Measurement of the sixth order cumulants of net-charge distributions in Au+Au collisions at the STAR experiment

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TAC seminer

Nov. 26, 2018

STAR

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Outline

- My activity
- C_6 analysis
  - Introduction
  - previous results and motivation
  - analysis method
  - results
- Volume fluctuation study
  - Introduction to VF
  - results by toy model
  - results by UrQMD
- Future plan

My activity

✦ **2014 - 2015** Master course ✦ **2016 - 2018** Ph.D course

Conference and workshop : 8 talk and 2 poster presentations

- CiRfSE workshop, Mar. 2015 talk
  @University of Tsukuba Jan. 2016 talk
- JPS @University of Miyazaki, Sep. 2016 talk
- QM2017 @Chicago, Feb. 2017 poster
- JPS @Osaka University, Mar. 2017 talk
- JPS @Utsunomiya University, Sep. 2017 talk
- QM2018 @Venice, May. 2018 poster
- HIP/HIC @ Nagoya University Jun. 2018 talk
- APS/JPS joint meeting @Hawaii, Oct. 2018 talk
- QNP2018 @Tsukuba, Nov. 2018 talk

STAR shift taking : (D1, D2)

Teaching experience and outreach

- Summer challenge TA @KEK - (M2, D1, D2)
- Poster presentation at school festival, etc.

Master thesis:

"$\Delta\eta$ dependence of net-charge fluctuations in Au+Au collisions at RHIC-STAR experiment"

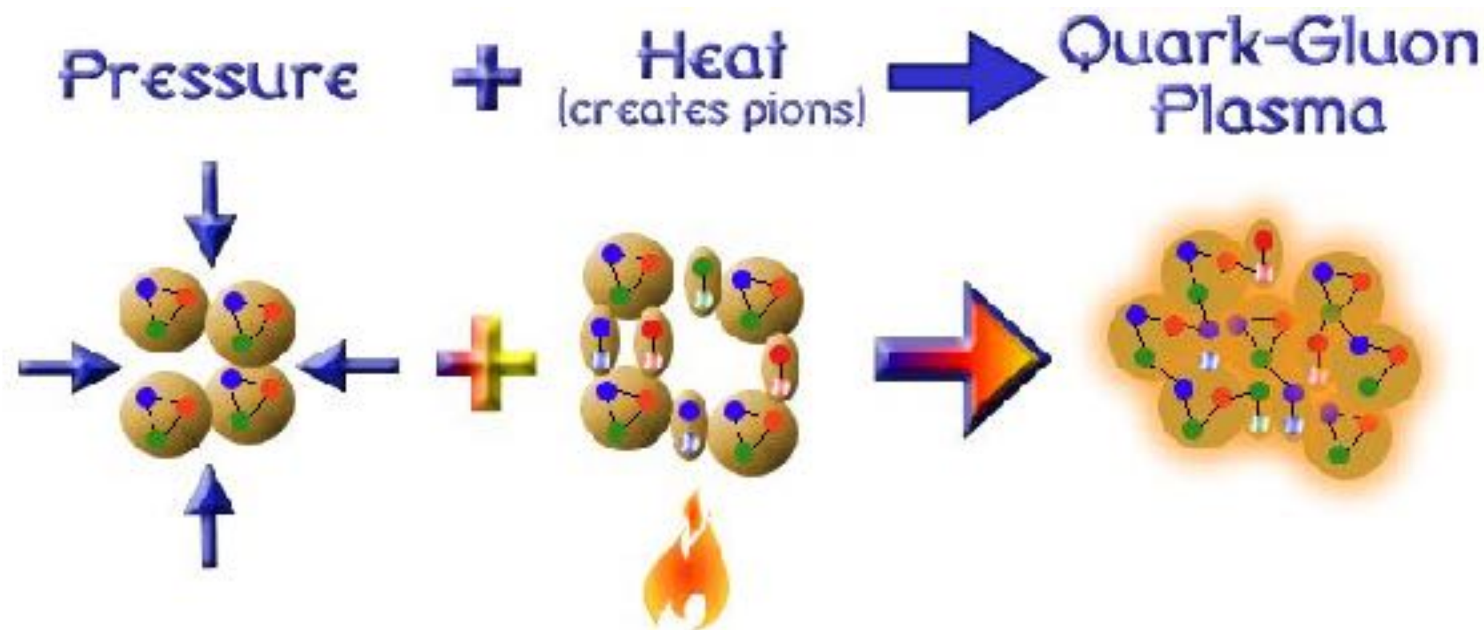
Paper: 1 paper contributed as 2nd author

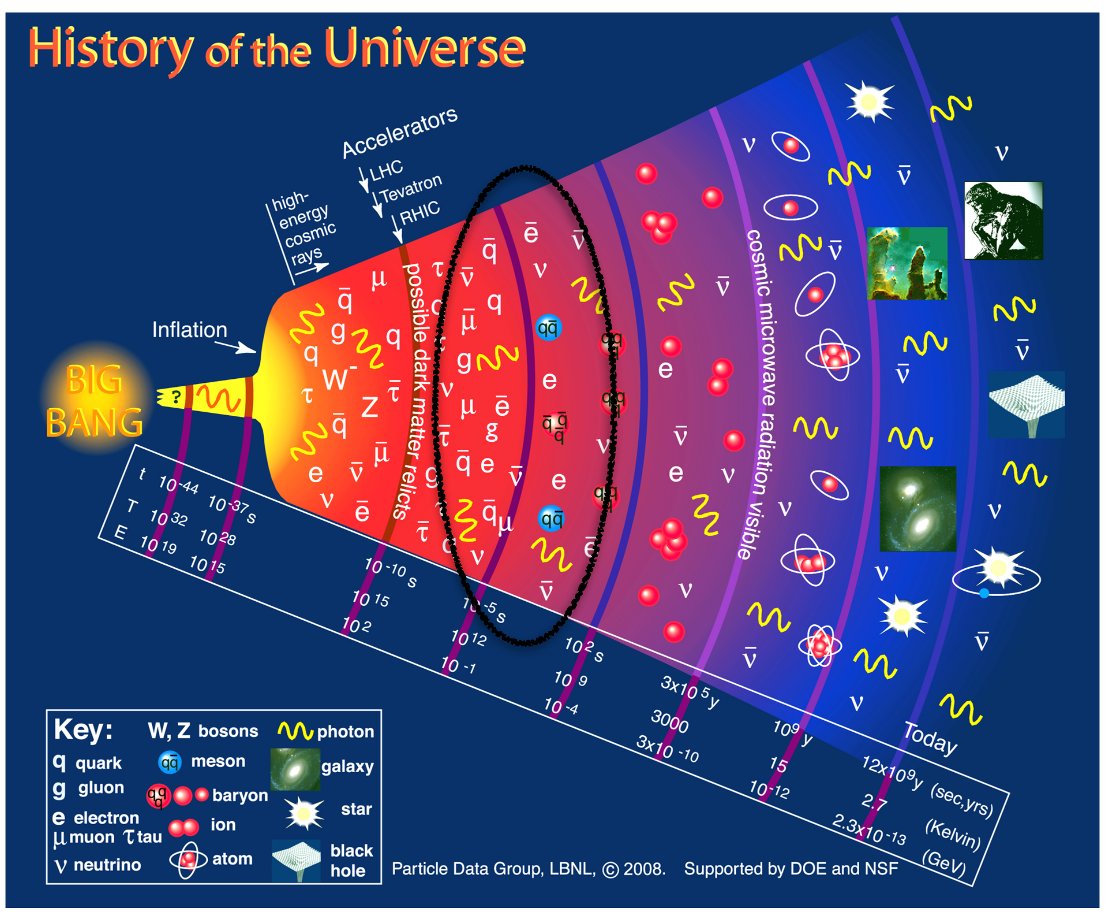
T. Nonaka, T. Sugiura, S. Esumi, H. Masui, X. Luo
"Importance of separated efficiencies between positively and negatively charged particles for cumulant calculations" PRC. 94. 034909, Sep. 2016

Quark Gluon Plasma (QGP)

- Quarks and gluons are confined inside of nucleons. **(Color confinement)**
- Interactions are very weak inside of nucleons. **(Asymptotic freedom)**

- Quarks are de-confined under high density or high temperature. **(Quark Gluon Plasma : QGP)**

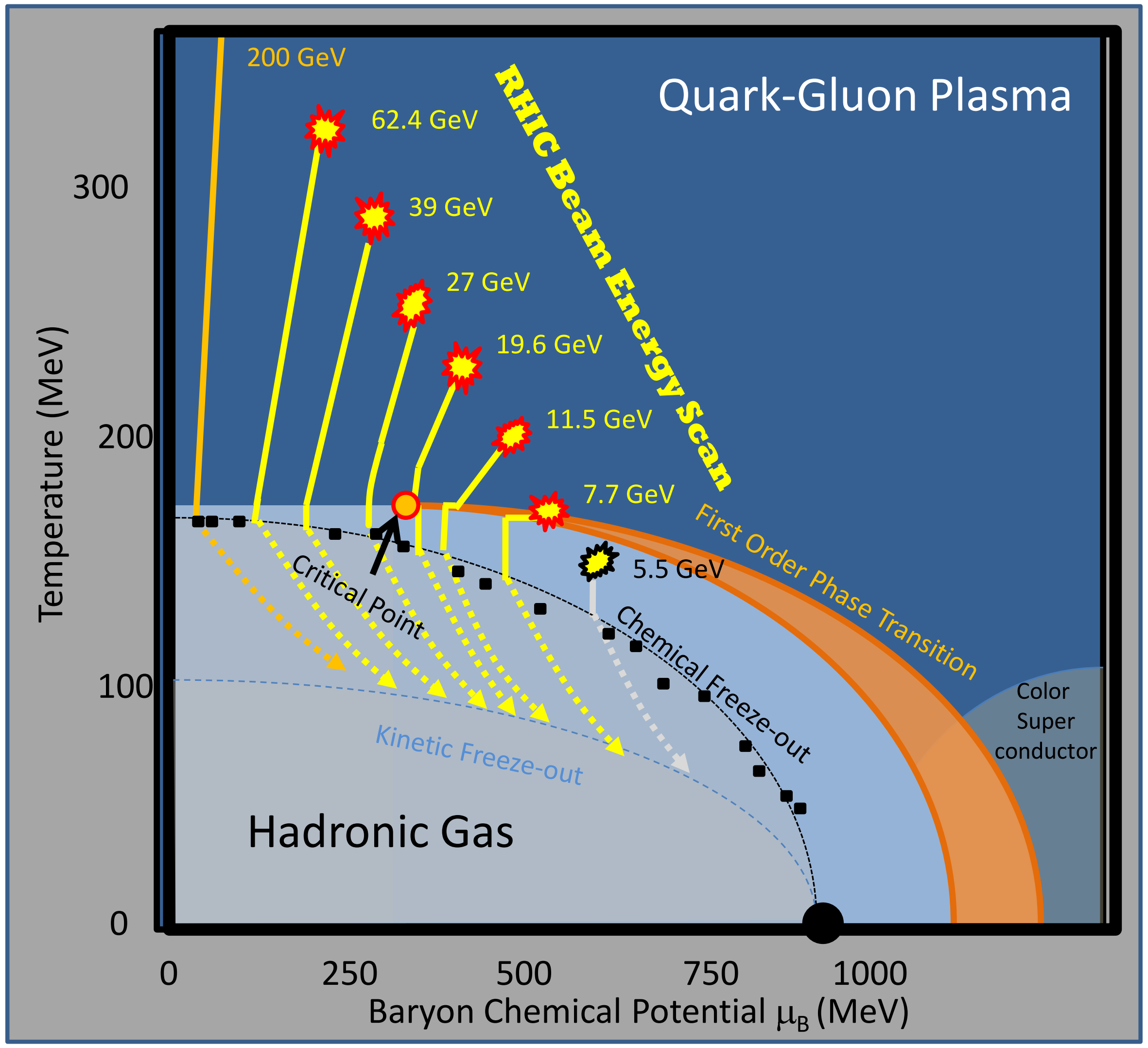




T_C=154 MeV

- QGP is considered to be existed in early universe, μs after big bang.

Phase transition and Bean Energy Scan



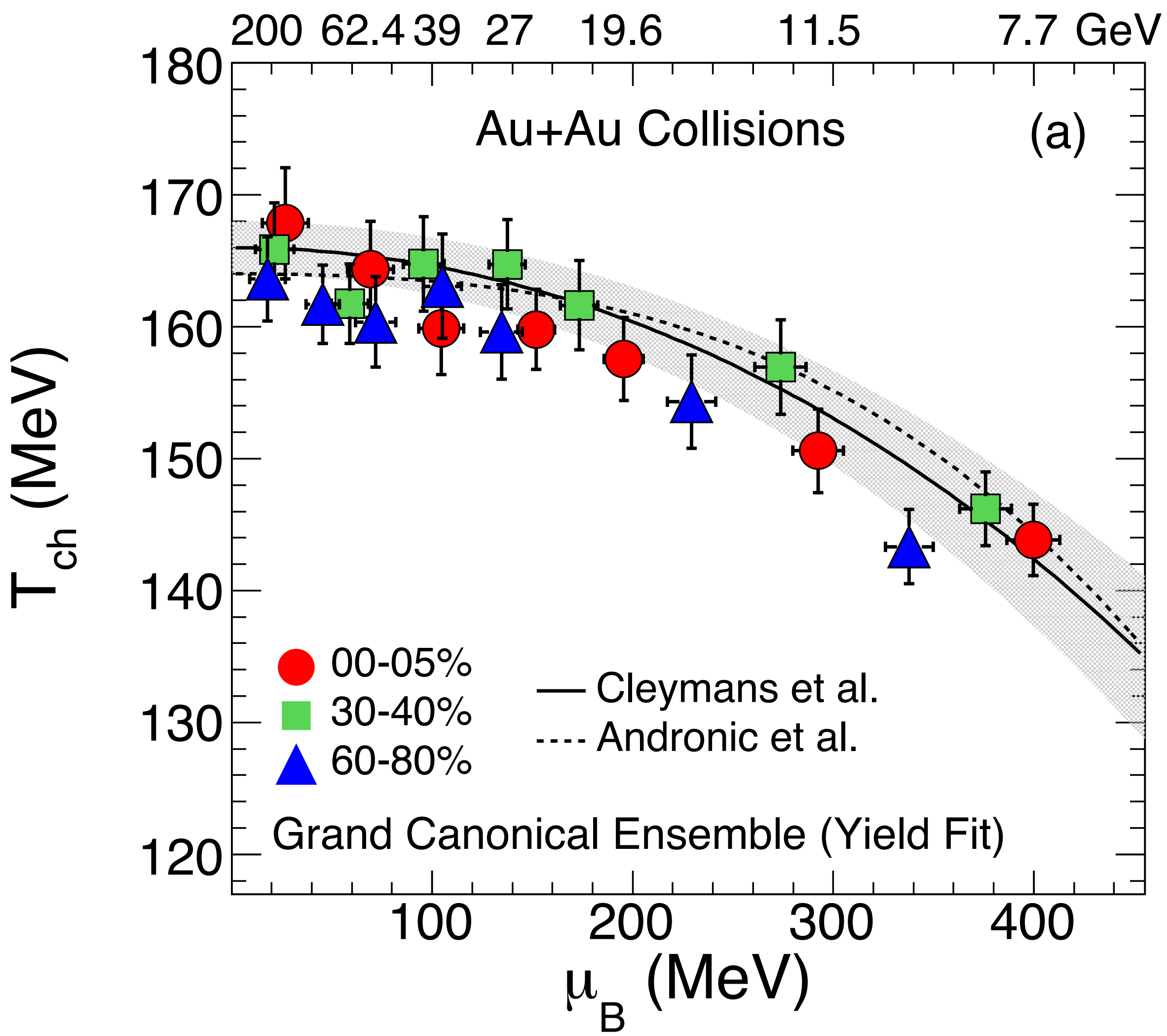
- **Cross over transition around μ_B=0** from Lattice QCD but there is no experimental evidence.
- Location of critical point is unknown.

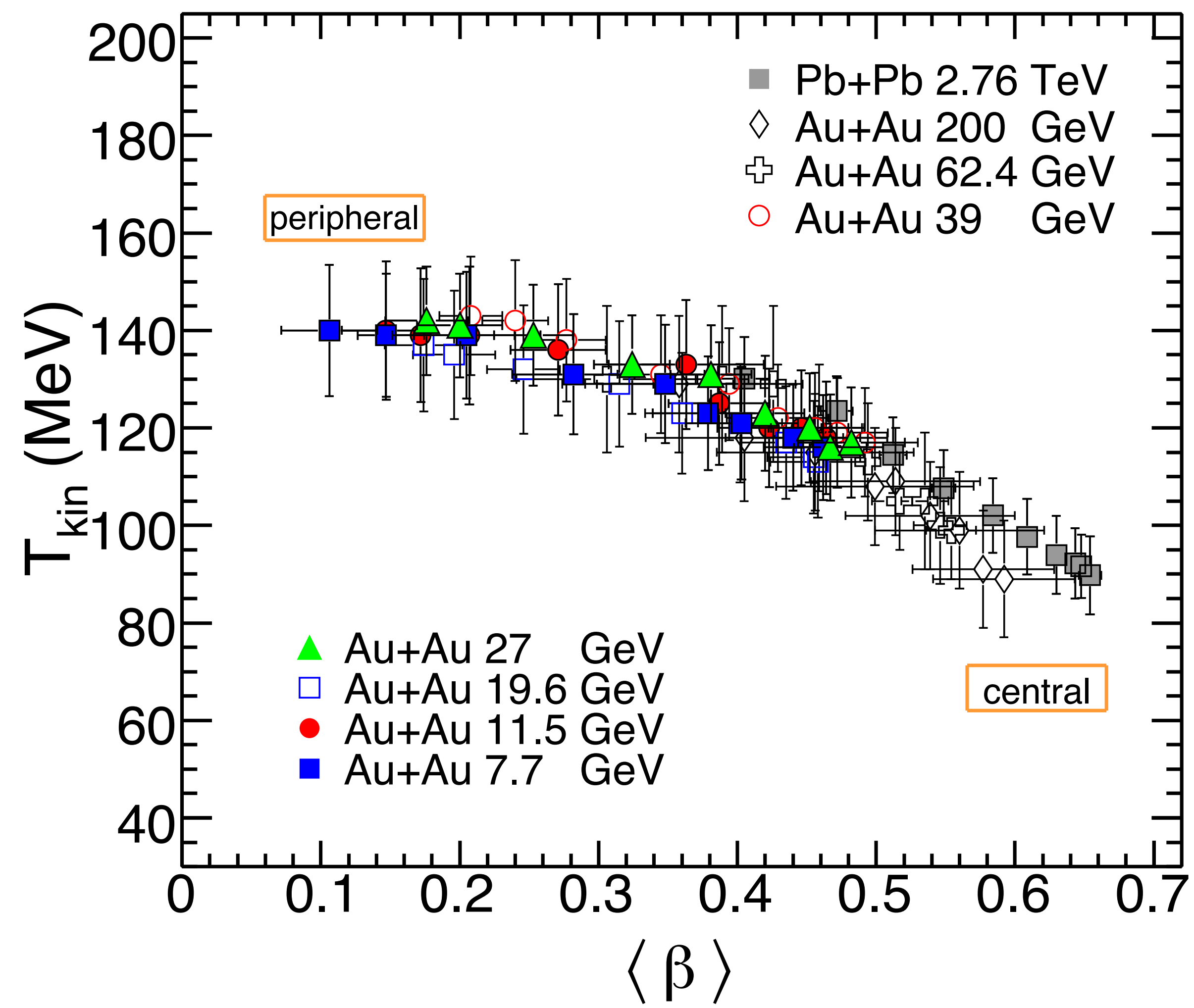
Detail structure of QCD diagram should be discovered experimentally

Beam Energy Scan I (BES-I) 2010-2014

: Varying the center of mass energy from 7.7 to 200 GeV in Au+Au collisions.

Freeze-out parameters





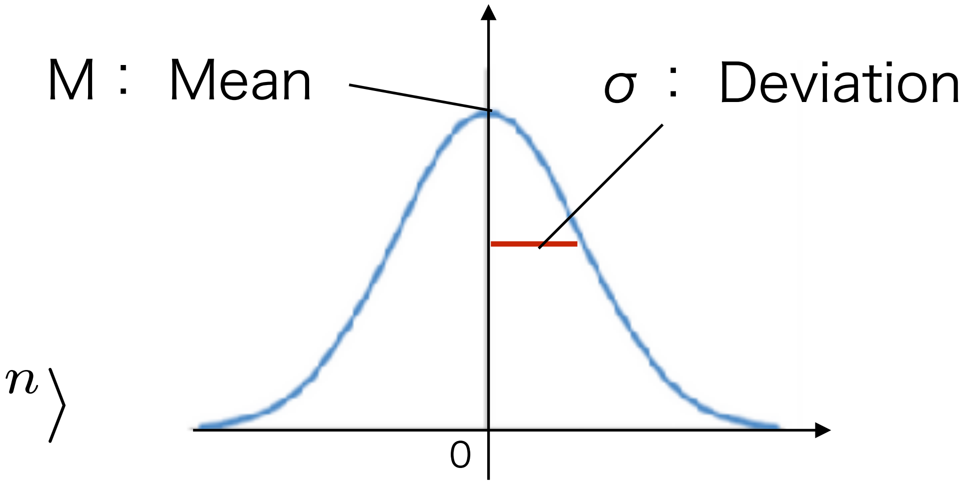
- Chemical and kinetic freeze-out and kinetic freeze-out parameters are obtained from particles yield and spectra.

What is the good observables which is sensitive to the phase transition?

Even by event fluctuations are considered as one of the powerful tool to discover the phase diagram.

Event-by-event fluctuations

Net-charge : $N_+ - N_-$



- nth-order moment is defined by

$$\langle m^n \rangle = \sum_m m^n P(m)\,, \quad \langle \delta m^n \rangle = \langle (m - \langle m \rangle)^n \rangle$$

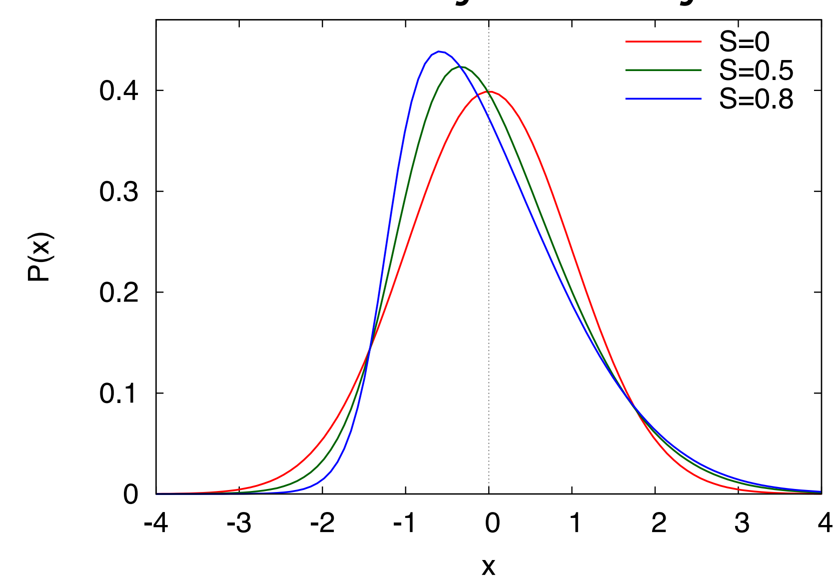
- Cumulants can be written by moments

Cumulants ⟷ Moments

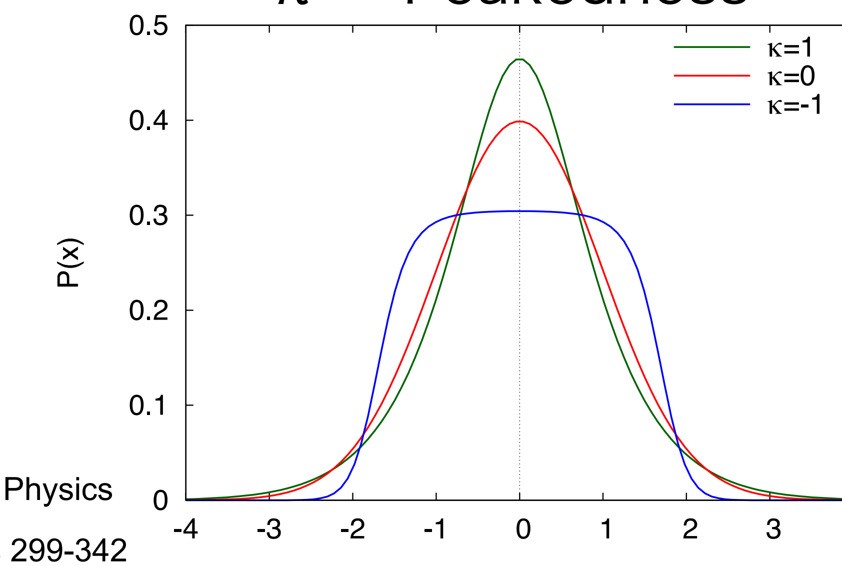
$$C_1 = \langle m \rangle$$
$$C_2 = \langle \delta m^2 \rangle$$
$$C_3 = \langle \delta m^3 \rangle$$
$$C_4 = \langle \delta m^4 \rangle - \langle \delta m^3 \rangle$$

$$M = C_1$$
$$\sigma^2 = C_2$$
$$S = \frac{C_3}{(C_2)^{3/2}}$$
$$\kappa = \frac{C_4}{(C_2)^2}$$

S : Asymmetry



κ : Peakedness



- Cumulants are proportional to the volume (**Additivity**)

Progress in Particle and Nuclear Physics
Volume 90, September 2016, Pages 299-342

Cumulant ratios

- By measuring the **cumulant ratios**, **effect from the volume can be canceled**.
- Cumulant ratios can be directly compared to the ratio of susceptibilities.

Cumulant ratios

$$S\sigma = \frac{C_3}{C_2} = \frac{\chi_3}{\chi_2}$$

$$\kappa\sigma^2 = \frac{C_4}{C_2} = \frac{\chi_4}{\chi_2}$$

- Higher order cumulants are more sensitive to the signal from the critical point.
- These relations are only valid for cumulants of **conserved quantities**.

$$C_2 \approx \xi^2 \quad C_5 \approx \xi^{9.5}$$

$$C_3 \approx \xi^{4.5} \quad C_6 \approx \xi^{12}$$

$$C_4 \approx \xi^7$$

ξ : Correlation length

Statistical baseline (Skellam distributions)

- If distribution of N_+ and N_- follow the Poisson distributions, net-charge distribution follow the **Skellam distribution.**

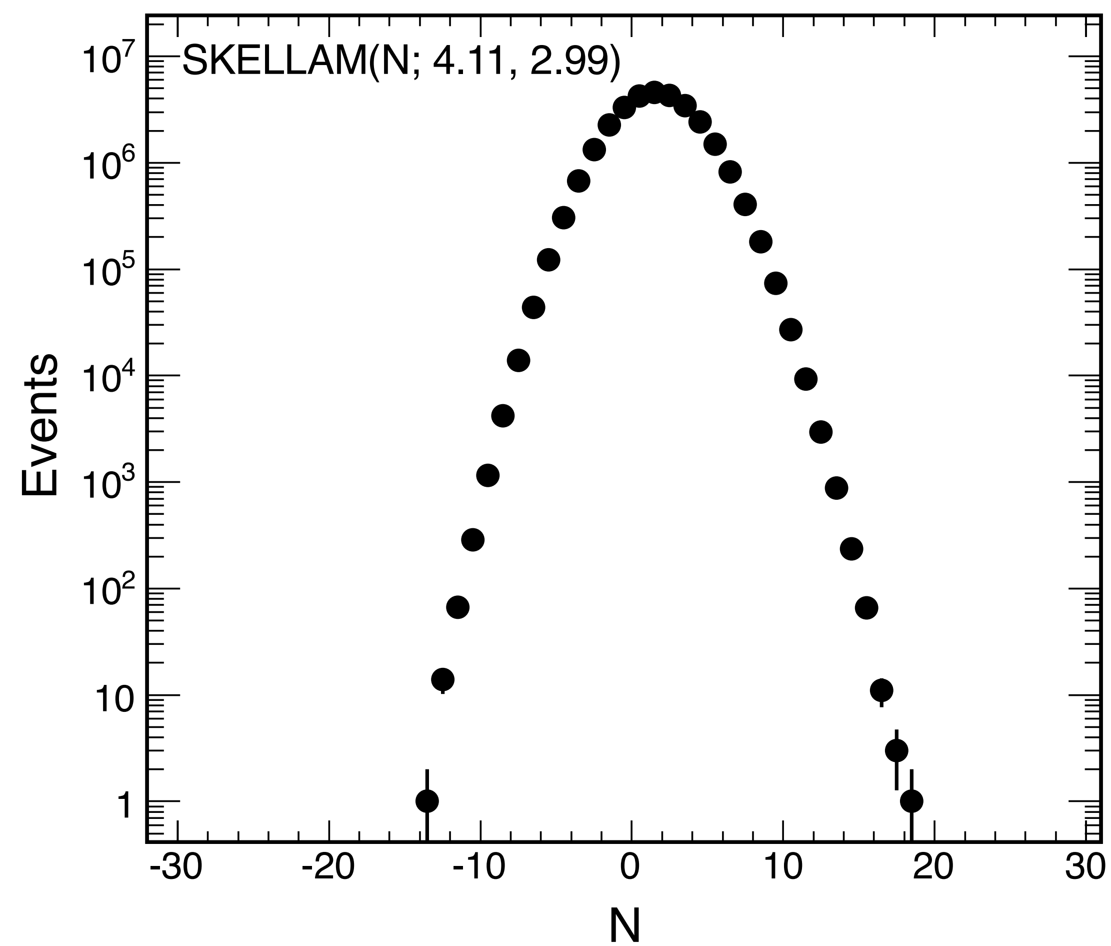
$$C_n = \lambda_1 + (-1)^n \lambda_2$$

λ_1 : Poisson parameter of N_+ distribution

λ_2 : Poisson parameter of N_- distribution

$$\frac{C_{2n}}{C_{2m}} = \frac{C_{2n+1}}{C_{2m+1}} = 1$$

$$\frac{C_{2n+1}}{C_{2m}} = \frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2}$$



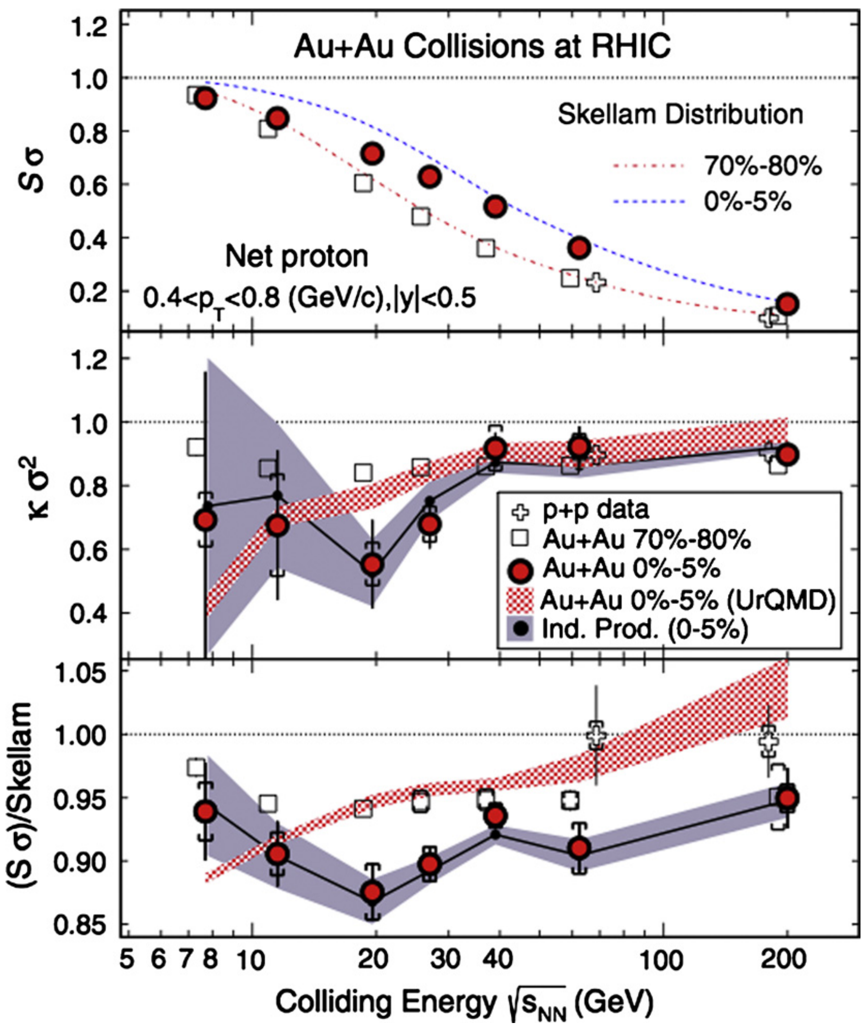
For example

$$\frac{C_6}{C_2} = \frac{C_4}{C_2} = \boxed{1}$$

This values can be used as a statistical baseline.

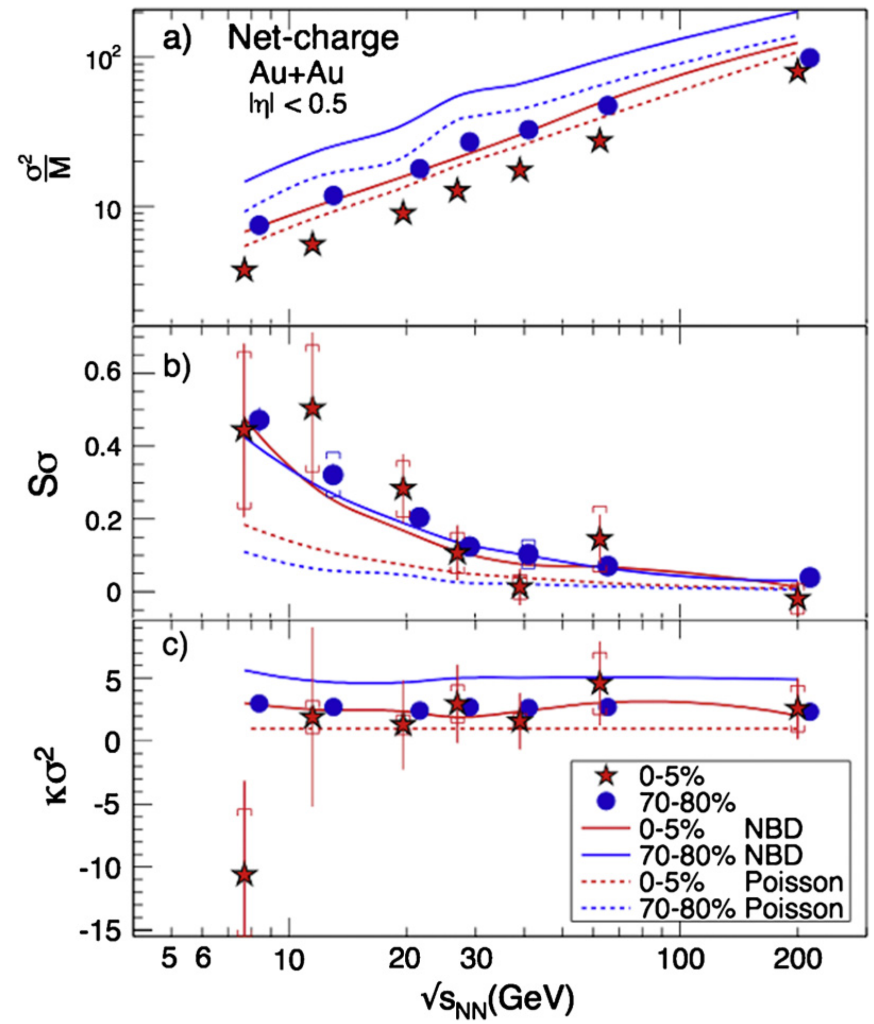
Published results

Net-proton



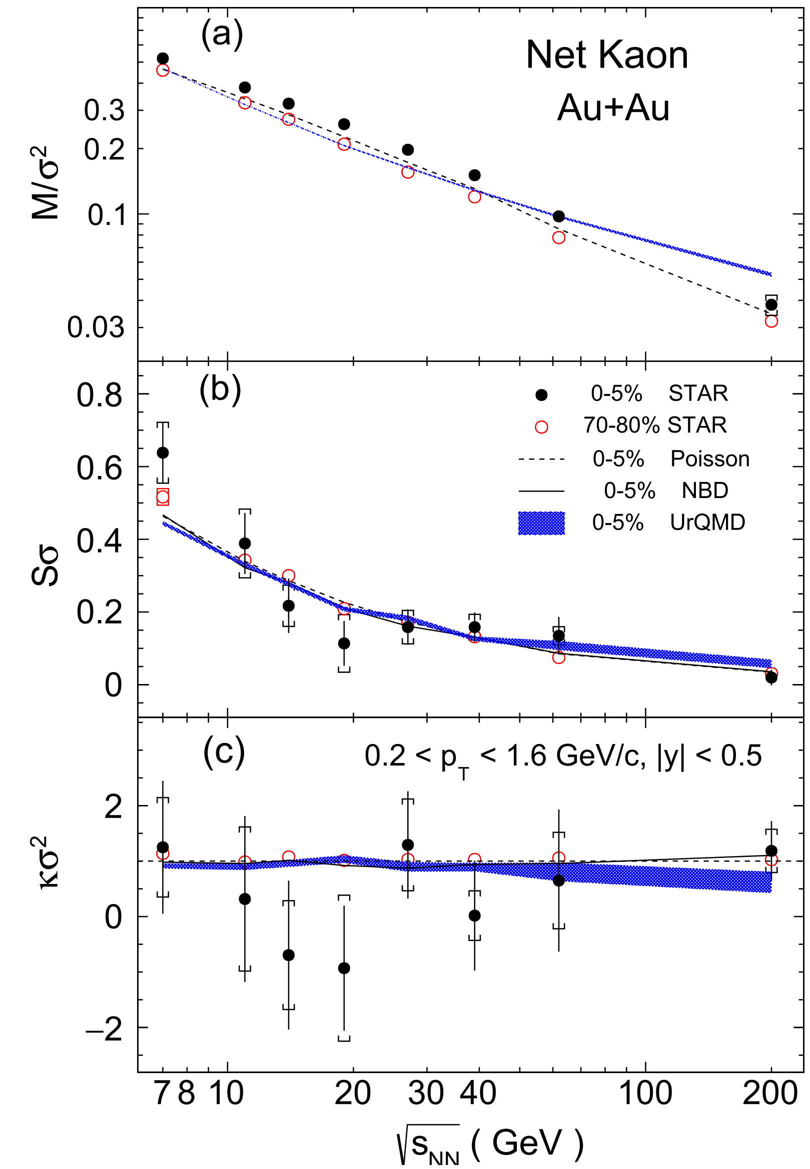
L. Adamczyk *et al.* (STAR Collaboration)
Phys. Rev. Lett. 112, 032302 (2014)

