HFT tracking will be hopeless.

Toy model with Skellam distribution assuming $\sqrt{s_{NN}}$ = 200 GeV, 0-10% by bootstrap