Net-charge



L. Adamczyk *et al.* (STAR Collaboration)
Phys. Rev. Lett. 113, 092301(2014)

Net-Kaon

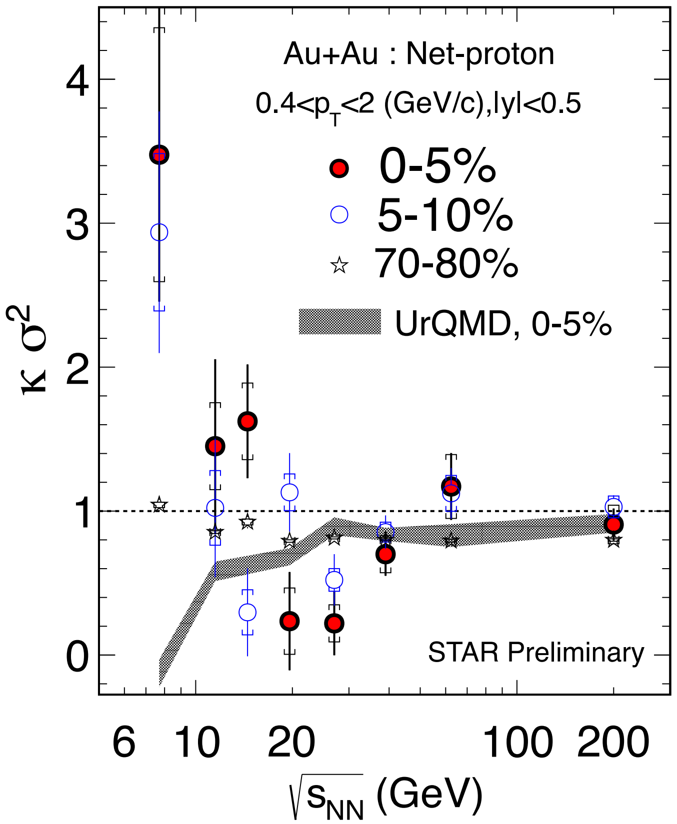


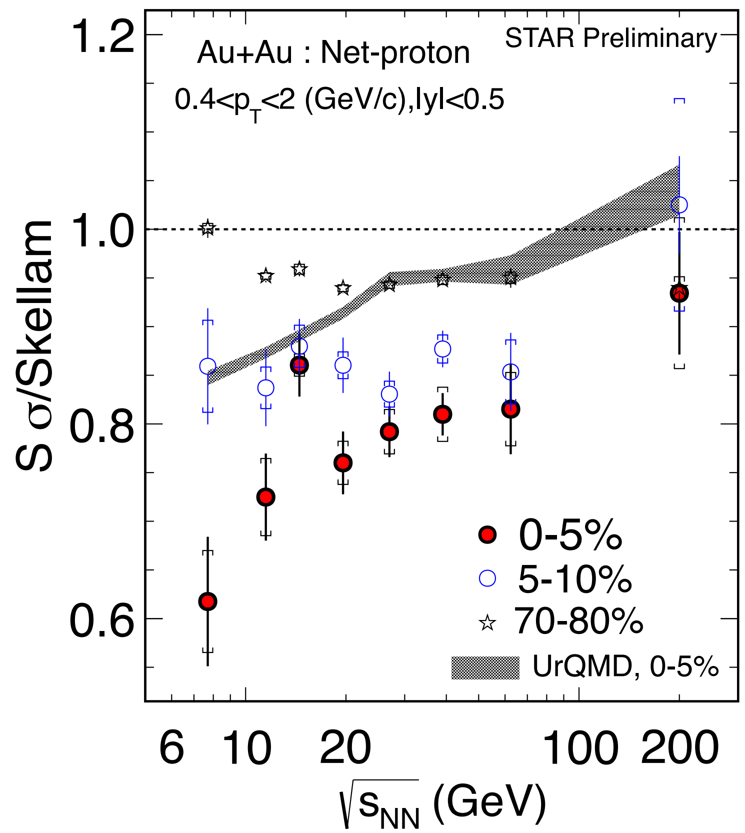
Physics Letters B Volume 785, 10 October 2018, Pages 551-560

- **STAR published up to 4-th order fluctuations** of net-proton, net-charge and net-kaon distributions.
- **Critical phenomena have not been observed yet.**

Recent results

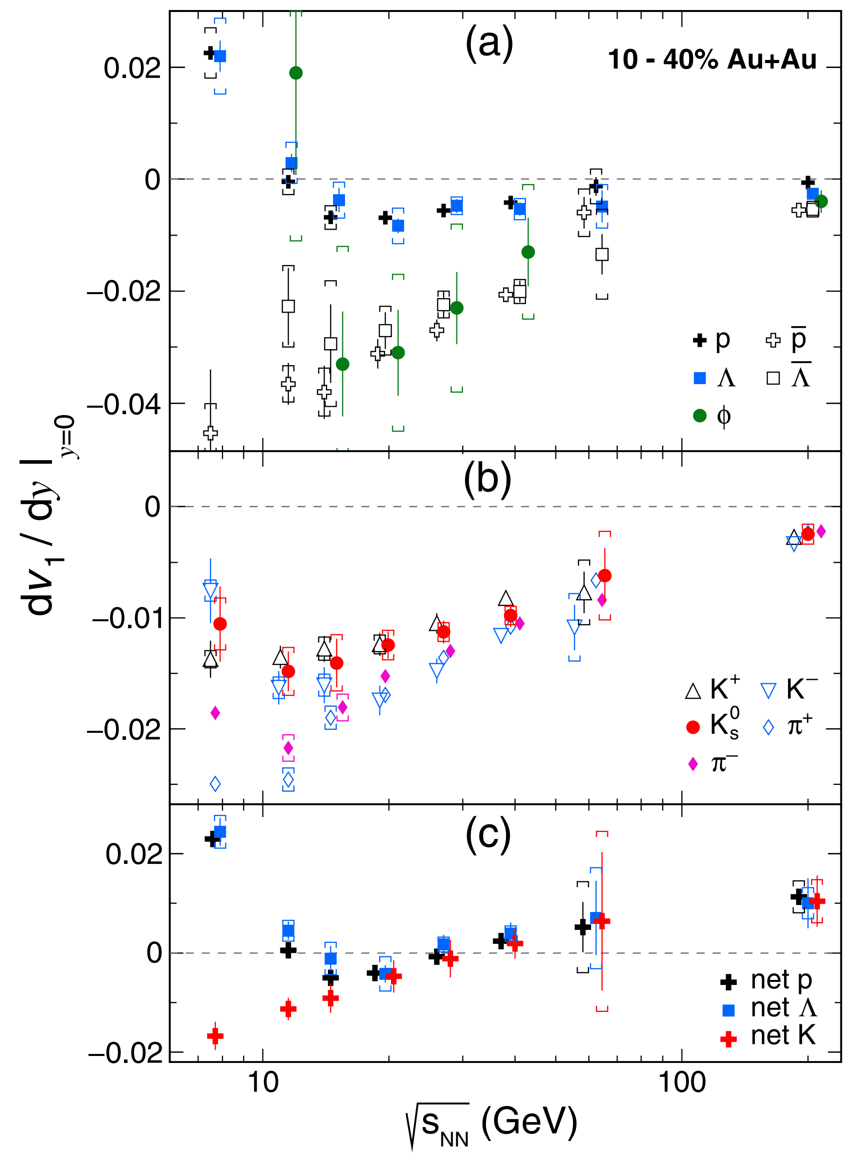
Net-proton, 0.4 <pT<2 (GeV/c)





X. Luo, Nucl. Phys. A956 (2016) 75–82, [1512.09215].

Published results of v1

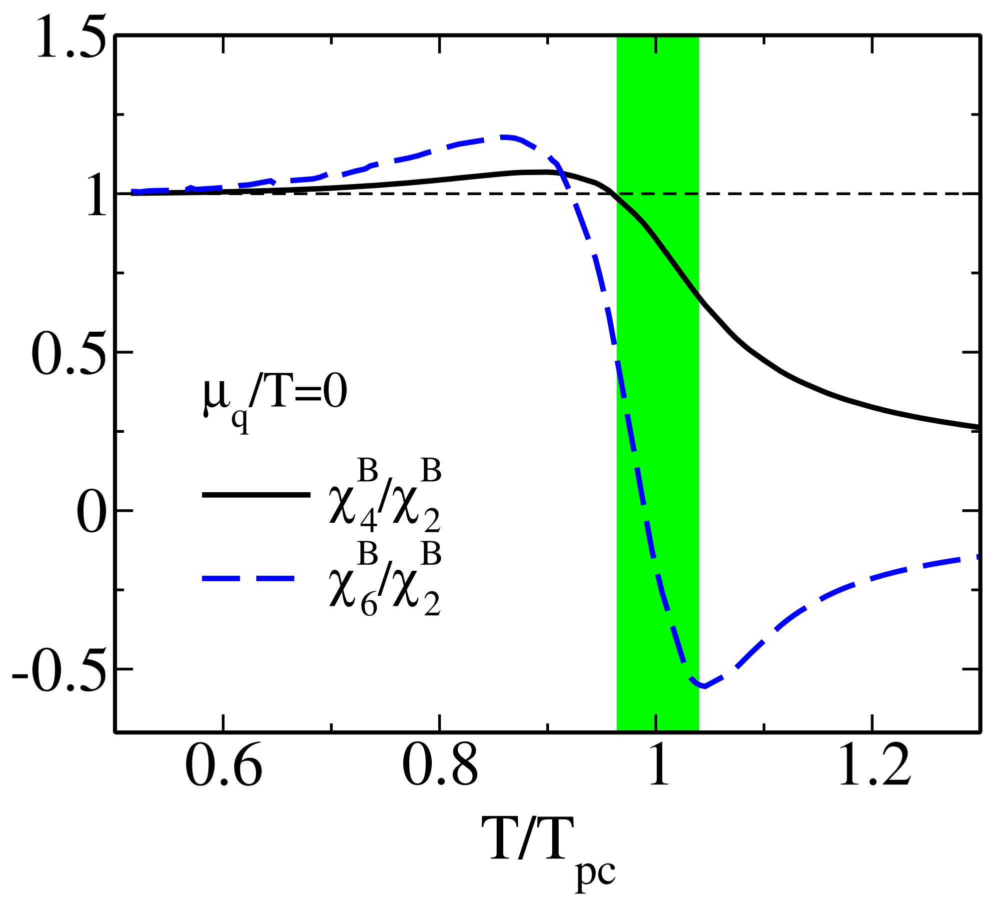


Phys. Rev. Lett. **120**, 062301 – Published 6 February 2018

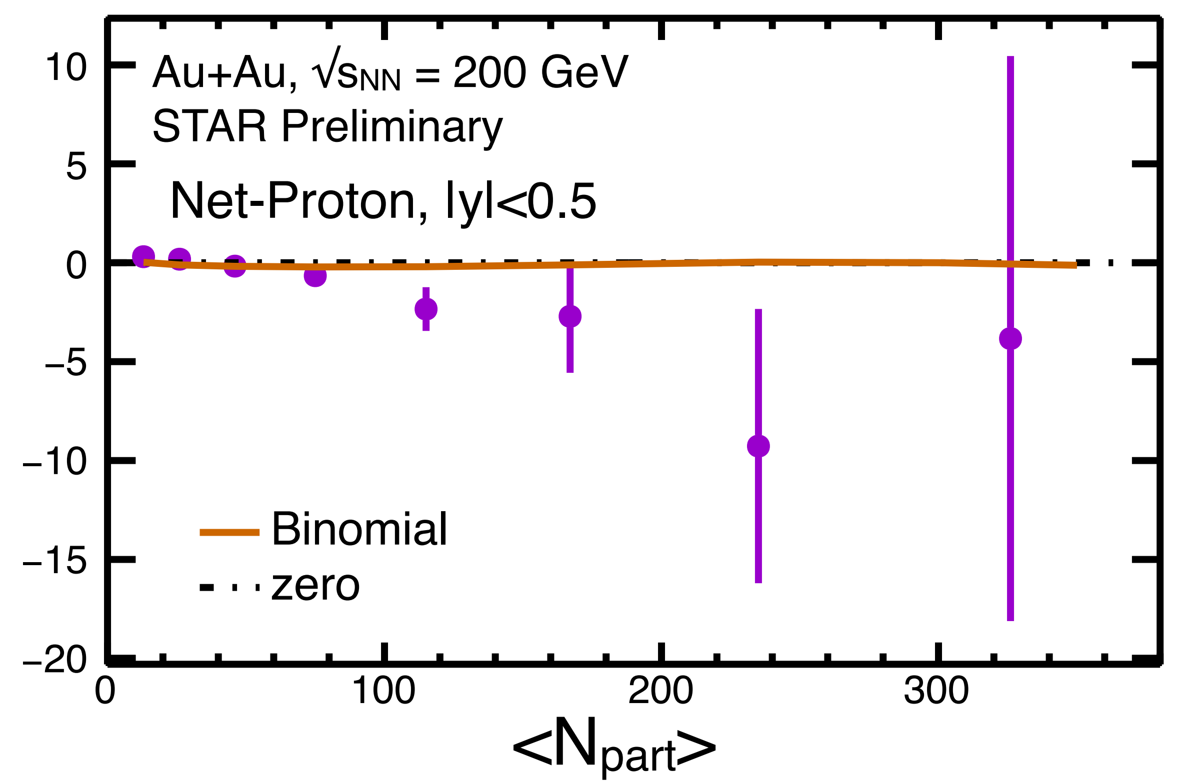
- Interesting energy dependences were observed at lower energy region at various measurement.

Motivation

Friman et al, Eur. Phys. J. C(2011) 71:1694



Quark Matter2017



freeze-out conditions	χ_4^B/χ_2^B	χ_6^B/χ_2^B	χ_4^Q/χ_2^Q	χ_6^Q/χ_2^Q
HRG	1	1	~ 2	~ 10
QCD: $T^{freeze}/T_{pc} \lesssim 0.9$	$\gtrsim 1$	$\gtrsim 1$	~ 2	~ 10
QCD: $T^{freeze}/T_{pc} \simeq 1$	~ 0.5	< 0	~ 1	< 0

- **C_6 of conserved value is predicted to be the signal of phase transition**. (C_6 <0)
- C_6 of net-proton were measured at QM2018 at $\sqrt{s_{NN}}$ = 200 GeV in Au+Au.
- **C_6 of net-charge results will be shown.**

Why net-charge?

- In addition to net-proton results, it is important to measure net-charge.
- Net-charge is conserved value whereas net-proton and net-Kaon are measured as a proxy of net-baryon and net-strangeness.

Difference between published and current method

- Earlier method

high p_T / low p_T

All positive charged particles	$|\eta|<0.5$	All negative charged particles
N_1, ε_1 $(w_{\pi^+}\epsilon_{\pi^+})+(w_{K^+}\epsilon_{K^+})+(w_p\epsilon_p)$		N_2, ε_2 $(w_{\pi^-}\epsilon_{\pi^-})+(w_{K^-}\epsilon_{K^-})+(w_{\bar{p}}\epsilon_{\bar{p}})$

Net-charge : N_1-N_2 **Efficiency : $(\varepsilon_1+\varepsilon_2)/2$**

- Current method

	π^+	K^+	p	π^-	K^-	pbar
high-p_T	n_1, ε_1	n_2, ε_2	n_3, ε_3	n_4, ε_4	n_5, ε_5	n_6, ε_6
low-p_T	n_7, ε_7	n_8, ε_8	n_9, ε_9	n_{10}, ε_{10}	n_{11}, ε_{11}	n_{12}, ε_{12}

Net-charge : $(n_1+n_2+n_3+n_7+n_8+n_9)-(n_4+n_5+n_6+n_{10}+n_{11}+n_{12})$

Efficiency : ϵ_1 ϵ_2 ϵ_3 $\cdots$ 12 efficiency bins were used

- **Earlier method** : Count all charged particle with averaged efficiency.
- **Current method** : Count π, K, p with separated efficiencies.

Efficiency correction for different phase space

It was suggested that efficiency correction using the average efficiency for different region of the phase space (different p_T bins for different particle species) would give artificial results when distributions do not follow Poisson.

2 distribution model

5% smaller cumulants than Poisson

n_A, ε_A | n_B, ε_B

$$\Delta\epsilon = \epsilon_A - \epsilon_B$$

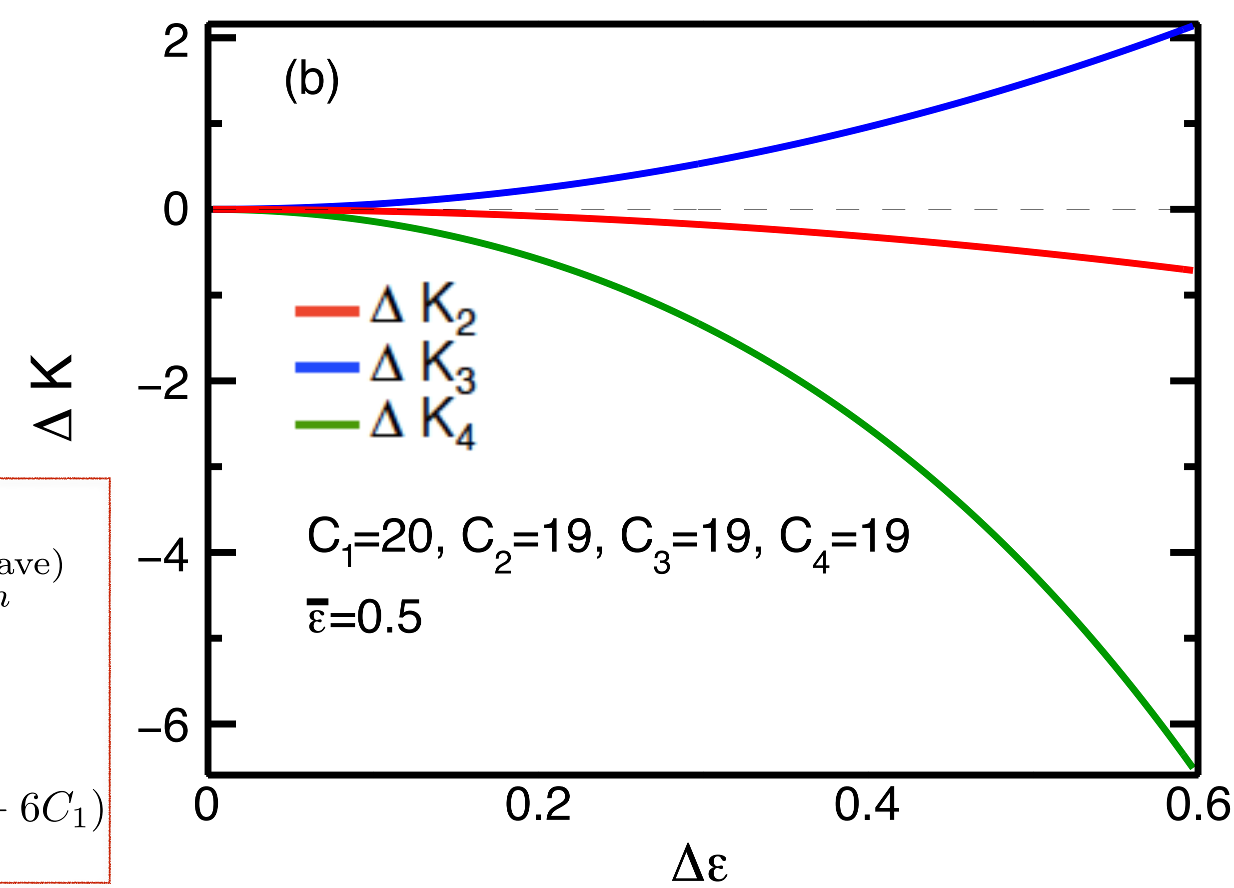
$$\Delta K_2 = \frac{1}{2}\left(\frac{\Delta\varepsilon}{\bar{\varepsilon}}\right)^2 (C_2 - C_1),$$

$$\Delta K_3 = \frac{3}{2}\left(\frac{\Delta\varepsilon}{\bar{\varepsilon}}\right)^2 (C_3 - 2C_2 + C_1),$$

$$\Delta K_4 = \frac{1}{2}\left(\frac{\Delta\varepsilon}{\bar{\varepsilon}}\right)^2 (6C_4 - 18C_3 + 19C_2 - 7C_1) + \frac{1}{8}\left(\frac{\Delta\varepsilon}{\bar{\varepsilon}}\right)^4 (C_4 - 6C_3 + 11C_2 - 6C_1)$$

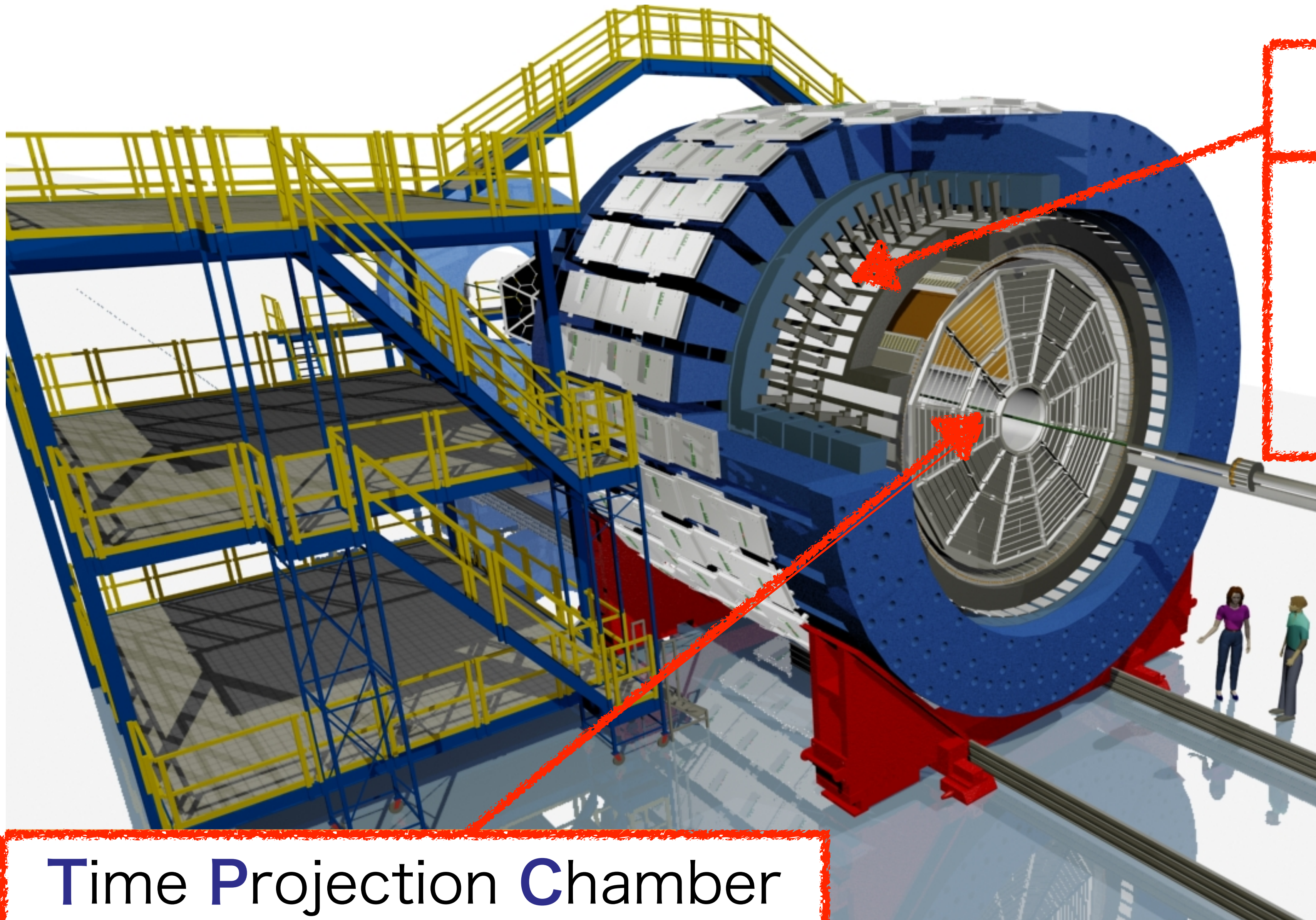
$$\Delta K_m = K_m - K_m^{(\mathrm{ave})} = 2C_m - K_m^{(\mathrm{ave})}$$

Phys. Rev. C **95**, 064912 T.Nonaka, M.Kitazawa and S.Esumi



<u>We can't estimate true corrected cumulant when uses average efficiencies if Δ ε is large.</u>

The STAR Detector



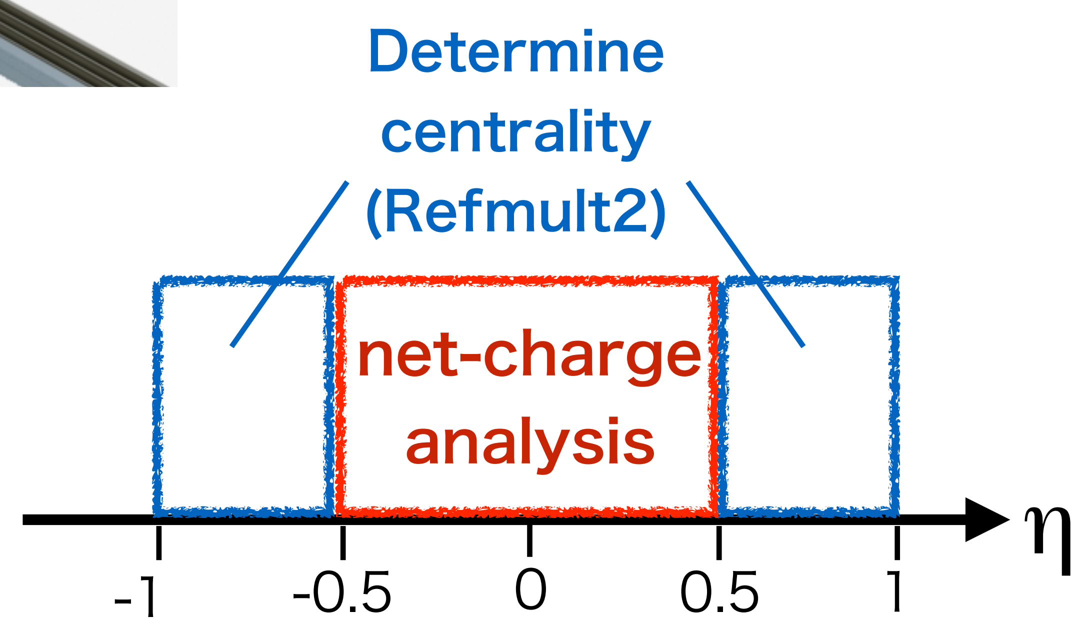
Time-Of-Flight

- PID in a broad p_T region.
- Used to remove pile-up events.

Time Projection Chamber

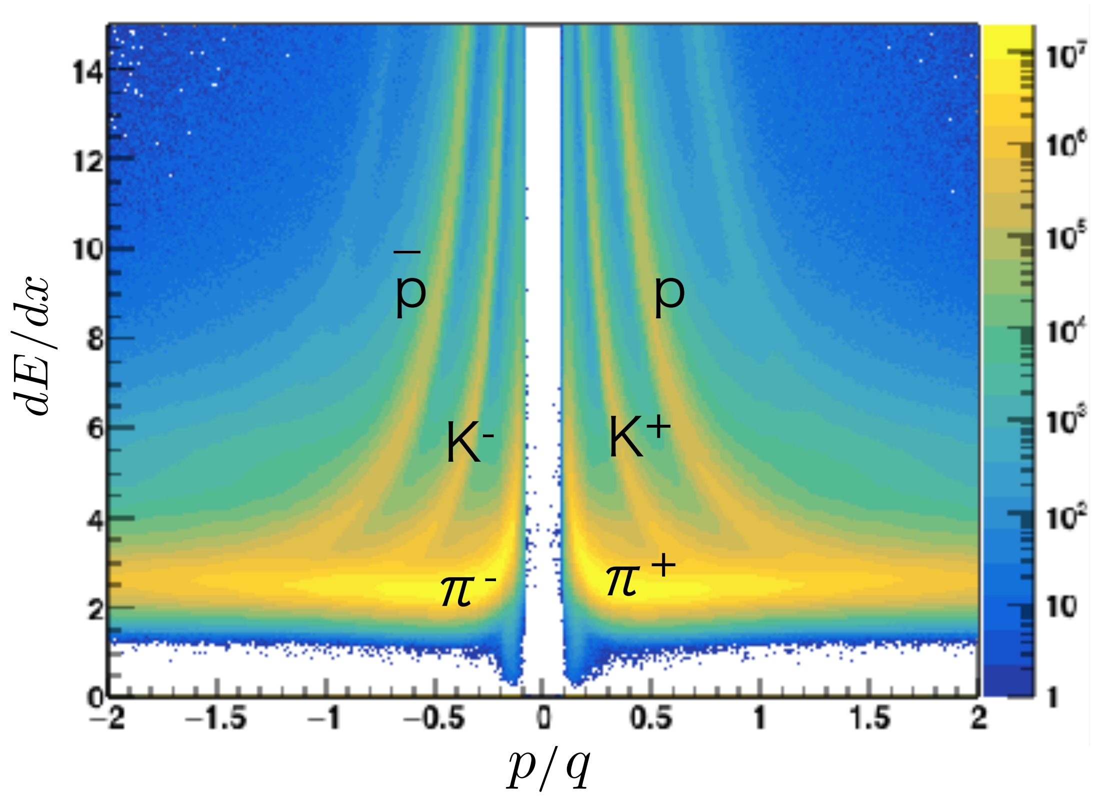
- Used for tracking, momentum determination and PID.
- Used to determine centrality and to calculate fluctuations.

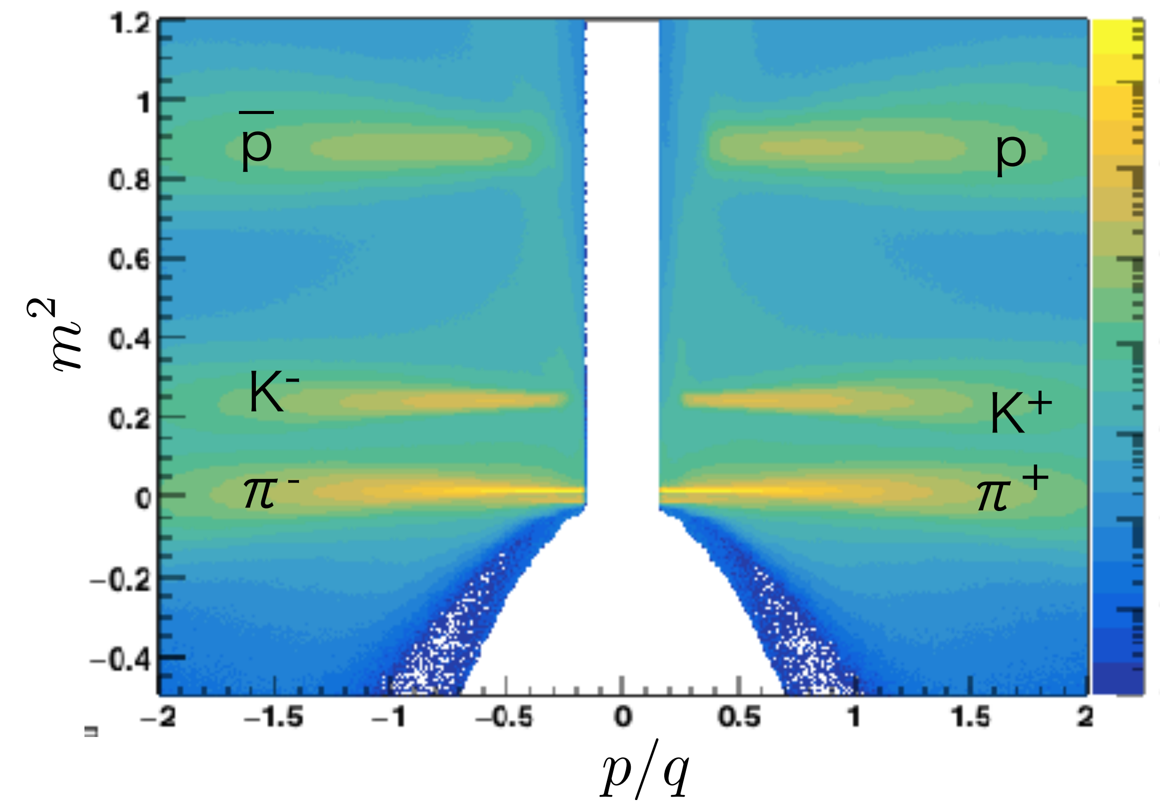
<u>Using different kinematic window to avoid auto-correlation.</u>



Particle Identification

Au+Au,$\sqrt{s_{NN}}$ = 200GeV
STAR Preliminary





- PID have been done by using dE/dx of TPC
- TOF is also used for PID at high p_T region

Data set

	Run11(0-80%)	Run10(10-80%)	Run10(0-10%)	UrQMD
NEvent	485M	211M	196M	45M

Event Selection

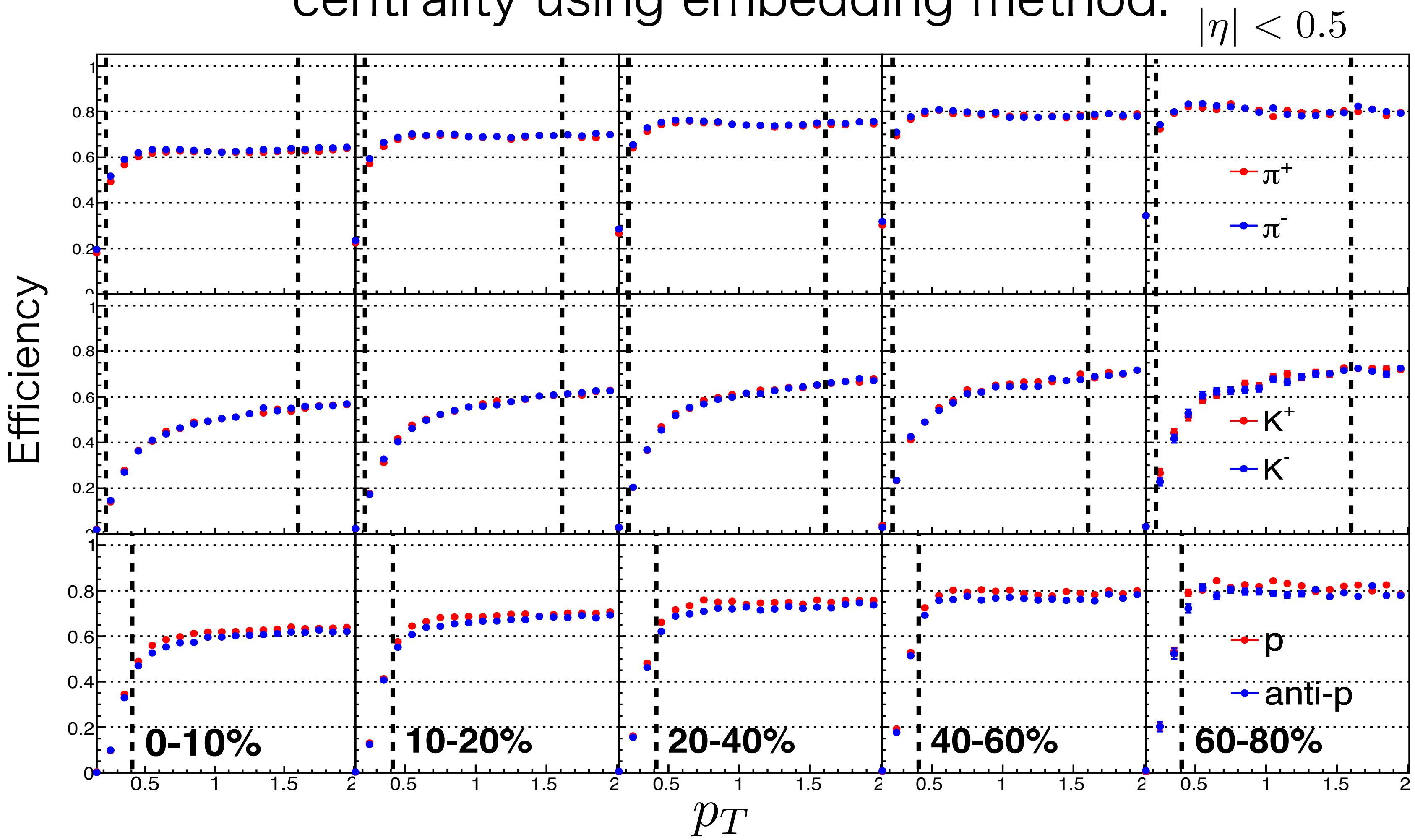
$\lvert v_z \rvert$	<30
$\lvert v_r \rvert$	<2
Pile up event cut	0.46 * nRefMult - 10 < nTOFMatch

Track cut	π	K	p
p_T (TPC only)	0.2 - 0.5 GeV/c	0.2 - 0.4 GeV/c	0.4 - 0.8 GeV/c
p_T (TPC+TOF)	0.5 - 1.6 GeV/c	0.4 - 1.6 GeV/c	0.8 - 2. GeV/c
$\lvert \eta \rvert$		< 0.5	
dca		<1	
nhitsfits		>20	
track quality		>0.52	
nhitsdedx		>10	
Centrality		Refmult2 ($\lvert \eta \rvert$ >0.5)	
PID (TPC)	nSigmaPion < 2	nSigmaKaon < 2	nSigmaProton < 2
PID (TOF)	-0.15< m^2 <0.14	0.14< $\lvert m^2 \rvert$ <0.4	0.6 < $\lvert m^2 \rvert$ <1.2

Same as net-charge published results

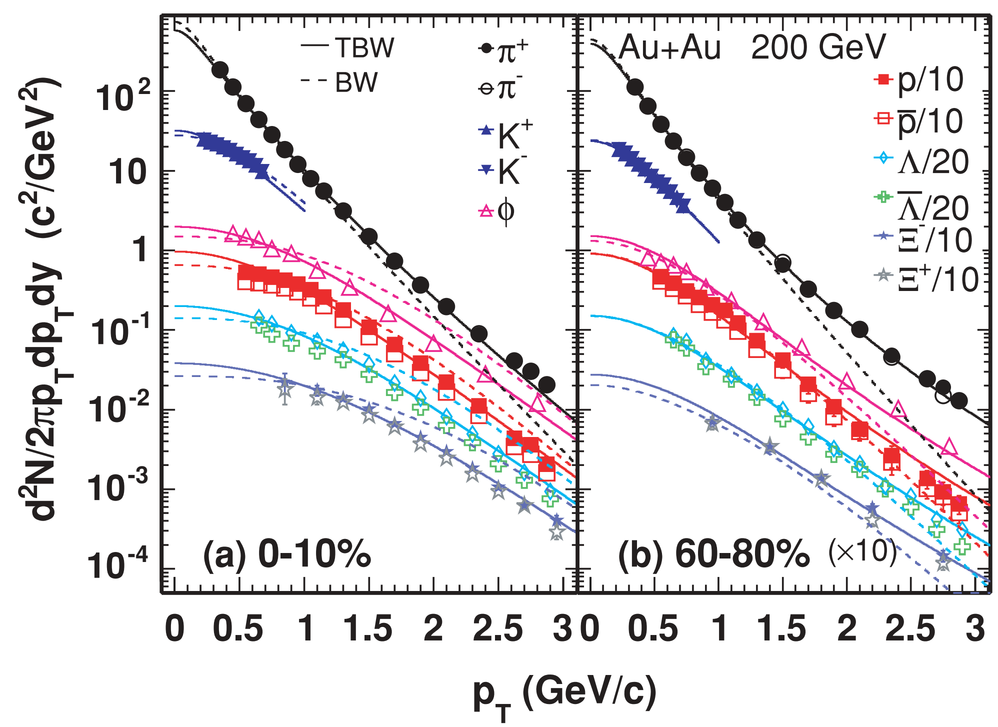
Efficiency estimation

①pT dependent efficiencies were estimated for each centrality using embedding method.

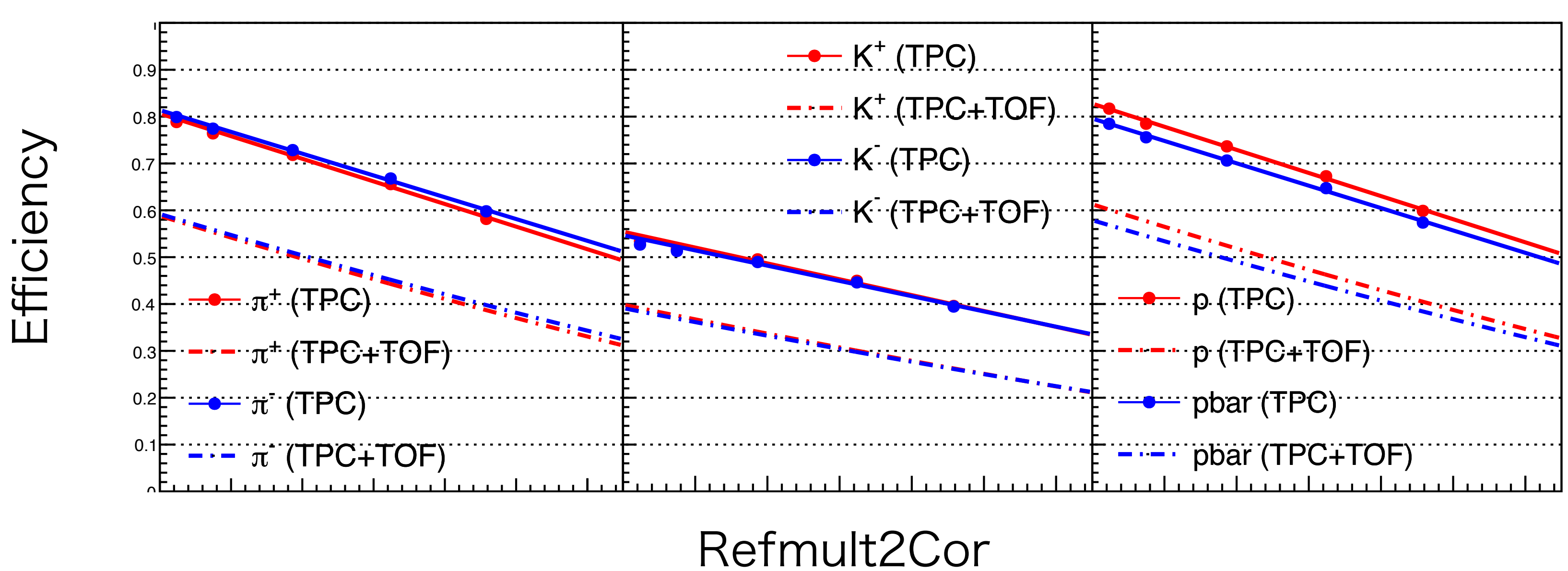


② pT integrated efficiencies were estimated using results of ① and spectra data.

PHYSICAL REVIEW C 79, 051901(R) (2009)



$$\epsilon_i = \frac{\int \epsilon_i p_T(p_T) f(p_T) dp_T}{\int f(p_T) p_T dp_T}$$



③ pol1 fit and estimate tracking efficiencies for each multiplicity bin.

Factorial cumulant method

Cumulant (measured) → Factorial Cumulant (measured)

↓ Efficiency correction

Cumulant (true) ← Factorial Cumulant (true)

1-4th-order cumulants (factorial cumulant formula)

Phys. Rev. C **95**, 064912

T.Nonaka, M.Kitazawa and S.Esumi

$$\langle Q\rangle_c = \langle q_{(1,1)}\rangle_c,$$

$$\langle Q^2\rangle_c = \langle q_{(1,1)}^2\rangle_c + \langle q_{(2,1)}\rangle_c - \langle q_{(2,2)}\rangle_c,$$

$$\langle Q^3\rangle_c = \langle q_{(1,1)}^3\rangle_c + 3\langle q_{(1,1)}q_{(2,1)}\rangle_c - 3\langle q_{(1,1)}q_{(2,2)}\rangle_c + \langle q_{(3,1)}\rangle_c - 3\langle q_{(3,2)}\rangle_c + 2\langle q_{(3,3)}\rangle_c,$$

$$\begin{aligned}\langle Q^4\rangle_c = {} & \langle q_{(1,1)}^4\rangle_c + 6\langle q_{(1,1)}^2 q_{(2,1)}\rangle_c - 6\langle q_{(1,1)}^2 q_{(2,2)}\rangle_c + 4\langle q_{(1,1)}q_{(3,1)}\rangle_c + 3\langle q_{(2,1)}^2\rangle_c \\ & +3\langle q_{(2,2)}^2\rangle_c - 12\langle q_{(1,1)}q_{(3,2)}\rangle_c + 8\langle q_{(1,1)}q_{(3,3)}\rangle_c - 6\langle q_{(2,1)}q_{(2,2)}\rangle_c \\ & +\langle q_{(4,1)}\rangle_c - 7\langle q_{(4,2)}\rangle_c + 12\langle q_{(4,3)}\rangle_c - 6\langle q_{(4,4)}\rangle_c,\end{aligned}$$

Cumulant (true)

Cumulant (measured)

$$q_{(r,s)} = q_{(a^r/p^s)} = \sum_{i=1}^{M}(a_i^r/p_i^s)n_i.$$

M: Number of phase space

n_i: Number of particle

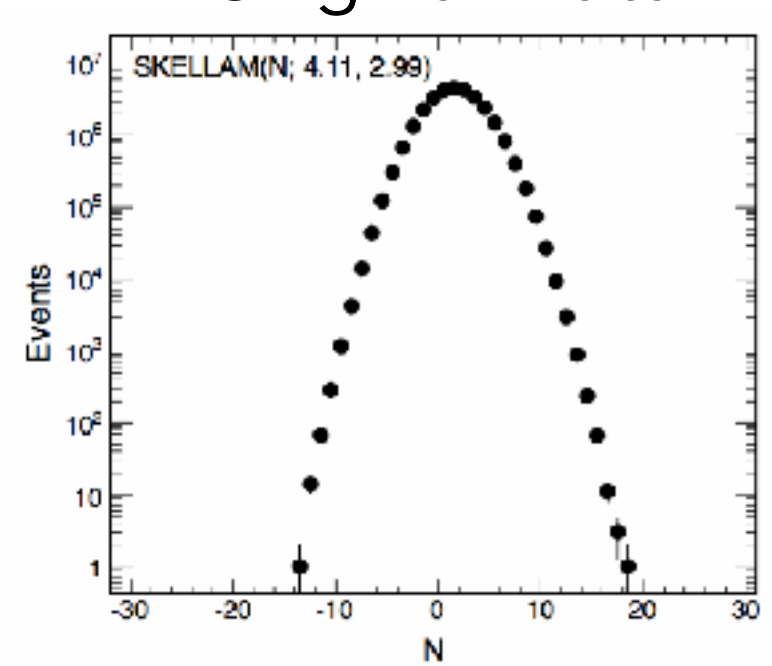
p_i: Efficiency

a_i: +1 (positive charge) or -1 (negative charge)

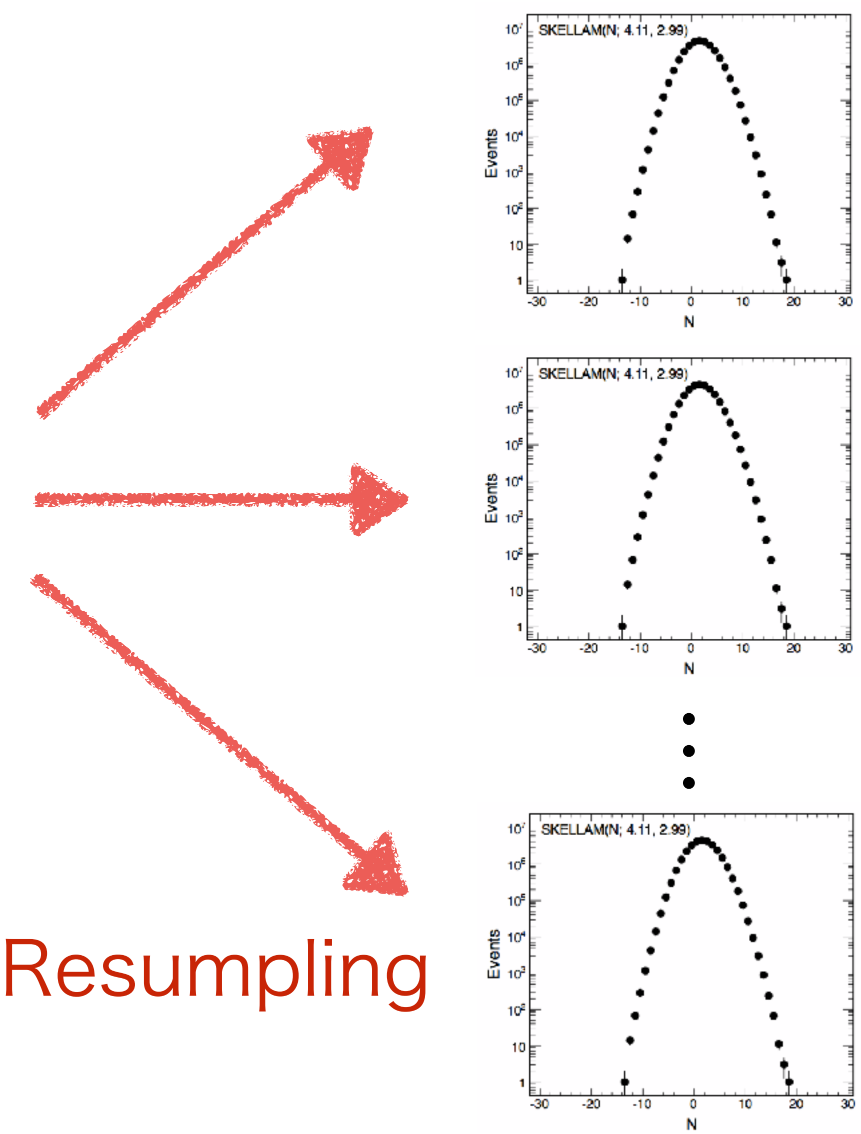
Statistical error estimation

- Statistical errors are estimated by **Bootstrap** method.

Original Data



Cumulant : C_n



Resumpling

Cumulant : C_n^{*1}

Cumulant : C_n^{*2}

Cumulant : C_n^{*100}

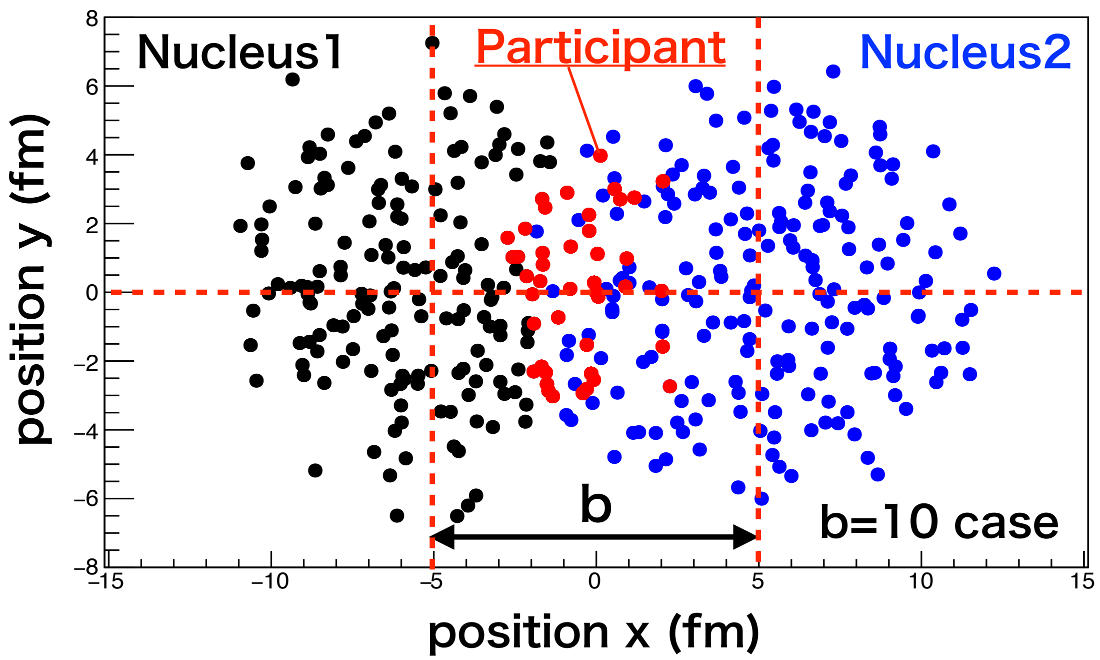
About 100 times

Error of C_n is estimated by calculating **standard error of C_n^*** distribution.

Glauber model

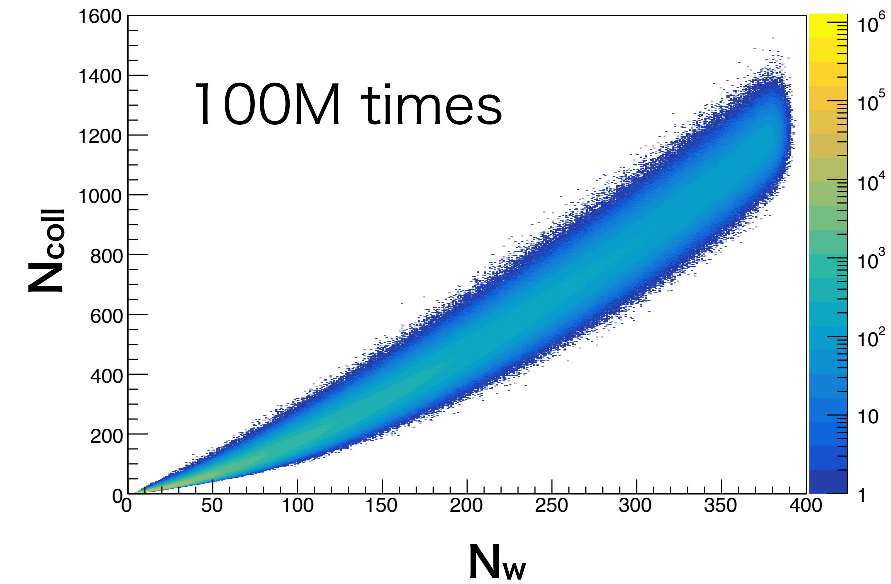
- N_W can't be measured directly by experiment so estimated by Glauber model.

1. Calculate N_W and N_{coll} by Monte Carlo simulation.



- Width : **0.535**
- Radius : **6.4**
- σ : **4.2**

Repeat



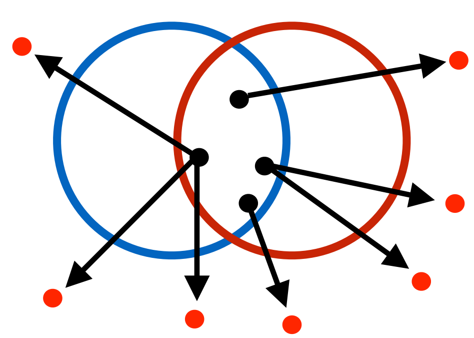
2. Calculate multiplicity by two component model and NBD.

Two component model

Number of source :
$(1-x)*(N_W/2.)+x*N_{coll}$

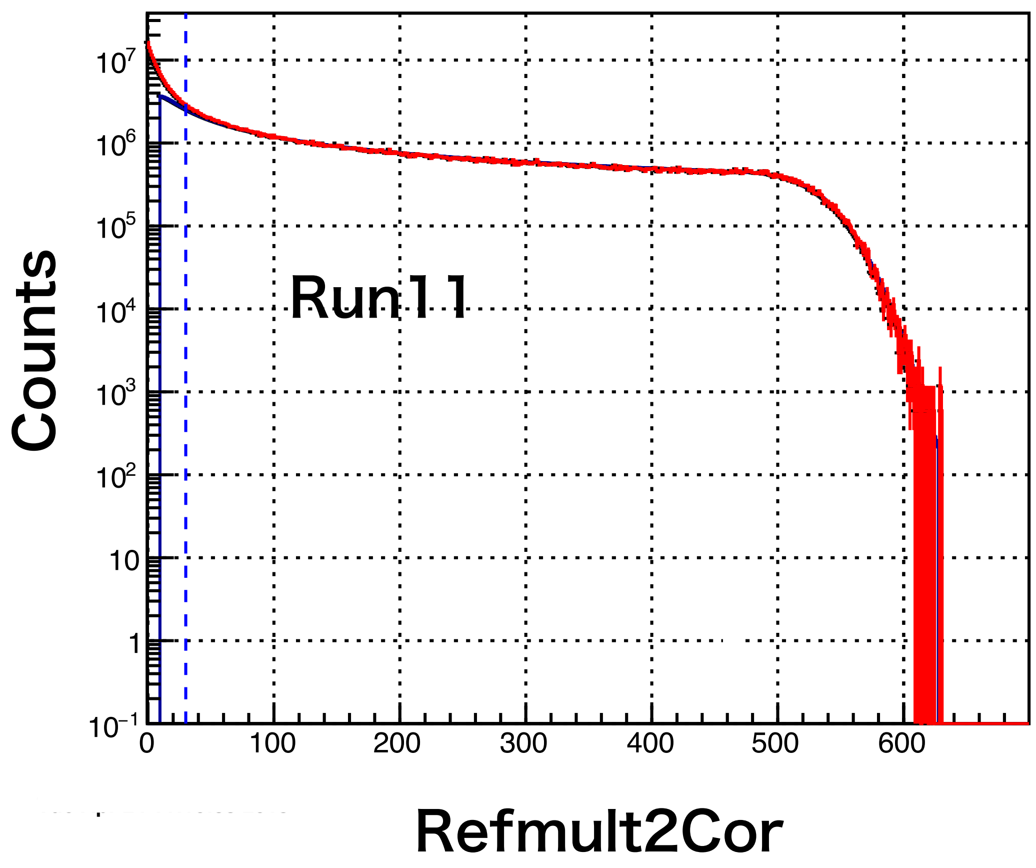
x=0.13

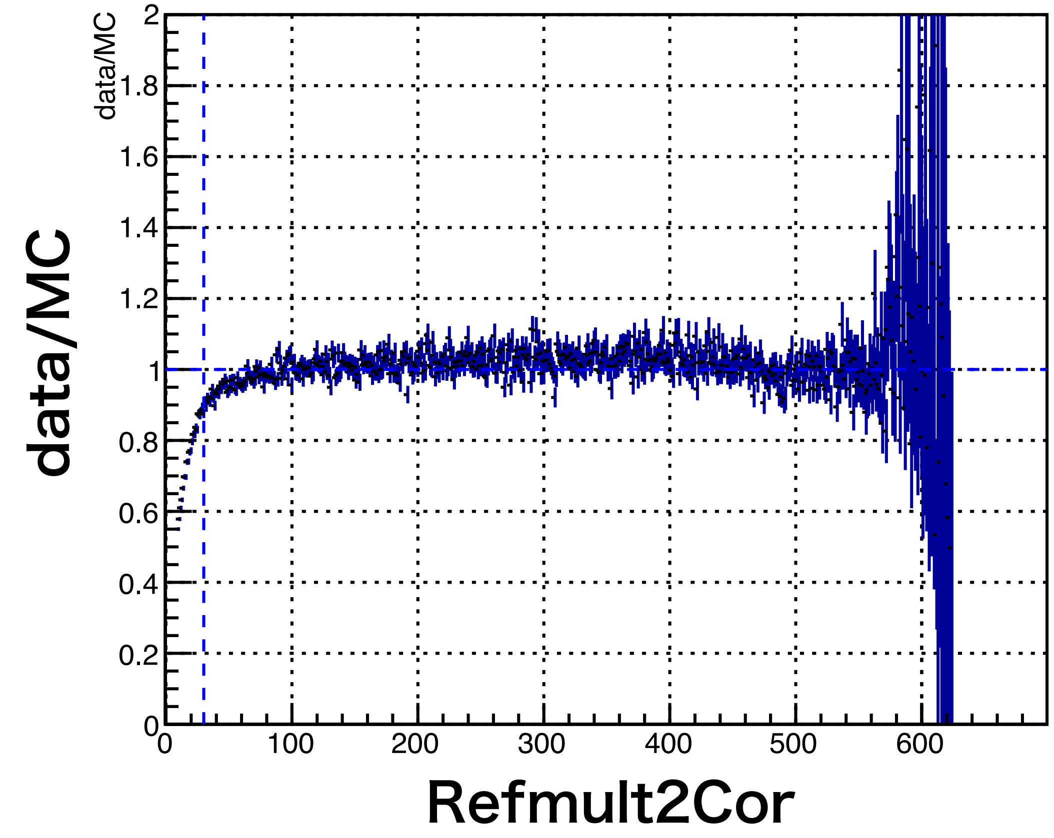
- : initial source
- : final state multiplicity



- Final state particles are generated from each source independently. (IPP)
- Number of particles from each source is fluctuating under the NBD.

Glauber fit results



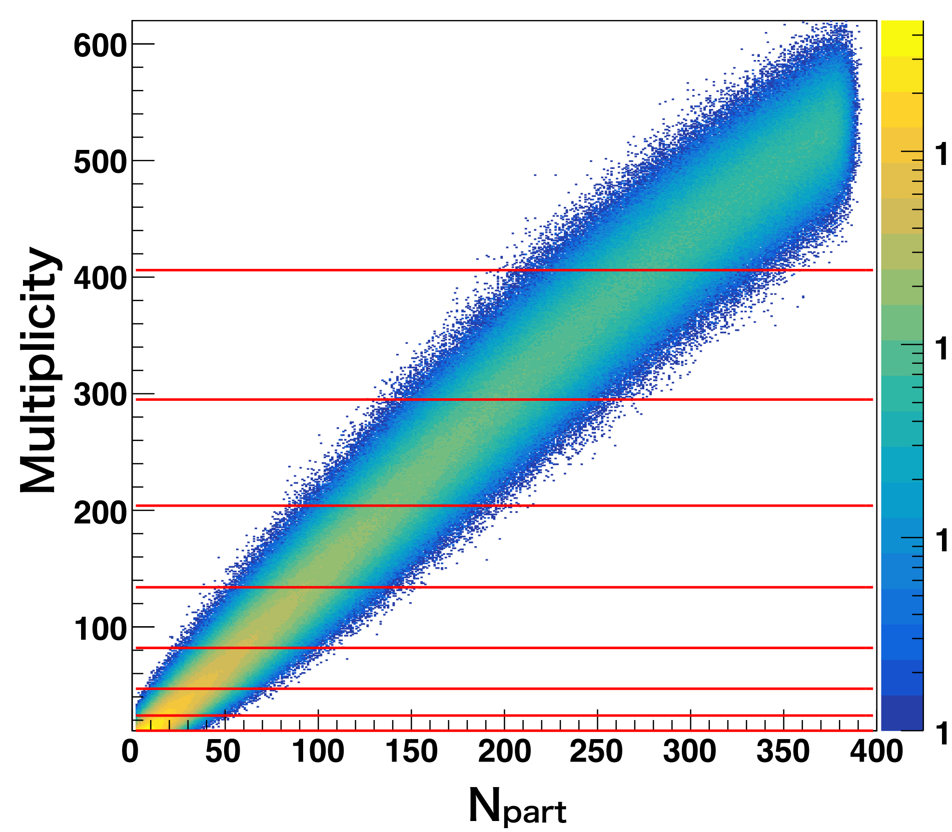


MC : 1M

n_{pp} : Mean number of generated particles from each source.

k : Parameters of NBD.

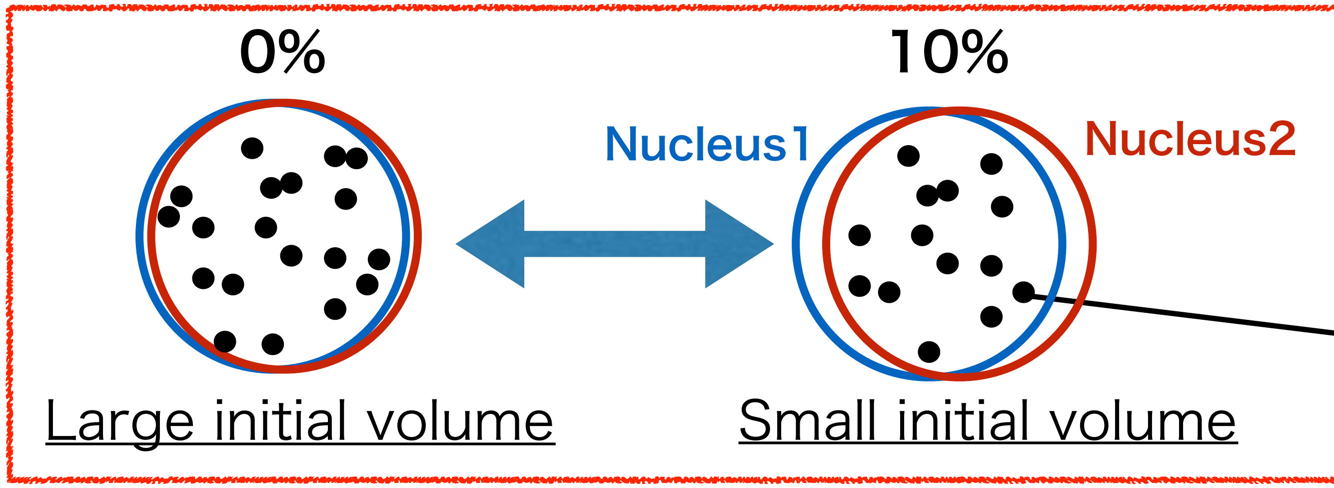
- n_{pp}=2.35
- k=0.9
- efficiency =1-Refmult2Cor*(0.206/560)



Centrality Bin Width Correction

- **Initial volume** which correspond to **number participant nucleons (Nw) are different** even in the same centrality bins.

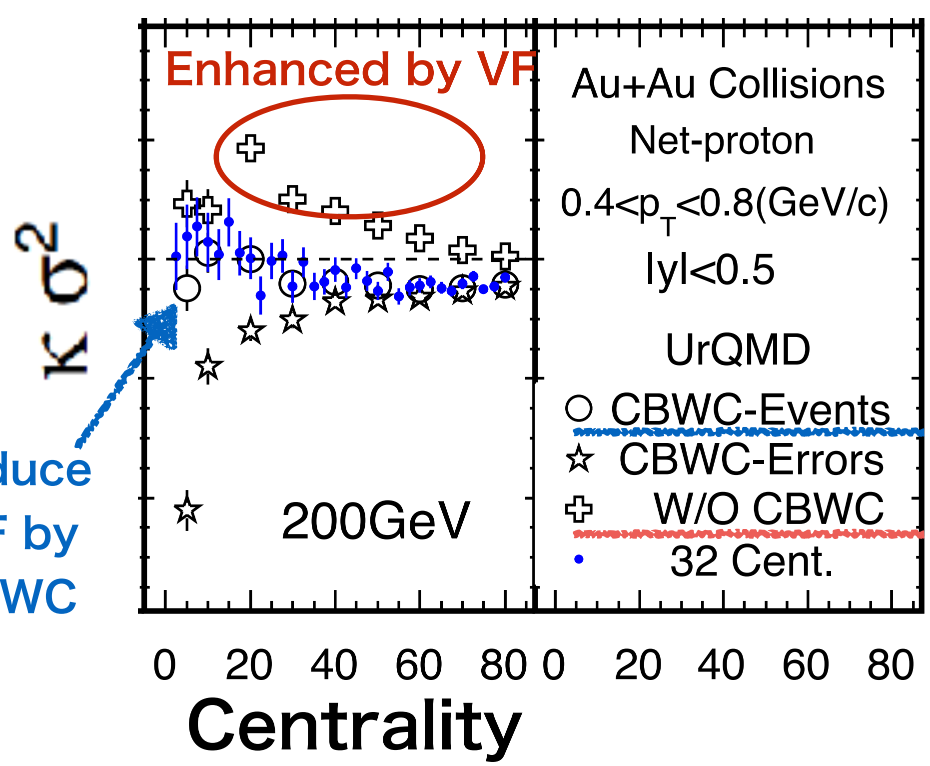
0-10% centrality bin



Participant

- Because of VF, cumulants are **artificially enhanced** which would be the background which should be eliminated.

Xiaofeng Luo, J. Phys. G: Nucl. Part. Phys. 40 105104 (2013)



- STAR applied **Centrality Bin Width Correction (CBWC)**.
- In this correction, cumulants are **calculated for each multiplicity bins**.

$$C_n = \sum_r w_r C_{(n,r)} \qquad w_r = \frac{N_r}{\sum_r N_r}$$

Cumulants for each centrality bins | Weight | Cumulants in r-th multiplicity bins

How to estimate Systematic uncertainty?

$$RMS = \sqrt{\frac{1}{n}\sum_{i}\left(\frac{Y_i - Y_{st.cut}}{Y_{st.cut}}\right)^2}$$

$$Sys.Err = Y_{st.cut}\sqrt{\sum_{j}(RMS)^2}$$

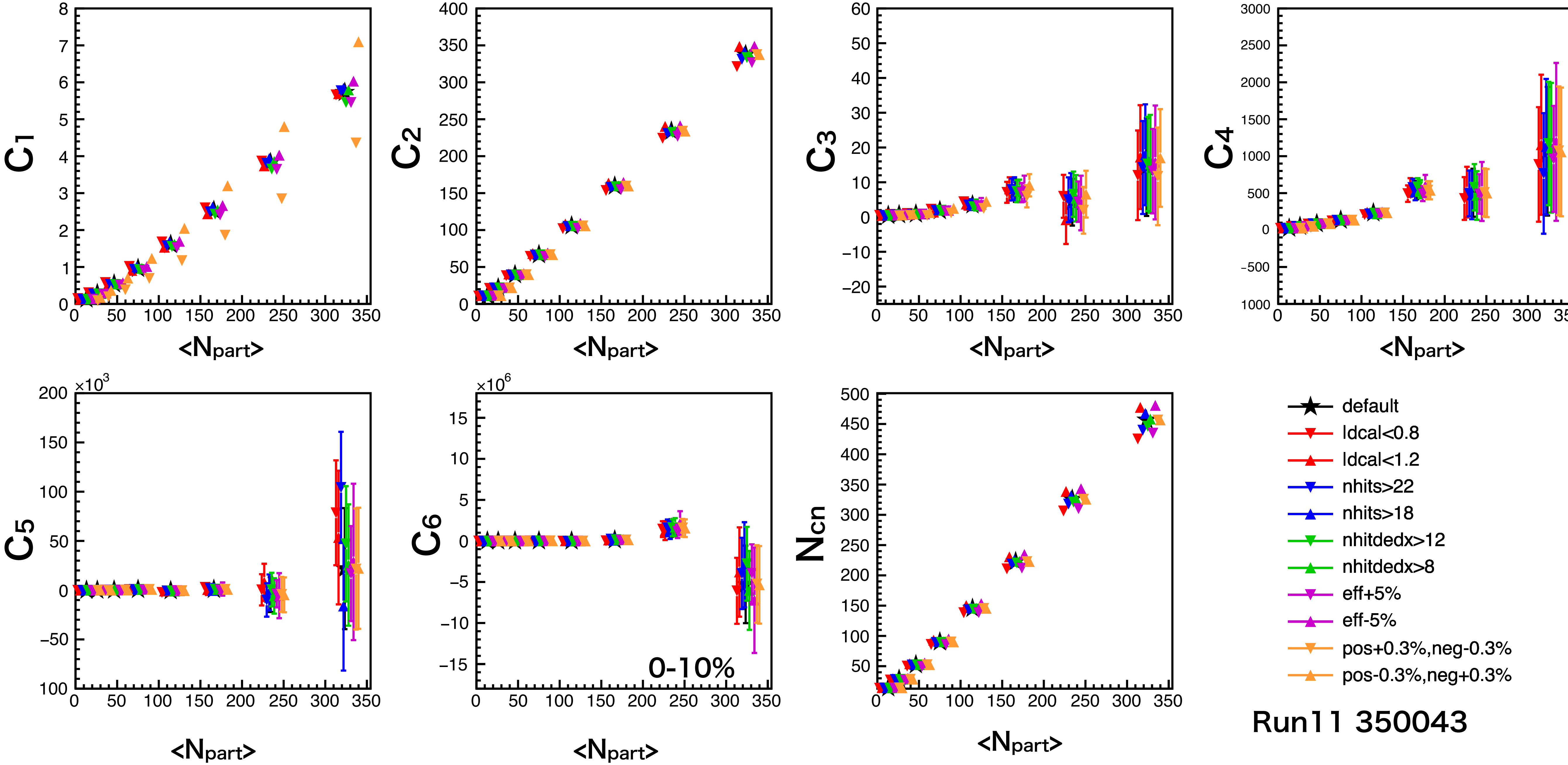
Net-charge (published)

dca	0.8, 1.0(default), 1.2
nFitPoints	-18, 20(default), 22
nhitsdedx	8, 10(default), 12
efficiency	±5% $+\alpha$?

- Efficiency of positively charged particle and negatively charged particle are changed ±5% at published results.
  (pos,neg) = (+5%, +5%) and (-5%, -5%)
- In this analysis, efficiencies are changed 0.3% separately in addition to conventional method.
  (pos,neg) =(+5%, +5%), (-5%, -5%), (+0.3%, -0.3%), (+0.3%, -0.3%)

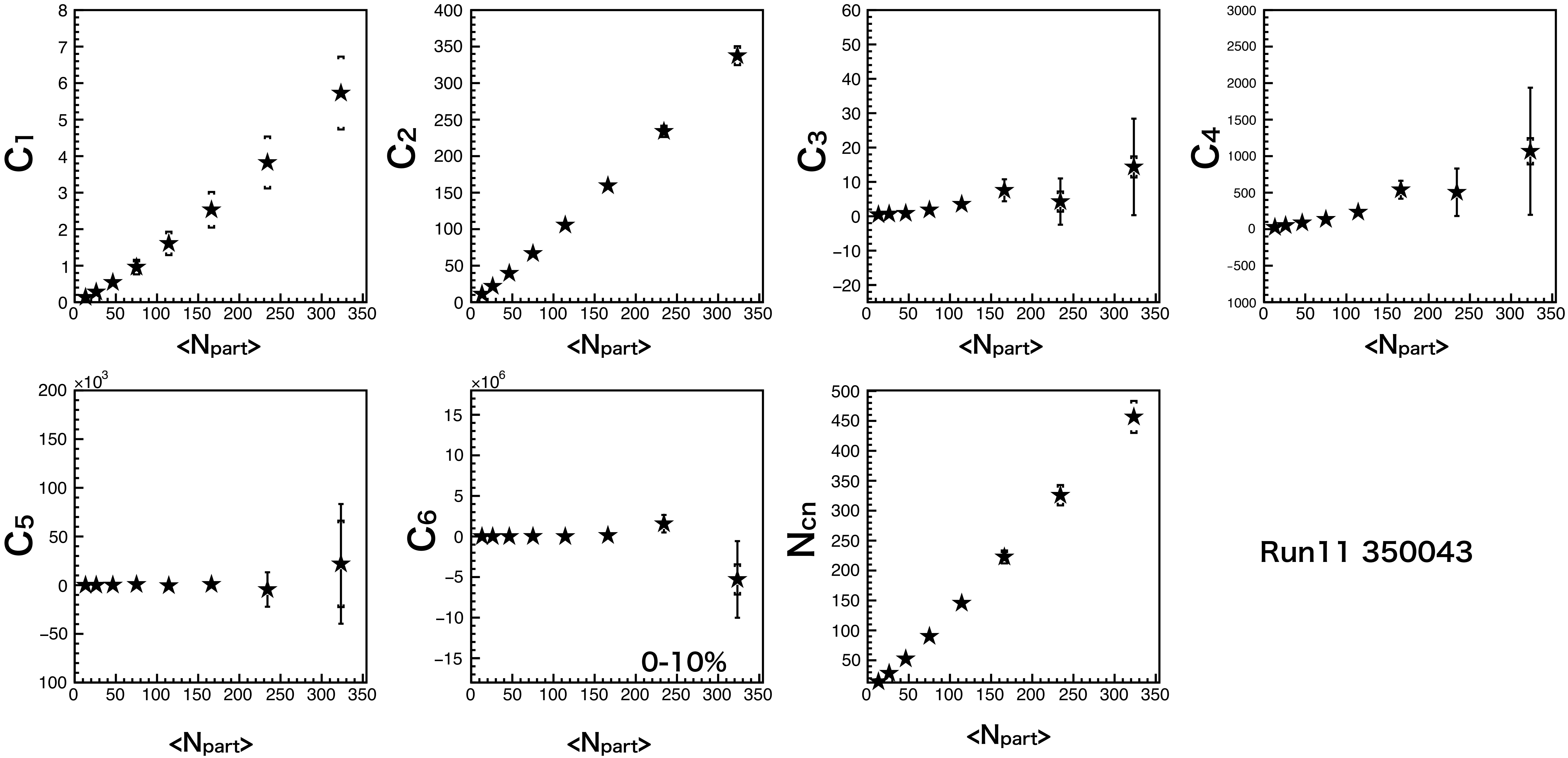
Results

Cumulants (Run11, 350043)



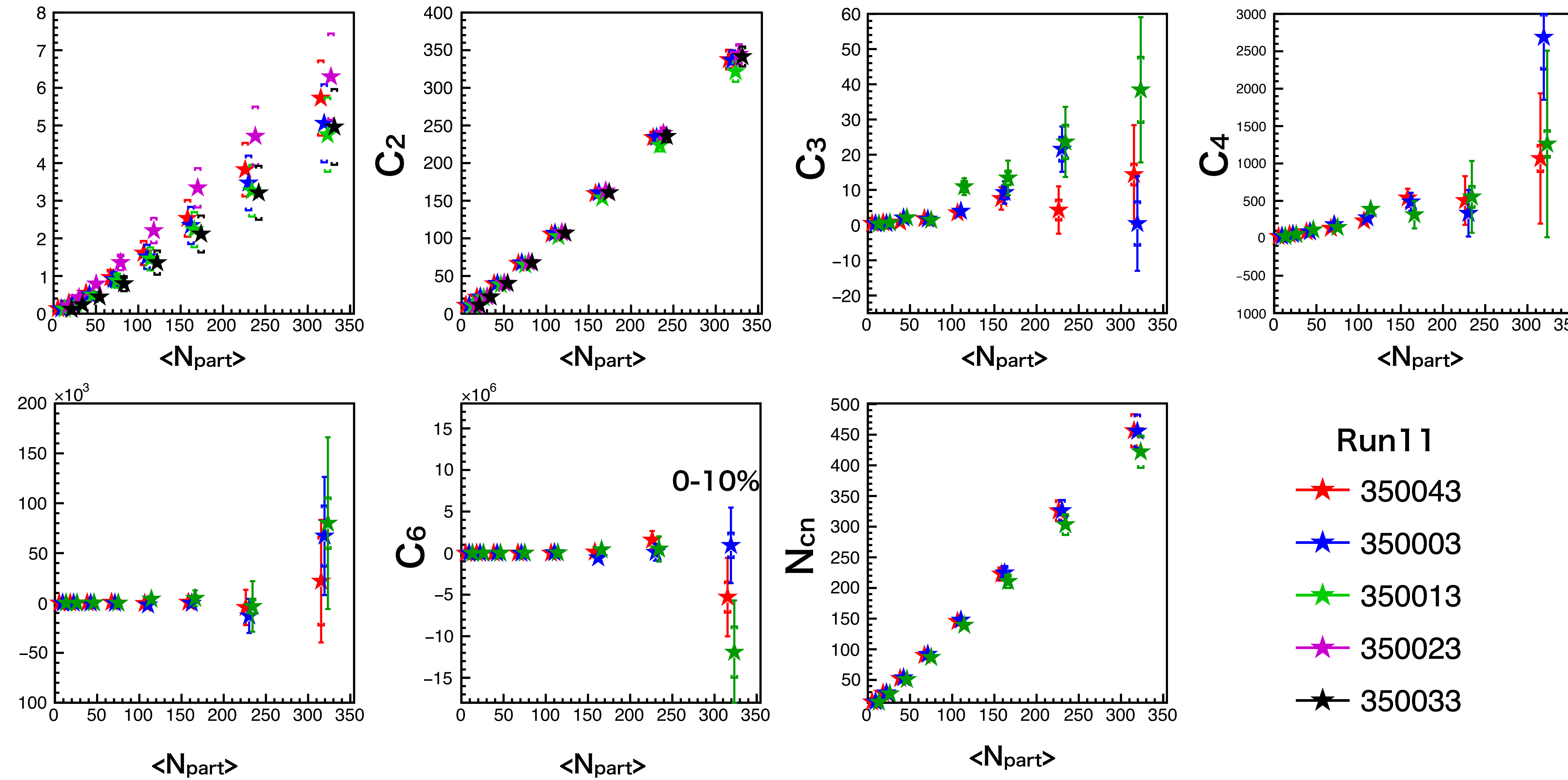
- (Effpos*1.003, Effneg*0.997) and (Effpos*0.997, Effneg*1.003) were calculated to estimate systematic uncertainty of the deviation between positively and negatively charged particle efficiency.

Cumulants (Run11, 350043)

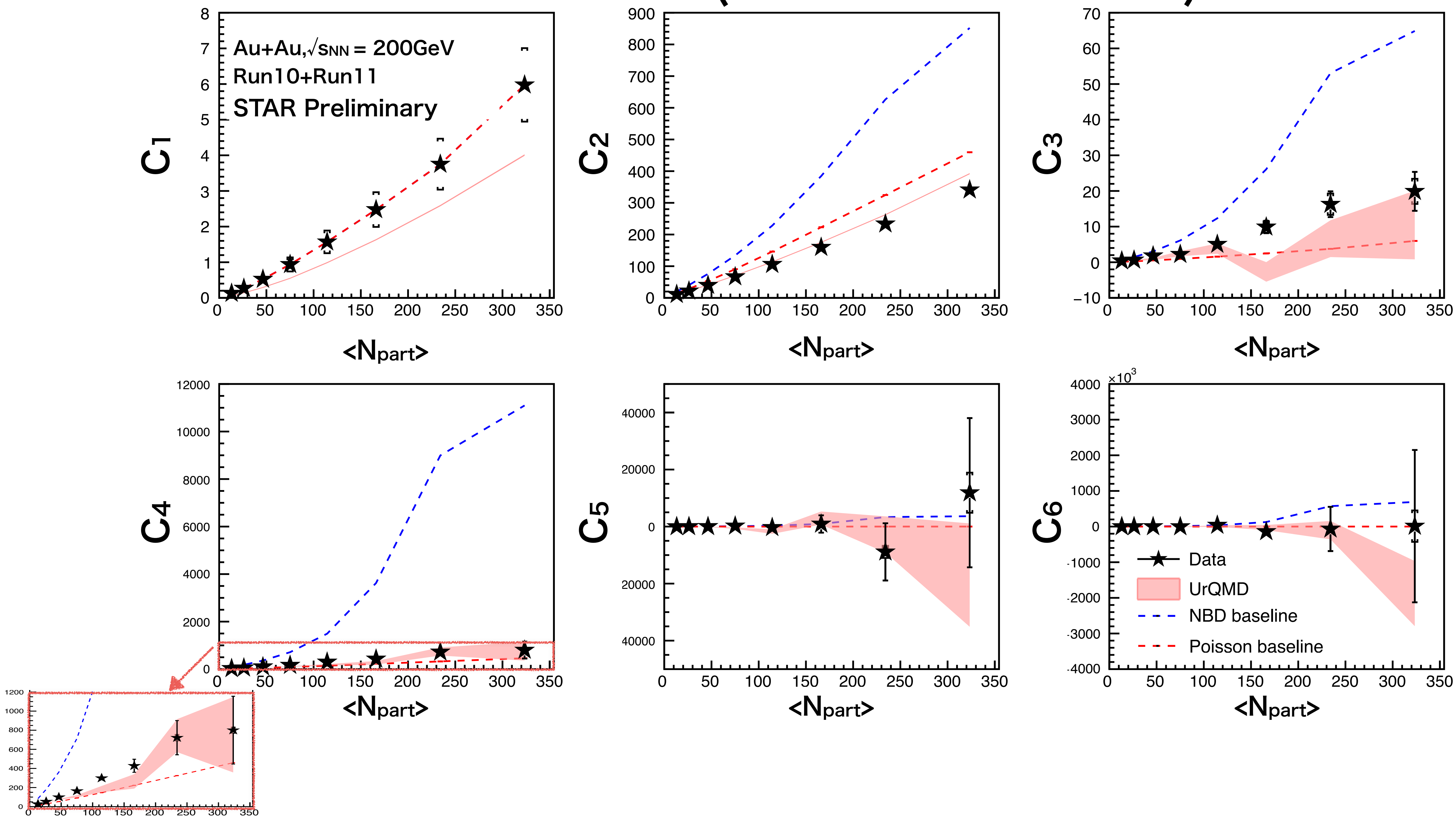


- (Effpos*1.003, Effneg*0.997) and (Effpos*0.997, Effneg*1.003) were calculated to
  estimate systematic uncertainty of the deviation between positively and negatively charged particle efficiency.

Cumulants (Run11, trigger by trigger)

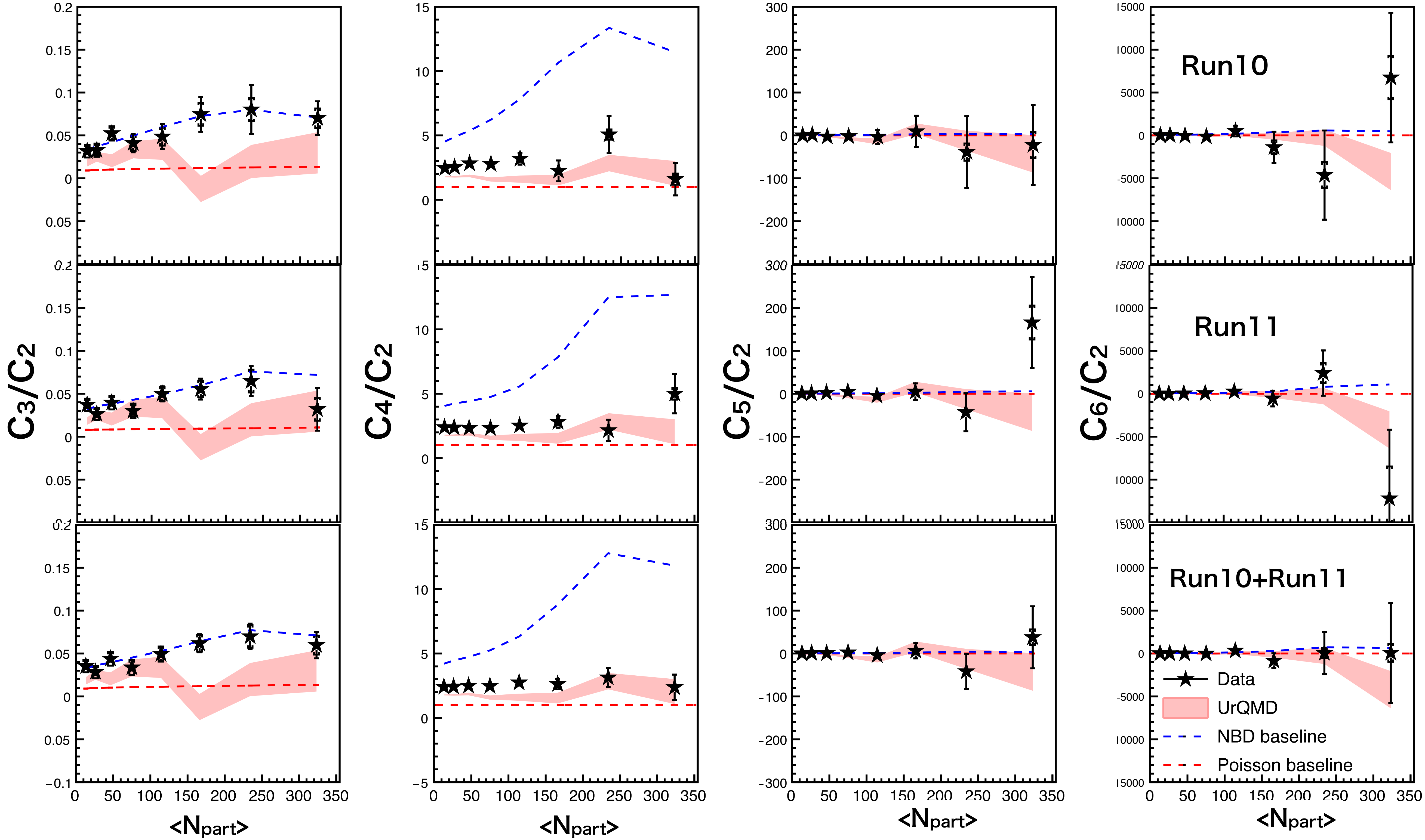


Cumulants (Run10+Run11)

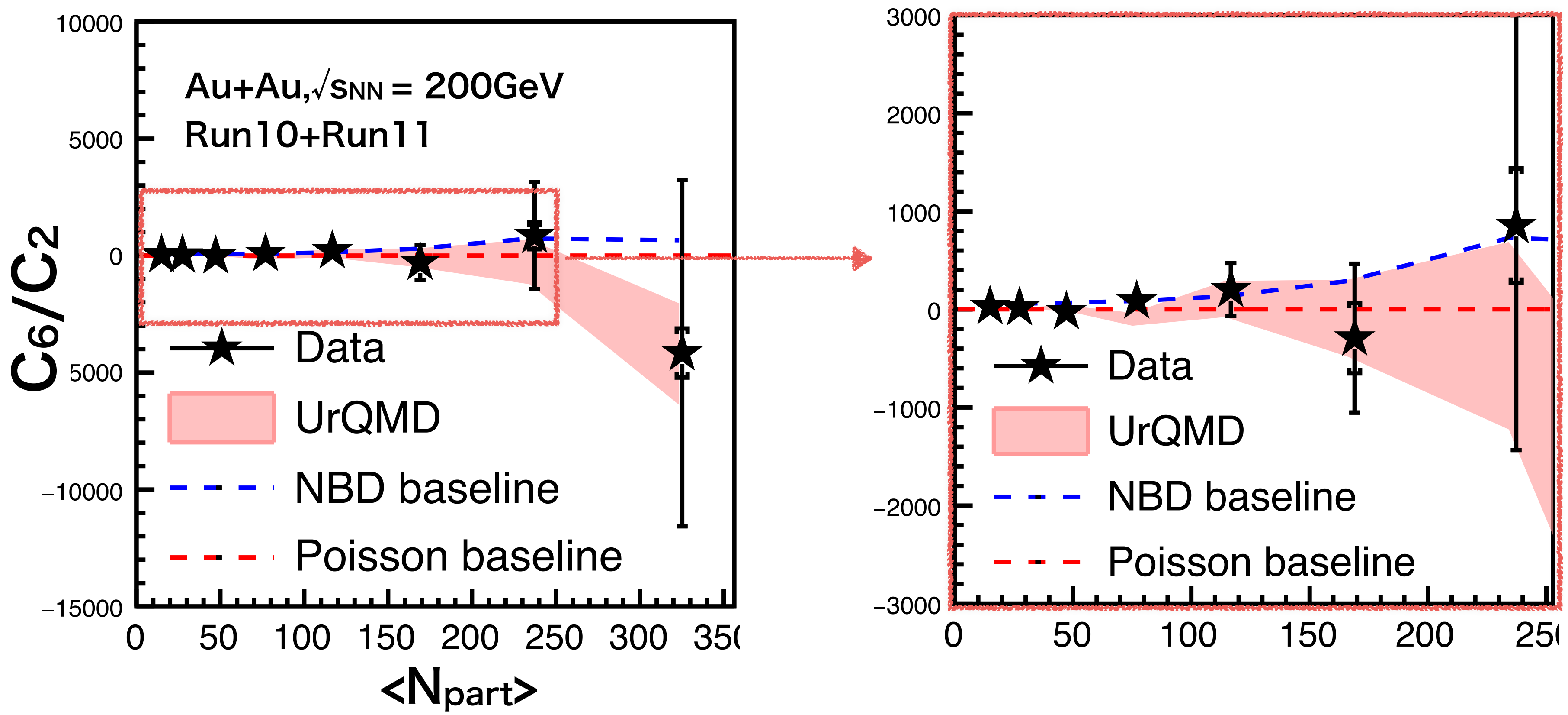


- From 1st- to 6th-order cumulants were measured as a function of <N_{part}>
- Linear trend can be seen.
- For higher-order cumulants, statistical uncertainties are larger than the systematic ones.
- NBD baseline becomes larger than experimental values especially at C_4.

Cumulant ratio (Run10+Run11)



Cumulant ratios



- C_6/C_2 is consistent with statistical baseline for all centralities.
- All statistical baseline and UrQMD results of C_6/C_2 are consistent within statistical uncertainties.

Summary1

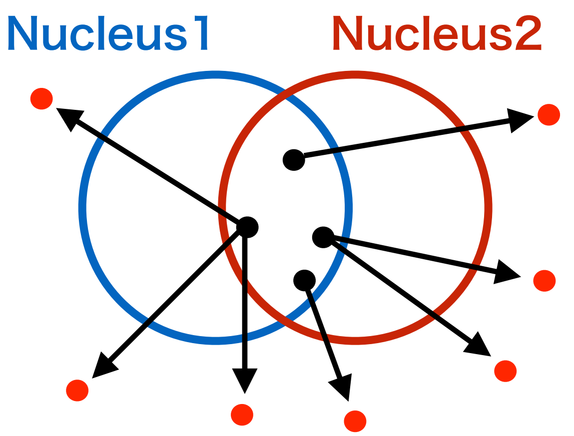
- Net-charge cumulants up to 6th-order in Au+Au collisions at $\sqrt{s_{NN}}$=200 GeV were reported.
- Cumulant ratios were compared to statistical baseline and UrQMD results.
- All statistical baselines and UrQMD results are consistent with experimental C_6/C_2 measurement within the statistical uncertainty.
  →With the current uncertainties crossover type of phase transition cannot be excluded.

Volume fluctuation correction

Volume fluctuation correction (VFC)

Nuclear Physics A Volume 960, April 2017, Pages 114-130

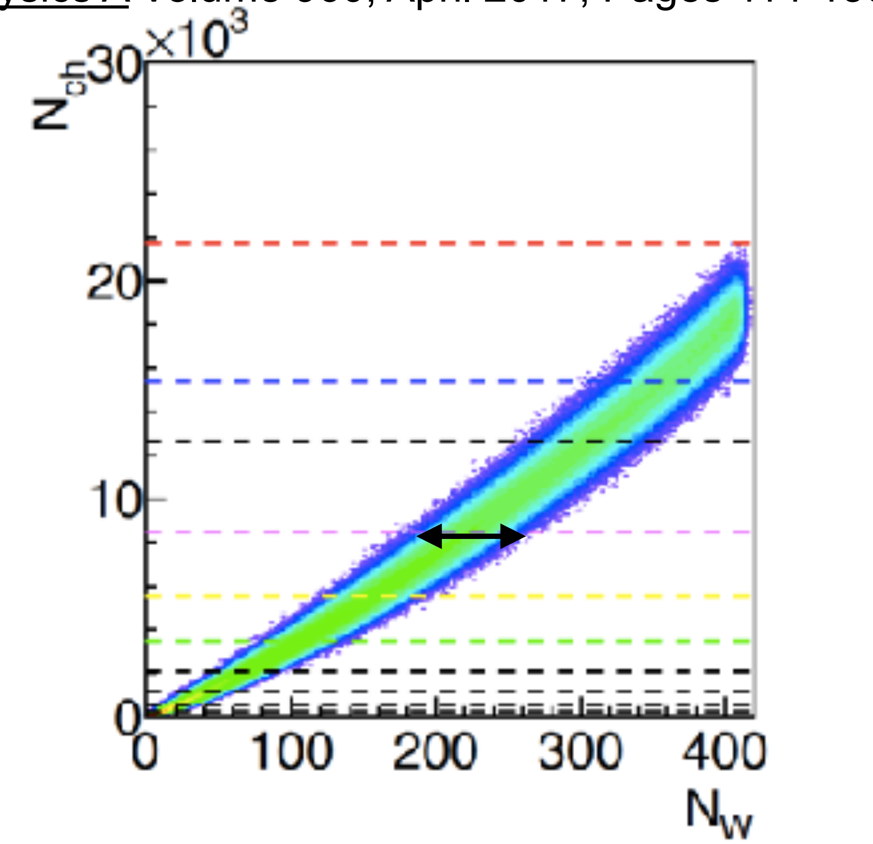
- Nw is fluctuating even if final state multiplicity is fixed.
  →**CBWC may not be enough to eliminate VF**



Independent particle production (IPP) model

● : initial source

● : final state multiplicity



- Under the IPP model, measured cumulants include additional term from VF which can be written by Nw cumulants.
- We can know **true cumulants by subtracting additional terms (VFC)**

Measured cumulant | True cumulant

Δn : number of net-particle

Additional term caused from N_W fluctuation

$$\kappa_1(\Delta N) = \langle N_W\rangle\kappa_1(\Delta n)$$

$$\kappa_2(\Delta N) = \langle N_W\rangle\kappa_2(\Delta n) + \langle\Delta n\rangle^2\kappa_2(N_W)$$

$$\kappa_3(\Delta N) = \langle N_W\rangle\kappa_3(\Delta n) + 3\langle\Delta n\rangle\kappa_2(\Delta n)\kappa_2(N_W) + \langle\Delta n\rangle^3\kappa_3(N_W)$$

$$\kappa_4(\Delta N) = \langle N_W\rangle\kappa_4(\Delta n) + 4\langle\Delta n\rangle\kappa_3(\Delta n)\kappa_2(N_W) + 3\kappa_2^2(\Delta n)\kappa_2(N_W) + 6\langle\Delta n\rangle^2\kappa_2(\Delta n)\kappa_3(N_W) + \langle\Delta n\rangle^4\kappa_4(N_W)$$

Toy model method

- Generate 2 independent Poisson distribution
- Parameter : λ_+, λ_-

Glauber : 500M events
Poisson : 100M events

$$\lambda_+ * \langle N_W \rangle = N_+$$
$$\lambda_- * \langle N_W \rangle = N_-$$
$$(\lambda_+ - \lambda_-) * \langle N_W \rangle = (N_+ - N_-)$$

Number of particles

Nw Calculated from Glauber

- λ_+ and λ_- are determined to N_+ and N_- describe the real experiment.

Nw fixed : Generate Poisson <Nw> times.
…No Nw fluctuation

Nw fluc : Generate Poisson Nw times.
…Nw fluctuation

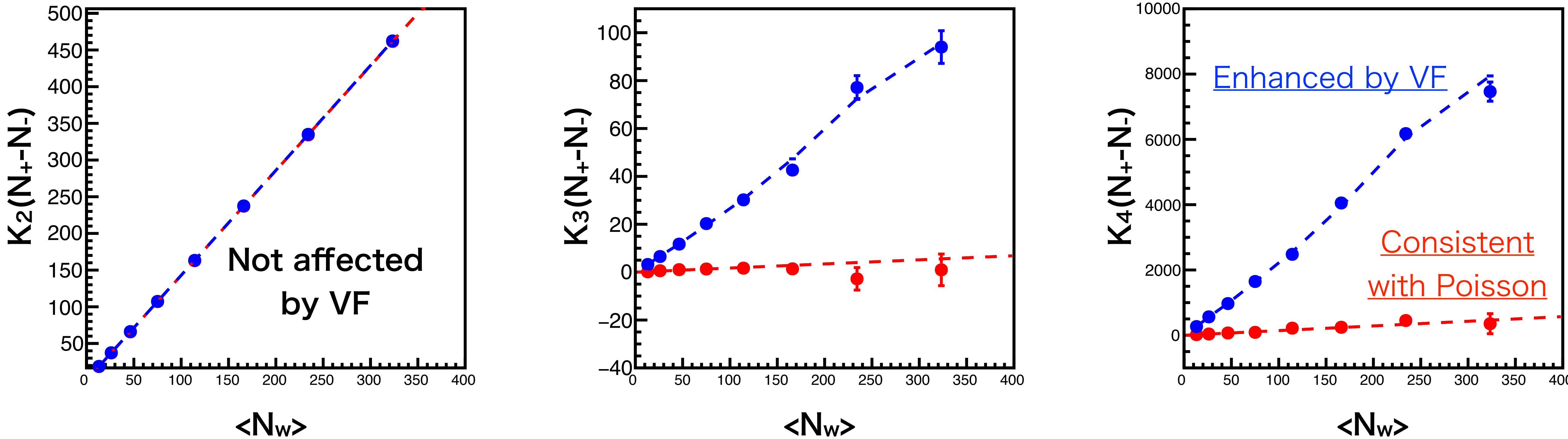
VFC : Subtract Nw fluctuation

- **Nw fixed** and **VFC** results should be same.
- Results from CBWC are also compared.

Should be same

- Nw fixed
- Poisson
- Nw fluc **VF(large)**
- Nw fluc Exp
- VFC
- CBWC+Nw fluc **VF(small)**

Toy model VFC check (10%step)

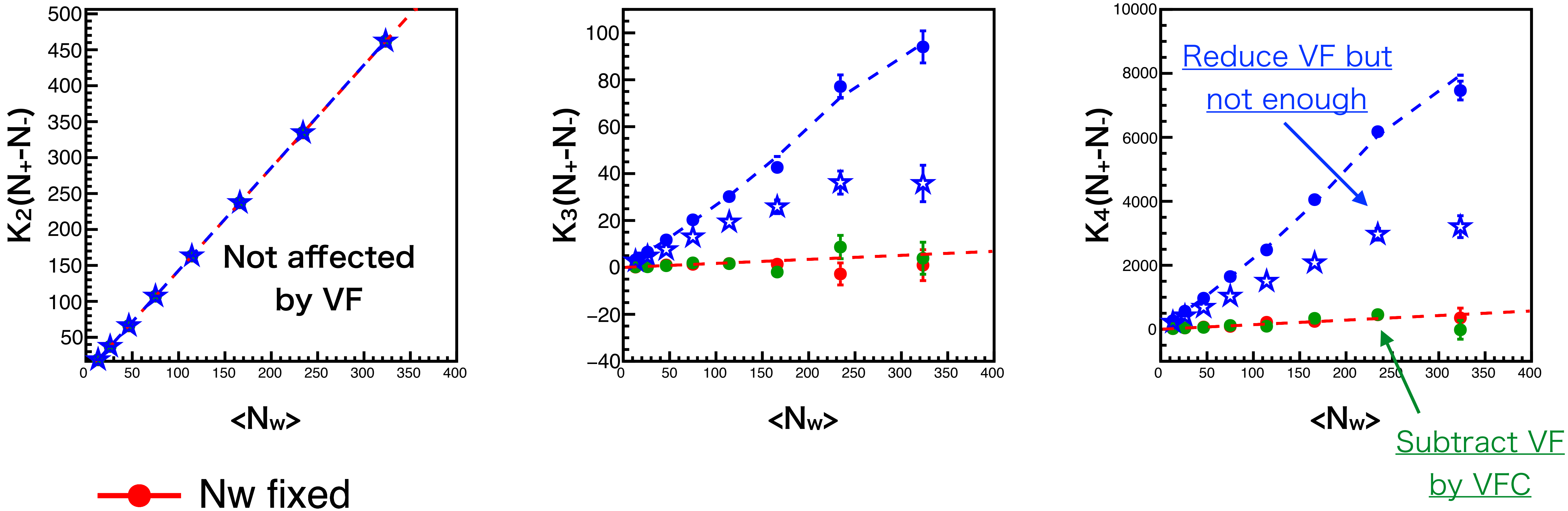


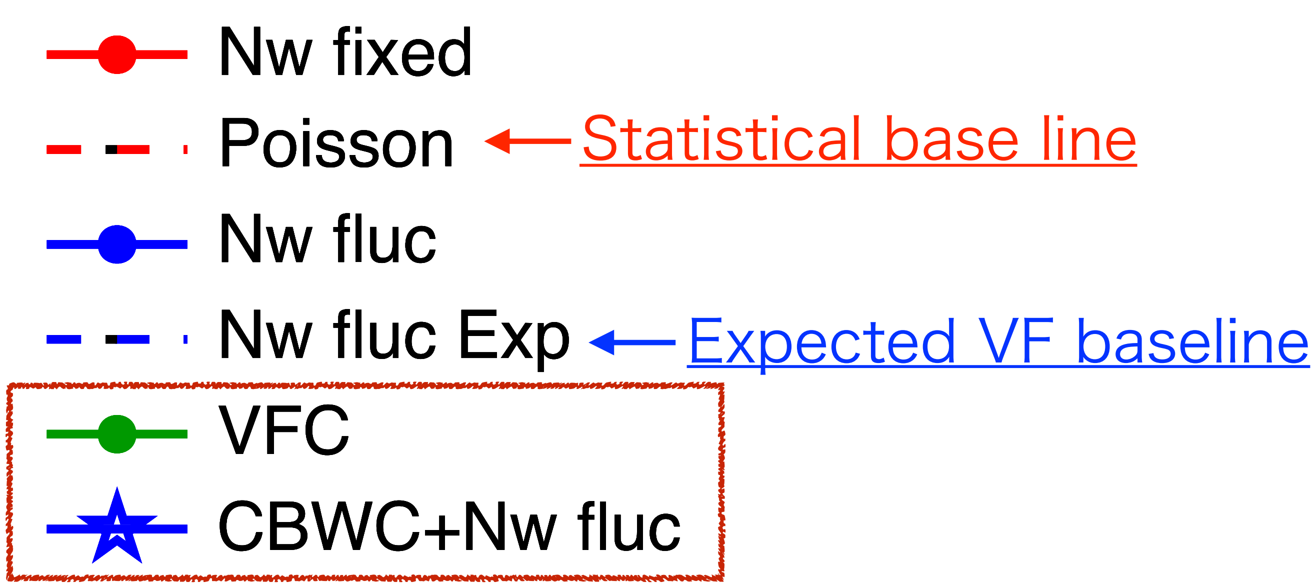
- Nw fixed
- Poisson ← Statistical base line
- Nw fluc
- Nw fluc Exp ← Expected VF baseline

Glauber : 500M
Poisson : 100M
bootstrap=100

- K_2 does not largely affected by participant fluctuation.
- Nw fixed cumulants are consistent with Poisson baseline.
- Nw fluc cumulants are enhanced by VF.

Toy model VFC check (10%step)





Glauber : 500M
Poisson : 100M
bootstrap=100

- VFC works well up to 4-th order cumulants.
- CBWC can reduce VF but not enough.

UrQMD model

- **VFC will be done by using UrQMD** model in which **IPP model might be broken**.
- In this model, **we can directly know N_w without Glauber fit**.

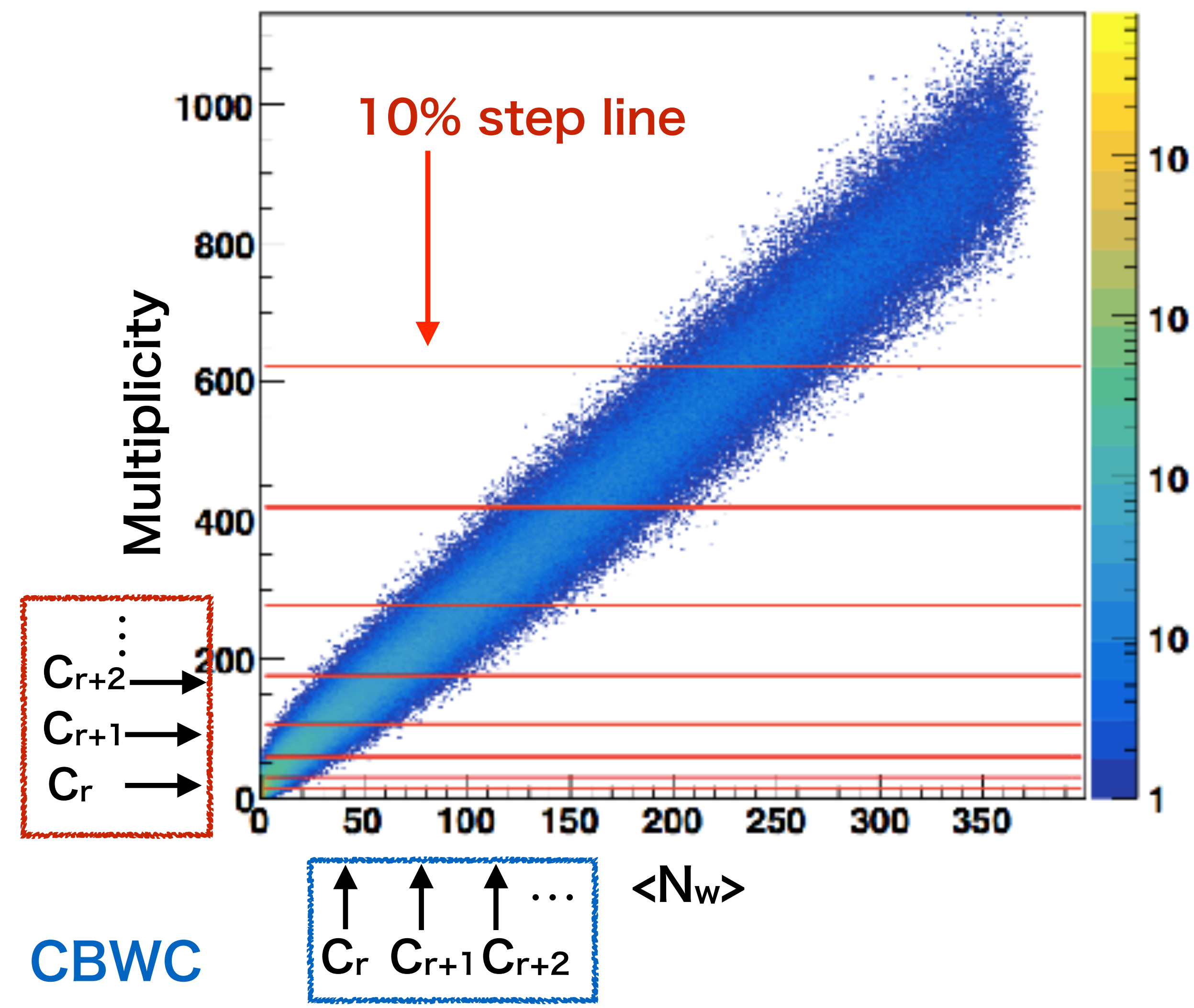
<u>How to know “true” cumulants?</u>

- In order to compare to CBWC and VFC results, we want to know “true” (no VF) cumulants.
- “True” cumulants can be calculated by CBWC using **N_w instead of multiplicities.**

$$C_n = \sum_{r} w_r C_{(n,r)}$$

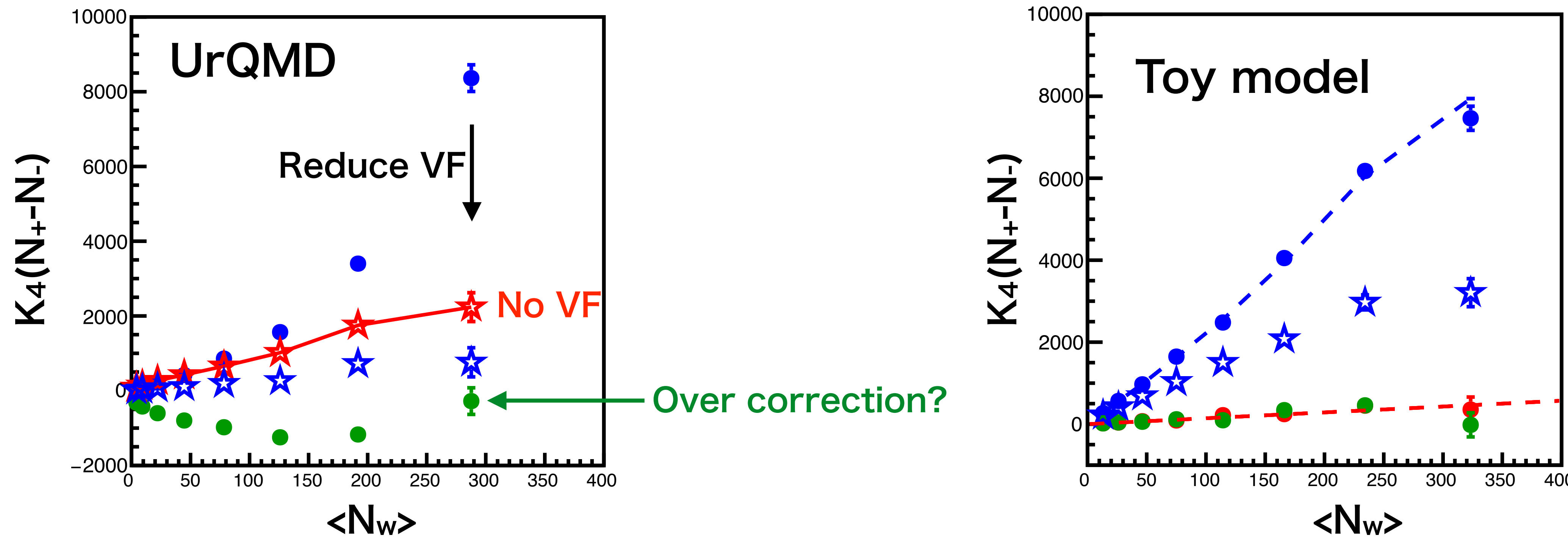
Cumulants in r-th Nw bins

Standatd CBWC



CBWC by Nw (no VF)

UrQMD results



Raw : No correction

VFC : VFC was applied

CBWC-Nw (no VF) : CBWC by Nw (true)

CBWC : CBWC by multiplicity (experimental CBWC)

- CBWC-Nw (no VF, true) cumulants are larger than CBWC results even though CBWC results include VF.
  - IPP assumption is expected to be broken in UrQMD.
- CBWC results are larger than VFC results and close to CBWC-Nw cumulants.
  - CBWC is better than VFC?

Summary2

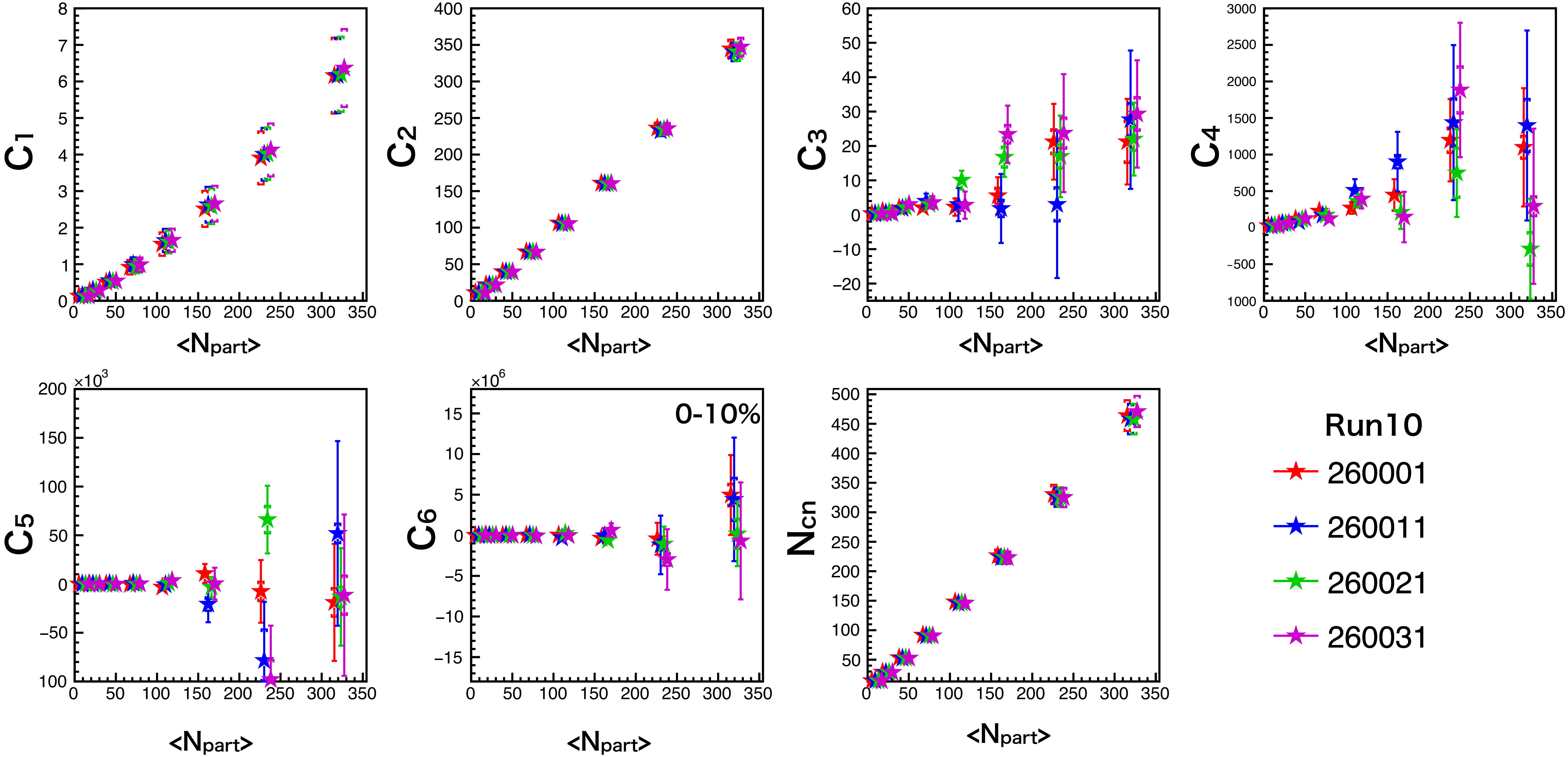
- It is important to eliminate VF in fluctuation analysis.
- 2.5% centrality division can reduce VF as well as CBWC.
- **Even though CBWC has applied, we can’t completely remove participant fluctuation under the IPP models**.
- However, **VFC does not works well in UrQMD model so IPP model is expected to be broken** in UrQMD.
- More detail studies for VFC are needed to apply VFC to real experiment.

Future plan

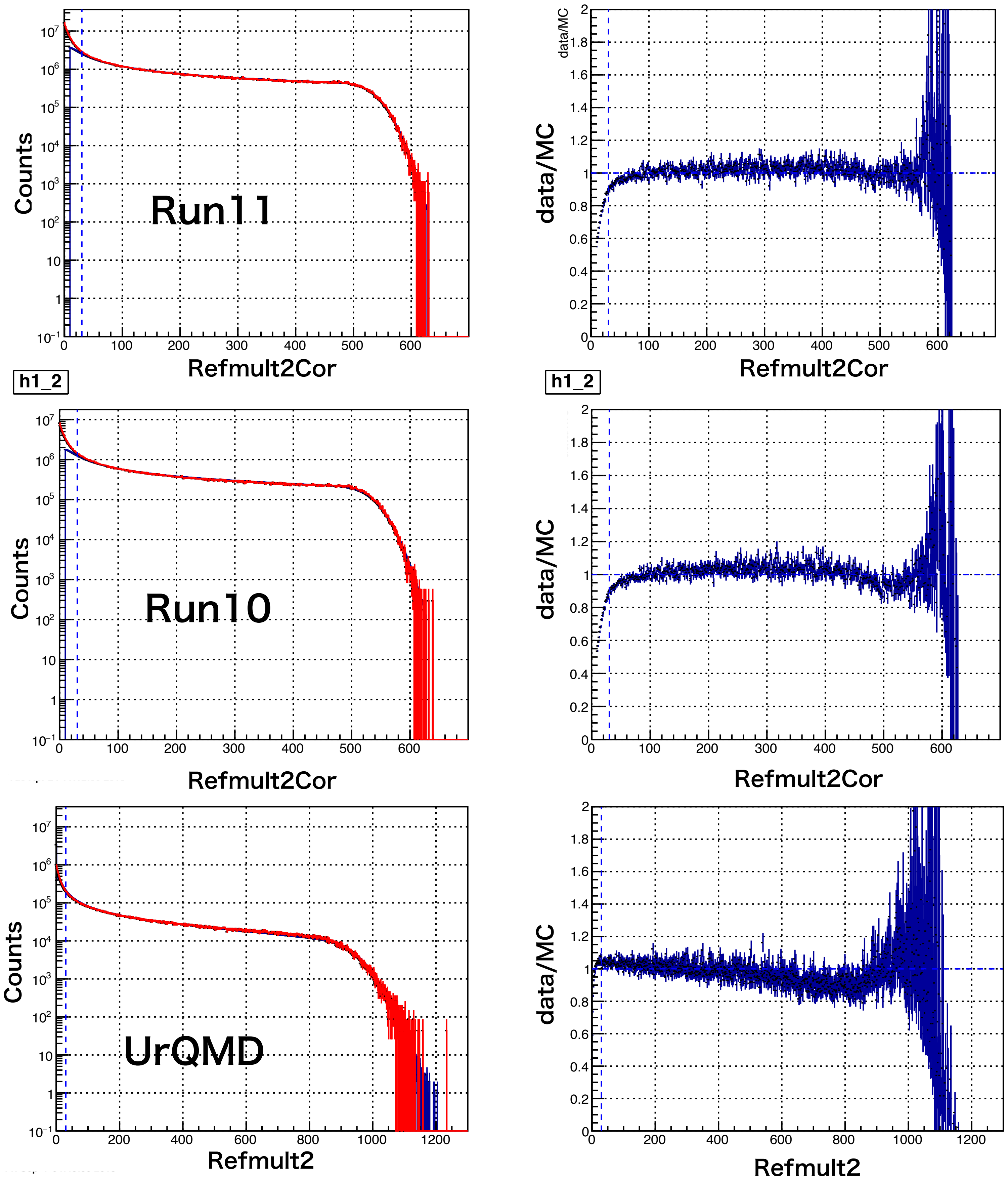
1. analyze 54 GeV net-charge
   - Recently net-proton C_6 of 54 GeV was reported.
2. Eta dependence
3. Develop VFC study
4. Apply VFC to experimental data

back up

Cumulants (Run10, trigger by trigger)



Glauber fit results



MC : 1M

Experiment

- n_{pp}=2.35
- x=0.13
- k=0.9
- efficiency
 =1-Refmult2Cor*(0.206/560)

n_{pp} : Mean number of generated particles from each source.
k : Parameters of NBD.
x : Parameter of two component model.
Number of source :
$(1-x)*(N_{part}/2.)+x*N_{coll}$

UrQMD

- n_{pp}=**2.90**
- x=0.13
- k=0.9
- efficiency=**1**

Centrality bins

8 bin

Centrality	bin	$\langle N_{part} \rangle$
0-10	406	323.41
10-20	295	234.16
20-30	204	166.14
30-40	134	114.3
40-50	82	75.09
50-60	47	46.32
60-70	24	26.14
70-80	11	13.15

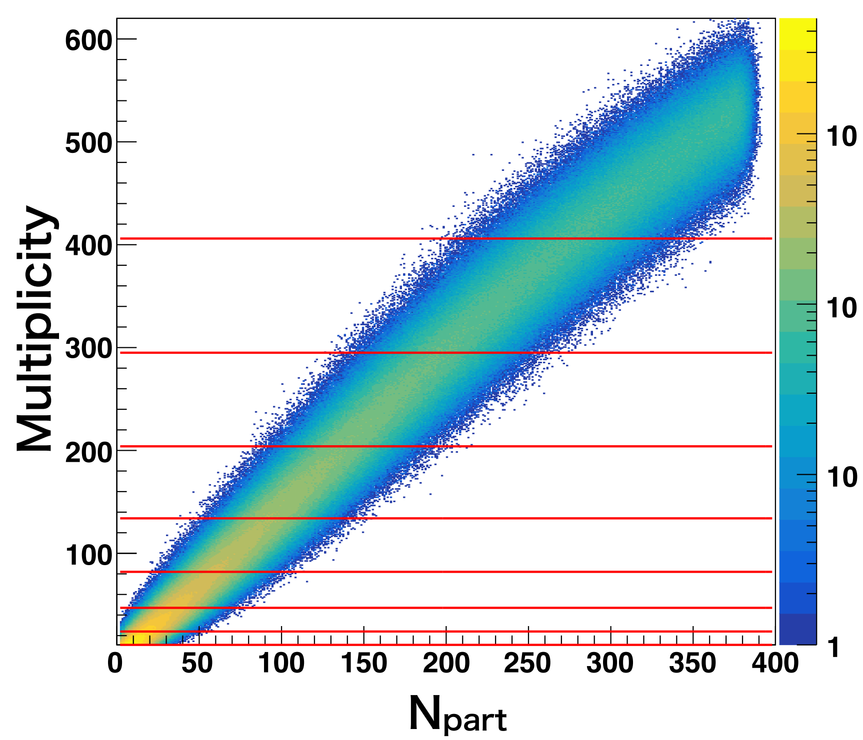
16 bin

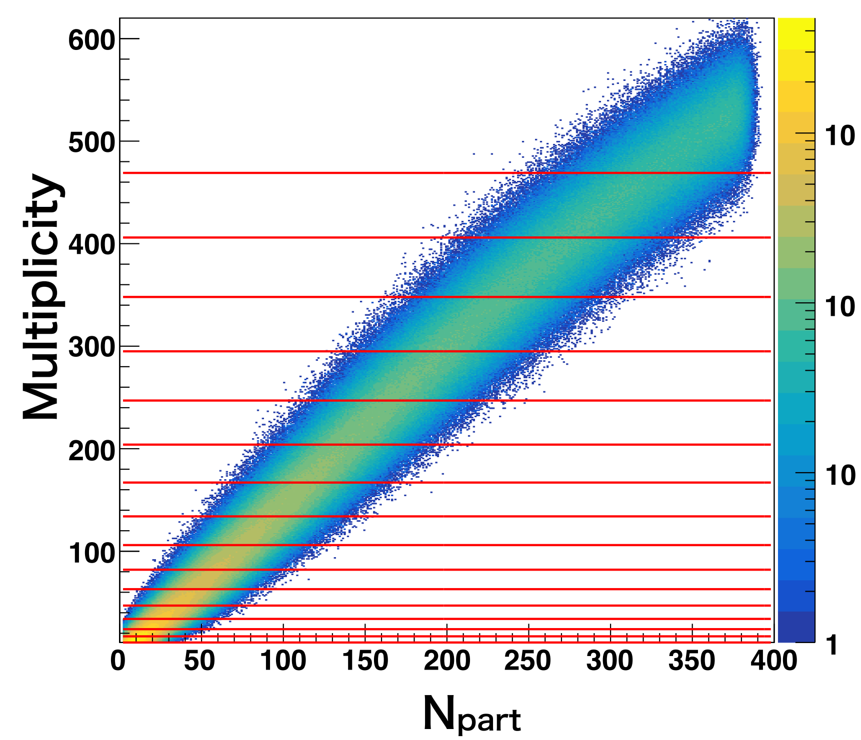
Centrality	bin	$\langle N_{part} \rangle$
0-5	469	347.47
5-10	406	298.9
10-15	348	253.64
15-20	295	214.79
20-25	247	180.99
25-30	204	151.34
30-35	167	125.669
35-40	134	103.2
40-45	106	83.61
45-50	82	66.65
50-55	63	52.38
55-60	47	40.46
60-65	34	30.28
65-70	24	22.0
70-75	17	15.73
75-80	11	10.93

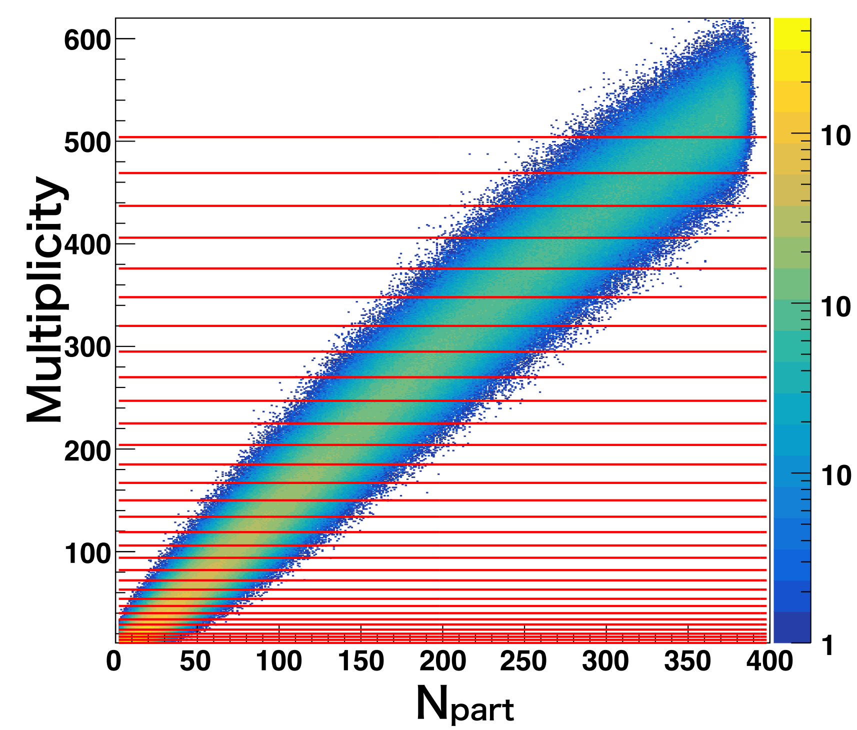
32 bin

Centrality	bin	$\langle N_{part} \rangle$
0-2.5	504	358.83
2.5-5	469	336.16
5-7.5	437	311.16
7.5-10	406	286.74
10-12.5	376	263.97
12.5-15	348	243.02
15-17.5	320	223.61
17.5-20	295	205.41
20-22.5	270	188.85
22.5-25	247	172.91
25-27.5	225	158.33
27.5-30	204	144.42
30-32.5	185	131.64
32.5-35	167	119.78
35-37.5	150	108.56
37.5-40	134	97.93
40-42.5	119	88.02

⋮







Factorial cumulant method

Example (the 1st-order net-charge fluctuation)

Cumulant (true)

$$\langle Q\rangle_c = \langle q_{(1,1)}\rangle_c,$$

$$q_{(r,s)} = q_{(a^r/p^s)} = \sum_{i=1}^{M}(a_i^r/p_i^s)n_i.$$

Number of phase space (M)

Number of particle (n_i)

Efficiency (p_i)

+1(positive charge) or -1(negative charge) (a_i)

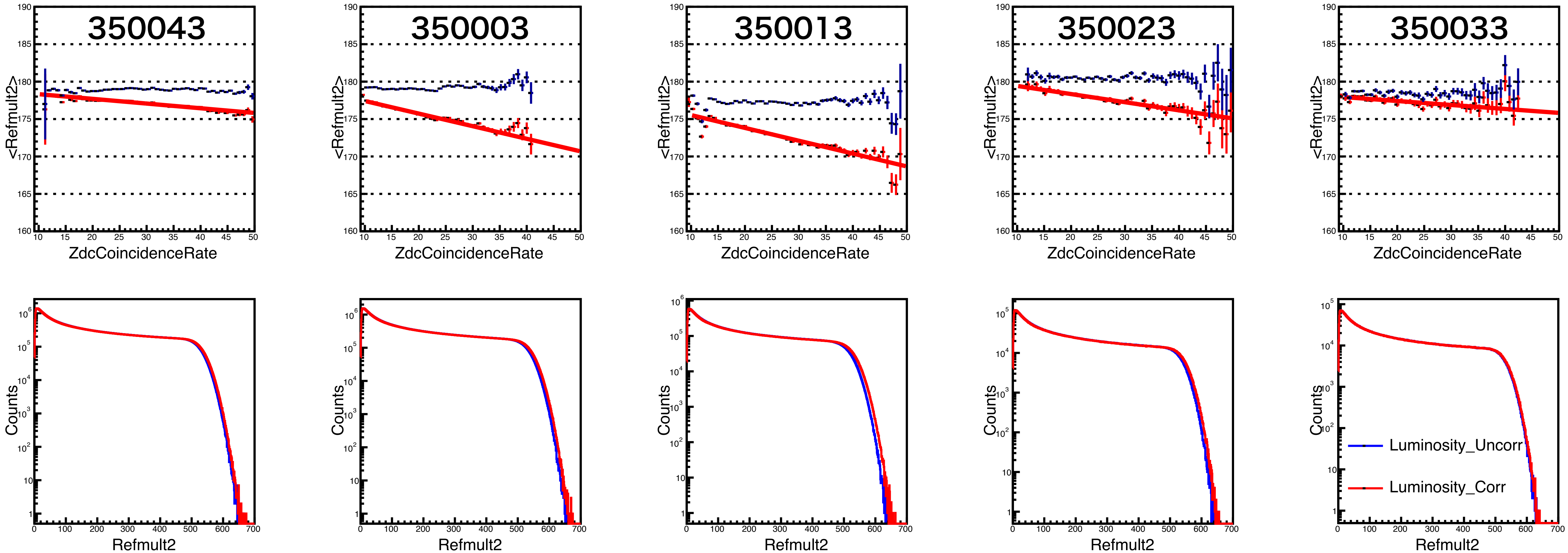
$$= \frac{n_{(\pi+,lowpt)}}{p_{(\pi+,lowpt)}} + \frac{n_{(\pi+,highpt)}}{p_{(\pi+,highpt)}} - \frac{n_{(K+,lowpt)}}{p_{(K+,lowpt)}} + \frac{n_{(K-,highpt)}}{p_{(K+,highpt)}} + \frac{n_{(pbar,lowpt)}}{p_{(pbar,lowpt)}} + \frac{n_{(p,highpt)}}{p_{(p,highpt)}}$$

$$- \frac{n_{(\pi-,lowpt)}}{p_{(\pi-,lowpt)}} - \frac{n_{(\pi-,highpt)}}{p_{(\pi-,highpt)}} - \frac{n_{(K-,lowpt)}}{p_{(K-,lowpt)}} - \frac{n_{(K-,highpt)}}{p_{(K-,highpt)}} - \frac{n_{(pbar,lowpt)}}{p_{(pbar,lowpt)}} - \frac{n_{(pbar,highpt)}}{p_{(pbar,highpt)}}$$

- 2(charge)*3(πKp)*2(highpt,lowpt)=12 efficiency bins are used at net-charge fluctuation analysis.

Correction parameter of Refmult2 (Run11)

- Refmult2 was used to determine centrality at net-charge analysis. (multiplicity of $0.5<|\eta|<1$)
- Refmult2 depend on z-vertex and luminosity and trigger ID.
- Calculate z-vertex and luminosity correction parameter for each trigger ID.

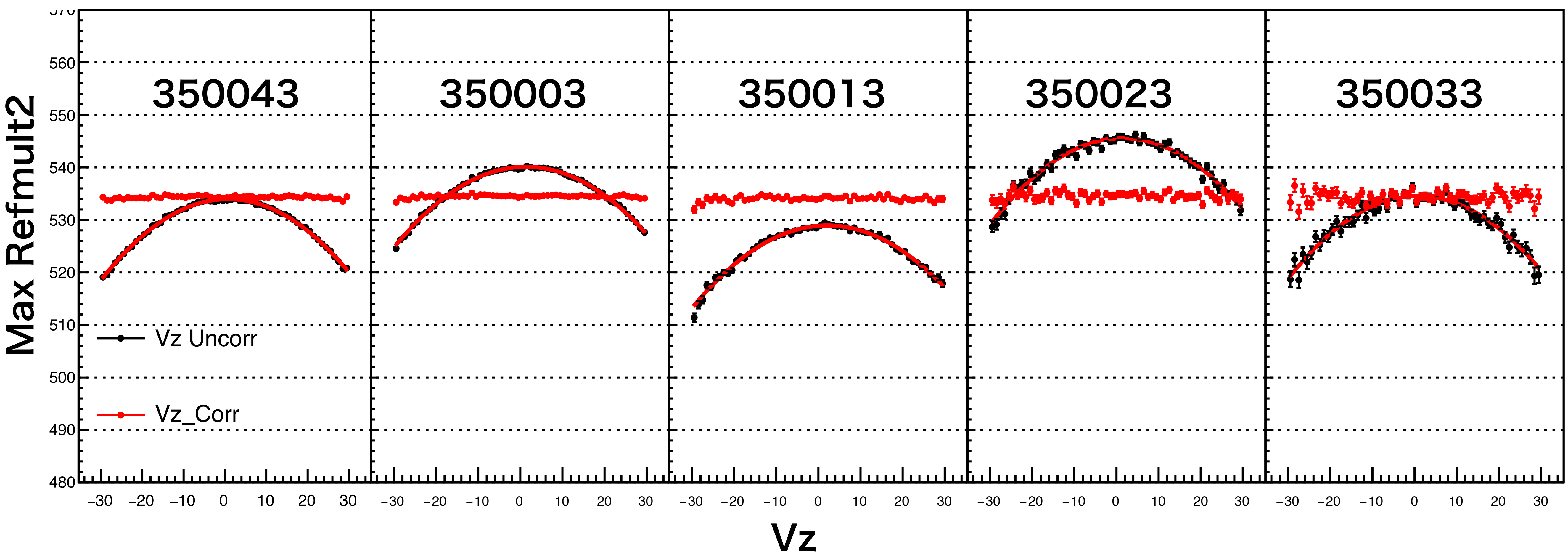


- Fit by pol1 ([0]+[1]x)

$$\text{correction\_luminosity} = \frac{1}{1 + [1]/[0] * \text{ZdcCoincidenceRate}}$$

luminosity correction factor

Z-vertex correction (Run11)



- Fit by pol2 ([0]+[1]x+[2]x^2)

$$\text{Hovno} = \frac{[0] + \text{par}}{[0] + [1] * \text{x} + [2] * \text{x}^2}$$

([0] of 350043) - [0] Scaled to 350043

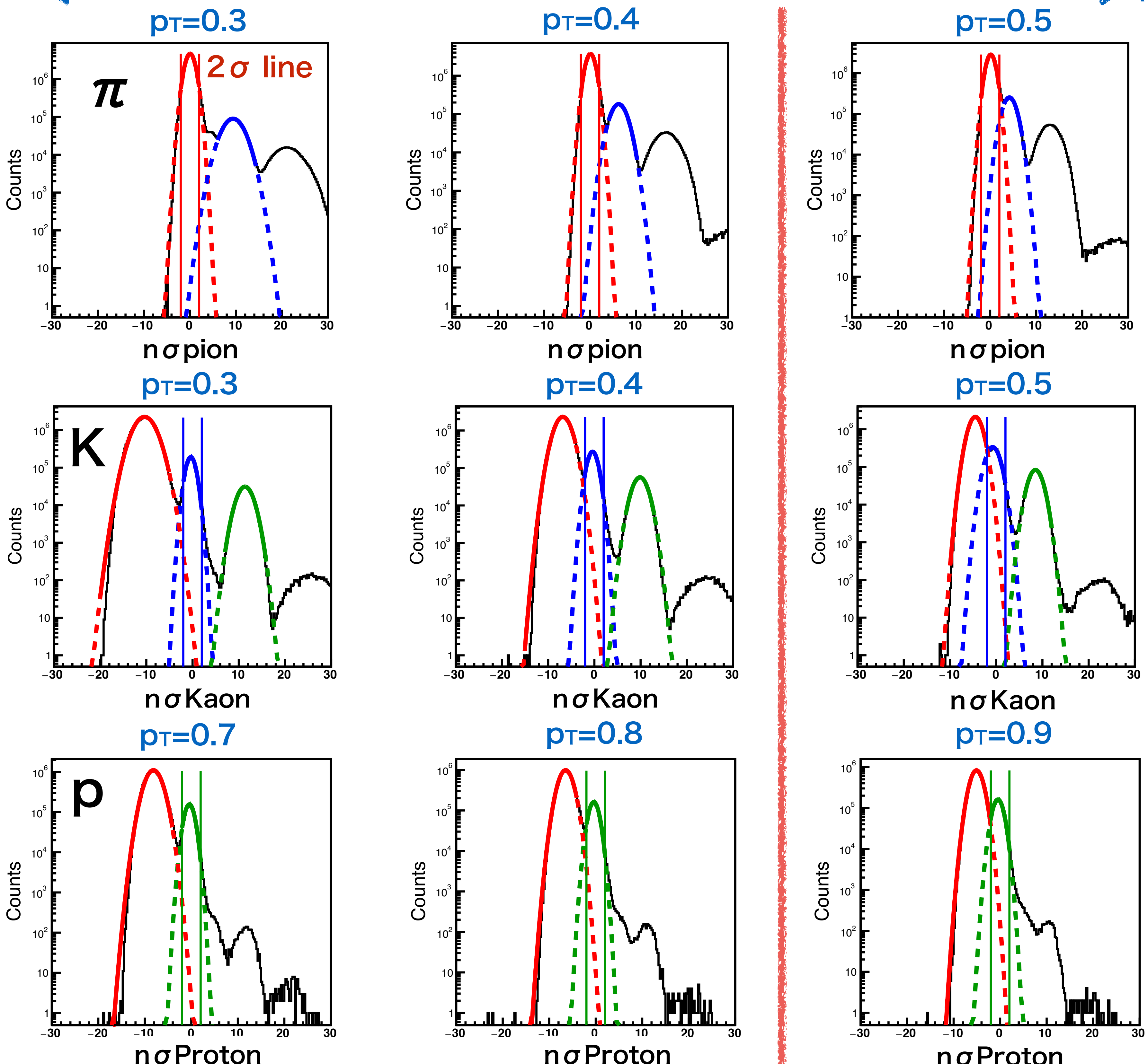
Vz correction factor

Refmult2Corr = (Refmult2+gRandom->Rndm())*correction_lunimosity*Hovno

Corrected Refmult2	Raw Remult2	0 to 1 random number to remove kink structure	Luminosity correction factor

Purity of π, K, p (TPC)

Low pT ⟷ High pT



Current boarder line

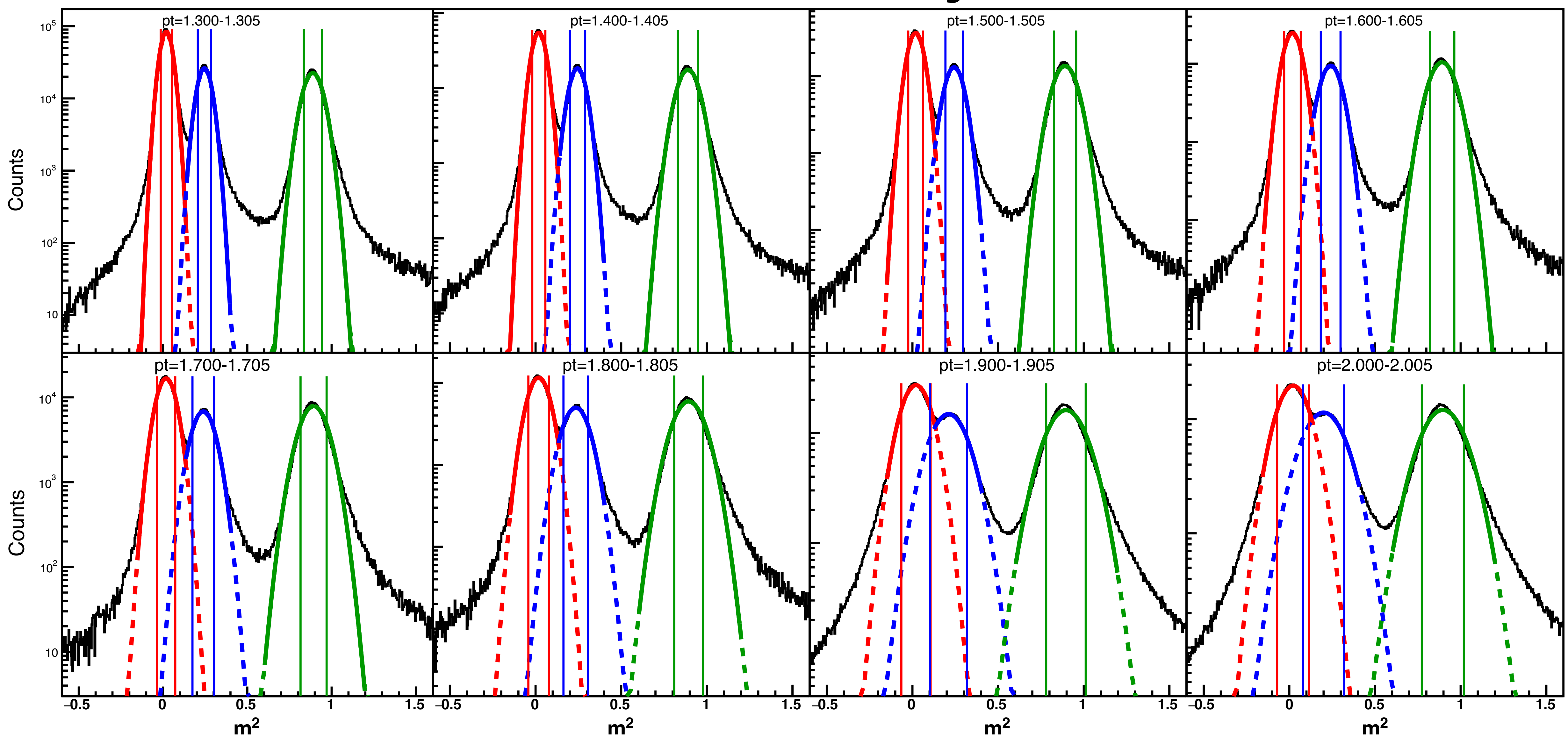
Purity	π	K
pT=0.4	99.97%	99.03%
pT=0.5	99.34%	85.04%
pT=0.6	90.11%	

Purity	p
pT=0.7	99.94%
pT=0.8	99.43%
pT=0.9	95.61%

- pT range of pion can be expand to 0.5 GeV/c. (before : 0.4GeV/c)
- Current pT range of Kaon and proton seems good. (99% purity)

Current boarder line

TOF Purity



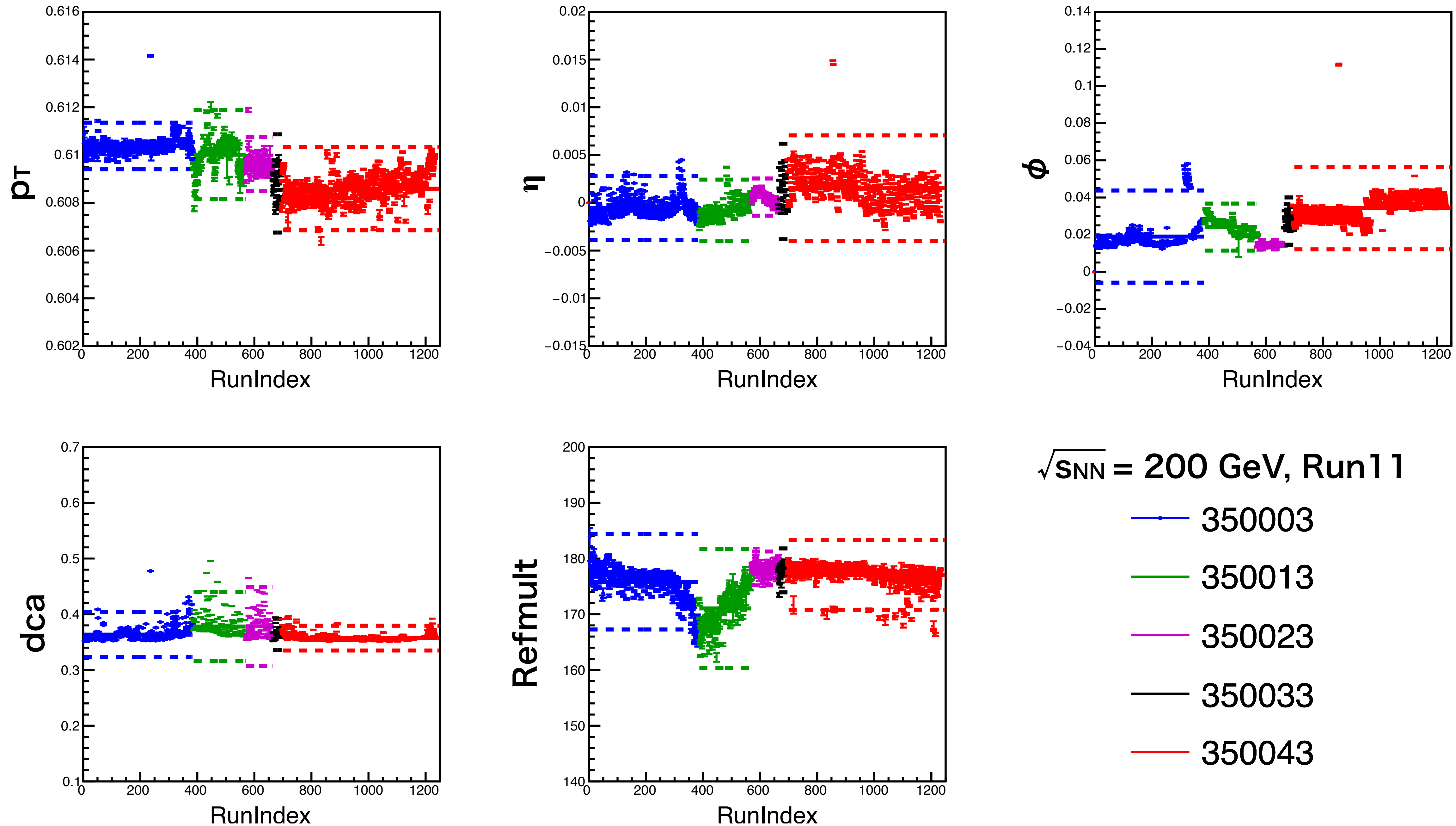
Purity	π	K
p_T=1.5	99%	100%
p_T=1.6	98%	99%
p_T=1.7	97%	97%
p_T=1.8	95%	96%
p_T=1.9	82%	88%

Border line

TOF	m^2
π	$-0.15 < m^2 < 0.14$
K	$0.14 < m^2 < 0.4$
p	$0.6 < m^2 < 1.2$

- Gaussian fit has been done but shape of m^2 distribution does not follow Gaussian exactly.
Please note this purity results are rough estimation.

RunQA

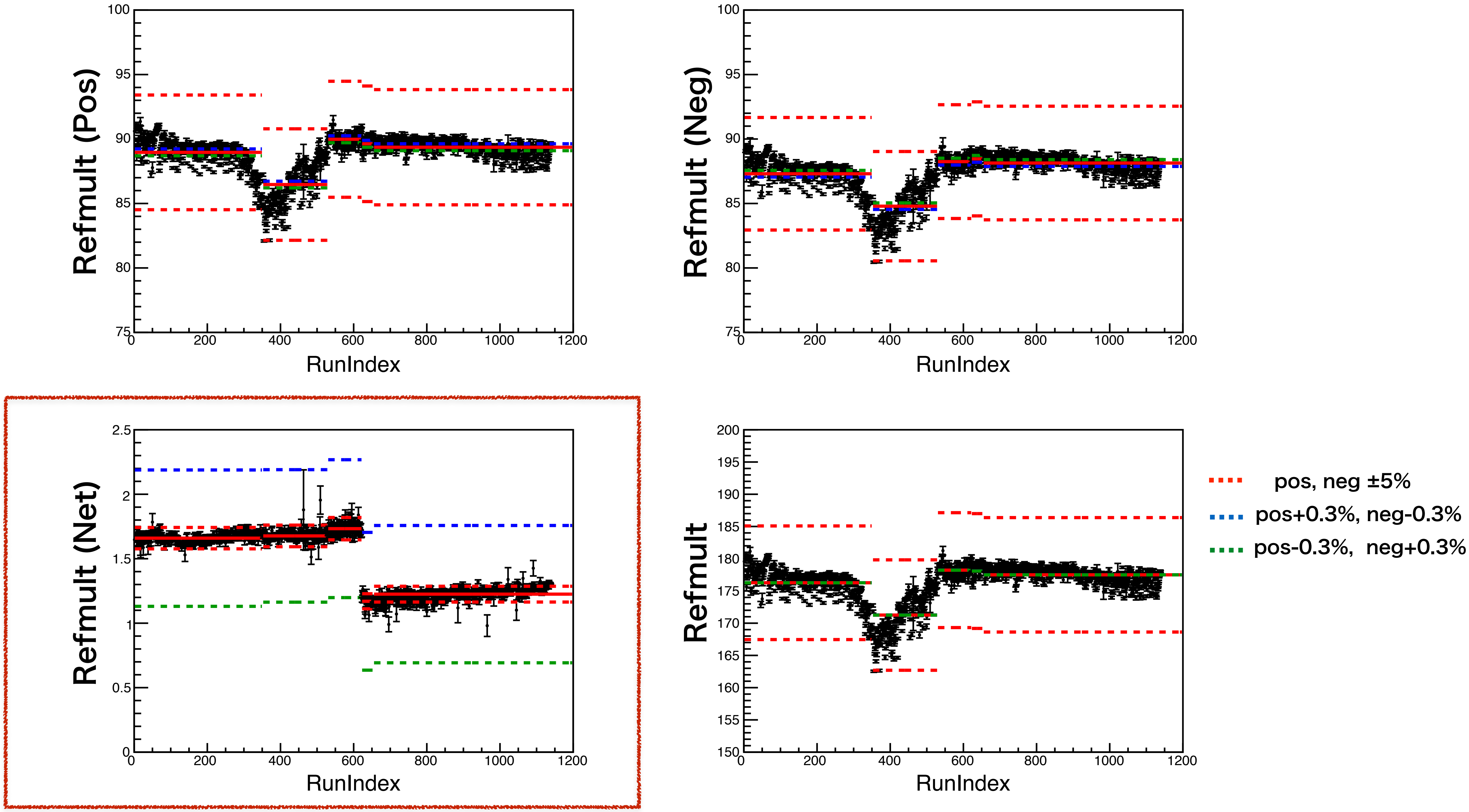


pT
RunIndex
η
RunIndex
ϕ
RunIndex
dca
RunIndex
Refmult
RunIndex
√SNN = 200 GeV, Run11
350003
350013
350023
350033
350043

Run by run check (Run11, 350043)

Only Good runs are shown

Refmult(Pos)+Refmult(Neg)=Refmult

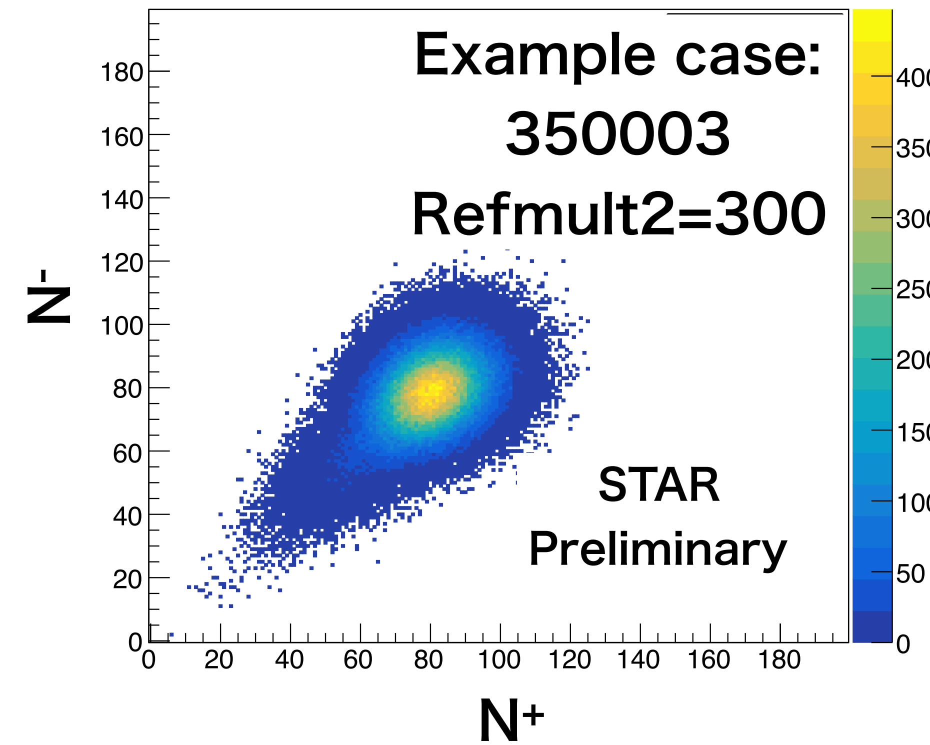


- When we change (pos,neg)=(+0.3%,-0.3%) and (+0.3%,-0.3%), run by run Refmult (Net) are fluctuate within blue and green line.
- **(pos+0.3%, neg-0.3%) and (pos-0.3%, neg+0.3%) seem enough to estimate systematic uncertainty of net-charge.**

NBD check

$$C_1 = \mu$$
$$C_2 = \mu\epsilon$$
$$C_3 = \mu\epsilon(2\epsilon - 1)$$
$$C_4 = \mu\epsilon(6\epsilon^2 - 6\epsilon + 1)$$
$$C_5 = \mu\epsilon(2\epsilon - 1)(12\epsilon^2 - 12\epsilon + 1)$$
$$C_6 = \mu\epsilon(120\epsilon^4 - 240\epsilon^3 + 150\epsilon^2 - 30\epsilon + 1)$$

$$C_n^{net-c} = C_n^+ + (-1)^n C_n^-$$



For example : 350003, Refmult2=300,

Efficiency Uncorrected case

C_1^+=79.53	C_1^-=77.81
C_2^+=122.1	C_2^-=117.93
$\varepsilon^+=C_2/C_1$=1.53	ε^-=1.51

used as NBD parameters.

C_3^+=253	C_3^+=240
C_4^+=724	C_4^+=671
C_5^+=2746	C_5^+=2486
C_6^+=13027	C_6^+=11525

NBD	Poisson
C_1^{net}=1.72	C_1^{net}=1.72
C_2^{net}=240	C_2^{net}=157
C_3^{net}=13.3	C_3^{net}=1.72
C_4^{net}=1395	C_4^{net}=157
C_5^{net}=260	C_5^{net}=1.72
C_6^{net}=24552	C_6^{net}=157

Odd: $C_1^+ - C_1^-$
Even: $C_1^+ + C_1^-$

- **NBD baseline is larger than Poisson because of large ε.**
- **If ε=1, NBD and Poisson baseline are exactly the same.**

NBD check

For example : 350003, Refmult2=300, Efficiency Corrected case

$C_1^+=139.44$	$C_1^-=136.54$
$C_2^+=265.5$	$C_2^-=256.26$
$\varepsilon^+=C_2/C_1=1.90$	$\varepsilon^-=1.87$

used as NBD parameters.

$C_3^+=745$	$C_3^+=705$
$C_4^+=3007$	$C_4^+=2786$
$C_5^+=16145$	$C_5^+=14640$
$C_6^+=108378$	$C_6^+=96183$

N^+

NBD	Poisson
$C_1^{net}=1.72$	$C_1^{net}=1.72$
$C_2^{net}=521$	$C_2^{net}=157$
$C_3^{net}=40.0$	$C_3^{net}=1.72$
$C_4^{net}=5794$	$C_4^{net}=157$
$C_5^{net}=1505$	$C_5^{net}=1.72$
$C_6^{net}=204561$	$C_6^{net}=157$

Odd: $C_1^+-C_1^-$

Even: $C_1^++C_1^-$

- **NBD baseline is larger than Poisson because of large ε.**
- **If $\varepsilon=1$, NBD and Poisson baseline are exactly the same.**

Error propagation

$$\boxed{A \pm \delta A} \qquad \delta A \sim \frac{1}{\sqrt{N_A}} \qquad\qquad \boxed{B \pm \delta B} \qquad \delta B \sim \frac{1}{\sqrt{N_B}}$$

① Suppose 2 different distribution

Sum: $A + B$ $\qquad E_{(A+B)} = \sqrt{\delta A^2 + \delta B^2}$

Wight $\quad w_A = \dfrac{N_A}{N_A + N_B} \qquad w_B = \dfrac{N_B}{N_A + N_B}$

Wighted sum: $w_A A + w_B B$ $\qquad E_{(w_A A + w_B B)} = \sqrt{(w_A \delta A)^2 + (w_B \delta B)^2}$

$$= \frac{\sqrt{N_A^2 \delta A^2 + N_B^2 \delta B^2}}{N_A + N_B} \sim \frac{1}{\sqrt{N_A + N_B}}$$

② Suppose 2 different distribution follow the same function

$$A = B, \delta A = \delta B$$

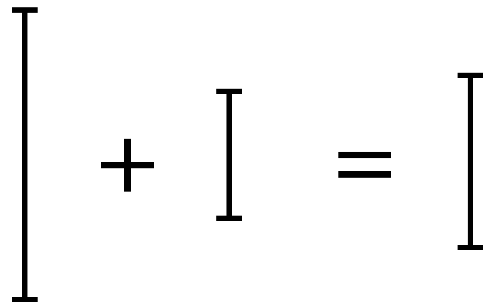
Wighted sum: $(w_A + w_B)A = A$ $\qquad E_{(w_A A + w_B B)} = \sqrt{(w_A \delta A)^2 + (w_B \delta B)^2}$

$$= \frac{\delta A}{\sqrt{N_A + N_B}}$$

Error propagation

$A \pm \delta A$ $B \pm \delta B$

① Suppose 2 different distribution



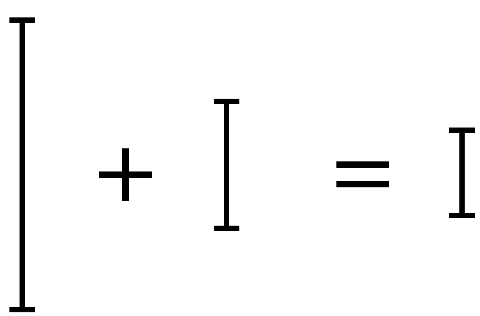
E_1 E_2 $E_3 = \dfrac{\sqrt{N_A^2 \delta A^2 + N_B^2 \delta B^2}}{N_A + N_B} \sim \dfrac{1}{\sqrt{N_A + N_B}}$

Example: CBWC, rebin centrality

E_3 proportional to $1/\sqrt{N_A+N_B}$

E_3 does not have to be smaller than E_2

② Suppose 2 different distribution follow the same function



E_1 E_2 $E_3 = \dfrac{\delta A}{\sqrt{N_A + N_B}}$

Example: Merge different trigger ID

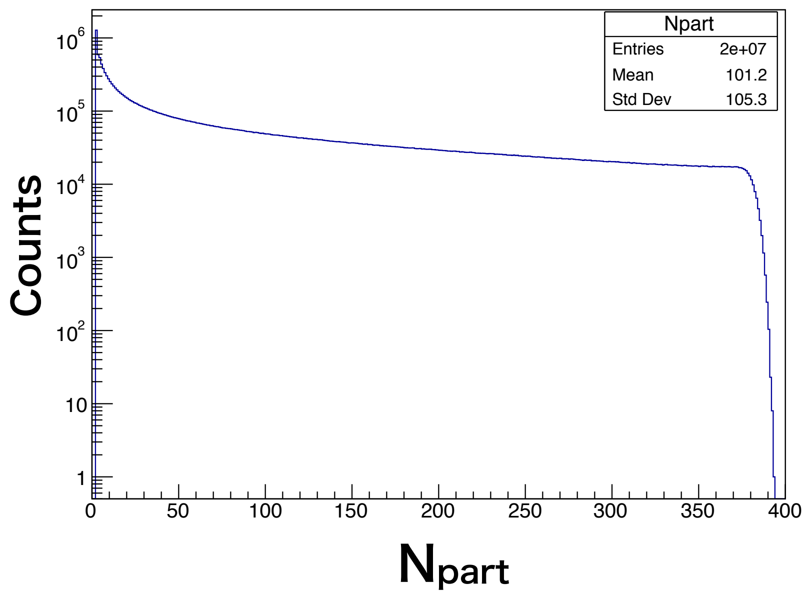
E_3 proportional to $1/\sqrt{N_A+N_B}$

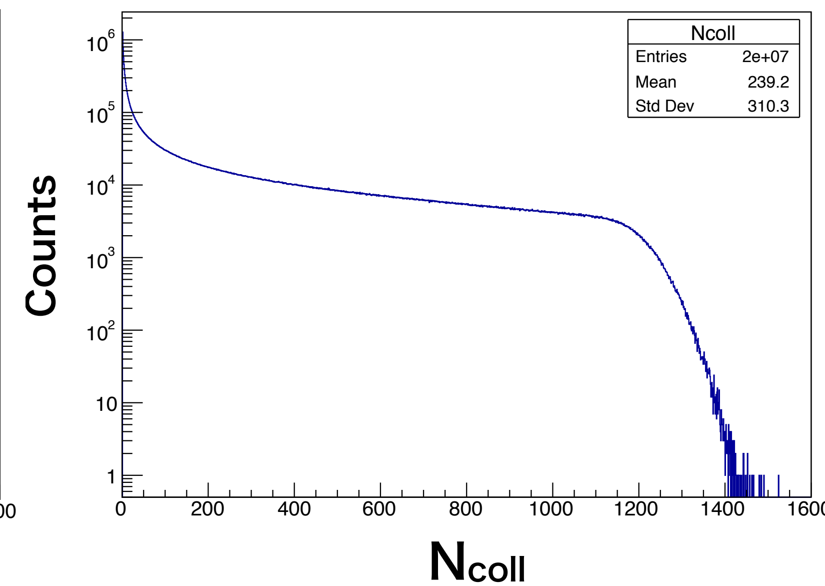
E_3 must be smaller than E_2

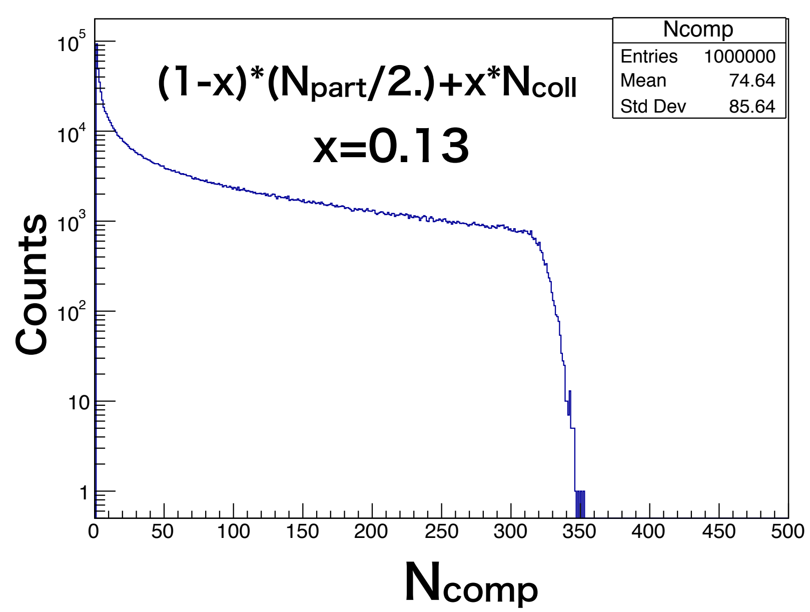
Glauber model

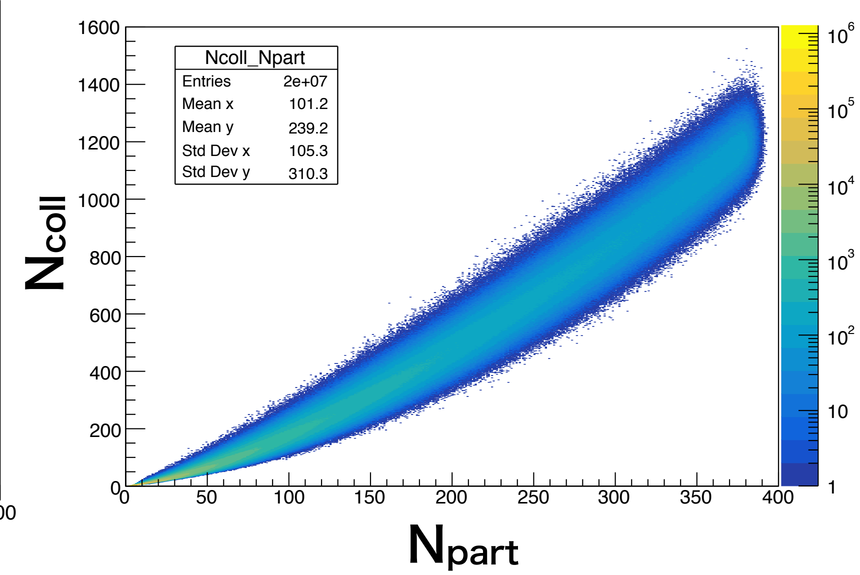
- Number of Nucleon : **197**
- Width (Parameter of Wood-saxon) : **0.535**
- Radius : **6.4**
- σ : **4.2**

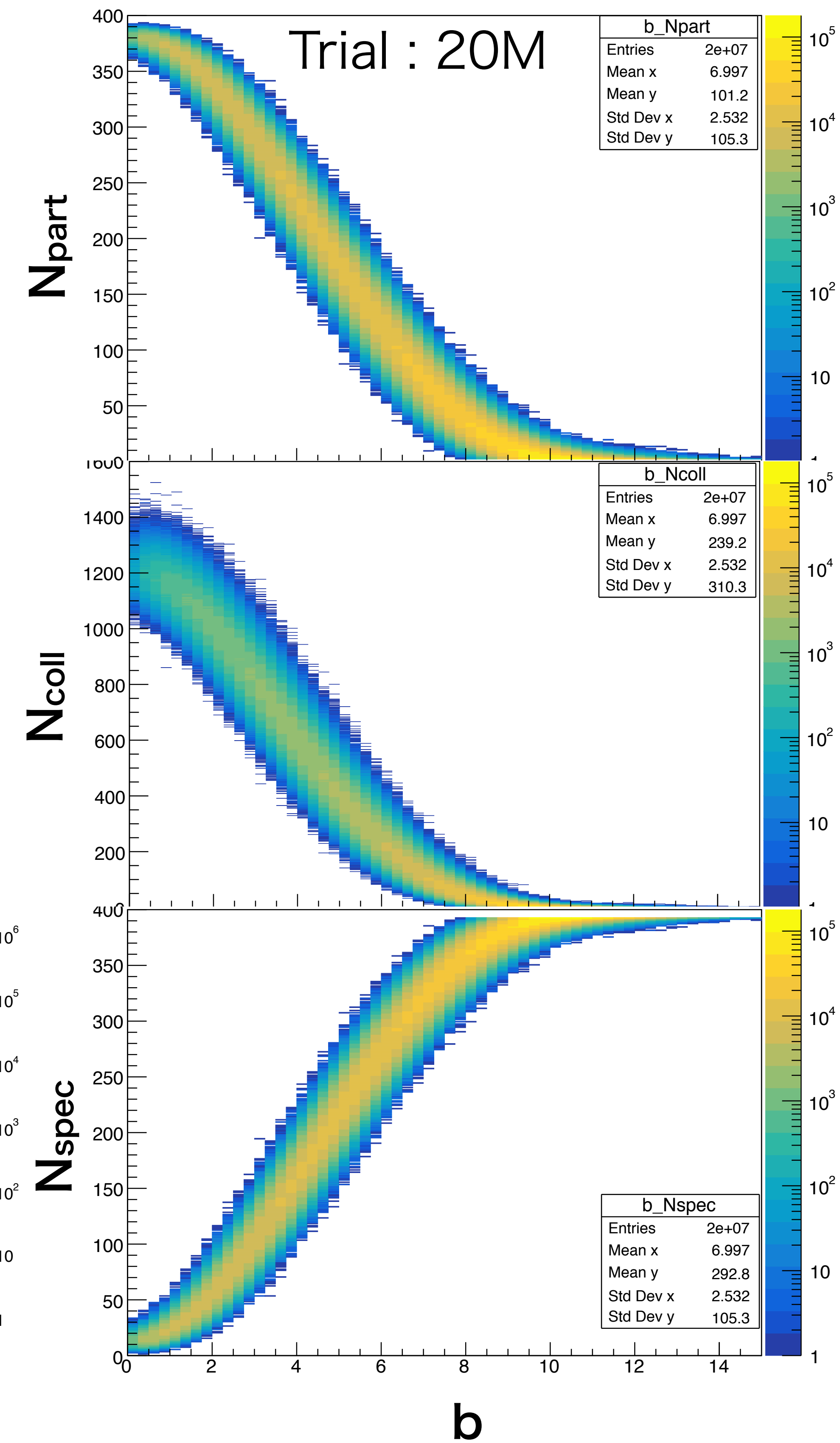
gRandom->Rndm() is used.
Linear congenital generator.
Periodicity = 2^{31}





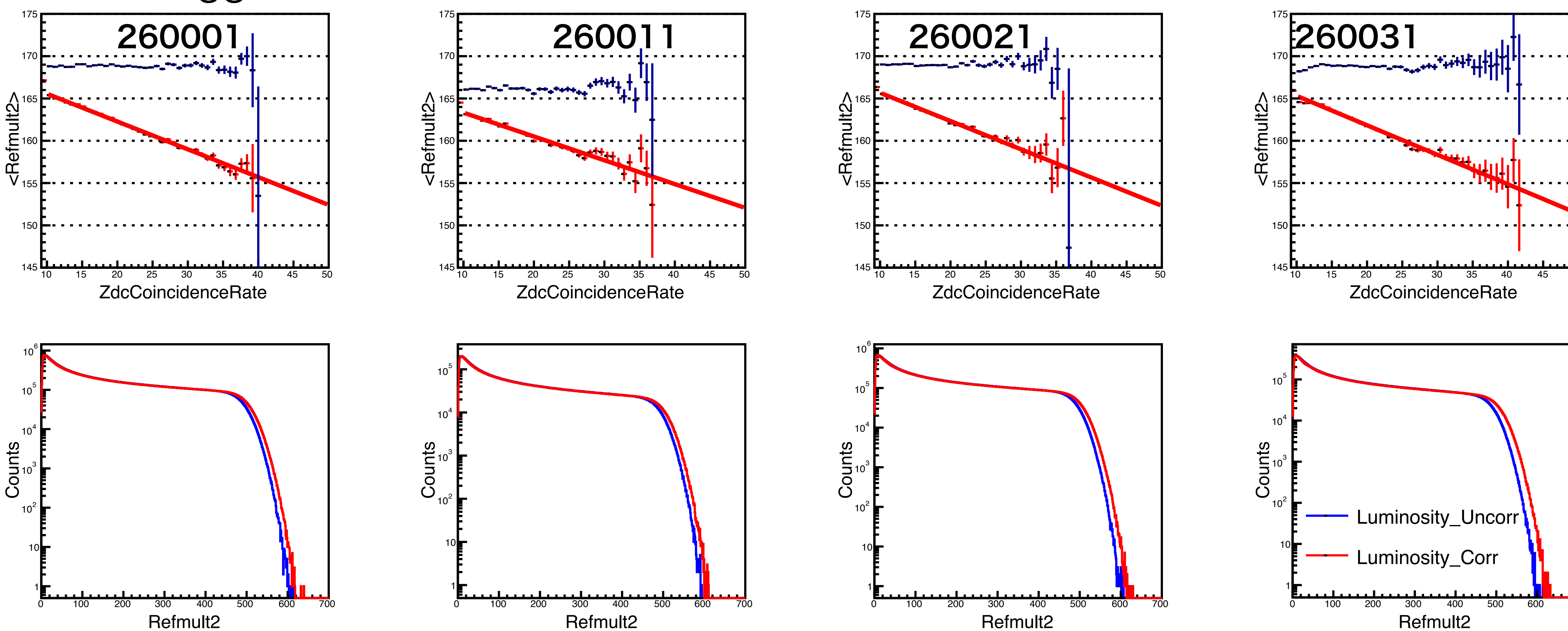






Correction parameter of Refmult2 (Run10)

- Refmult2 was used to determine centrality at net-charge analysis. (multiplicity of $0.5<|\eta|<1$)
- Refmult2 depend on z-vertex and luminosity and trigger ID.
- Calculate z-vertex and luminosity correction parameter for each trigger ID.



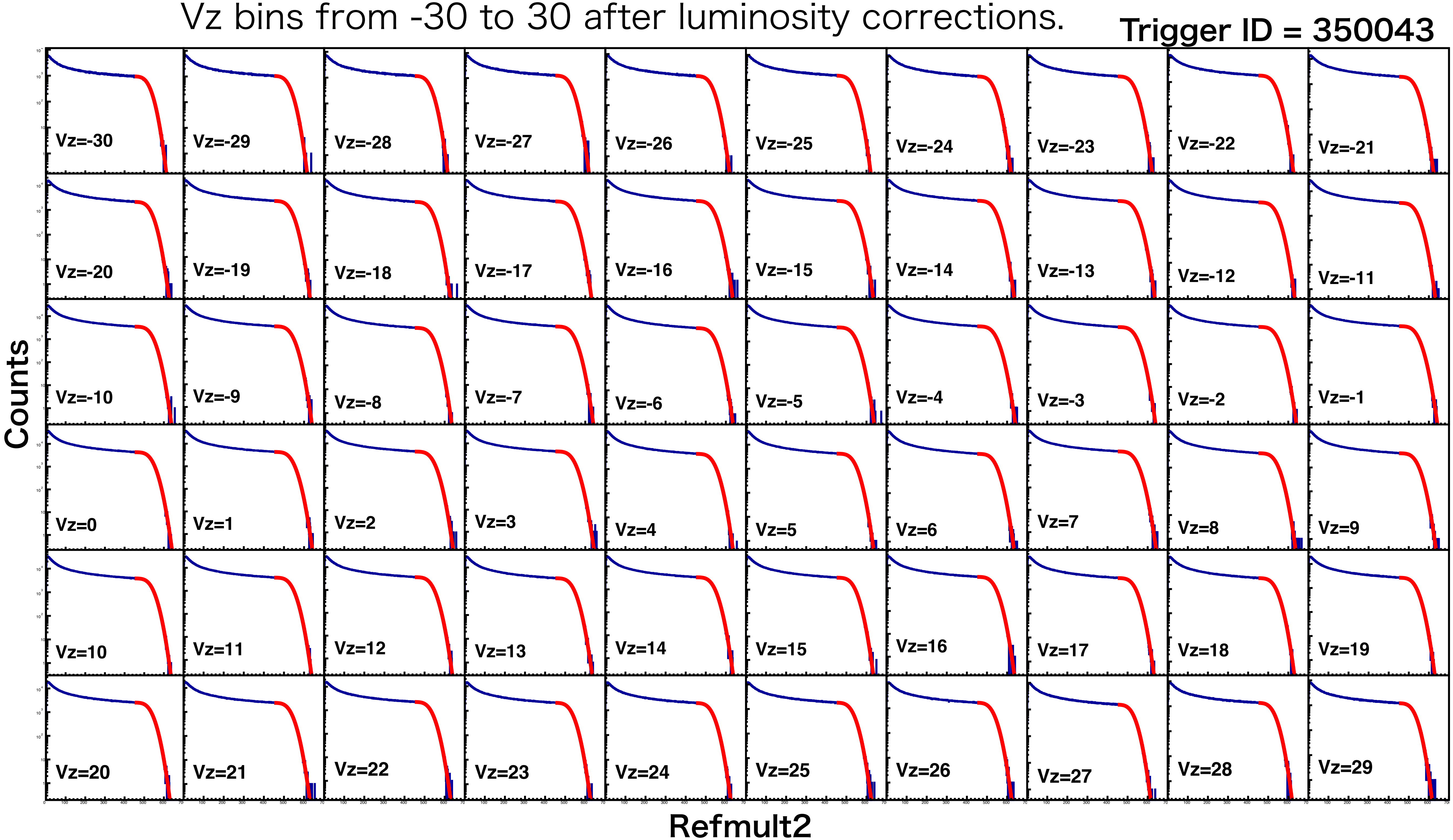
- Fit by pol1 ([0]+[1]x)

$$\text{correction\_luminosity} = \frac{1}{1 + [1]/[0] * \text{ZdcCoincidenceRate}}$$

luminosity correction factor

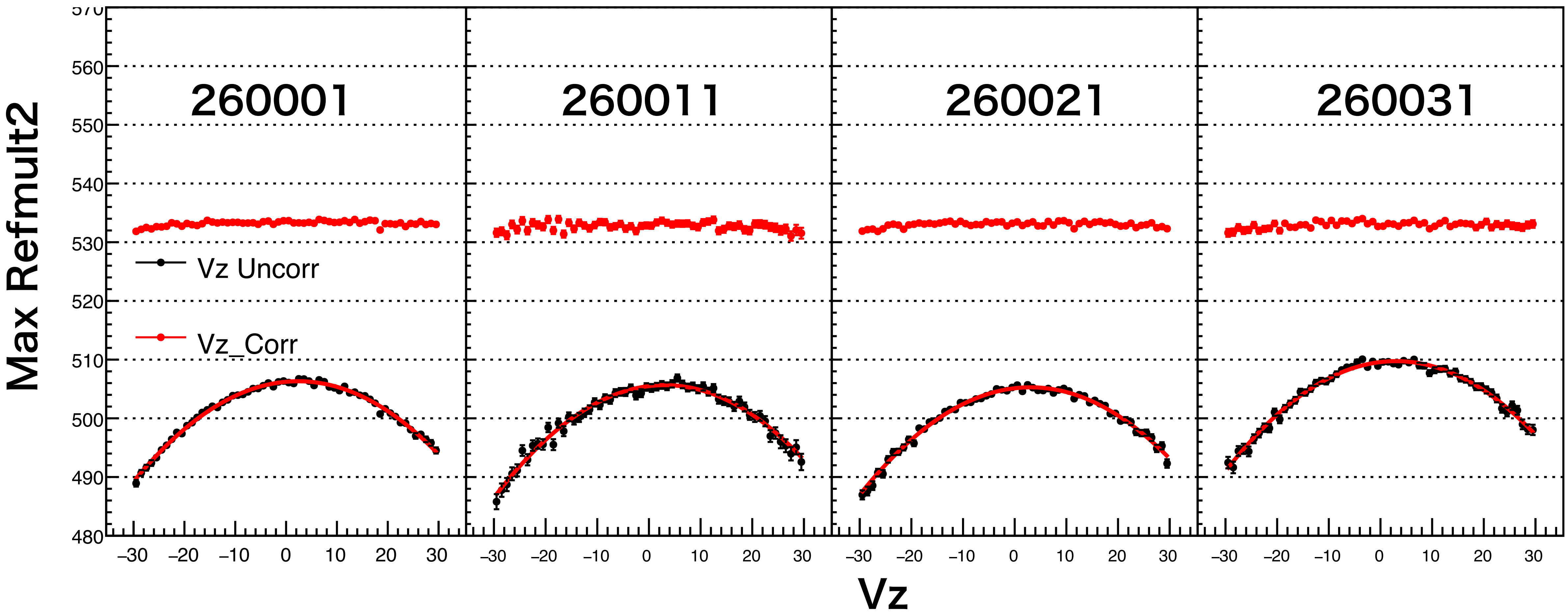
Refmult2 distributions of each Vz

- Maxrefmult2 was measured that is defined by error function for each Vz bins from -30 to 30 after luminosity corrections.



- Fit by ([1]*TMath::Erf(-[1]*(x-[2]))+[0]) → **[2] : Maxrefmult2**

Z-vertex correction (Run10)



- Fit by pol2 ([0]+[1]x+[2]x^2)

$$\text{Hovno} = \frac{[0] + \text{par}}{[0] + [1] * \text{x} + [2] * \text{x}^2}$$

([0] of 350043) - [0]
Scaled to 350043

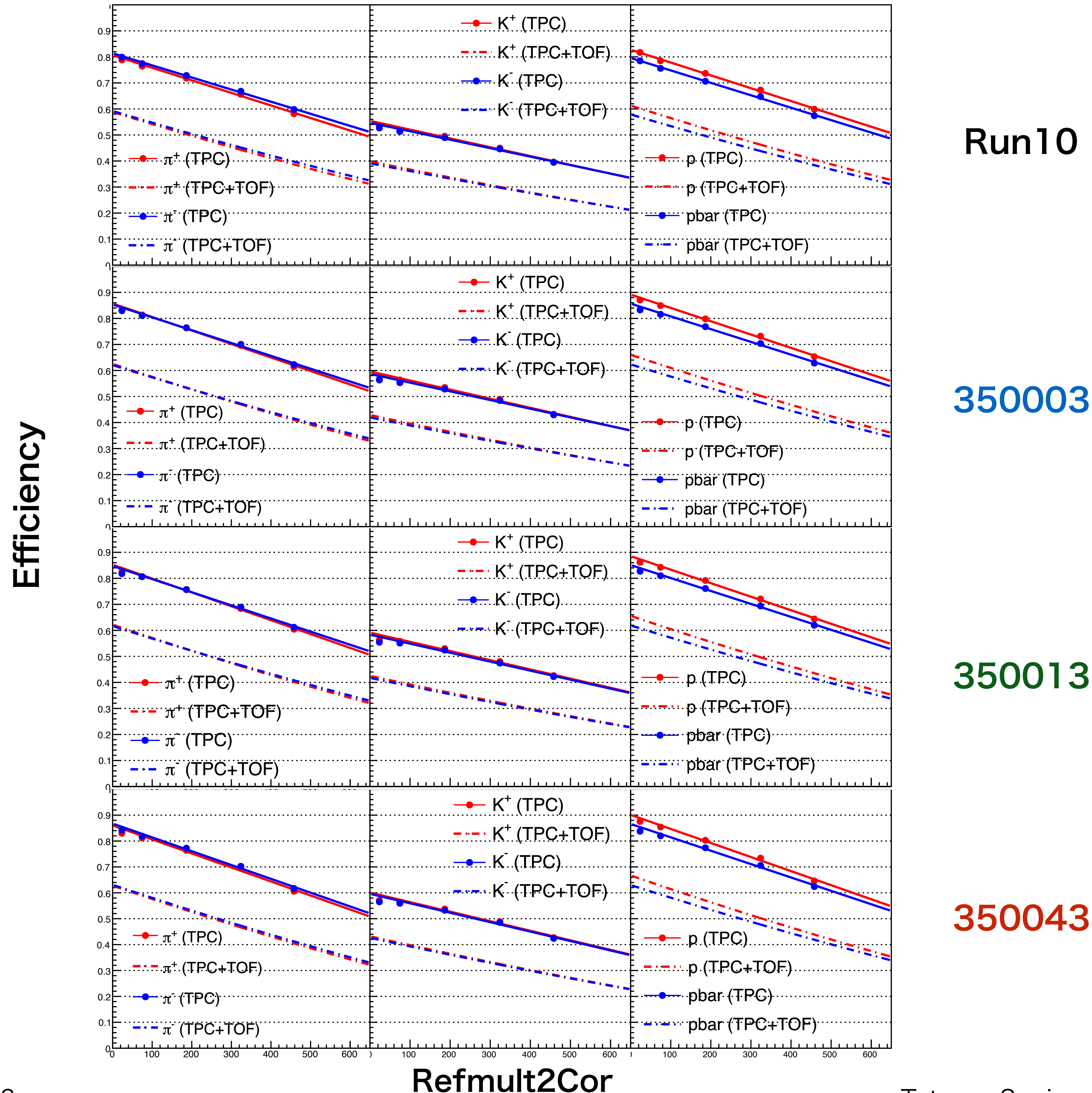
Vz correction factor

Refmult2Corr = (Refmult2+gRandom->Rndm())*correction_lunimosity*Hovno

Corrected Refmult2 | **Raw Remult2** | **0 to 1 random number to remove kink structure** | **Luminosity correction factor** | **Vz correction factor**

Using StRefmultCorr class

Tracking efficiency



Number of Events

NEvent	350003	350013	350023	350033	350043	Total
Run11 (0-80%)	200M	74M	15M	9M	187M	**485M**

NEvent	260001	260011	260021	260031	Total
Run10 (10-80%)	80M	21M	70M	40M	**211M**
Run10 (0-10%)	53M	20M	76M	47M	**196M**

NEvent	Total
UrQMD	**45M**

Negative binomial distribution

Binomial distribution

r success , given n trials

$$f(r;n,p) = \binom{n}{r} p^r (1-p)^k$$

r : success

k : failures

n : trials

p : probability of success

Negative binomial distribution (NBD)

r success before k failures

$$f(r;k,p) = \binom{r+k-1}{r} p^r (1-p)^k$$

number of trials

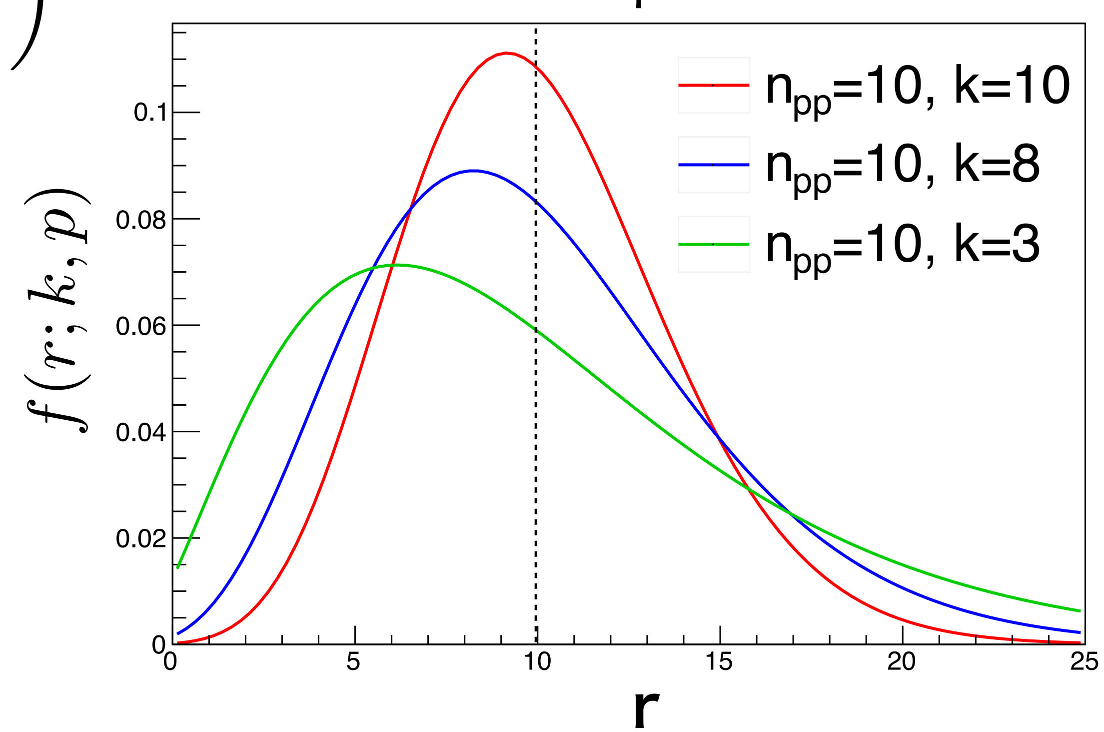
$$\frac{\langle r \rangle}{p} = \frac{k}{1-p} = (k+r)$$

$$\rightarrow \langle r \rangle = \frac{pk}{1-p} \equiv n_{pp}$$

Expand k and r to positive real values.

$$f(r;k,p) = \frac{\Gamma(r+k)}{\Gamma(r+1)\Gamma(k)} \left(\frac{n_{pp}}{k+n_{pp}}\right)^r \left(\frac{k}{k+n_{pp}}\right)^k$$

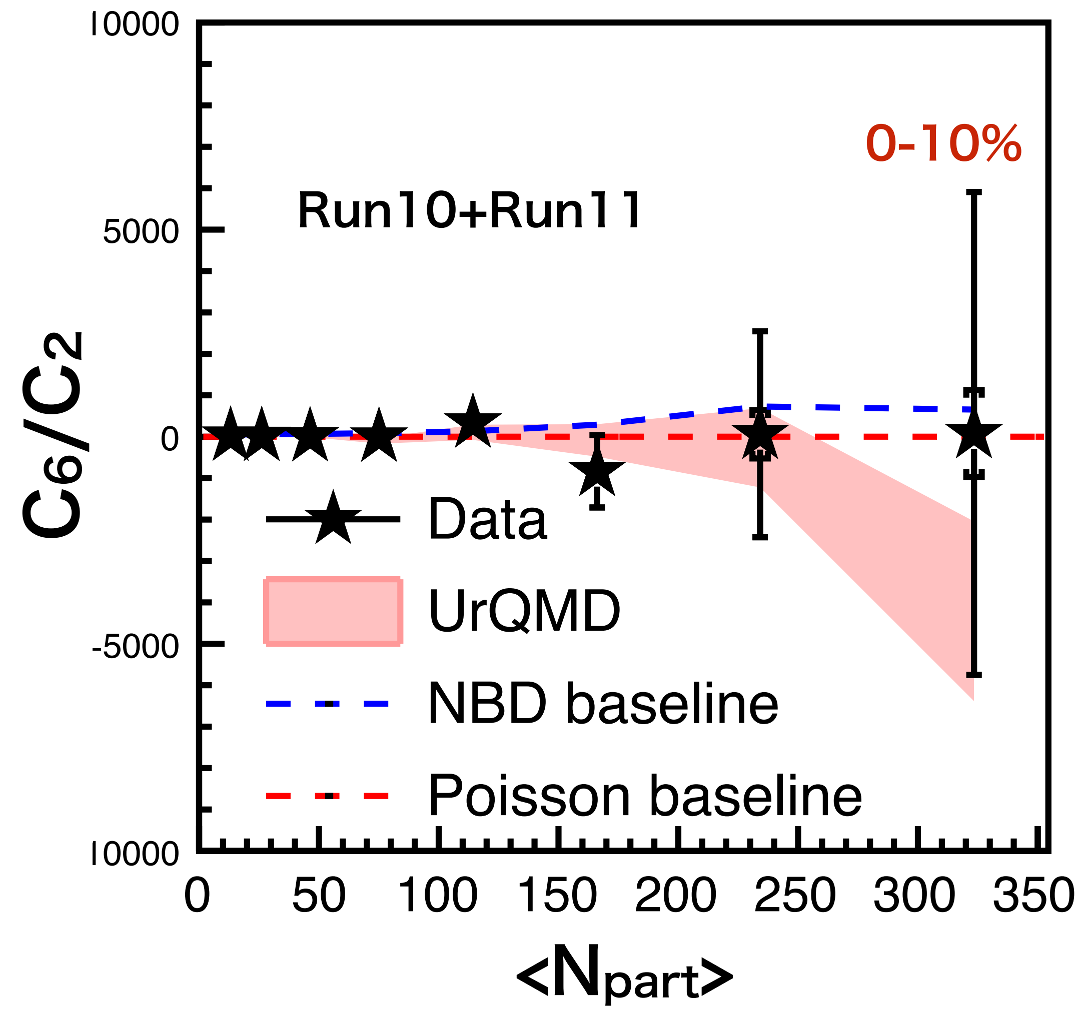
Example

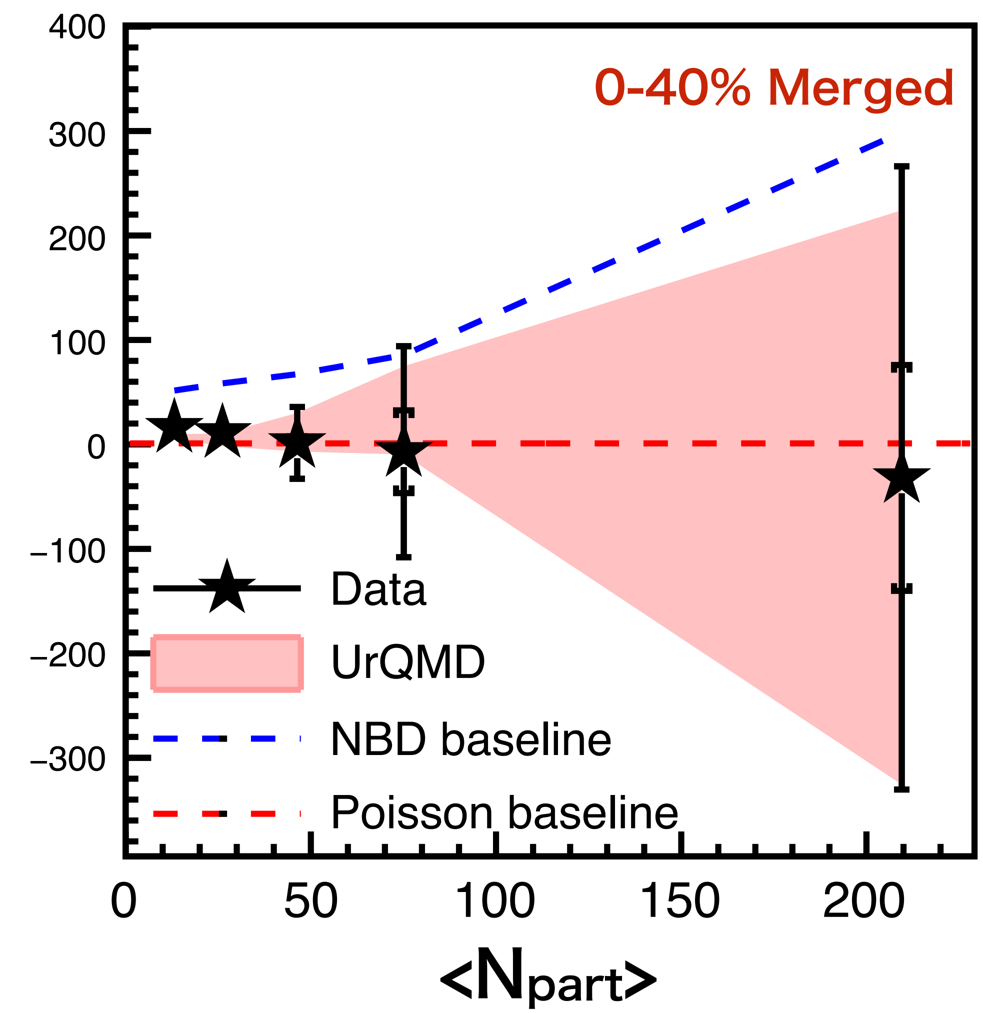


n_{pp} : Mean number of generated particles from each ancestors.

k : Parameters of NBD.

Cumulant ratio (C_6/C_2)





Weights are calculate by Number of Events

$$\left(\frac{C_6}{C_2}\right)_{merged} = \sum_{i}^{Ntrig} w_i \left(\frac{C_6}{C_2}\right)_i$$

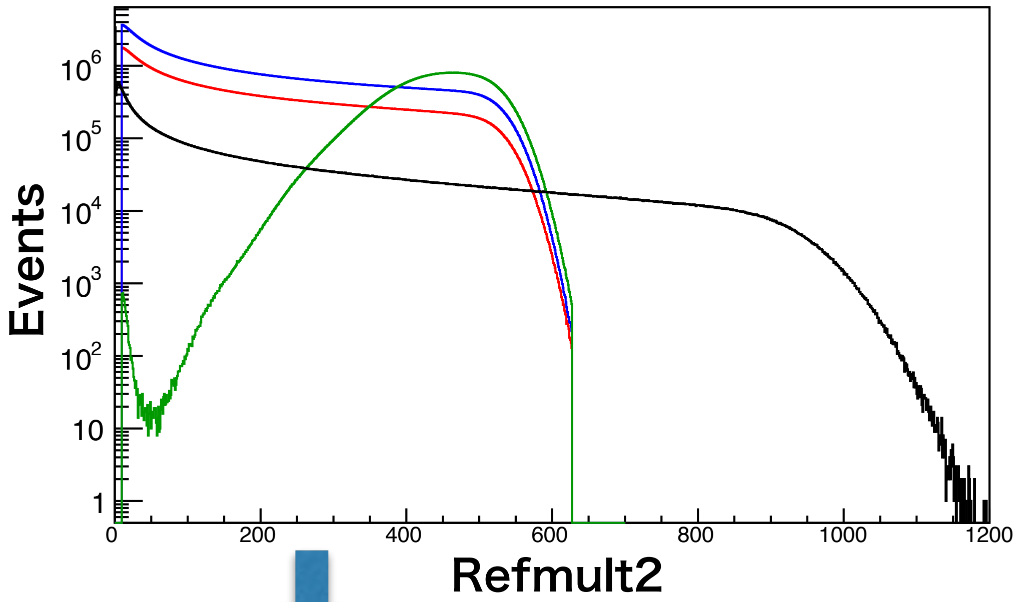
$$Err\left(\frac{C_6}{C_2}\right)_{merged} = \sqrt{\sum_{i}^{Ncent} {w_i}^2 \left(\frac{C_6}{C_2}\right)_i^2}$$

Weight is calculated by number of event for each trigger ID and centralities.

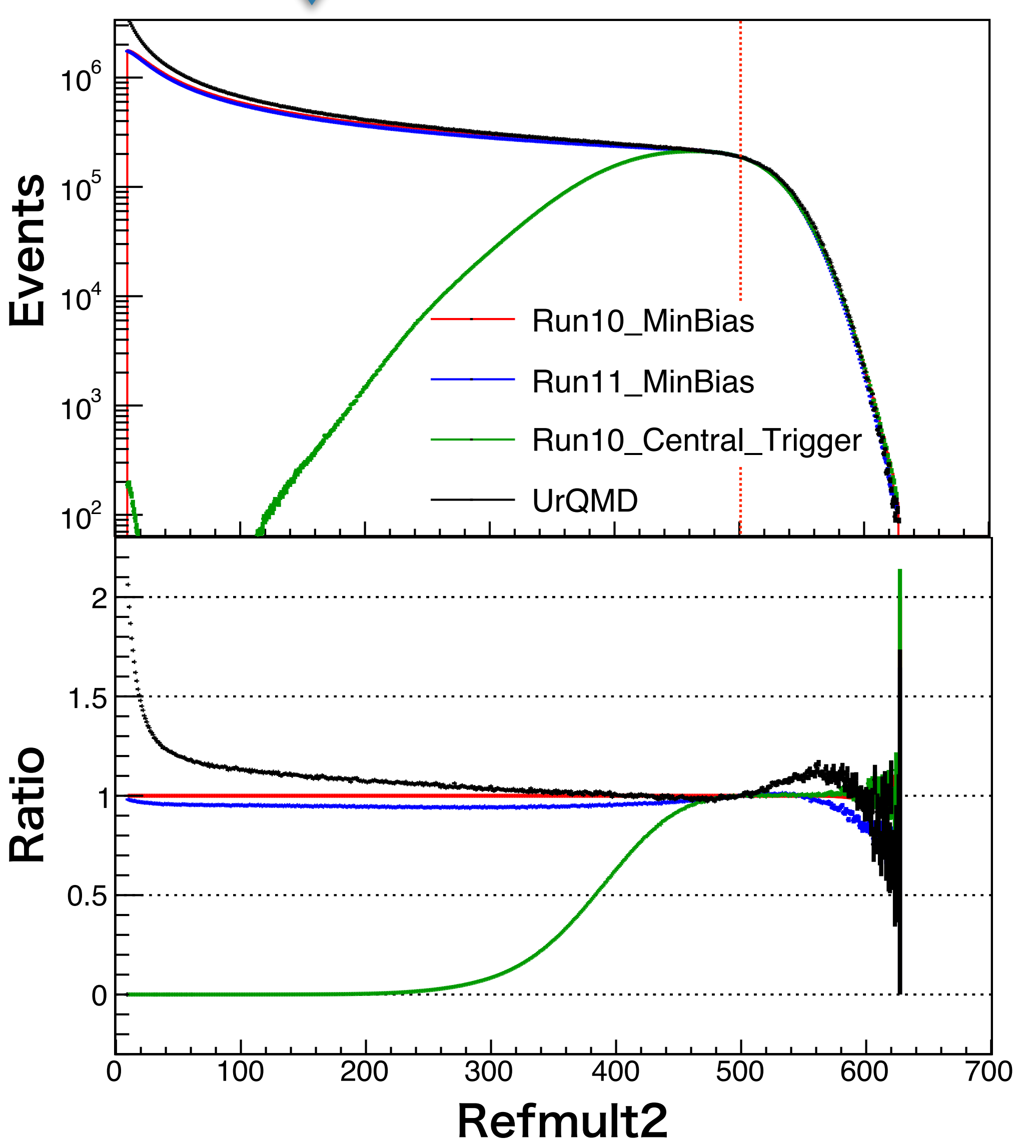
$$w_i = \frac{N_i}{\sum_i^{Ntrig} N_i}$$

- C_6/C_2 is consistent with statistical baseline in all centralities.

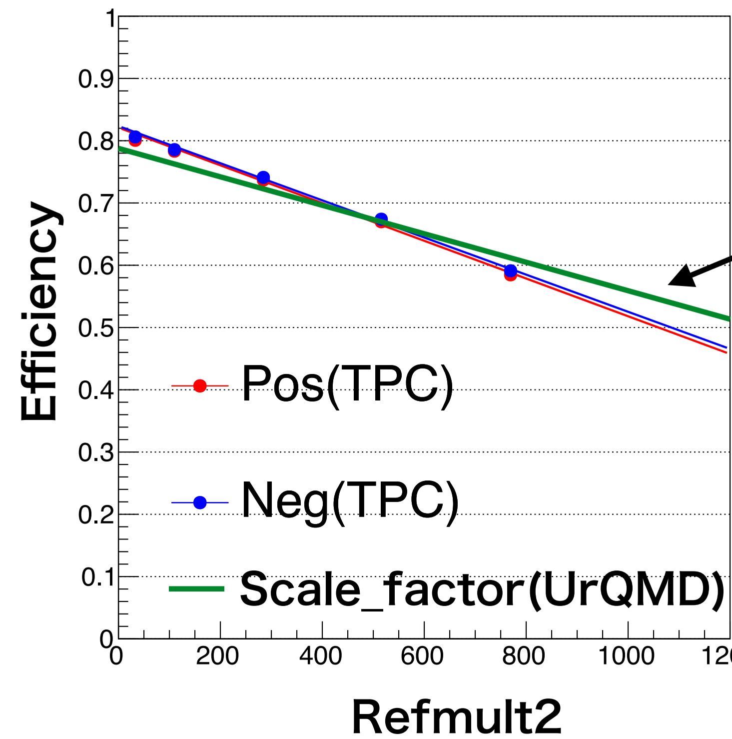
② Scale Refmult2 distributions



Scaled to Run10_MinBias



- y-axis of all results are scaled at Refmult2=500
- x-axis of UrQMD is scaled to Run10_MinBias by randomly sampling and consistent within ±20% except most peripheral collisions.

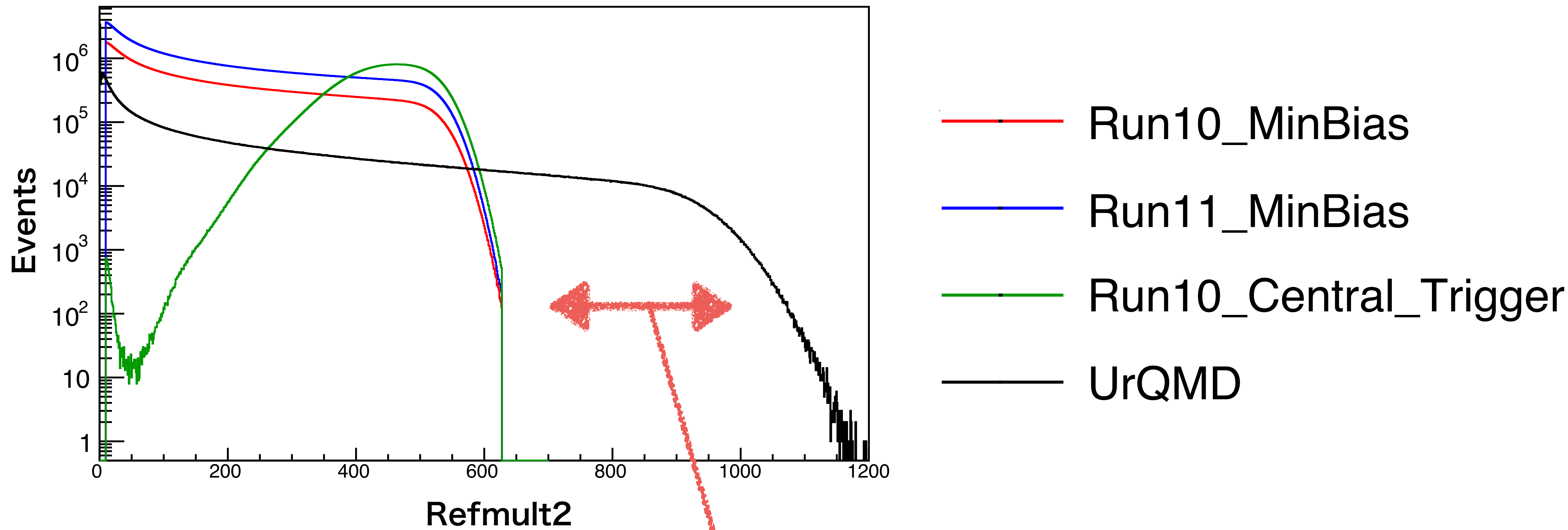


$$\text{Scale\_factor} = \frac{2.28}{2.9}\left(1 - \frac{0.29}{1020} * \text{Refmult2}\right)$$

- Refmult2 distribution of UrQMD can be scaled to experimental data by multiplying scale factor
- Refmult2 dependence of scale factor is similar to charged particle efficiencies as a function of Refmult2.

Differences between UrQMD and experimental refmult2 distributions can be roughly described by the tracking efficiency.

How to treat UrQMD?



Scale is different between UrQMD and experimental refmult2 because of tracking efficiencies.

① **Glauber fit and apply CBWC**

…Participant fluctuations of UrQMD are smaller than experimental data.

② **Scale to experimental data and apply CBWC**

…Participant fluctuations of UrQMD are same as experimental data.

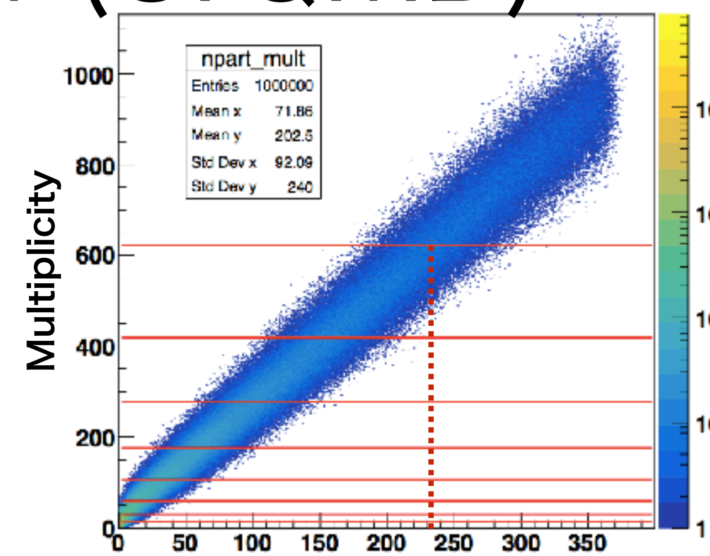
→ ① was applied tentatively.

CBWC by Nw (UrQMD)

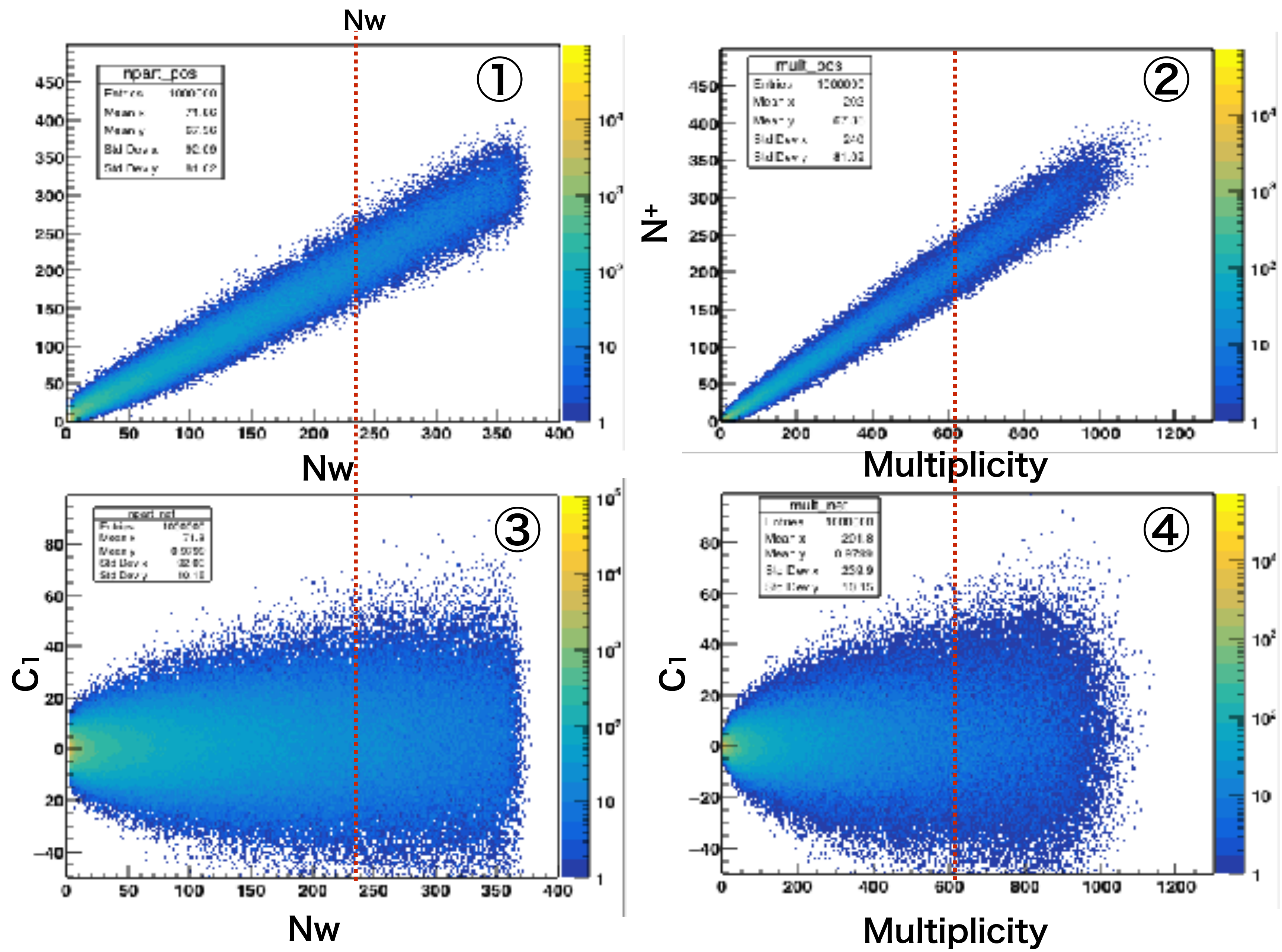
- If CBWC is done by using Nw instead of multiplicities, can we reduce VF?

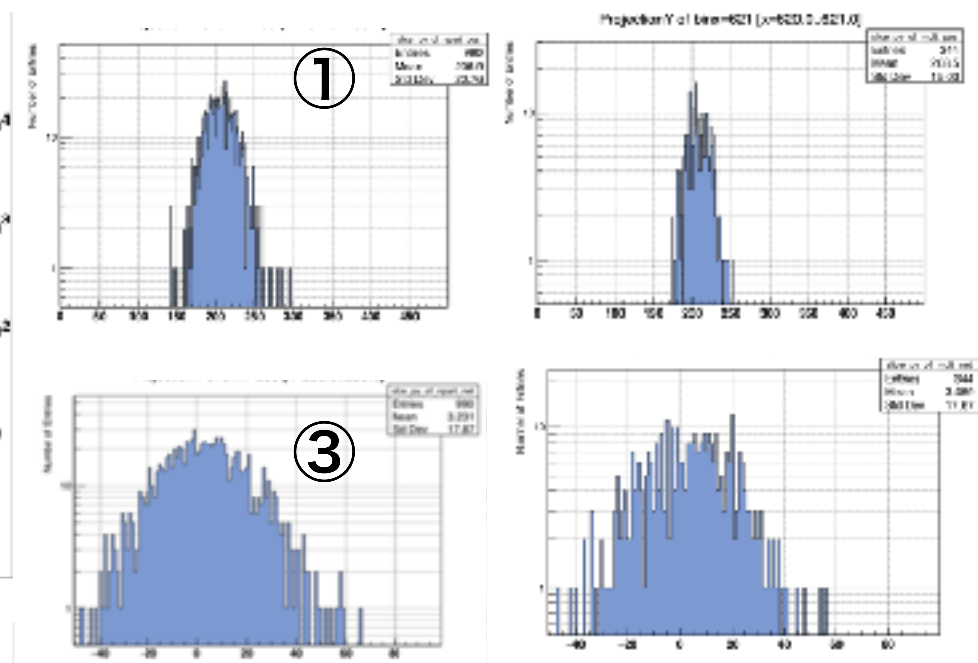
$$C_n = \sum_r w_r C_{(n,r)}$$

Cumulants in r-th Nw bins



Multiplicity=621
<Nw=233>



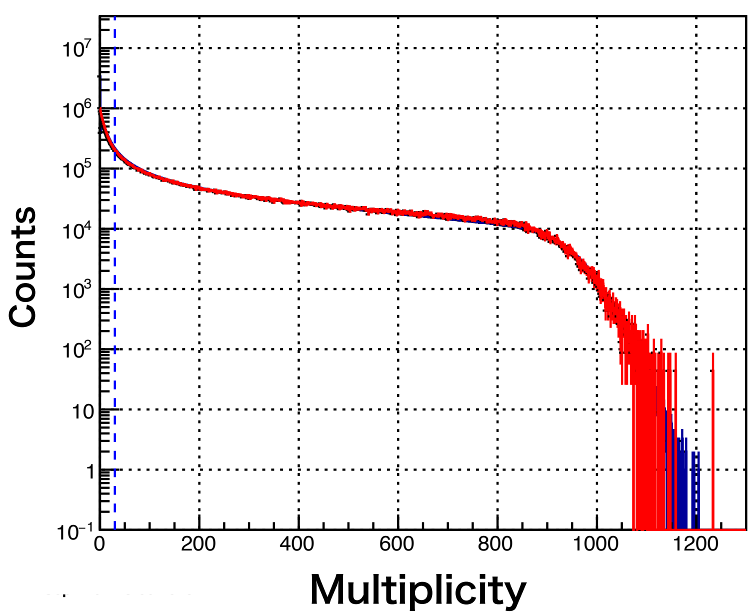


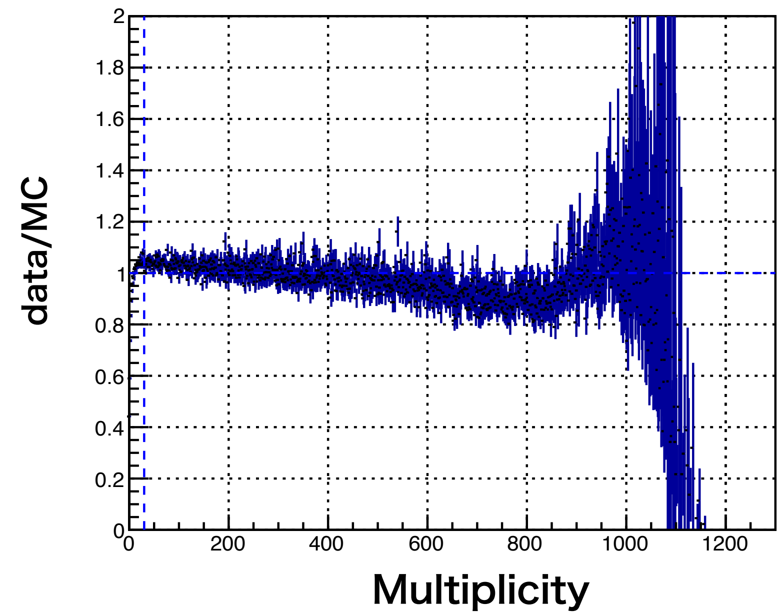
①C_2=430 > ②C_2=225
③C_2=319 ~ ④C_2=312

- CBWC C_2 by Nw is larger than that by multiplicity.

UrQMD model

- VFC works well in Toy model case under the IPP model assumption consist.
- Next, **VFC will be done by using UrQMD** model in which **IPP model might be broken** and discuss the validity of VFC.



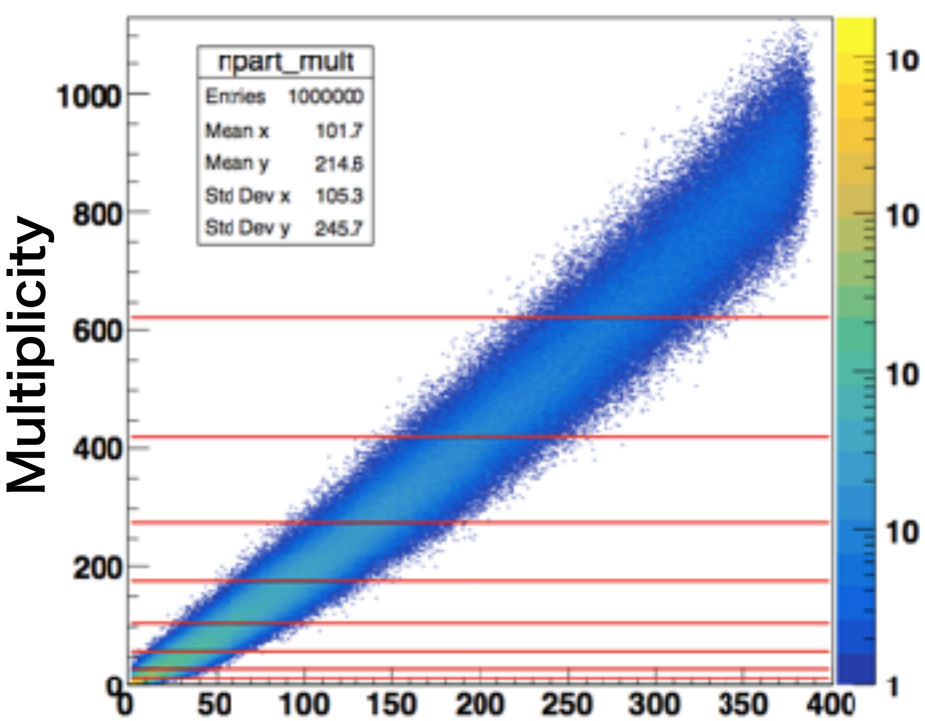


- We can **directly know true N_W** by using UrQMD model.
- N_W can also be obtained by Glauber fit like previous Toy model.

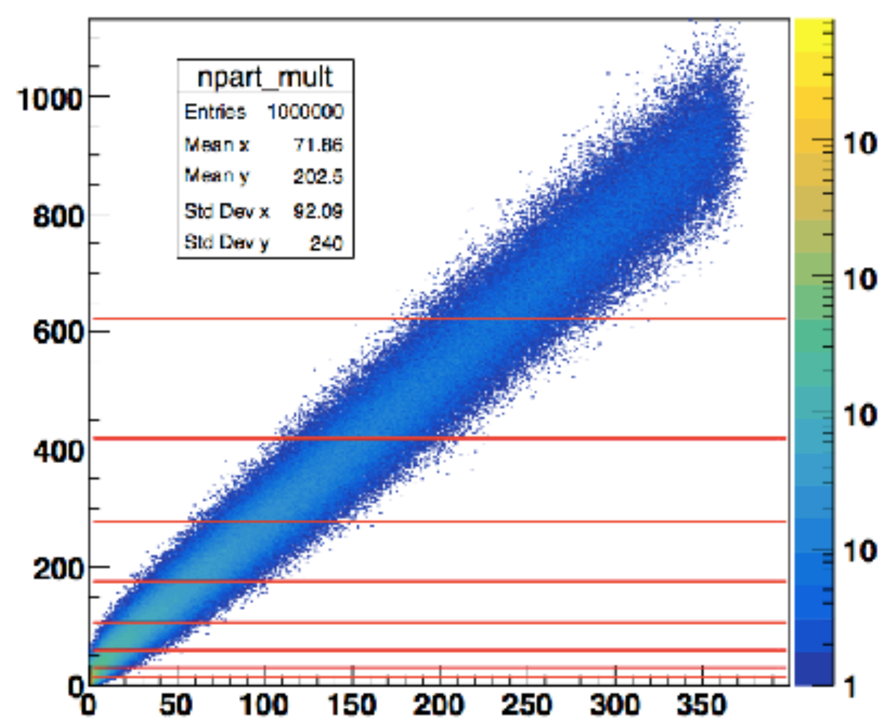
Fit parameters

- n_{pp}=**2.90,** x=0.13
- k=0.9, efficiency=**1**

Glauber



UrQMD

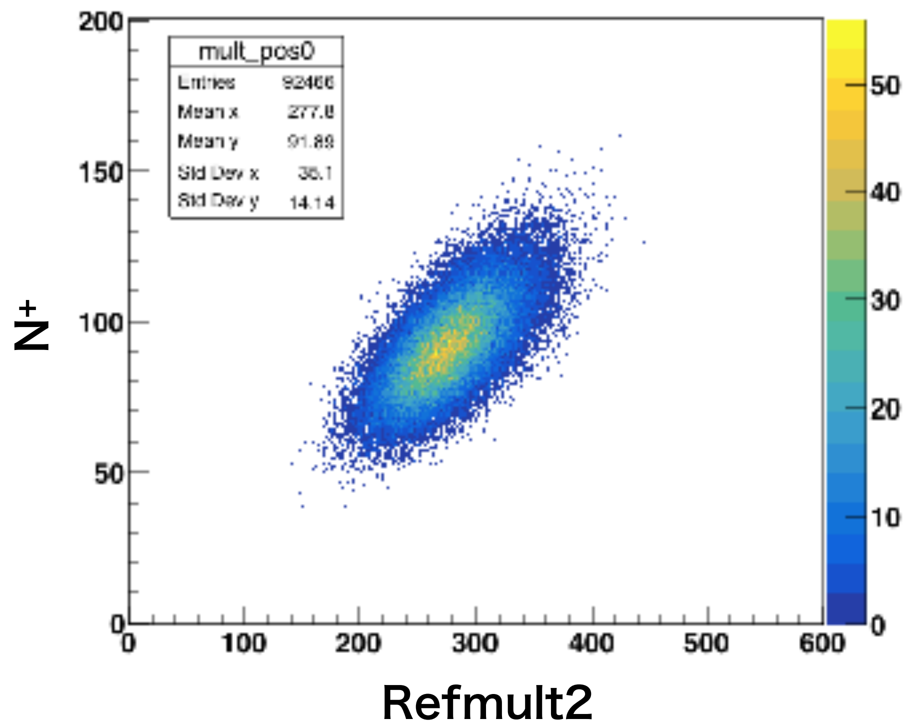


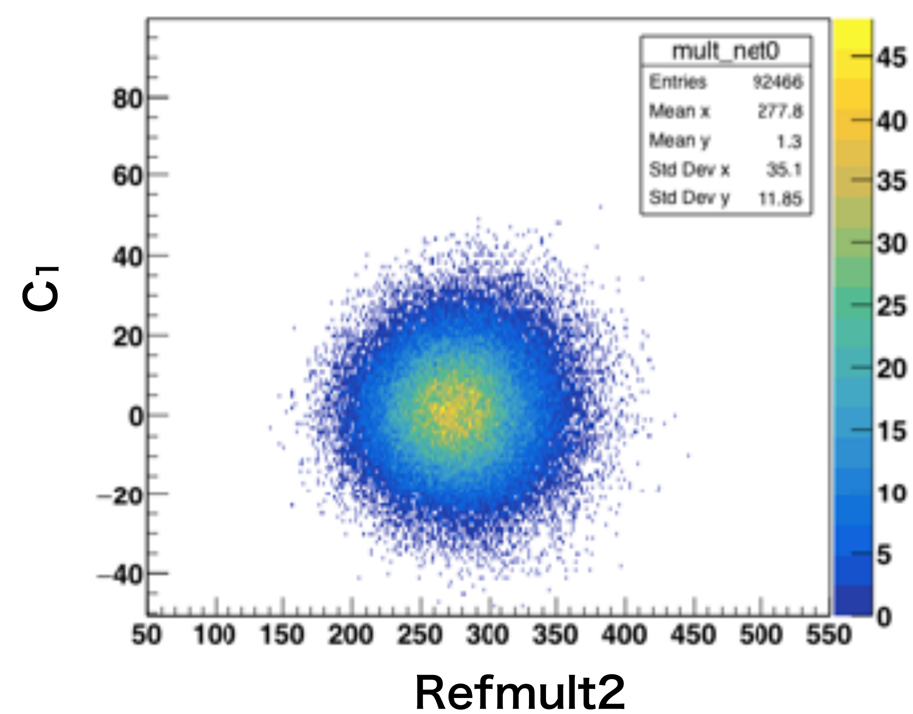
<N_W>

- Even though multiplicity distributions are the same, **Nw distributions are different** between Glauber model and UrQMD.
- **N_W from UrQMD is used** for following VFC calculation.

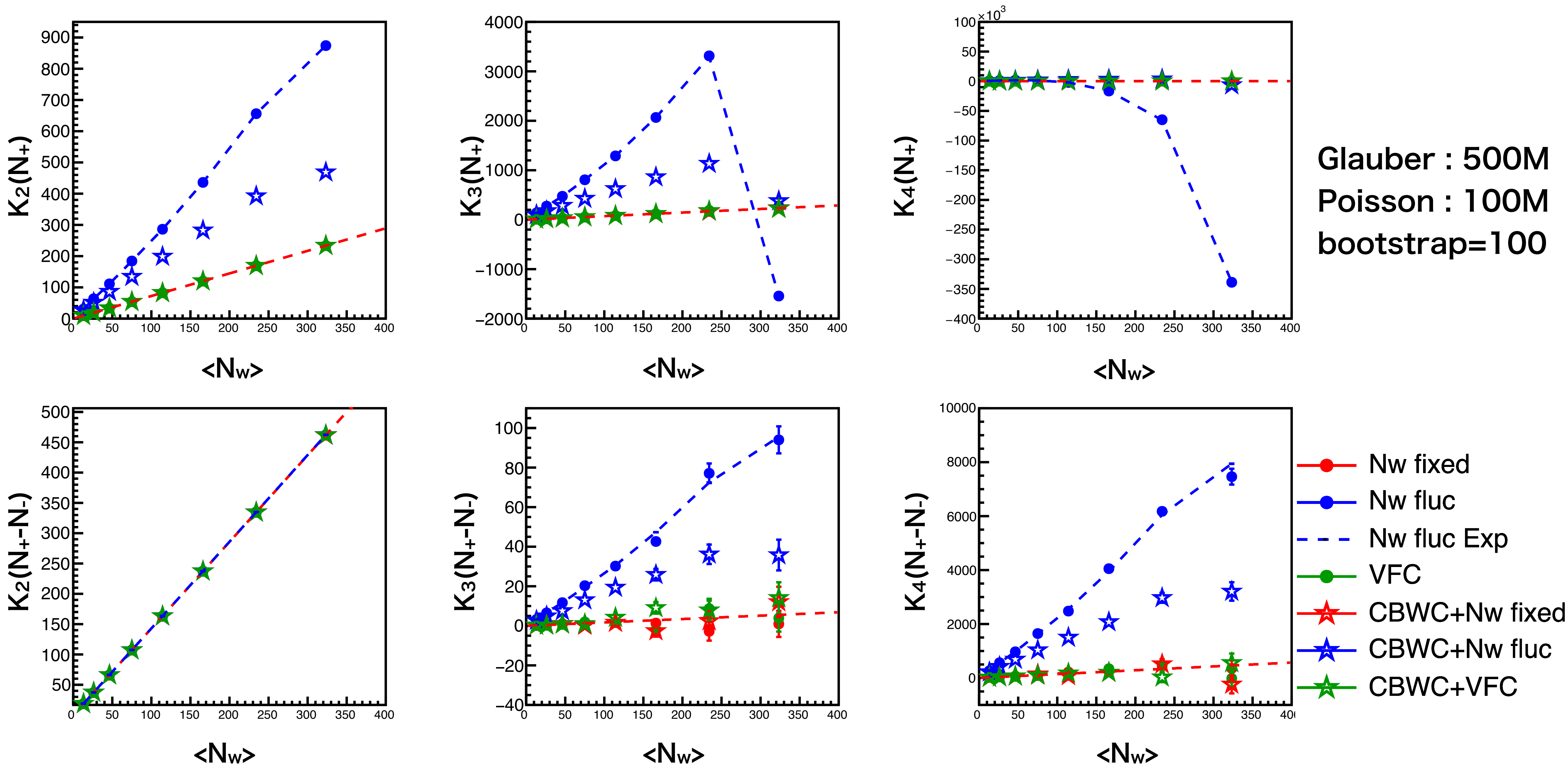
CBWC by Nw

Nw=100 (Fixed)



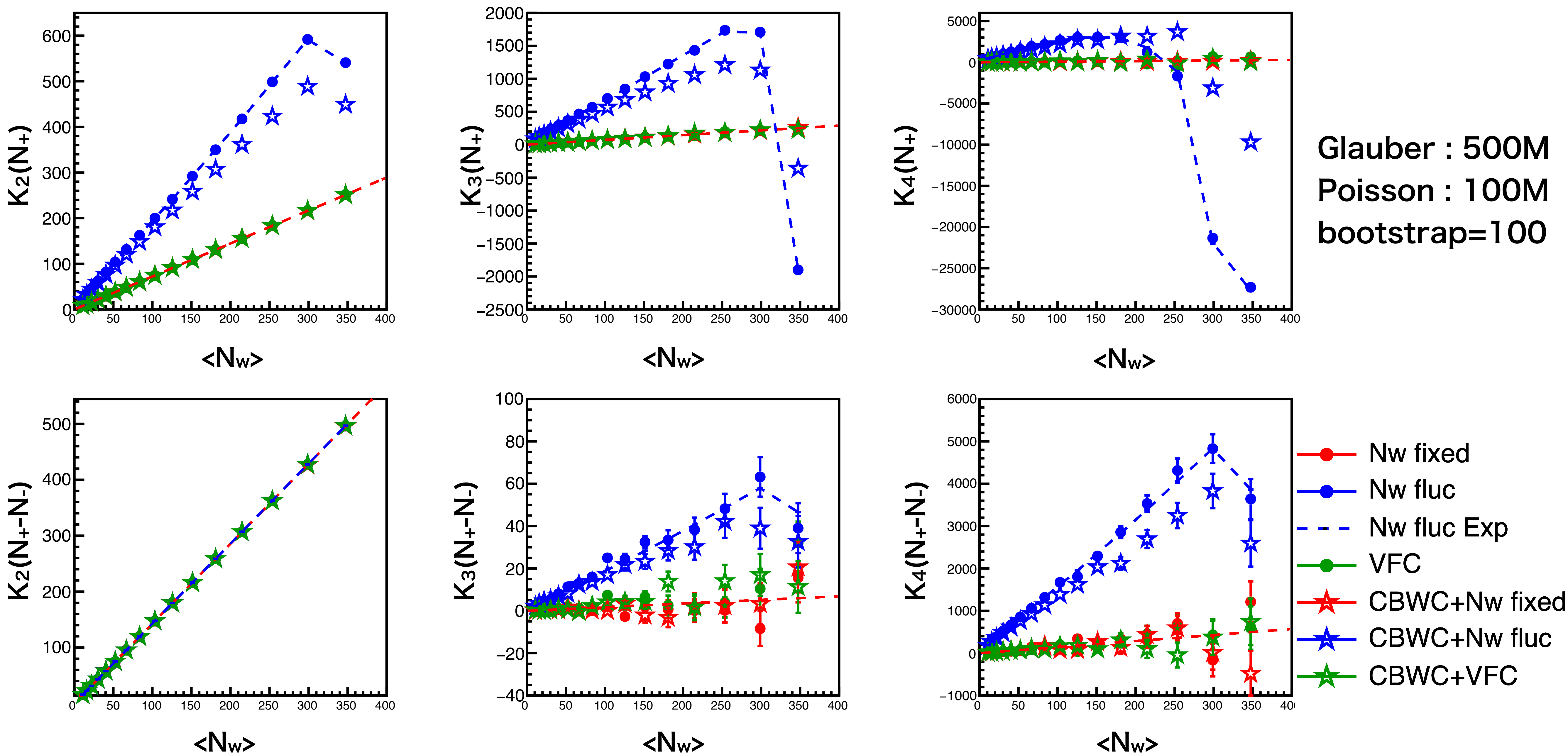


Toy model VFC check (8bin)



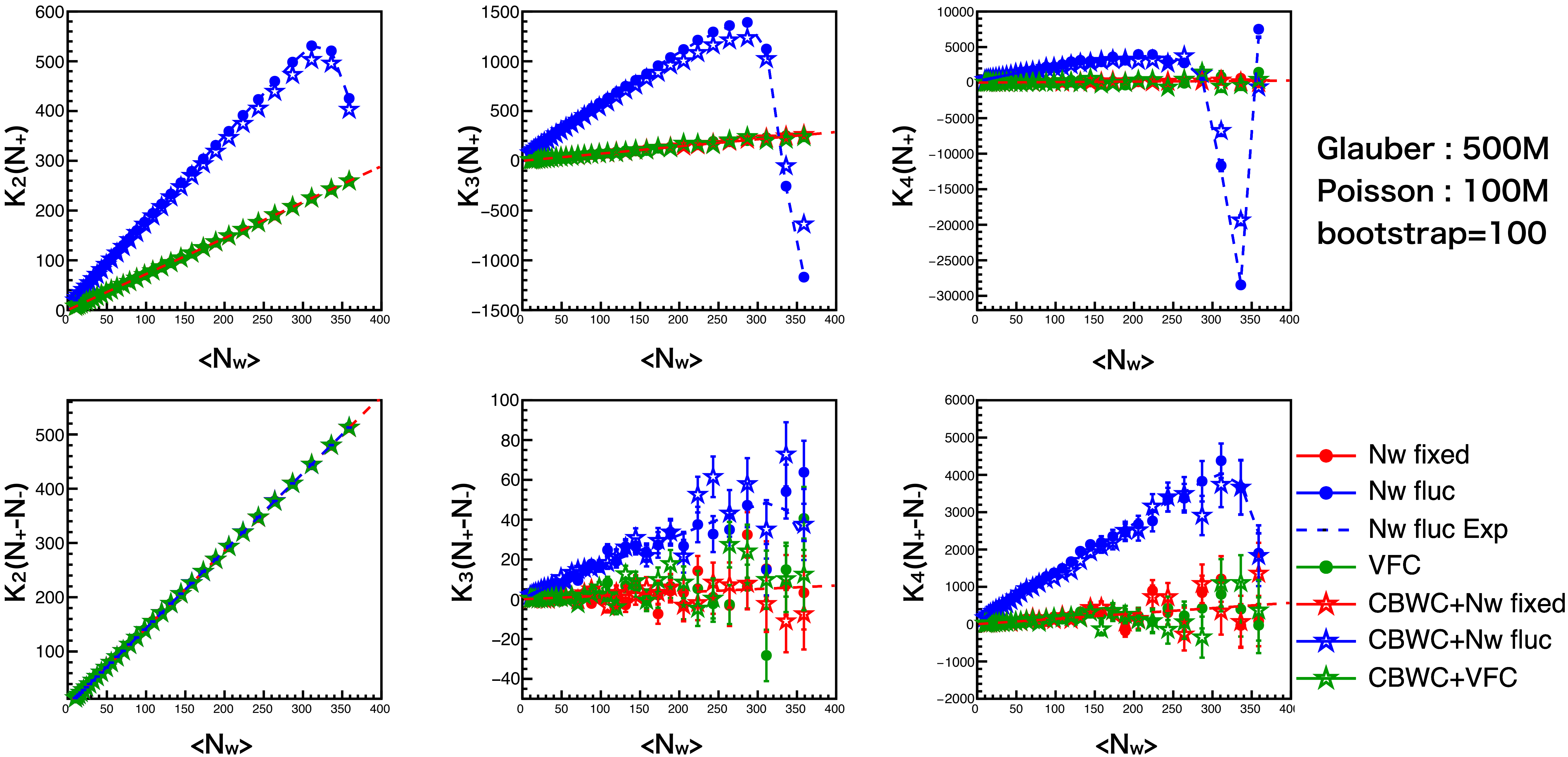
- It seems VFC works well.
- K_1 does not affected by participant fluctuation.
- In net-charge fluctuation, effect from participant fluctuation is smaller than N_+ cumulants because Δn of net-charge is smaller than N_+.

Toy model VFC check (16bin)



- It seems VFC works well.
- K_1 does not affected by participant fluctuation.
- In net-charge fluctuation, effect from participant fluctuation is smaller than N_+ cumulants because Δn of net-charge is smaller than N_+.

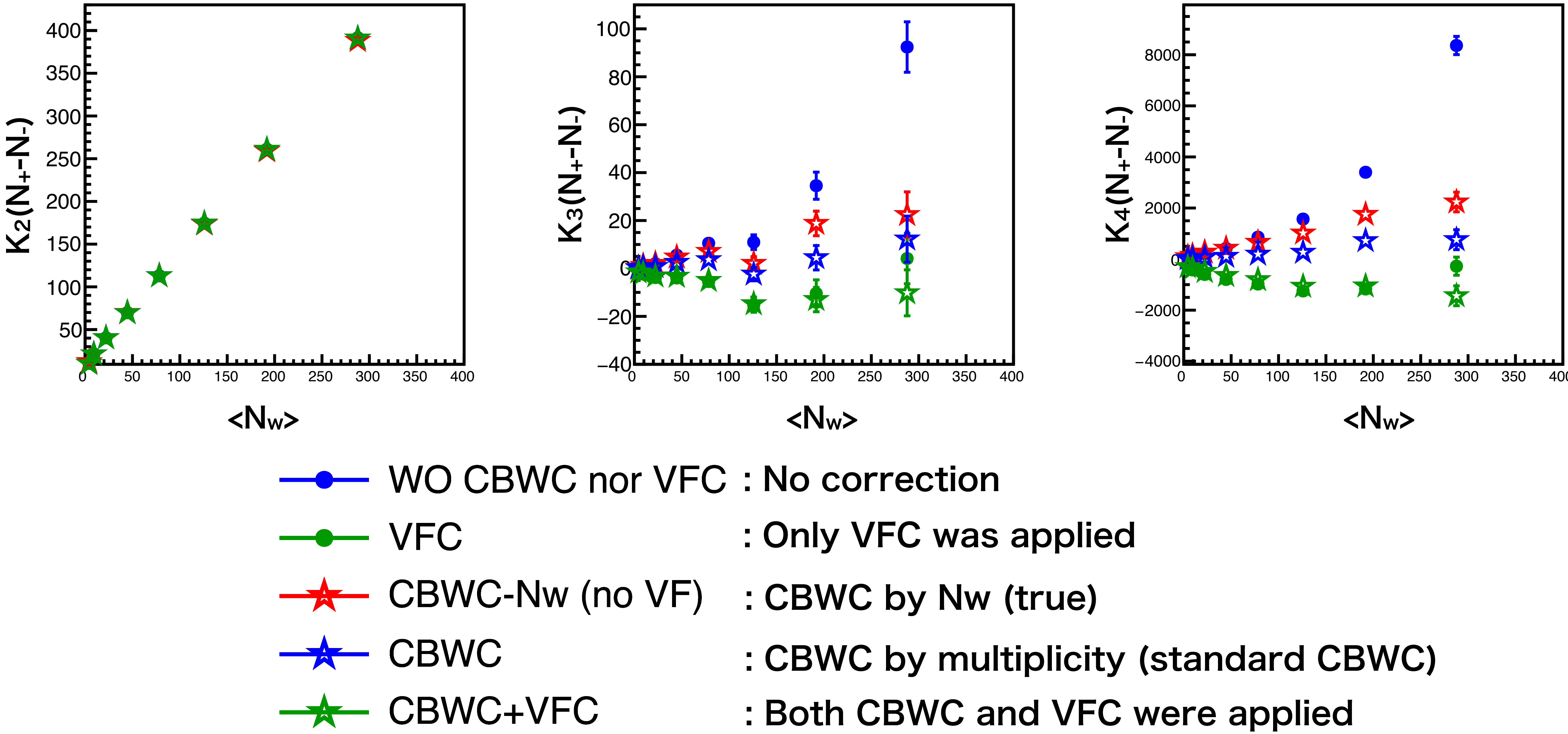
Toy model VFC check (32bin)



Glauber : 500M
Poisson : 100M
bootstrap=100

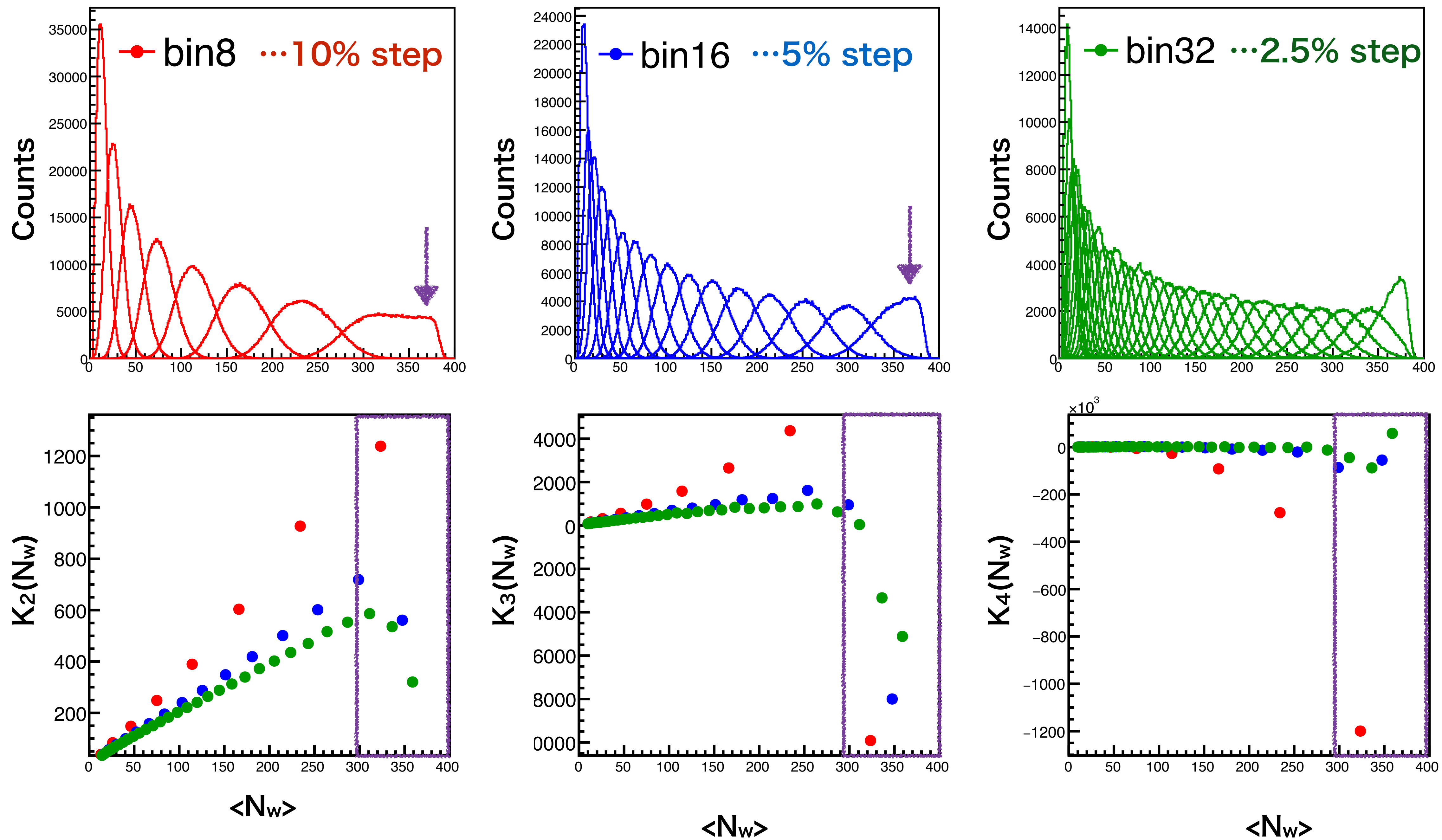
- It seems VFC works well.
- K_1 does not affected by participant fluctuation.
- In net-charge fluctuation, effect from participant fluctuation is smaller than N_+ cumulants because Δn of net-charge is smaller than N_+.

UrQMD results



- CBWC-Nw (no VF, true) cumulants are larger than CBWC results even though CBWC results include VF.
  - IPP assumption is broken in UrQMD?
- CBWC results are larger than VFC results and close to CBWC-Nw (true) cumulants.
  - CBWC is better than VFC?

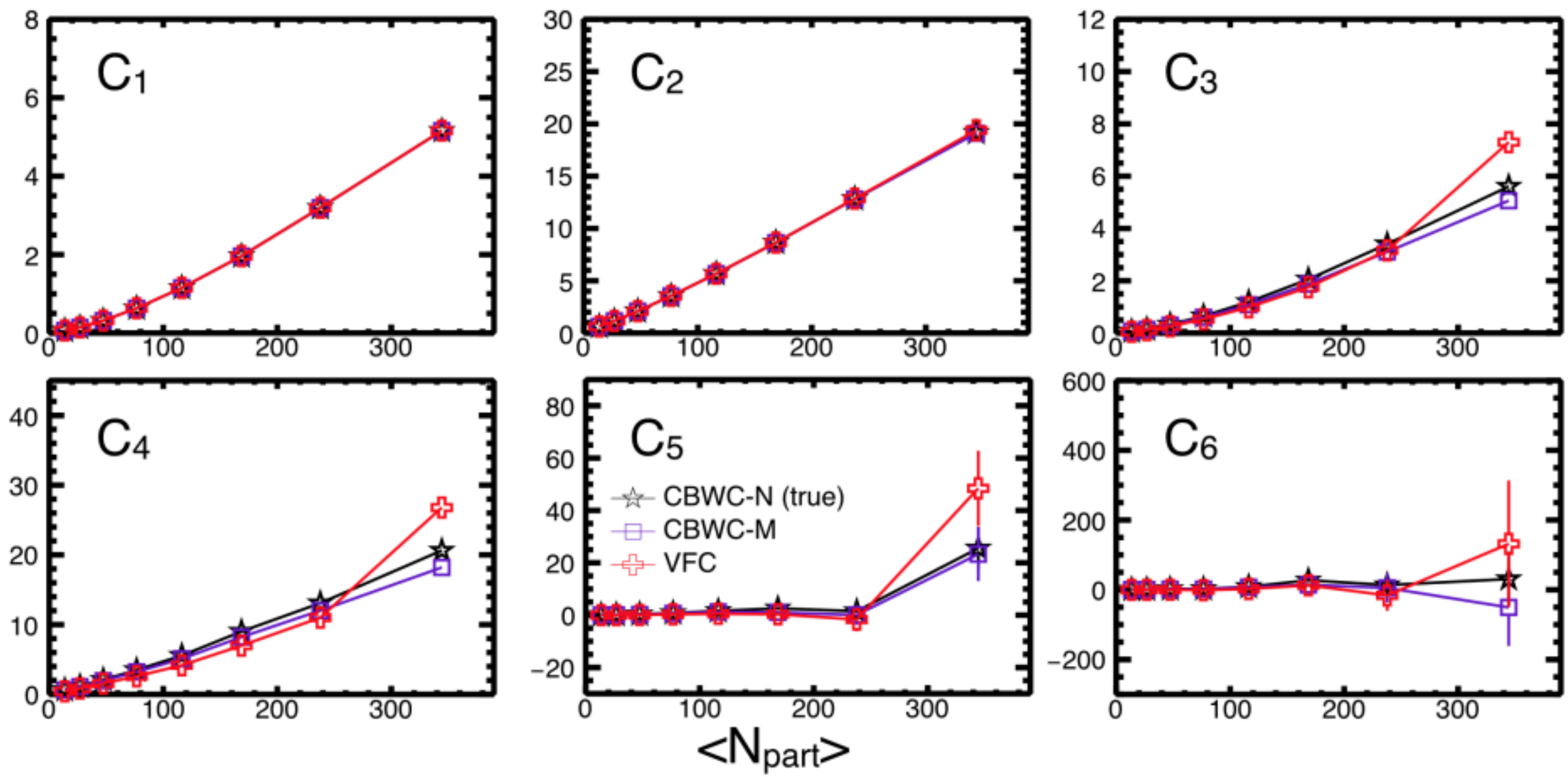
Participant fluctuation



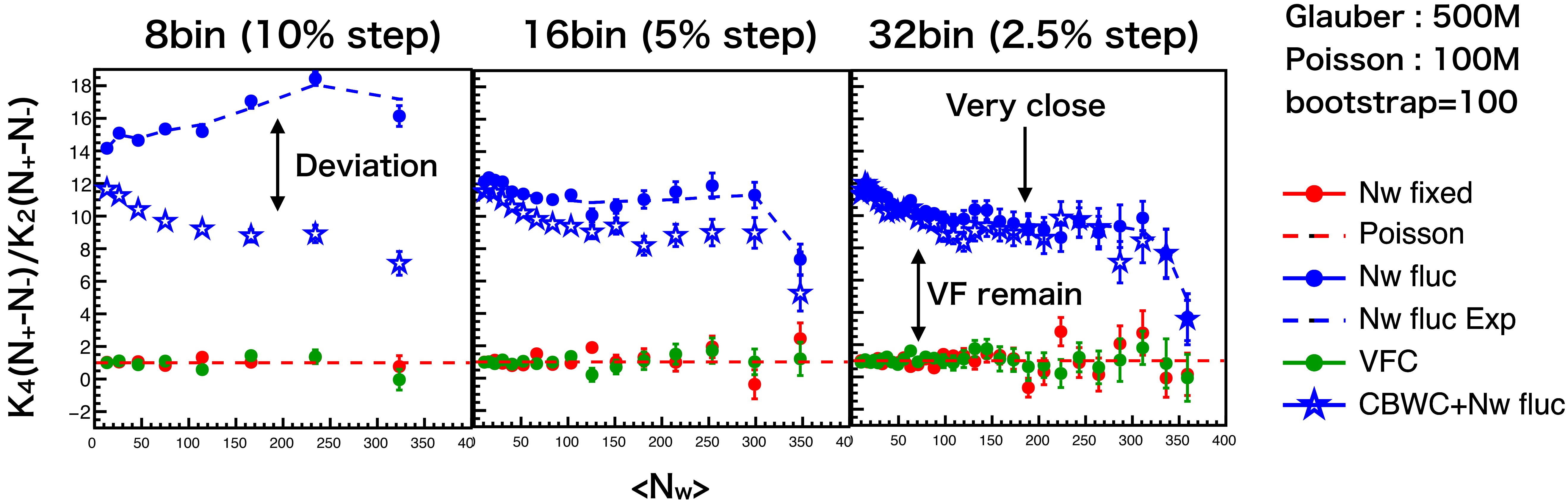
- **<u>Trends are changed around central collision</u>** because maximum value of N_w is fixed.
- Participant fluctuation become larger with number of bin divisions become small.

Net-proton case

T.Nonaka, Doctoral thesis



Toy model VFC check (bin dependence)



- CBWC cumulants and Without CBWC cumulants become very close if centrality step become 2.5%.

Volume fluctuation correction (5,6-th order)

True cumulant

$$\kappa_5(\Delta N) = \langle N_W\rangle\kappa_5(\Delta n) + \left\{5\kappa_4(\Delta n)\kappa_1(\Delta n) + 10\kappa_3(\Delta n)\kappa_2(\Delta n)\right\}\kappa_2(N_W)$$
$$+\left\{10\kappa_3(\Delta n)\kappa_1^2(\Delta n) + 15\kappa_2^2(\Delta n)\kappa_1(\Delta n)\right\}\kappa_3(N_W) + 10\kappa_2(\Delta n)\kappa_1^3(\Delta n)\kappa_4(N_W)$$
$$+\kappa_1^5(\Delta n)\kappa_5(N_W)$$

Additional term caused from N_W fluctuation

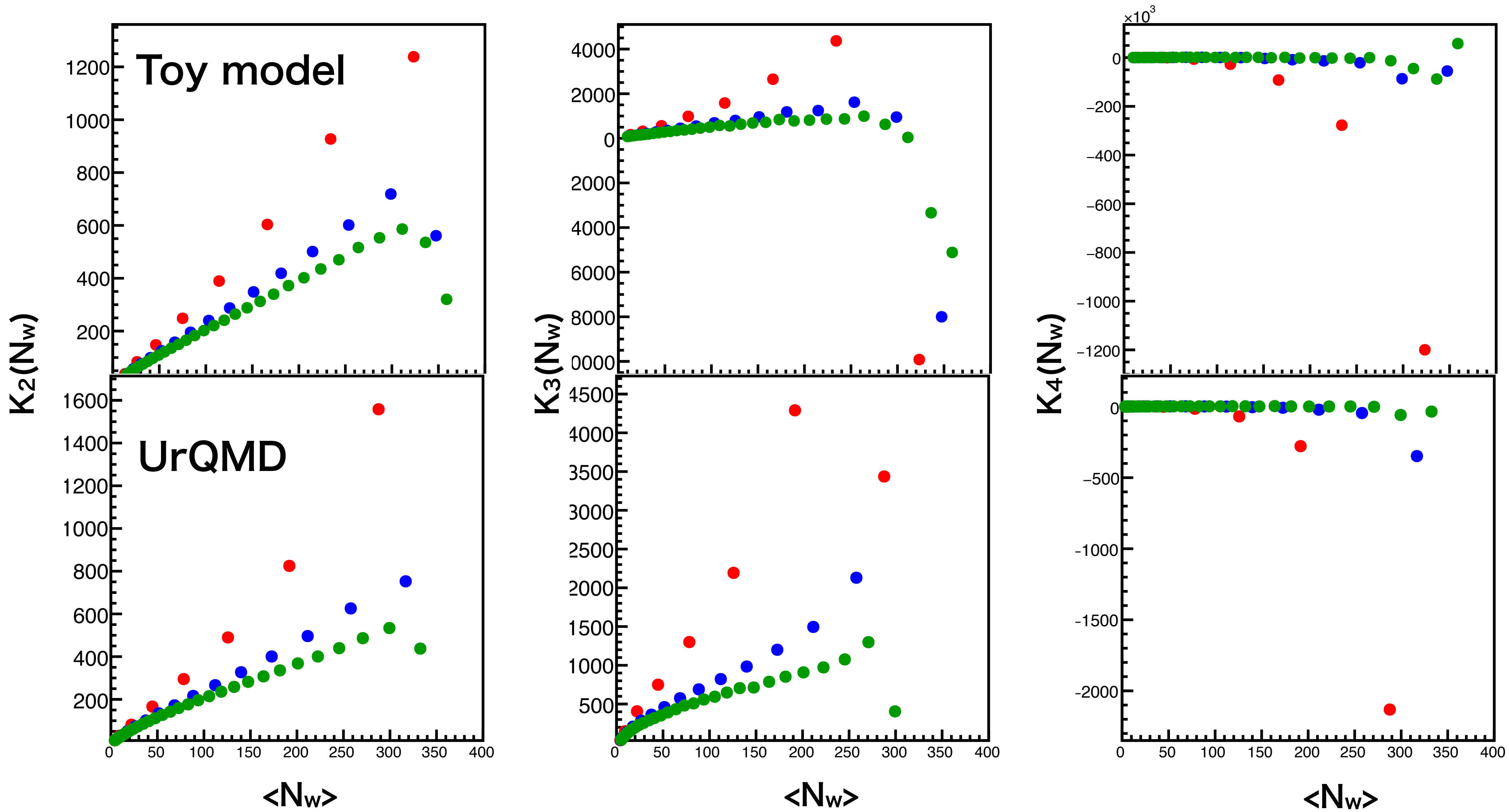
T.Nonaka, Doctoral thesis

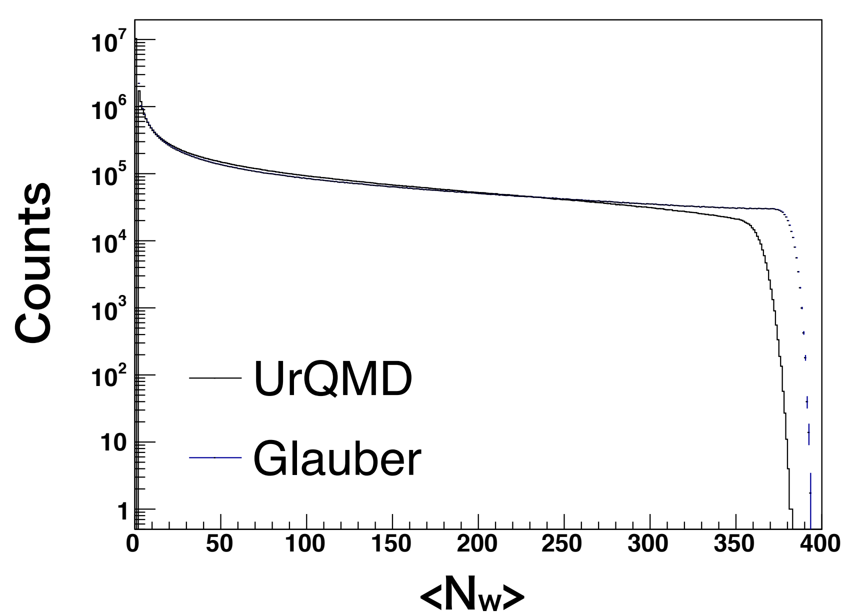
$$\kappa_6(\Delta N) = \langle N_W\rangle\kappa_6(\Delta n)$$
$$+\left\{6\kappa_5(\Delta n)\kappa_1(\Delta n) + 15\kappa_4(\Delta n)\kappa_2(\Delta n) + 10\kappa_3^2(\Delta n)\right\}\kappa_2(N_W)$$
$$+\left\{15\kappa_4(\Delta n)\kappa_1^2(\Delta n) + 60\kappa_3(\Delta n)\kappa_2(\Delta n)\kappa_1(\Delta n) + 15\kappa_2^3(\Delta n)\right\}\kappa_3(N_W)$$

Δn : number of net-particle

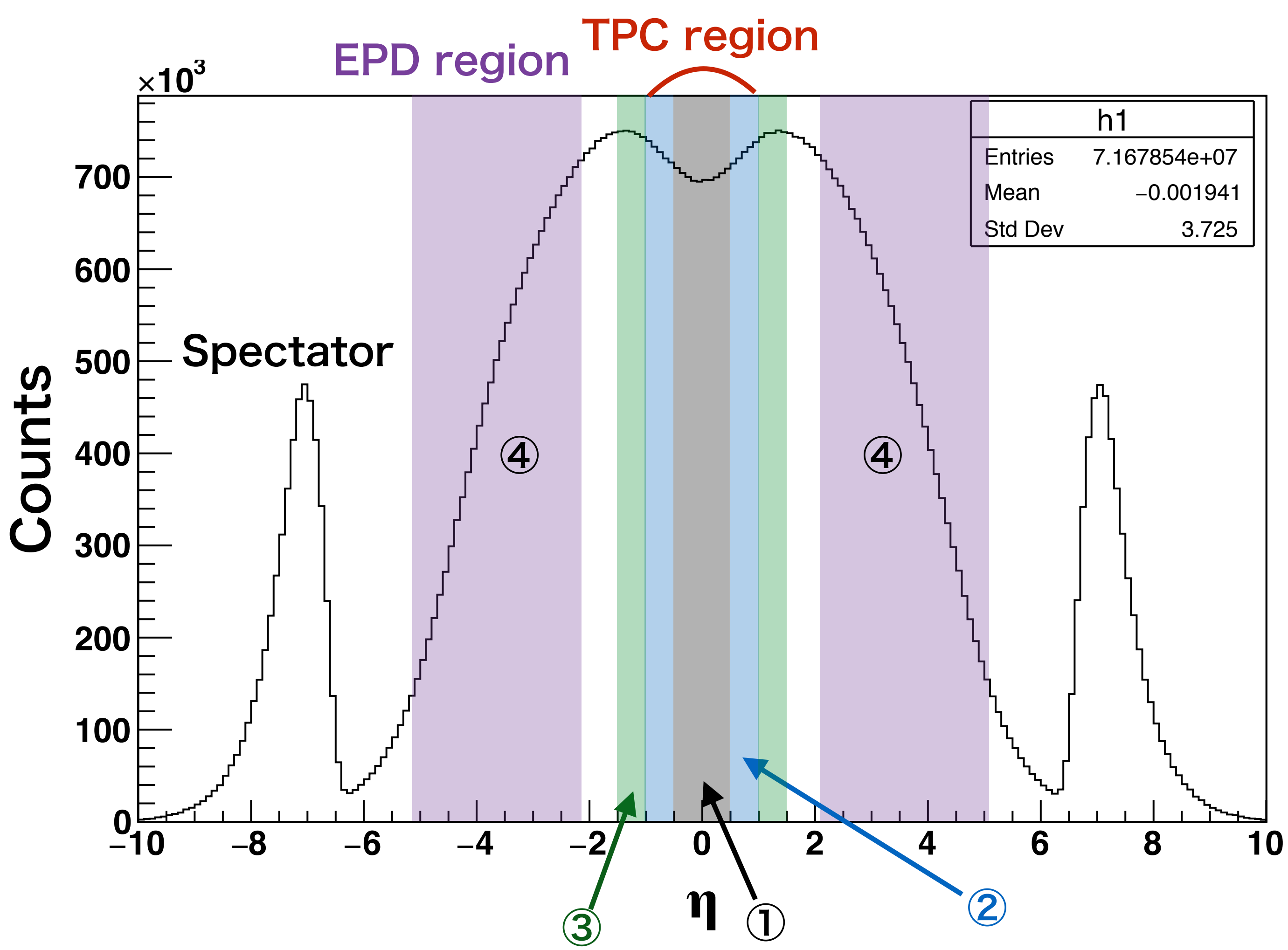
Measured cumulant

Participant fluctuation (UrQMD)





UrQMD model



① |η|<0.5 (Refmult)

② 0.5<|η|<1 (Refmult2)

- experimentally used for net-c analysis

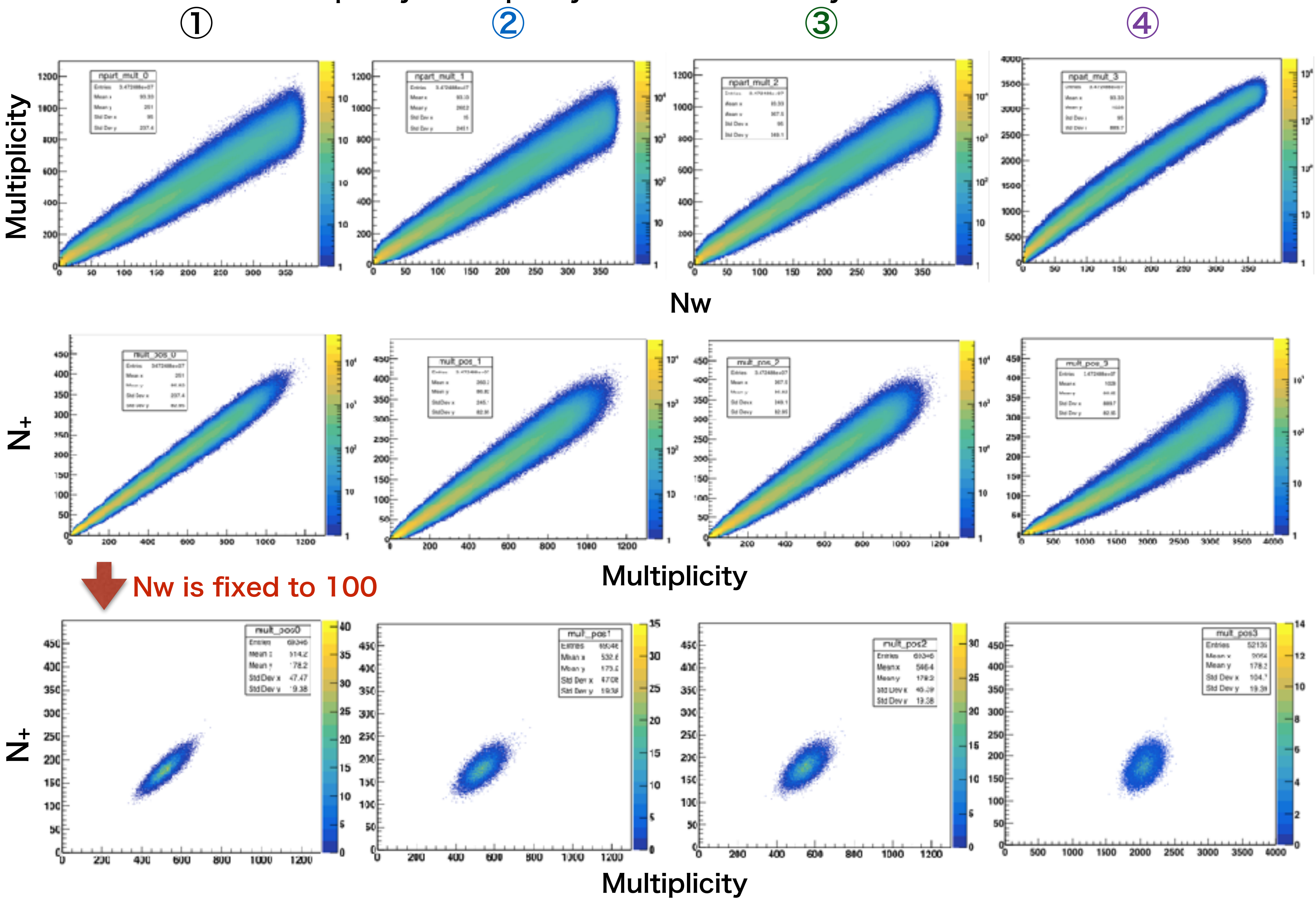
③ 1<|η|<1.5

④ 2.1<|η|<5.1 (EPD region)

- centrality determination from BES II

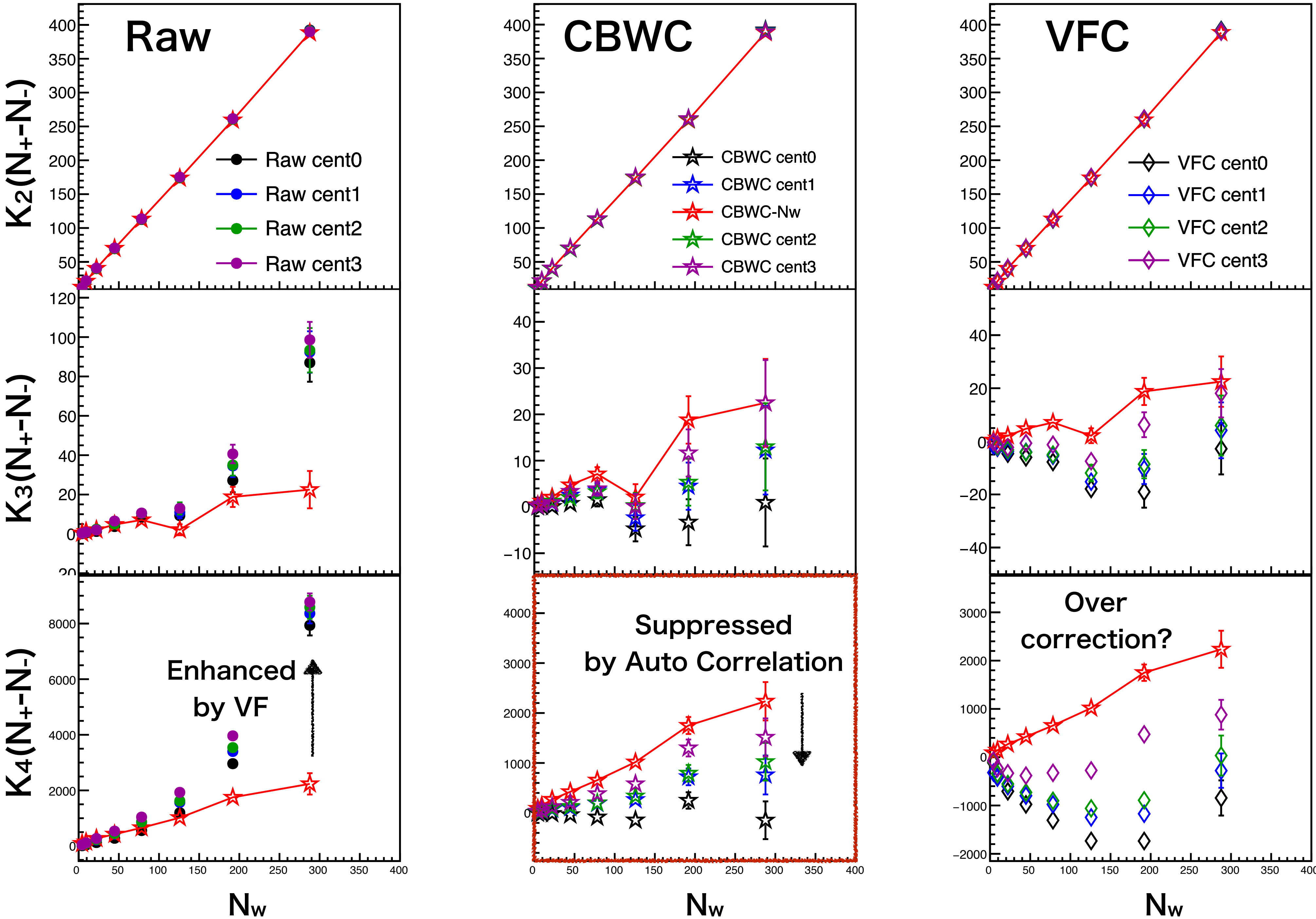
UrQMD model

Multiplicity : multiplicity used for centrality determination



Tetsuro Sugiura Univ. of Tsukuba

UrQMD model



cent0 |η|<0.5

cent2 0.5<|η|<1

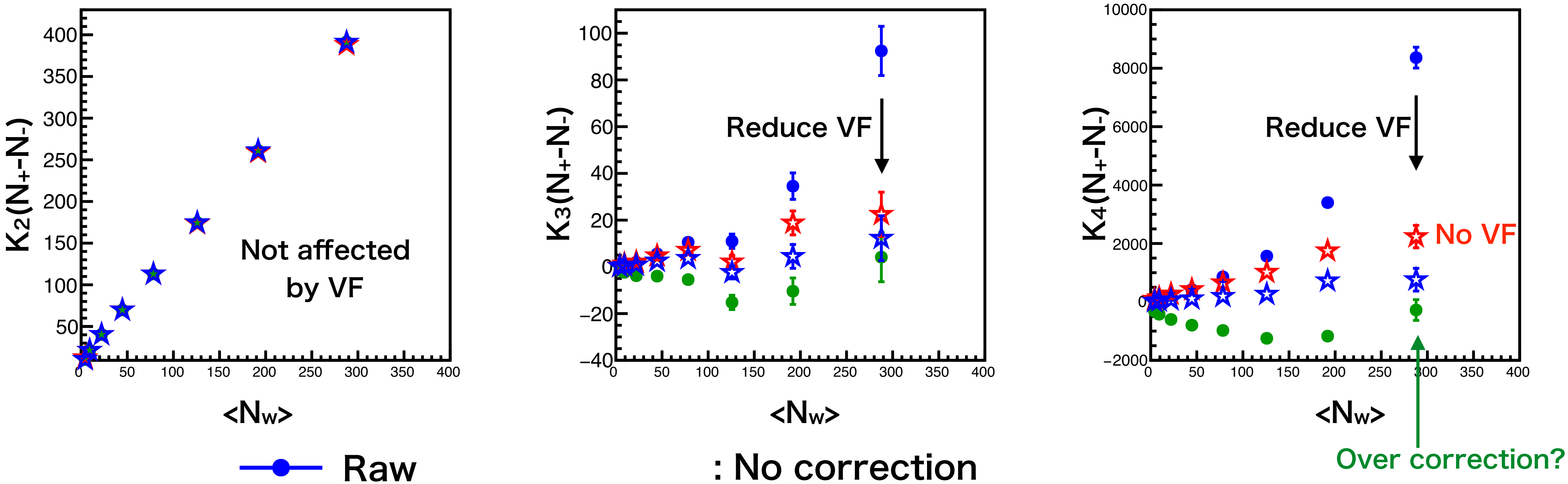
cent3 1<|η|<1.5

cent4 2.1<|η|<5.1

CBWC-Nw :
No VF and written
in all plot

- Suppressions are observed because of autocorrelation
  - Largest in refmult region and smallest in EPD region
- VF are observed in raw results but not observed in CBWC results
  - NOT consistent with Toy model. IPP is broken?
- If CBWC-Nw results are “true”, VFC results are over correction

UrQMD results



- **Raw** : **No correction**
- VFC : **VFC was applied**
- CBWC-Nw (no VF) : **CBWC by Nw (true)**
- CBWC : **CBWC by multiplicity (experimental CBWC)**

- CBWC-Nw (no VF, true) cumulants are larger than CBWC results even though CBWC results include VF.
  - IPP assumption is expected to be broken in UrQMD.
- CBWC results are larger than VFC results and close to CBWC-Nw cumulants.
  - CBWC is better than VFC?
- K_2 is not affected by VF which trends are same as Toy model.